

Linear Algebra

***Vector Part 2:
Vector Extension Concept***

Automotive Intelligence Lab.



Contents

- **Vector set**
- **Linear weighted combination**
- **Linear independence**
- **Subspace and span**
- **Basis**
- **Summary**

Vector set

Vector Set

■ Definition of vector set

- ▶ A **collection** of vectors.

■ Notation of vector set

- ▶ Vector set is indicated as S or V , represented in **capital italics letters**.
- ▶ Mathematical representation of a vector set: $V = \{v_1, \dots, v_n\}$.

■ Characteristics of vector set

- ▶ Vector set can contain a **finite** or an **infinite** number of vectors.
- ▶ Vector set can also be **empty** which is indicated as $V = \{ \}$

Linear weighted combination

Linear Weighted Combination

■ What is **linear weighted combination**?

- ▶ A way of mixing information from multiple variables, with some variables contributing more than others.
- ▶ It is also sometimes called **linear mixture** or **weighted combination** when linear part is assumed.
- ▶ Weight can also be expressed as **coefficient**.
- ▶ Linear weighted combination simply means **scalar-vector multiplication** and **addition** as Eq 1.
 - It is assumed that all vectors v_i have the **same dimensionality**; otherwise, the addition is invalid.
 - The λ_i can be any real number, including zero.
 - Subtraction can be handled by setting a λ_i to be negative.

$$\mathbf{w} = \lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2 + \dots + \lambda_n \mathbf{v}_n$$

Eq 1. Standard form of linear weighted combination

Scalar

Subtraction of Linear Weighted Combination

■ In Eq 2., λ_3 shows an example of subtraction.

$$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = -3, \mathbf{v}_1 = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$
$$\mathbf{w} = \lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2 + \lambda_3 \mathbf{v}_3 = \begin{bmatrix} -7 \\ -4 \\ -13 \end{bmatrix}$$

Eq 2. Example of linear weighted combination

Code Exercise of Linear Weighted Combination using Matlab

Code Exercise (03_01)

► Linear weighted combination.

```
% Define vectors
v1 = [1; 2]; % example vector 1
v2 = [-2; 1]; % example vector 2

% Define weights
alpha = 0.4;
beta = 0.2;

% Compute the linear weighted combination
resultant_vector = alpha * v1 + beta * v2;

% Visualize vectors
figure;
hold on;
% Plot vectors
quiver(0, 0, v1(1), v1(2), 'Color', 'b', 'LineWidth', 2, 'MaxHeadSize', 1,
'AutoScale', 'off');
quiver(0, 0, v2(1), v2(2), 'Color', 'r', 'LineWidth', 2, 'MaxHeadSize', 1,
'AutoScale', 'off');
quiver(0, 0, resultant_vector(1), resultant_vector(2), 'Color', 'g',
'LineWidth', 2, 'MaxHeadSize', 1, 'AutoScale', 'off');

% Set axis equal, xlim, and ylim
axis equal;
xlim([-3, 3]);
ylim([-1, 3]);

% Add labels
xlabel('X-axis');
ylabel('Y-axis');

% Add legend
legend('Vector v1', 'Vector v2', 'Resultant Vector');

% Set title
title('Linear Weighted Combination of Vectors');

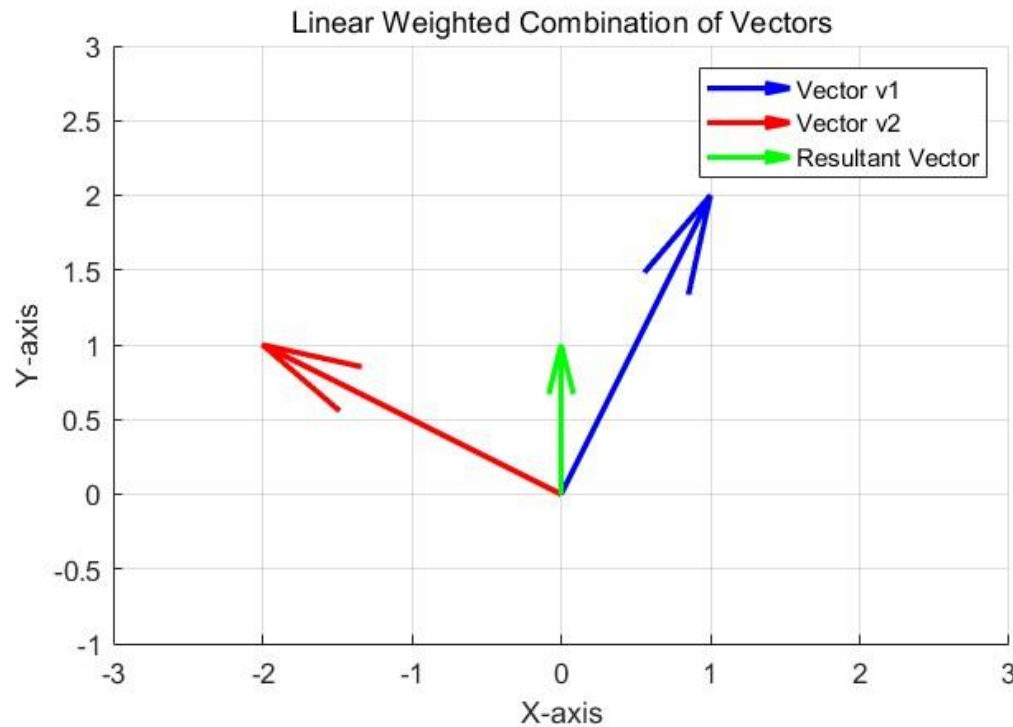
% Show grid
grid on;
hold off;
```

Source code

Visualization Result of Linear Weighted Combination using Matlab

■ Code Exercise (03_01)

- ▶ Linear weighted combination.



Source code result

Applications of Linear Weighted Combination

■ The **predicted data** from a statistical model

- ▶ The linear weighted combination of **regressors** (predictor variables) and **coefficients** (scalars) is used to generate predictions.
- ▶ These regressors and coefficients are computed via the **least squares algorithm**.

■ Dimension-reduction procedures

- ▶ The linear weighted combination of the data channels and weights.
- ▶ The weights selected to maximize the variance of the component.
 - Such as **PCA (Principal Components Analysis)**.

