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**Theorem.** (*Existence - Uniqueness Thrm,  $\exists!$  Thrm*). If  $I$  is an interval,  $a \in I$ , and  $p_u$  for all  $1 \leq k \leq n-1$ , and  $f$  are cts on  $I$ , then  $\forall b_i$ , for  $0 \leq i \leq n-1$ , (then initial value problem)

$$(1) \ y^{(n)} + \sum_{k=1}^n y^{(n-k)} p_k(x) = f(x)$$

&  $y^{(i)}(a) = b_i$ , has a unique soln on  $I$ .

Note: the solns to linear ODEs are unique on the whole interval  $I$ .

**Theorem.** *Thrm.* If  $w = \{y \in \mathcal{D}^{(n)}(I) \mid y \text{ satisfies (1)}\}$  then  $\dim W \geq n$ .

*Proof.* put  $y_j^{(i)}(a) = \delta_{ij}$ , where

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j; \\ 0, & \text{if } i \neq j \end{cases}$$

for  $1 \leq i, j \leq n$ . By the  $\exists!$  Thrm, for each  $j$  there is a unique soln to (1) on  $I$ . Now,

$$\begin{aligned} W(a) &= (y_j^{(i)}(a)) \in \text{Mat}_n(\mathbb{R}) \\ &= I_n \end{aligned}$$

$$\therefore |W(a)| = 1 \neq 0$$

whence,  $y_1, \dots, y_n$  are linearly independent. Hence,

$$\dim W \leq n$$

□

**Theorem.** *Thrm (Strong wronskian converse)* If  $y_1, \dots, y_n$  are linearly independent (in  $\mathcal{D}^{(n)}(I)$ ) and  $y_1, \dots, y_n$  are solns to

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$$y^{(n)} + \sum_{k=1}^n y^{(n-k)} p_k(x) = 0$$

where  $p_k$  are cts on  $I$  for  $k = 1, \dots, n$ , then  $|W(a)| \neq 0$  for all  $a \in I$ .

*Proof.* Assume that there is an  $a \in I$  st  $|W(a)| = 0$ , then the linear system

$$(1) \quad W(a)\vec{x} = \vec{0}$$

has a nontrivial soln, say  $\vec{x} = \vec{c} \in \mathbb{R}^n$   
 $(W(a) \in \text{Mat}_n(\mathbb{R}))$ . Denote:  $\vec{c} = (c_1, \dots, c_n) \neq \vec{0}$ .

Put  $y = \sum_{j=1}^n c_j y_j$ , then  $y$  satisfies (\*) and

$$y(a) = \sum_{j=1}^n c_j y_j(a) = 0 \implies$$

$$y'(a) = \sum_{j=1}^n c_j y_j'(a) = 0 \implies$$

$$y''(a) = \sum_{j=1}^n c_j y_j''(a) = 0 \implies$$

$$\vdots$$

$$y^{(n-1)}(a) = \sum_{j=1}^n c_j y_j^{(n-1)}(a) = 0 \quad (\text{by (1)})$$

Notice that  $y \equiv 0$  on  $I$  also satisfies (\*) and is such that  $y^{(k)}(a) = 0$  for  $1 \leq k \leq n-1$ .  $\therefore$  by  $\exists!$  thrm,

$$\sum_{j=1}^n c_j y_j \equiv 0 \text{ on } I$$

Thus, since  $y_1, \dots, y_n$  are lin ind, all  $c_j = 0$ , which is a contradiction.  $\square$

**Theorem.** *If*

$$W = \{y \in \mathbb{R}^{(n)}(I) \mid y \text{ satisfies } (*) \}$$

*then*  $\dim W \leq n$

*Proof.* let  $y_1, \dots, y_n$  be lin ind solns of (\*); we show that there are scalars  $c_j \in \mathbb{R}$  st  $y = \sum_{j=1}^n c_j y_j$  on  $I$ .

