

# Code Explanation: Phytoplankton Strain Dynamics

## 0.1 Why is $2\delta$ instead of $\delta$ inside the loop?

The  $2\delta$  inside the loop (for  $j = 1$  to  $M_s - 1$ ) and the  $\delta$  at the boundaries ( $j = 0$  and  $j = M_s$ ) in the strain update equation are due to how we model the probabilistic shift of biomass between neighboring strains. Let's illustrate with an example:

Imagine we have 5 strains ( $M_s = 4$ ), each represented by a circle:

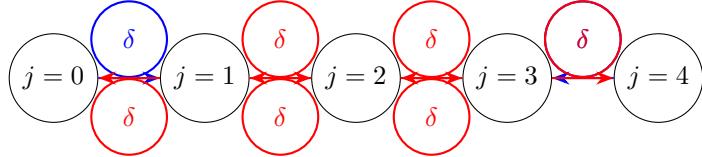


Figure 1: Biomass transfer between strains. Blue arrows (boundaries): transfer probability  $\delta$ . Red arrows (inner strains):  $\delta$  to each neighbor, totaling  $2\delta$  transfer from each inner node.

Let  $\delta = 0.2$  represent the probability that biomass shifts from one strain to a neighboring strain.

\* **Strain 0 ( $j = 0$ ):** This strain is at the boundary. It can only lose biomass to strain 1. The fraction of biomass that remains in strain 0 is  $(1 - \delta) = (1 - 0.2) = 0.8$ .

\* **Strain 1 ( $j = 1$ ):** This strain is in the interior. It can lose biomass to \*both\* strain 0 and strain 2. It loses a fraction  $\delta$  to strain 0 and a fraction  $\delta$  to strain 2. The total fraction of biomass lost by strain 1 is  $2\delta = 2 \times 0.2 = 0.4$ . Therefore, the fraction of biomass that \*remains\* in strain 1 is  $(1 - 2\delta) = (1 - 0.4) = 0.6$ .

\* **Strain 2 ( $j = 2$ ):** Similar to strain 1, it loses  $\delta$  to strain 1 and  $\delta$  to strain 3, for a total loss of  $2\delta$ . The remaining fraction is  $(1 - 2\delta)$ .

\* **Strain 4 ( $j = 4$ ):** This strain is also at a boundary. It only loses biomass to strain 3. The remaining fraction is  $(1 - \delta)$ .

This explains why the update equation uses  $(1 - \delta)$  for the boundary strains and  $(1 - 2\delta)$  for the inner strains. The  $2\delta$  accounts for the biomass transferred to \*both\* neighboring strains.

## 0.2 Why only one delta at the ends and two deltas in the beginning? (Question and Answer)

**Question:** Why is the biomass transfer probability  $\delta$  used for the boundary strains ( $j = 0$  and  $j = M_s$ ) while  $2\delta$  is used for the inner strains ( $j = 1$  to  $M_s - 1$ ) in the update equation? It seems like there are "more deltas at the beginning."

**Answer:** The difference between  $2\delta$  inside the loop and  $\delta$  at the boundaries is not about "more deltas at the beginning" but about the \*number of neighbors\* each strain has.

- **Inner Strains ( $j = 1$  to  $M_s - 1$ ):** Each inner strain has \*two\* neighbors. When biomass shifts, each inner strain loses some biomass to \*both\* of these neighbors. If the probability of shifting to \*one\* neighbor is  $\delta$ , then the total probability of shifting biomass \*away\* from an inner strain is  $2\delta$ . That's why the update equation for inner strains uses  $(1 - 2\delta)$ : it represents the fraction of biomass that \*remains\* in the strain after transfers to \*both\* neighbors.

- **Boundary Strains ( $j = 0$  and  $j = M_s$ ):** The strains at the boundaries only have \*one\* neighbor. Strain  $j = 0$  only has a neighbor to its right (strain  $j = 1$ ), and strain  $j = M_s$  only has a neighbor to its left (strain  $j = M_s - 1$ ). They can't transfer biomass "outside" the array. Therefore, they only lose biomass to \*one\* neighbor. The fraction of biomass lost is just  $\delta$ . The update equation uses  $(1 - \delta)$  because only one transfer is possible.

**Analogy:** Imagine a row of people. Each person represents a strain. They're throwing balls (representing biomass) to their neighbors.

- **People in the middle:** Each person in the middle throws balls to *\*both\** the person on their left and the person on their right. They lose a total of 2 balls (if each throw has a probability of 1).
- **People at the ends:** The people at the ends only have *\*one\** neighbor. They can only throw a ball to that one neighbor. They lose only 1 ball.

**In summary:** The difference between  $2\delta$  inside the loop and  $\delta$  at the boundaries is not about "more deltas at the beginning" but about the *\*number of neighbors\** each strain has. Inner strains have two neighbors, so they lose biomass to both, leading to a total loss probability of  $2\delta$ . Boundary strains have only one neighbor, so they lose biomass to only that one neighbor, leading to a loss probability of  $\delta$ . This difference is essential for correctly modeling the biomass transfer at the edges of the system.