BIG DATA ANALYTICS

WEEK-08 | Supervised Learning - Regression

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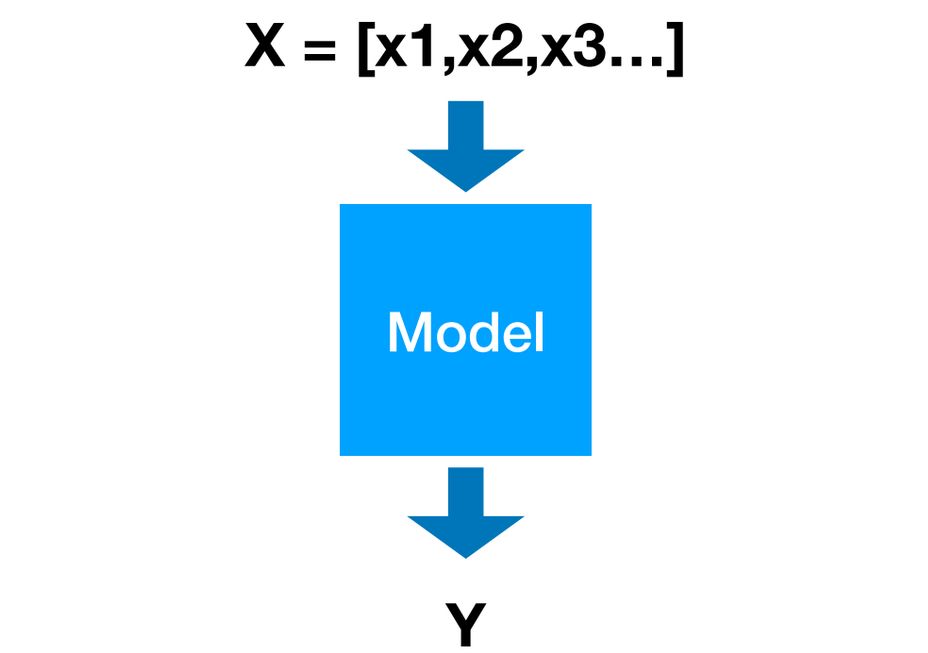
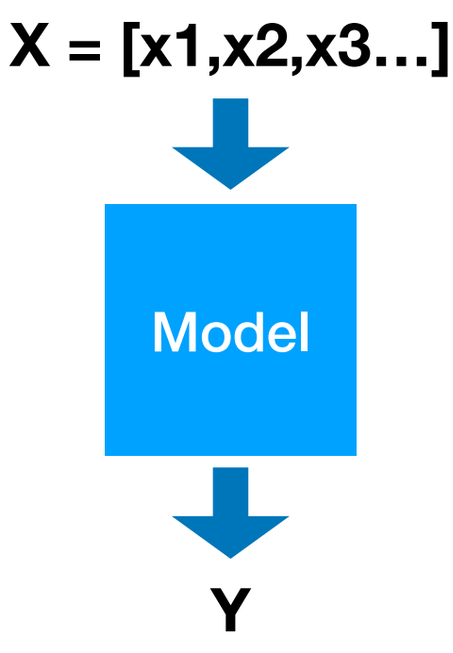
Classification vs. Regression



이때 Y는 범주형
e.g., (1,0), (True,False), (A, B, C, D)

