Linear Algebra

Matrix:
Matrix Inverse

Automotive Intelligence Lab.

Contents

The matrix inverse

Inverse of Matrix A

- **Multiply A to produce identity matrix.**
  - Which is written as A^{-1}.
  - Cancel a matrix into identity matrix.

- **Linearly transform a matrix into identity matrix.**
  - Matrix inverse
    - Contains linear transformation.
  - Matrix multiplication
    - Mechanism of applying transformation to the matrix.

$$A^{-1}A = I$$

Inverse of matrix

Why We Need to Invert Matrices?

- **In the form $\boldsymbol{Ax} = \boldsymbol{b}$**
  - Already know $\boldsymbol{A}$ and $\boldsymbol{b}$.
  - Need to cancel matrix to solve $\boldsymbol{x}$.

$$\boldsymbol{Ax} = \boldsymbol{b}$$
$$\boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{b}$$
$$\boldsymbol{Ix} = \boldsymbol{A}^{-1}\boldsymbol{b}$$
$$\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b}$$

General form to solve x

Types of inverses and conditions for invertibility

Type 1: Full Inverse

- **Means $A^{-1}A = AA^{-1} = \square$**

- **Two conditions**
  - Square
  - Full-rank

Type 2: One Sided Inverse

- **Inverse of non-square matrix**
  - Can transform a rectangular matrix into identity matrix.
    - Works only for one multiplication order.
  - A tall matrix $\boldsymbol{T}$ can have a left-inverse.
    - $\boldsymbol{LT} = \boldsymbol{I}$ but $\boldsymbol{TL} \neq \boldsymbol{I}$.
  - A wide matrix $\boldsymbol{W}$ can have a $\boxed{}$-inverse.
    - $\boldsymbol{WR} = \boldsymbol{I}$ but $\boldsymbol{RW} \neq \boldsymbol{I}$.

- **Conditions of one-sided inverse**
  - Only if it has maximum possible rank.
  - Non-square matrix size: $M \times N$
    - Tall matrix's rank should be N.
    - Wide matrix's rank should be $\boxed{}$

Type 3: Pseudoinverse

- **Inverse for every matrix**
  - Every matrix has a pseudoinverse.
    - Regardless of its shape and rank.
  - Square full rank matrix
    - Its pseudoinverse equals its full inverse.
  - Nonsquare and its maximum possible rank
    - Tall matrix: pseudoinverse equals its $\boxed{}$-inverse.
    - Wide matrix: pseudoinverse equals its $\boxed{}$-inverse.
  - Reduced-rank matrix
    - Has a pseudoinverse matrix.
    - Pseudoinverse transforms the singular into another matrix.
      - Close but not equal to the identity matrix.

- **Matrices that do not have a full or one-sided inverse are called singular or noninvertible.**
  - Same thing as labeling a matrix reduced-rank or rank-deficient.

Computing the inverse

Inverse of a 2×2 Matrix

How to invert 2×2 matrix?

- 1. Swap the diagonal elements.
- 2. Multiply the off-diagonal elements by -1.

$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \begin{bmatrix} d & b \\ c & a \end{bmatrix} \rightarrow \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- 3. Divide by the determinant.

$$\boldsymbol{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\boldsymbol{AA}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$= \frac{1}{ad - bc} \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Proof of inverse matrix

$$\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 7 & -4 \\ -2 & 1 \end{bmatrix} \frac{1}{7-8}$$
$$= \begin{bmatrix} (7-8) & (-4+4) \\ (14-14) & (-8+7) \end{bmatrix} \frac{1}{-1}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Numerical example

Code to Inverse of a 2×2 Matrix

Computing the inverse in MATLAB.

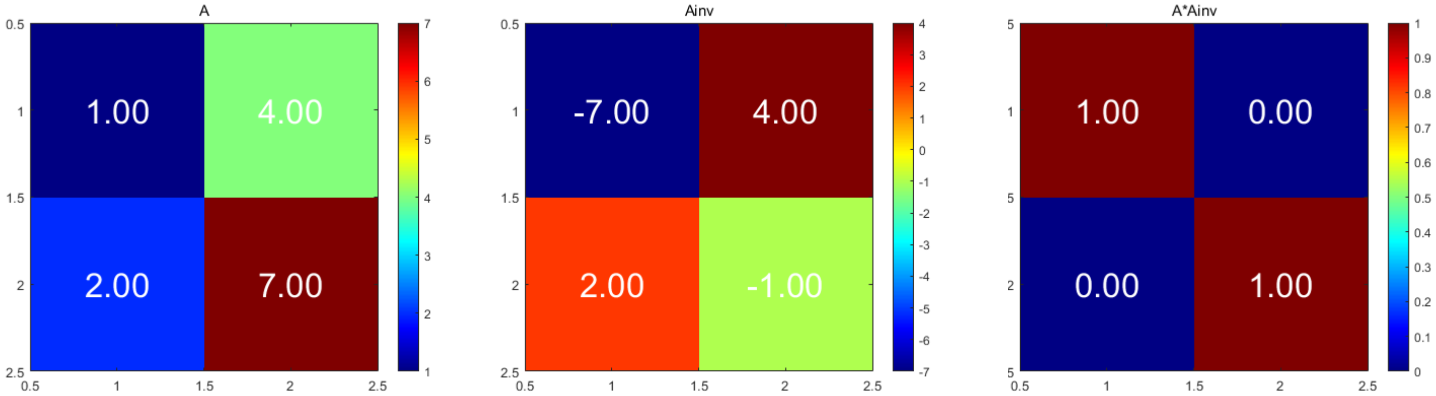
- A * Ainv gives the identity matrix.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Create a 2x2 matrix
A = [[1 4]; [2 7]];

