Deep Learning and Applications

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Today's Outline

- Feedforward Neural Nets
- Convolutional Neural Nets
  - Convolution
  - Pooling

Feedforward Neural Network

- Linear model $f(x, W, b) = Wx + b$
- A feedforward neural network model will include nonlinearities
- Two layer model
  - $f(x, W_1, b_1, W_2, b_2) = W_2 \max(0, W_1 x + b_1) + b_2$
  - Say x is d dimensional
  - The number of hidden nodes is q
  - The number of labels is p
  - The notion of layer is for vectorizing/is conceptual

Nonlinearities (I)

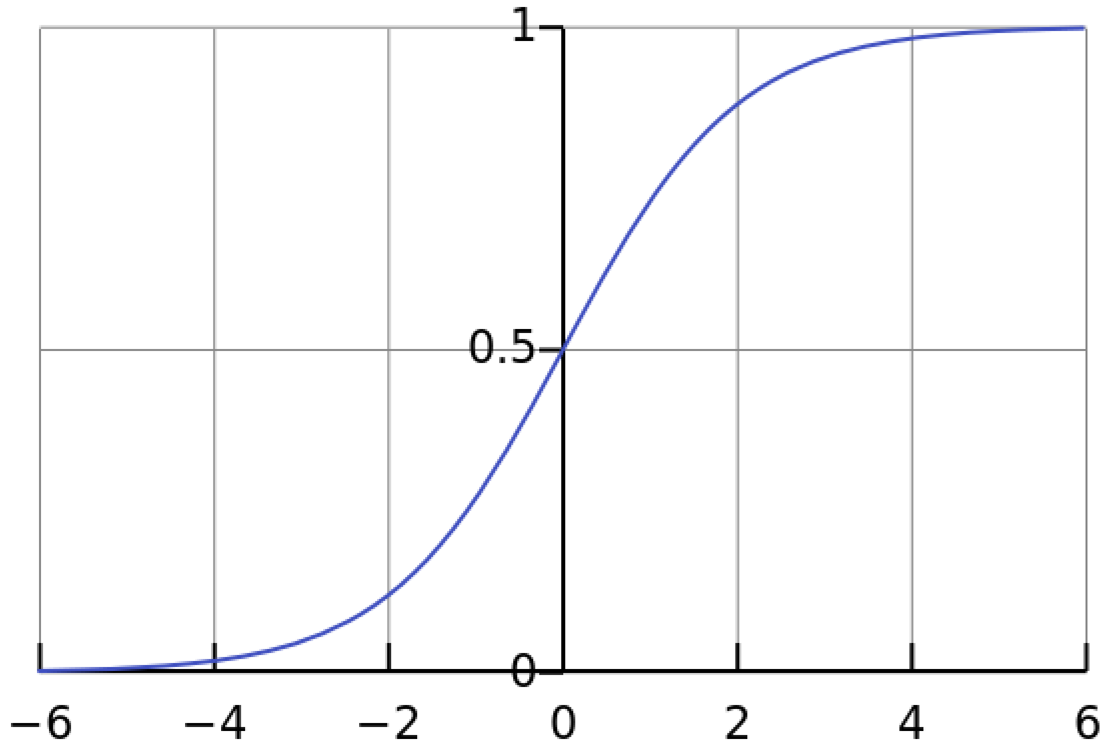
Name	Formula	Year
none	$y = x$	-
sigmoid	$y = \frac{1}{1+e^{-x}}$	1986
tanh	$y = \frac{e^{2x}-1}{e^{2x}+1}$	1986
ReLU	$y = \max(x, 0)$	2010
(centered) SoftPlus	$y = \ln\left(e^x + 1\right) - \ln 2$	2011
LReLU	$y = \max(x, \alpha x),\ \alpha \approx 0.01$	2011
maxout	$y = \max(W_1 x + b_1, W_2 x + b_2)$	2013
APL	$y = \max(x,0) + \sum_{s=1}^{S} a_i^s \max\left(0, -x + b_i^s\right)$	2014
VLReLU	$y = \max(x, \alpha x),\ \alpha \in 0.1, 0.5$	2014
RReLU	$y = \max(x, \alpha x),\ \alpha = \text{random}(0.1,\ 0.5)$	2015
PReLU	$y = \max(x, \alpha x),\ \alpha \text{ is learnable}$	2015
ELU	$y = x, \text{ if } x \geq 0, \text{ else } \alpha(e^x - 1)$	2015

- How to pick the nonlinearity/activation function?

[1] Systematic evaluation of CNN advances on the ImageNet, arxiv:1606.02228

Nonlinearities (II)

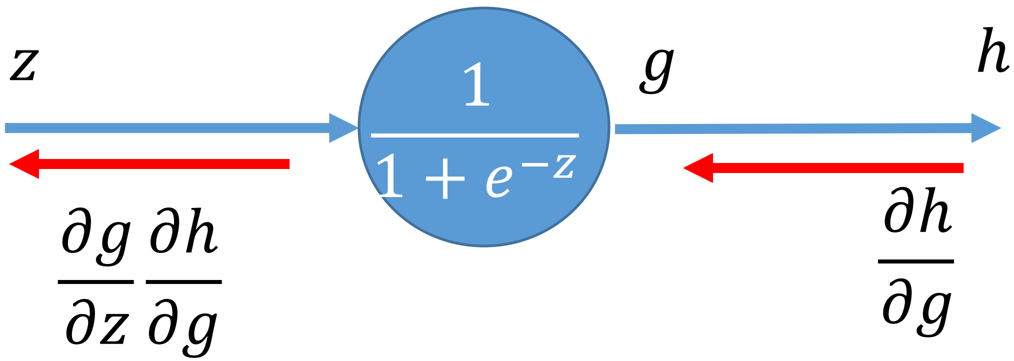
- Sigmoid
  - Is a map whose range is [0,1]

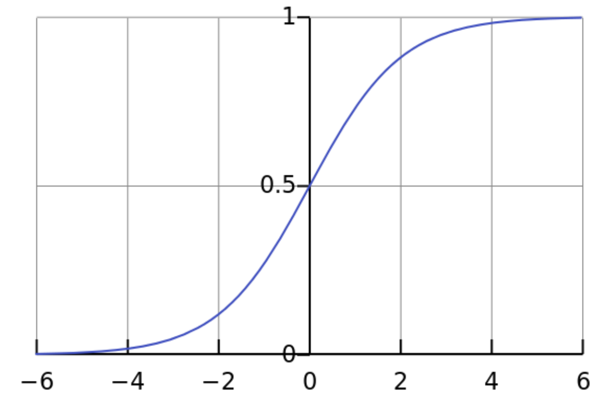


[1]Figure: Qef, Public Domain, https://commons.wikimedia.org/w/index.php?curid=4310325

Nonlinearities (III)

- Saturated node/neuron makes gradients vanish



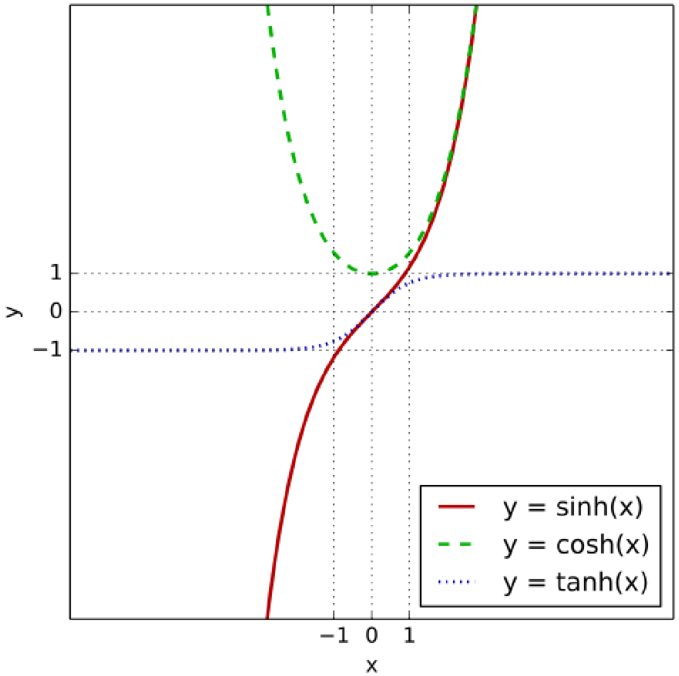


- Not zero-centered
  - Empirically may lead to slower convergence

[1]Figure: Qef, Public Domain, https://commons.wikimedia.org/w/index.php?curid=4310325

Nonlinearities (IV)

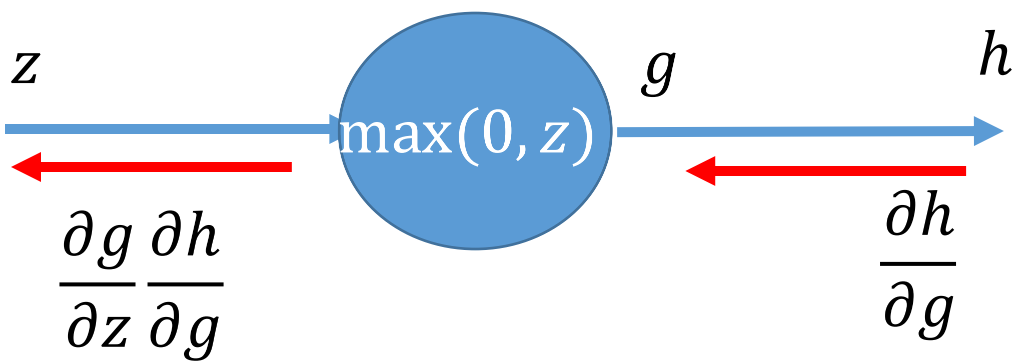
- tanh() addresses the zero-centering problem. So will typically give better results
- Still gradients vanish



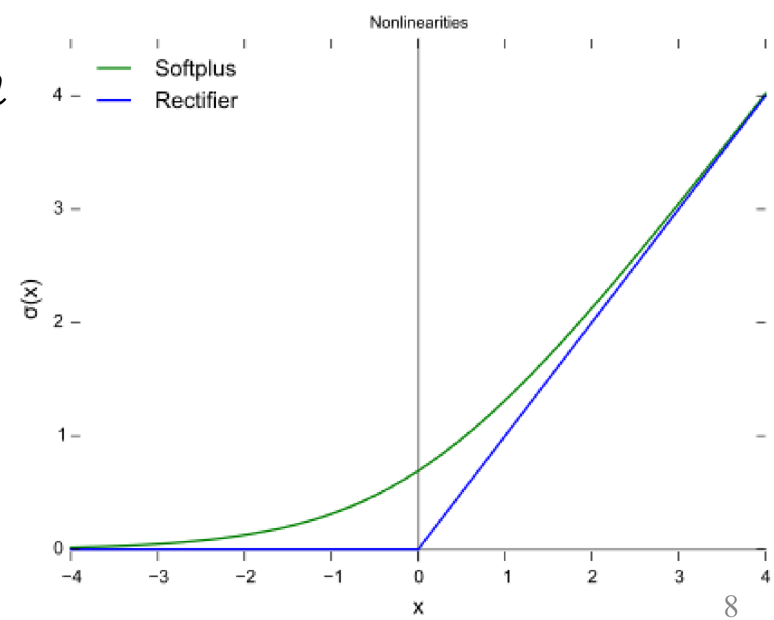
[1]Figure: Fylwind, Public Domain, https://commons.wikimedia.org/w/index.php?curid=1642946

