Available online at www.sciencedirect.com

SciVerse ScienceDirect

journal homepage: www.elsevier.com/locate/acme

Original Research Article

An empirical model for shear capacity of RC deep beams using genetic-simulated annealing

A.H. Gandomi[a,*], A.H. Alavi[b], D. Mohammadzadeh Shadmehri[c], M.G. Sahab[d]

[a]Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, USA
[b]Department of Civil and Environmental Engineering, Engineering Building, Michigan State University, East Lansing, MI 48824, USA
[c]Civil Engineering Department, Ferdowsi University of Mashhad, Mashhad, Iran
[d]College of Civil Engineering, Tafresh University, Iran

ARTICLE INFO

Article history:
Received 20 November 2012
Accepted 24 February 2013
Available online 1 March 2013

Keywords:
Shear capacity
RC deep beam
Genetic-simulated annealing
Empirical formula

ABSTRACT

This paper presents an empirical model to predict the shear strength of RC deep beams. A hybrid search algorithm coupling genetic programming (GP) and simulated annealing (SA), called genetic simulated annealing (GSA), was utilized to develop mathematical relationship between the experimental data. Using this algorithm, a constitutive relationship was obtained to make pertinent the shear strength of deep beams to nine mechanical and geometrical parameters. The model was developed using an experimental database acquired from the literature. The results indicate that the proposed empirical model is properly capable of evaluating the shear strength of deep beams. The validity of the proposed model was examined by comparing its results with those obtained from American Concrete Institute (ACI) and Canadian Standard Association (CSA) codes. The derived equation is notably simple and includes several effective parameters.

1. Introduction

Reinforced concrete (RC) deep beams may be used in pile caps, bunkers, some shear walls, floor slabs under horizontal loads, and many different types of structures. In deep beams, the bending strain distribution through the depth of transverse sections of beam considerably deviates from the linear distribution, predicted by elementary bending theory of beams. Consequently, the transverse plane sections before bending do not remain plane after bending. As the ratio of depth to span of a beam becomes less than 2 for simply supported beams and 2.5 for continuous beams, the simple bending theory cannot be basically used for determination of the bending and shear stresses [29,39]. The failure mode of deep beams is usually dominated by the shear stresses; hence, shear in deep beams is a major consideration in their design. Many research efforts have been performed to formulate the shear strength of RC deep beams [28,32,34,40,45]. Some of the researchers have employed strut-and-tie model (STM) to determine the shear strength of RC deep beams [7,31,37,48]. STM has been also adopted by American Concrete Institute (ACI) [1] and the Canadian Standard Association (CSA) [13].

Empirical modeling by heuristic and modern search techniques such as artificial neural networks (ANNs) is a different approach to determine the shear strength of beams. ANNs have been widely applied to assess different characteristics

*Corresponding author. Tel.: +1 234 788 0619.
E-mail addresses: a.h.gandomi@gmail.com, ag72@uakron.edu (A.H. Gandomi), alavi@msu.edu, ah_alavi@hotmail.com (A.H. Alavi).

1644-9665/$ - see front matter
http://dx.doi.org/10.1016/j.acme.2013.02.007

of RC slender beams such as prediction of shear capacity [35], torsional strength [44], and deflection analysis [15]. Sanad and Saka [42] utilized ANNs to predict the shear strength of RC deep beam. Despite the acceptable performance of ANNs, they usually do not give a certain function to calculate the outcome using the input values. Fuzzy logic (FL) [47] is well suited to implementing control rules that can only be expressed verbally, or systems that cannot be modeled with linear differential equations. Recently, Choi et al. [11] used this method for the modeling of the shear strength of slender RC beams. Similar to ANNs, the FL approach is not basically able to provide the mathematical expression of the model. In addition, determination of the fuzzy rules is a difficult task in FL procedure. The ANN and FL approaches are mostly appropriate to be used as a part of a computer program [23].

Genetic programming (GP) [30] can be regarded as a new alternative approach for the modeling of concrete behavior. GP may generally be defined as a specialization of genetic algorithms (GA) where the solutions are computer programs rather than binary strings. The main advantage of the GP-based approaches is their ability to generate prediction equations without assuming prior form of the existing relationship [22]. The developed equations can be easily manipulated in practical circumstances. Pérez et al. [38], Gandomi et al. [21,24] and Ashour et al. [8] employed GP to derive complex relationships among the RC beam experimental data.