Consider the linear system

$$(2) \quad W(a)\vec{x} = \begin{pmatrix} y(a) \\ y'(a) \\ \vdots \\ y^{(n-1)}(a) \end{pmatrix}$$

where  $W(a)$  is the Wronskian matrix of  $y_1, \dots, y_n$ . By the SWC,  $|W(a)| \neq 0$ . Thus, (2) has a unique, not-trivial soln, say  $\vec{0} \pm \vec{x} = \vec{c} \neq \vec{0} \in \mathbb{R}^n$ . Thus

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$$W(a)\vec{c} = \begin{pmatrix} y(a) \\ y'(a) \\ \vdots \\ y^{(n-1)}(a) \end{pmatrix}$$

has rows

$$\begin{aligned} \sum_{j=1}^n c_j y_j(a) &= y(a) \\ \sum_{j=1}^n c_j y'_j(a) &= y'(a) \\ \sum_{j=1}^n c_j y''_j(a) &= y''(a) \\ &\vdots \\ \sum_{j=1}^n c_j y_j^{(n-1)}(a) &= y^{(n-1)}(a) \end{aligned}$$

Finally, since both  $y$  and  $\sum_{j=1}^n c_j y_j$  are solns to the hom nth order linear ODE (\*) (the ODE being hom and  $y_j$ , for  $1 \leq j \leq n$ , being solns, so is any linear combo of  $y_j$ s since the soln space to a linear hom ODE is linear) and both  $y$  and  $\sum_{j=1}^n c_j y_j$  satisfy all the same  $n$  many initial ?condos?, by the  $\exists!$  thrm,  $y = \sum_{j=1}^n c_j y_j$  on  $I$ .  $\therefore y \in \text{Span} \{y_j \in W \mid 1 \leq j \leq n\}$ . Thus,  $\dim W \leq n$ .  $\square$

**Corollary.** *If*

$$W = \{y \in D^{(n)}(I) \mid y \text{ satisfies } (x) \}$$

where

$$(*) \quad y^{(n)} + \sum_{k=1}^n y^{(n-k)} p_k(x) = 0$$

then  $\dim W = n$ .

*Proof.* We showed that  $n \leq \dim W \leq n$ .  $\square$

**Example.**  $y'' - y = 0$

Note that both  $y_1 = \cosh x$  and  $y_2 = \sinh x$  are solns on  $\mathbb{R}$ . The soln space is 2 - dim'l. Not also that if

$$a \cosh x + b \sinh x = 0 \text{ for all } x \in \mathbb{R}$$

then in particular,  $x = 0 \implies a = 0$ .

$\therefore b \sinh x = 0$  for all  $x$ .

However, if  $x \neq 0$  then  $\sinh x \neq 0$ ; whence,  $b = 0$ . Hence,  $B = \{\cosh x, \sinh x\}$  are linearly independent solns to  $y'' = y$ , and the soln space has dim'n 2,  $B$  is a basis for the soln space.  $\therefore$  every soln to  $y'' = y$  has the form

$$y = a \cosh x + b \sinh x$$

for some  $a, b \in \mathbb{R}$ . Likewise,  $B' = \{e^x, e^{-x}\}$  is also a basis. Note:

$$|W(x)| = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = e^x \begin{vmatrix} 1 & e^{-x} \\ 1 & -e^{-x} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2 \neq 0$$

$\therefore$  by Wronsky's Thrm,  $B'$  is lin ind.

**Example.** Example:  $y'' + y = 0$

Here  $B = \{\cos x, \sin x\}$  is a basis.

nth order hom linear ODEs with constant coefficients:

$$(1) \sum_{k=0}^n a_k y^{(k)} = 0$$

consider

$$y = e^{rx}$$

then

$$\begin{aligned} y' &= r e^{rx} = r y \implies \\ y'' &= r^2 e^{rx} = r^2 y \implies \\ &\vdots \\ y^{(k)} &= r^k y \end{aligned}$$

thus,  $y = e^{rx}$  yields

$$\begin{aligned} 0 &= \sum_{k=0}^n a_k y^{(k)} \\ &= \sum_{k=0}^n a_k r^k y \\ &= y \sum_{k=0}^n a_k r^k \implies \end{aligned}$$

$$\sum_{k=0}^n a_k r^{(k)} = 0$$

$\therefore y = e^{rx}$  is a soln to (1) if  $r$  is a root (zero) of the char poly of  $\sum_{k=0}^n a_k y^{(k)} = 0$ ,

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$$\rho(x) = \sum_{k=0}^n a_k x^k,$$

then  $y = e^{rx}$  is a soln to  $\sum_{k=0}^n a_k y^{(k)} = 0$ .