Nonlinearities (V)

- ReLU (2012 Krizhevsky et al.)
- No vanishing gradient on the positive side
- Empirically observed to be very good
- Initialization/high learning rate may lead to permanently dead ReLUs (diagnosable)



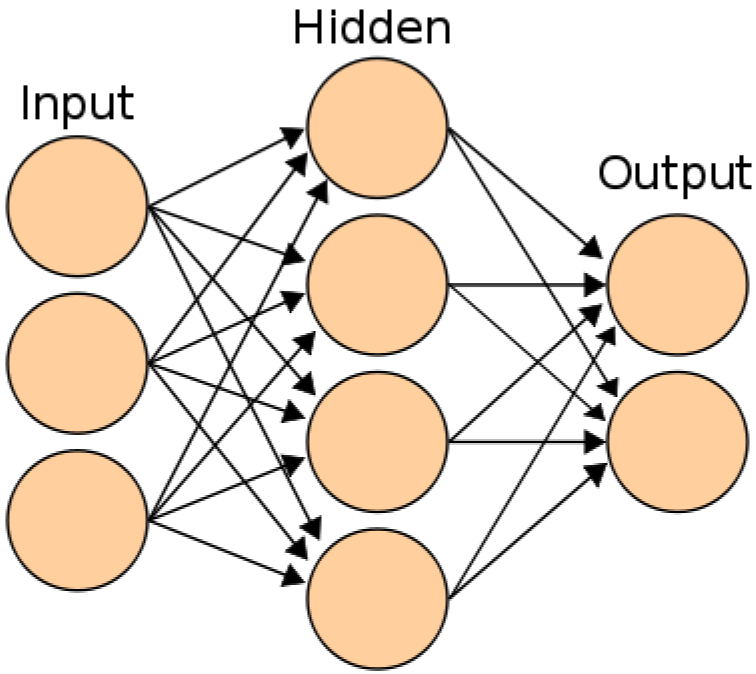
Is a gradient gate!



[1]Figure: CC0, https://en.wikipedia.org/w/index.php?curid=48817276

Feedforward Neural Net

- Lets focus on a 2-layer net
- Layers
  - Input
  - Hidden
  - Output
- Node
- Nonlinearity
  - Activation



$$f(x, W_1, b_1, W_2, b_2) = W_2 \max(0, W_1 x + b_1) + b_2$$

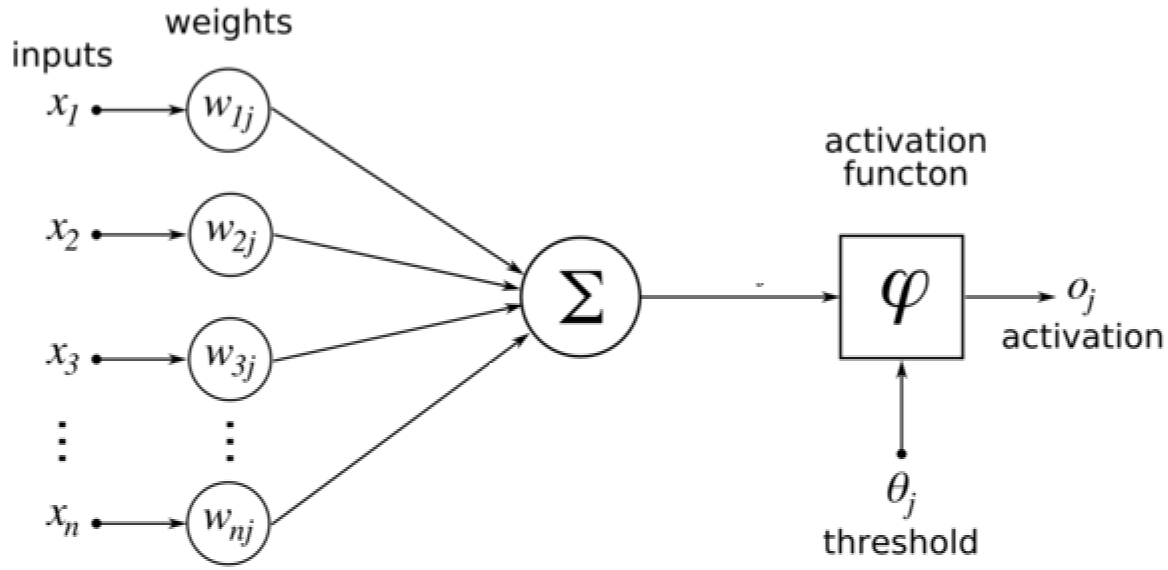
[1] Figure: https://en.wikibooks.org/wiki/Artificial_Neural_Networks/Print_Version

Neuron (I)

- Historical
- Let $f(x) = w \cdot x + b$
- Perceptron from 1957: $h(x) = \begin{cases} 0, & f(x) < 0 \\ 1, & \text{otherwise} \end{cases}$
- Update rule was $w_{k+1} = w_k + \alpha(y - h(x))x$ similar to gradient update rules we see today
- Passing the score through a sigmoid was likened to how a neuron fires
  - Firing rate $= \frac{1}{1+e^{-yf(x)}}$

Neuron (II)

- Get a vector x_i and transform it to a score vector by passing through a sequence of hidden layers
- Each hidden layer has neurons
- Each neuron is fully connected to previous layer

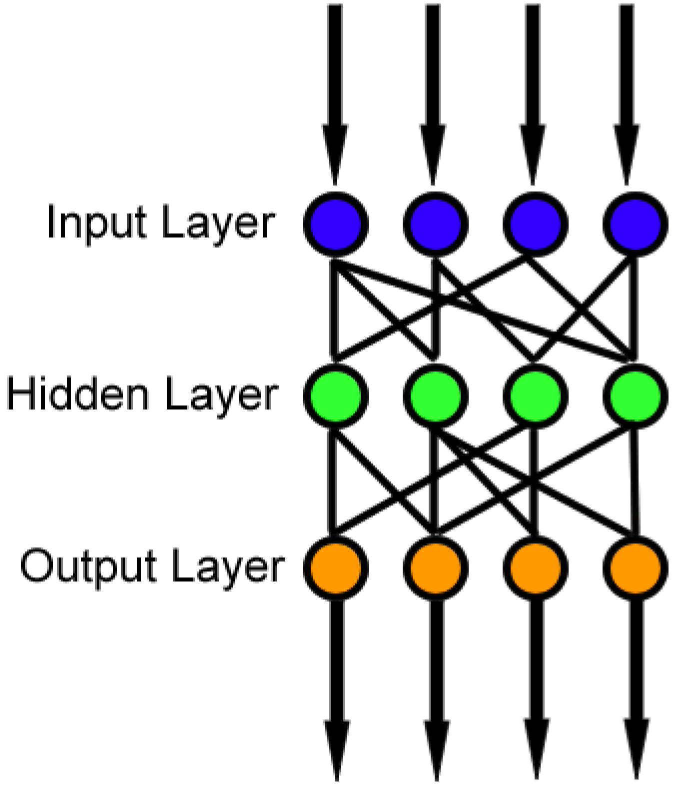


[1]Figure: https://en.wikibooks.org/wiki/Artificial_Neural_Networks/Print_Version

Feedforward Net: Two Layer Model

- Number of layers is the number of W, b pairs

- Some questions to think about:
  - How to pick the number of layers?
  - How to pick the number of hidden units in each layer?



[1]CC BY-SA 3.0, https://en.wikipedia.org/w/index.php?curid=8201514

Feedforward Net and Backprop

- Choose a mini-batch (sample) of size B
- Forward propagate through the computation graph
  - Compute losses $L_{i_1}, L_{i_2}, \ldots L_{i_B}$ and $R(W_1, b_1, W_2, b_2)$
  - Get loss L for the batch
- Backprop to compute gradients with respect to W_1, b_1, W_2 and b_2
- Update parameters W_1, b_1, W_2 and b_2
  - In the direction of the negative gradient

Feedforward Net in Python

```
# Feedforward neural net model


# Start with an initial set of parameters randomly
h = 100 # size of hidden layer
W = 0.01 * np.random.randn(D,h)
b = np.zeros((1,h))
W2 = 0.01 * np.random.randn(h,K)
b2 = np.zeros((1,K))



# Initial values from hyperparameter
reg = 1e-3 # regularization strength

#For simplicity, we will not optimize this using grid search here.
```

Feedforward Net in Python

```
#Perform batch SGD using manual backprop


#For simplicity we will take the batch size to be the same as number of examples
num_examples = X.shape[0]

#Initial value for the Gradient Descent Parameter
step_size = 1e-0 #Also called learning rate

#For simplicity, we will not hand tune this algorithm parameter as well.


# gradient descent loop
for i in xrange(10000):

  # evaluate class scores, [N x K]
  hidden_layer = np.maximum(0, np.dot(X, W) + b) # note, ReLU activation
  scores = np.dot(hidden_layer, W2) + b2

  # compute the class probabilities
  exp_scores = np.exp(scores)
  probs = exp_scores / np.sum(exp_scores, axis=1, keepdims=True) # [N x K]

  # compute the loss: average cross-entropy loss and regularization
  corect_logprobs = -np.log(probs[range(num_examples),y])
  data_loss = np.sum(corect_logprobs)/num_examples
  reg_loss = 0.5*reg*np.sum(W*W) + 0.5*reg*np.sum(W2*W2)
  loss = data_loss + reg_loss
  if i % 1000 == 0:
    print "iteration %d: loss %f" % (i, loss)
```

Feedforward Net in Python

```python
# compute the gradient on scores
dscores = probs
dscores[range(num_examples),y] -= 1
dscores /= num_examples

# backpropate the gradient to the parameters
# first backprop into parameters W2 and b2
dW2 = np.dot(hidden_layer.T, dscores)
db2 = np.sum(dscores, axis=0, keepdims=True)
# next backprop into hidden layer
dhidden = np.dot(dscores, W2.T)
# backprop the ReLU non-linearity
dhidden[hidden_layer <= 0] = 0
# finally into W,b
dW = np.dot(X.T, dhidden)
db = np.sum(dhidden, axis=0, keepdims=True)

# add regularization gradient contribution
dW2 += reg * W2
dW += reg * W

# perform a parameter update
W += -step_size * dW
b += -step_size * db
W2 += -step_size * dW2
b2 += -step_size * db2
```

