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## Phase 1 Proof of Concept

### 2D Least Squares Position Estimation - Theory

The 2D coordinates of the user's position was determined using least squares estimation. The parametric functional model developed for the trilateration of range observations was determined as  $f(x) - l = 0$ :

$$\sqrt{(x_{user} - x_{node})^2 + (y_{user} - y_{node})^2} - d_{node,user} = 0$$

This model was then linearized with respect to the unknown parameters  $x_0 = [x_{user}, y_{user}]^T$ . Linearizing the functional model was done by deriving the form:

$$A\delta + v + w = 0$$

Where:

$$A = \text{Design Matrix with respect to the unknowns} = \frac{\partial f}{\partial x^0}$$

$\delta$  = Correction to unknowns

$v$  = residual of the observations

$w$  = misclosure evaluated at  $x^0$  and  $l^{obs}$

Therefore, the design matrix  $A$  becomes:

$$A_{nx2} = \begin{bmatrix} \frac{\partial f}{\partial x_{user}^0} & \frac{\partial f}{\partial y_{user}^0} \\ \vdots & \vdots \end{bmatrix} = \begin{bmatrix} \frac{x_{user}^0 - x_{node}}{d_{node,user}^0} & \frac{y_{user}^0 - y_{node}}{d_{node,user}^0} \\ \vdots & \vdots \end{bmatrix}$$

Where  $n$  is the number of range observations and  $d_{node,user}^0 = \sqrt{(x_{user}^0 - x_{node})^2 + (y_{user}^0 - y_{node})^2}$ .

The initial estimate of the unknown coordinates of the user ( $x_0$ ) were determined as the mean of all node coordinates. The misclosure vector can then be determined as:

$$w_{nx1} = \begin{bmatrix} \sqrt{(x_{user}^0 - x_{node})^2 + (y_{user}^0 - y_{node})^2} - d_{node,user}^{obs} \\ \vdots \end{bmatrix}$$

The weight matrix for used to complete the least squares estimation was determined to be the identity matrix, as all measurements are assumed to be weighted the same. The correction to the unknown parameters  $\delta$  is then determined as:

$$\delta = -(A^T P A)^{-1} A^T P w$$

The unknown parameters can then be calculated as:

$$\hat{x} = x_0 + \delta$$

Iterations are then performed, updating  $x_0 = \hat{x}$ , until the correction to the unknown parameters is smaller than a millimetre:  $\delta_{max} < 0.0001\text{m}$ . Please see the estimation code developed for the full implementation and results.

## 2D Least Squares Position Estimation – Initial Results

The following figure shows the results obtain from initial data collection.

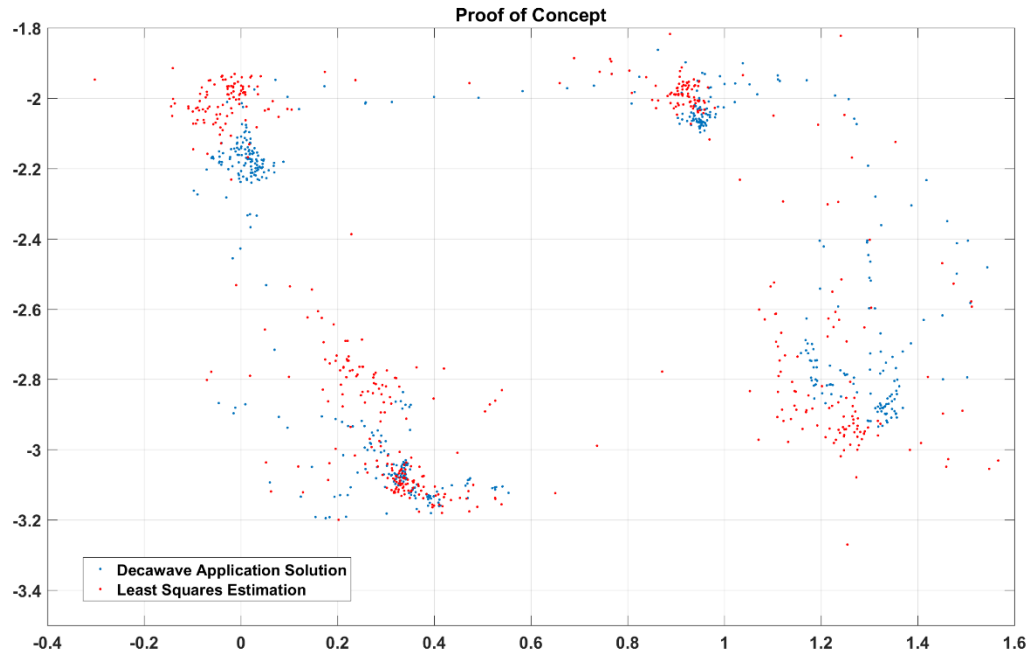


Figure 1: Least Squares Results

As seen in the figure above, the positions obtained from the least squares estimation software are agree with that of current Decawave position application. This proves that using the technique of least squares estimation is a valid method to determine the users position. Moving forward, both solutions will be compared with true coordinates to determine the accuracy of the solution, in order to determine that the position obtained from the least squares estimation is better then the current estimation software. The software developed will then be updated to support 3D position estimation, as well as data streaming for real time position determination.