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QUESTION:

Maze game is a well-known problem, where we are given a grid of 0’s and 1’s, 0’s correspond to a place that can be traversed, and 1 corresponds to a place that cannot be traversed (i.e. a wall); the problem is to find a path from bottom left corner of grid to top right corner; immediate right, immediate left, immediate up and immediate down only are possible (no diagonal moves). We consider a variant of the maze problem where a cost is attached to visiting each location in the maze, and the problem is to find a path of least cost through the maze.

Solution

Consider a maze given by a cost c_{ij} such that c_{ij} takes a finite value for traversable locations and ∞ for walls (non-traversable locations), with n rows and m columns.

We convert the maze to a weighted graph G(V, E) as follows: each location (i, j) in the maze corresponds to a node in the graph v_{ij}, and $V = \bigcup v_{ij}$ and E = { (u, v) | u, v ∈ V }. Each node v_{ij} is connected to other nodes with weights in the following manner:

$d(v_{ij}, v_{i,j+1}) = c_{i,j+1}$
$d(v_{ij}, v_{i+1,j}) = c_{i+1,j}$
$d(v_{ij}, v_{i,j-1}) = c_{i,j-1}$
$d(v_{ij}, v_{i-1,j}) = c_{i-1,j}$
$d(v_{ij}, v_{rs}) = \infty$ otherwise

This also means that the weight for an adjacent location in the maze (e.g. $d(v_{ij}, v_{i,j+1})$) takes the value ∞ if the destination ($v_{i,j+1}$) represents a wall. Now, consider a path t_0, t_1, t_2, … t_k in the maze, where $t_i \in Z_+ \times Z_+$ is a 2D tuple indicating the location (row, col) in the maze); this has corresponding nodes in G(V, E) (say) u_0, u_1, u_2, … u_k. Then, the cost of the path t_0, t_1, t_2, … t_k in the maze is

c0,0+i=0k-1d(ui, ui+1)

where $c_{0,0}$ remains the same for all paths and may be ignored. So, the optimal path minimizes i=0k-1d(ui, ui+1) where $u_0 = v_{n,1}$ and $u_k = v_{1,m}$. This can be accomplished by computing the optimal path from $v_{n,1}$ to $v_{1,m}$ using Djikstra’s algorithm on the graph G(V, E).

Hints: The problem can have multiple solutions. Students can use any design technique such as greedy method, backtracking, dynamic programming. Students can choose their own conditions, positive or negative costs for the graph.

AIM:-To find minimum cost and reach the destination (i.e. from bottom left to top right) in a maze of 0’s and 1’s with cost attached to each cell and moving in 4 possible directions.

Approach:-Backtracking

Given a grid of n*n filled with either 0(safe) or 1(un-safe) and a cost matrix with cost to move to respective square in the grid we use 4 recursive calls (up, down, left, right) with two base conditions

1-if the path go’s out of bound or it tries to revisit the visited cell or if the cell has 1 which is a wall we simply return the function

2-If the path reaches the destination then we push the path and the cost into a data structure and simply return the function

Once the path is completed we make the visited vertices un-visited

This recursive approach helps us to find the minimum cost of the path from source to destination

ALGORITHM: DFS

- Create a recursive function that takes the index of node and a visited array.
- Mark the current cell as visited .
- Traverse all the adjacent and unmarked nodes and call the recursive function with index of adjacent node and append to the path and sum up the cost.