Feedforward Net in Python

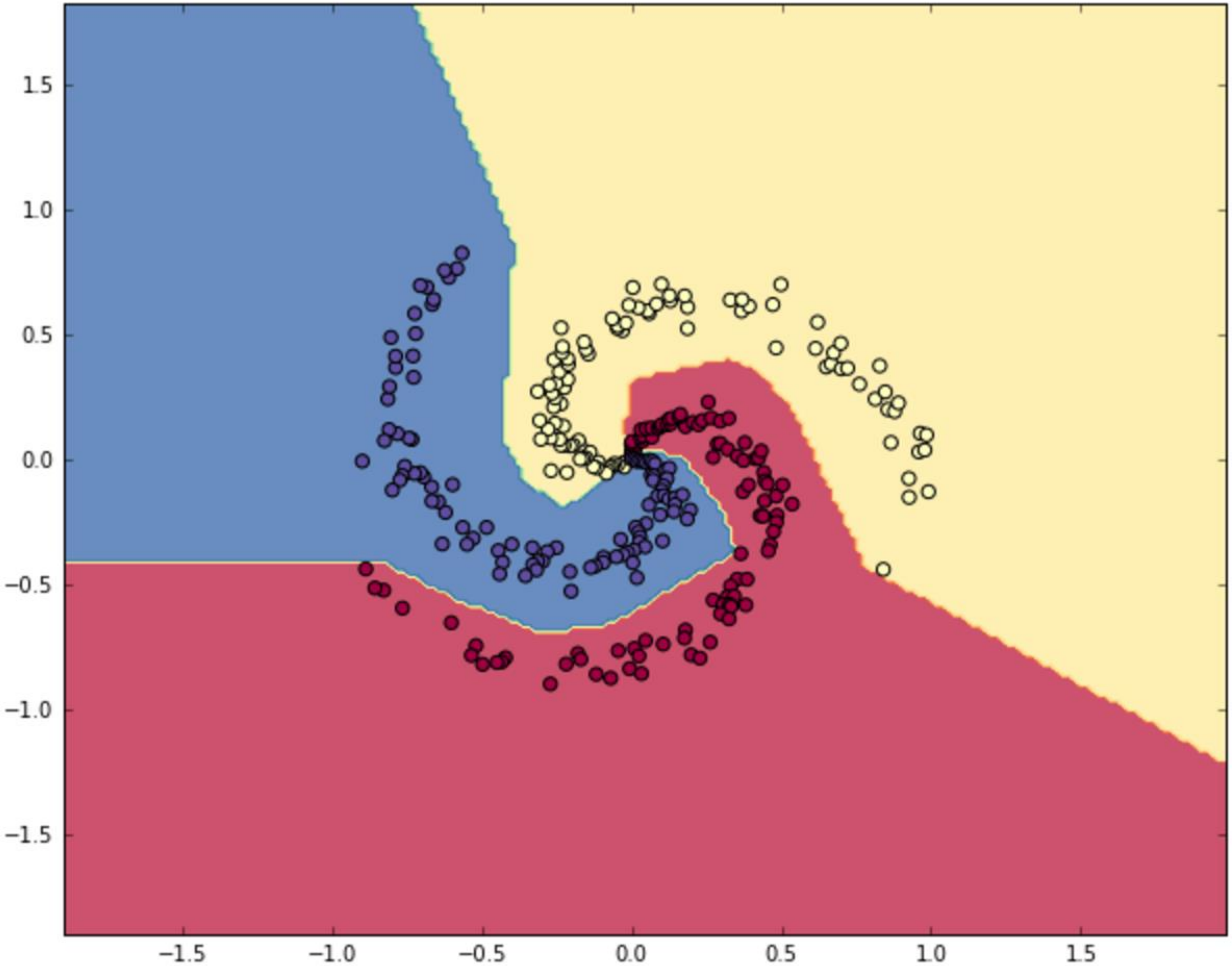
Post Training

```
# Post-training: evaluate test set accuracy

#For simplicity, we will use training data as proxy for test. Do not do this.
X_test = X
y_test = y


hidden_layer = np.maximum(0, np.dot(X_test, W) + b)
scores = np.dot(hidden_layer, W2) + b2
predicted_class = np.argmax(scores, axis=1)
print 'test accuracy: %.2f' % (np.mean(predicted_class == y_test))
```

Feedforward Net in Python



Questions?

Today's Outline

- Feedforward Neural Nets
- Convolutional Neural Nets
  - Convolution
  - Pooling

Convolutional Neural Network

Similar to Feedforward NN

- Similar to feedforward neural networks
- Each neuron/node is associated with weights and a bias
- Node receives input
  - Performs dot product of vectors
  - Applies non-linearity

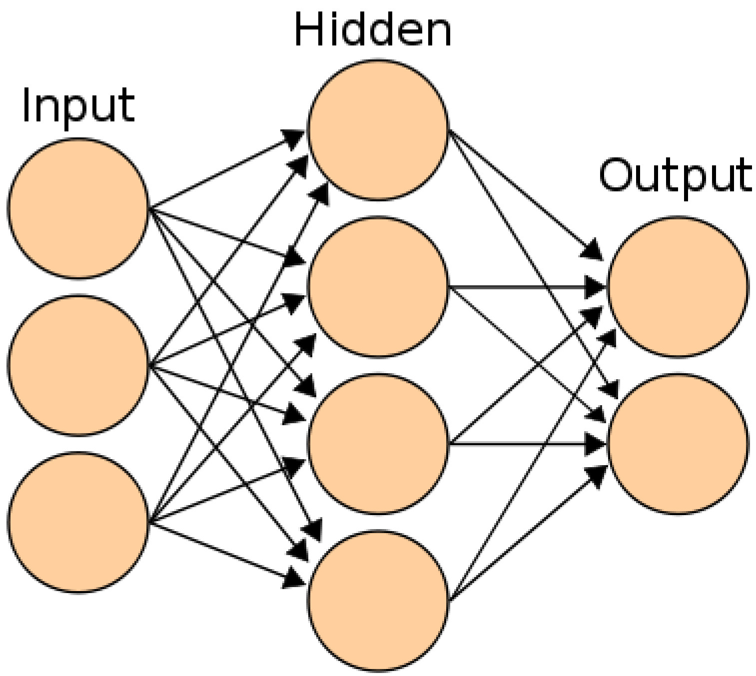
- The difference:
  - Number of parameters is reduced!

How? That is the content of this lecture!

[1]Reference: http://cs231n.github.io/convolutional-networks/

Towards CNNs (I)

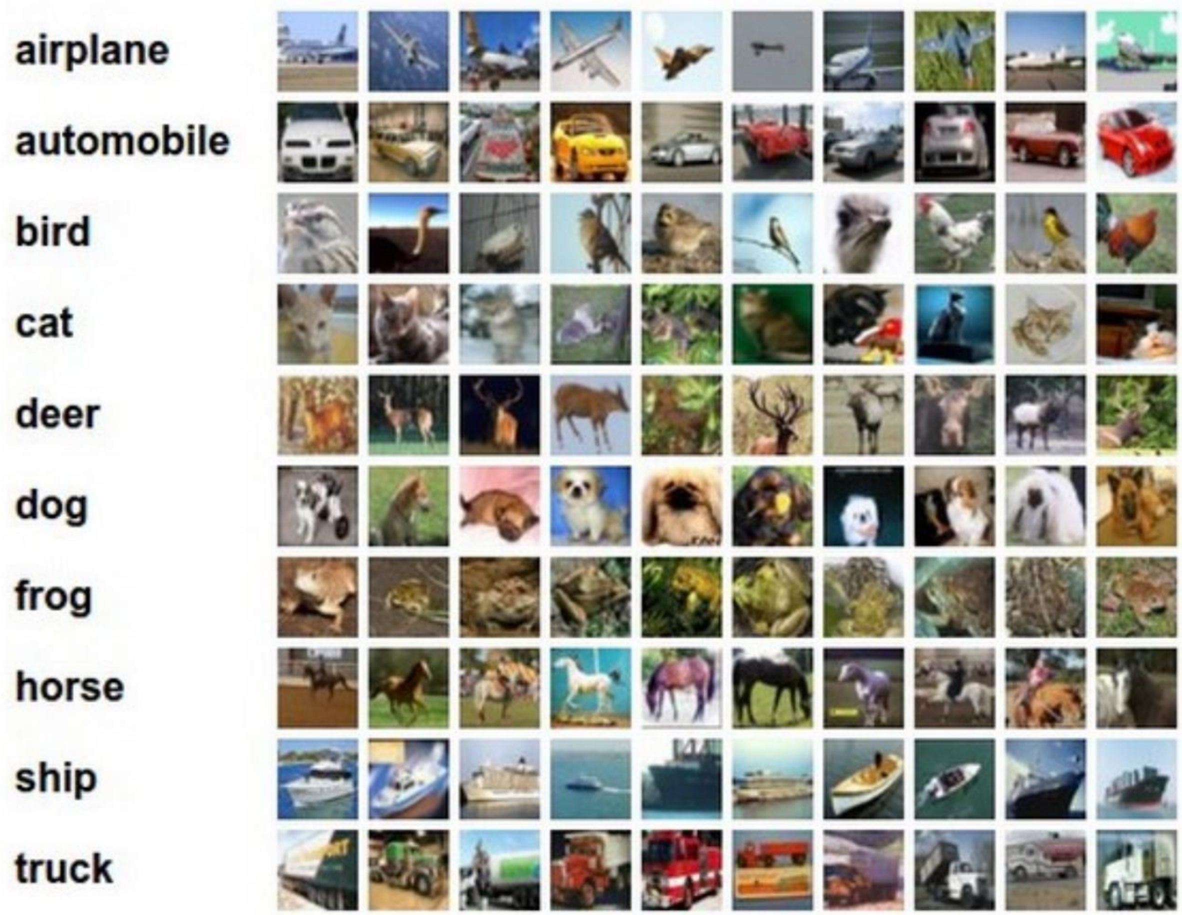
- Feedforward net:
  - Can you visualize the connections for an arbitrary neuron here?



[1]Figure: https://en.wikibooks.org/wiki/Artificial_Neural_Networks/Print_Version

Towards CNNs (II)

- Consider the CIFAR-10 Dataset. Images are 32*32*3 in size



[1]Figure: http://cs231n.github.io/classification/

Towards CNNs (III)

- First fully connected feedforward neuron would have 32*32*3 weights associated with it (+1 bias parameter)
- What if the images were 1280*800*3?

- Clearly, we also need many neurons in each hidden layer. This leads to explosion in the total number of parameters (or the dimension of Ws and bs)

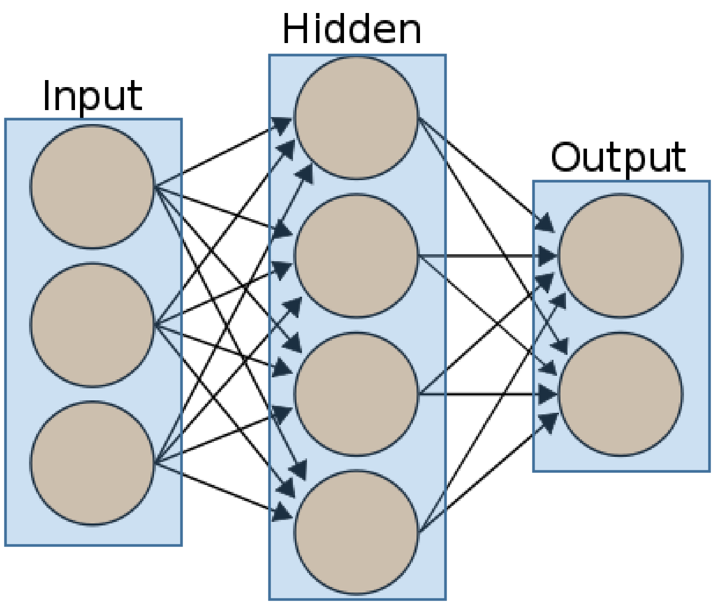
CNN Architecture

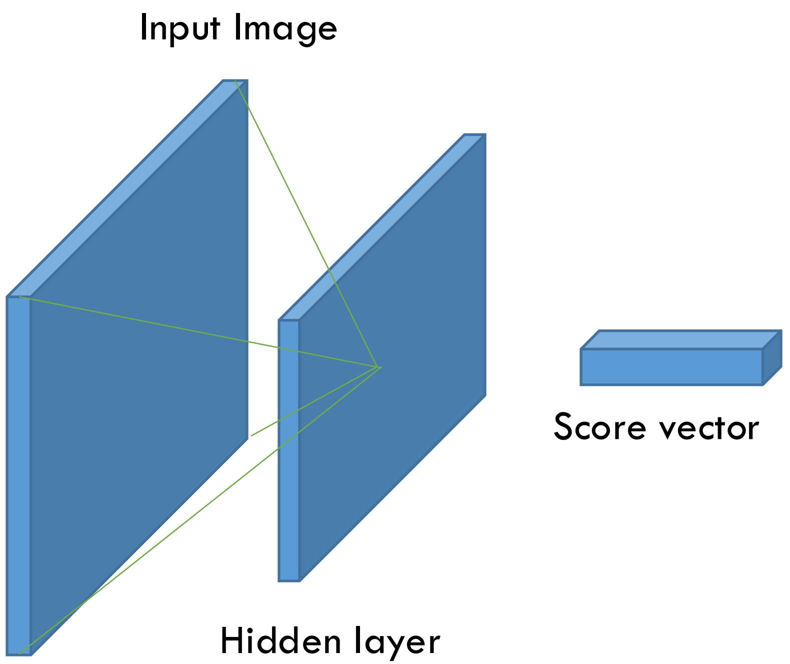
- We will look at it from layers point of view

