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Shear strength of reinforced concrete deep beams – A review with improved model by genetic algorithm and reliability analysis

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ABSTRACT

This paper reviews various models for the shear strength prediction of reinforced concrete (RC) deep beams with and without web reinforcement. A database of 381 tests on deep beams was utilized to conduct a comparative study among the analytical models and the code equations. The accuracy of each model was evaluated based on the statistical analysis and the performance test. It was found that code equations for the shear strength of RC deep beams are too conservative to estimate the shear strength of deep beams. Therefore, simplified improved shear equations for RC deep beams with and without web reinforcement were proposed from genetic algorithm (GA). The most significant parameters and their interactions that affect the shear strength of deep beams were identified by factorial design. The proposed equations were calibrated by reliability analysis which can be used for future design purposes. The resistance factors for the shear design equations were calculated at a target reliability index, β_T of 3.5 in order to achieve an acceptable level of structural safety.

1. Introduction

A predominant failure mode in deep beams is the shear failure that can lead to catastrophic consequences. The Bernoulli-Navier's hypothesis for slender beams states that the distribution of strain through the depth of the beam is linearly proportional to the distance from the neutral axis. Their hypothesis was based on the assumption that shear deformation in a slender beam is negligible compared to flexural deformation. However, this is not applicable in case of deep beams. The strain distribution along the depth of a section in deep beam becomes non-linear with the decrease of the shear span to depth ratio [1]. Therefore, their theory cannot accurately predict the behavior of deep beams since the assumption of 'plane sections remain plane' does not apply and therefore, the theory will underestimate the shear strength which is unacceptable. This phenomenon in deep beam was investigated by many researchers and concluded that the sectional approach failed to predict the accurate behavior in case of a beam having $a/d < 2.0$ [1,2].

A beam is generally regarded as a deep beam when its shear span (a) to depth ratio (d) is < 2.5 [3]. The main difference between a slender and deep beam is that in case of a slender beam the shear deformation is negligible and could be ignored while it must be considered in the analysis and design of a deep beam. An extensive experimental investigation was reported in the literature focused on the shear behaviour of deep beams with different parameter [4–21]. Previous research also focused on the shear behaviour of deep beams under static and dynamic loading [22]. The contribution of various parameters on the shear strength of deep beams has been discussed in separate section of this paper. Schlaich et al. [23] and Marti [24] developed strut and tie model (STM) from the failure pattern of deep beams under static loading. The STM method analyzes concrete members with a plastic truss analogy that transfers the forces from the loading point to the supports using concrete struts acting in compression and steel reinforcing ties acting in tension [23,24]. Lim and Hwang [25] presented a macro model using STM to predict the shear strength of deep beams. The model was verified against experiments and identified several parameters such as definition of a shear element with force discontinuity, width of strut, dimension of nodal zone and failure modes. Deng et al. [26] proposed modified STM method for the application of RC and fiber reinforced concrete deep beams. The contribution of longitudinal reinforcement and concrete tensile strength was modified in the proposed model which showed good agreement with the test results.

The STM model has been incorporated in many design codes including ACI 318 [27] and CSA A23.3 [28]. In 1984, CSA A23.3 was the first design code adopted the STM model as a design standard for RC members considering the compression field theory [29]. ACI building code incorporated the STM method in Table A1 of the section "Building

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code requirements for structural concrete and commentary [27]". The drawback of the STM method is the complexity of the calculation required to calculate the shear capacity. The STM method allows designer free to choose the truss dimensions (strut and tie) that carry the load through the disturbed region also called "D-region" to the supports. The complexity arises when applying the STM approach as there is no single solution and it is recommended to assume more than one STM model to determine the capacity of a deep beam. Previous experimental investigations showed a substantial variation with the STM method and test results when calculating the shear strength of RC deep beams [18,30,31].

In this paper, the shear strength of deep beams predicted by various models proposed in the literature and the codes were compared to experimental results. The comparative study was performed to identify the accuracy of the available models. The performance of each equation was evaluated by calculating the inverse of the slope of a linear least-squares regression of the calculated shear strength (V_{cal}) to the experimental shear strength (V_{exp}) of deep beams (i.e., χ factor), the coefficient of variation (*CoV*), the standard deviation (*SD*), the sample variance (*VAR*) and the absolute error (*AAE*). The performance index (*PI*) of each model was also calculated and compared based on their total penalty (*p*). Additionally, this research also developed an improved analytical model to predict the shear strength of RC deep beams by genetic algorithm (GA). For design purposes, the proposed equations were calibrated and the resistance factors were estimated from the reliability analysis.

2. Database

A total of 381 tests on RC deep beams with and without reinforcement were constructed from literature (Table 1). The test results were first compiled in order to establish relationships among various factors affecting the shear strength of deep beams. The variables used in the experiments are beam width (*b*), height (*h*), effective depth (*d*), shear span to effective depth ratio (*a/d*), the compressive strength of concrete ($f_c^{'}$), longitudinal reinforcement ratio (ρ), horizontal shear reinforcement ratio (ρ_h) and vertical shear reinforcement ratio (ρ_v) where, Table 1 shows the range of the variables. The database has a wide range of *a/d* ratio from 0.13 to 2.5. The longitudinal, horizontal and vertical shear reinforcement ratios (ρ, ρ_h, and ρ_v) were from 0.1 to 4.1%, 0–2.5% and 0–2.7%, respectively. The database was divided into two sections: deep beams with shear reinforcement and deep beams without any shear reinforcement.

3. Factors affecting the shear strength of deep beams

The important factors that affected the shear capacity of the deep beams are shear span to depth ratio, compressive strength of concrete, longitudinal reinforcement, horizontal shear reinforcement and vertical shear reinforcement [12]. Fig. 1 shows the scattered plot of the normalized shear strength, $\hat{V}$, versus different parameters where the normalized shear strength is calculated using Eq. (1).

$$\hat{V} = \frac{V_u}{\sqrt{f'^{'}_{c}}bh} \tag{1}$$

where, $\hat{V}$is the normalized shear strength, V_u is the ultimate shear strength, $f_c^{'}$ is the concrete compressive strength, *b* is the width of the beam, and *h* is the height of the beam. The contribution of each factor on the shear strength of RC deep beams is discussed below.

3.1. Shear span to depth ratio (a/d)

The shear strength of a deep beam largely depends on its span to depth ratio, *a/d*. This has been established after Kani's investigations in the 1960s [32]. Later other researchers e.g. Collins [1] and Rogowsky and MacGregor [2] also investigated the size effect on deep beams and reported the same findings. A recent numerical study by Chen et al. [33] showed that the size effect is highly dependent on the depth of deep beams which was not inherently considered in ACI STM model and hence, resulted in lower safety for large depth deep beams. Previous experimental investigations on RC deep beams showed that *a/d* is the main parameter that affects their shear strength which increases with the decrease of *a/d* ratio [8,12,17,21,34,35]. This is because as the *a/d* ratio decreases, the shear force is transferred by the concrete strut directly to the supports, the mechanism is called the strut and tie action in deep beam. The normalized shear strength versus *a/d* ratio plot (Fig. 1a) shows that the shear strength of a deep beam is linearly proportional with *a/d* ratio.

3.2. Beam span to depth ratio (l_n/d)

Manuel et al. [8] performed 12 experiments on deep beams with different span to depth ratio and commented that, similar to *a/d* ratio, l_n/d ratio has a significant influence on the shear strength of deep beam where the shear strength is inversely proportional to l_n/d ratio (Fig. 1b). This is because as the l_n/d ratio increased, a longer arch is required to transfer the load to the support and, at the same time, the mid-span

Table 1
Database of RC deep beams.

Ref.	*b* [mm] min	max	*h* [mm] min	max	*d* [mm] min	max	*a/d* [–] min	max	f_c' [Mpa] min	max	ρ [%] min	max	ρ_h [%] min	max	ρ_v [%] min	max
[4]	203	203	457	457	390	390	1.2	2.3	14	48	1.6	3.1	0.0	0.0	0.3	1.2
[5]	51	102	178	330	152	305	0.7	1.7	20	39	0.5	2.6	0.0	0.0	0.0	1.4
[6]	76	81	381	762	350	731	0.3	0.6	11	28	0.1	0.7	0.0	0.0	0.0	0.0
[7]	76	76	254	762	216	724	0.2	0.7	19	25	0.5	1.7	0.5	2.5	0.5	2.5
[8]	100	100	460	460	410	410	0.3	1.0	30	45	1.0	1.0	0.0	0.0	0.0	0.0
[9]	152	152	330	330	272	272	1.5	2.0	30	36	1.0	2.4	0.0	0.0	0.0	0.2
[10]	76	76	254	762	216	724	0.2	0.7	21	28	0.5	1.7	0.5	2.5	0.5	2.5
[11]	75	75	300	650	225	575	0.3	1.0	11	16	0.6	1.4	0.0	0.0	0.0	0.0
[12]	102	102	356	356	305	305	1.0	2.1	16	23	1.9	1.9	0.2	0.9	0.2	1.3
[13]	50	100	500	900	450	850	0.4	1.5	28	47	0.2	1.2	0.3	0.4	0.2	0.4
[14]	100	100	500	900	450	850	0.1	1.3	43	45	0.2	1.5	0.3	1.0	0.2	0.6
[15]	203	203	508	508	425	425	2.2	2.2	28	43	2.7	2.7	0.0	0.0	2.7	2.7
[16]	110	110	500	500	463	463	0.3	2.5	41	59	1.2	1.2	0.0	0.0	0.5	0.5
[17]	140	140	500	1750	444	1559	0.6	1.1	31	49	2.6	2.6	0.0	0.1	0.0	0.1
[18]	120	130	560	560	500	500	0.5	2.0	24	74	1.3	1.6	0.0	0.9	0.0	0.4
[19]	50	75	400	600	380	560	1.0	2.5	31	48	0.1	1.7	0.5	0.8	0.5	0.8
[20]	100	150	460	460	370	380	0.8	1.6	22	50	2.0	4.1	0.0	0.2	0.0	0.7
[21]	100	100	150	400	125	375	1.1	2.3	24	37	0.3	2.4	0.0	0.0	0.0	2.3