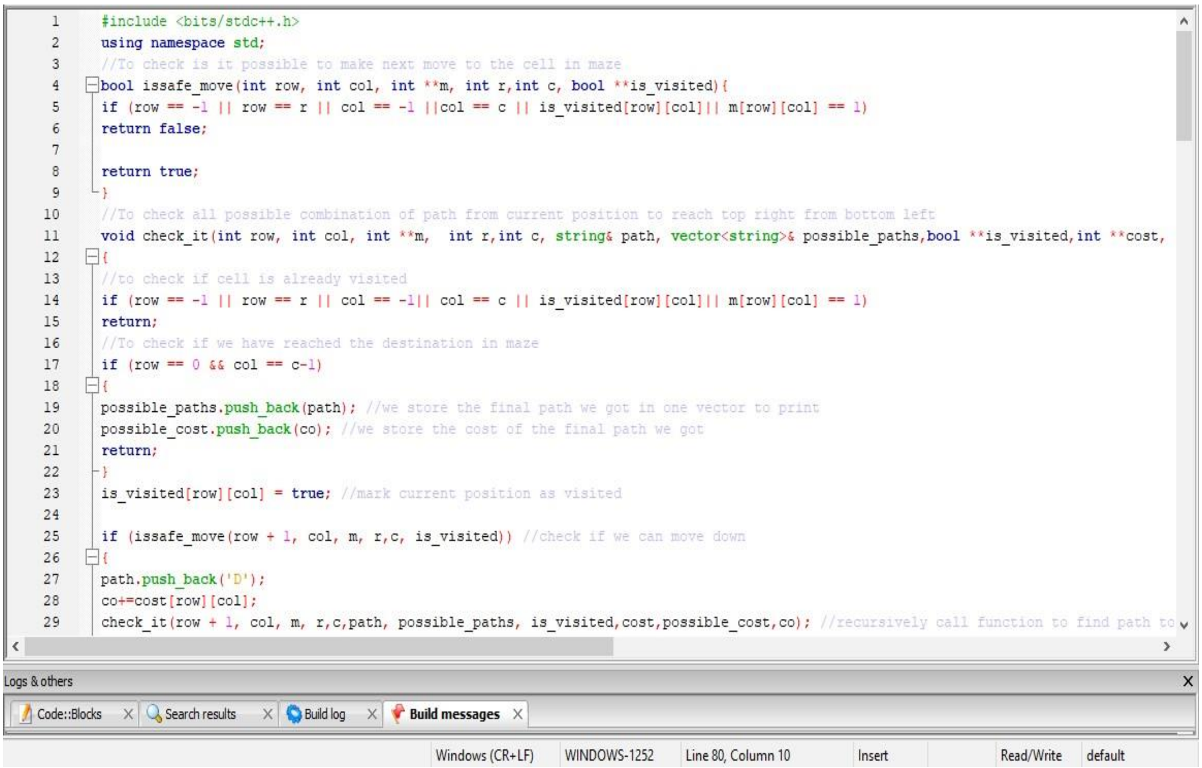
CODE:

```
#include <bits/stdc++.h>
using namespace std;
//To check is it possible to make next move to the cell in maze
bool issafe_move(int row, int col, int **m, int r,int c, bool **is_visited){
if (row == -1 || row == r || col == -1 ||col == c || is_visited[row][col]|| m[row][col]
== 1)
return false;

return true;
```

