

Optimal State Estimation

Practice 3. Propagation of state and covariance

Please make a m file for the Example 4.1. Submit the m file to draw the Figure 4.1.

■ EXAMPLE 4.1

A linear system describing the population of a predator $x(1)$ and that of its prey $x(2)$ can be written as

$$\begin{aligned}x_{k+1}(1) &= x_k(1) - 0.8x_k(1) + 0.4x_k(2) + w_k(1) \\x_{k+1}(2) &= x_k(2) - 0.4x_k(1) + u_k + w_k(2)\end{aligned}\quad (4.9)$$

In the first equation, we see that the predator population causes itself to decrease because of overcrowding, but the prey population causes the predator population to increase. In the second equation, we see that the prey population decreases due to the predator population and increases due to an external food supply u_k . The populations are also subject to random disturbances (with respective variances 1 and 2) due to environmental factors. This system can be written in state-space form as

$$\begin{aligned}x_{k+1} &= \begin{bmatrix} 0.2 & 0.4 \\ -0.4 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k + w_k \\w_k &\sim (0, Q) \quad Q = \text{diag}(1, 2)\end{aligned}\quad (4.10)$$

Equations (4.2) and (4.4) describe how the mean and covariance of the populations change with time. Figure 4.1 depicts the two means and the two diagonal elements of the covariance matrix for the first few time steps when $u_k = 1$ and the initial conditions are set as $\bar{x}_0 = [10 \ 20]^T$ and $P_0 = \text{diag}(40, 40)$. It is seen that the mean and covariance eventually reach steady-state values given by

$$\begin{aligned}\bar{x} &= (I - F)^{-1}Gu \\&= [2.5 \ 5]^T \\P &\approx \begin{bmatrix} 2.88 & 3.08 \\ 3.08 & 7.96 \end{bmatrix}\end{aligned}\quad (4.11)$$

The steady-state value of P can also be found directly (i.e., without simulation) using control system software.¹ Note that since F for this example is stable and Q is positive definite, Theorem 21 guarantees that P has a unique positive definite steady-state solution.

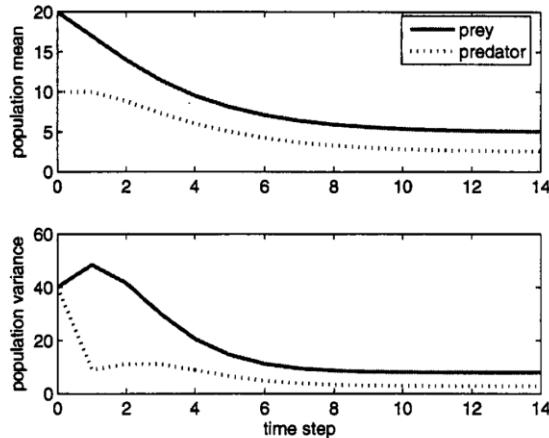


Figure 4.1 State means and variances for Example 4.1.