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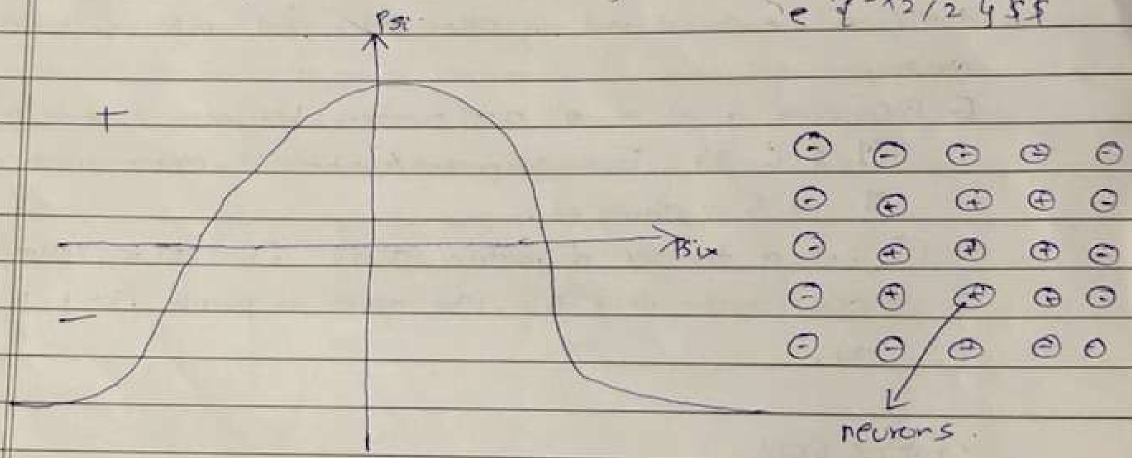
SOFT COMPUTING TQ — UNIT 3..

Q. What is an Mexican Hat? Draw and explain it?

ANS. Mexican hat function also known as the Ricker wavelet, is a function used in various applications, such as signal processing and neural networks, It's name comes from its shape, which resembles as an hat.

Mathematically, the Mexican Hat is given as : $$\psi(x)=(1-x^2) e^{-x^2/2}$$



It is used to identify edges and features in data. It's unique shape make a deal for detecting details while reducing noice.

Ex: image processing an image to detect the edges of the objects. By applying the ~~magic~~ mexican hat wavelet to the image, the sharp transmission (transition (edges)) becomes prominent, while the smoother areas (noises) are minimized. This helps in better object recognition and feature extraction.

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Q. Classical sets & The Fuzzy Sets

Ans • CLASSICAL SETS

Classical set theory is all about crisp, clear boundaries. Each element either belongs to the set or doesn't. There is no ambiguity or partial membership.

Think of a classroom. If the defined as "student who have passed the exam", each student is either set or not, with no middle ground.

Ex: (i) Consider a set of all even numbers between 1 and 10 {2, 4, 6, 8}. Each component/number is either a set or not. The no. 5 is clearly out.

(ii) Think of the set of primary colors: {Red, Blue, Yellow}. Any color outside this trio, like green or purple, isn't in the set.

• FUZZY SETS

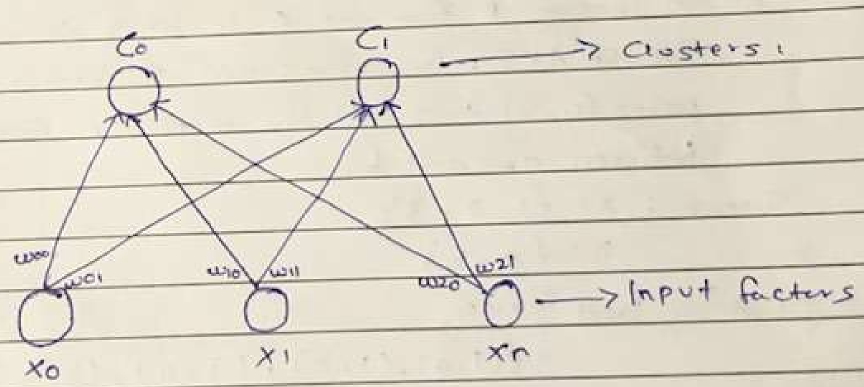
Fuzzy set theory, introduces the concept of partial membership. Elements can belong to a set ~~of~~ to a certain degree, ranging between 0 (not in set) and 1 (completely in set). It's like a dimmer switch, allowing for the shades of grey

For instance, the fuzzy set of "tall people" someone who is 180cm tall might belong to the set of membership value of 0.7, while someone who is 170cm tall might belong with a value of 0.4.