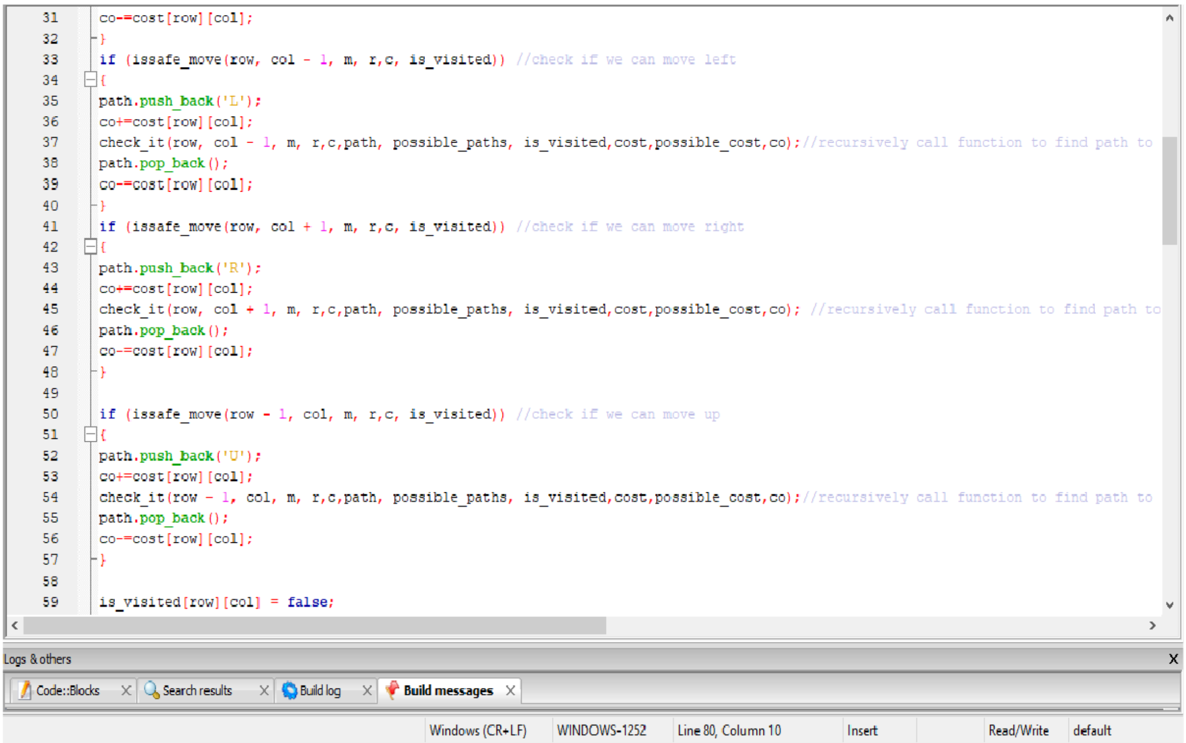
```
}
//To check all possible combination of path from current position to reach top right
from bottom left
void check_it(int row, int col, int **m,  int r,int c, string& path, vector<string>&
possible_paths,bool **is_visited,int **cost, vector<int>& possible_cost, int co)
{
//to check if cell is already visited
if (row == -1 || row == r || col == -1|| col == c || is_visited[row][col]|| m[row][col]
== 1)
return;
//To check if we have reached the destination in maze
if (row == 0 && col == c-1)
{
possible_paths.push_back(path); //we store the final path we got in one vector to
print
possible_cost.push_back(co); //we store the cost of the final path we got
return;
}
is_visited[row][col] = true; //mark current position as visited

if (issafe_move(row + 1, col, m, r,c, is_visited)) //check if we can move down
{
path.push_back('D');
co+=cost[row][col];
check_it(row + 1, col, m, r,c,path, possible_paths, is_visited,cost,possible_cost,co);
//recursively call function to find path to reach destination
path.pop_back(); //we delete the last path and search for other possible path
co-=cost[row][col];
}
if (issafe_move(row, col - 1, m, r,c, is_visited)) //check if we can move left
{
path.push_back('L');
co+=cost[row][col];
check_it(row, col - 1, m, r,c,path, possible_paths,
is_visited,cost,possible_cost,co);//recursively call function to find path to reach
destination
path.pop_back();
co-=cost[row][col];
}
if (issafe_move(row, col + 1, m, r,c, is_visited)) //check if we can move right
{
path.push_back('R');
co+=cost[row][col];
check_it(row, col + 1, m, r,c,path, possible_paths, is_visited,cost,possible_cost,co);
//recursively call function to find path to reach destination
path.pop_back();
```

```
co-=cost[row][col];
}

if (issafe_move(row - 1, col, m, r,c, is_visited)) //check if we can move up
{
path.push_back('U');
co+=cost[row][col];
check_it(row - 1, col, m, r,c,path, possible_paths,
is_visited,cost,possible_cost,co);//recursively call function to find path to reach
destination
path.pop_back();
co-=cost[row][col];
}

is_visited[row][col] = false;
}

void solve(int **m, int r,int c, int **cost)
{
vector<string> possible_paths;
vector<int> possible_cost;
int co=0;
string path;
//bool is_visited[n][n];
bool **is_visited=new bool *[r];
for(int i=0;i<r;i++)
is_visited[i]=new bool[c];
//memset(is_visited, false, sizeof(is_visited));
for(int i=0;i<r;i++)
for(int j=0;j<c;j++)
is_visited[i][j]=false; //we initialize the is_visited array to false initially as it is not
visited still

check_it(r-1, 0, m, r,c, path,possible_paths, is_visited,cost,possible_cost,co); //call
function to find all possible path
if(possible_paths.size()==0) //check if there is no path
{
cout<<"Oops Sorry,There is no path available to reach the destination !!:(";
}
else  //if multiple path exists
{
int minp=0;
cout<<"\nPaths available: "<<possible_paths.size()<<" \tcost "<<endl;
for (int i = 0; i < possible_paths.size(); i++)
{
cout << possible_paths[i] << "\t\t\t "<<possible_cost[i]+cost[0][c-1]<<endl;
if(possible_cost[i]<possible_cost[minp])
{
minp=i;  //finding the index of path which has min cost
}
}
cout<<"\nPath with minimum cost is : ";
cout<<"("<<possible_paths[minp]<<") "<<possible_cost[minp]+cost[0][c-1];
string st=possible_paths[minp];
char s[r][c];
for(int i=0;i<r;i++)
for(int j=0;j<c;j++)  // To display the path containing min cost in seperate matrix
s[i][j]='.';
s[r-1][0]='@';
int j=r-1,k=0;
for(int i=0;i<st.size();i++){
if(st[i]=='U')
s[--j][k]='@';
else if(st[i]=='R')
s[j][++k]='@';
else if(st[i]=='L')
s[j][--k]='@';
else
s[j++][k]='@';
}
cout<<"\n\nPATH HAVING MIN. COST:\n";
for(int  i=0;i<r;i++){
for(int j=0;j<c;j++){
cout<<s[i][j]<<"  ";}
cout<<endl;}
}
}
int main()
{
int i,j;
cout<<"Enter the number of rows and columns for maze :";
int r,c;
cin>>r>>c; //input no.of rows and columns
int **m=new int *[r];
for(int i=0;i<r;i++)
m[i]=new int[c];
int **cost=new int *[r];
for(int i=0;i<r;i++)
cost[i]=new int[c];
//int m[n][n],cost[n][n];
cout<<"\nEnter the values of each cell in matrix in 0's and 1's:\n";
for(i=0;i<r;i++)
```

