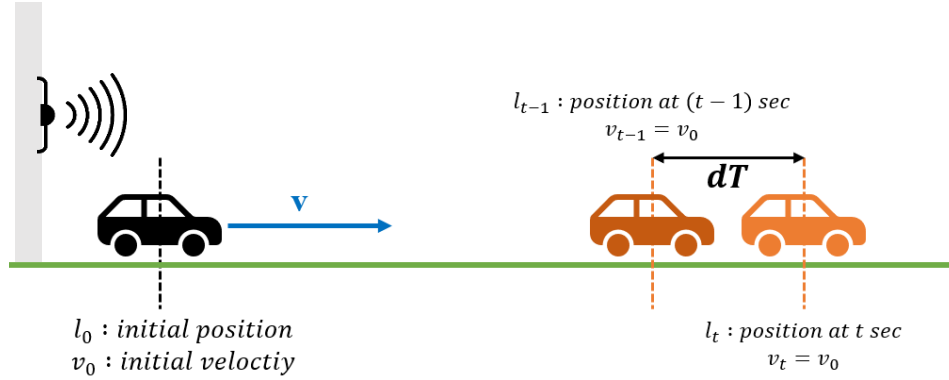# Optimal State Estimation

Practice 4. The discrete-time Kalman filter

The Kalman filter is a set of mathematical equations that provides an efficient computational estimate of the state of a process (e.g., the position and orientation of car) given a time-varying sequence of noisy measurements (e.g., speed, position, etc). The filter is a popular mathematical estimator due to its efficiency and robustness. Our goal is to develop a simulation program with MATLAB to help develop the intuition and insight of novice users regarding the behavior of the Kalman filter. We would have the ability to change various input parameters and then see how the Kalman filter responds to a given set of noisy measurements.

The simulation targets a dynamic state vector (position, velocity) of a constant velocity model, and a range sensor that can measure the position of a vehicle.

l_{t-1} : position at $(t-1)$ sec
$v_{t-1} = v_0$
dT

l_0 : initial position
v_0 : initial veloctiy

l_t : position at t sec
$v_t = v_0$

[Practice 1: State simulation]

A constant velocity discrete model can be represented as follow:

$$l_{t+1} = l_t + dT * v_t$$

$$v_{t+1} = v_t$$

, where l_t is a position of the vehicle at time t, v_t is the velocity of the vehicle at time t, and dT is a sampling time.

State consists of position and velocity, therefore discrete state space description of this system can be written as

$$\begin{bmatrix} l \\ v \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & dT \\ 0 & 1 \end{bmatrix} \begin{bmatrix} l \\ v \end{bmatrix}_t$$

, we can get measurement description of this system can be written as

$$y_{t+1} = [1 \; 0] \begin{bmatrix} l \\ v \end{bmatrix}_{t+1}$$
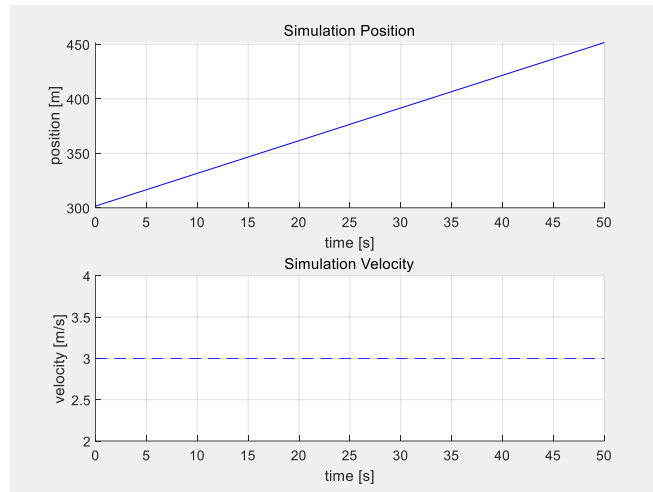
We have the sampling time and initial information as below:

1) $\Delta t = 0.1$

2) initial velocity $v_0 = 3m/s$

3) initial range $l_0 = 1m$

Please simulate the l_t, v_t from time $t = 0$ to $t = 50$. Plot the simulation (x-axis is time and y-axes are l_t, v_t. Submit the figure for the simulation.
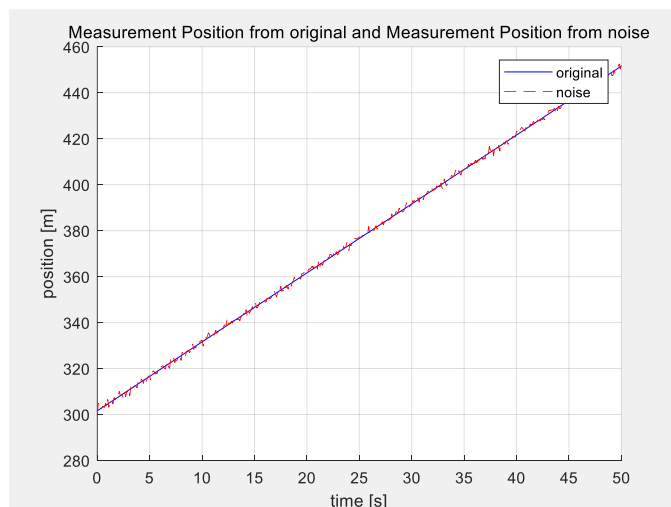
example)

[Practice 2 : Measurement simulation]

Please simulate y_t, which is a range sensor simulation data for the previous simulation. The standard deviation of the sensor noise is **one meter**. Plot the simulation results and save to m file. Submit the simulation figure and m. file.

(Hint: using gaussian random variable "normrnd" in Matlab)
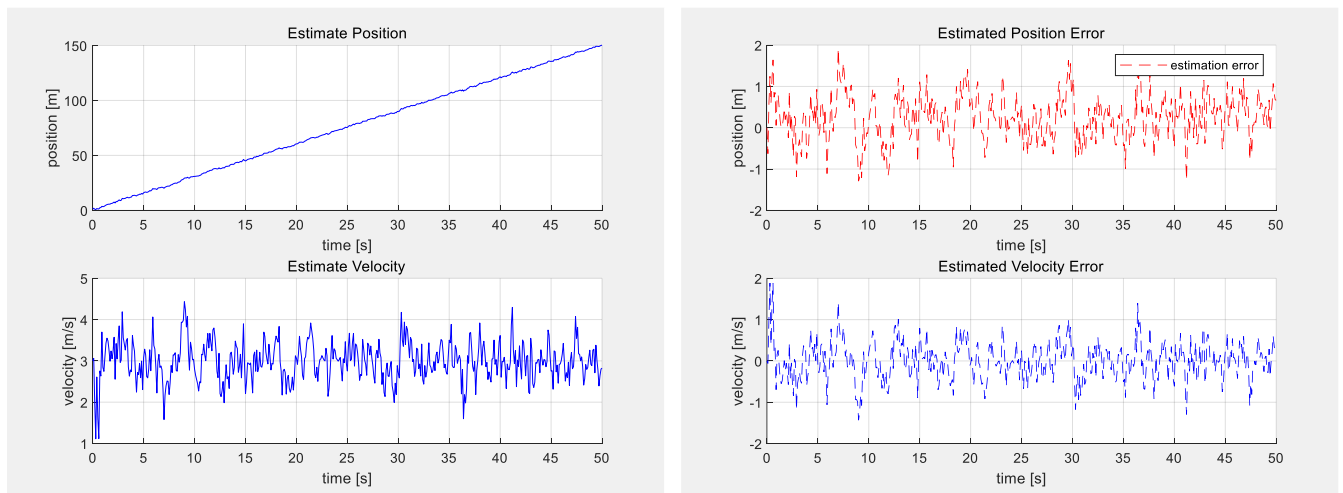
example)

[Practice 3 : State estimation]

Please estimate l_t, v_t and covariance matrix P using Kalman filter for below cases:

- Case1: $Q = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, R = 0.5$

- Case2: $Q = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, R = 3$

- Case3: tuning the optimal Q, R

- Case 4: $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, P_0 = \begin{bmatrix} 10000 & 0 \\ 0 & 10000 \end{bmatrix}$

- Case 5: $x_0 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, P_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

[Practice 3-1] Plot the estimated state and the error between estimated state and true value in three cases (x-axis is time and y-axes are l_t, v_t) for each case. Submit the figures for the estimation.

example)

[Practice 3-2] Get the RMSE between estimated state and true value for each case.

example)

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[Practice 3-2]
RMSE position: 0.603008, RMSE velocity: 0.458276
>>
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[Practice 3-3] Plot the P in three cases (x-axis is time and y-axes are $P_{00} \sim P_{11}$) for each case. Submit the figure for the P.

example)