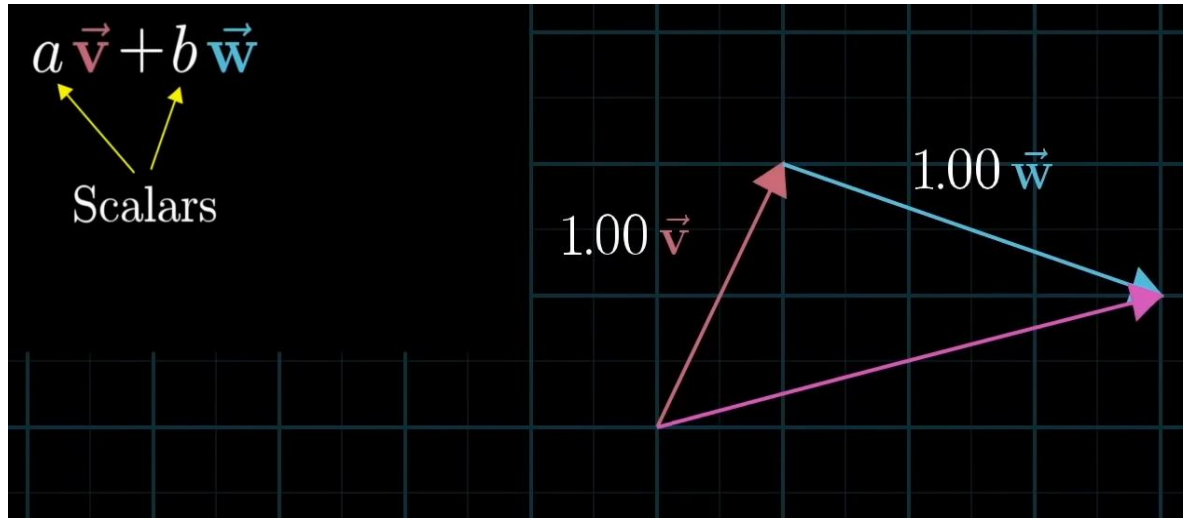
■ Artificial neural networks

- ▶ Two operations: Linear weighted combination of input data and nonlinear transformation.
- ▶ The weights are learned by minimizing a cost function.
 - Cost function: difference between the model prediction and the real-world target variable.

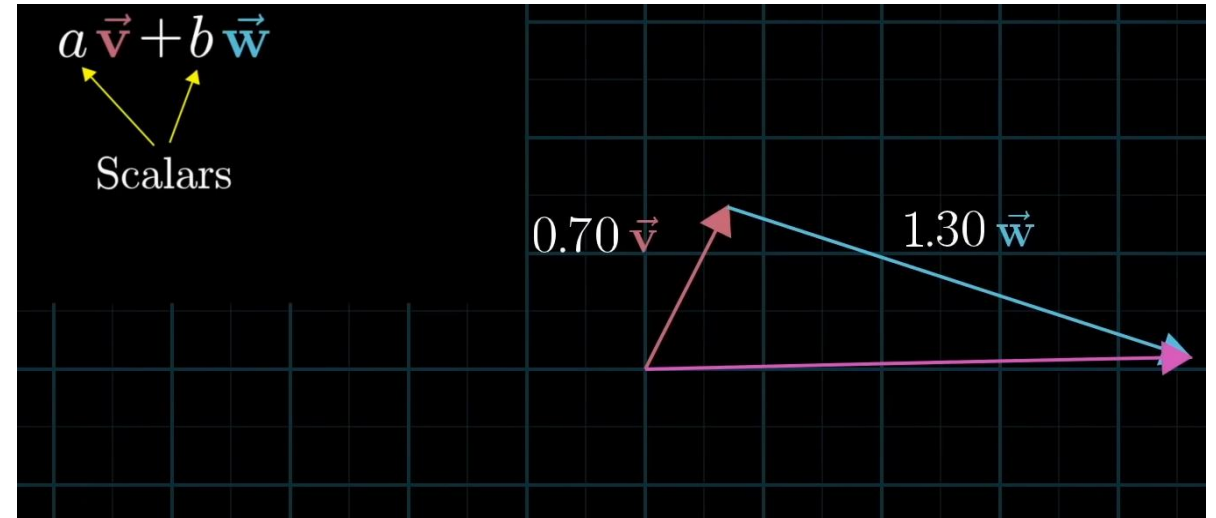
Visual Materials

■ Geometric representation of linear weighted combination.

- ▶ Linear combination (3:00 - 3:46)
- ▶ https://youtu.be/k7RM-ot2NwY?si=Cc5H_I_8cmfEoDsK&t=180 (English)
- ▶ <https://youtu.be/2CcCOgDilO8?feature=shared&t=180> (Korean)



Visualization of linear combination \vec{v} and \vec{w} (when scalar a is 1.0, and scalar b is 1.0)



Visualization of linear combination \vec{v} and \vec{w} (when scalar a is 0.7, and scalar b is 1.3)

Linear independence

Definition of Linear Independence

■ Linearly dependent

► At least one vector in vector set can be expressed as a linear weighted combination of other vectors in that set.

- Infinite number of such combinations, two of which are $s_1 = 0.5s_2$ and $s_2 = 2s_1$

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \end{bmatrix} \right\}$$

Vector set S

■ Linearly independent

► No vector in vector set can be expressed as a linear weighted combination of other vectors in that set.

- No possible scalar λ for which $v_1 = \lambda v_2$.

$$V = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix} \right\}$$

Vector set V

Linear Independence in Complex Vector

■ How about below vector set T ?

- ▶ The sum of the first three vectors equals twice the fourth vector → **Linearly dependent**

■ Determine whether linearly independent

- ▶ It is hard to figure out just from visual inspection.
 - In Ch.5, we will learn **matrix rank** for determine independence of vector set.
 - Create a matrix from the vector set.
 - Compute the rank of the matrix rank.
 - Compare the rank to the smaller of the number of rows or columns.

$$T = \left\{ \begin{bmatrix} 8 \\ -4 \\ 14 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 14 \\ 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 13 \\ 2 \\ 9 \\ 8 \end{bmatrix} \right\}$$

$$2 * t_4 = t_1 + t_2 + t_3$$

$$t_4 = \frac{1}{2} (t_1 + t_2 + t_3)$$

Example of linearly dependent

Property of Linear Independence

Independent sets

- ▶ Independence is a property of a **set** of vectors, not individual vector within a set.
 - A set of vectors can be linearly independent or linearly dependent
- ▶ In case of below vector set V , is it linearly independent?
- ▶ If then, each of v_1, v_2, v_3, v_4 is linearly independent?

$$V = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 12 \\ 7 \end{bmatrix} \right\} \quad \text{when } v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \end{bmatrix}, v_4 = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 7 \end{bmatrix}$$

Vector set V

The Math of Linear Independence

■ Formal mathematical definition of Linear dependence

- ▶ Define **some linear weighted combination of the vectors** in the set to produce the zeros vector.
 - If there are **some λ s** that make the Eq. 1. **true**, set of vectors is linearly **dependent**
 - Conversely, If **no possible way** to linearly **combine the vectors to produce zeros vector**, set of vectors is linearly **independent**

■ Why do we care about the zeros vector regarding the question of Linear dependence?