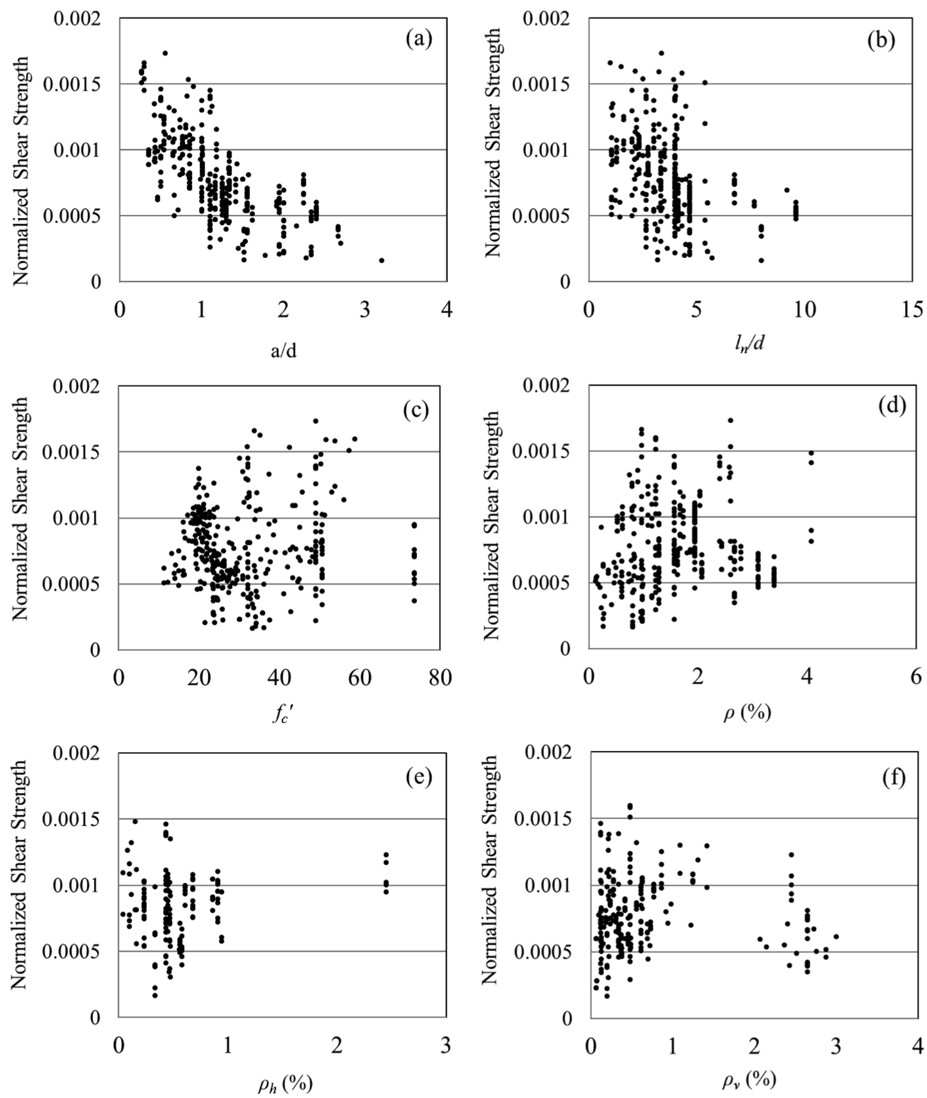


Fig. 1. Scatter plots of normalized shear strength of deep beams versus: (a) shear span-to-depth, (b) beam span-to-depth, (c) concrete compressive strength, (d) longitudinal reinforcement ratio, (e) horizontal shear reinforcement ratio and (f) vertical shear reinforcement ratio.

deflection increases which results in wider flexural crack and therefore, the shear strength decreases [17].

3.3. Compressive strength of the concrete (f_c')

The shear strength is a function of the compressive strength, $f_c^{'}$. El-Sayed et al. [36] showed that the shear strength increased by 10.7% when $f_c^{'}$ increased by 44.5% ($f_c^{'}$ from 43.6 MPa to 63 MPa). However, the increase of shear strength is not proportional because, in the case of high strength concrete ($f_c^{'} > 60$ MPa), the fractured aggregates at ultimate load will generate less friction compared to normal strength concrete (Fig. 1c). Similarly, Smith and Vantsiotis's [12] investigation on deep beams showed that $f_c^{'}$ has a great influence on the shear strength. Their results showed that the strength is higher in the case of a deep beam with high $f_c^{'}$ and low web reinforcement compared to a beam with low $f_c^{'}$ and high web reinforcement. However, their tests were limited to only normal strength concrete ($f_c^{'}$ of 16 to 23 MPa). On the contrary, Londhe [21] showed that the compressive strength of concrete ($f_c^{'}$ of 24 to 37 MPa) has small effect on the shear increase of deep beams.

3.4. Longitudinal reinforcement (ρ)

Mau and Hsu [35] conducted 64 tests on deep beams and found that with the increase of longitudinal reinforcement, the shear strength of deep beam increased significantly. Similar studies by Ashour [34] and Londhe [21] found that the longitudinal reinforcement has a linear correlation with the shear strength up to a certain limit for deep beams without shear reinforcement and beyond that it has no effect. Longitudinal reinforcement increases the shear strength of deep beams by reducing the crack width, by improving the interface shear transfer mechanism and by increasing the dowel action [21]. Fig. 1d shows that the average shear strength increases linearly as the longitudinal reinforcement ratio increases up to 1.5% and beyond that it reaches a plateau.

3.5. Horizontal shear reinforcement (ρ_h)

Although the reason to provide the horizontal shear reinforcement is to improve the shear capacity, some previous studies showed that it has no effect on the shear strength [10]. This can be observed in Fig. 1e, where the average shear strength remains almost constant with the change in horizontal shear reinforcement. Other researchers found a

Table 2
Statistical analysis of deep beams equations with web reinforcement.

Ref.	*PF*	χ	*SD*	*VAR*	*CoV (%)*	*AAE (%)*
ACI 318 [27]	1.78	1.62	0.71	0.51	40.1	40.7
CSA 23.3 [28]	1.74	1.29	0.75	0.57	43.4	38.9
Ramakrishnan [6]	0.94	1.01	0.29	0.09	31.0	32.4
Kong et al [10]	1.03	1.05	0.35	0.12	34.0	32.9
Selvam [11]	1.02	0.53	0.49	0.24	48.3	61.2
Mau & Hsu [35]	0.81	0.76	0.17	0.03	20.5	30.4
Matamoros [31]	1.16	0.82	0.42	0.18	36.4	40.0
Arabzadeh [19]	1.28	1.17	0.49	0.24	38.6	29.1
Londhe [21]	1.45	1.33	0.76	0.59	52.8	42.1
Proposed [Eq. (5)]	0.99	1.02	0.19	0.04	19.2	13.8
Proposed [Eq. (6)]	1.01	1.07	0.26	0.07	25.7	22.2

small increase in shear strength with the increase in horizontal shear reinforcement [12]. This is especially the case in presence of low vertical shear reinforcement, where adding horizontal shear reinforcement ratio in deep beams have no further contribution to shear strength [12]. On the contrary, Ashour [34] reported that horizontal shear reinforcement is more effective compared to vertical shear reinforcement in case of $a/d < 0.75$.

3.6. Vertical shear reinforcement (ρ_v)

Vertical web reinforcement is one of the most important parameters that affect the shear strength of deep beams. The primary purpose of vertical web reinforcement is to provide confinement to the concrete which helps to improve the shear capacity of deep beams. In addition to this, it is more effective in improving the shear strength compared to horizontal shear reinforcement and in case of shear failure, it makes the beam fail in ductile manner. All previous studies showed that the shear strength of a deep beams increases linearly with the increase of the vertical shear reinforcement [4,10,12,16,18,20,32]. However, Smith and Vantsiotis [12] found that the contribution of the vertical shear reinforcement diminishes as the a/d decreases ($a/d < 1.0$). A similar study by Ashour [34] confirmed that with a higher the a/d ratio ($a/d > 0.75$), the contribution of the vertical web reinforcement was observed higher. On the contrary, Londhe [21] reported that the shear strength increase was observed up to 1.25% of vertical shear reinforcement ratio, a similar pattern is can be seen in Fig. 1f, where it shows that the shear strength increases up to a vertical shear reinforcement ratio of 1.42%, the average shear strength does not change much beyond 2% of vertical shear reinforcement ratio.

4. Review of shear strength prediction

This section presents a brief overview and comparison of previous shear equations proposed by Ramakrishnan and Ananthanarayana [6], Kong et al. [10], Selvam [11], Mau and Hsu [35], Matamoros and Wong [31], Arabzadeh [19], Londhe [21]. The shear strength of a deep beam was also calculated by both ACI 318 [27] and CSA A23.3 [28] equations. The shear equations are listed in Table A1.

The internal force system in the RC deep beam is very complex. Moreover, concrete is a non-homogeneous material and its stress–strain distribution is highly non-linear. Therefore, it is difficult to predict a theoretical solution for the shear strength of deep beams. Ramakrishnan and Ananthanarayana [6] proposed an equation to calculate the shear strength of deep beam from test results. A similar approach was adopted by Kong et al. [10] considering concrete cylinder splitting tensile strength as the main variable to predict the shear strength. Their proposed equation was a function of the shear span to depth ratio and the longitudinal and web reinforcement ratio. The equation was derived using 135 tests on deep beams. Selvam [11] proposed an equation from the equilibrium conditions considering the strength of concrete in compression and tension. The contribution of longitudinal reinforcement was also included. Mau and Hsu [35] derived a non-dimensional shear equation to predict the shear strength in deep beams considering the equilibrium condition of the effective shear element in the shear span. The equation is expressed in terms of four variables: shear span to depth ratio, compressive strength of concrete, horizontal shear reinforcement and vertical shear reinforcement. Matamoros and Wong [31] developed a design equation for the shear strength of deep beams based on the STM method. The authors developed a simplified model considering three load transfer mechanisms in the strut and tie and proposed a correction factor which was calibrated using 175 tests on deep beams. Similarly, Arabzadeh [19] developed an STM model for the ultimate shear strength of deep beams. He considered two load transfer mechanisms- diagonal concrete strut action by STM and resisting equivalent force perpendicular to the diagonal crack by shear reinforcement. Londhe [21] proposed an analytical model based on 27 experimental results on deep beams.