Simulated annealing (SA) is a general stochastic search algorithm, which introduces the concept of evolution into the annealing process. SA was first presented by Metropolis et al. [33] to mimic the natural process of metals annealing. This algorithm is independently applied to optimization problems by Kirkpatrick et al. [27]. SA is very useful for solving several types of optimization problems with nonlinear functions and multiple local optima. Deschaine et al. [14] and Folino et al. [16] combined GP and SA to make a hybrid algorithm with better efficiency. The SA strategy was used to decide the acceptance of a new individual. It was shown that introducing this strategy into the GP process improves the simple GP profitably [16]. This genetic-simulated annealing (GSA) method has rarely been applied to engineering problems [2,6,25].

In this study, the GSA approach was utilized to estimate the shear capacity of RC beams with stirrups. A generalized relationship was obtained between the load capacity and the amount of longitudinal and shear reinforcement, concrete compressive strength, effective depth, web width, beam span, and shear span to depth ratio. The proposed model was developed based on several shear test results of RC beams reported in the literature. A comparative study was performed between the obtained results and those of the ACI and CSA code models. A parametric study was further carried out to justify the results.

2. Method

2.1. *Genetic programming*

GP is a symbolic optimization technique that creates computer programs to solve a problem using the principle of Darwinian natural selection. GP was introduced by Koza [30] as an extension of GA. In GP, programs are represented as tree structures and expressed in the functional programming language [23]. The main difference between the GA and GP approaches is that in GP the evolving programs (individuals) are parse trees rather than fixed-length binary strings. The traditional optimization techniques, like GA, are generally used in parameter optimization to evolve the best values for a given set of model parameters. GP, on the other hand, gives the basic structure of the approximation model together with the values of its parameters. In fact, GP evolves population of computer programs according to their fitness determined by a program ability to perform a given computational task [3,46].

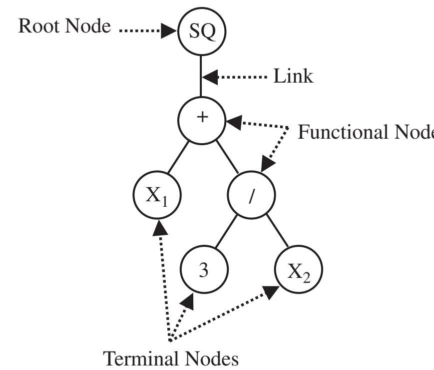

Fig. 1 – Typical tree representation for $(X_1+3/X_2)^2$ in GP.

A random population of individuals (trees) is created to achieve high diversity at the beginning of the GP procedure. The symbolic optimization algorithms present the potential solutions by structural ordering of several symbols. A population member in GP is a hierarchically structured tree comprising functions and terminals. The functions and terminals are selected from a set of functions and a set of terminals. For example, the function set, F, can contain the basic arithmetic operations (+, −, ×, /, etc.), Boolean logic functions (AND, OR, NOT, etc.), or any other mathematical functions. The terminal set, T, contains the arguments for the functions and can consist of numerical constants, logical constants, variables, etc. The functions and terminals are chosen at random and constructed together to form a computer model. The evolved model has a tree-like structure with a root point with branches extending from each function and ending in a terminal. An example of a simple tree representation of a GP model is illustrated in Fig. 1.

Once a population of individuals (models) has been created at random, the GP algorithm evaluates the fitness value of each computer program (individual), selects individuals for reproduction, and generates new individuals by reproduction, crossover and mutation [30]. The fitness value is usually calculated using a function named fitness function. This function is defined so that its value reflects how good the result of a computer program in the population can be matched with the experimental data. The reproduction operation gives a higher probability of selection to more successful individuals. They are copied into the matting pool without any change [2]. The crossover operation ensures the exchange of genetic material between the evolved programs. During the crossover procedure, a point on a branch of each solution (program) is selected at random and the set of terminals and/or functions from each program are then swapped to create two new programs. Fig. 2 shows a typical crossover operation of two computer programs consisting of several function and terminal genes. Two new child computer programs (Child I, Child II) are generated from two parental computer programs (Parent I, Parent II). In Fig. 2, the randomly generated crossover points are shown by dotted lines. It can be seen that both child organisms include the genetic material from their parents. It is necessary to preserve syntactic structure of the programs during the crossover process [2].

During the mutation process, the GP algorithm occasionally selects a function or terminal from a model at random and mutates it. The mutation operation can be applied to the function or terminal nodes. A node in the tree is selected at random. If the selected node is a terminal, it is replaced by another terminal. If the node is a function and point mutation is to be applied, it is replaced by a new function with the same parity [2]. If a tree mutation is to be performed, a new function node, which is not necessarily with the same parity, is chosen. Then, the original node together with its relative sub-tree is replaced by a new randomly created sub-tree. Fig. 3 illustrates a typical mutation operation in GP. Through the above steps, a new generation of computer programs is created. The fitness value for all of the individuals in the new generation is calculated. If one of the termination or convergency conditions is satisfied the process is terminated. Otherwise another round of evaluation using genetic operators is repeated [2,30].