- ▶ how about expressing equation like Eq. 2.?

$$0 = \lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2 + \cdots + \lambda_n \mathbf{v}_n, \quad \lambda \in \mathbb{R}$$

Eq 1. The mathematical definition of linear dependence

$$\lambda_1 \mathbf{v}_1 = \lambda_2 \mathbf{v}_2 + \cdots + \lambda_n \mathbf{v}_n, \quad \lambda \in \mathbb{R}$$

Eq 2. Another definition of linear dependence

Express Zero Vector for Linear Independence

■ Equation with zeros vector on the left-hand side

- ▶ Setting the equation to zero helps reinforce the principle that the **entire set** is dependent or independent.
- ▶ No individual vector has the privileged position of being “dependent vector”.
 - When independence, vector sets are purely egalitarian (= 평등주의).

Definition and Constraint of Trivial Solution

■ Trivial solution

- ▶ Set **all λ 's to zero**, and the equation reads $0 = 0$, regardless of the vectors in the set.

■ Mathematical definition of linear dependence with constraint

- ▶ Trivial solutions involving zeros are often ignored in linear algebra, so **the constraint that at least one $\lambda \neq 0$** is added.
- ▶ This constraint can be **incorporated into the equation** by dividing through by one of the scalars.
 - v_1 and λ_1 can refer to any vector/scalar pair in the set.

$$0 = v_1 + \cdots + \frac{\lambda_n}{\lambda_1} v_n, \quad \lambda \in \mathbb{R}, \lambda_1 \neq 0$$

The mathematical definition of linear dependence with constraint

Independence and the Zeros Vector

■ Linear independence and the zeros vector

- ▶ Any vector set that includes the zeros vector is automatically a linearly dependent set.
 - Because any scalar multiple of the zeros vector is still the zeros vector

■ Linear dependence

- ▶ As long as $\lambda_0 \neq 0$, it has a **nontrivial solution**, and the set fits with the definition of linear dependence.

■ Nontrivial solution

- ▶ Solution to a homogeneous equation that is not the zero solution.
 - Any solution in which at least one variable has a nonzero value.

$$\lambda_0 0 = 0\mathbf{v}_1 + 0\mathbf{v}_2 + \cdots + 0\mathbf{v}_n$$

Equation of Linear dependence with zero vector

Question about Nonlinear Independence

■ About nonlinear independence

- ▶ Linear algebra is all about, well, linear operations.
- ▶ If you can express one vector as [a nonlinear combination of other vectors](#), then those vectors still form a linearly independent set.

■ Reason for the linearity constraint

- ▶ For express transformations as matrix multiplication.

■ Nonlinear systems can be well approximated using linear functions!

Subspace and span

Concept of Subspace and Span

■ Subspace

- ▶ The space formed by **infinitely linear combination of vectors** within a vector set, where each vector is multiplied by different weights.

■ Span

- ▶ The **mechanism** of combining all possible linear weighted combination.

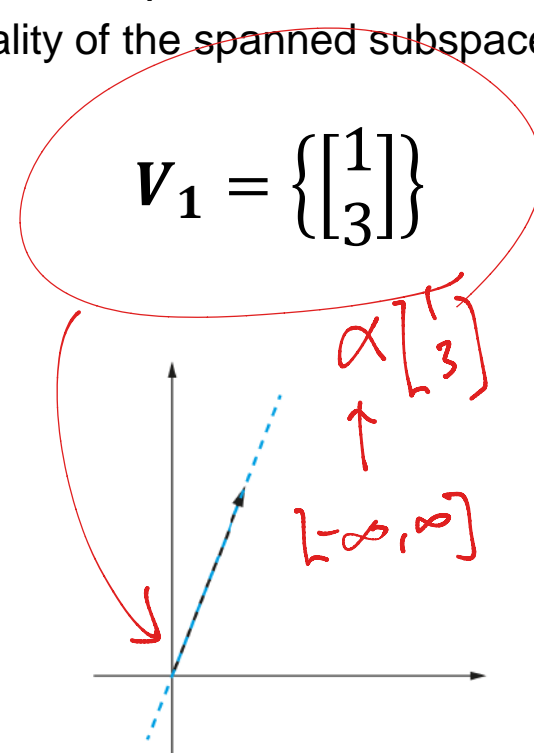
■ Difference between subspace and span

- ▶ Think of span as **verb** and subspace as **noun**.
 - A set of vectors **spans**, and the result of their spanning is a **subspace**.
- ▶ If using span as a noun, span and subspace refer to the same infinite vector set.

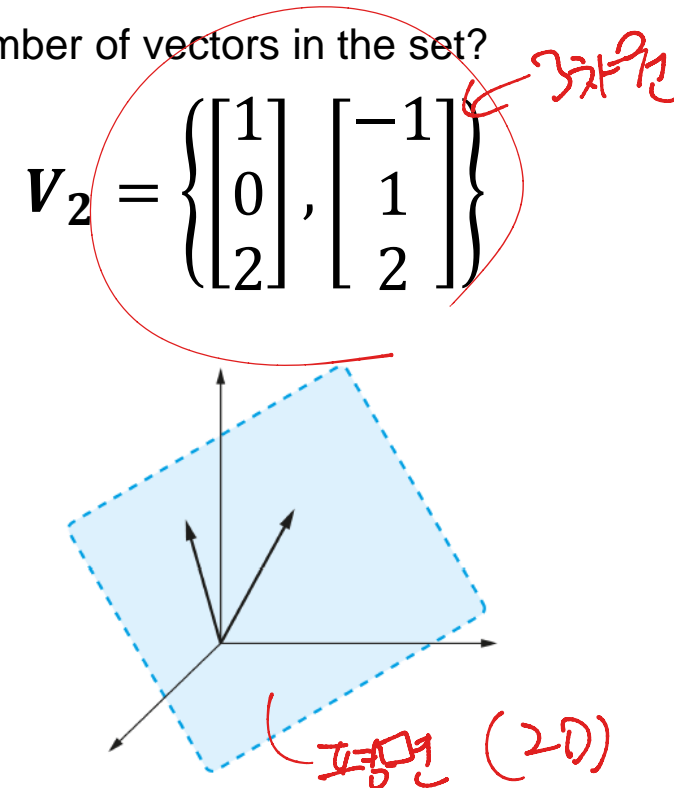
Subspace Spanned by Vector Set

■ Example of subspace of a set of linearly independent vector set refer to below figures

- ▶ The blue color represents the subspace generated by the black colored vector.
- ▶ V_1 has one vector and its span is 1D subspace, V_2 has two vectors and their span is 2D subspace, is there a pattern?
- Dimensionality of the spanned subspace and the number of vectors in the set?