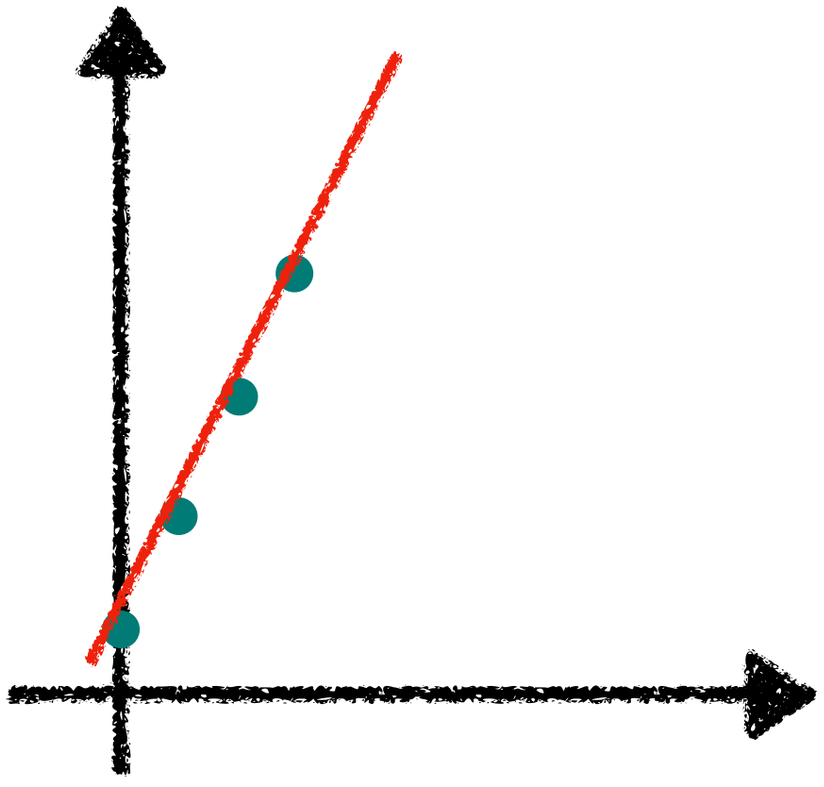


이때 Y는 숫자형
e.g., 133, 0.11, 103030

기본 원리

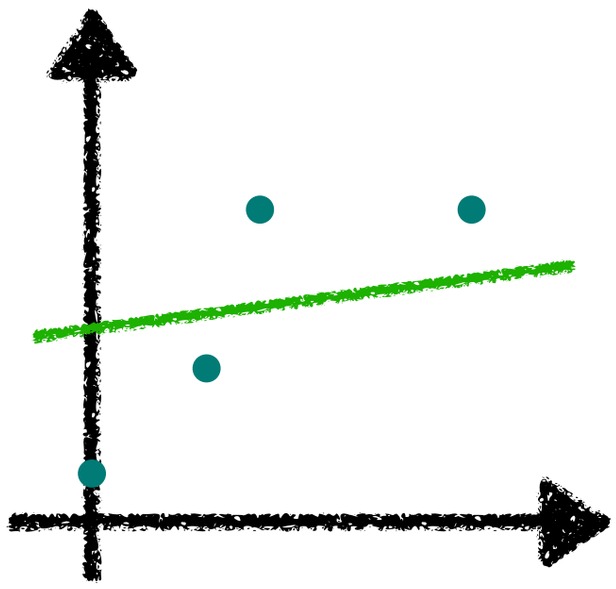
$$y= 3x+1$$

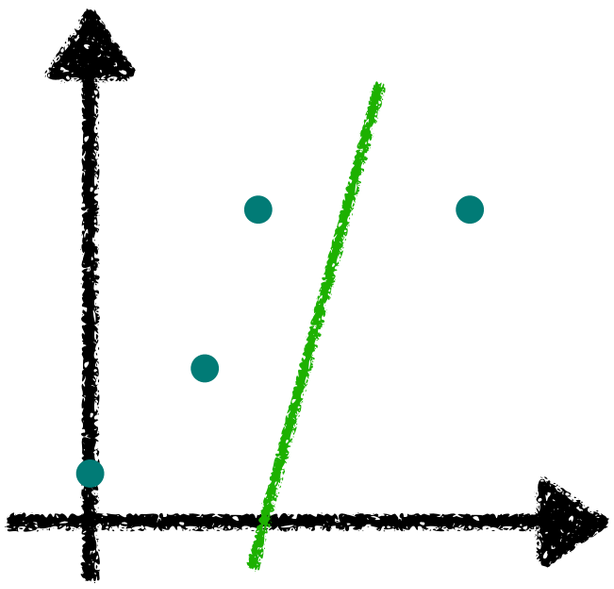
x	y
1	4
2	7
3	10
4	?