Ex: Consider a temperature in celcius. A classical set might define "warm days" as those exactly between 20° and 25°c. In a fuzzy set, temperatures like 18°c or 27°c can still be somewhat warm so, 22°c might have a membership value of 1, 18°c might be 0.5 and 15°c could be 0.2 in the "warm days" set.

Q. Kohen - self Organizing Maps

ANS: Self Organizing Maps or Kohenen Maps is atype of Artificial Neural Network which is also inspired by biological models of neural systems from 1970's. It follows an unsupervised learning approach and trained its network through a competetive learning algorithm. SOM is used for clustering and mapping techniques to map multidimensional data onto a lower-dimensional which allows people to reduce complex problems for easy interpretation. SOM has two layers, one is input and another is output layer. Architecture of SOP with two clusters and n input features of any sample is given



It Simply States that whichever input facter carries the shortest distance is the winning Custers.

Q. Cartesian Product of Relation

ANS: The Cartesian product of relation is the same as relation accross two sets. Generally, the cartesian product is represented for a set and not for an relation. The cartesian Product is an fundamental concept in set theory, providing an foundational Structre for ~~bo~~ defining relation between sets. By

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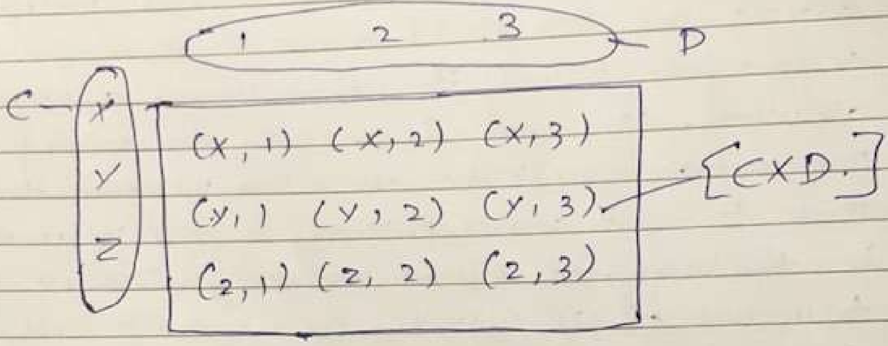
forming all possible ordered pairs, it allows for creation of meaningfull subsets that represents specific relationships.

Ex:



Ex: A Relation R from set A to set B is an subset of cartesian Product A×B. This means that R consists of certain ordered pairs (a, b) that satisfy a specific condition or relation between a and b.

Suppose : $A = \{1, 2, 3\}$

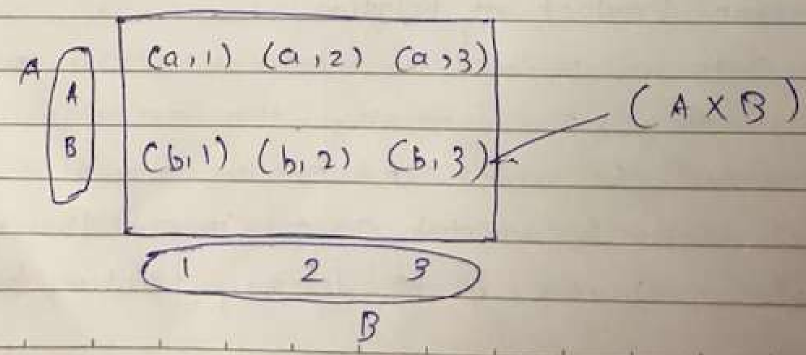
$B = \{a, b\}$

Cartesia Prod $= A \times B$

$= \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$R = \{(1,a)\ (2,b)\}$

Here, R is a subset of A×B where specific pairs satify the relation criteria.



elf-organizing maps

..lize the weight v_{ij}. Radom values may be assumed. Initialize learning Rate α.