% Check if the matrix is invertible by determining if the
determinant is non-zero
if det(A) == 0
    error('The original matrix is singular and does not have an
inverse.');
end

% Compute the inverse of the matrix
Ainv = inv(A);

% Compute the product of the original matrix and its inverse
(should be the identity matrix)
AAinv = A * Ainv;

% Visualize the original matrix with its values
visualizeMatrixWithValues(A, 'A');

% Visualize the inverse matrix with its values
visualizeMatrixWithValues(Ainv, 'Ainv');

% Visualize the product of the original matrix and its inverse
with its values
visualizeMatrixWithValues(AAinv, 'A*Ainv');

% Function definition : visualize matrix and value
function visualizeMatrixWithValues(matrix, titleStr)
    figure;
    imagesc(matrix);
    colorbar;
    title(titleStr);
    colormap jet;
    axis square;
    % Add text annotations for each element
    for i = 1:size(matrix, 1)
        for j = 1:size(matrix, 2)
            text(j, i, num2str(matrix(i, j), '%.2f'), ...
                'HorizontalAlignment', 'center', ...
                'Color', 'white', 'FontSize', 25);
        end
    end
end
```

Code of inverting 2×2 matrix

Result of Inverse of a 2×2 Matrix

- **Result of computing the inverse matrix in MATLAB.**
  - A * Ainv gives the identity matrix.



Visualization of inverting matrix

Another Example of Inverse of a 2×2 Matrix

Severe problems with this example.

- Matrix multiplication gives us 0 instead of ΔI.
  - Determinant is zero.
  - Cannot divide by zero.

$$\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} 8 & -4 \\ -2 & 1 \end{bmatrix} \frac{1}{0} = \begin{bmatrix} (8-8) & (-4+4) \\ (16-16) & (-8+8) \end{bmatrix} \frac{1}{0} = ???$$

Another example of inverse of 2×2 matrix

What's different about the second example?

- It is a reduced-rank matrix($rank = \square$).
- Shows numerical example that reduced-rank matrices are not invertible.

MATLAB about Second Example

- **MATLAB won’t even try to calculate the result like we did.**
  - It gives an error with following message.
  - Reduced-rank matrices do not have an inverse.
    - Programs like MATLAB won’t even try to calculate it.

```
% Create a 2x2 matrix
A = [[1 4]; [2 8]];

% Check if the matrix is invertible by determining if the determinant is non-zero
if det(A) == 0
    error('The original matrix is singular and does not have an inverse.');
end

% Compute the inverse of the matrix
Ainv = inv(A);
```

MATLAB code of second case

