Linear Algebra

Vector Part 2:
Vector Extension Concept

Automotive Intelligence Lab.

Contents

■ **Vector set**

■ **Linear weighted combination**

■ **Linear independence**

■ **Subspace and span**

■ **Basis**

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Vector set

Vector Set

■ **Definition of vector set**

▶ A collection of vectors.

■ **Notation of vector set**

▶ Vector set is indicated as S or V, represented in capital italics letters.

▶ Mathematical representation of a vector set: $V = \{v_1, \dots, v_n\}$.

■ **Characteristics of vector set**

▶ Vector set can contain a finite or an infinite number of vectors.

▶ Vector set can also be empty which is indicated as $V = $ | $\{\}$ |

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Linear weighted combination

Linear Weighted Combination

■ **What is linear weighted combination?**

▶ A way of mixing information from multiple variables, with some variables contributing more than others.

▶ It is also sometimes called **linear mixture** or **weighted combination** when linear part is assumed.

▶ Weight can also be expressed as **coefficient**.

▶ Linear weighted combination simply means **scalar-vector multiplication** and **addition** as Eq 1.
  ● It is assumed that all vectors v_i have the same | dimensionality |; otherwise, the addition is invalid.
  ● The λ_i can be any real number, including zero.
  ● Subtraction can be handled by setting a λ_i to be negative.

$$\mathbf{w} = \lambda_1 \mathbf{v_1} + \lambda_2 \mathbf{v_2} + ... + \lambda_n \mathbf{v_n}$$

Eq 1. Standard form of linear weighted combination

Subtraction of Linear Weighted Combination

■ **In Eq 2., λ_3 shows an example of subtraction.**

$$\lambda_1 = 1 \,, \lambda_2 = 2 \,, \lambda_3 = -3 \,, \ \mathbf{v_1} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} , \mathbf{v_2} = \begin{bmatrix} -4 \\ 0 \\ -4 \end{bmatrix} , \mathbf{v_3} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\mathbf{w} = \lambda_1 \mathbf{v_1} + \lambda_2 \mathbf{v_2} + \lambda_3 \mathbf{v_3} = \begin{bmatrix} -7 \\ -4 \\ -13 \end{bmatrix}$$

Eq 2. Example of linear weighted combination

Code Exercise of Linear Weighted Combination using Matlab

Code Exercise (03_01)

▶ Linear weighted combination.

```matlab
% Define vectors
v1 = [1; 2]; % example vector 1
v2 = [-2; 1]; % example vector 2


% Define weights
alpha = 0.4;
beta = 0.2;


% Compute the linear weighted combination
resultant_vector = alpha * v1 + beta * v2;


% Visualize vectors
figure;
hold on;
% Plot vectors
quiver(0, 0, v1(1), v1(2), 'Color', 'b', 'LineWidth', 2, 'MaxHeadSize', 1,
'AutoScale', 'off');
quiver(0, 0, v2(1), v2(2), 'Color', 'r', 'LineWidth', 2, 'MaxHeadSize', 1,
'AutoScale', 'off');
quiver(0, 0, resultant_vector(1), resultant_vector(2), 'Color', 'g',
'LineWidth',2, 'MaxHeadSize', 1, 'AutoScale', 'off');
```

```matlab
% Set axis equal, xlim, and ylim
axis equal;
xlim([-3, 3]);
ylim([-1, 3]);


% Add labels
xlabel('X-axis');
ylabel('Y-axis');


% Add legend
legend('Vector v1', 'Vector v2', 'Resultant Vector');


% Set title
title('Linear Weighted Combination of Vectors');


% Show grid
grid on;
hold off;
```
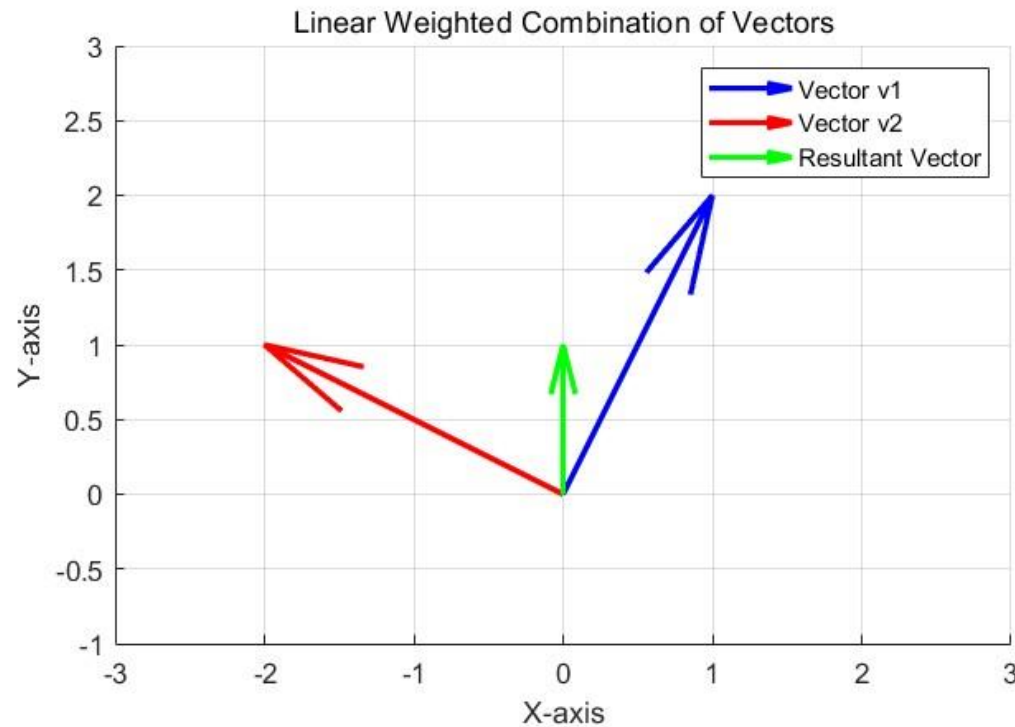
Source code

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Visualization Result of Linear Weighted Combination using Matlab

■ Code Exercise (03_01)

▶ Linear weighted combination.

Source code result

Applications of Linear Weighted Combination

■ **The predicted data from a statistical model**

▶ The linear weighted combination of regressors (predictor variables) and coefficients (scalars) is used to generate predictions.