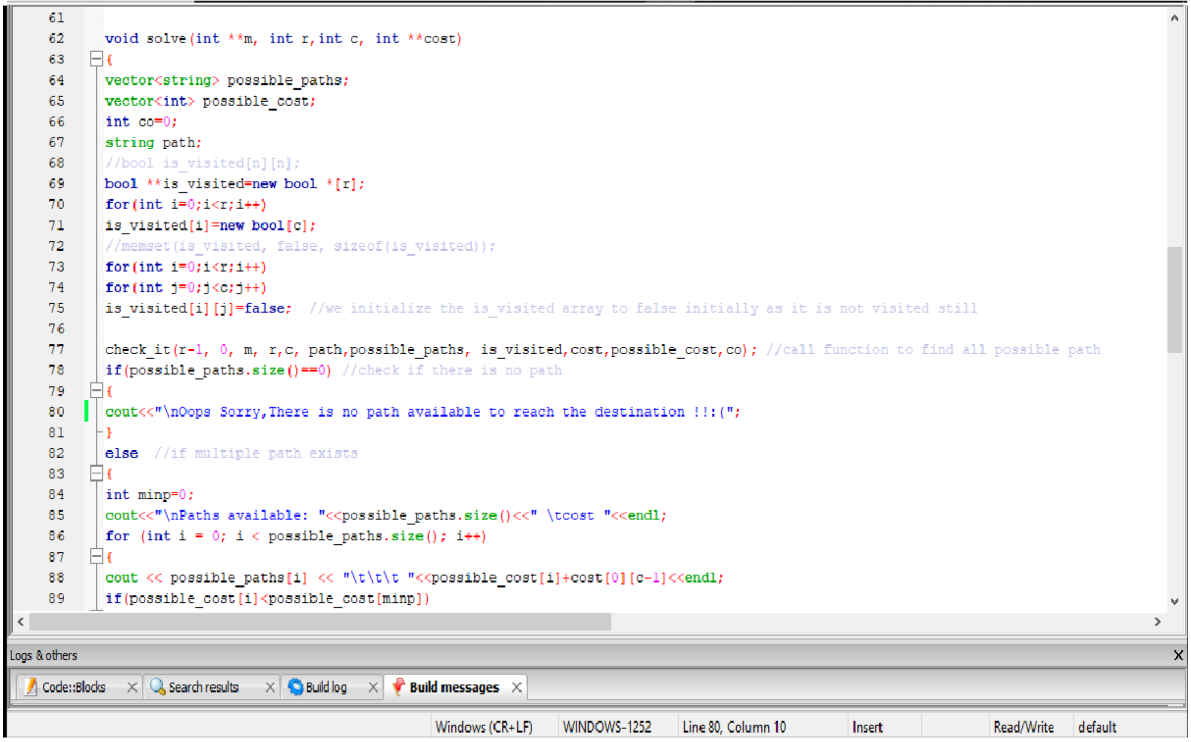
```
{
for(j=0;j<c;j++)
{
cin>>m[i][j];  //input the 1's and 0's of maze
}
}
cout<<"\nEnter cost for the maze :\n";
for(i=0;i<r;i++)
{
for(j=0;j<c;j++)
{
cin>>cost[i][j];  //input the cost of each cell
}
}

solve(m, r,c, cost); //solve
cout<<endl;
return 0;
}
```



