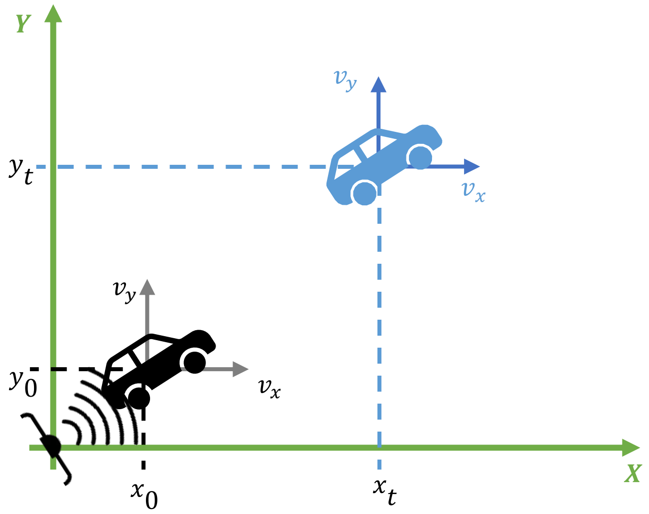
Optimal State Estimation

Practice 5. The 2D discrete-time Kalman filter

[Practice 1: simulation of state and measurement of a 2D vehicle]

The simulation targets are a dynamic state vector (position, velocity) of a vehicle on two-dimension space (X and Y), and a range sensor located at origin, which can measure the position of a vehicle.



A discrete vehicle motion model with **acceleration inputs** can be represented as follow:

$$x_{t+1} = x_t + v_{x_t} dT + \frac{1}{2} a_{x_t} dT^2$$

$$y_{t+1} = y_t + v_{y_t} dT + \frac{1}{2} a_{y_t} dT^2$$

$$v_{x_{t+1}} = v_{x_t} + a_{x_t} dT$$

$$v_{y_{t+1}} = v_{y_t} + a_{y_t} dT$$

,where x_t is a X position of the vehicle at time t, y_t is a Y position of the vehicle at time t, v_{x_t} is the velocity of the vehicle in the x axis at time t, v_{y_t} is the velocity of the vehicle in the y axis at time t, and dT is a sampling time. The a_{x_t} and a_{y_t} are the acceleration input of the vehicle.

State consists of X, Y position and velocity in the X, Y axes, therefore discrete state space description of this system can be written as

$$\begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 0 & dT & 0 \\ 0 & 1 & 0 & dT \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_t + \begin{bmatrix} \frac{dT^2}{2} & 0 \\ 0 & \frac{dT^2}{2} \\ dT & 0 \\ 0 & dT \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

$$x' = Fx + Ga.$$

The measurement is the 2D position of vehicle, so the measurement model can be described as follow:

$$y_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_{t+1}$$

$$y = Hx.$$

We have the sampling time and initial information as below:

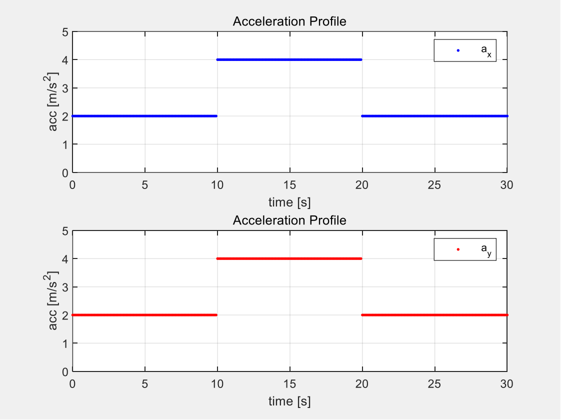
1) $\Delta t = 0.1$
2) initial velocity $v_x = 0m/s,\ v_y = 0m/s$
3) initial position $x = 1m, y = 1m$

[Practice 1-1]

Please simulate the $x_t, y_t, v_{x_t}, v_{y_t}$ from time $t = 0$ to $t = 30$ using given acceleration data:

$$a_x = \begin{cases} 2 & (simulation\ time < 10) \\ 4 & (10 \le simulation\ time < 20) \\ 2 & (20 \le simulation\ time) \end{cases}$$

$$a_y = \begin{cases} 2 & (simulation\ time < 10) \\ 4 & (10 \le simulation\ time < 20) \\ 2 & (20 \le simulation\ time) \end{cases}$$



Plot the simulation and submit the figure (each state and 2D vehicle position) for the simulation.

[Practice 1-2]

Please simulate y_t, which is a position sensor simulation data for the previous simulation. The standard deviation of the sensor noise is **one meter**. Plot the simulation results and save to m file. Submit the simulation figure and m. file.

(Hint: using gaussian random variable “normrnd” in Matlab)

[Practice 2: Kalman filter with Constant Velocity (CV) Model]

A constant velocity (CV) process model can be written as:

$$\begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 0 & dT & 0 \\ 0 & 1 & 0 & dT \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}_t$$

[Practice 2-1]

Estimate the state and the error between estimated state and true value using Kalman filter for below cases:

- $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P_0 = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix}.$

Tuning the Q, R to get the best estimate! Submit the figures (x-axis is time and y-axes are $x_t, y_t, v_{x_t}, v_{y_t}$) for the estimated state and error.

*** Hint! process covariance matrix for acceleration uncertainty σ_{ax} and σ_{ay}!**

- $Q = E[vv^T] = E[Gaa^T G^T] = GE[vv^T]G^T = G\begin{pmatrix} \sigma_{ax}^2 & \sigma_{axy} \\ \sigma_{axy} & \sigma_{ay}^2 \end{pmatrix}G^T = GQ_vG^T$

$$= \begin{pmatrix} \frac{dT^4}{2}\sigma_{ax}^2 & 0 & \frac{dT^3}{2}\sigma_{ax}^2 & 0 \\ 0 & \frac{dT^4}{2}\sigma_{ay}^2 & 0 & \frac{dT^3}{2}\sigma_{ay}^2 \\ \frac{dT^3}{2}\sigma_{ax}^2 & 0 & dT^2\sigma_{ax}^2 & 0 \\ 0 & \frac{dT^3}{2}\sigma_{ay}^2 & 0 & dT^2\sigma_{ay}^2 \end{pmatrix}$$

[Practice 2-2]

Get the RMSE between estimated state and true value for each case.

[Practice 2-3]

Plot the P in three cases (x-axis is time and y-axes are P_{00}~P_{33}) for each case. Submit the figure for the P.

[Practice 3: Kalman filter with Constant Acceleration (CA) Model]

A state of constant acceleration (CA) model can be written as:

$$\begin{bmatrix} x \\ y \\ v_x \\ v_y \\ a_x \\ a_y \end{bmatrix}$$

[Practice 3-1]

Derive the process model for the CA model!

[Practice 3-2]

Plot the estimated state and the error between estimated state and true value (x-axis is time and y-axes are $x_t, y_t, v_{x_t}, v_{y_t}$) for each case. Submit the figures for the estimation for the best *Q, R*

[Practice 3-3] Get the RMSE between estimated state and true value for each case.

[Practice 3-4] Plot the P in three cases (x-axis is time and y-axes are P_{00}~P_{55}) for each case. Submit the figure for the P.

[Practice 3-5] Please describe the performance difference between CV and CA Kalman filter!