4.1. Model comparison

The models described in the previous section including ACI and CSA building code equations for deep beams were compared with the test results. The comparison was assessed with various statistical parameters such as: a) performance factor (*PF*): the ratio of the experimental shear strength to the calculated shear strength (V_{exp}/V_{cal}), b) χ factor: the inverse of the slope of a linear least-squares regression of the calculated shear strength (V_{cal}) versus the experimental shear strength (V_{exp}) plot, which represents the degree of over or underestimation of V_{cal} compared to V_{exp}, c) standard deviation (*SD*), d) sample variance (*VAR*), e) coefficient of variation (*CoV*), and f) average absolute error (*AAE*). The parameters are shown in Tables 2 and 3. In addition, a performance test was performed based on the performance factor (*PF*). A weighted penalty classification system was applied based on the demerit points classification model proposed by Collins [37]. The data points were categorized by their *PF* and a penalty (p) was applied to each of them. The penalties were chosen based on the structural safety. For example, high penalty value (p) is assigned on data points with PF less than one and those points are categorized as extremely dangerous ($p = 5$) and dangerous ($p = 3$). This is because data points with PF less than one is unacceptable in terms of safety. Similarly, high penalty value is also assigned for extremely conservative points ($p = 4$). The penalty value (p) for different *PF* is shown in Tables 4 and 5. The performance of each model was determined in terms of its performance index (*PI*). The *PI* is the summation of multiplying the number of data points in each category with their assigned penalty value. In order to better understand the shear strength, the database was divided into two groups, one with web reinforcement and the other without web reinforcement. The accuracy and performance of different models and code equations are reported in Tables 2–5.

4.1.1. Shear strength with web reinforcement

Table 2 shows the statistical comparison between the experimental

Table 3
Statistical analysis of deep beams equations without web reinforcement.

Ref.	*PF*	χ	*SD*	*VAR*	*CoV (%)*	*AAE (%)*
ACI 318 [27]	1.66	1.72	0.97	0.95	58.8	40.7
CSA 23.3 [28]	1.51	1.39	0.49	0.24	32.2	29.8
Ramakrishnan [6]	0.95	1.29	0.59	0.35	61.9	76.9
Kong et al [10]	1.11	1.37	0.69	0.48	62.6	59.7
Selvam [11]	0.84	0.63	0.42	0.17	49.7	68.2
Mau & Hsu [35]	0.95	1.01	0.23	0.05	24.0	22.0
Matamoros [31]	1.22	1.15	0.38	0.14	30.9	24.2
Arabzadeh [19]	0.96	1.11	0.30	0.09	31.7	29.6
Londhe [21]	1.75	2.01	1.02	1.04	58.3	39.6
Proposed [Eq. (7)]	0.99	1.07	0.23	0.05	23.2	19.2

Table 4
Performance index for deep beams equations with web reinforcement.

PF	Classification	p	[27]	[28]	[6]	[10]	[11]	[35]	[31]	[19]	[21]
< 0.75	Extremely dangerous	5	10	11	57	60	89	93	51	25	45
0.75–1.00	Dangerous	3	5	24	98	62	56	144	30	47	43
1.00–1.25	Reduced safety	0	39	43	81	80	38	27	66	83	38
1.25–1.75	Appropriate safety	1	108	79	28	56	66	2	104	71	69
1.75–3.00	Conservative	2	80	93	2	8	17	0	15	40	61
> 3.00	Extremely conservative	4	24	16	0	0	0	0	0	0	10

and calculated shear strength for deep beams with web reinforcement. In the case of shear strength prediction with web reinforcement, both ACI and CSA codes were more conservative compared to the analytical models. Although the *PF* value was close for both of them, ACI had higher χ value (1.62) than CSA (χ = 1.29) which implies that the ACI code is more conservative. Moreover, both equations had very high *SD* and *CoV* compared to other models.

The average *PF* of Selvam [11] model was only 1.02 which seemed to be a good prediction, but, the equation produced very low χ value (0.53) which indicates that the calculated shear strength is overpredicted. This resulted in a high *AAE* (61.2%). Therefore, Selvam [11] model was the least accurate model. A similar conclusion can be made for Ramakrishnan and Ananthanarayana [6] and Mau and Hsu [35] models. Although these models had very low *SD*, *VAR*, *CoV* and *AAE*, the χ value (χ < 1) made the model unsafe to be used in the design since it is overpredicted the experimental shear strength.

Among the analytical models, Londhe's [21] model was the most conservative model since *PF, χ, SD, CoV* values were quite high. The equations proposed by Kong et al. [10], Matamoros and Wong [31] and Arabzadeh [19] predicted the shear strength quite accurately and had low SD, COV, and AAE values compared to the code equations. Among these three models, the χ value of the model proposed by Matamoros and Wong [31] seems to be unsafe (χ = 0.82). Therefore, the remaining proposed models by Kong et al. [10] and Arabzadeh [19] seem to be better and safe to be used in the design since they had low *SD*, *CoV* and *AAE* values. Table 2 shows that the Kong et al. [10] model produced much lower *PF*, χ, *SD* and *CoV* values compared to the Arabzadeh [19] model. On the contrary, Arabzadeh [19] model produced lower *AAE* and higher *CoR* values than the Kong et al. [10] model which makes the model more accurate.

The performance index based on weighted penalty classification could give an indication of the safety of the predicted model. The weighted penalty on each model was applied based on “Demerits Point Classification” proposed by Collins [37]. The penalty (*p*) value was assigned based on the PF value from the statistical analysis where the classifications of penalty are from extremely dangerous (*p* = 5) to extremely conservative (*p* = 4). Table 3 shows the performance test analysis of each model where a lower performance index (*PI*) value indicates a better model prediction. From statistical analysis, it was observed that both ACI and CSA codes are too conservative. But comparing the performance index with different equations, it was observed that the codes equations were better model since the *PI* was the lowest. Among the code equations, ACI equation produced low a *PI* of 429 compared to the CSA of *PI* = 456 and it was the second-best predicted model where only few points were in the dangerous zones. The average *PF* for most of the models [6,10,11,35,21] was found close to one and below the code equations. Comparing the *PI* values, most of the models [6,10,11,35,21] showed poor performance in predicting the shear strength since many predicted points remained below the safe zone. Only Matamoros and Wong [31] and Arabzadeh [19] proposed models showed very good prediction and had the lowest *PI* values of 479 and 417, respectively compared to other analytical models. However, observing the χ value, it can be concluded that Matamoros and Wong [31] was not a safe model. Therefore, the model proposed by Arabzadeh [19] was the most accurate and safe model that could predict the shear strength of deep beams with web reinforcement.

The predicted shear strength with web reinforcement is plotted against the experimental shear strength and presented as seen in Figs. 2a–10a. Comparing the ACI and CSA models (Figs. 2a and 3a), CSA model has more points in the dangerous zone i.e., above the 45^0 line. Therefore, it is safer to use the ACI equation for predicting the shear strength of deep beams with web reinforcement which produced relatively low *PI*. The shear equation models by Selvam [11], Mau and Matamoros [31] (Figs. 6a, 7a, and 8a) showed that most points were in the danger zone and therefore these models are unacceptable for design purposes. Among the analytical models, Arabzadeh [19] (Fig. 9a) showed good agreement with the experimental results and has fewer points in the dangerous zone.

4.1.2. Shear strength without web reinforcement

Table 3 shows the statistical comparison between the experimental and calculated shear strength for deep beams without web reinforcement. The STM code equations for predicting shear strength without web reinforcement were more conservative than the analytical models except for Londhe [21]. Comparing ACI and CSA equations, ACI was more conservative which is reflected in the χ value (1.72) that was 24% higher than the CSA value. In addition, ACI equation had very high *SD* (0.97), *VAR* (0.95), *CoV* (59%), *AAE* (41%) which were respectively 100%, 296%, 82% and 36% higher than the CSA values.

Selvam's [11] proposed model was found the least accurate since it had the lowest χ value (0.63) and a high *AAE* value (68.15%). On the contrary, Londhe [21] model was the most conservative model where it's average *PF* and χ values were 1.75 and 2.01, respectively. Moreover, the *SD*, *VAR,* and *CoV* values were found very high, therefore, it was observed that the calculated versus the experimental shear strength has too much scattered on the V_{cal} vs V_{exp} plot (Fig. 6b). In case of Mau and Hsu [35] and Arabzadeh [19] models, in spite of low *SD*, *CoV*, *VAR*, and *AAE* values, the models still overpredicted the shear strength since

Table 5
Performance index for deep beams equations without web reinforcement.