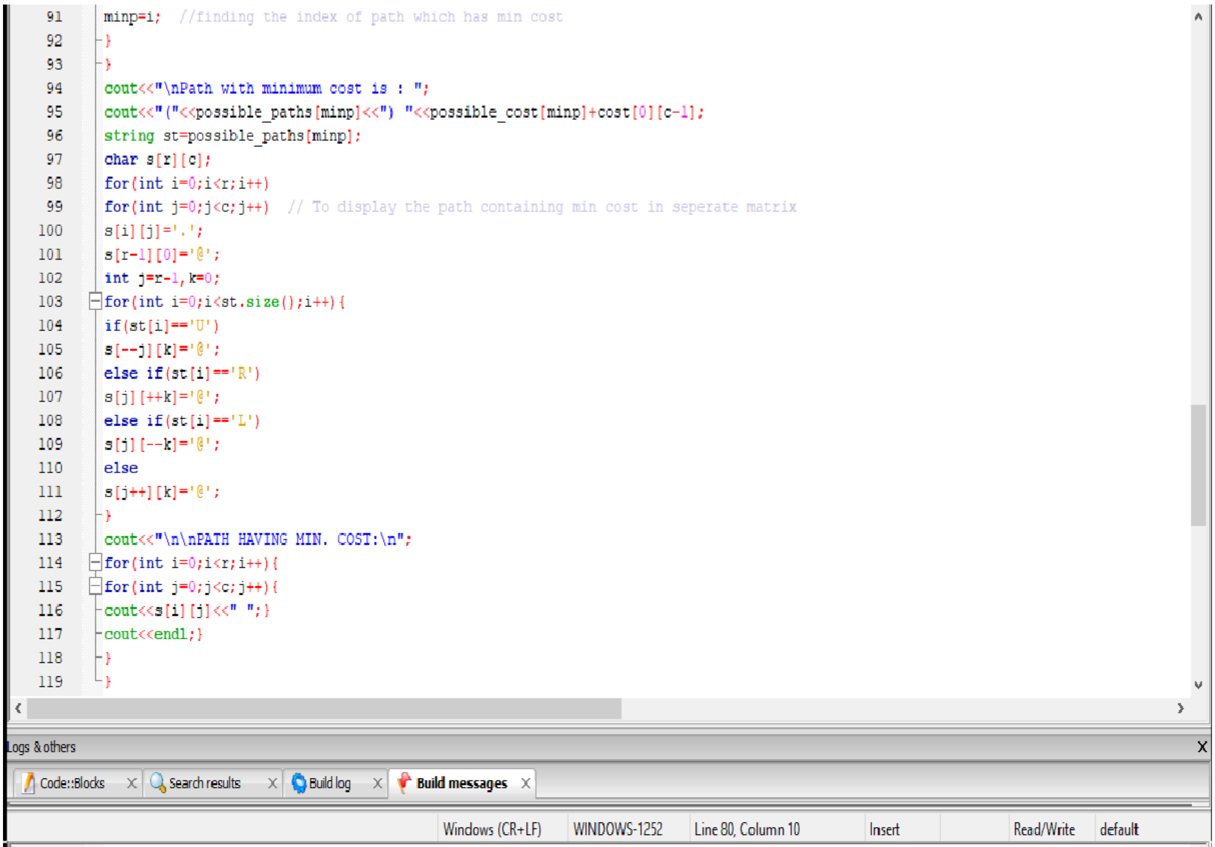
```
#include <bits/stdc++.h>
using namespace std;
//To check is it possible to make next move to the cell in maze
bool issafe_move(int row, int col, int **m, int r,int c, bool **is_visited){
if (row == -1 || row == r || col == -1 ||col == c || is_visited[row][col]|| m[row][col] == 1)
return false;

return true;
}
//To check all possible combination of path from current position to reach top right from bottom left
void check_it(int row, int col, int **m,  int r,int c, string& path, vector<string>& possible_paths,bool **is_visited,int **cost,
{
//to check if cell is already visited
if (row == -1 || row == r || col == -1|| col == c || is_visited[row][col]|| m[row][col] == 1)
return;
//To check if we have reached the destination in maze
if (row == 0 && col == c-1)
{
possible_paths.push_back(path); //we store the final path we got in one vector to print
possible_cost.push_back(co); //we store the cost of the final path we got
return;
}
is_visited[row][col] = true; //mark current position as visited

if (issafe_move(row + 1, col, m, r,c, is_visited)) //check if we can move down
{
path.push_back('D');
co+=cost[row][col];
check_it(row + 1, col, m, r,c,path, possible_paths, is_visited,cost,possible_cost,co); //recursively call function to find path to
```