```
다음 사용 중 오류가 발생함: AILAB_LA_figure_inverse_matrix_error (9번 라인)
The original matrix is singular and does not have an inverse.
```

MATLAB code error

Inverse of a Diagonal Matrix

- **Product of two diagonal matrices is scalar multiplied diagonal elements.**
  - Simply invert each diagonal element.
    - Ignoring off-diagonal zeros.
  - Diagonal matrix with at least one zero on the diagonal has no inverse.
    - You'll have $1/0$.
    - Diagonal matrix is full-rank only if all diagonal elements are $\boxed{}$

$$\begin{bmatrix} 1/b & 0 & 0 \\ 0 & 1/c & 0 \\ 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} b & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example of inverting a diagonal matrix

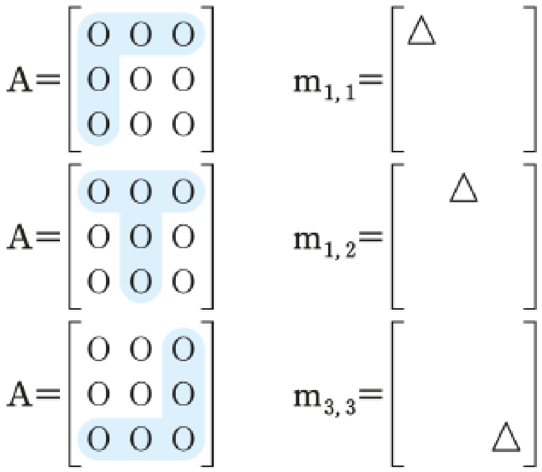
Algorithms to Compute Inverse

- **Involves four intermediate matrices.**
- **Minors matrix**
  - Comprise determinants of submatrices.
  - Element $m_{i,j}$ of the minors matrix
    - Determinant of submatrix
    - Created by excluding $i\ th$ row and $j\ th$ column.

The adjugate of a matrix $\mathbf{A}$ can be used to find the inverse of $\mathbf{A}$ as follows:

If $\mathbf{A}$ is an invertible matrix, then

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\operatorname{adj}(\mathbf{A}).$$



Overview of procedure for 3 × 3 matrix

Other Algorithms to Compute Inverse

Grid matrix

- Checkerboard of alternating $+1s$ and $-1s$.

$$g_{i,j} = -1^{i+j}$$

Formula of grid matrix

Cofactors matrix

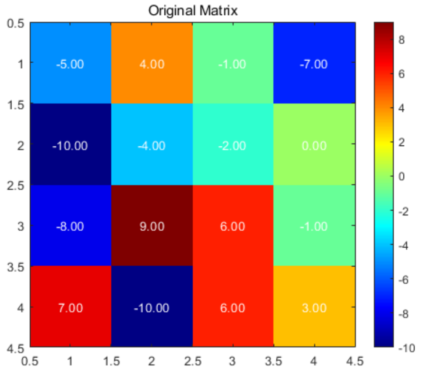
- Hadamard multiplication of the minors matrix with grid matrix.

Adjugate matrix

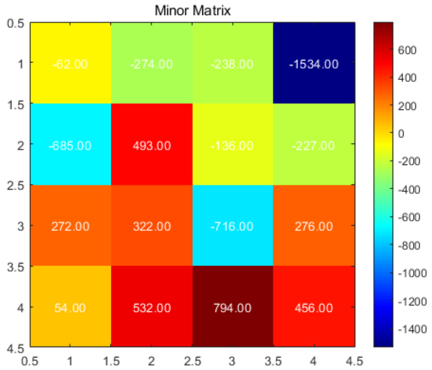
- Transpose of cofactors matrix.
- Scalar multiplied by the inverse of the determinant of the original matrix.
- Adjugate matrix is inverse of the original matrix.

Visualization Result of Computing Various Matrices

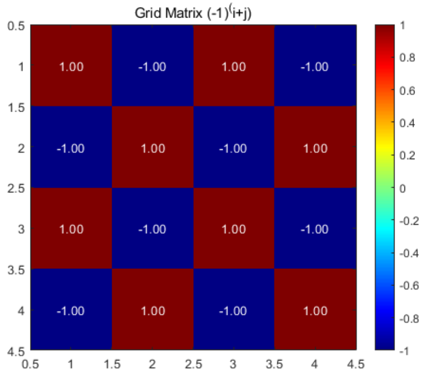
- **Implements of these matrices are Homework!**



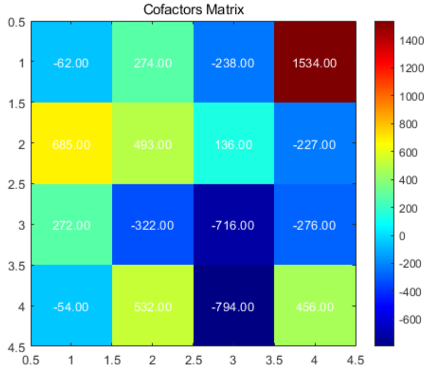
Original matrix



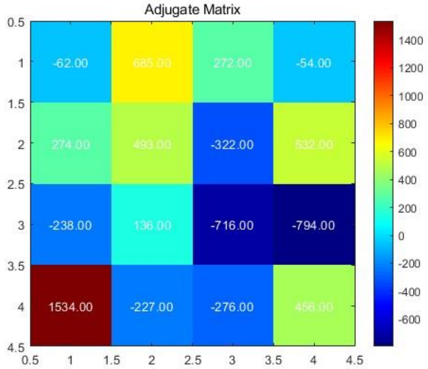
Minors matrix



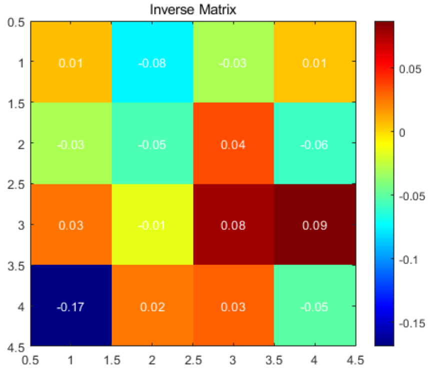
Grid matrix



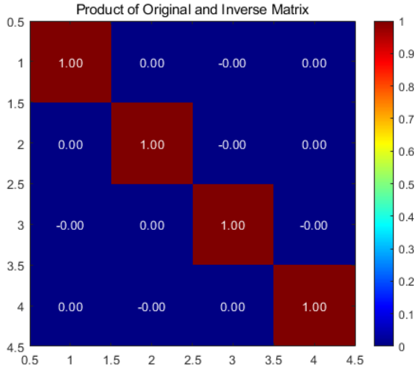
Cofactors matrix



Adjugate matrix



Inverse matrix



Original * Inverse

One-Sided Inverse

- **Tall matrix $\boldsymbol{T}$ of size $M > N$ doesn't have a full inverse.**
  - No tall matrix $\boldsymbol{T}^{-1}$ such that $\boldsymbol{T}\boldsymbol{T}^{-1} = \boldsymbol{T}^{-1}\boldsymbol{T} = \boldsymbol{I}$.
  - There is matrix $\boldsymbol{L}$ such that $\boldsymbol{L}\boldsymbol{T} = \boldsymbol{I}$.

- **Goal is to find matrix L.**
  - Making matrix T square.
    - Multiplying it by its transpose.
    - Question: $\boldsymbol{T}^{\boldsymbol{T}}\boldsymbol{T}$ $(\boldsymbol{N} \times \boldsymbol{N})$ or $\boldsymbol{T}\boldsymbol{T}^{\boldsymbol{T}}$ $(\boldsymbol{M} \times \boldsymbol{M})$?
      - Both are square.
      - But $\boldsymbol{T}^{\boldsymbol{T}}\boldsymbol{T}$ is $\boxed{}$ if $\boldsymbol{T}$ is full column-rank.
      - All square full-rank matrices have an inverse.
  - Look for a matrix that left-multiplies $\boldsymbol{T}$ to produce the identity matrix.
    - $(\boldsymbol{T}^{\boldsymbol{T}}\boldsymbol{T})^{-1}\left(\boldsymbol{T}^{\boldsymbol{T}}\boldsymbol{T}\right) = \boldsymbol{I}$
    - $\boldsymbol{L} = (\boldsymbol{T}^{\boldsymbol{T}}\boldsymbol{T})^{-1}\boldsymbol{T}^{\boldsymbol{T}}$
    - $\boldsymbol{L}\boldsymbol{T} = \boldsymbol{I}$
  - Matrix $\boldsymbol{L}$ is the left-inverse of matrix $\boldsymbol{T}$.

Code Exercise of Calculating Left Inverse

Code Exercise (08_01)

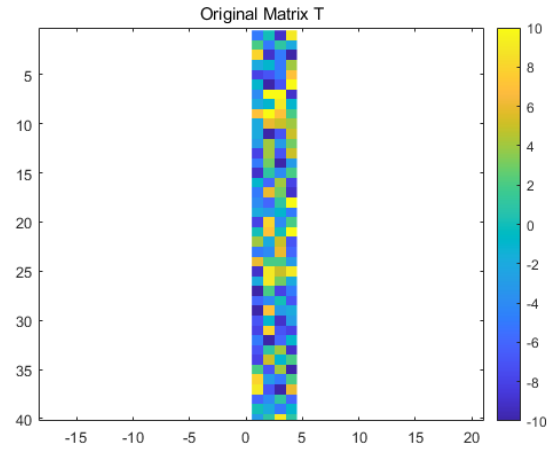
- Get the left inverse matrix.
- Check whether left multiplying the original tall matrix(TL) produces the identity matrix.
- Check the result of post multiplying(LT).

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Generate a random 40x4 matrix with integer elements from -10 to 10
T = randi([-10, 10], 40, 4);

% Compute the product of T transpose and T
TtT = T' * T;

% Compute the inverse of the product
TtT_inv = inv(TtT);

% Compute the Left inverse matrix
L = TtT_inv * T';

% Multiply the inverse with the original product to get the identity matrix
LT = L * T;
TL = T * L;

% Visualize the matrices
visualizeOriginalMatrix(T, 'Original Matrix T');
visualizeLeftInverseMatrix(L, 'L');
visualizeMatrix(LT, 'LT ');
visualizeMatrix(TL, 'TL ');

% Function definition : visualize matrix
function visualizeMatrix(matrix, titleStr)
    figure;
    imagesc(matrix); % Display the matrix as a color image
    colorbar;        % Show a color scale
    title(titleStr);
    axis square;     % Make axes square
end

% Function definition : visualize original matrix
function visualizeOriginalMatrix(matrix, titleStr)
    figure;
    imagesc(matrix); % Display the matrix as a color image
    colorbar;        % Show a color scale
    title(titleStr);
    axis([0.5 40 0 40]); % Make axes square
end

% Function definition : visualize left inverse matrix
function visualizeLeftInverseMatrix(matrix, titleStr)
    figure;
    imagesc(matrix); % Display the matrix as a color image
    colorbar;        % Show a color scale
    title(titleStr);
    axis([0 40 0.5 40]); % Make axes square
end
```