- The new idea is that layers have width and depth!
  - (In contrast, Feedforward NN layers only had height)
  - (depth here does NOT correspond to number of layers of a network)

CNN Architecture

- View FFN layers as having width and height





[1]Left figure: https://en.wikibooks.org/wiki/Artificial_Neural_Networks/Print_Version

CNN Architecture

- The new idea is that CNN layers have depth!
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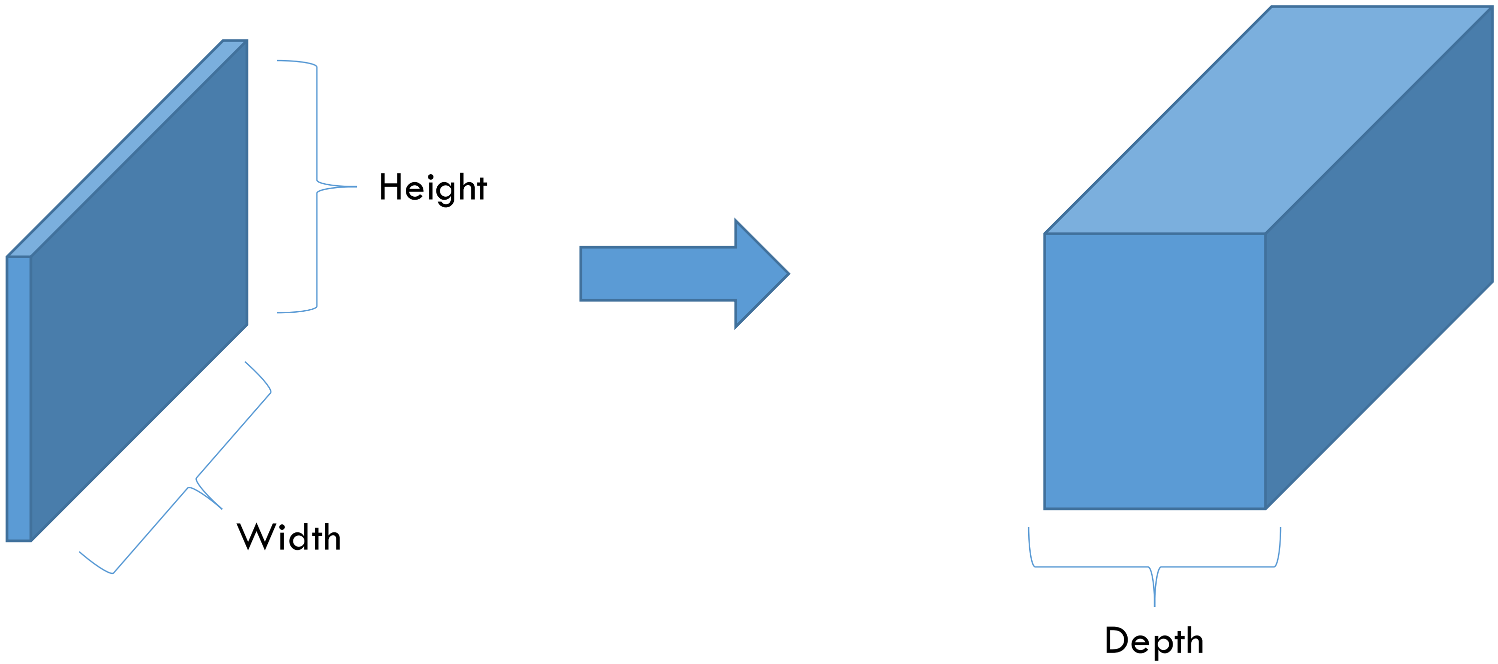
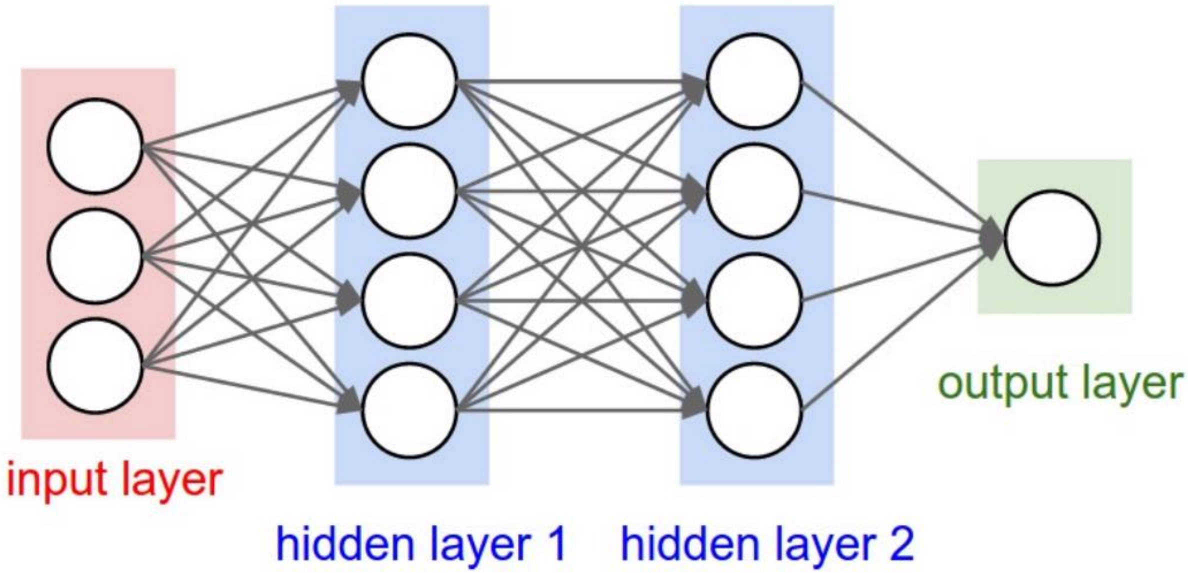


3D Volumes of Neurons

- Input has dimension 32*32*3 (for CIFAR-10 dataset)
- Final output has dimension 1*1*10 (10 classes)

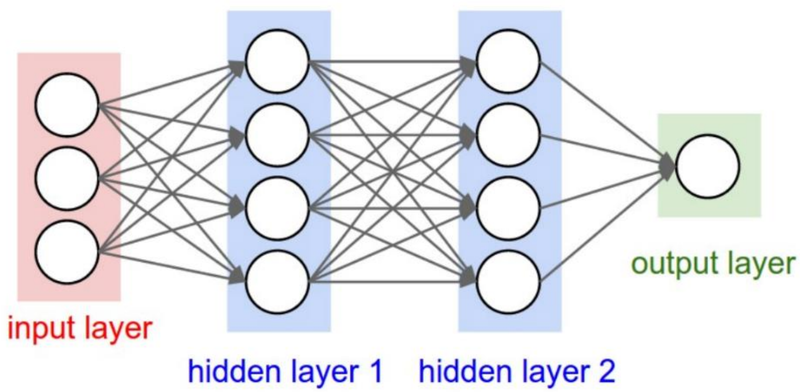
- Previously,

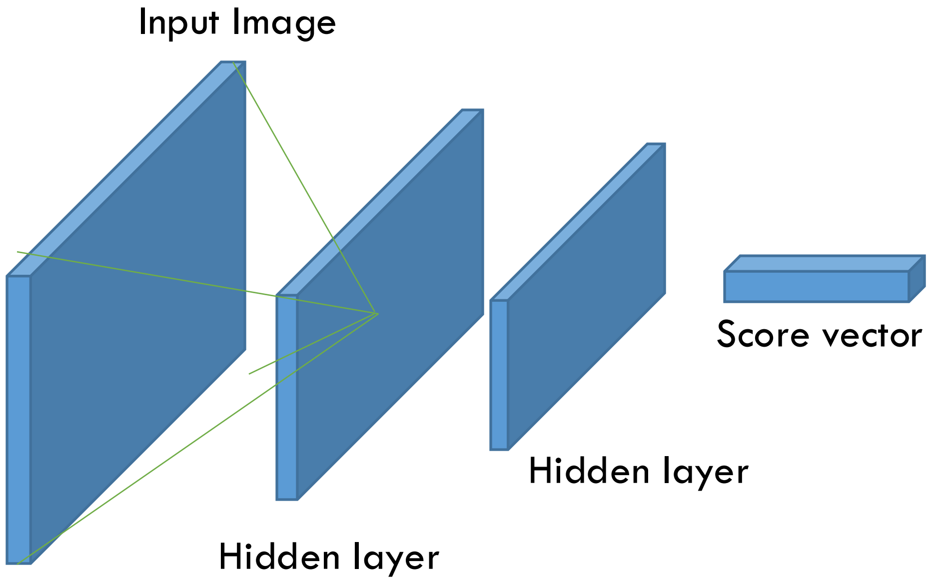


3D Volumes of Neurons

- Input has dimension 32*32*3 (for CIFAR-10 dataset)
- Final output has dimension 1*1*10 (10 classes)

- So assuming 2 hidden layers, previously we had,

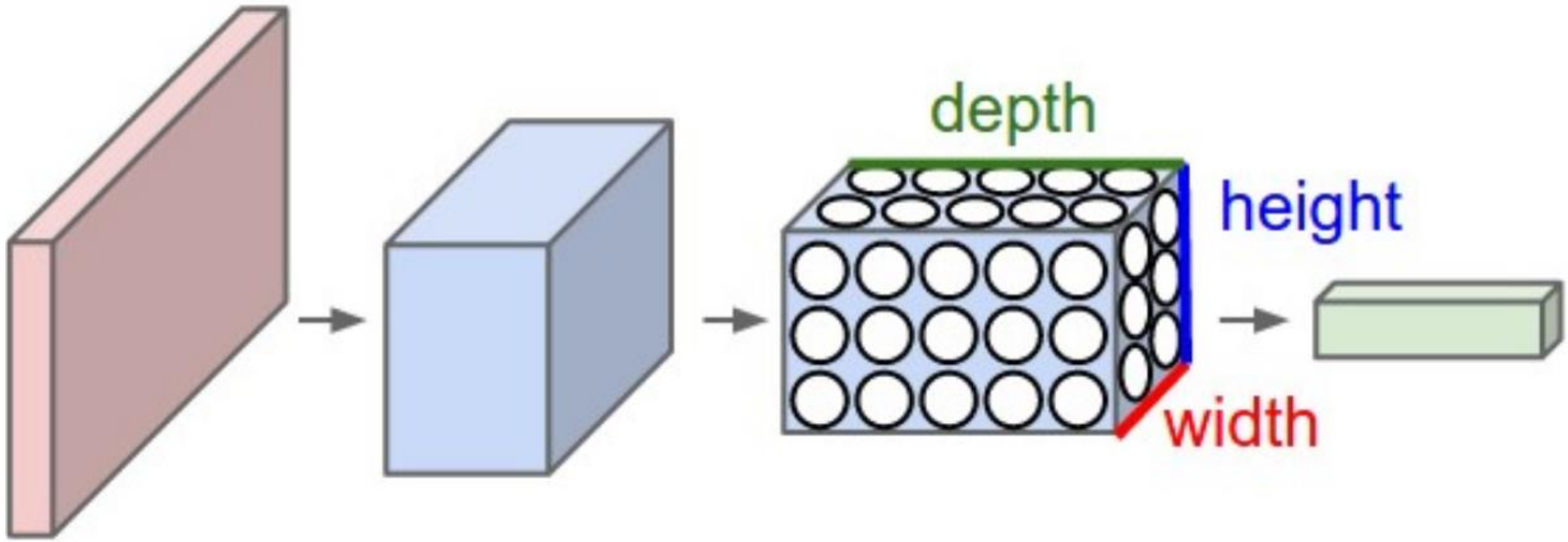




[1]Left figure: https://en.wikibooks.org/wiki/Artificial_Neural_Networks/Print_Version