**Example.** If  $r_1, \dots, r_n \in \mathbb{R}$  are pairwise distinct then  $e^{r_1 x}, \dots, e^{r_n x}$  are linearly independent (in  $\mathcal{F}(\mathbb{R})$ ). we use the Wronskian:

$$|W(x)| = \begin{vmatrix} e^{r_1 x} & e^{r_2 x} & \dots & e^{r_n x} \\ r_1 e^{r_1 x} & r_2 e^{r_2 x} & \dots & r_n e^{r_n x} \\ \vdots & \vdots & \ddots & \vdots \\ r_1^{n-1} e^{r_1 x} & r_2^{n-1} e^{r_2 x} & \dots & r_n^{n-1} e^{r_n x} \end{vmatrix}$$

$$e^{(r_1 + r_2 + \dots + r_n)x} = \begin{vmatrix} 1 & 1 & \dots & 1 \\ r_1 & r_2 & \dots & r_n \\ \vdots & \vdots & \ddots & \vdots \\ r_1^{n-1} & r_2^{n-1} & \dots & r_n^{n-1} \end{vmatrix}$$

$$e^{(r_1 + r_2 + \dots + r_n)x} \prod_{1 \leq i < j \leq n} (r_j - r_i) \neq 0$$

(for all  $x$ ). therefore by the wronskian thrm,  $e^{r_1 x}, e^{r_2 x}, \dots, e^{r_n x}$  are lin ind,

**Example.** (Vandermonds)  
consider

$$V_n(x) = \begin{pmatrix} 1 & x & x^2 & \dots & x^{n-1} \\ 1 & r_2 & r_2^2 & \dots & r_2^{n-1} \\ 1 & r_3 & r_3^2 & \dots & r_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_n & r_n^2 & \dots & r_n^{n-1} \end{pmatrix}$$

which is an  $n \times n$  Vandermonde Matrix.

Put

$$f(x) = \det V_n(x) \in P_{n-1}[\mathbb{R}],$$

then  $f(r_j) = \det(V_n(r_j)) = 0$  for all  $2 \leq j \leq n$ , by the alternating property of determinates. therefore by the factor Thrm,

$$f(x) = a \prod_{2 \leq j \leq n} (x - r_j),$$

where

$$a = (-1)^{n-1} \begin{vmatrix} 1 & x & x^2 & \cdots & x^{n-1} \\ 1 & r_2 & r_2^2 & \cdots & r_2^{n-1} \\ 1 & r_3 & r_3^2 & \cdots & r_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & r_n & r_n^2 & \cdots & r_n^{n-1} \end{vmatrix},$$

which is an  $(n-1) \times (n-1)$  det. therefore by the inductive hypothesis,

$$a = (-1)^{n-1} \prod_{2 \leq i < j \leq n} (r_j - r_i)$$

therefore

$$f(x) = (-1)^{n-1} \prod_{2 \leq i < j \leq n} (r_j - r_i) \prod_{2 \leq j \leq n} (x - r_j)$$

Notice that

$$\prod_{2 \leq j \leq n} (x - r_j) = (-1)^{n-1} \prod_{2 \leq j \leq n} (r_j - x).$$

Therefore

$$\begin{aligned} f(x) &= (-1)^{n-1} \prod_{2 \leq i < j \leq n} (r_j - r_i) (-1)^{n-1} \prod_{2 \leq j \leq n} (r_j - x) \\ &= \prod_{2 \leq j \leq n} (r_j - x) \prod_{2 \leq i < j \leq n} (r_j - r_i). \end{aligned}$$

In particular,

$$\begin{aligned} \det V_n(r_1) &= f(r_1) \\ &= \prod_{2 \leq j \leq n} (r_j - r_1) \prod_{2 \leq j < i \leq n} (r_j - r_i) \\ &= \prod_{1 \leq i < j \leq n} (r_j - r_i). \end{aligned}$$

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Recall: the characteristic poly of  $\sum_{k=0}^n a_k y^{(k)}$  is

$$\rho(x) = \sum_{k=0}^n a_k x^{(k)},$$

and we showed that if  $r_1, \dots, r_n$  are pairwise distinct then  $B = \{e^{r_1 x}, \dots, e^{r_n x}\}$  is linearly independent (in  $\mathcal{C}^{(\infty)}(\mathbb{R})$ ). We also showed that if  $r$  is a zero of

$\rho(x)$  then  $y = e^{rx}$  is a soln to  $\sum_{k=0}^n a_k y^{(k)} = 0$ . This all immediatly implies the following:

**Theorem.** *If  $\rho(x) \sum_{k=0}^n a_k x^{(k)}$  has  $n$  many distinct real zeros then the soln space of*

$$(*) \sum_{k=0}^n a_k y^{(k)} = 0$$

*has basis  $\{e^{r_1 x}, \dots, e^{r_n x}\}$ , i.e., every soln of  $*$  has the form*

$$\sum_{j=1}^n c_j e^{r_j x}.$$

**Example.** (Revisited)

Consider  $y'' - y = 0$ . This has char poly

$$\rho(x) = x^2 - 1 = (x - 1)(x + 1),$$

which has zeros  $\pm 1$ .

Thus,  $B = \{e^x, e^{-x}\}$  is a basis for the soln space of

$$y'' - y = 0$$

Q: What are the mising basis elements if a char poly has repeating zeros?

Recall the "ring" of polynomials

$$\mathbb{R}[x] = \left\{ \sum_{k=0}^n a_k x^k \mid n \in \mathbb{N} \ \& \ a_k \in \mathbb{R} \right\},$$

where

$$\sum_{k=0}^n a_k x^{(k)} + \sum_{k=0}^n b_k x^{(k)} = \sum_{k=0}^n (a_k + b_k) x^{(k)}$$

$$\sum_{i=0}^m a_i x^{(i)} \sum_{j=0}^m b_j x^{(j)} = \sum_{k=0}^{m+n} c_k x^{(k)}, \text{ where}$$

$$c_k = \sum_{i+j=k} a_i b_j.$$

Notation. Put  $D = \frac{d}{dx} : \mathcal{C}^{(\infty)}(I) \rightarrow \mathcal{C}^{(\infty)}(I)$   
 $D^0 = i$  (identity on  $\mathcal{C}^{(\infty)}(I)$ ), and

$$D^n = D D^{n-1} \text{ for } n \in \mathbb{Z}^+.$$

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where  $Dy = D(y) = \frac{dy}{dx}$

Consider  $L = \sum_{k=0}^n a_k D^k$ , where  $a_k \in \mathbb{R}$  and  $D^k = \frac{d^k}{dx^k}$ . Note that  $L : \mathcal{C}^{(\infty)}(I) \rightarrow \mathcal{C}^{(\infty)}(I)$ , where

$$L_y = L(y) = \sum_{k=0}^n a_k D^k y =$$

$$a_0 y + a_1 y' + a_2 y'' + \cdots + a_n y^{(n)}.$$

we show that  $L$  is a linear operation:

$$\begin{aligned} L(y_1 + y_2) &= \sum_{k=0}^n a_k D^k (y_1 + y_2) \\ &= \sum_{k=0}^n a_k (D^k y_1 + D^k y_2) \\ &= \sum_{k=0}^n (a_k D^k y_1 + a_k D^k y_2) \\ &= L_{y_1} + L_{y_2} \end{aligned}$$

Also,

$$\begin{aligned} L(cy) &= \sum_{k=0}^n a_k D^k (cy) \\ &= \sum_{k=0}^n a_k (c D^k y) \\ &= c \sum_{k=0}^n a_k D^k y \\ &= c L_y. \end{aligned}$$

Put

$$\mathbb{R}[D] = \left\{ \sum_{k=0}^n a_k D^k \mid a_k \in \mathbb{R} \text{ \& } n \in \mathbb{N} \right\}.$$

We show that  $\mathbb{R}[x] = R[D]$ , isomorphic as "rings," which includes multiplication. Addition in  $\mathbb{R}[D]$  is defined as

$$L_1, L_2 \in \mathbb{R}[D], (L_1 + L_2)y = L_1 y + L_2 y;$$

Multiplication in  $\mathbb{R}[D]$  is defined as



$$L_1, L_2 \in \mathbb{R}[D] , (L_1 L_2)y = (L_1 \cdot L_2)$$

Define  $h : \mathbb{R}[x] \rightarrow \mathbb{R}[D]$  Such that

$$h \left( \sum_{k=0}^n a_k x^k \right) = \sum_{k=0}^n a_k D^k$$

For example,  $1 \mapsto D^0 = \text{Identify function in } \mathcal{C}^{(\infty)}(I),$