Calculate square of ~~equatio~~ Eclidean distance. i.e. for each $j = 1$ to n

$$D(j) = \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i - w_{ij})^2$$

S3: Find winning unit index J. Such that $D(j)$ is minimum

S4: For all unit j within a specific neighborhood of J and for all i, calculate new weights

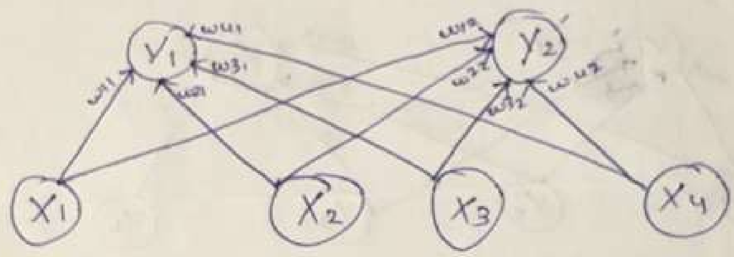
$$w_{ij}(new) = w_{ij}(old) + \alpha [x_i - w_{ij}(old)]$$

S5: update Learning Rate α using the formula α

Q. Construct KSOFM to cluster For Given vector [0 0 1 1], [1,0 0 0] [0 1 1 0] and [0 0 0 1]. No of cluster: to be formed is 2. Assume an initial learning rate of 0.5.

Soln: No. of input vector, n = 4

No. of cluster, m = 2



Intializing weight Randomly btwn 0 & 1

$$w_{ij} = \begin{bmatrix} 0.2 & 0.9 \\ 0.4 & 0.7 \\ 0.6 & 0.5 \\ 0.8 & 0.3 \end{bmatrix}$$

Rep w Connected with Y_1 — with Y_2

First Input Vector

$x = [0\ 0\ 1\ 1]$

a Calculate Euclidean Dist

$$D(j) = \sum_i (w_{ij} - x_i)^2$$

$$D(1) = (0.2-0)^2 + (0.4-0)^2 + (0.6-\phi)^2 + (0.8-\phi)^2$$

$$= 0.04 + 0.16 + 0.16 + 0.04$$

$$= 0.4$$

$$D(2) = \sum_{i=1} (w_{i2} - x_i)^2$$

$$= (0.9-0)^2 + (0.7-0) + (0.5-1)^2 + (0.3-1)^2$$

$$= 0.81 + 0.49 + 0.25 + 0.49$$

$$= 2.04$$

$D(1) < D(2)$. Therefore winning Custor is $J = 1$ i.e Y_1

Update weight on winning Cluster where $J = 1$

$$w_{ij}(new) = w_{ij}(old) + \alpha [x_i - w_{ij}(old)]$$

$$w_{i1}(new) = w_{i1}(old) + \alpha [x_i - w_{i1}(old)]$$

$$w_{11}(n) = w_{11}(0) + 0.5 [x_1 - w_{11}(0)]$$

$$= 0.2 + 0.5 [0 - 0.2]$$

$$= 0.1$$

$w_{21}(n) = w_{21}(old) + 0.5\,[x_2 - w_{21}(old)]$

$= 0.4 + 0.5\,(0 - 0.4)$

$= 0.2$

$w_{31}(n) = 0.6 + 0.5\,(1 - 0.6)$

$= 0.8$

$w_{41}(n) = 0.8 + 0.5\,(1 - 0.8)$

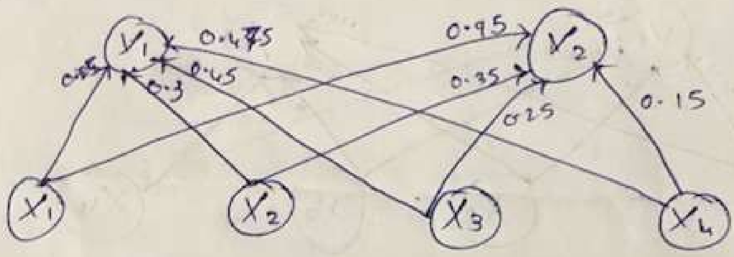
$= 0.9$

Updated Weight Matrix

$$w_{ij} = \begin{bmatrix} 0.1 & 0.9 \\ 0.2 & 0.7 \\ 0.8 & 0.5 \\ 0.9 & 0.3 \end{bmatrix}$$

Repeating the same steps for all the vector

Final Answer will be



$$w_{ij} = \begin{bmatrix} 0.025 & 0.95 \\ 0.3 & 0.35 \\ 0.45 & 0.25 \\ 0.475 & 0.15 \end{bmatrix}$$

Relation

Fuzzy Cartesian Product.