Subspace of V_1



Subspace of V_2

Relation Between Dimensionality of Subspace and Number of Vectors

■ Example of subspace of a set of linearly dependent vector set refer to below figure

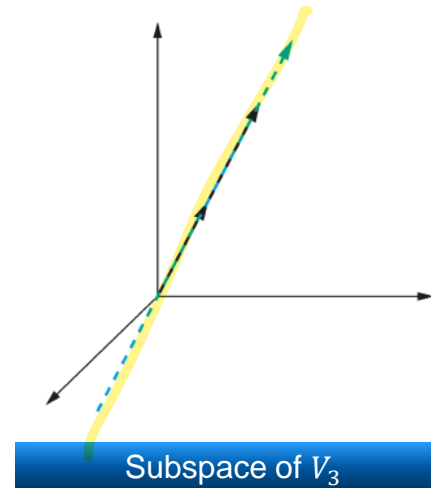
- ▶ Two vectors in \mathbb{R}^3 , but subspace of V_3 is still only a 1D subspace.
 - One vector in the set is already in the space of the other vector.

■ Relation between dimensionality of spanned subspace and the number of vectors in vector set

- ▶ Related to linear independence.

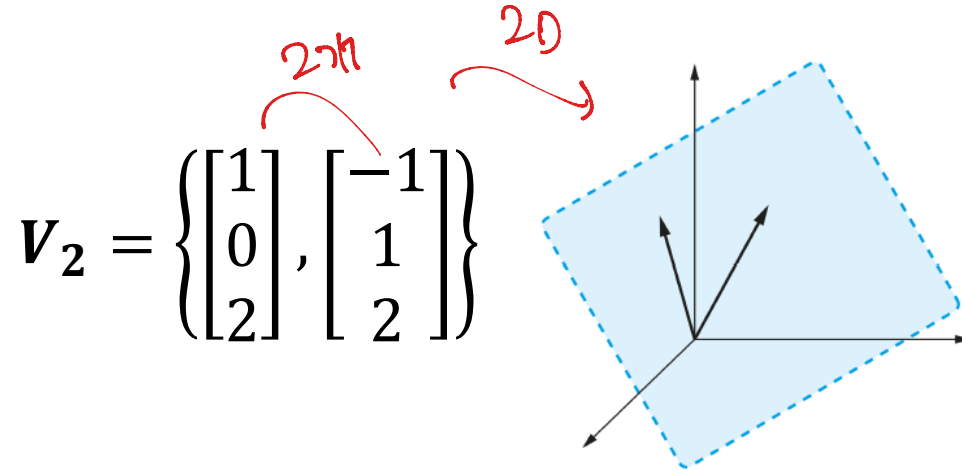
Linear independent인 2개의 벡터만
Sub space를 형성함

$$V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$

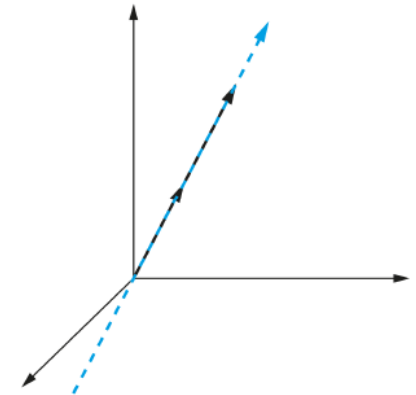


Dimension of Subspace and Linear Independence

- If the vectors in a vector set are linearly independent,
 - ▶ the dimension of the subspace equals the number of vectors in the set like V_2 .
- If the vectors in a vector set are linearly dependent,
 - ▶ the dimension of the subspace is less than the number of vectors in the set like V_3 .

Subspace of V_2

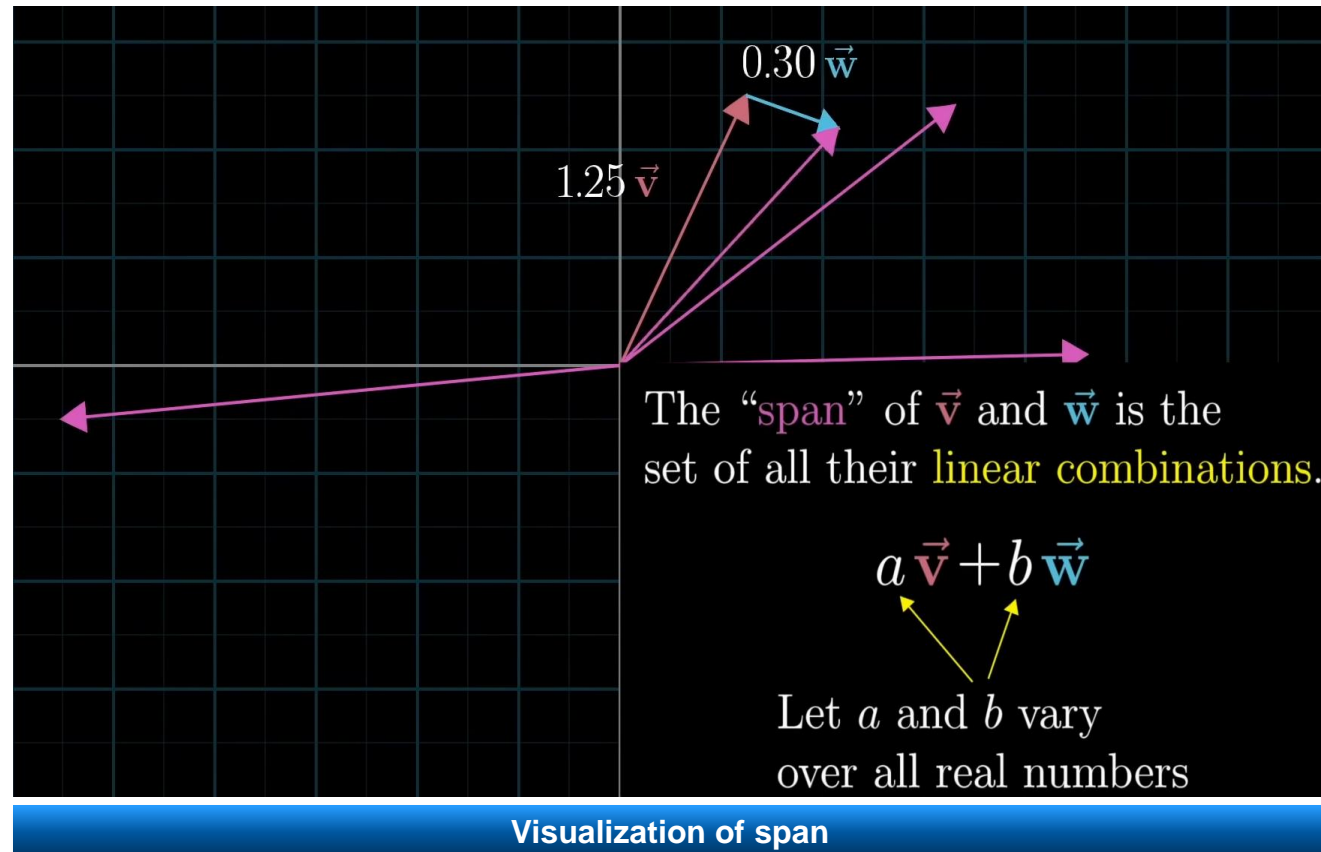
$$V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$