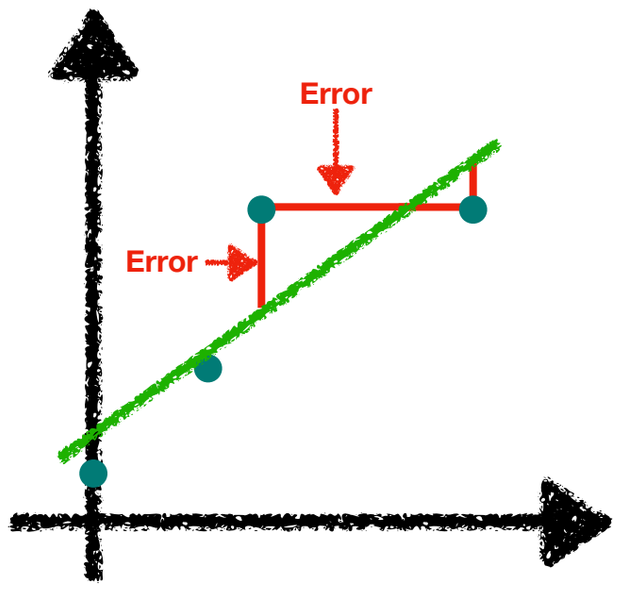


Regression

- Error를 최소화 할 수 있는 방정식(Equation)을 찾기







Linear Regression

- 단순 선형회귀
  - $y = wx + b$

- 다중 선형회귀 (n개의 feature)
  - $y = w_1x_1 + w_2x_2 + \cdots + w_nx_n + b$

- 가설 (Hypothesis)
  - 변수간의 관계를 유추하기 위해 수학적으로 나타낸 식
  - $H(x) = wx + b$

Loss function (손실함수)

- 실제값과 예측값과의 차이를 측정 할 수 있는 함수
- Regression Loss Functions
  - Mean Squared Error Loss =⟩ $\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$
  - Mean Squared Logarithmic Error Loss
  - Mean Absolute Error Loss =⟩ $\mathrm{MAE} = \frac{\sum_{i=1}^{n}\left|y_i - x_i\right|}{n} = \frac{\sum_{i=1}^{n}\left|e_i\right|}{n}$
- Binary Classification Loss Functions
  - Binary Cross-Entropy
  - Hinge Loss
  - Squared Hinge Loss
- Multi-Class Classification Loss Functions
  - Multi-Class Cross-Entropy Loss
  - Sparse Multiclass Cross-Entropy Loss
  - Kullback Leibler Divergence Loss