PF	Classification	p	[27]	[28]	[6]	[10]	[11]	[35]	[31]	[19]	[21]
< 0.75	Extremely dangerous	5	9	1	34	35	44	16	8	22	7
0.75–1.00	Dangerous	3	8	8	20	13	14	34	12	32	11
1.00–1.25	Reduced safety	0	20	18	12	6	12	23	34	14	17
1.25–1.75	Appropriate safety	1	18	29	9	14	12	11	23	16	21
1.75–3.00	Conservative	2	21	28	9	14	2	0	7	0	21
> 3.00	Extremely conservative	4	8	0	0	2	0	0	0	0	7

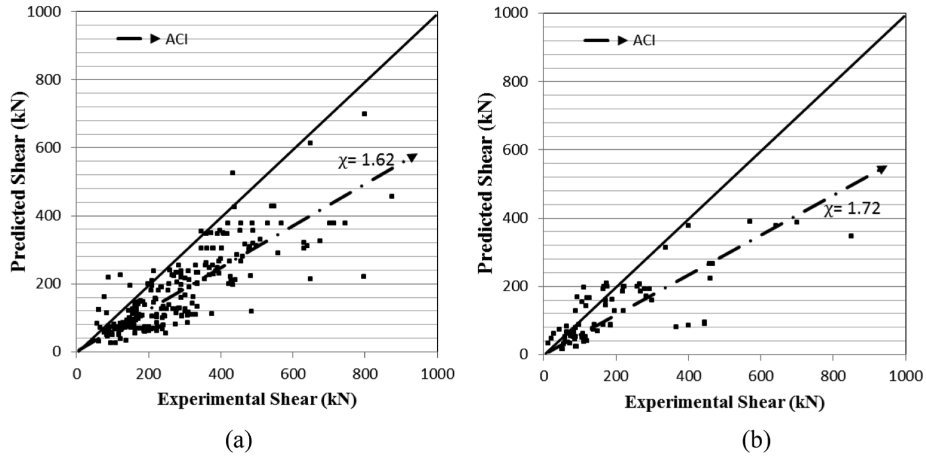


Fig. 2. Shear strength prediction by ACI [27]: (a) with (b) without web reinforcement.

their average PF is just below one. Comparing all the models, it could be concluded that Matamoros and Wong [30] analytical model predicted the shear strength more accurately than the other models since both *PF* (1.22) and χ value (1.15) are above one and the values were less conservative than the code equations. Moreover, except for Mau and Hsu's [35] model, *SD*, *CoV*, *VAR* and *AAE* values of the Matamoros and Wong [30] model were lower than the other models.

The performance index (*PI*) analysis for shear strength prediction of RC deep beams without web reinforcement is shown in Table 5. The *PI* analysis showed that the ACI STM model was highly conservative compared to the CSA code equation (*PI* of ACI = 161; *PI* of CSA = 114) which was already proven from the statistical analysis. The analytical models were primarily developed to predict the shear strength with web reinforcement. Therefore, from the performance index, it was clearly observed that these models [6,10,11,35,19] were poor in predicting the shear strength without web reinforcement. Consequently, they produced very high *PI* value (Table 5). As a result, > 50% of the predicted points were below the safe zone. Only two models, Matamoros and Wong [31] and Londhe [21], had low *PI* values (113 and 159, respectively). However, from the statistical analysis, it was clear that Londhe [21] model is highly conservative where 33% of the points were in the conservative zone. In summary, it could be concluded that Matamoros and Wong [31] proposed an analytical model that could predict the shear strength of deep beams without web reinforcement accurately and safely. Along with the Matamoros and Wong [31] model, CSA 23.3 [28] model was another good method for accurately predicting the nominal strength of deep beams.

The predicted shear strength without web reinforcement was plotted against the experimental shear strength and presented in Figs. 2b–10b. Unlike deep beams with web reinforcement, the CSA predicted shear strength showed better performance than the ACI where only 10% of the points are falling in the danger zone. Except Selvam [11] (Fig. 6a), linear regression lines for all the analytical models were below the 45^0 line. Fig. 7b, 8b, 9b showed that Mau and Hsu [35], Matamoros and Wong [31] and Arabzadeh [19] were better models since the predicted points were less scattered and within the narrow range of the 45^0 line.

5. Improved shear equations for deep beams

From the discussion in the previous section, it was observed the existing shear equations for deep beams either overly conservative or unsafe to be used when compared to experimental results. Therefore, the present research proposed an improved model to predict the shear strength of deep beams developed by genetic algorithm (GA). The parameters affecting the shear strength of deep beams also identified from factorial design.

5.1. Factorial design

Factorial designs are widely used to identify the important parameters affecting the experimental results [38]. Moreover, it examines the interaction effect among the variables. The present study only considered the 2^k factorial design in which there are two levels of each factor that affects the experimental response. These levels are quantitative values and each represents a high and a low side of the data

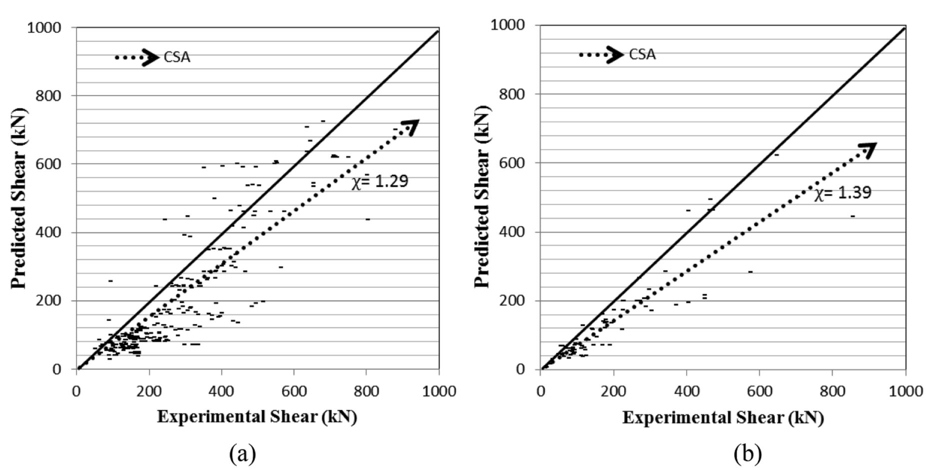


Fig. 3. Shear strength prediction by CSA [28]: (a) with (b) without web reinforcement.

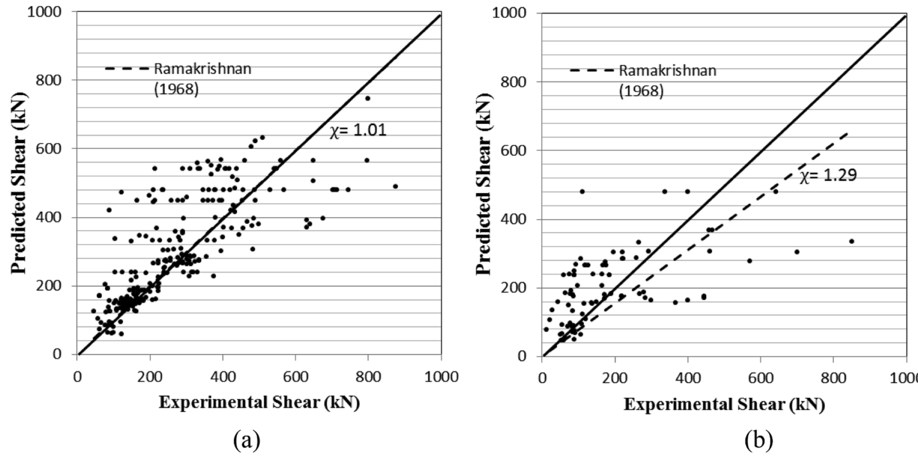


Fig. 4. Shear strength prediction by Ramakrishnan [6]: (a) with (b) without web reinforcement.

range. For the factorial design, the response of the output was considered linear over the ranges of the variables with a 5% risk considered in the factorial design to reject the null hypothesis i.e., the probability of 'Type I' error, $\alpha = 0.05$ [39].

In this study, the response was the shear strength of a deep beam, while from the experimental results reported in the literature, there were 8 variables that influenced this response, namely, b, h, f_c', f_y, a/d, ρ, ρ_h, ρ_v (Table 1). In order to reduce the number of variables, the response and the main variables were normalized following Eq. (1). The reinforcement ratios were normalized by multiplying them by the ratio of the steel yield strength to the concrete compressive strength (f_y/f_c') in order to include the variation of both f_y and f_c'in the proposed equation, while the shear strength was normalized by f_c' and the cross-section of the tested beams ($b \times h$). With the reduced number of variables, the general shear equation for a deep beam with and without web reinforcement is presented in Eqs. (3) and (4).

$$\begin{cases} \hat{V}_{test} = \frac{V_{test}}{f'_c bh} \\ \hat{\rho} = \rho\left(\frac{f_y}{f'_c}\right) \\ \hat{\rho}_h = \rho_h\left(\frac{f_y}{f'_c}\right) \\ \hat{\rho}_v = \rho_v\left(\frac{f_y}{f'_c}\right) \end{cases} \tag{2}$$

$$\hat{V} = f\left(\frac{a}{d},\ \hat{\rho},\hat{\rho}_h,\ \hat{\rho}_v\right) \tag{3}$$

$$\hat{V} = f\left(\frac{a}{d},\ \hat{\rho}\right) \tag{4}$$

The percent contribution and the P-value for each factor were calculated to identify the most significant factors as well as the interactions among the factors. The P-value is the probability of obtaining a test statistics at least as extreme as the observed value when the null hypothesis is true [38]. Two-way, three-way and four-way interactions were considered. The interactions helped include the higher-order nonlinear terms in the prediction of shear equations which eventually improved the accuracy of the equations.

5.1.1. Deep beam with web reinforcement

Table 6 presents the results from the 2^4 factorial design for a deep beam with web reinforcement. The P-value for the main parameters a/d, $\hat{\rho}$, $\hat{\rho}_h$ and $\hat{\rho}_v$ were found to be 0.0, 0.029, 0.192 and 0.018, respectively. Comparing with $\alpha = 0.05$, it can be concluded that the risk to reject the null hypothesis for a/d, $\hat{\rho}$ and $\hat{\rho}_v$ was < 5%. This indicated that among the four variables, a/d, $\hat{\rho}$ and $\hat{\rho}_v$ are the most significant parameters that affect the shear strength of the RC deep beams. Beside the P-value, the percent contribution of a/d ratio was around 56%, which indicates that a/d has the highest influence on the shear strength of deep beams with web reinforcement.