$$x \mapsto D, x^2 \mapsto D^2, \dots, x^n \mapsto D^n.$$

We show that  $h$  is a "ring" homomorphism, i.e., if  $f_1 = \sum_{k=0}^m a_k x^k$ ,  $f_2 = \sum_{k=0}^n b_k x^k \in \mathbb{R}[x]$ , and  $L_1 = \sum_{k=0}^m a_k D^k$ ,  $L_2 = \sum_{k=0}^n b_k D^k \in \mathbb{R}[D]$ , then

$$(i) \ h(f_1 + f_2) = L_1 + L_2;$$

$$(ii) \ h(f_1 f_2) = L_1 L_2;$$

To see (i), notice that

$$\begin{aligned} f_1 + f_2 &= \sum_{k=0}^n a_k x^k + \sum_{k=0}^n b_k x^k \\ &= \sum_{k=0}^n (a_k + b_k) x^k \end{aligned}$$

So,

$$\begin{aligned} h(f_1 + f_2) &= \sum_{k=0}^n (a_k + b_k) D^k \\ &= \sum_{k=0}^n (a_k D^k + b_k D^k) \\ &= \sum_{k=0}^n a_k D^k + \sum_{k=0}^n b_k D^k \\ &= L_1 + L_2 \end{aligned}$$

Also, notice that

$$f_1 f_2 = \sum_{k=0}^n c_k x^k, \text{ where } c_k = \sum_{i+j=k} a_i b_j$$

So,

$$h(f_1 f_2) = \sum_{k=0}^n c_k D^k.$$

Now,

$$\begin{aligned}
c_k D^k &= \left( \sum_{i+j=k} a_i b_j \right) D^k \\
&= \sum_{i+j=k} (a_i b_j D^k) \\
&= \sum_{i+j=k} a_i D^i b_j D^j
\end{aligned}$$

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$$\begin{aligned}
L_1 L_2 y &= (L_1 \circ L_2)(y) \\
&= L_1(L_2(y)) \\
&= \sum_{i=0}^m a_i D^i L_2(y) \\
&= \sum_{i=0}^m a_i D^i \left( \sum_{j=0}^n b_j D^j y \right) \\
&= \sum_{i=0}^m \sum_{j=0}^n a_i b_j D^{i+j} y.
\end{aligned}$$

therefore

$$L_1 L_2 = \sum_{i=0}^m \sum_{j=0}^n a_i b_j D^{i+j}$$

Now, notice that from above

$$L_1 L_2 = \sum_{k=0}^{m+n} c_k D^k, \quad c_k = \sum_{i+j=k} a_i b_j.$$

therefore

$$h(f_1 f_2) = \sum_{k=0}^{m+n} c_k D^k = L_1 L_2 = h(f_1) h(f_2),$$

$c_k = \sum_{i+j=k} a_i b_j$ . Hence,  $h$  is a "ring" homomorphism. Clearly,  $h$  is onto, We show that  $h$  is 1-1.

Firstly, recall that

$$\ker(h) = \{f \in \mathbb{R}[x] \mid h(f) = 0 \in \mathbb{R}[D]\}$$

Also,  $h$  is 1-1  $\iff \ker(h) = \{0\}$ .

Secondly, if  $n = 0$  then

$$\begin{aligned} a_0 D^0 &= 0 \in \mathbb{R}[D] \implies \\ a_0 D^0 y &= 0 \in \mathcal{C}^\infty(I) \text{ for all } y \in \mathcal{C}^\infty(I) \\ \implies a_0 y &= 0 \text{ for all } y \in \mathcal{C}^\infty(I) \end{aligned}$$

However, if  $y = 1$  on  $I$  then

$$a_0 = 0$$

$$\therefore a_0 x^0 \in \mathbb{R}[x].$$

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Now, assume that

$$\sum_{k=0}^n a_k x^k \mapsto 0 \in \mathbb{R}[D] \implies$$

$a_k = 0$  for  $0 \leq k \leq n$  ( inductive hypothesis). If

$$h \left( \sum_{k=0}^n a_k x^k \right) = 0 \in \mathbb{R}[D],$$

then

$$\begin{aligned} \sum_{k=0}^n a_k D^k &= 0 \in \mathbb{R}[D] \implies \\ (1) \sum_{k=0}^n a_k y^k &= 0 \in \mathcal{C}^{(\infty)} \forall y \in \mathcal{C}^{(\infty)}(I) \end{aligned}$$

Thus, if  $a_{n+1} \neq 0$  then

Take  $y \equiv 1 \in \mathcal{C}^{(\infty)}(I)$ , then (1) becomes  $a_0 y \equiv 0 \in \mathcal{C}^{(\infty)}(I)$ ; thus,  $a_0 = 0$ .