Subspace of V_3

Visual Materials of Span

■ Geometric representation of span

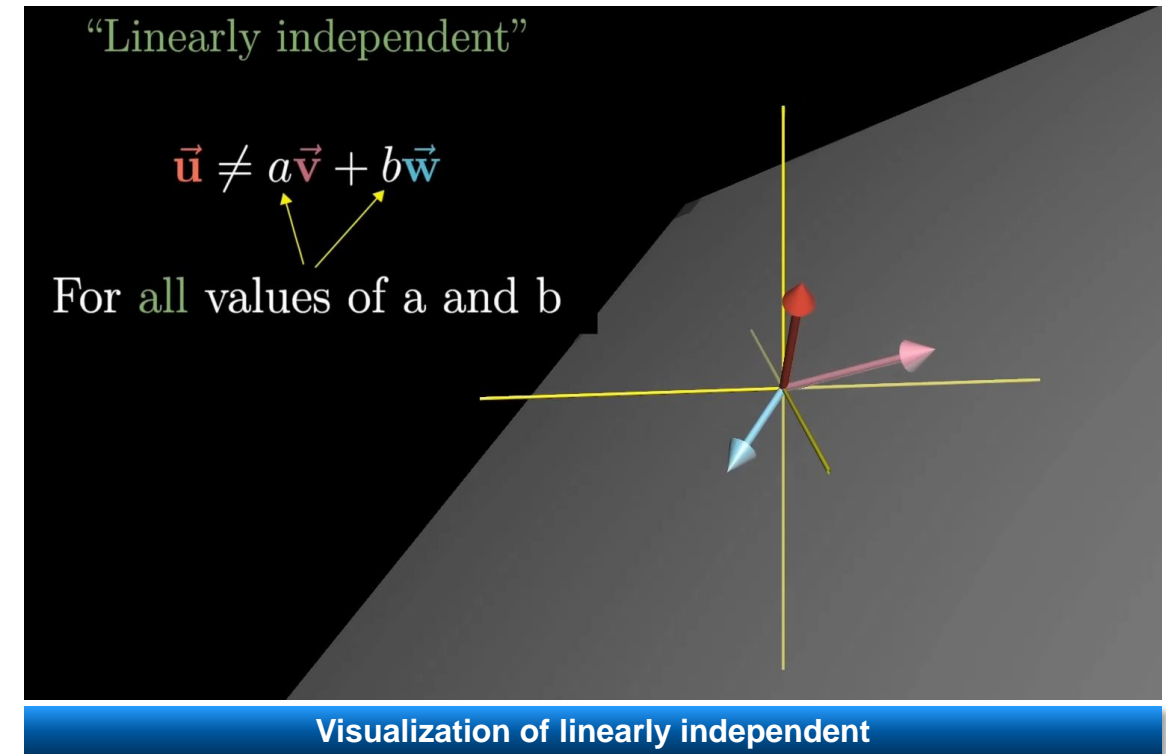
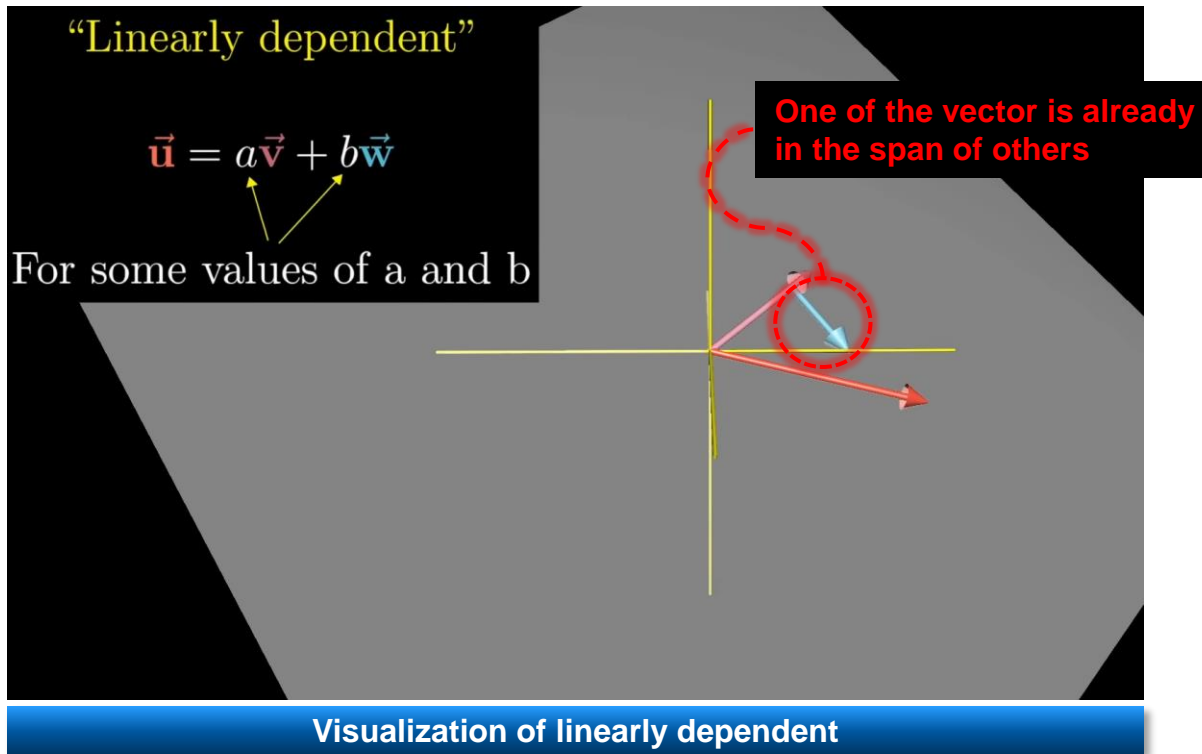
- ▶ Span (3:47 ~ 7:54)
- ▶ https://youtu.be/k7RM-ot2NWY?si=Zm_LRnsDlnohsn1y&t=227 (English)
- ▶ <https://youtu.be/2CcCOgDilO8?si=CIE6vVzhC5qCGMLI&t=227> (Korean)