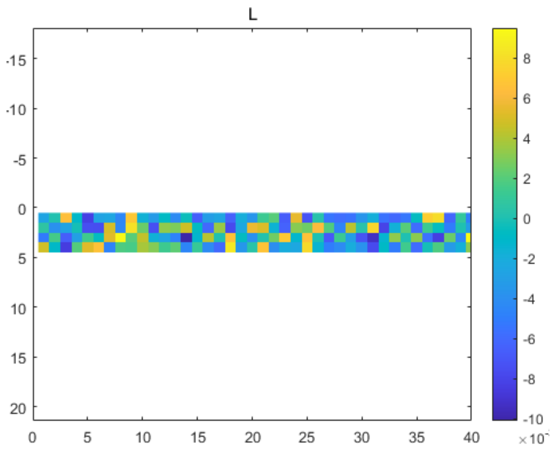
MATLAB code example of calculating left inverse

Code to Calculate Left Inverse

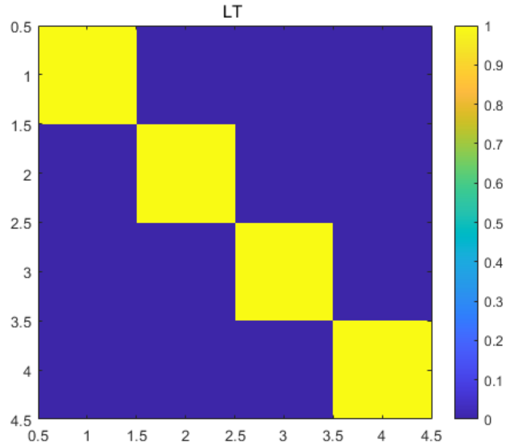
- **Following figure illustrates a tall matrix, its left inverse, and two ways of multiplying left inverse by the matrix.**
  - Left matrix is not a
  - Left multiplying by the left inverse is the identity matrix.
  - Right multiplying by the left inverse is not the identity matrix.



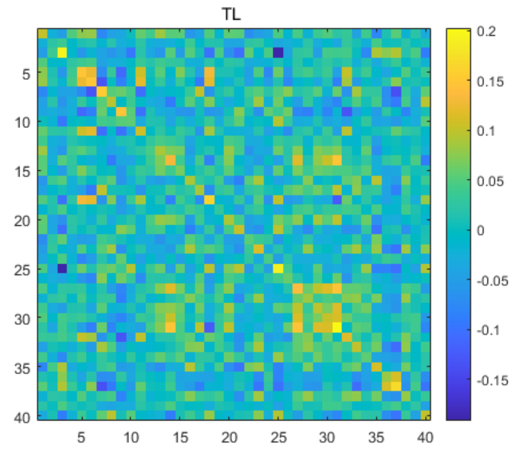
Tall matrix



Left inverse



$L * T$



$T * L$

About Left Inverse of matrix T

- **Left inverse is a one-sided inverse.**
  - $\boldsymbol{LT}$ is identity matrix. But $\boldsymbol{TL}$ is not.