Fuzzy relation is the cartesian product of mathematical fuzzy sets. Two fuzzy sets are taken as input, fuzzy relation is then equal to cross product of sets which is created by vector multiply

Q. FCP : $A = \left\{ \frac{0.4}{a_1} + \frac{0.6}{a_2} + \frac{0.1}{a_3} \right\}$

$B = \left\{ \frac{0.3}{b_1} + \frac{0.5}{b_2} \right\}$

$$R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{bmatrix} 0.3 & 0.4 \\ 0.3 & 0.5 \\ 0.1 & 0.1 \end{bmatrix} \end{array}$$

Sol:

$R = M_R(x, y) = M_{x \times y}(x, y) = \min \{M_x(x), M_Y(y)\}$

To find the R we need to compare the value btwn A(a) & B(b)

$M_R(a_1, b_1) = \min [M_A(a_1), M_B(b_1)] = \min (0.4, 0.3) = 0.3$

$M_R(a_1, b_2) = \min [M_A(a_1), M_B(b_2)] = \min (0.4, 0.5) = 0.4$

$M_R(a_2, b_1) = \min [M_A(a_2), M_B(b_1)] = \min (0.6, 0.3) = 0.3$

$M_R(a_2, b_2) = \min [M_A(a_2), M_B(b_2)] = \min (0.6, 0.5) = 0.5$

$M_R(a_3, b_1) = \min [M_A(a_3), M_B(b_1)] = \min (0.1, 0.3) = 0.1$

$M_R(a_3, b_2) = \min [M_A(a_3), M_B(b_2)] = \min (0.1, 0.5) = 0.1$

(II) Fuzzy Relation

A fuzzy Relation is a concept that represent relations between the elements of fuzzy sets.

Fuzzy Relation between two sets X and Y is called binary fuzzy relation and is denoted by R(X, Y).

$X = \{x_1, x_2, \dots x_n\}$ & $Y = \{y_1, y_2, \dots y_m\}$

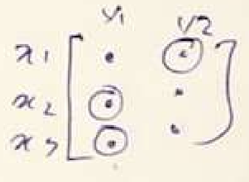
$$R(X, Y) = \begin{bmatrix} M_R(x_1, y_1), & M_R(x_1, y_2) & \dots & M_R(x_1, y_m) \\ M_R(x_2, y_1), & M_R(x_2, y_2) & \dots & M_R(x_2, y_m) \\ \vdots & & & \\ M_R(x_n, y_1), & M_R(x_n, y_2) & \dots & M_R(x_n, y_m) \end{bmatrix}$$

Fuzzy Matrix

Q.

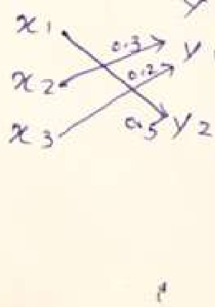
$X = \{x_1, x_2, x_3\}$ & $Y = \{y_1, y_2\}$

$R = \frac{0.5}{x_1, y_2} + \frac{0.3}{x_2, y_1} + \frac{0.2}{x_3, y_1}$



Corresponding fuzzy matrix R will be

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0 & 0.5 \\ 0.3 & 0 \\ 0.2 & 0 \end{bmatrix} \end{array} \Rightarrow$$



– Graph Repr

(iii) Operation on Fuzzy Relations

Let R and S be two fuzzy Relation on A×B

① Union : $\mu_{R\cup S}(x,y) = \max[\mu_R(x,y), \mu_S(x,y)]$
This operation combines two relations by taking the maximum membership value for each pair.

② Intersection : $\mu_{R\cap S}(x,y) = \min[\mu_R(x,y), \mu_S(x,y)]$
This operation finds the community between two relations by taking the minimum membership value for each pair

③ Compliment
$\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$
This operation reverses the membership degree, converting high membership values to low and vice versa.

④ Containment : $R \subset S = \mu_R(x,y) \leq \mu_S(x,y)$

⑤ Inverse : $R^{-1}(y,x) = R(x,y)$

⑥ Projection : $\mu_{[R \downarrow Y]}(x,y) = \max \mu_R(x,y)$ $\begin{bmatrix} 0.1 & 0.6 & 0.7 \\ 0.3 & 0.2 & 0.8 \\ 0.1 & 0.5 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.8 \\ 0.5 \end{bmatrix}$

(iv) PROPERTIES OF FUZZY RELATION

- Commutavity, associativity, distributivity, idempotency & identity, Demorgan's law →(holds good prop for fuzzy except two given below)
- Excluded Middle Law is not satisfied & Law of Contradiction is not satisfied

$R \cup \bar{R} \neq U$
$R \cap \bar{R} \neq \phi$

(v) FUZZY EQUIVALENCE RELATION.