Moreover, from the factorial design, it was observed that the parameters were highly interacting with each other which eventually contributed non-linearly to the shear strength. Among the two-way interactions, $\hat{\rho}_h$ and $\hat{\rho}_v$ produced the lowest P-value of 0.027 ($\alpha = 0.05$). The parameters $\hat{\rho}$ and $\hat{\rho}_h$ were also interacting with each other and

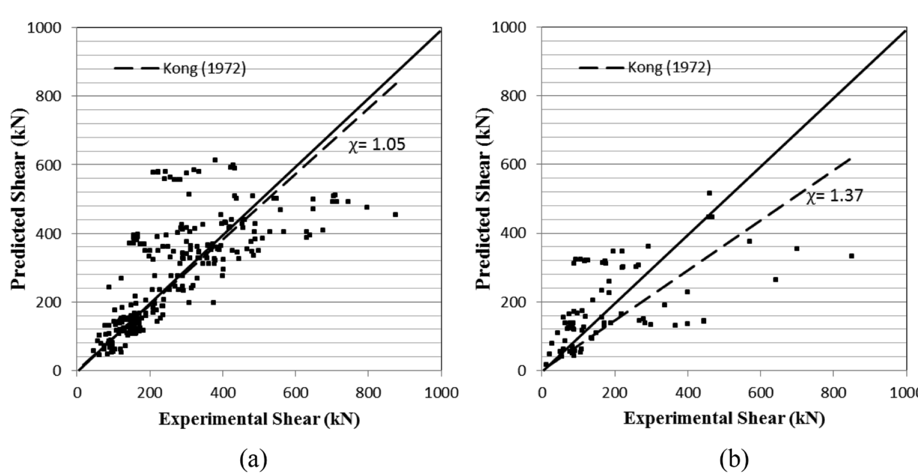


Fig. 5. Shear strength prediction by Kong [10]: (a) with (b) without web reinforcement.

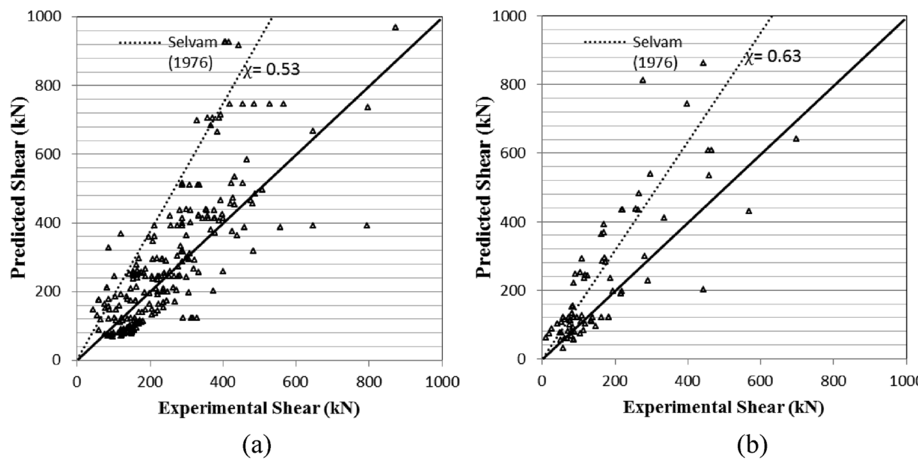


Fig. 6. Shear strength prediction by Selvam [11]: (a) with (b) without web reinforcement.

produced a *P*-value of 0.07, but this interaction was ignored since it exceeded the value for $\alpha = 0.05$. Three-way and four-way interactions were also observed in the analysis. Among the three-way interactions, $\hat{\rho}$, $\hat{\rho}_h$ and $\hat{\rho}_v$ produced a *P*-value of 0.001 and their total percent contribution was 8.15% which was the second-highest after the contribution of a/d ratio. The parameters a/d, $\hat{\rho}_h$ and $\hat{\rho}_v$ were also highly interacting with each other and therefore, a low *P*-value of 0.009 was obtained. All the parameters were found to interact four-way (four-way interaction) and produced a *P*-value of 0.001. The significant and non-significant parameters and their interactions were plotted in normal plot (Fig. 11).

5.1.2. Deep beam without web reinforcement

Similar to the shear strength of deep beam with web reinforcement, the statistical analysis results from the factorial design for a deep beam without web reinforcement are presented in Table 6. The *P*-values for the main parameters a/d and $\hat{\rho}$ are 0.001 and 0.006, respectively. Therefore, comparing with $\alpha = 0.05$, it could be concluded that both of them are important parameters which were also confirmed from the normal plot of significant and non-significant parameters (Fig. 12). The percent contribution of a/d ratio is approximately 55%, which was found similar to the previous analysis for a deep beam with web reinforcement. The other important parameter was the main longitudinal reinforcement $\hat{\rho}$, which has a percent contribution of 38%. The interaction between a/d and $\hat{\rho}$ was found not to be significant (*P*-value of 0.25).

5.2. Shear equations by genetic algorithm

Genetic Algorithm (GA) was first developed by John Holland [40]. The basic mechanism of the GAs was based on biological evolution. GA has a wide area of applications which is used to understand the processes in natural systems. Previous research showed that GA techniques are very effective and accurate for developing analytical model [39,41–43]. The present study used the GA technique to develop an equation to predict the shear strength of deep beams. The same two groups from the experimental database were used to develop the shear equations for deep beams with and without web reinforcement. The analysis from the factorial design was incorporated in the proposed equations by GA. The commercial software program Eureqa [44] was used to run the GA program. The primary population size used was 1000. The mean square error was used as a fitness function to search for a solution. The mathematical operators $\{+, -, *, /, \sqrt{x}, x^2, x^a\}$ were considered for the program runs.

5.2.1. Shear equation for deep beam with web reinforcement

From the factorial design, it was found that a/d, $\hat{\rho}$ and $\hat{\rho}_v$ were the most significant parameters that contribute to the shear strength. In addition, 2-way, 3-way and 4-way interactions among the parameters were observed between a/d, $\hat{\rho}$, $\hat{\rho}_h$ and $\hat{\rho}_v$. Therefore, in addition to the main parameters, these high order non-linear interaction terms were included in the proposed equation in order to achieve the highest accuracy. The shear equation developed by the GA technique with all the important parameters and their interactions is presented in Eq. (5).

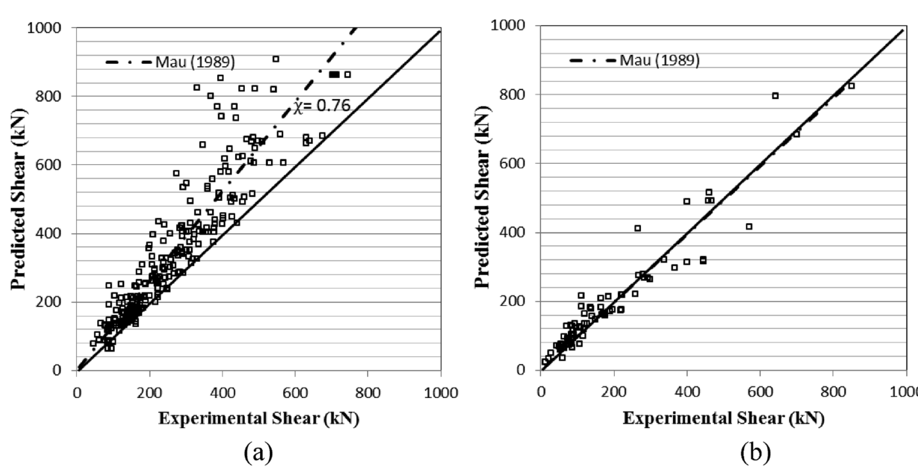
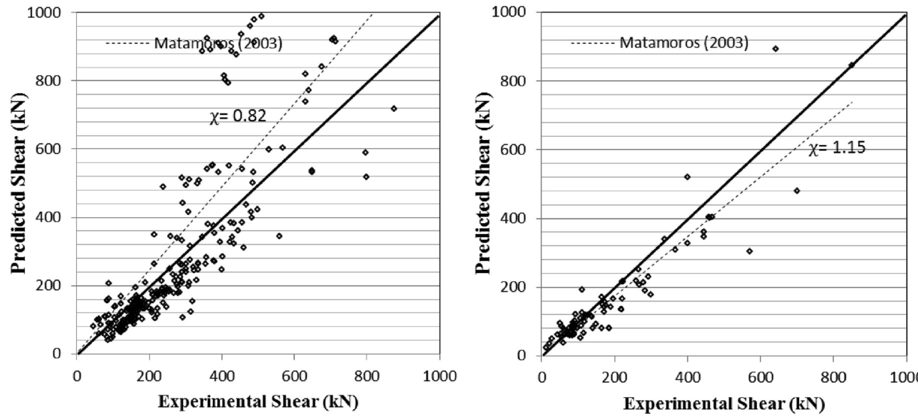


Fig. 7. Shear strength prediction by Mau and Hsu [35]: (a) with (b) without web reinforcement.