▶ These regressors and coefficients are computed via the least squares algorithm.

■ **Dimension-reduction procedures**

▶ The linear weighted combination of the data channels and weights.

▶ The weights selected to maximize the variance of the component.

● Such as PCA (Principal Components Analysis).

■ **Artificial neural networks**

▶ Two operations: Linear weighted combination of input data and nonlinear transformation.

▶ The weights are learned by minimizing a cost function.

● Cost function: difference between the model prediction and the real-world target variable.
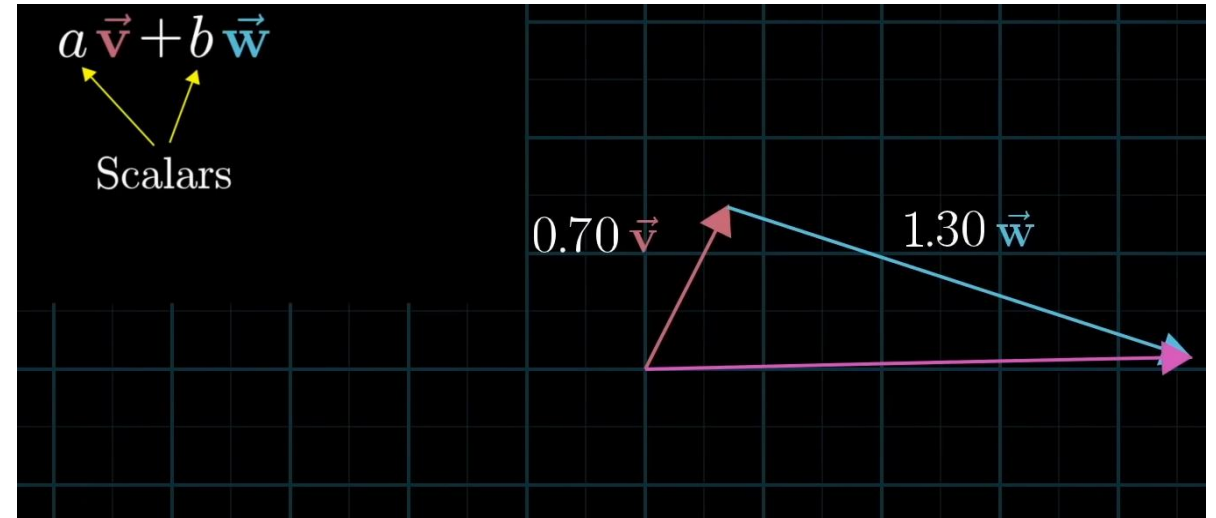
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Visual Materials

- **Geometric representation of linear weighted combination.**
  - ▶ Linear combination (3:00 - 3:46)
  - ▶ https://youtu.be/k7RM-ot2NWY?si=Cc5H_I_8cmfEoDsK&t=180 (English)
  - ▶ https://youtu.be/2CcCOgDilO8?feature=shared&t=180 (Korean)
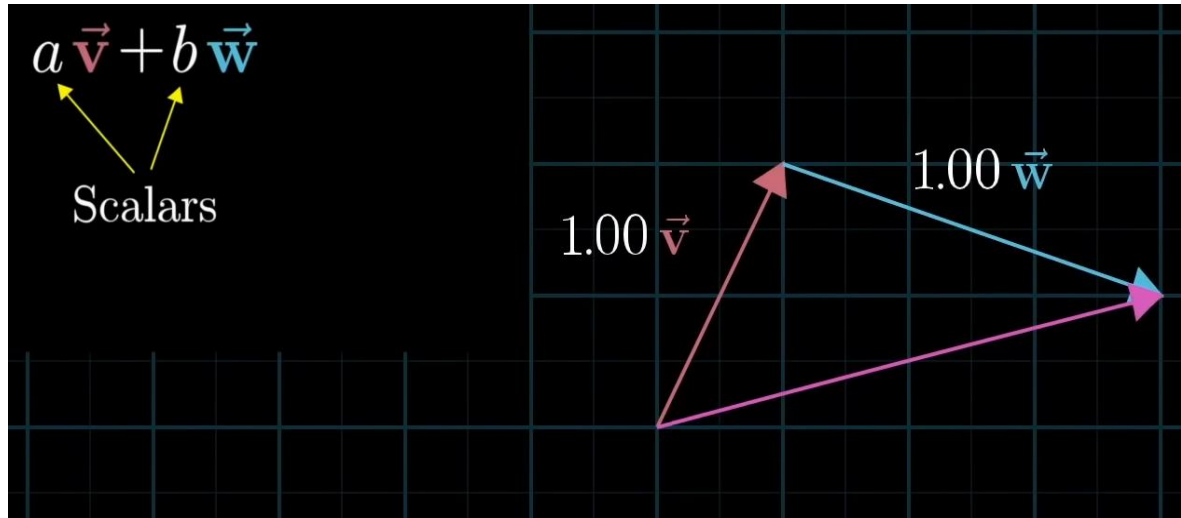
Visualization of linear combination v and w (when scala a is 1.0, and scala b is 1.0)

Visualization of linear combination v and w (when scala a is 0.7, and scala b is 1.3)

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Linear independence

Definition of Linear Independence

■ Linearly **dependent**

▶ At least one vector in vector set can be expressed as a linear weighted combination of other vectors in that set.

● Infinite number of such combinations, two of which are $s_1 = 0.5s_2$ and ⬜

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \end{bmatrix} \right\}$$

Vector set S

■ Linearly **independent**

▶ No vector in vector set can be expressed as a linear weighted combination of other vectors in that set.

● No possible scalar λ for which $v_1 = \lambda v_2$.

$$V = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix} \right\}$$

Vector set V

Linear Independence in Complex Vector

■ **How about below vector set T?**

▶ The sum of the first three vectors equals twice the fourth vector → **Linearly dependent**

■ **Determine whether linearly independent**

▶ It is hard to figure out just from visual inspection.

● In Ch.5, we will learn **matrix rank** for determine independence of vector set.

▪ Create a matrix from the vector set.

▪ Compute the rank of the matrix rank.

▪ Compare the rank to the smaller of the number of rows or columns.

$$T = \left\{ \begin{bmatrix} 8 \\ -4 \\ 14 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 14 \\ 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 13 \\ 2 \\ 9 \\ 8 \end{bmatrix} \right\}$$

$$2 * t_4 = t_1 + t_2 + t_3$$
$$t_4 = \frac{1}{2}(t_1 + t_2 + t_3)$$

Example of linearly dependent

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Property of Linear Independence

Independent sets

▶ Independence is a property of a **set** of vectors, not individual vector within a set.