3D Volumes of Neurons

- Now,



- Each layer simply does this: transforms an input tensor (3D volume) to an output tensor using some function

[1]Figure: http://cs231n.github.io/convolutional-networks/

3D Volumes of Neurons

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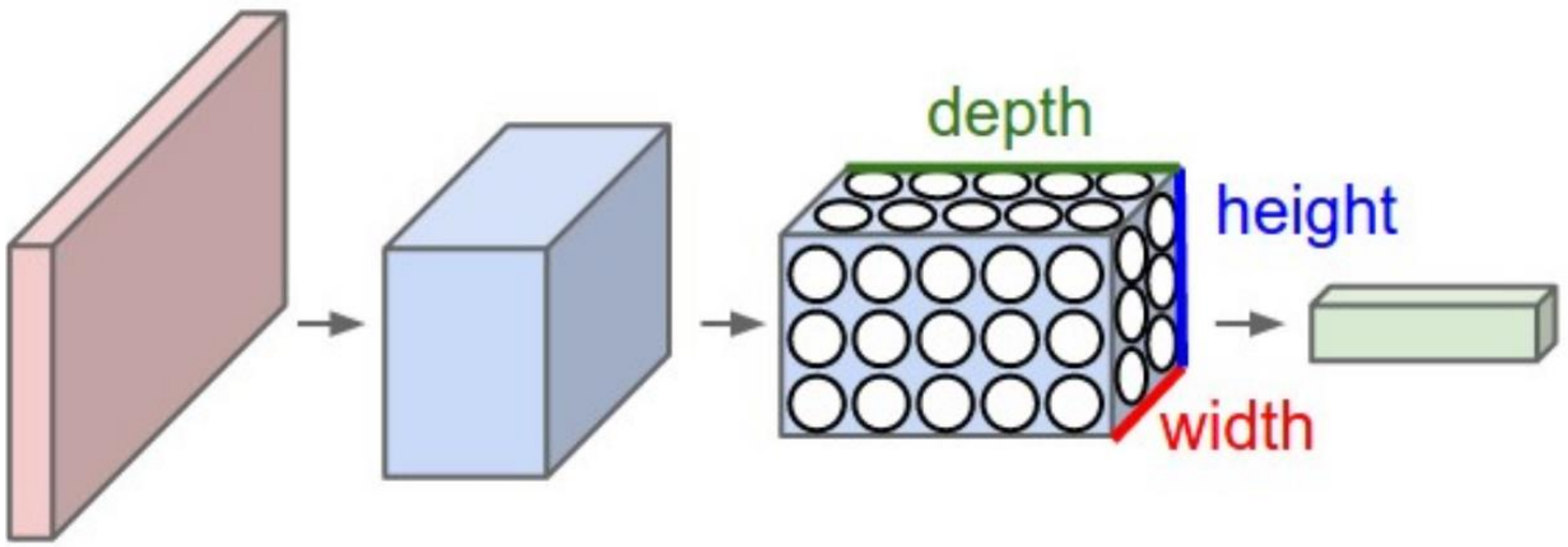

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CNN Layers

- Three types
  - Convolutional Layer (CONV)
  - Pooling Layer (POOL)
  - Fully Connected Layer (same as Feedforward neural network, i.e., 1*1*#Neurons is the layer's output tensor)

- Stack these in various ways

CNN Example Architecture

- Say our classification dataset is CIFAR-10
- Let the architecture be as follows:
  - INPUT -> CONV -> POOL -> FC
- INPUT:
  - This layer is nothing but 32*32*3 in dimension (width*height*3 color channels)

CNN Example Architecture

- Say our classification dataset is CIFAR-10
- Let the architecture be as follows:
  - INPUT -> CONV -> POOL -> FC
- INPUT:
  - This layer is nothing but 32*32*3 in dimension (width*height*3 color channels)
- CONV:
  - Neurons compute like regular feedforward neurons (sum the product of inputs with weights and add bias).
  - May output a different shaped tensor, say with dimension 32*32*12

CNN Example Architecture

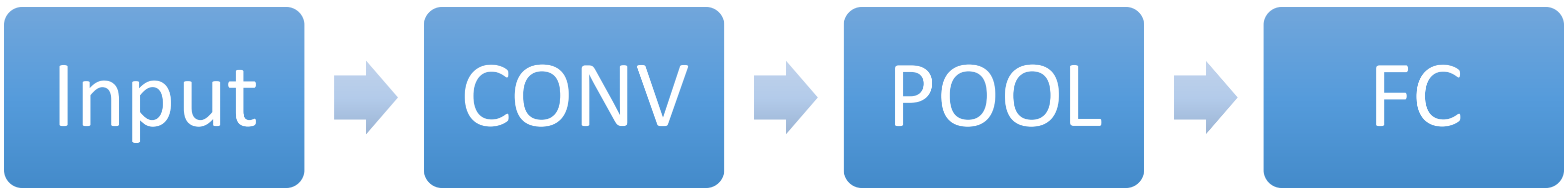
- POOL:
  - Performs a down-sampling in the spatial dimension
  - Outputs a tensor with the depth dimension the same as input
  - If input is 32*32*12, then output could be 16*16*12

CNN Example Architecture

- POOL:
  - Performs a down-sampling in the spatial dimension
  - Outputs a tensor with the depth dimension the same as input
  - If input is 32*32*12, then output could be 16*16*12
- FC:
  - This is the fully connected layer. Input can be any tensor (say 16*16*12) but the output will have only one effective dimension (1*1*10 since this is the last layer and CIFAR-10 has 10 classes)

CNN Example Architecture

- So we went from pixels (32*32 RGB images) to scores (10 in number)



- Some layers have parameters (CONV and FC), other layers do not (POOL)

- Optimization of these parameters still for achieving scores consistent with image labels

The Convolution Layer (CONV)

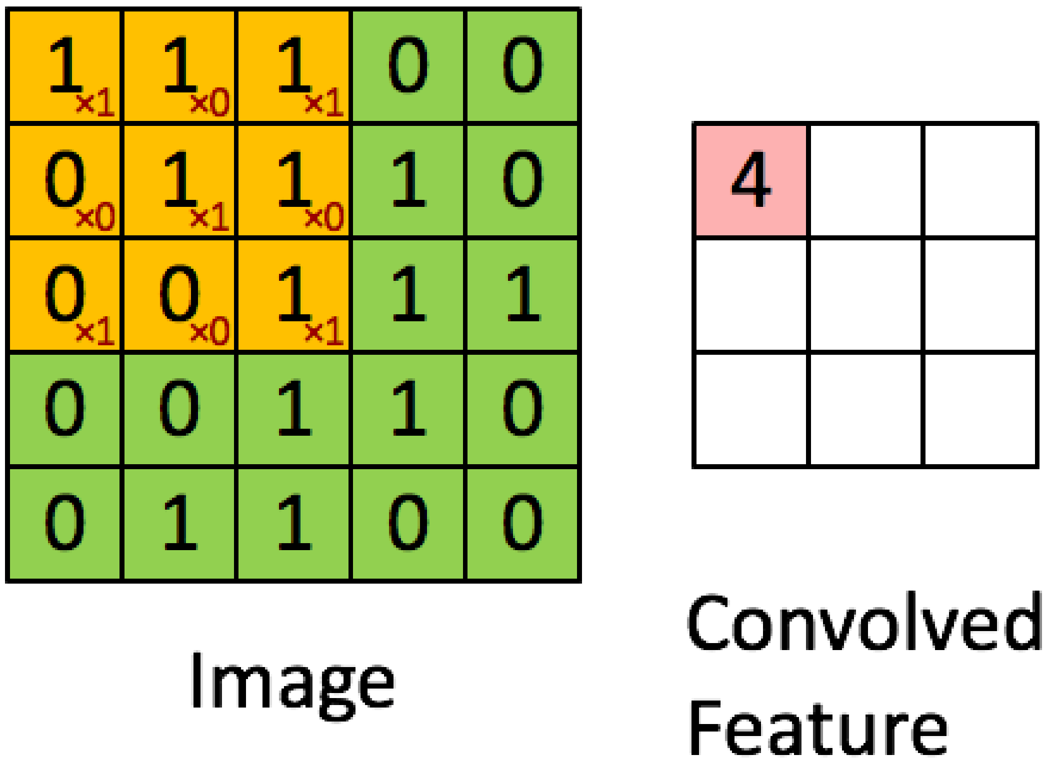
- Layer's parameters correspond to a set of filters
- What is a filter?
  - A linear function parameterized by a tensor
  - Outputs a scalar
  - The parameter tensor is learned during training

- Example
  - First layer filter may be of dimension 3*3*3
    - 3 pixels wide
    - 3 pixels high
    - 3 unit filter-depth for three color channels

- We slide (convolve) the filter across the width and height of the input volume and compute the scalar output to be passed into the nonlinearity

CONV: Sliding/Convolving

- We slide (convolve) the filter across the width and height of the input volume and compute the scalar output to be passed into the nonlinearity



Also see http://setosa.io/ev/image-kernels/

[1]Figure: http://deeplearning.stanford.edu/wiki/index.php/Feature_extraction_using_convolution

The Convolution Layer (CONV)

- Three things to notice
  - Filters are small along width and height
  - Same filter-depth as the input tensor (3D volume)
    - If the input is $x * y * z$, then filter could be $3 * 3 * z$
  - As we slide, we produce a 2D activation map

The Convolution Layer (CONV)

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  - Filters are small along width and height
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  - As we slide, we produce a 2D activation map

- Filters (i.e., filter parameters) will be learned during training that ‘detect’ certain visual features
  - Example:
    - Oriented edges, colors, etc. at the first layer
    - Specific patterns in higher layers