(a) (b)

Fig. 8. Shear strength prediction by Matamoros [31]: (a) with (b) without web reinforcement.

$$V = 0.1976 - 0.5897(a/d)^{0.1625} + 0.6215\hat{\rho}^{0.09661} + 0.4631\hat{\rho}_v^{1.185} - 320.3(\hat{\rho}_h\hat{\rho}_v)^{2.391} + 20.63(\hat{\rho}\hat{\rho}_h\hat{\rho}_v)^{1.884} + 20.14((a/d)\hat{\rho}_h\hat{\rho}_v)^{1.073} - 21.04((a/d)\hat{\rho}\hat{\rho}_h\hat{\rho}_v)^{0.9088} \tag{5}$$

For design purposes, Eq. (5) was simplified without losing accuracy. For the simplified equation, only the parameters and interactions which have the percent contribution higher than 5% were considered (Table 6). The GA was performed again and the simplified equation was proposed (Eq. (6)). By ignoring some parameters and their interactions, a small amount of accuracy was sacrificed, but eventually, the equation became quite simple.

$$V = \frac{2}{5} - \frac{1}{4}(a/d)^{0.23} + 0.85(\hat{\rho}\hat{\rho}_h\hat{\rho}_v)^{1/10} - \frac{3}{5}((a/d)\hat{\rho}_h\hat{\rho}_v)^{1/16} - 200((a/d)\hat{\rho}\hat{\rho}_h\hat{\rho}_v)^{2.65} \tag{6}$$

5.2.2. Shear equation for deep beam without web reinforcement

Similarly, from the factorial design for deep beams without web reinforcement, it was observed that a/d and $\hat{\rho}$ were the most significant parameters that affected the shear strength and there was very little interaction between them. Therefore, only these two terms were included in the GA solution for the shear equation. The equation from the GA analysis for deep beam without web reinforcement is presented in Eq. (7).

$$V = 1.74 - 2(a/d)^{0.044} + \frac{1}{2}\rho^{0.14} \tag{7}$$

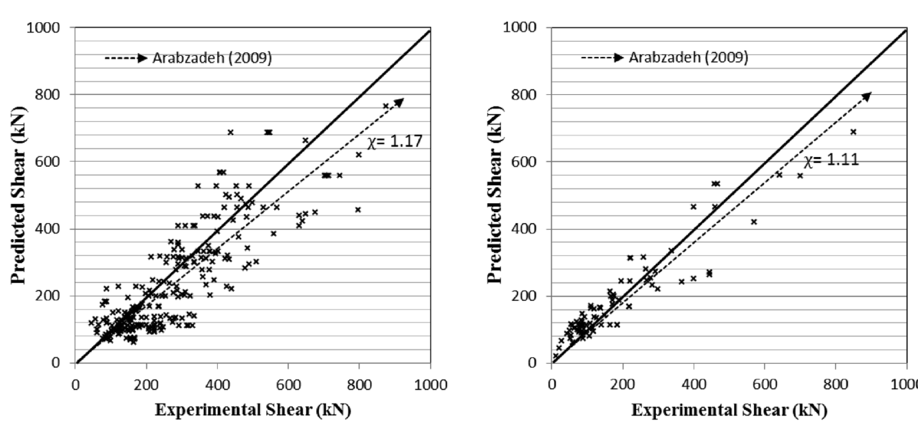
5.3. Performance of the proposed shear equations

The performance of the proposed shear equations was compared with the ACI and CSA codes against various statistical parameters as described in Section 5 e.g. *PF, χ, SD, VAR, CoV,* and *AAE*.

5.3.1. Shear equation for deep beam with web reinforcement

The statistical analysis and comparison between the proposed equations and the ACI and CSA STM models are presented in Table 2. The statistical analysis showed that both the ACI and CSA STM model were too conservative. The average PF for ACI and CSA equations was 1.78 and 1.74. On the other hand, the proposed equations (general and simplified) have an average PF close to 1 which is also reflected in the χ value. The statistical parameters e.g. standard deviation (*SD*), variance (*VAR*) and the coefficient of variation (*CoV*) were reduced significantly in the case of the proposed equation compared to the code equations. The *CoV* for the original ACI and CSA STM equation was > 40% whereas it was reduced to less than one-half (19%) in the case of the proposed shear equation. The AAE of the proposed equation was only one-third of that of the original ACI and CSA STM model. Fig. 13a illustrates the comparison between the proposed shear equations with the ACI and CSA STM model. The linear least square regression line along with the χ value indicated the improvement in accuracy achieved using the proposed shear equations compared to the code equations.

As expected, comparing the simplified equation to the general proposed equation, the accuracy was reduced. The standard deviation of the simplified equation increased from 0.19 to 0.26. Similarly, the *CoV* increased from 19.19% to 25.74% and the average absolute error



(a) (b)

Fig. 9. Shear strength prediction by Arabzadeh [19]: (a) with (b) without web reinforcement.

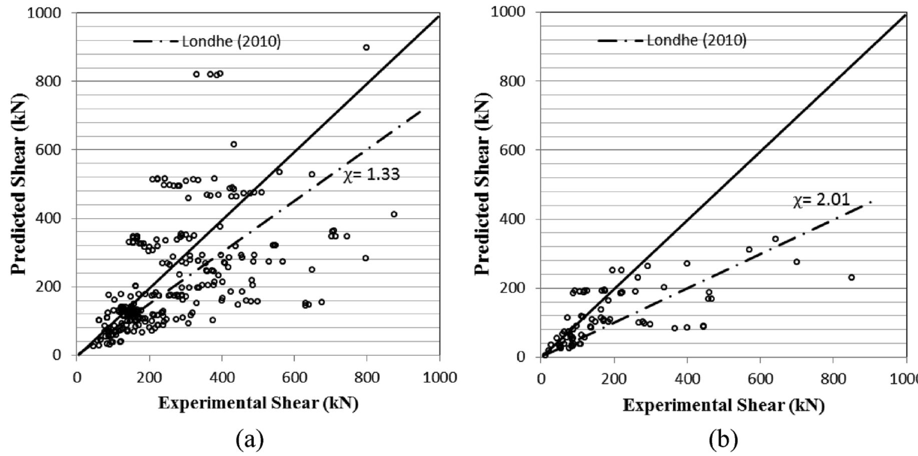


Fig. 10. Shear strength prediction by Londhe [21]: (a) with (b) without web reinforcement.

Table 6
Factorial analysis chart.

Deep beam with web reinforcement		
Parameters	% Contribution	P
Main effects		
(a/d)	55.52	0
$\hat{\rho}$	3.60	0.029
$\hat{\rho}_h$	1.28	0.192
$\hat{\rho}_v$	4.28	0.018
2-way Interactions		
$(a/d) * \hat{\rho}$	1.88	0.115
$(a/d) * \hat{\rho}_h$	0.01	0.897
$(a/d) * \hat{\rho}_v$	0.03	0.839
$\hat{\rho} * \hat{\rho}_h$	2.48	0.07
$\hat{\rho} * \hat{\rho}_v$	1.98	0.106
$\hat{\rho}_h * \hat{\rho}_v$	3.70	0.027
3-way Interactions		
$(a/d) * \hat{\rho} * \hat{\rho}_h$	2.53	0.068
$(a/d) * \hat{\rho} * \hat{\rho}_v$	1.27	0.195
$(a/d) * \hat{\rho}_h * \hat{\rho}_v$	5.15	0.009
$\hat{\rho} * \hat{\rho}_h * \hat{\rho}_v$	8.15	0.001
4-way Interactions		
$(a/d) * \hat{\rho} * \hat{\rho}_h * \hat{\rho}_v$	8.14	0.001
Deep beam without web reinforcement		
Parameters	% Contribution	P
(a/d)	55.20	0.001
$\hat{\rho}$	38.30	0.006
$(a/d) * \hat{\rho}$	6.51	0.249

increased from 14% to 22%. Despite this, the statistical indicators (*PF, χ, SD, VAR, COV, AAE*) for the simplified equation were lower than the code equations. Finally, it is worth mentioning that the proposed simplified equation was much simpler to implement and use compared to the code equations.

5.3.2. Shear equation for deep beam without web reinforcement

Table 3 shows the comparison between the proposed equation and the ACI and CSA STM models. Similar to the equation with web reinforcement, both the ACI and CSA STM models were highly conservative. The average *PF* for ACI and CSA equation was 1.66 and 1.51, respectively. On the other hand, the average *PF* for the proposed equation was only 0.99. Moreover, the χ value of the proposed equation was only 1.07 lower than the ACI and the CSA STM models (1.72 and

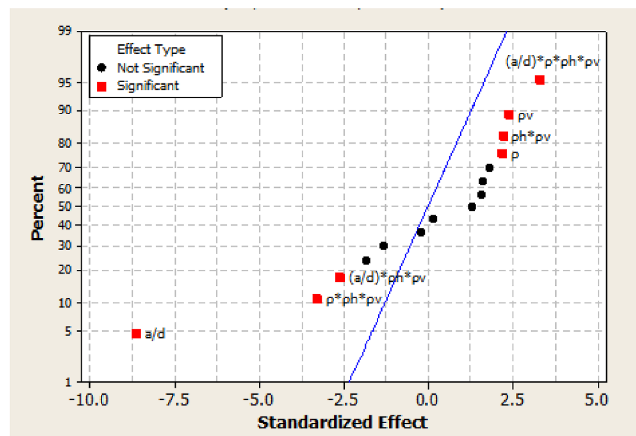


Fig. 11. Normal plot for significant and non-significant factors for deep beams with web reinforcement.

1.39, respectively). Besides this, the descriptive statistical parameters i.e., *SD, VAR,* and *CoV* were reduced significantly in the case of the proposed equation compared to the ACI and the CSA STM models. The *CoV* for the ACI and CSA STM was quite high and among them CSA STM model produced the lower *CoV* compared to ACI. The *CoV* for the proposed predicted equation was reduced by 60% and 28% compared to the ACI and CSA model, respectively. The average error of the proposed equation was reduced to 53% compared to the ACI model and 36% compared to the CSA model. The linear least-square regression line is plotted in Fig. 13b which shows the improvement made by the proposed shear equation in terms of accuracy.