● A set of vectors can be ⬚ or ⬚

▶ In case of below vector set V, is it linearly independent?

▶ If then, each of v_1, v_2, v_3, v_4 is linearly independent?

$$V = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 12 \\ 7 \end{bmatrix} \right\}$$

$when\ v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \end{bmatrix}, v_4 = \begin{bmatrix} 4 \\ 8 \\ 12 \\ 7 \end{bmatrix}$

Vector set **V**

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The Math of Linear Independence

■ **Formal mathematical definition of Linear dependence**

▶ Define some linear weighted combination of the vectors in the set to produce the zeros vector.

- If there are some λs that make the Eq. 1. true, set of vectors is linearly ☐
- Conversely, If no possible way to linearly combine the vectors to produce zeros vector, set of vectors is linearly ☐

■ **Why do we care about the zeros vector regarding the question of Linear dependence?**

▶ how about expressing equation like Eq. 2.?

$$0 = \lambda_1 \boldsymbol{v}_1 + \lambda_2 \boldsymbol{v}_2 + \cdots + \lambda_n \boldsymbol{v}_n, \qquad \lambda \in \mathbb{R}$$

Eq 1. The mathematical definition of linear dependence

$$\lambda_1 \boldsymbol{v}_1 = \lambda_2 \boldsymbol{v}_2 + \cdots + \lambda_n \boldsymbol{v}_n, \qquad \lambda \in \mathbb{R}$$

Eq 2. Another definition of linear dependence

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Express Zero Vector for Linear Independence

■ **Equation with zeros vector on the left-hand side**

▶ Setting the equation to zero helps reinforce the principle that the entire set is dependent or independent.

▶ No individual vector has the privileged position of being "dependent vector".
- When independence, vector sets are purely egalitarian (= 평등주의).

Definition and Constraint of Trivial Solution

■ Trivial solution

▶ Set all λ's to zero, and the equation reads $0 = 0$, regardless of the vectors in the set.

■ Mathematical definition of linear dependence with constraint

▶ Trivial solutions involving zeros are often ignored in linear algebra,
so the constraint that at least one $\lambda \neq 0$ is added.

▶ This constraint can be incorporated into the equation by dividing through by one of the scalars.

● $\boldsymbol{v}_1$ and λ_1 can refer to any vector/scalar pair in the set.

$$0 = \boldsymbol{v}_1 + \cdots + \frac{\lambda_n}{\lambda_1}\boldsymbol{v}_n, \qquad \lambda \in \mathbb{R}, \lambda_1 \neq 0$$

The mathematical definition of linear dependence with constraint

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Independence and the Zeros Vector

■ **Linear independence and the zeros vector**

▶ Any vector set that includes the zeros vector is automatically a linearly dependent set.

● Because any scalar multiple of the zeros vector is still the ⬚

■ **Linear dependence**

▶ As long as $\lambda_0 \neq 0$, it has a **nontrivial solution**, and the set fits with the definition of linear dependence.

■ **Nontrivial solution**

▶ Solution to a homogeneous equation that is not the zero solution.

● Any solution in which at least one variable has a nonzero value.

$$\lambda_0 0 = 0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n$$

Equation of Linear dependence with zero vector

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Question about Nonlinear Independence

About nonlinear independence

▶ Linear algebra is all about, well, linear operations.

▶ If you can express one vector as a nonlinear combination of other vectors, then those vectors still form a linearly independent set.

Reason for the linearity constraint

▶ For express transformations as matrix multiplication.

Nonlinear systems can be well approximated using linear functions!

Subspace and span

Concept of Subspace and Span

■ **Subspace**

▶ The space formed by infinitely linear combination of vectors within a vector set, where each vector is multiplied by different weights.

■ **Span**

▶ The **mechanism** of combining all possible linear weighted combination.
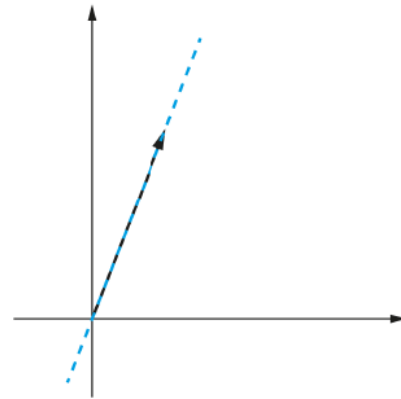
■ **Difference between subspace and span**

▶ Think of span as **verb** and subspace as **noun**.

● A set of vectors spans, and the result of their spanning is a subspace.

▶ If using span as a noun, span and subspace refer to the same infinite vector set.

Subspace Spanned by Vector Set

■ **Example of subspace of a set of linearly independent vector set refer to below figures**

▶ The ⬜ color represents the subspace generated by the ⬜ colored vector.

▶ V_1 has one vector and its span is 1D subspace, V_2 has two vectors and their span is 2D subspace, is there a pattern?

● Dimensionality of the spanned subspace and the number of vectors in the set?

$$V_1 = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\} \qquad\qquad V_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

Subspace of V_1 Subspace of V_2

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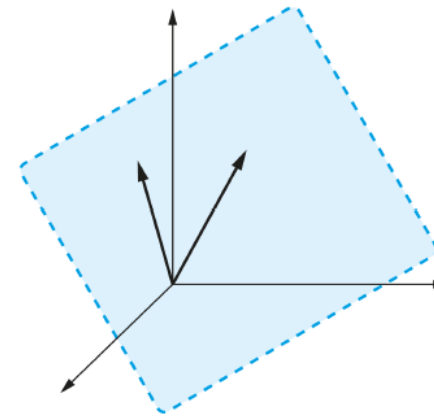
Relation Between Dimensionality of Subspace and Number of Vectors

■ **Example of subspace of a set of linearly dependent vector set refer to below figure**

    ▶ Two vectors in $\mathbb{R}^3$, but subspace of V_3 is still only a ⬚ subspace.

        ● One vector in the set is already in the space of the other vector.