2.2. Simulated annealing

SA makes use of the Metropolis algorithm [33] for the computer simulation of annealing. Annealing is a process in which a metal is heated to a high temperature and then is gradually cooled to relieve thermal stresses. During the cooling process, each atom takes a specific position in the crystalline structure of the metal. By changing the temperature, this crystalline structure changes to a different configuration. An internal energy, E, can be measured and assigned to each state of crystalline structure of the metal which is achieved during the annealing process [2].

At each step of the cooling process, if the temperature does not decrease quickly the atoms are allowed to adjust to a stable equilibrium state of least energy. It is evident that changing of the crystalline structure of a metal, through annealing, is associated with a changing of the internal energy as ΔE. However, as the metal temperature drops down gradually, the overall trend of changing internal energy follows a decreasing process but sometimes the energy may increase by chance [2]. The probability of acceptance of an increase in internal energy by ΔE is given by Boltzmann's probability distribution function as follows:

$$P(\Delta E) = e^{-\Delta E/KT} \tag{1}$$

where T is the temperature of the metal in Kelvin's temperature scale and K is the Boltzmann's constant. The crystalline structure of a metal achieves near global minimum energy states during the process of annealing. This process is simulated by SA to find the minimum of a function in a certain design space. The objective function corresponds to the energy state and moving to any new set of design variables corresponds to a change of the crystalline structural state [2].

2.3. Hybrid genetic programming–simulated annealing algorithm

In this paper, a modified algorithm of GP with a SA-based selection strategy is employed for developing the prediction models. In this coupled algorithm, the SA strategy is used to select new individuals [16,17,25]. The GP system used in this study is linear genetic programming (LGP) [10]. LGP is a new subset of GP with a linear structure similar to the DNA molecule in biological genomes. The main characteristic of LGP in comparison with the traditional tree-based GP is that expressions of a functional programming language (like LISP) are substituted by programs of an imperative language (like C/C++) [9]. Fig. 4 presents a comparison of the program structures in LGP and tree-based GP. Fig. 4(a) shows a linear

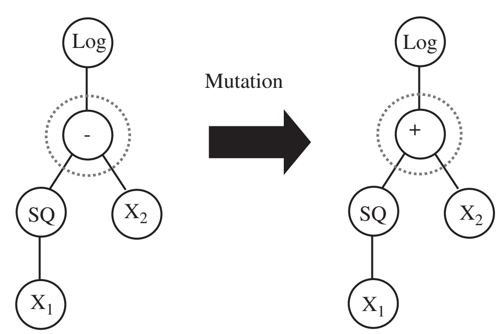

Fig. 3 – Typical mutation operation in GP.

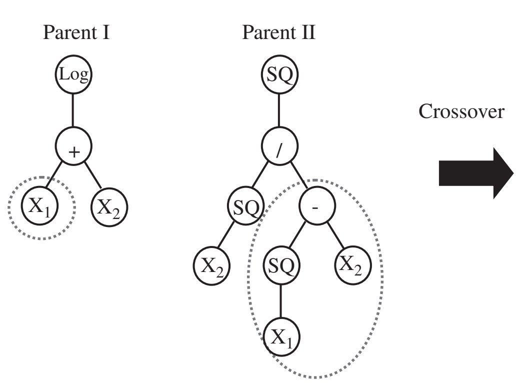

Fig. 2 – Typical crossover operation in GP.

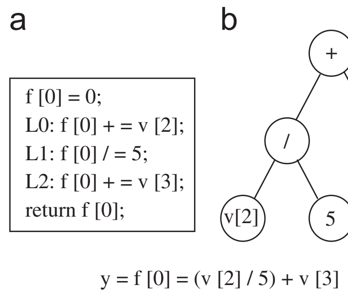

Fig. 4 – Comparison of the GP program structures: (a) LGP and (b) Tree-based GP.

genetic program that can be seen as a data flow graph generated by multiple usage of register content. That is, on the functional level the evolved imperative structure denotes a special directed graph. As observed in Fig. 4(b), in tree-based GP, the data flow is more rigidly determined by the tree structure of the program [9]. In the LGP system utilized here, an individual program is interpreted as a variable-length sequence of simple C instructions [4,5].