- **Left inverse is defined only for $\boxed{}$ matrices that have full column-rank.**
  - A matrix size $M > N$ with rank $r < N$ doesn't have a left-inverse.
  - $\boldsymbol{T}^{\boldsymbol{T}}\boldsymbol{T}$ is reduced-rank.
  - Thus cannot be inverted.

- **How about right inverse?**
  - Try yourself.

The inverse is unique

Proof of Uniqueness of Inverse

- **Cannot be $AB = I$ and $AC = I$ while $B \boxed{} C$.**
  - Several proofs of this claim.
    - One is proof by negation.
      - Try but fail to prove a false claim.
      - Thereby proving the correct claim.
  - Proof by negation: start with three assumptions.
    - 1. Matrix A is $\boxed{}$
    - 2. Matrices B and C are inverses of A.
    - 3. Matrices B and C are distinct.
      - Meaning $B \neq C$.
    - Follow each expression from left to right.

$$C = CI = CAB = IB = B$$

Proof by negation

    - Assumption of $B \neq C$ is false.
  - Conclusion: invertible matrix has exactly one inverse.

Moore-Penrose pseudoinverse

Pseudoinverse for Singular Matrices

- **Reduced-rank matrices do not have a $\boxed{}$ inverse.**
  - Impossible to transform a reduced-rank matrix into the identity matrix.
    - By matrix multiplication.

- **But singular matrices do have pseudoinverses.**

- **Pseudoinverses?**
  - Transformation matrices that bring a matrix close to the identity matrix.
  - Plural pseudoinverses was not a typo.
    - Reduced-rank matrix has an infinite number of pseudoinverse.

- **Best pseudoinverse: Moore-Penrose pseudoinverse**
  - Sometimes abbreviated as the MP pseudoinverse.

Moore-Penrose Pseudoinverse

- **Best pseudoinverse: Moore-Penrose pseudoinverse**
  - Can always assume that pseudoinverse refers to it.
  - Following matrix is pseudoinverse of the singular matrix.
    - First line: pseudoinverse of the matrix.
    - Second line: product of the matrix and its pseudoinverse.
    - Scaling for 85: to facilitate visual inspection of the matrix.
    - Indicating pseudoinverse using dagger, plus sign, or asterisk.
      - $\boldsymbol{A}^{\dagger}, \boldsymbol{A}^{+}$, or $\boldsymbol{A}^{*}$.

$$\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}^{\dagger} = \frac{1}{85}\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

$$\frac{1}{85}\begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} .2 & .4 \\ .4 & .8 \end{bmatrix}$$

Example of pseudoinverse matrix

Code: Moore-Penrose Pseudoinverse

- **Use function [ ] in MATLAB.**

```
A = [[1 4]; [2 8]];
Apinv = pinv(A);
AApinv = A*Apinv;

disp("Apinv");
disp(Apinv);
disp("AApinv");
disp(AApinv);
```

MATLAB code to pseudoinverse

- **How is the pseudoinverse computed?**
  - Take [ ] of a matrix.
  - Invert nonzero singular values without changing singular vectors.
  - Reconstruct matrix by multiplying $U\Sigma^{+}\boldsymbol{V}^{\boldsymbol{T}}$.

  - If you don't understand it, don't worry.
    - It will be intuitive by the end of Lecture 14.

Numerical stability of the inverse

Complexity of Computing Matrix Inverse

- **Computing matrix inverse involves a lot of**
  - Matrix inverse includes many determinants.
  - Computing many determinants can lead to numerical inaccuracies.
    - Accumulate and cause significant problems when working with large matrices.
  - Low level libraries generally strive to avoid explicitly inverting matrices.
    - Or decompose matrices into the product of other matrices that are more numerically stable.

- **Matrices that have numerical values in roughly the same range tend to be more stable.**
  - Why random-numbers matrices are easy to work with.
  - Matrices with a large range of numerical values.
    - Have a high risk of numerical instability.
  - Range of numerical values?
    - Formally captured as the condition number of a matrix.
    - Condition number
      - Ratio of the largest to smallest singular value.
      - A measure of the spread of numerical values in a matrix.

Hilbert Matrix

Numerically unstable matrix

- Hilbert matrix is defined by the following formula.
  - i and j are row and column indices.

$$h_{i,j} = \frac{1}{i+j-1}$$

Formula to create a Hilbert matrix

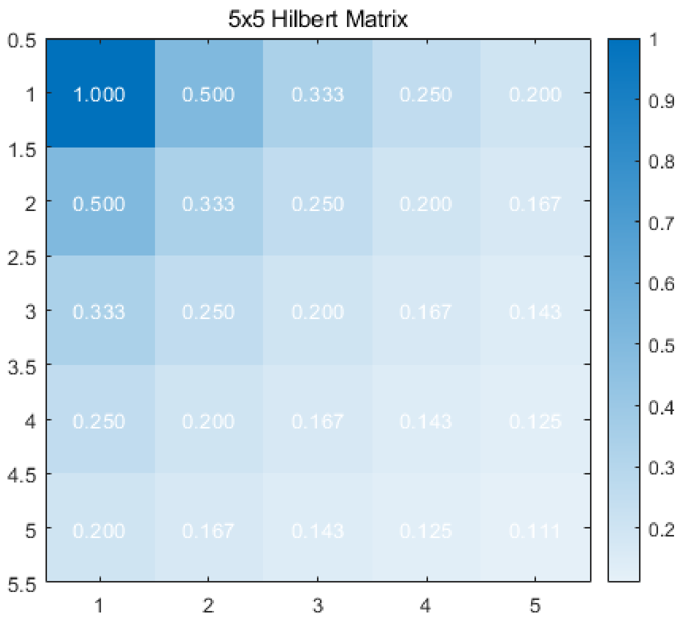
$$\begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$$