■ **Relation between dimensionality of spanned subspace and the number of vectors in vector set**

    ▶ Related to **linear independence.**

$$V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$

Subspace of V_3

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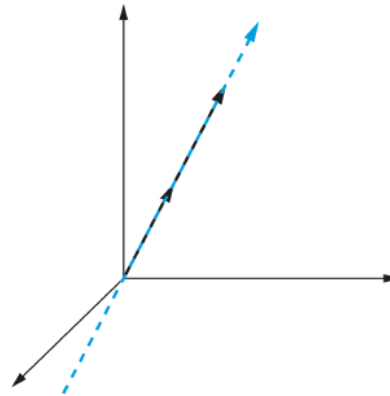
Dimension of Subspace and Linear Independence
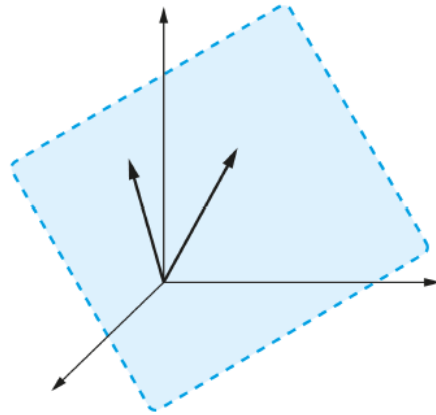
■ **If the vectors in a vector set are linearly independent,**

▶ the dimension of the subspace [   ] the number of vectors in the set like V_2.

■ **If the vectors in a vector set are linearly dependent,**
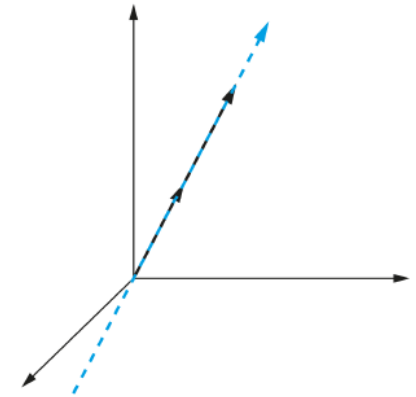
▶ the dimension of the subspace is [   ] the number of vectors in the set like V_3.

$$V_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

Subspace of V_2

$$V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$$
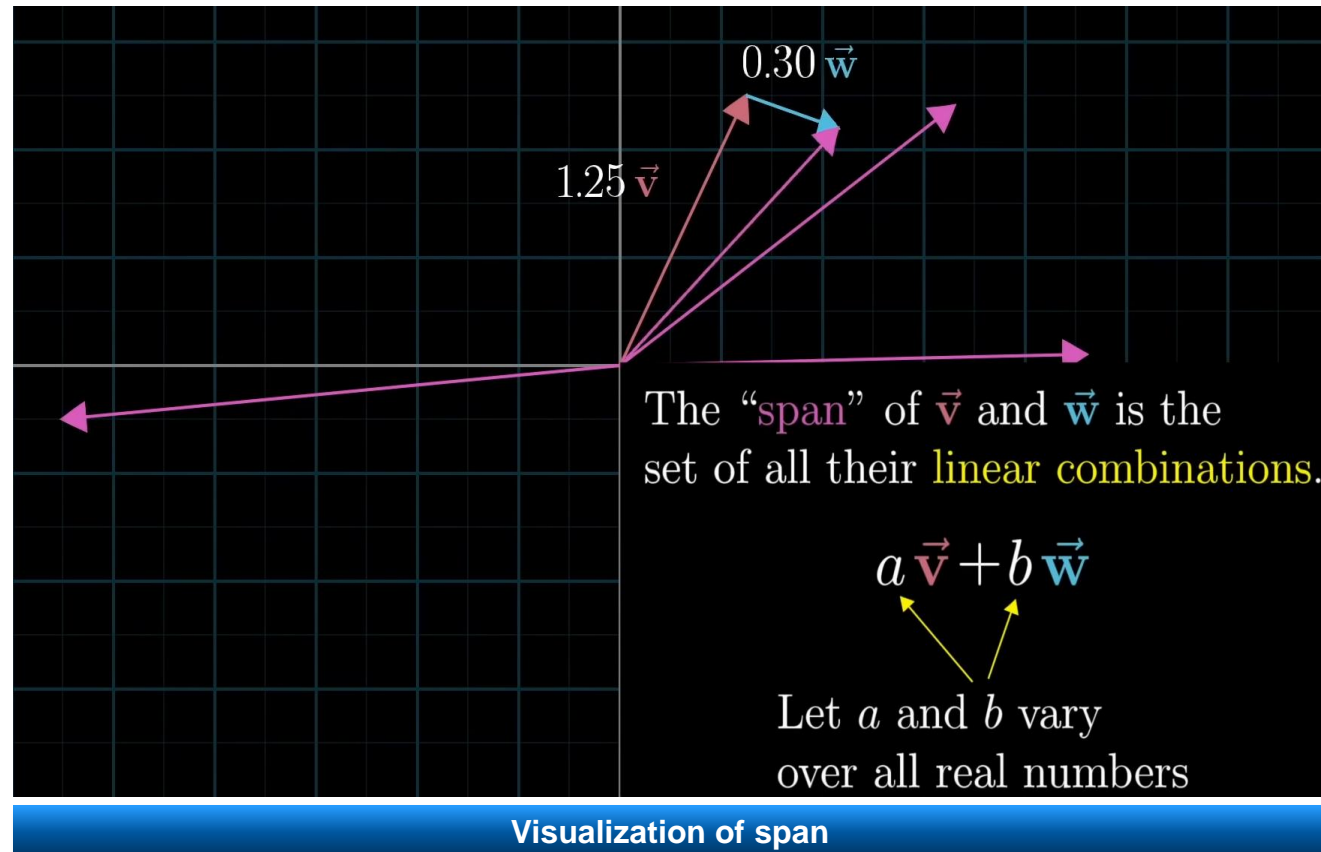
Subspace of V_3

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Visual Materials of Span

■ **Geometric representation of span**

▶ Span (3:47 ~ 7:54)

▶ https://youtu.be/k7RM-ot2NWY?si=Zm_LRnsDlnohsn1y&t=227 (English)

▶ https://youtu.be/2CcCOgDilO8?si=CIE6vVzhC5qCGMLI&t=227 (Korean)

Visualization of span

Visual Materials of Linear Dependence

■ **Geometric representation of linear dependence and linear independence**

► Linear dependence (7:55 ~)