CONV: Filters

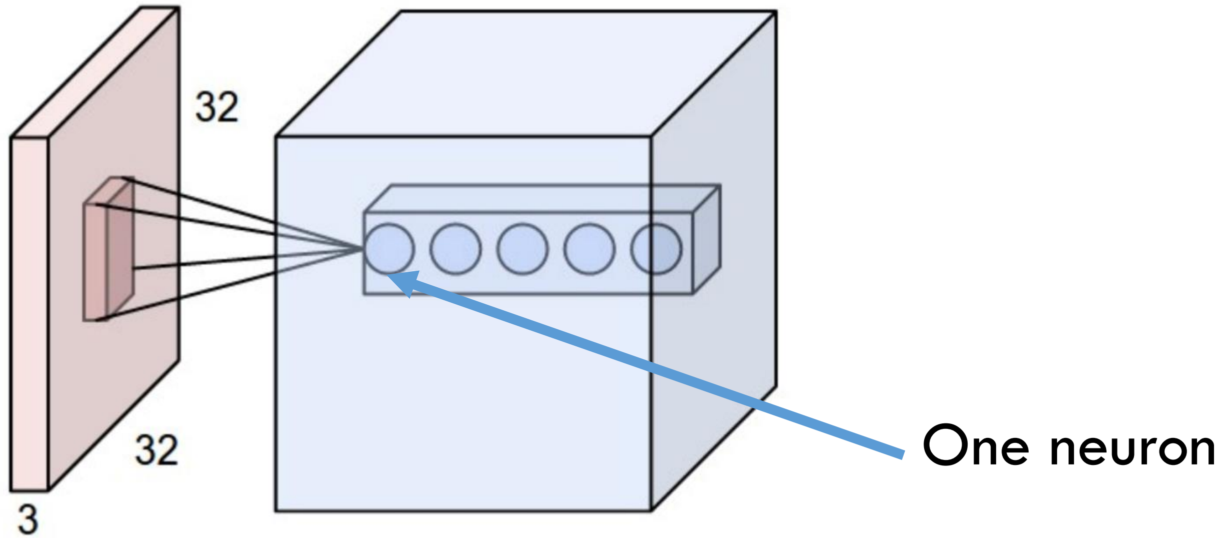
- Before we look at the patterns …

- Lets now look at the neurons themselves
  - How are they connected?
  - How are they arranged?
  - How can we get reduced parameters?

CONV: Local Connectivity

- Connect each neuron to a local (spatial) region of the input tensor
- Spatial extent of this connectivity is called receptive field
- Depth connectivity is the same as input depth

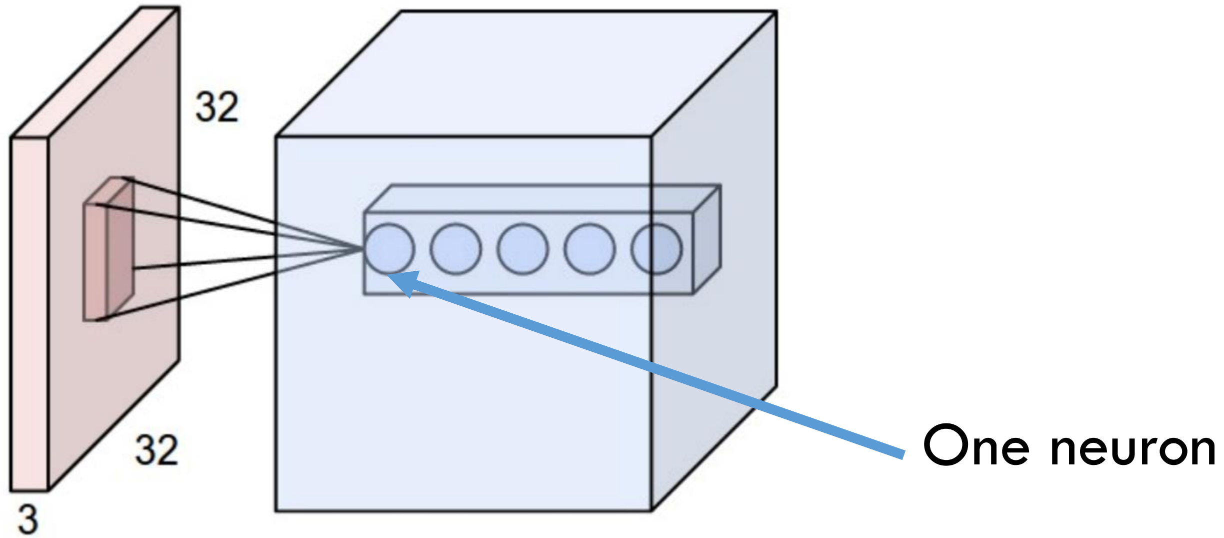
CONV: Local Connectivity



- Example: If input tensor is 32*32*3 and filter is 3*3*3 then
  - the number of weight parameters is 27, and
  - there is 1 bias parameter

[1]Figure: http://cs231n.github.io/convolutional-networks/

CONV: Local Connectivity



- All 5 neurons are looking at the same spatial region
- Each neuron belongs to a different filter

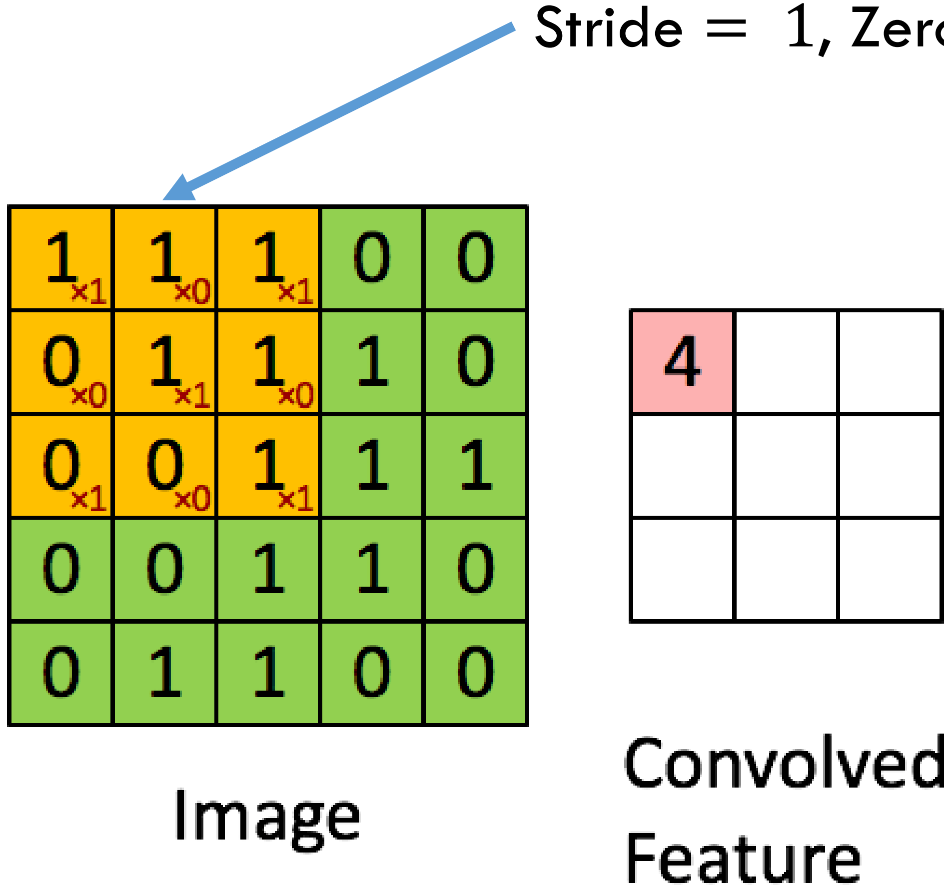
[1]Figure: http://cs231n.github.io/convolutional-networks/

CONV: Spatial Arrangement

- Back to layer point of view
- Size of output tensor depends on three numbers:
  - Layer Depth
    - Corresponds to the number of filters
  - Stride (how much the filter is moved spatial)
    - Example: If stride is 1, then filter is moved 1 pixel at a time
  - Zero-padding
    - Deals with boundaries (is usually 1 or 2)

CONV: Stride/Zero-pad

Stride $=$ 1, Zero-padding $=$ 0

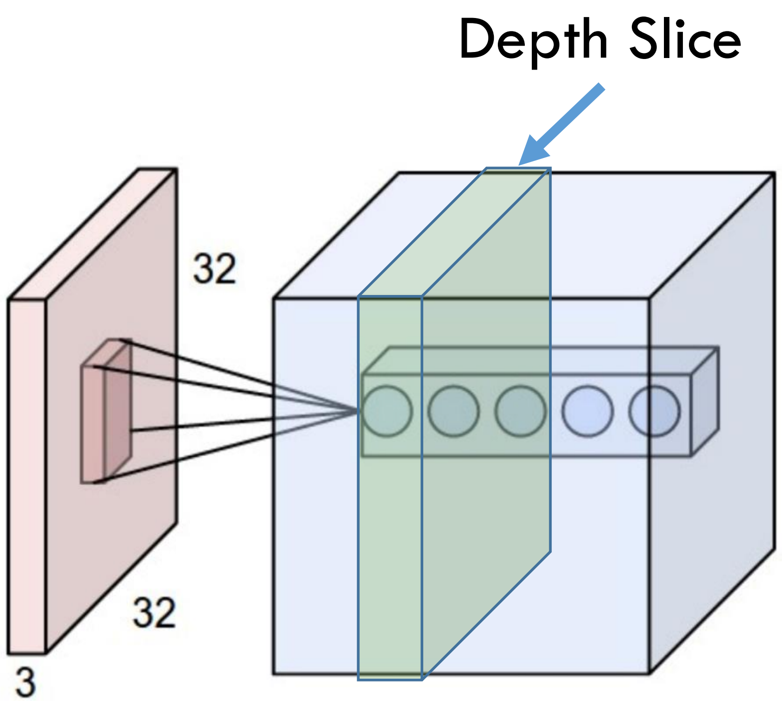


[1]Figure: http://deeplearning.stanford.edu/wiki/index.php/Feature_extraction_using_convolution

CONV: Parameter Sharing

- Key assumption: If a filter is useful for one region, it should also be useful for another region

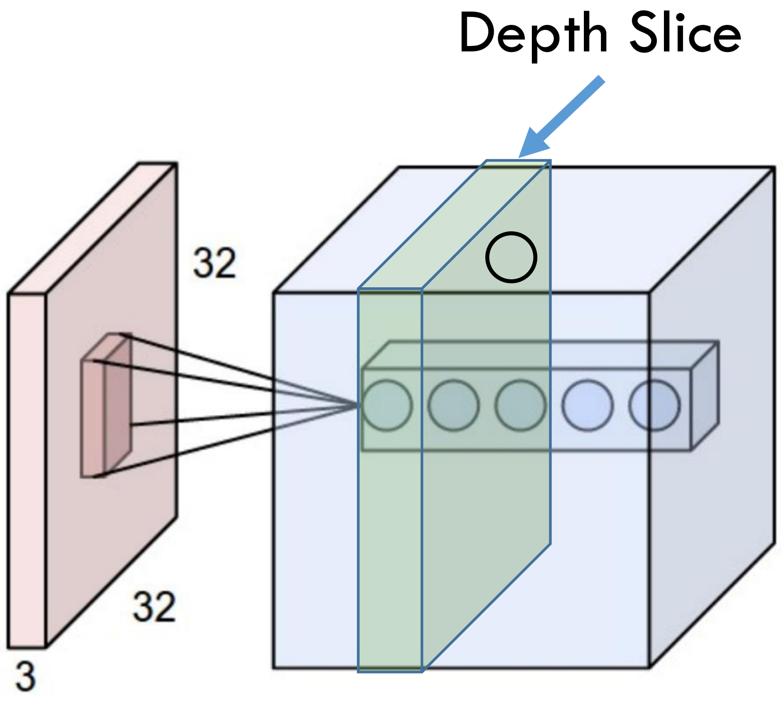
- Denote a single 2D slice of depth of a layer as depth slice



[1]Figure: http://cs231n.github.io/convolutional-networks/

CONV: Parameter Sharing

- Then, all neurons in each depth slide use the same weight and bias parameters!