Optimizer

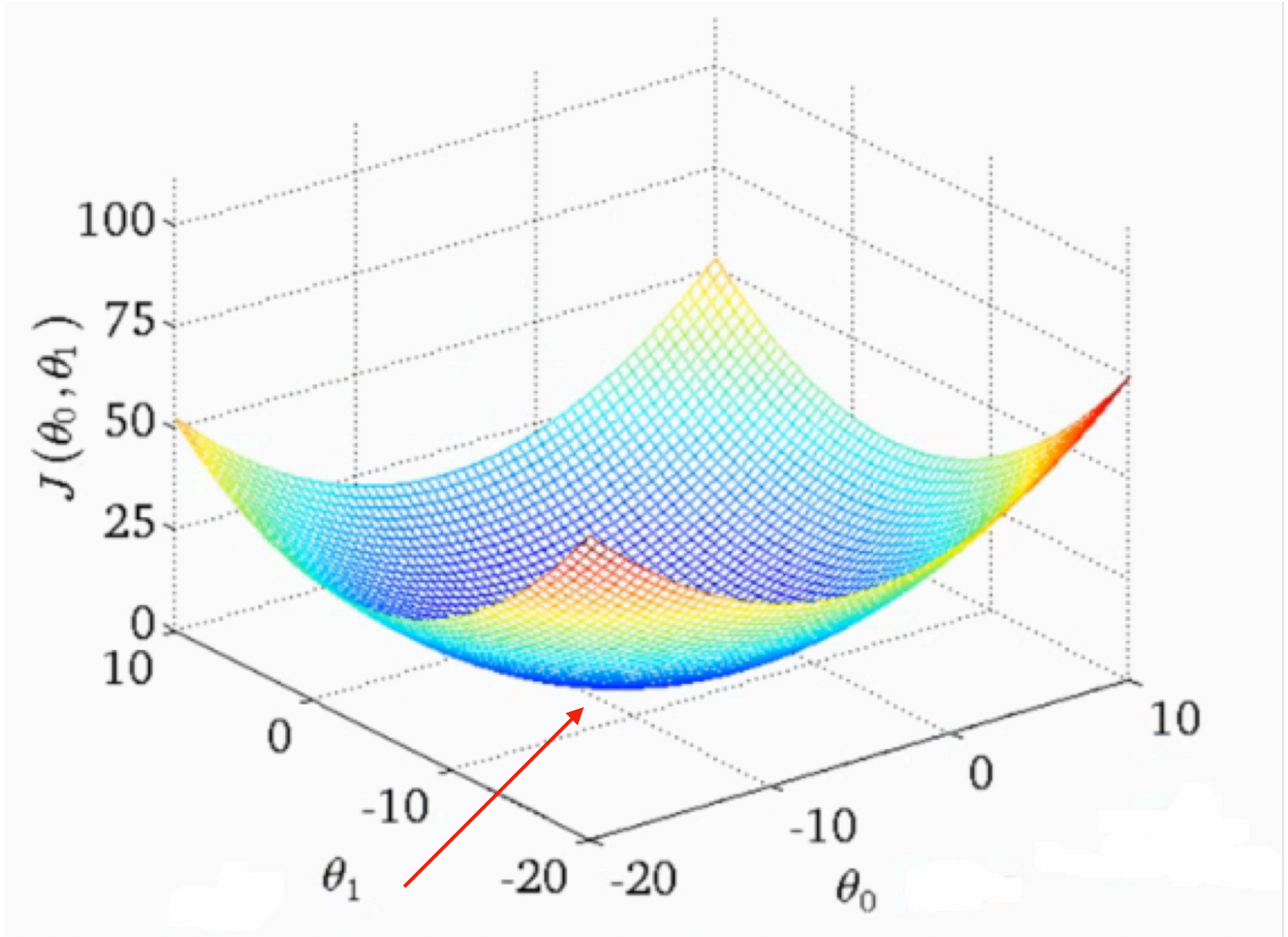
- Loss function을 이용해서 error 값을 측정을 한다면, 가장 최소의 error를 빠르게 찾는 알고리즘은 없을까?

- Hypotheis 재정의
  - $h_\theta(x) = \theta_0 + \theta_1 x$

- Cost Function 재정의
  - $J(\theta_1, \theta_2) = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i)^2$