5.4. Reliability analysis

Reliability is a measure of the likelihood of failure. In structural engineering, it is the margin of safety of a structural system at the ultimate limit state (i.e., just before collapse). The relevant specifications of concrete components are generally conservative in order to maintain structural safety. At the same time, it should not be over-conservative to ensure economic design. Hence, it is necessary to reliably estimate safety factors for new models or systems up to a target of reliability level which is introduced in the design codes. In a reliability model, *Q* represents the load effect on the structural system and *R* represents the resistance of the structural system. Both *Q* and *R* are the random

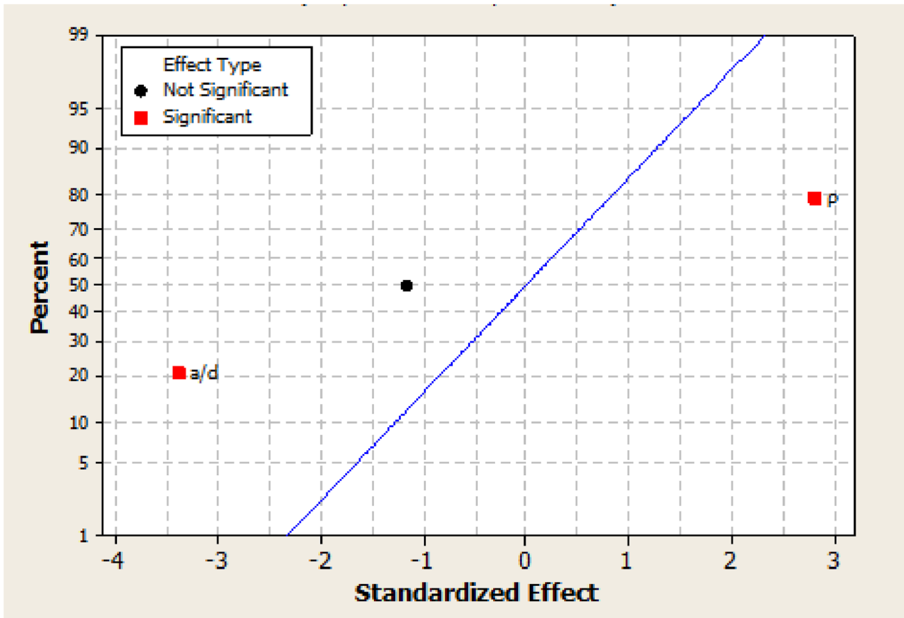
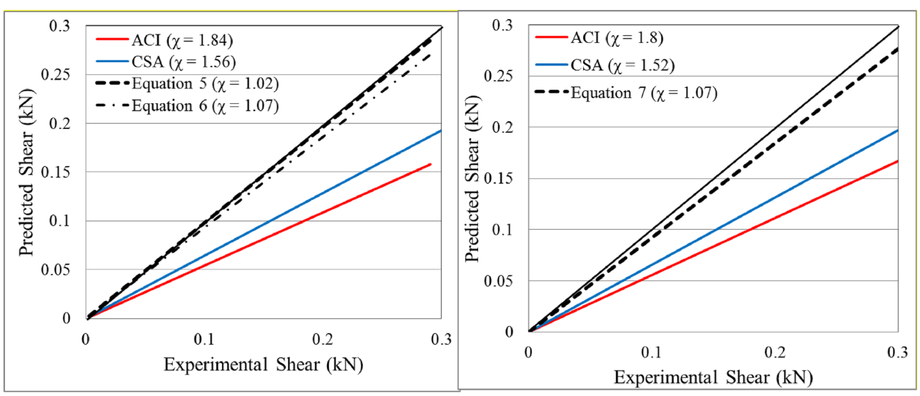
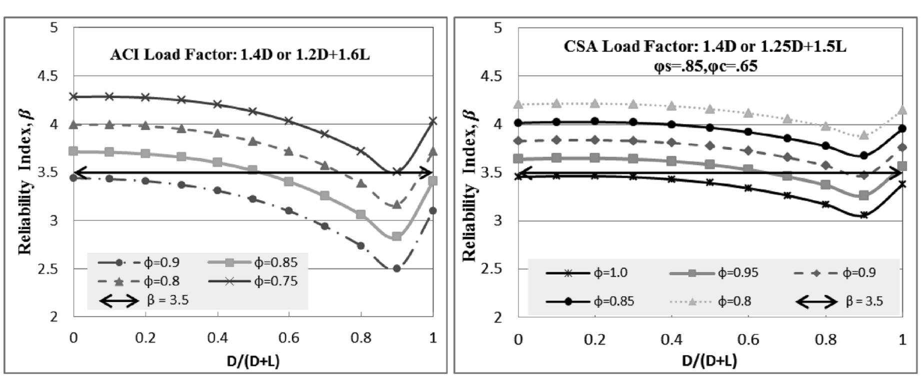


Fig. 12. Normal plot for significant and non-significant factors for deep beams without web reinforcement.



(a) (b)

Fig. 13. Comparison of proposed shear equations: predicted vs experimental:(a) with and (b) without web reinforcement.



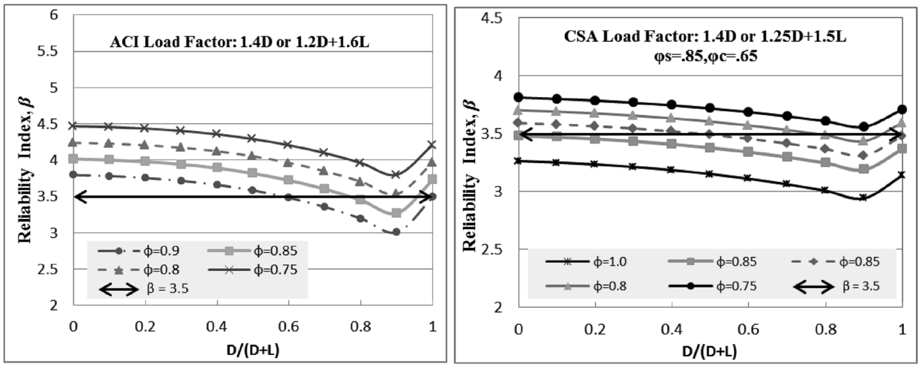
(a) (b)

Fig. 14. Reliability index for the proposed shear Eq. (5) for deep beams with web reinforcement: using (a) ACI load factors and (b) CSA load factors.

variables which can be determined from probability density functions. Nowak and Collins [45] described various methods to calculate reliability. In general, reliability can be expressed by a 'limit state function' or 'performance function' g such as

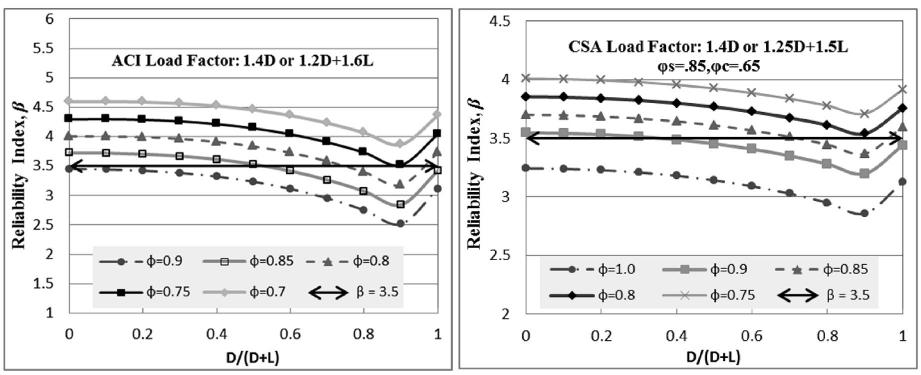
$$g(R, Q) = R - Q \tag{8}$$

where, g is the safety margin. If $g \geqslant 0$, the structure is safe (desired performance); if $g < 0$, the structure is unsafe (undesired performance).



(a) (b)

Fig. 15. Reliability index for the proposed simplified shear Eq. (6) for deep beams with web reinforcement: using (a) ACI load factors and (b) CSA load factors.



(a) (b)

Fig. 16. Reliability index for the proposed shear Eq. (7) for deep beams without web reinforcement: using (a) ACI load factors and (b) CSA load factors.

Table 7
Reliability index and resistance factors.

Proposed equation	Load Cases	Reliability Index, β				Recommended φ
		φ = 0.9	φ = 0.85	φ = 0.8	φ = 0.75	
Eq. (5)	ACI	2.50	2.83	3.16	3.50	0.75
	CSA	3.47	3.68	3.89	4.10	0.85
Eq. (6)	ACI	3.01	3.27	3.53	3.80	0.8
	CSA	3.19	3.31	3.44	3.56	0.75
Eq. (7)	ACI	2.52	2.84	3.18	3.52	0.75
	CSA	3.20	3.37	3.54	3.71	0.8

The probability of failure is equal to the probability that the undesired performance will occur. The probability of failure, P_f can be expressed as,

$$P_f = P(R - Q < 0) = P(g < 0) \quad (9)$$

If R and Q are normally distributed, then the reliability index (β) can be expressed as [45],

$$\beta = \frac{m_R - m_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \quad (10)$$

where, m_R, m_Q are the mean values of resistance and total load effect, respectively and σ_R, σ_Q are the standard deviation of resistance and total load, respectively. The reliability index can be considered as a function of failure P_f.

$$\beta = -\Phi^{-1}(P_f) \quad (11)$$

where, Φ^{-1} is the inverse standard normal distribution function.

5.4.1. Resistance models

The resistance of a structure can be expressed as,

$$R = R_n \times M \times F \times P_r \quad (12)$$

where, R_n is the nominal resistance, M is the material factor, F is the fabrication factor, and P_r is the professional factor. The bias factor and the coefficient of variation of the resistance, R, is given by,

$$m_R = R_n \times \lambda_M \times \lambda_F \times \lambda_P \quad (13)$$

$$V_R = (V_M^2 + V_F^2 + V_P^2)^{1/2} \quad (14)$$

where, λ_M, λ_F, λ_P are the bias factors of M, F, P_r and V_M, V_F, V_P are the coefficients of variation of M, F, P_r, respectively.

For the present study, the bias factor and the coefficient of variation for material and fabrication were taken from the previous research performed by Nowak and Szerszen [46]. The professional factor, P_r, is the ratio of the experimental shear strength to the shear strength predicted by the optimized shear equations. Statistics of the professional factors such as the bias factor, standard deviation, and coefficient of variation for each optimized shear equations were computed separately.