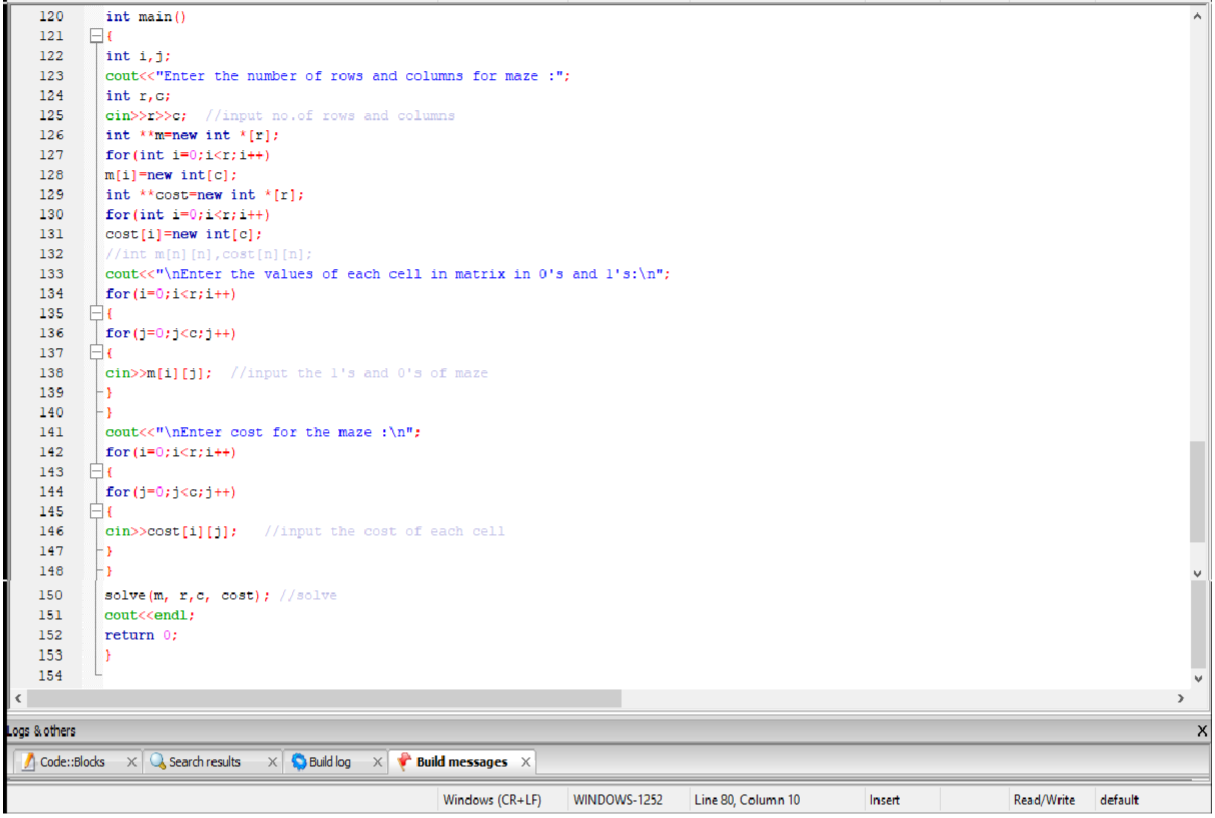
```
co-=cost[row][col];
}
if (issafe_move(row, col - 1, m, r,c, is_visited)) //check if we can move left
{
path.push_back('L');
co+=cost[row][col];
check_it(row, col - 1, m, r,c,path, possible_paths, is_visited,cost,possible_cost,co);//recursively call function to find path to
path.pop_back();
co-=cost[row][col];
}
if (issafe_move(row, col + 1, m, r,c, is_visited)) //check if we can move right
{
path.push_back('R');
co+=cost[row][col];
check_it(row, col + 1, m, r,c,path, possible_paths, is_visited,cost,possible_cost,co); //recursively call function to find path to
path.pop_back();
co-=cost[row][col];
}

if (issafe_move(row - 1, col, m, r,c, is_visited)) //check if we can move up
{
path.push_back('U');
co+=cost[row][col];
check_it(row - 1, col, m, r,c,path, possible_paths, is_visited,cost,possible_cost,co);//recursively call function to find path to
path.pop_back();
co-=cost[row][col];
}

is_visited[row][col] = false;
```



```

void solve(int **m, int r,int c, int **cost)
{
vector<string> possible_paths;
vector<int> possible_cost;
int co=0;
string path;
//bool is_visited[n][n];
bool **is_visited=new bool *[r];
for(int i=0;i<r;i++)
is_visited[i]=new bool[c];
//memset(is_visited, false, sizeof(is_visited));
for(int i=0;i<r;i++)
for(int j=0;j<c;j++)
is_visited[i][j]=false;  //we initialize the is_visited array to false initially as it is not visited still

check_it(r-1, 0, m, r,c, path,possible_paths, is_visited,cost,possible_cost,co); //call function to find all possible path
if(possible_paths.size()==0) //check if there is no path
{
cout<<"\nOops Sorry,There is no path available to reach the destination !!:(";
}
else  //if multiple path exists
{
int minp=0;
cout<<"\nPaths available: "<<possible_paths.size()<<" \tcost "<<endl;
for (int i = 0; i < possible_paths.size(); i++)
{
cout << possible_paths[i] << "\t\t\t "<<possible_cost[i]+cost[0][c-1]<<endl;
if(possible_cost[i]<possible_cost[minp])
```