Thus,

$$(2) \sum_{k=1}^{n+1} a_k y^{(k)} = 0 \in \mathcal{C}^{(\infty)}(I)$$

take  $y \equiv x \in \mathcal{C}^{(\infty)}(I)$ , then (2) becomes  $a_1 y' \equiv 0 \in \mathcal{C}^{(\infty)}(I)$ ; thus,  $0 = a_1$ ,  $y'(x) = a_1$  for all  $x \in I$ . therefore  $a_1 = 0$ . Thus,

$$(3) \sum_{k=2}^{n+1} a_k y^{(k)} = 0 \in \mathcal{C}^{(\infty)}(I)$$

Take  $y = x^2$ , etc, then  $(n+1) a_{n+1} y^{n+1} = 0 \in \mathcal{C}^{(\infty)}(I)$

Take  $y \equiv x^{n+1}$ , then as above,  $a_{n+1} = 0$ , a contradiction.  $\therefore a_{n+1} = 0$ . As such, (1) becomes

$$\sum_{k=0}^n a_k y^k = 0 \in \mathcal{C}^{(\infty)}(I)$$

Hence, by the inductive hypo,  $a_k = 0$  for  $0 \leq k \leq n$ . therefore  $\ker(h) = (0)$ ; whence,  $h$  is an isomorphism. therefore  $\mathbb{R}[x] \equiv \mathbb{R}[D]$  as "rings"

Date: 04.07.15

Was not at school this day

Date: 04.08.15

Recall:  $B = \{x^k e^{rx} \in \mathcal{C}^{(\infty)}(\mathbb{R}) \mid 0 \leq k \leq m-1\}$

where  $y \in B$  are solns to  $(D-r)^m y = 0$ .

$B$  is linearly independent since

$$\sum_{k=0}^{m-1} c_k x^k e^{rx} = 0 \implies \sum_{k=0}^{m-1} c_k x^{k-1} = 0 \implies$$

$c_k = 0$  for all  $0 \leq k \leq m-1$  this is because  $\{1, x, x^2, \dots, x^{m-1}\} \subset \mathbb{R}[x]$  is linearly independent. therefore we have found the missing basis elements. In summary, if

$$\sum_{k=0}^n a_k y^{(k)} = 0 \text{ \& } \rho p(x) = a_n \prod_{k=0}^m (x - r_k)^{m_k}$$

where  $r_k \in \mathbb{R}$  for  $1 \leq k \leq m$ , then the soln space of  $\sum_{k=0}^n a_k y^{(k)} = 0$  has basis

$$B = \{x^j e^{r_k x} \in \mathcal{C}^{(\infty)}(\mathbb{R}) \mid 0 \leq j \leq m-1 \text{ \& } 1 \leq k \leq m\}$$

Note: since  $\sum_{k=1}^m m_k = n$ ,  $\text{card} B = n$

**Example.** say  $\sum_{k=0}^n a_k y^{(k)} = 0$  has the factored form

$$(D-2)^3(D+1)(D-4)^2 y = 0$$

then basis elements are :

$$e^{2x}, x e^{2x}, x^2 e^{2x}, e^{-x}, e^{4x}, x e^{4x}.$$

### Exam 3

hw up through 3.3, excluding problems involving  $\mathbb{C}$  - valued zeros

1a Given functions, prove they are lin independent

1b Verify these are solns to a given lin ODE

1c Solve the linear ODE initial value prob

2 given a 4th order lin hom ODE find a basis for its soln space, find the gen soln

3 Find a basis for a 2nd order hom ODE; find the gen soln 3cont Given a particular soln to the linear non-hom ODE, find the gen soln to the non-hom lin ODE

$$Ly = 0$$

$$\sum a_k y^{(k)} = 0$$

$$Ly = f$$

## Books

Hausen and sullivan "Real Analysis"

Reference: I.N. Herstein's "Topics in Abstract Algebra" (2nd ed), "Abstract Algebra" (3rd ed)

Artin's "Algebra"

(Formal poly : See hungerfords springer GTM.)