► https://youtu.be/k7RM-ot2NWY?si=NX_fK1ppEP5aGmPU&t=475 (English)

► https://youtu.be/2CcCOgDilO8?si=AqHObsQ2v2zLbfrz&t=475 (Korean)

Visualization of linearly dependent

Visualization of linearly independent

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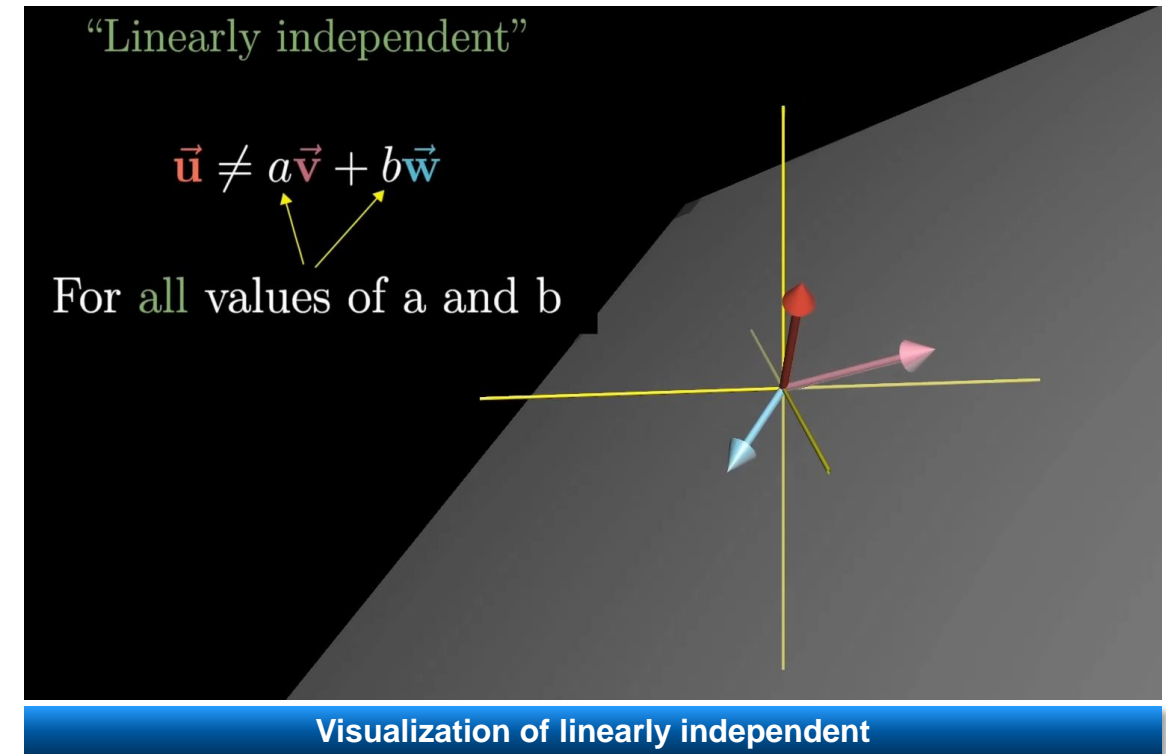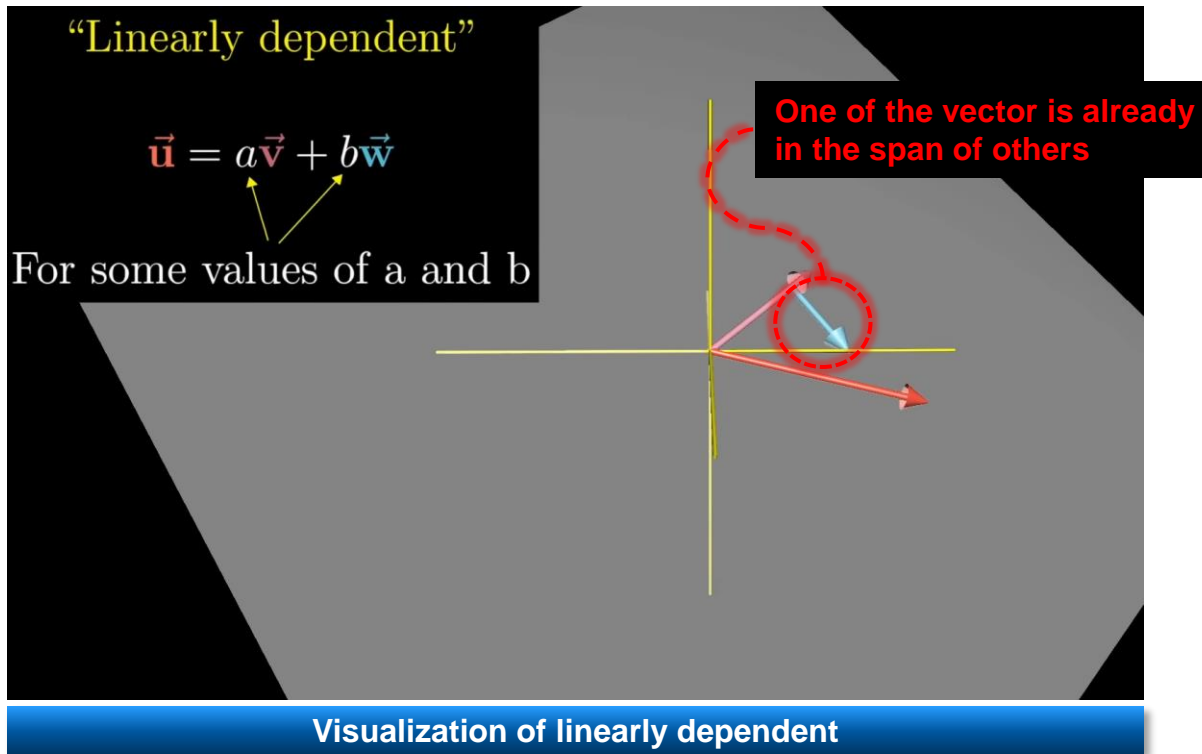
Code Exercise of Vector Span using Matlab