- $J(\theta_1, \theta_2)$ 의 최솟값 찾기

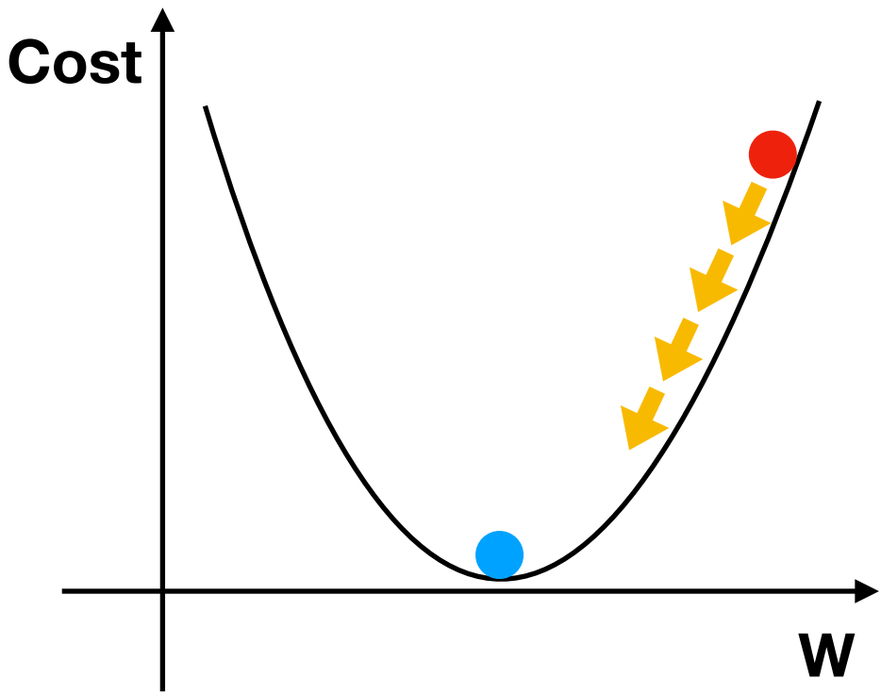
$$J(\theta_1, \theta_2)$$

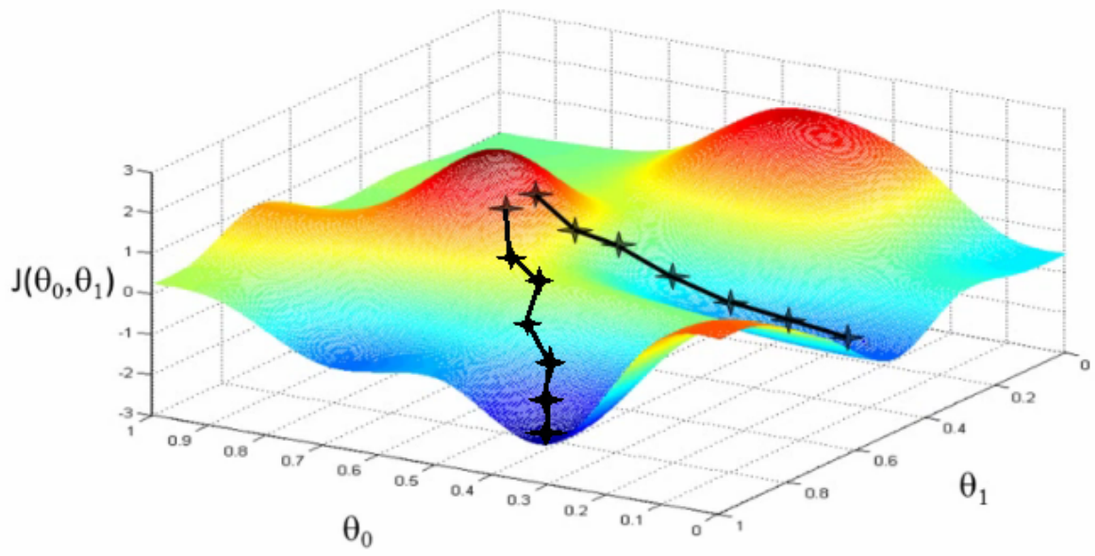


Gradient Descent

- 경사하강법

- 함수를 미분해서, 기울기가 음수인 곳을 계속 찾아간다면 언젠간 그 함수의 최솟값에 도달 하지 않을까?





GD Step by Step

- Cost function 정의

$$J(\theta_1, \theta_2) = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^i) - y^i)^2$$