A fuzzy equivalence relation is a way to say that elements are partially equivalent to each other, with degree of equivalence from 0 to 1. It satifies three properties: reflexivity (every element is fully equivalent to itself), symmetry (equivalence is mutual) and transitivity (if an element is related to second, which is related to third then the first is also related to third.

Further explanation

...er Relation R will be equivalance relation of all three properties. R - S - i

Reflexivity : $M_R(x_i, x_i) = 1$ OR $(x_i, x_i) \in R$

$$\begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 1 & 1 \end{array}$$

All x_i should be same

(ii) Symmetry : $M_R(x_i, x_j) = M_R(x_j, x_i)$

i.e. $(x_i, x_j) \in R \Rightarrow (x_j, x_i) \in R$

$$\begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ x_1 & 0.8 & 0.1 & 0.7 \\ x_2 & 0.1 & 1 & 0.6 \\ x_3 & 0.7 & 0.6 & 0.5 \end{array}$$

if ~~we are~~ should be same
ie if a = b, then b should be equal to i.e b = a.

(iii) Transitivity :

$M_R(x_i, x_j) = \lambda_1$ or $M_R(x_j, x_k) = \lambda_2$

so $M_R(x_i, x_k) = \lambda$

~~i.e. x_i, x_j R so $(x_i, x_k) \in R$~~

where $\lambda = \min[\lambda_1, \lambda_2]$

(VI) Tolerance Relation

A fuzzy tolerance relation is a way to describe that elements are partia similar to each other with varying degrees. It needs to satisfy 2 propert reflexivity (each element is fully similar to itself) and symmetry (similarity btwn any 2 elements is mutual).

Ex: Tolerance Relation R_1 on universe x is one where the only the 2 property of reflexivity & symmetry are satisfied otherwise called proximity relation

$$R_1^{n-1} = R_1 \cdot R_1 \cdot \ldots R$$

Q. DEFUZZIFICATION METHODS
Process of converting a fuzzy set into a crisp set

① Lambda - Cut Method
(For sets)

Consider a fuzzy set A. The set A_λ $(0<\lambda<1)$ called the lambda (λ)-cut (or alpha [α]-cut) set is a crisp set of the ~~follow~~ fuzzy set & is defined as follow:

$$A_\lambda = \{x \mid \mu_A(x) \geq \lambda\};\ \lambda \in (0,1]$$

Q: Consider 2 fuzzy sets A & B, both defined on X, given as follows

	x_1	x_2	x_3	x_4	x_5
A	0.2	0.3	0.4	0.7	0.1
B	0.4	0.5	0.6	0.8	0.9

Express foll λ cuts sets using Zadins notations
a) $(\bar{A})_{0.7}$ b) $(B)_{0.2}$ (c) $(A\cup B)_{0.6}$
d) $(A\cap B)_{0.5}$

$$A = \left\{\frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5}\right\}$$

$$B = \left\{\frac{0.4}{y_1} + \frac{0.5}{y_2} + \frac{0.6}{y_3} + \frac{0.8}{y_4} + \frac{0.9}{y_5}\right\}$$

$$A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}$$

a) $(\bar{A}) = 1 - \mu_A(x)$

$$= \left\{\frac{0.8}{x_1} + \frac{0.1}{x_2} + \frac{0.6}{x_3} + \frac{0.3}{x_4} + \frac{0.9}{x_5}\right]$$

$(\bar{A})_{0.7} = (x_1, x_2, x_3]$ x ≥ value

b) $(B)_{0.2} = \{x_1, x_2, x_3, x_4, x_5)$. All are big

c) $A\cup B = \max\{\mu_A(x), \mu_B(x)\}$
Greater value b/w A & B
$$\left\{\frac{0.4}{x_1} + \frac{0.5}{x_2} + \frac{0.6}{x_3} + \frac{0.9}{x_4} + \frac{0.9}{x_5}\right]$$

$(A\cup B)_{0.6} = \{x_3, x_4, x_5)$ x ≥ value

d) $A\cap B = \min(\mu_A(x), \mu_B(x)]$
$$= \left\{\frac{0.2}{x_1} + \frac{0.3}{x_2} + \frac{0.4}{x_3} + \frac{0.7}{x_4} + \frac{0.1}{x_5}\right]$$