Visual Materials of Linear Dependence

■ Geometric representation of linear dependence and linear independence

- ▶ Linear dependence (7:55 ~)
- ▶ https://youtu.be/k7RM-ot2NWX?si=NX_fK1ppEP5aGmPU&t=475 (English)
- ▶ <https://youtu.be/2CcCOgDilO8?si=AqHObsQ2v2zLbfrz&t=475> (Korean)



Code Exercise of Vector Span using Matlab

Code Exercise (03_02)

► Vector Span.

```
% Define two 3D vectors
v1 = [1; -3; -2];
v2 = [2; 4; -1];

% Calculate the normal vector (cross product of the two vectors)
normal = cross(v1, v2);

% Choose a point on the plane (for example, using v1)
point = v1;

% Equation of the plane: ax + by + cz + d = 0
% where a, b, c are components of the normal vector, and d is the constant term
of the plane equation
a = normal(1);
b = normal(2);
c = normal(3);
d = -dot(normal, point);

% Code for visualization
[x, y] = meshgrid(-10:1:10, -10:1:10); % Create a grid to represent the plane
z = (-d - a*x - b*y) / c; % Calculate z values of the plane

% Draw the plane
figure;
mesh(x, y, z);
hold on;

% Draw the two vectors
quiver3(0, 0, 0, v1(1), v1(2), v1(3), 'r', 'LineWidth', 2, 'AutoScale', 'off',
'MaxHeadSize', 2);
quiver3(0, 0, 0, v2(1), v2(2), v2(3), 'b', 'LineWidth', 2, 'AutoScale', 'off',
'MaxHeadSize', 2);

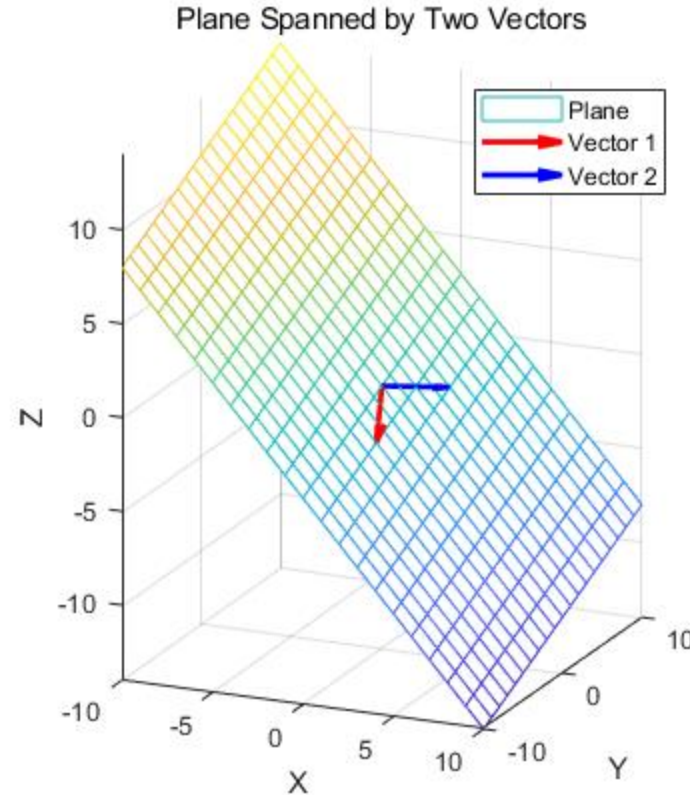
xlabel('X'); ylabel('Y'); zlabel('Z');
title('Plane Spanned by Two Vectors');
legend('Plane', 'Vector 1', 'Vector 2');
axis equal;
grid on;
```

Source code

Visualization Result of Vector Span using Matlab

■ Code Exercise (03_02)

► Vector Span.



Source code result

Basis

Concept of Basis

■ In linear algebra, a basis is a **set of** filters that describes the information in the matrix.

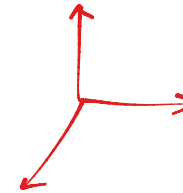
■ Most common basis set is the **Cartesian axis**.

► The Cartesian basis set comprises vectors that are mutually **orthogonal and unit length**.

- It's why the Cartesian basis sets are so ubiquitous.
- They are called the **standard basis set**.

dot product
확인

$$S_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad S_3 = \left\{ \alpha \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \beta \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \gamma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$



Basis sets for 2D and 3D Cartesian graphs

Determining Basis

■ Cartesian axis is not the only basis sets

- ▶ Basis set **S**, **T** both span the same subspace.

- Subspace : all of R^2

$$\mathbf{S} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad \left| \quad \mathbf{T} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$$

$$\mathbf{s}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{s}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \left| \quad \mathbf{t}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{t}_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Basis set of **S** and **T**

- ▶ Let's represent point p and q in terms of the basis **S** and **T**.
 - In basis **S**, point p is (3,1), point q is (-6,2).
 - Because $p = 3\mathbf{s}_1 + \mathbf{s}_2$, $q = -6\mathbf{s}_1 + 2\mathbf{s}_2$
 - In basis **T**, point p is (1,0), point q is (0,2).
 - Because $p = 1\mathbf{t}_1 + 0\mathbf{t}_2$, $q = 0\mathbf{t}_1 + 2\mathbf{t}_2$
- ▶ Data points p and q are the same regardless of the basis set, but **T** provide a **compact** and **orthogonal** description.