- 임의의 점에서 시작
  - 예: (0,0), (10,3) 등

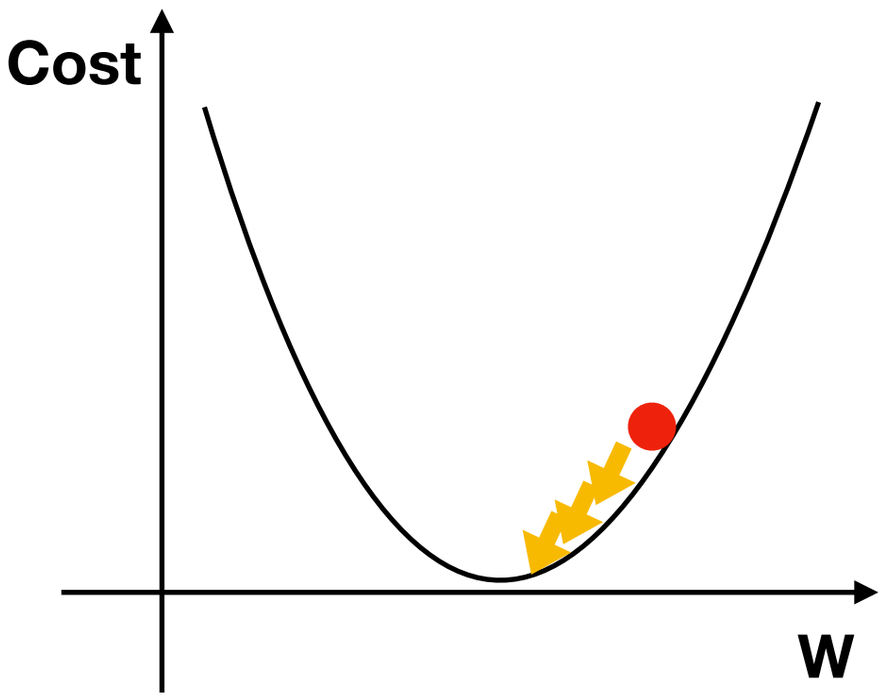
- J를 줄이는 θ_1, θ_2 로 업데이트

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad (\text{for } j = 0 \text{ and } j = 1)$$

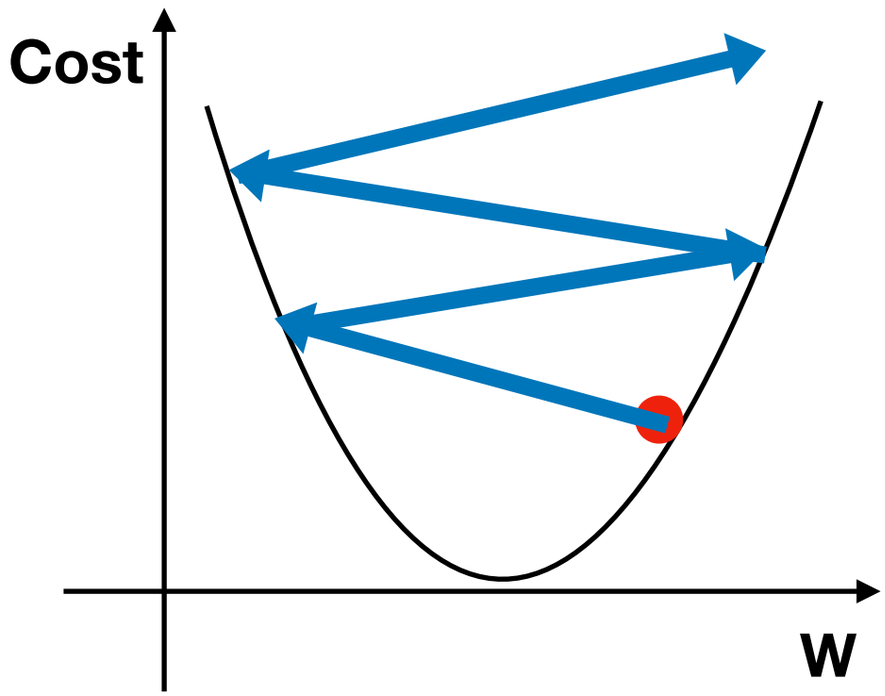
α : learning rate

- 수렴 할 때까지 반복!

Learning Rate



$\alpha = 0.0001$



$\alpha = 0.1$

α는 작으면 작을 수록 좋을까?

모델의 검증은 어떻게?

예측 값들 [1,0,1,1,1]

정답 값들 [1,1,0,1,0]

→ $\frac{\text{맞춘 값 수}}{\text{전체 값 수}}$ → $\frac{3}{5}$ → 60%

높을 수록 좋다

<분류문제 - Accuracy>

예측 값들 [100, 99, 39, 22]

정답 값들 [100, 89, 49, 30]

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 .$$

$$MSE = \frac{1}{4}((100-100)^2 + (99-89)^2 + (39-49)^2 + (22-30)^2)$$

=66 0에 가까울 수록 좋다

<회귀문제 - Mean Squared Error>

Regression 모델의 종류

- Linear Regression
- Polynomial Regression
- Support Vector Regression
- Decision Tree Regression
- Random Forest Regression
- Ridge Regression
- Lasso Regression

E.O.D