$(A\cap B)_{0.5} = \{x_1, x_2, x_3\}$ x ≤ value

For RELATION

$$R_\lambda = \{(x,y) \mid \mu_R(x,y) \geq \lambda\}$$

Q: Determine the crisp λ-cut relation when $\lambda = 0.1$, 0^+, 0.9 for foll Relation

$$R = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0.3 & 0.7 & 0.1 \\ 0.8 & 0.9 & 1.0 \end{bmatrix}$$

$$R = \{(x,y) \mid \mu_R(x,y) \geq \lambda\}$$
$$= \{1 \mid \mu_R(x,y) \geq \lambda\ ;\ 0 \mid \mu_R(x,y) < \lambda\}$$

If value ≥ (Gr) lambda = 1, V < lambda = 0

a) $\lambda = 0.1$

$$R_{0.1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

b) $0^+ = \lambda$ [Taking only value which are Gr than 0]

$$R_{0^+} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

c) $\lambda = 0.9$ [value less then 0.9 will be 0]

$$R_{0.9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Q: ~~Fuzzy~~ Relation:

ANS. ~~Fuzzy Relations are the mapping of variable from one fuzzy set to another.~~

Q. Membership Function

ANS. A membership function in fuzzy logic defines how each element in a set is mapped to a membership value btwn 0 & 1. This value indicates the degree to which the elements belong to the fuzzy set.

Think of it like a curve or a shape that shows how true a statement is in different scenarios. The shape of membership fun can vary (trian, trapezodial, etc) based on application of data.

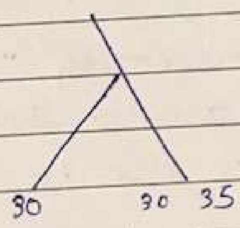
Ex: Imagine we're talking about the concept of a "hot day" in degree Celsius.

Membership Function.

- Below 20°c : Not hot (mf = 0 1)
- 20° c to 30° c : Getting hotter (mf = 0 to 1)
- Above 30°c : Defintely hot (mf = 1

Defining mf $u(x)$ for temp x

$$u(x) = \begin{cases} 0 & \text{if } x < 20 \\ \dfrac{x-20}{3-20} & \text{if } 20 \le x \le 30 \\ 1 & \text{if } x > 30 \end{cases}$$



15°c = Not consd hot (m = 0)

25° c = Partially hot (m = 0.5)

33° c = Definetly hot (m = 1)

= 0.1

Features of Membership Function:

(i) Range:
The value range from 0 to 1, indicating degree of Membership

(ii) Shape:
Membership functions can have various shapes such as triangular, bell-shaped etc, depending upon specific application and nature of data

(iii) Continuity
They can be either continous or discrete, meaning they can either smoothly transition.

(iv) Normality
A mf is normal if atleast one element is set has mem value of 1

Q. Measures of fuzziness & Fuzzy Measures
Ans. Fuzziness measures are all about quantifying the degree of uncertainty or imprecision with a system.

(i) Fuzzy Entropy
This measure quantifies the amount of uncertainty or randomness in a fuzzy set. Higher entropy indicates more fuzziness, meaning less definiteness about membership
Ex: Imagine you have a fuzzy set representing "tall people". Each person's height is assigned a value between 0 & 1. If everyone's membership value is close to 0.5, the

entrophy is high, indicating high fuzziness. If most value are either 0 or 1, entrophy is low, indicating low fuzziness.

(ii) Membership Function:

These determines how each point in the input space is mapped to a membership value between 0 and 1. A membership value of 0 means no membership, 1 means full membership and values in between represent partial membership.

Ex: Marks High → 1

low → 0

Mid → 0.5

(iii) Cardinanlity

Cardianality means that the sum of membership values of all elements in fuzzy set. Like a set of (a, b, c) = a+b+c

Set A = abc

Ex: A fuzzy set of elements with membership value of

{(0.2), (0.3), (0.5)}

A = 1

(iv) Choquet Integral

A type of integral used to aggregate function information with fuzzy measures, considering the interaction btwn criteria.

Ex: If choosing a vacation spot, criteria include budget (0.7), Climate (0.9), and activities (0.8). The Choquet integral Combines these values, factoring in their interactions.

= 0.1