5.4.2. Load models

The present study only considered gravity loads, namely the dead and live load combinations in the load model. The statistical parameters for the dead and live loads were obtained from the previous research by Nowak [47]. For example, the bias factor for cast-in-place concrete is 1.05 with a coefficient of variation of 0.10. The bias factor for a 50-year live load is 1.0 with a coefficient of variation of 0.18. Both the ultimate limit state load cases specified by ACI 318 and CSA A23.3 building codes were considered. Therefore, the shear equations were calibrated for ACI 318 and CSA A23.3 ultimate limit states and two sets of resistance factors were calculated. The ACI 318 and CSA A23.3 ultimate limit state are described in Eqs. (15) and (16), respectively:

$$\begin{gathered} 1.4D < \phi R \\ 1.2D + 1.6L < \phi R \end{gathered} \tag{15}$$

$$\begin{gathered} 1.4D < \phi R \\ 1.25D + 1.5L < \phi R \end{gathered} \tag{16}$$

where, D is the dead load, L is the live load and ϕ is the resistance factor

Reliability indexes for the shear equations were calculated for the full range of $D/(D + L)$ ratio. The resistance factor for each shear equation was recommended based on the target reliability index $\beta_T$of 3.5. In the case of CSA A23.3, the resistance factors were calculated by keeping the original resistance for concrete ($\phi_c = 0.65$)and steel ($\phi_s = 0.85$) recommended in the code. For design purposes, an additional resistance factor, ϕ was recommended in addition to $\phi_c$and ϕ_s.

5.4.3. Results from the reliability analysis

The reliability index values for the proposed shear equations with and without web reinforcement are presented in Figs. 14, 15 and 16. The resistance factors were reported to the nearest of 0.05. The analysis showed that the reliability index skewed at $D/(D + L)$ ratio of 0.9 which was considered as a critical point. This is because, at $D/(D + L)$ ratio of 0.9, the mean values of the resistance (m_R) were found to be minimum and therefore, the difference between the mean value of the resistance (m_R) and the total load effect (m_Q) was lower than at other load ratios. The recommended resistance factors (ϕ) for the ACI and CSA load factors were 0.75 and 0.85, respectively for the proposed general equation with web reinforcement. On the other hand, the recommended ϕ values for the ACI and CSA load factors were 0.8 and 0.75, respectively for the simplified shear equation. Similarly, the resistance factors (ϕ) for the proposed shear equation without web reinforcement were found to be 0.75 and 0.8 for the ACI and CSA load factors, respectively. The reliability index and the recommended resistance factors are presented in Table 7. The design shear equations with and without web reinforcement are presented in Eqs. (17) and (18) where, Eq. (17) is the simplified shear equation for deep beam with web reinforcement.

$$\begin{gathered} V_u = \frac{2}{5} - \frac{1}{4}(a/d)^{0.23} + 0.\,85(\hat{\rho}\hat{\rho}_h\hat{\rho}_v)^{1/10} - \frac{3}{5}((a/d)\hat{\rho}_h\hat{\rho}_v)^{1/16} - 200((a/d)\hat{\rho}\hat{\rho}_h\hat{\rho}_v)^{2.65} \\ \text{ACI: } \phi = 0.8,\ \ \text{CSA: } \phi = 0.75 \text{ (in addition to } \phi_c = 0.65 \text{ and } \phi_s = 0.85) \end{gathered} \tag{17}$$

$$\begin{gathered} V = 1.\,74 - 2(a/d)^{0.044} + \frac{1}{2}\rho^{0.14} \\ \text{ACI: } \phi = 0.75,\ \ \text{CSA: } \phi = 0.8 \text{ (in addition to } \phi_c = 0.65 \text{ and } \phi_s = 0.85) \end{gathered} \tag{18}$$

6. Summary

This paper presented a comparative study of the shear strength prediction models of RC deep beams with and without web reinforcement. The accuracy of the models was compared against 381 tests on deep beams. Simplified and improved shear equations were proposed for deep beams with and without web reinforcement by genetic algorithm. A factorial design was conducted to identify the important parameters affecting the shear strength of deep beams. Finally, the resistance factors for the proposed equations were calculated based on reliability analysis. The following conclusions can be drawn from the study:

- Based on the statistical analysis it was observed that both the ACI and CSA STM models underestimated the experimental shear strength.
- Among the previous 7 analytical models compared in this research, it was clear from the that Londhe's [21] proposed model was the most conservative model for predicting the shear strength of deep beams both with and without web reinforcement.
- The results from factorial design showed that shear span-to-depth ratio (a/d), horizontal and vertical shear reinforcement ratio are the most important parameters affecting the shear strength where the percent contribution of a/d ratio was found the highest (56%). Moreover, two-way, three-way and four-way interactions among the parameters were also observed and their combined contribution was > 25%. Therefore, the inclusion of these higher-order nonlinear terms for the interacted parameters in the proposed shear equation improved accuracy.
- A comparative study among the proposed shear equations with the ACI and CSA models was presented. The performance factor and the χ value of the STM models were quite high, however, they were found close to 1.0 in the case of proposed equations. Moreover, from the descriptive statistical analysis (e.g., *SD, VAR, COV,* and *AAE*), it was found that the accuracy of the proposed shear equation was significantly higher when compared to the ACI and CSA STM model.
- The resistance factors for the proposed equations were recommended from a reliability analysis. In the case of CSA load factors, the resistance factor(ϕ)for the ultimate shear strength was calculated in addition to the current material resistance factors for concrete ($\phi_c = 0.65$)and steel ($\phi_s = 0.85$). Therefore, in order to calculate the ultimate shear strength according to the ACI code, the proposed model (Eqs. (17) and (18)) need to be multiplied with only the resistance factor(ϕ). On the other hand, in the case of CSA code, the proposed model (Eqs. (17) and (18)) need to multiply with the resistance factor(ϕ)along with the material resistance factors recommended in the code.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Table A1
Shear Equations.

Ref.	Shear Equation
ACI 318 [26]	$F_{ns} = f_{ce}A_{cs}$ $f_{ce} = 0.85\beta_s f'_c$ $F_{nt} = A_{ts}f_y$ $F_{nn} = f_{ce}A_{nz}$ $f_{ce} = 0.85\beta_n f'_c$ $V_u = \min\begin{cases} F_{ns}\sin\theta \\ F_{nn}\sin\theta \end{cases}$
CSA A23.3 [27]	$F_{ns} = \phi f_{cu}A_{cs}$ $f_{cu} = \frac{f'_c}{0.8+170\varepsilon_1} \leqslant 0.85f'_c$ $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2\theta_s$ $F_{nt} = \phi_s f_y A_{st}$ $V_u = F_{ns}\sin\theta_s$
Ramakrishnan & Ananthanarayana [6]	$V_n = \beta K f_t bh \beta = 2$for one point and two point load$K = \pi/2$ for cylinder split test$f_t = 7.2\sqrt{f'_c}$; f'_cin psi
Kong et al. [10]	$V_n = C_1\left(1 - 0.35\frac{x}{h}\right)f_{sp}bh + C_2\sum^{n} A\frac{y}{h}\sin^2\alpha C_1 = 1.4$and $C_2 = 130MPa$for plain bar and 300 MPa for deformed bar
Selvam [11]	$P_u = \psi_s f'_{sp}bh$ $\psi_s = \frac{[\xi m^2 - (1-m^2) + \alpha^2 + 2\xi m(1-m)]}{\alpha}$ $m = \frac{m_0\exp(1-\beta)}{\pi\alpha\beta}$; $m_0 = \frac{1}{1+\xi}$; $\xi = \frac{0.85f'_c}{f'_{sp}}$
Mau and Hsu [34]	$\frac{v}{f'_c} = \frac{1}{2}[K(\omega_h + C) + \sqrt{K^2(\omega_h + C)^2 + 4(\omega_h + C)(\omega_v + C)}]$ $K = \frac{2d_v}{h}; \quad 0 < a/h \leqslant 0.5$ $= \frac{d_v}{h}\left[\frac{h}{a}\left(\frac{4}{3} - \frac{2}{3}\frac{a}{h}\right)\right]; \quad 0.5 < a/h \leqslant 2$ $=0; \quad a/h > 2$ $\omega_h = \frac{\rho_h f_y}{f'_c}$; $\omega_v = \frac{\rho_v f_y}{f'_c}$; $C = \frac{\sigma_l}{f'_c}$ $V_n = vbd_v$
Matamoros and Wong [30]	$V = \frac{0.3f'_c\ b\ w_{st}}{a/d} + \frac{\rho_{wv}\ b\ a\ f_{yv}}{3} + (1 - a/d)\rho_{wh}\ b\ d\ A_{th}\ f_{yh}$ $w_{st} = l_b\sin\theta + h_a\cos\theta$
Arabzadeh [19]	$V_u = \frac{(f'_c)^{0.7}}{0.5 + 0.1\left(\frac{a}{d}\right)^2}A_{str}\sin\theta + 0.09(\rho_p)^{-0.35}A_{wp}\cos\theta$ $A_{wp} = A_v\cos\theta + A_h\sin\theta$
Londhe [21]	$V = V_c + V_{ms} + V_{wh} + V_{wv}$ $V_c = \alpha_1[(1 - 0.3a/d)\sqrt{0.8f_{ck}}bd]$; $V_{ms} = \alpha_2\left(\frac{100A_s d\sin^2\theta_i}{D}\right)$ $V_{wh} = \alpha_2\left[\sum_{i=1}^{n}\frac{100A_{iwh}y_i\sin^2\theta_i}{D}\right]$; $V_{wv} = \alpha_2\left[\sum_{i=1}^{n}\frac{100A_{iwv}y_i\sin^2\theta_i}{D}\right]$ $\alpha_1 = \left(\frac{0.375C_1}{\gamma_{mc}}\right)$; $\alpha_2 = \left(\frac{0.75C_2}{100\gamma_{mc}}\right)$

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