Example of a 3×3 Hilbert matrix

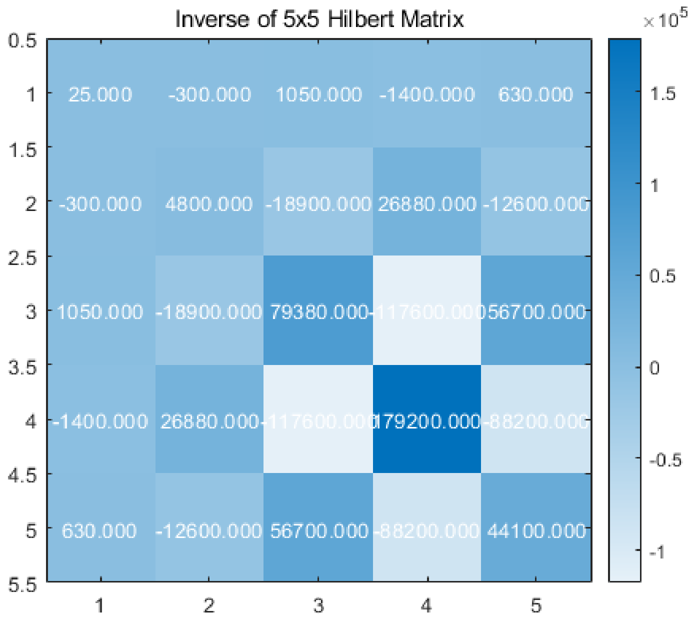
- As the matrix gets larger, ranges of numerical values $\boxed{}$.
  - Computer calculated Hilbert matrix quickly becomes rank-deficient.

Inverse of Hilbert Matrix

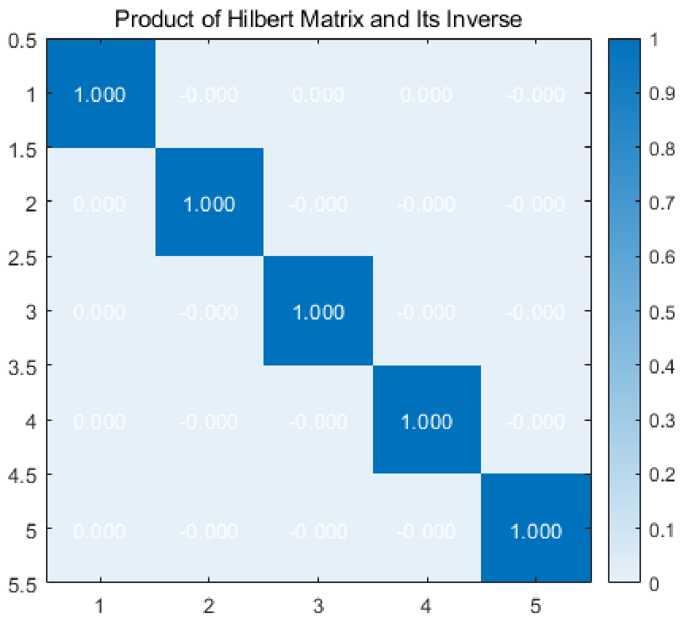
- **Full-rank Hilbert matrices have inverse in a very different numerical range.**
  - Illustrated them below.
  - Product matrix certainly looks like the identity matrix.
    - In reality, rounding error increases rapidly.
      - With the size of the matrix.



Hilbert matrix



Inv(Hilbert)



Their product

Geometric interpretation of the inverse

Geometric Transformation of Inverse

Matrix inverse: undoing geometric transformation imposed by multiplication.

- This geometric effect is unsurprising.
  - Following equations
    - P is $2 \times N$ matrix of original geometric coordinates.
    - T is transformation matrix.
    - Q is matrix of transformed coordinates.
    - U is matrix of back-transformed coordinates.
  - Interpretation of this equation: undoing the transform imposed by the matrix.

$$\boldsymbol{Q} = \boldsymbol{TP}$$
$$\boldsymbol{U} = \boldsymbol{T}^{-1}\boldsymbol{Q}$$
$$\boldsymbol{U} = \boldsymbol{T}^{-1}\boldsymbol{TP}$$

Math of geometric effect

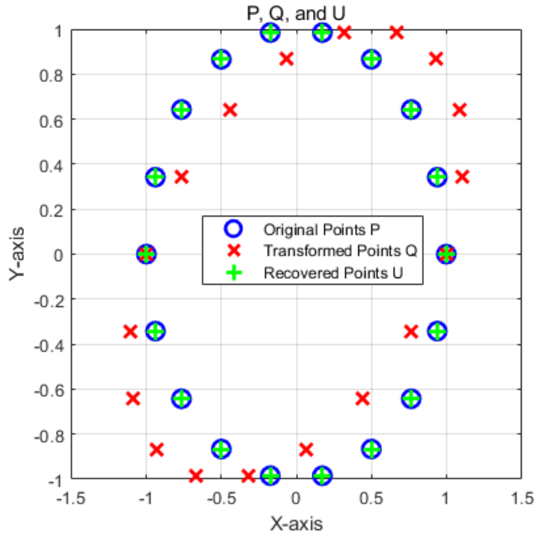


Figure of geometric effect

Code Exercise of Geometric Transformation of Inverse

Code Exercise (08_02)