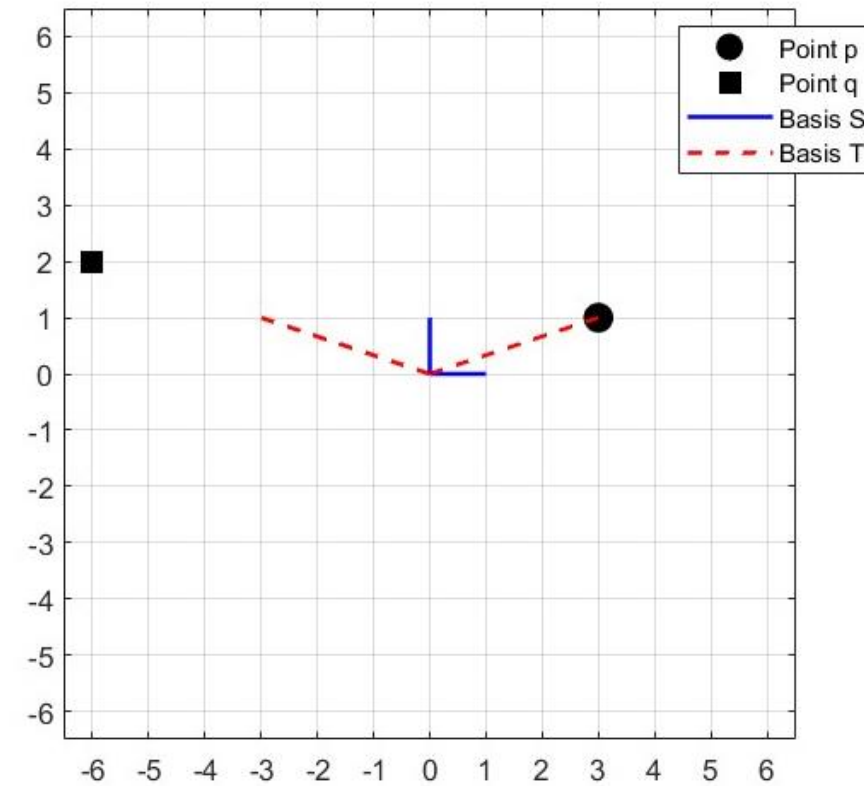


Figure 1. The same points can be described by different basis set

Standard Basis

■ Basis in vector space

- ▶ A set of vectors $\{v_1, v_2, \dots, v_n\}$ within a vector space V is called a basis of V if it satisfies the following two conditions.
 - The set $\{v_1, v_2, \dots, v_n\}$ is linearly independent
 - The set $\{v_1, v_2, \dots, v_n\}$ spans V , meaning it generates the whole space V .

Applications of Basis

■ Application of basis in data science and machine learning

▶ Many problems in applied linear algebra can be conceptualized as finding the best set of basis vectors to describe some subspace.

- Dimension reduction
- Feature extraction
- Principal components analysis
- Independent components analysis
- Factor analysis
- Singular value decomposition
- Linear discriminant analysis
- Image approximation
- Data compression
- ...

■ All of those analyses are essentially ways of identifying optimal basis vectors for a specific problem.

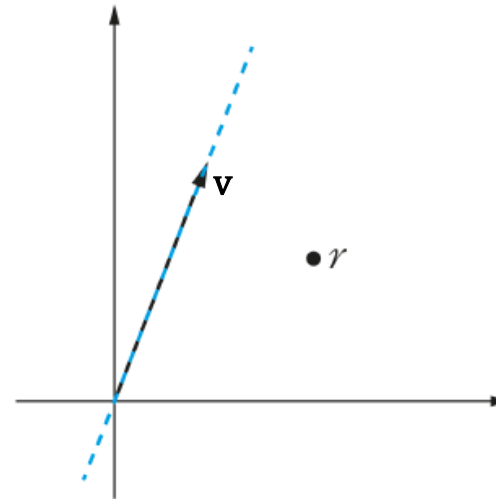
Definition of Basis

■ Formal definition of basis

- ▶ A basis is the combination of Span and independence
 - 1. Spans a certain subspace.
 - 2. Independent set of vectors.

■ The basis needs to span a subspace for it to be used as a basis for that subspace

- ▶ A basis set can measure only what is contained its span.
 - A basis vector for blue colored subspace cannot measure point r which is not in subspace of vector v .



Basis and Subspace

Linearly Independency of Basis

■ Why a basis set must be linearly independent?

- ▶ All vectors in a subspace **must have unique coordinates** with respect to that basis.
- ▶ To ensure this uniqueness, the basis vectors must not be linearly combinable in any way.
 - For example, U is a valid set of vectors.
 - But if you want to describe point (3,1), the coefficients for the linear weighted combination of the three vectors in U could be (3, 0, 1) or (0, 1.5, 1) or ... a bajillion other possibilities.
 - It's so confusing, and so mathematicians decided that a **vector must have unique coordinates within a basis set**.

$$U = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

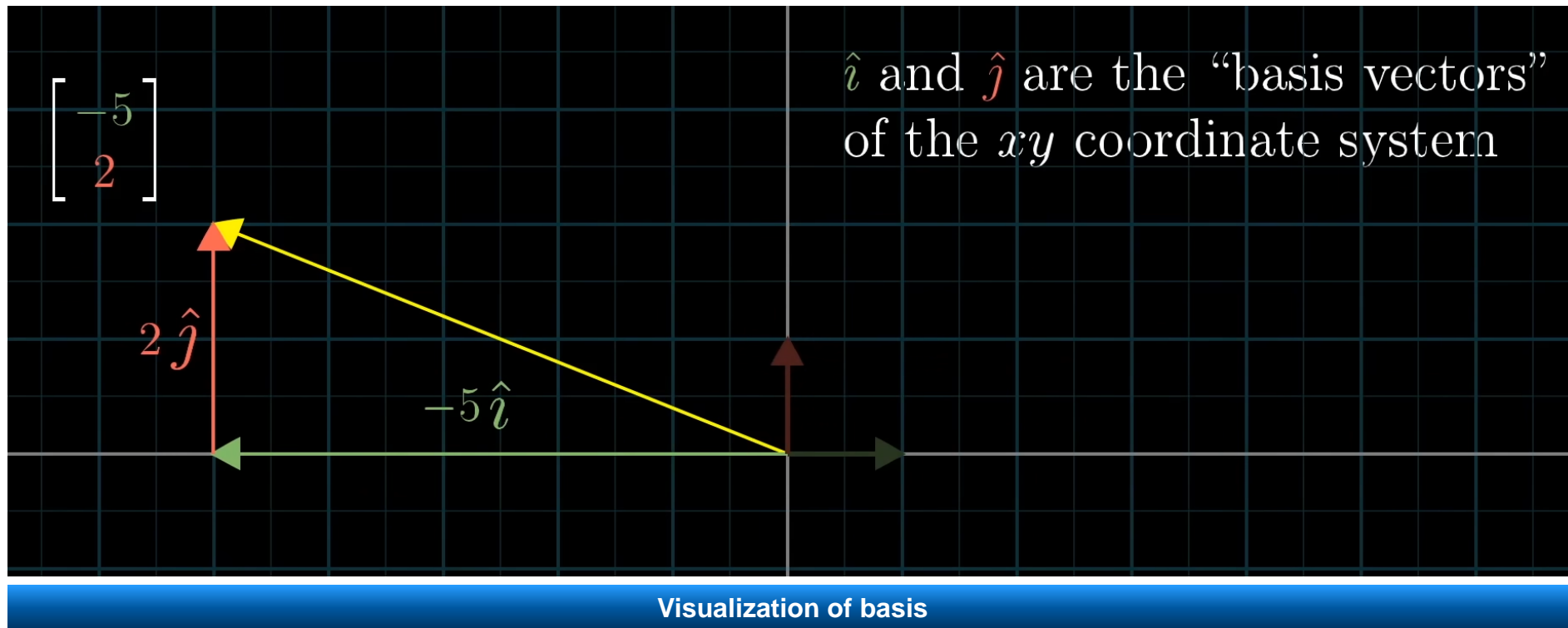
Vector set of U

- ▶ To be clear, any point can be described using an infinite numbers of basis sets. Within a basis set, a point is defined by exactly one linear weighted combination.