■ **Code Exercise (03_02)**

▶ Vector Span.

```matlab
% Define two 3D vectors
v1 = [1; -3; -2];
v2 = [2; 4; -1];

% Calculate the normal vector (cross product of the two vectors)
normal = cross(v1, v2);

% Choose a point on the plane (for example, using v1)
point = v1;

% Equation of the plane: ax + by + cz + d = 0
% where a, b, c are components of the normal vector, and d is the constant term
of the plane equation
a = normal(1);
b = normal(2);
c = normal(3);
d = -dot(normal, point);

% Code for visualization
[x, y] = meshgrid(-10:1:10, -10:1:10); % Create a grid to represent the plane
z = (-d - a*x - b*y) / c; % Calculate z values of the plane

% Draw the plane
figure;
mesh(x, y, z);
hold on;
```

```matlab
% Draw the two vectors
quiver3(0, 0, 0, v1(1), v1(2), v1(3), 'r', 'LineWidth', 2, 'AutoScale', 'off',
'MaxHeadSize', 2);
quiver3(0, 0, 0, v2(1), v2(2), v2(3), 'b', 'LineWidth', 2, 'AutoScale', 'off',
'MaxHeadSize', 2);

xlabel('X'); ylabel('Y'); zlabel('Z');
title('Plane Spanned by Two Vectors');
legend('Plane', 'Vector 1', 'Vector 2');
axis equal;
grid on;
```
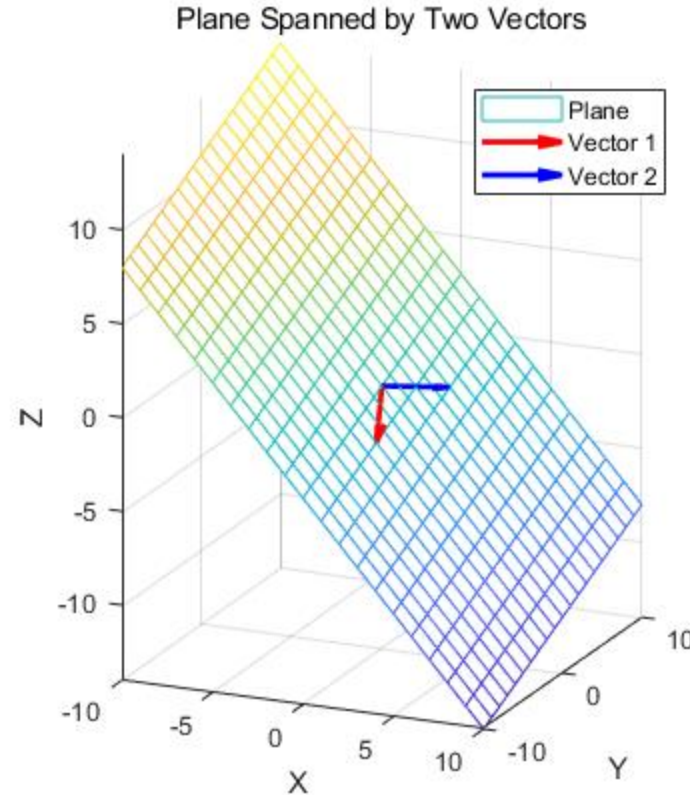
Source code

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Visualization Result of Vector Span using Matlab

■ **Code Exercise (03_02)**

▶ Vector Span.

Source code result

Basis

Concept of Basis

■ **In linear algebra, a basis is a set of** ⬚ **that describes the information in the matrix.**

■ **Most common basis set is the Cartesian axis.**

▶ The Cartesian basis set comprises vectors that are mutually orthogonal and unit length.

● It's why the Cartesian basis sets are so ubiquitous.

● They are called the standard basis set.

$$S_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \qquad S_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Basis sets for 2D and 3D Cartesian graphs

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Determining Basis

■ **Cartesian axis is not the only basis sets**

▶ Basis set **S**, **T** both span the same subspace.

● Subspace : all of R^2

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad \middle| \quad T = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$$

$$s_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, s_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \middle| \quad t_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, t_2 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

Basis set of S and T

▶ Let's represent point p and q in terms of the basis S and T.

● In basis **S**, point p is (3,1), point q is (-6,2).
- Because $p = 3s_1 + s_2$, $q = -6s_1 + 2s_2$

● In basis **T**, point p is (1,0), point q is (0,2).
- Because $p = 1t_1 + 0t_2$, $q = 0t_1 + 2t_2$

▶ Data points p and q are the same regardless of the basis set, but **T** provide a **compact** and **orthogonal** description.
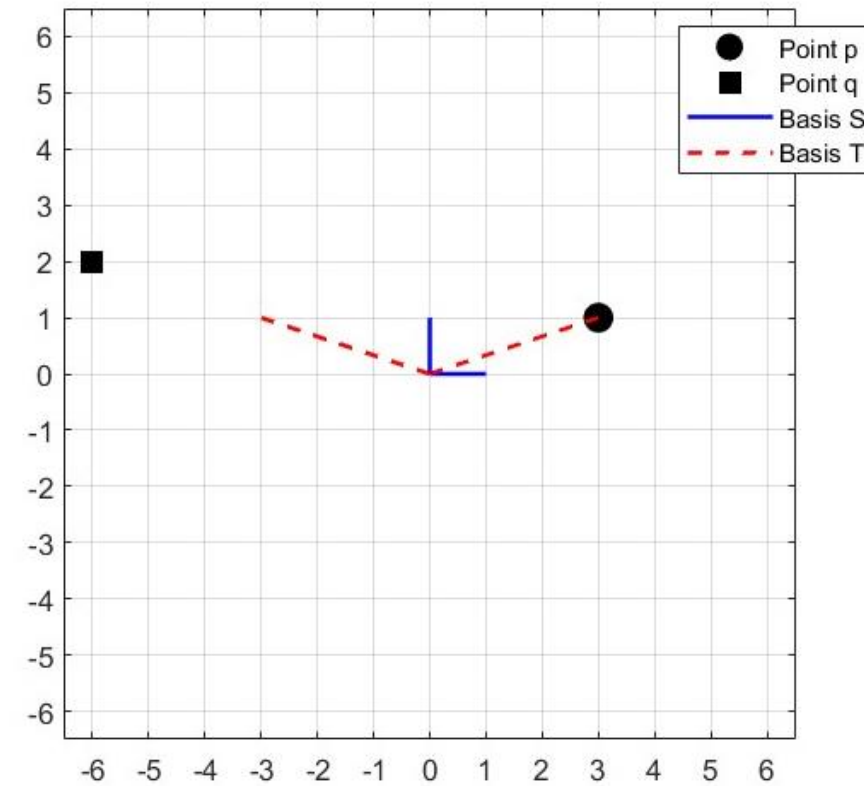

Figure 1. The same points can be described by different basis set

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Standard Basis

■ Basis in vector space

▶ A set of vectors $\{v_1, v_2, \cdots, v_n\}$ within a vector space V is called a basis of V if it satisfies the following two conditions.