- Generate the points on a circle.
- Define the Transformation matrix T and inverse matrix T^{-1}.
- Check the result of TP and $T^{-1}TP$.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Define angles 0 to 340 degrees in 20 degree increments
angles = 0:20:340;

% Convert degrees to radians for computation
radians = deg2rad(angles);

% Create points on a circle with radius 1
P = [cos(radians); sin(radians)];

% Define a transformation matrix T
T = [1 0.5; 0 1];

% Apply the transformation to create Q
Q = T * P;

% Calculate U, which should be the same as P
U = inv(T) * Q;

% Visualize the points P, Q, and U on the same plot for comparison
figure;
plot(P(1, :), P(2, :), 'bo', 'LineWidth', 2, 'MarkerSize', 10, 'DisplayName',
'Original Points P');
hold on; % Hold on to plot additional points
plot(Q(1, :), Q(2, :), 'rx', 'LineWidth', 2, 'MarkerSize', 10, 'DisplayName',
'Transformed Points Q');
plot(U(1, :), U(2, :), 'g+', 'LineWidth', 2, 'MarkerSize', 10, 'DisplayName',
'Recovered Points U');
legend show;
axis square;
grid on;
title('P, Q, and U');
xlabel('X-axis');
ylabel('Y-axis');
hold off;
```

MATLAB code example of geometric transformation of inverse

Application of Geometric Transformation of Inverse

- **Come in handy when you learn about diagonalizing a matrix through eigen-decomposition.**

- **Provide intuition for why has no inverse.**
  - Geometric effect of transforming by singular matrix
    - At least one dimension is flattened.
    - Once a dimension flattened, it cannot be unflattened.
      - Like you cannot see your back when facing a mirror.

Summary

Summary

- **Matrix inverse**
  - A matrix that transforms a maximum-rank matrix into the identity matrix.
    - Through matrix multiplication.
  - Has many purposes.
    - Including moving matrices around in an equation(e.g., solve for $\boldsymbol{x}$ in $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$).
- **Square full-rank matrix**
  - Has a full inverse.
- **Tall full column-rank matrix**
  - Has a left-inverse.
- **Wide full row-rank matrix**
  - Has a right-inverse.
- **Reduced-rank matrices cannot be linearly transformed into the identity matrix.**
  - But they do have a pseudoinverse.
  - Pseudoinverse transforms matrix into another matrix that is closer to the identity matrix.

Summary

- **Inverse is unique.**
  - If a matrix can be linearly transformed into the identity matrix, there is only one way to do it.
- **Some tricks for computing the inverses of some kinds of matrices.**
  - Including 2×2 and diagonal matrices.
  - These shortcuts are simplifications of the full formula for computing a matrix inverse.
- **Due to the risk of numerical precision errors, production-level algorithms try to avoid explicitly inverting matrices or will decompose a matrix into other matrices that can be inverted with greater numerical stability.**

Code exercises

Hand Made Inverse Algorithm

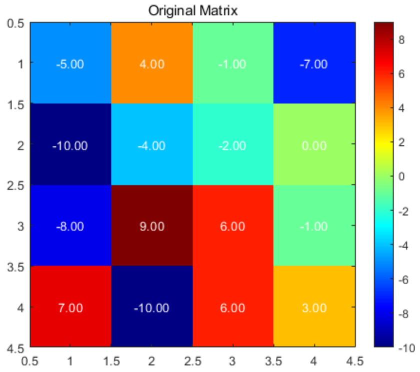
- Implement the full-inverse algorithm for a 2×2 matrix using matrix elements $a, b, c,$ and d. (not using inverse function)
- Hint: Remember that the determinant of a scalar is its absolute value.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Assume that input value is [a,b;c,d] (2x2 matrix)
% then make a function that return a inverse matrix
% NOTE: Do not use inverse function in MATLAB

% input1
a1 = 2;
b1 = 1;
c1 = 6;
d1 = 8;

% input2
a2 = 4;
b2 = 10;
c2 = 2;
d2 = 5;

% Write here (start)
% Create error when inverse matrix does not exist
function inverse_matrix = handMadeInverseMatrix(a, b, c, d)
    % Compute the determinant

    % Check error
    if
       error('Matrix is not invertible');
    end
    % Compute the inverse manually

end
% Write here (end)

% Call the function and store the inverse matrix
% Display the result
inv1 = handMadeInverseMatrix(a1,b1,c1,d1);
disp('Inverse matrix1:');
disp(inv1);
inv2 = handMadeInverseMatrix(a2,b2,c2,d2);
```