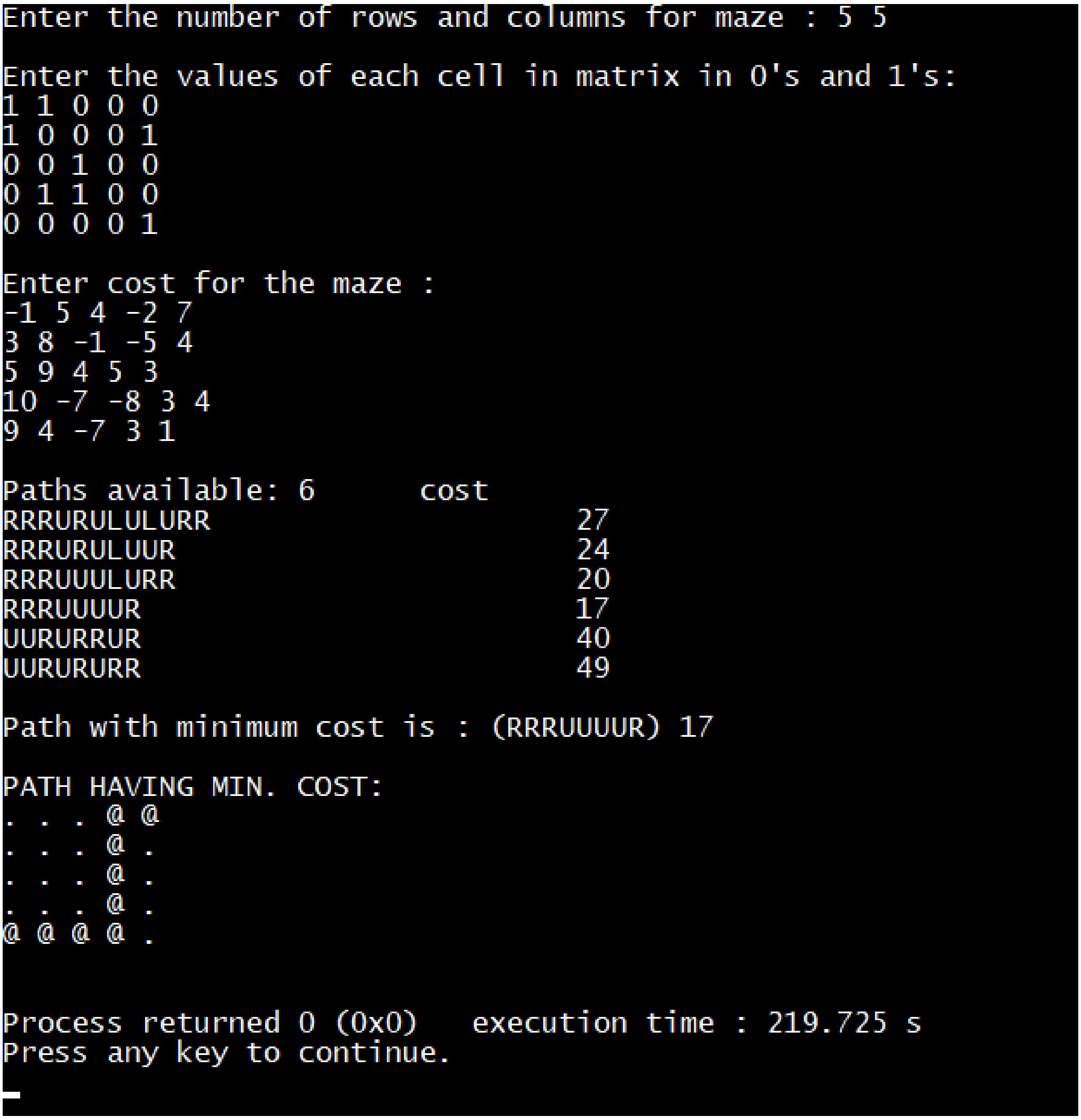
```
minp=i;  //finding the index of path which has min cost
}
}
cout<<"\nPath with minimum cost is : ";
cout<<"("<<possible_paths[minp]<<") "<<possible_cost[minp]+cost[0][c-1];
string st=possible_paths[minp];
char s[r][c];
for(int i=0;i<r;i++)
for(int j=0;j<c;j++)  // To display the path containing min cost in seperate matrix
s[i][j]='.';
s[r-1][0]='@';
int j=r-1,k=0;
for(int i=0;i<st.size();i++){
if(st[i]=='U')
s[--j][k]='@';
else if(st[i]=='R')
s[j][++k]='@';
else if(st[i]=='L')
s[j][--k]='@';
else
s[j++][k]='@';
}
cout<<"\n\nPATH HAVING MIN. COST:\n";
for(int i=0;i<r;i++){
for(int j=0;j<c;j++){
cout<<s[i][j]<<" ";}
cout<<endl;}
}
}
```



```
int main()
{
int i,j;
cout<<"Enter the number of rows and columns for maze :";
int r,c;
cin>>r>>c;  //input no.of rows and columns
int **m=new int *[r];
for(int i=0;i<r;i++)
m[i]=new int[c];
int **cost=new int *[r];
for(int i=0;i<r;i++)
cost[i]=new int[c];
//int m[n][n],cost[n][n];
cout<<"\nEnter the values of each cell in matrix in 0's and 1's:\n";
for(i=0;i<r;i++)
{
for(j=0;j<c;j++)
{
cin>>m[i][j];  //input the 1's and 0's of maze
}
}
cout<<"\nEnter cost for the maze :\n";
for(i=0;i<r;i++)
{
for(j=0;j<c;j++)
{
cin>>cost[i][j];   //input the cost of each cell
}
}
solve(m, r,c, cost); //solve
cout<<endl;
return 0;
}
```