[1]Figure: http://cs231n.github.io/convolutional-networks/

CONV: Parameter Sharing

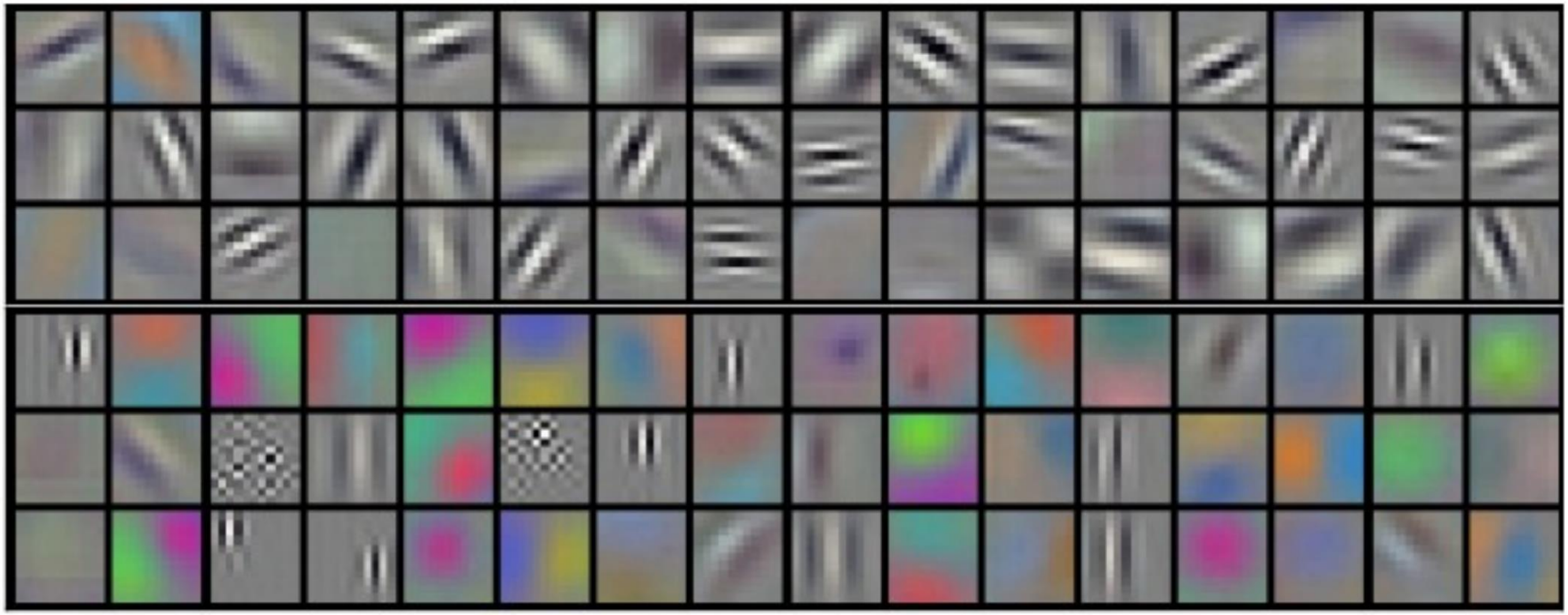
- Number of parameters is reduced!

- Example:
  - Say the number of filters is M (= Layer Depth)
  - Then, this layer will have $M * (3 * 3 * 3 \ + 1)$ parameters

- Gradients will get added up across neurons of a depth slice

CONV: Parameter Sharing

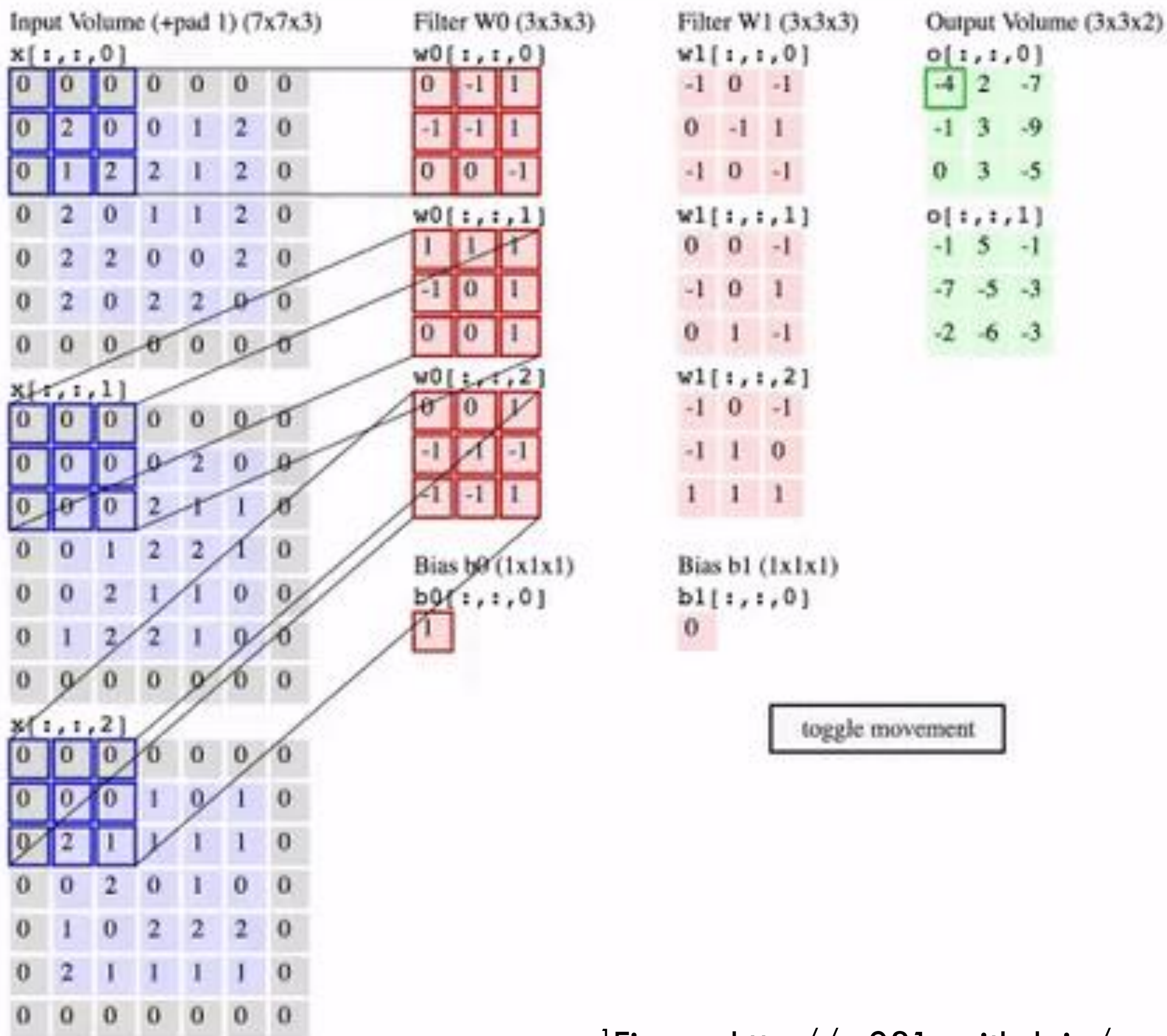
- AlexNet's first layer has 11*11*3 sized filters 96 in number. The filter weights are plotted below:



- Intuition: If capturing an edge is important, then important everywhere

[1]Figure: http://cs231n.github.io/convolutional-networks/

Example: CONV Layer Computation



The name 'convolution' comes from the convolution operation in signal processing that is essentially a matrix matrix product.