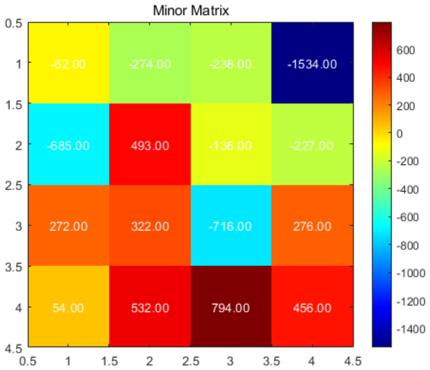
Sample code

Implements of Various Matrices

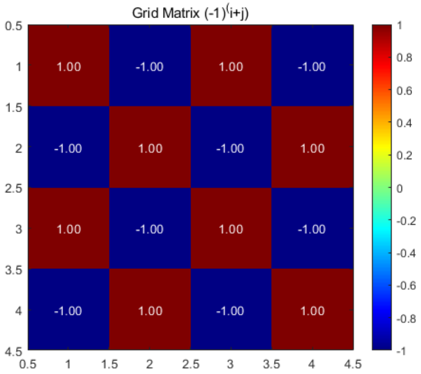
■ Implements of these matrices



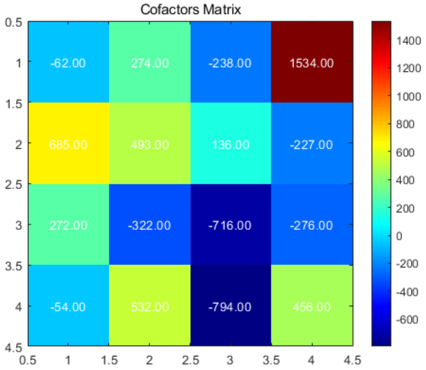
Original matrix



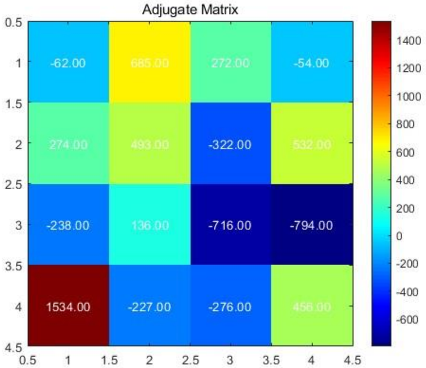
Minors matrix



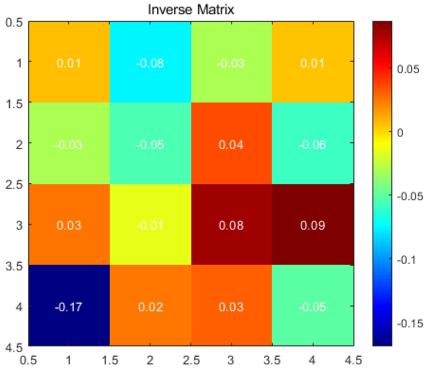
Grid matrix



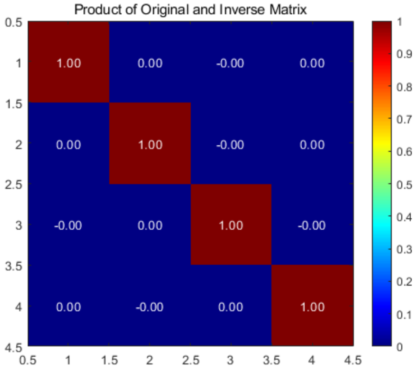
Cofactors matrix



Adjugate matrix



Inverse matrix



Original * Inverse

Implements of Various Matrices

- You can use any MATLAB of custom function.
- Just create these matrices using any method.

```
% Clear workspace, command window, and close all figures
clc; clear; close all;

% Create a 4x4 original matrix with integer elements
originalMatrix = [-5,4,-1,-7;
                 -10,-4,-2,0;
                 -8,9,6,-1;
                 7,-10,6,3];

% Create a 4x4 grid matrix where elements are (-1)^(i+j)
[i, j] = meshgrid(1:4, 1:4);
gridMatrix = (-1).^(i+j);

% Compute the minor matrix
minorMatrix = zeros(4);
for row = 1:4
   for col = 1:4
   subMatrix = originalMatrix;
   subMatrix(row, :) = [];
   subMatrix(:, col) = [];
   minorMatrix(row, col) = det(subMatrix);
   end
end

%%%%%%% TODO %%%%%%%
% Refer to the lecture slide 43
% Compute the cofactors matrix
% Use '.*' function to compute the cofactorsMatrix
cofactorsMatrix = ;

% Check if the matrix is invertible
determinant = det(originalMatrix);
if determinant == 0
   error('The original matrix is singular and does not have an inverse.');
end

% Compute the adjugate matrix and inverse matrix of the original matrix
% Use 'transpose(matrix)' to compute the adjugate matrix
% Use adjugateMatrix to compute inverse matrix
adjugateMatrix = ;
inverseMatrix = ;

% Compute the product of the original matrix and its inverse
identityMatrix = ;
%%%%%%%%%%%%%%%%%%%%

% Visualize all matrices
visualizeMatrix(originalMatrix, 'Original Matrix');
visualizeMatrix(minorMatrix, 'Minors Matrix');
visualizeMatrix(gridMatrix, 'Grid Matrix (-1)^(i+j)');
visualizeMatrix(cofactorsMatrix, 'Cofactors Matrix');
visualizeMatrix(adjugateMatrix, 'Adjugate Matrix');
visualizeMatrix(inverseMatrix, 'Inverse Matrix');
visualizeMatrix(identityMatrix, 'Product of Original and Inverse Matrix');

% Function definition : visualize matrix
function visualizeMatrix(matrix, titleStr)
   figure;
   imagesc(matrix);
   colorbar;
   title(titleStr);
   colormap jet;
   axis square;
   % Add text annotations for each element
   for i = 1:size(matrix, 1)
      for j = 1:size(matrix, 2)
      text(j, i, num2str(matrix(i, j), '%.2f'), ...
      'HorizontalAlignment', 'center', ...
      'Color', 'white', 'FontSize', 10);
      end
   end
end
```

Sample code

THANK YOU
FOR YOUR ATTENTION