TEST CASE-1

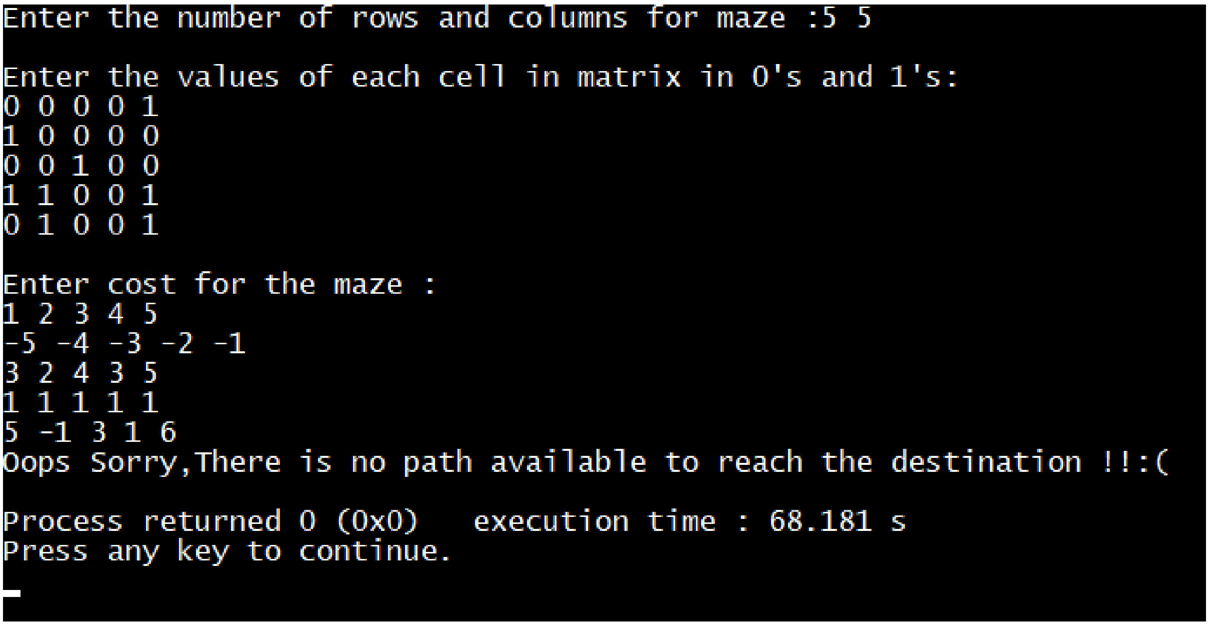
Input & Output:



```
Enter the number of rows and columns for maze : 5 5

Enter the values of each cell in matrix in 0's and 1's:
1 1 0 0 0
1 0 0 0 1
0 0 1 0 0
0 1 1 0 0
0 0 0 0 1

Enter cost for the maze :
-1 5 4 -2 7
3 8 -1 -5 4
5 9 4 5 3
10 -7 -8 3 4
9 4 -7 3 1

Paths available: 6      cost
RRRURULULURR                    27
RRRURULUUR                      24
RRRUUULURR                      20
RRRUUUUR                        17
UURURRUR                        40
UURURURR                        49

Path with minimum cost is : (RRRUUUUR) 17

PATH HAVING MIN. COST:
. . . @ @
. . . @ .
. . . @ .
. . . @ .
@ @ @ @ .


Process returned 0 (0x0)   execution time : 219.725 s
Press any key to continue.
```

TEST CASE-2

Input & Output:



```
Enter the number of rows and columns for maze :5 5

Enter the values of each cell in matrix in 0's and 1's:
0 0 0 0 1
1 0 0 0 0
0 0 1 0 0
1 1 0 0 1
0 1 0 0 1

Enter cost for the maze :
1 2 3 4 5
-5 -4 -3 -2 -1
3 2 4 3 5
1 1 1 1 1
5 -1 3 1 6
Oops Sorry,There is no path available to reach the destination !!:(

Process returned 0 (0x0)   execution time : 68.181 s
Press any key to continue.
```

COMPLEXITY ANALYSIS:

TIME COMPLEXITY:

1. Number of total cells=SIZE^2

2. Each cell has a maximum of 3 unvisited neighbouring cells

3. Therefore, time complexity= O(3^(SIZE^2))=O(3^(N^2))

SPACE COMPLEXITY:

1. As there can only be a maximum of 3 unvisited cells for each cell

2. The space complexity=O(3^(N^2))

RESULT:

Using the backtracking method, we determined the optimized solution for the maze problem with the cost that will be incurred for the whole traversal.