Visual Materials of Basis

■ Geometric representation of basis

- ▶ Basis (0:12 ~ 2:27)
- ▶ <https://youtu.be/k7RM-ot2NwY?si=-MomPGwuSEOwwvew&t=12> (English)
- ▶ <https://youtu.be/2CcCOgDilO8?si=c00bqWCvnhEuVwrr&t=12> (Korean)



Code Exercise of Basis using Matlab

Code Exercise (03_03)

► Basis

```
% Define basis vectors
i = [1; 0; 0];
j = [0; 1; 0];
k = [0; 0; 1];

% Define an arbitrary 3D vector v
v = [3; 2; 5];

% Visualization
figure;
hold on;
grid on;
axis equal;

% Draw the basis vectors
quiver3(0, 0, 0, i(1), i(2), i(3), 'r', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'i');
quiver3(0, 0, 0, j(1), j(2), j(3), 'g', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'j');
quiver3(0, 0, 0, k(1), k(2), k(3), 'b', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'k');

% Draw the vector v
quiver3(0, 0, 0, v(1), v(2), v(3), 'm', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'v');

% Visualize each multiplication of basis vector
plot3([0 v(1)], [0 0], [0 0], 'k--', 'LineWidth', 1);
plot3([v(1) v(1)], [0 v(2)], [0 0], 'k--', 'LineWidth', 1);
plot3([v(1) v(1)], [v(2) v(2)], [0 v(3)], 'k--', 'LineWidth', 1);

% Visualize the values of multiplication
text(v(1)/2, 0, 0, sprintf('%0.1f \\iti', v(1)), 'VerticalAlignment', 'bottom',
'FontSize', 15);
text(v(1), v(2)/2, 0, sprintf('%0.1f \\itj', v(2)), 'VerticalAlignment',
'bottom', 'FontSize', 15);
text(v(1), v(2), v(3)/2, sprintf('%0.1f \\itk', v(3)), 'VerticalAlignment',
'bottom', 'FontSize', 15);

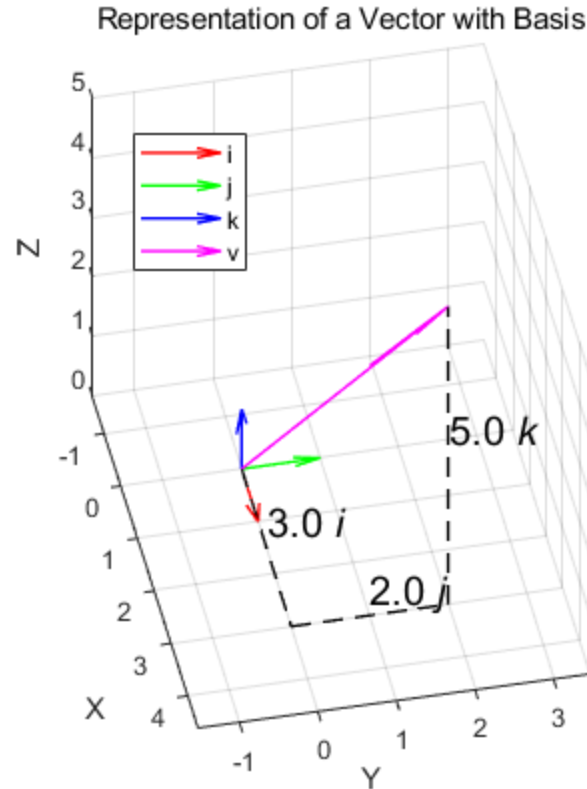
xlabel('X');
ylabel('Y');
zlabel('Z');
title('Representation of a Vector with Basis');
legend('i','j','k','v','Location', 'northeastoutside');
view(45, 45); % Adjust the 3D view angle
hold off;
```

Source code

Visualization Result of Basis using Matlab

■ Code Exercise (03_03)

► Basis



Source code result

Summary

Summary

■ A vector set

- ▶ The set can contain either a finite or an infinite number of vectors.

■ Linear weighted combination

- ▶ Multiply scalar and add vectors in a set.
- ▶ One of the single most important concepts in linear algebra.

■ Linearly dependent vs linearly independent

- ▶ If a vector can be expressed as a linear weighted combination, the set is linearly dependent.
- ▶ If no such linearly weighted combination, the set is linearly independent.

■ A subspace

- ▶ The infinite set of all possible linearly weighted combination of vector set.

■ A basis

- ▶ If vector set (1) spans a certain subspace and (2) is linearly independent, it can be a basis for subspace.

Exercises (1)

1. Rewrite the Original code for linear weighted combination, but put the scalars in array and the vectors as elements in an array
 - ▶ you will have two arrays, one of the **scalars** and one of **vectors**.
2. Then use a for loop to implement the linear weighted combination operation.
 - ▶ Initialize the output vector using zeros().
3. Display var *linCombo2*
 - ▶ *linCombo1* & *linCombo2* display same result.

```
% Scalars
l1 = 1;
l2 = 2;
l3 = -3;

% Vectors
v1 = [4, 5, 1];
v2 = [-4, 0, -4];
v3 = [1, 3, 2];

% Linear weighted combination
linCombo1 = l1 * v1 + l2 * v2 + l3 * v3;
disp(linCombo1);
```

Original code

```
% Scalars
l1 = 1;
l2 = 2;
l3 = -3;

% Vectors
v1 = [4, 5, 1];
v2 = [-4, 0, -4];
v3 = [1, 3, 2];

% Scalars and vectors organized into arrays
scalars =
vectors =

% Initialize the linear combination
linCombo2 =

% Implement linear weighted combination using a loop
for i = 1:length(scalars)
    linCombo2 =
end

% Confirm it's the same answer as above
disp(linCombo2);
```

Generated code

Exercises (2)

1. **Create a scalar list and vector list like the original code in Exercise (1), but length of scalar is different**
 - ▶ Scalar
 - length : 4
 - ▶ Vector
 - dimension : 3
 - length : 3
 - ▶ You can use any number in the list.

2. **Write code for linear weighted combination and execute code**
 - ▶ If code run successfully, write comment the result.
 - ▶ If code isn't run successfully, write comment the reason of error.



**THANK YOU
FOR YOUR ATTENTION**