● The set $\{v_1, v_2, \cdots, v_n\}$ is [   ]

● The set $\{v_1, v_2, \cdots, v_n\}$ spans V, meaning it generates the whole space V.

Applications of Basis

- **Application of basis in data science and machine learning**
  - ▶ Many problems in applied linear algebra can be conceptualized as finding the best set of basis vectors to describe some subspace.
    - Dimension reduction
    - Feature extraction
    - Principal components analysis
    - Independent components analysis
    - Factor analysis
    - Singular value decomposition
    - Linear discriminant analysis
    - Image approximation
    - Data compression
    - …

- **All of those analyses are essentially ways of identifying optimal basis vectors for a specific problem.**
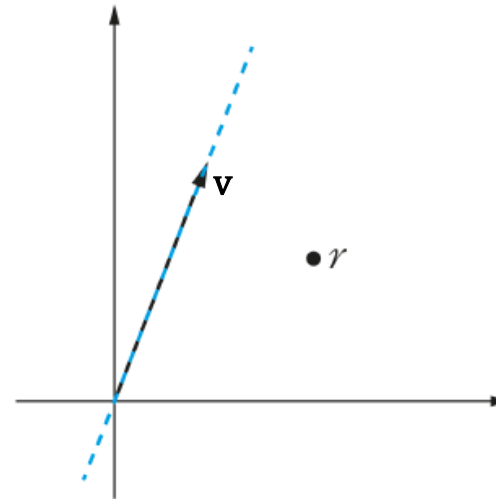
Definition of Basis

■ **Formal definition of basis**

▶ A basis is the combination of ☐ and ☐

- 1. Spans a certain subspace.
- 2. Independent set of vectors.

■ **The basis needs to span a subspace for it to be used as a basis for that subspace**

▶ A basis set can measure only what is contained its span.

- A basis vector for ☐ colored subspace cannot measure point r which is not in subspace of vector **v**.

Basis and Subspace

Linearly Independency of Basis

■ **Why a basis set must be linearly independent?**

▶ All vectors in a subspace must have unique coordinates with respect to that basis.

▶ To ensure this uniqueness, the basis vectors must no be linearly combinable in any way.

● For example, U is a valid set of vectors.

● But if you want to describe point (3,1) , the coefficients for the linear weighted combination of the three vectors in U could be (3, 0, 1) or (0, 1.5, 1) or … a bajillion other possibilities.

● It's so confusing, and so mathematicians decided that a vector must have unique coordinates within a basis set.

$$U = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

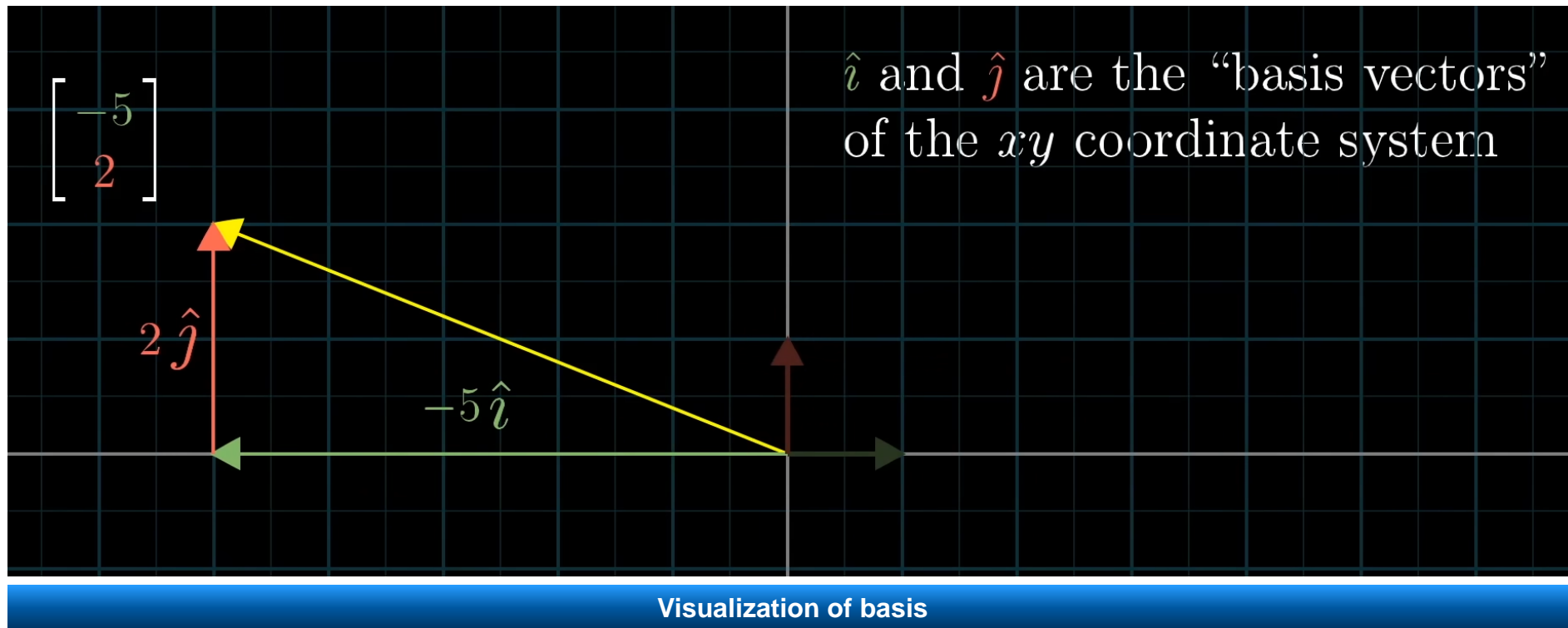**Vector set of U**

▶ To be clear, any point can be described using an infinite numbers of basis sets. Within a basis set, a point is defined by exactly one linear weighted combination.