[1]Figure: http://cs231n.github.io/convolutional-networks/

Example: CONV Layer Computation

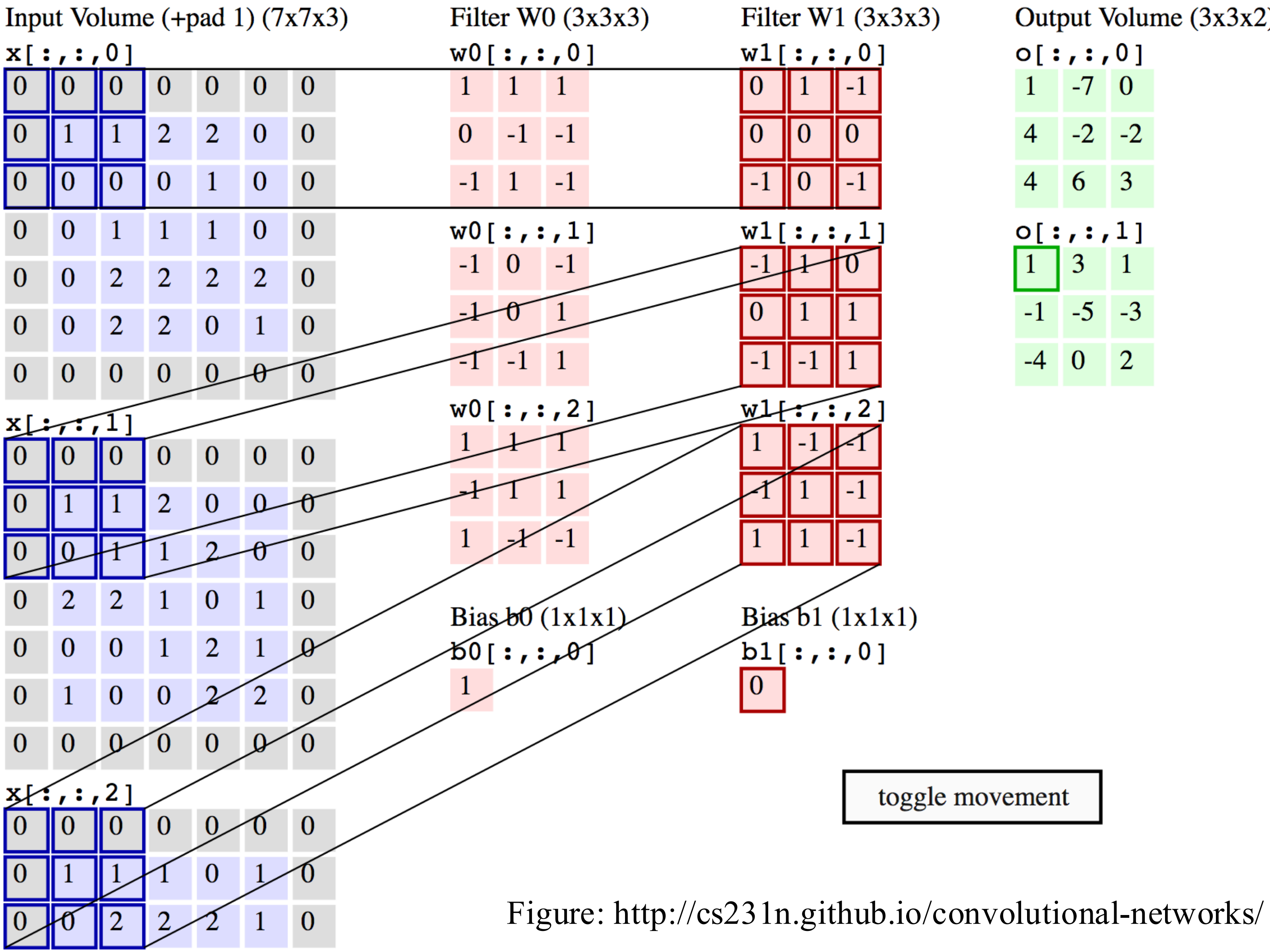


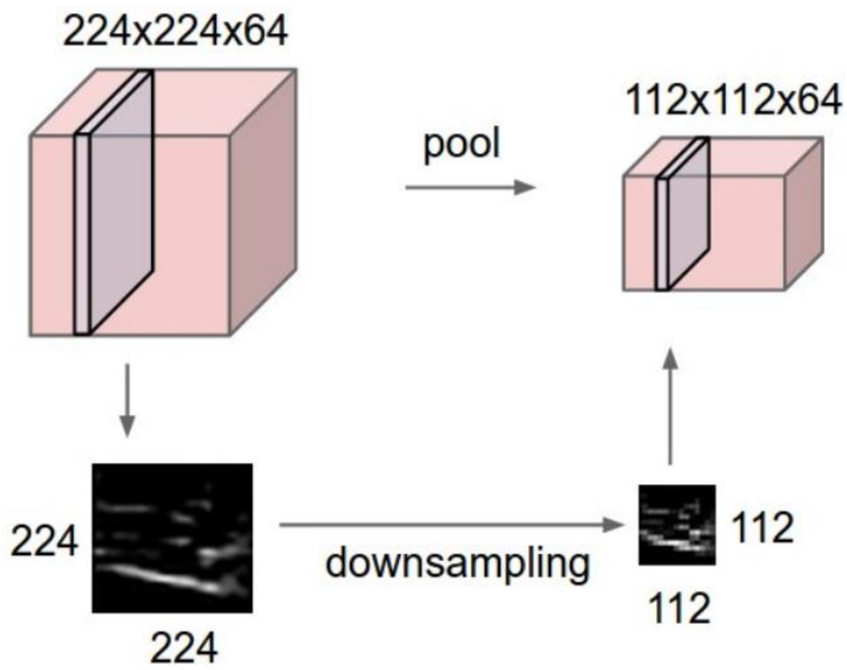
Figure: http://cs231n.github.io/convolutional-networks/

The Pooling Layer: POOL

- Vastly more simpler than CONV
- Reduce the spatial size by using a MAX or similar operation
- Operate independently for each depth slice

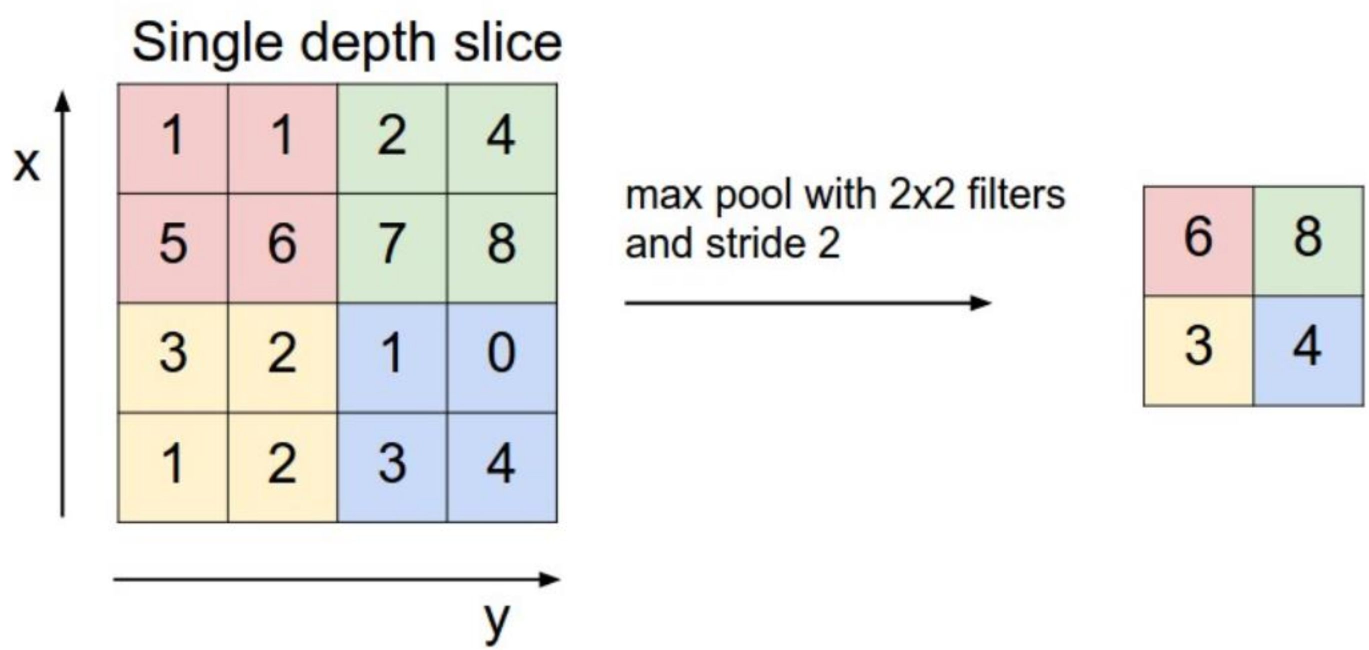
POOL: Example

- Input depth is retained



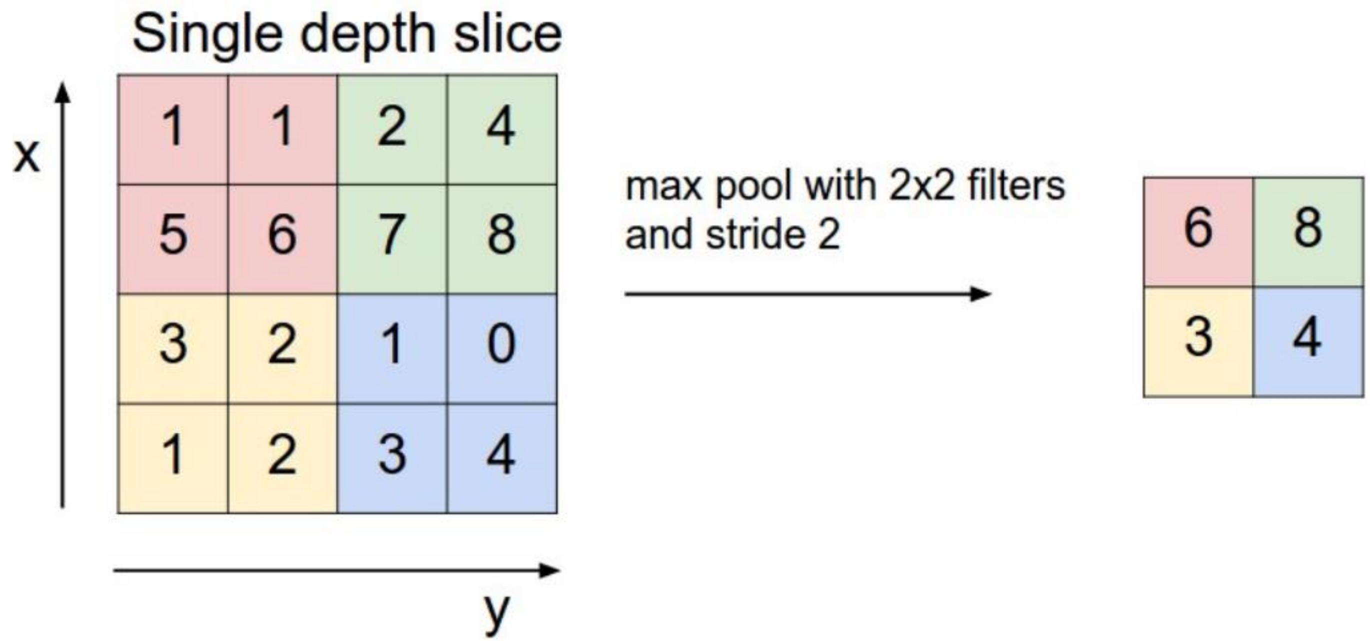
[1]Figure: http://cs231n.github.io/convolutional-networks/

POOL: Example



[1]Figure: http://cs231n.github.io/convolutional-networks/

POOL: Example



- Recent research is showing that you may not need a pooling layer

Fully Connected Layer: FC

- Essentially a fully connected layer
- Already seen while discussing feedforward neural networks

Summary

- Feedforward neural nets can do better than linear classifiers (saw this for a low-dimensional small synthetic example)
- CNN have been very effective in image related applications.
- Exploit specific properties of images
  - Hierarchy of features
  - Locality
  - Spatial invariance
- Lots of design choices that have been empirically validated and are intuitive. Still, there is room for improvement.

Appendix

Naming: Why 'Convolution'

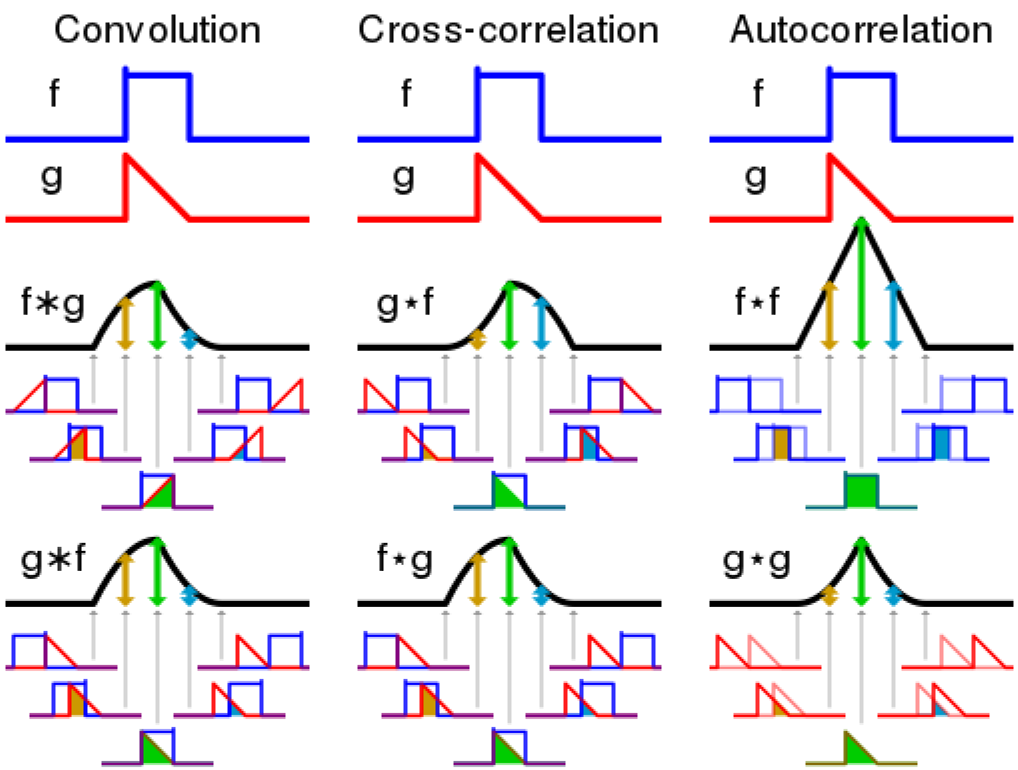


Figure:https://en.wikipedia.org/wiki/Convolution#/media/File:Comparison_convolution_correlation.svg