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Visual Materials of Basis

■ **Geometric representation of basis**

▶ Basis (0:12 ~ 2:27)

▶ https://youtu.be/k7RM-ot2NWY?si=-MomPGwuSEOwvwev&t=12 (English)

▶ https://youtu.be/2CcCOgDilO8?si=c00bqWCvnhEuVwrr&t=12 (Korean)

Visualization of basis

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Code Exercise of Basis using Matlab

■ **Code Exercise (03_03)**

▶ Basis

```matlab
% Define basis vectors
i = [1; 0; 0];
j = [0; 1; 0];
k = [0; 0; 1];

% Define an arbitrary 3D vector v
v = [3; 2; 5];

% Visualization
figure;
hold on;
grid on;
axis equal;

% Draw the basis vectors
quiver3(0, 0, 0, i(1), i(2), i(3), 'r', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'i');
quiver3(0, 0, 0, j(1), j(2), j(3), 'g', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'j');
quiver3(0, 0, 0, k(1), k(2), k(3), 'b', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'k');

% Draw the vector v
quiver3(0, 0, 0, v(1), v(2), v(3), 'm', 'LineWidth', 1, 'AutoScale', 'off',
'MaxHeadSize', 1, 'DisplayName', 'v');
```

```matlab
% Visualize each multiplication of basis vector
plot3([0 v(1)], [0 0], [0 0], 'k--', 'LineWidth', 1);
plot3([v(1) v(1)], [0 v(2)], [0 0], 'k--', 'LineWidth', 1);
plot3([v(1) v(1)], [v(2) v(2)], [0 v(3)], 'k--', 'LineWidth', 1);

% Visualize the values of multiplication
text(v(1)/2, 0, 0, sprintf('%0.1f \\iti', v(1)), 'VerticalAlignment', 'bottom',
'FontSize', 15);
text(v(1), v(2)/2, 0, sprintf('%0.1f \\itj', v(2)), 'VerticalAlignment',
'bottom','FontSize', 15);
text(v(1), v(2), v(3)/2, sprintf('%0.1f \\itk', v(3)), 'VerticalAlignment',
'bottom', 'FontSize', 15);

xlabel('X');
ylabel('Y');
zlabel('Z');
title('Representation of a Vector with Basis');
legend('i','j','k','v','Location', 'northeastoutside');
view(45, 45); % Adjust the 3D view angle
hold off;
```
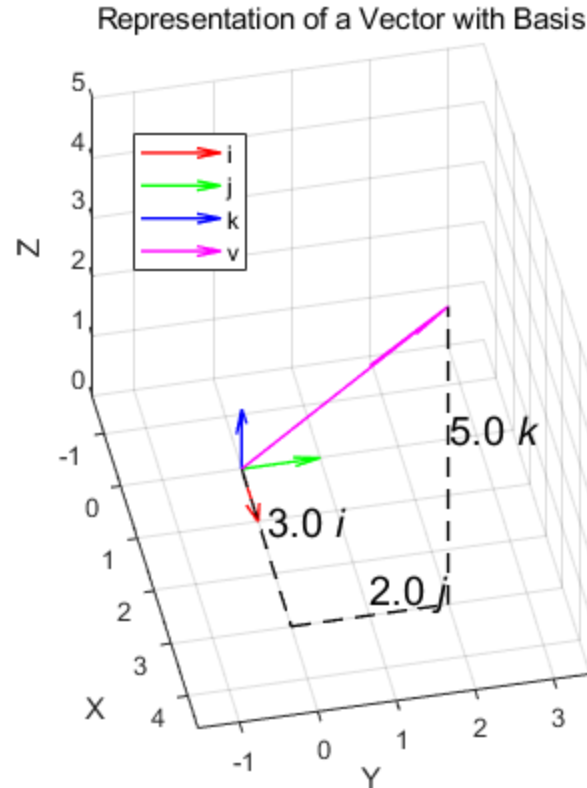
Source code

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Visualization Result of Basis using Matlab

■ **Code Exercise (03_03)**

▶ Basis

Source code result

Summary

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- **A vector set**
  - ▶ The set can contain either a finite or an infinite number of vectors.
- **Linear weighted combination**
  - ▶ Multiply scalar and add vectors in a set.
  - ▶ One of the single most important concepts in linear algebra.
- **Linearly dependent vs linearly independent**
  - ▶ If a vector can be expressed as a linear weighted combination, the set is linearly dependent.
  - ▶ If no such linearly weighted combination, the set is linearly independent.
- **A subspace**
  - ▶ The infinite set of all possible linearly weighted combination of vector set.
- **A basis**
  - ▶ If vector set (**1**)spans a certain subspace and (**2**)is linearly independent, it can be a basis for subspace.

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Exercises (1)

1. **Rewrite the Original code for linear weighted combination, but put the scalars in array and the vectors as elements in an array**

   ▶ you will have two arrays, one of the **scalars** and one of **vectors.**

2. **Then use a for loop to implement the linear weighted combination operation.**

   ▶ Initialize the output vector using zeros().

3. **Display var** *linCombo2*

   ▶ *linCombo1* & *linCombo2* display same result.

```matlab
% Scalars
l1 = 1;
l2 = 2;
l3 = -3;

% Vectors
v1 = [4, 5, 1];
v2 = [-4, 0, -4];
v3 = [1, 3, 2];

% Scalars and vectors organized into arrays
scalars =
vectors =

% Initialize the linear combination
linCombo2 =

% Implement linear weighted combination using a loop
for i = 1:length(scalars)
    linCombo2 =
end

% Confirm it's the same answer as above
disp(linCombo2);
```

```matlab
% Scalars
l1 = 1;
l2 = 2;
l3 = -3;

% Vectors
v1 = [4, 5, 1];
v2 = [-4, 0, -4];
v3 = [1, 3, 2];

% Linear weighted combination
linCombo1 = l1 * v1 + l2 * v2 + l3 * v3;
disp(linCombo1);
```

Original code

Generated code

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Exercises (2)

1. **Create a scalar list and vector list like the original code in Exercise (1), but length of scalar is different**

   ▶ Scalar
   - length : 4

   ▶ Vector
   - dimension : 3
   - length : 3

   ▶ You can use any number in the list.

2. **Write code for linear weighted combination and execute code**

   ▶ If code run successfully, write comment the result.

   ▶ If code isn't run successfully, write comment the reason of error.

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THANK YOU
FOR YOUR ATTENTION

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