2.3.1. *The genetic-simulated annealing algorithm*

Considering the above explanations for GP and SA, the coupled GSA algorithm uses the following main steps to evolve a computer program [2,14,16]:

I. A single program is initially created at random. This is the "parent" program for the first repetition of the learning cycle.
II. The parent program is copied.
III. A search operator, crossover or mutation, transforms the copy of the parent program. The transformed copy is called "child" program or "offspring" program. The crossover operator produces two children programs. But only one of these programs is compared with the parent as a candidate to replace the parent program.
IV. The fitness value of the both parent and child program is calculated.
V. Based on the fitness value of the child and parent program, the SA algorithm decides whether to replace the parent program with the child program. If the child has better fitness than the parent, the child always replaces its parent. If the child has worse fitness than the parent, the child replaces the parent probabilistically. The probability of replacement depends on how much worse the fitness of the child is than the parent and also on the SA temperature, *T*. As the annealing process continues, *T* is gradually reduced at each *n*th iteration. This means that, for the program, the probability of replacing a worse child to a better parent gets lower and lower as the run continues. If the child program replaces the parent program then the child program becomes the new parent for the next cycle. Alternatively, if the parent program is not replaced by the child, it remains as the parent program for the next cycle.
VI. If the termination or convergency conditions are satisfied the process is terminated. Otherwise, the process is continued from step III.

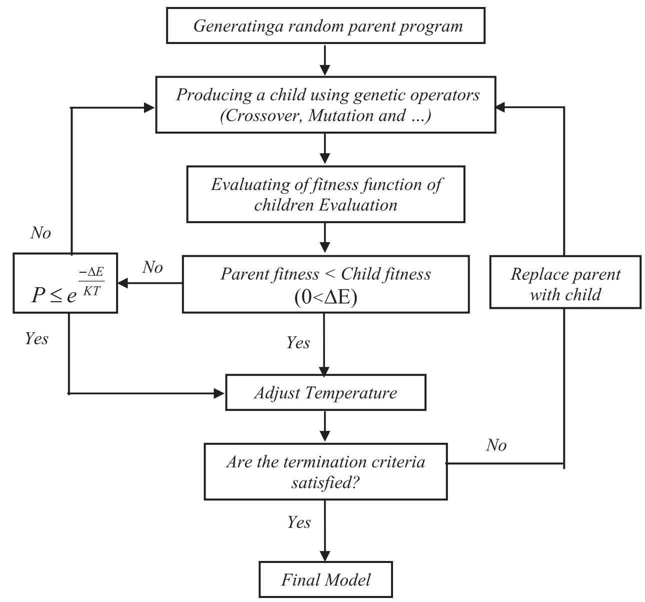

Fig. 5 – A basic representation of the GSA algorithm.

A basic representation of the GSA algorithm is presented in Fig. 5.

A description of the basic parameters used by the GSA algorithm to direct a search for a program is given in [17].

3. Model development and results

3.1. *The proposed GSA model*

In order to provide accurate assessment of the shear strength of RC deep beams, the effects of several factors were incorporated into the developed model. In this study, the GSA approach was utilized to obtain a meaningful relationship between the load capacity of deep beams and the influencing variables as follows:

$$v = f\left(\frac{a}{h}, f_c, \rho, \rho_h, \rho_v\right) \tag{2}$$

where

- V=ultimate strength of RC beam ($v = V/bd$) (MPa),
- b=web width (mm),
- d=effective depth (mm),
- a/d=shear span to depth,
- f_c=concrete compressive strength (MPa),
- ρ=main reinforcement ratio (%),
- ρ_h=horizontal shear reinforcement ratio (%) and
- ρ_v=vertical shear reinforcement ratio (%).

The above variables were chosen on the basis of recommended values in the literature [37,42] and a trial study. The geometrical parameters are shown in Fig. 6.

Various parameters involved in the GSA algorithm are shown in Table 1. The parameter selection will affect the model generalization capability of GSA. They were selected based on some previously suggested values [2,25] and also after a trial study. Several runs were conducted to tune the GSA parameters. In order to find models with minimum error, each run was performed with large number of temperature levels and iterations. Three levels were set for the number of temperature levels parameter. At the low level the crossover rate was 50% and at the high level it was 95%. The success of the GSA algorithm usually increases by increasing the initial and maximum program size parameters. In this case, the complexity of the evolved functions increases and the speed of the algorithm decreases. The initial program size was set to 80. Two optimal levels were also set for the maximum program size as tradeoffs between the running time and the complexity of the evolved solutions. Start and stop temperatures were set to 5 and 0.01, respectively. There are $3\times2\times2=12$ different combinations of the parameters. All of these combinations were tested and 10 replications for each combination were performed. Therefore, the total number of runs is amounted to 120. The GSA algorithm was implemented using the Discipulus Lite software [12].

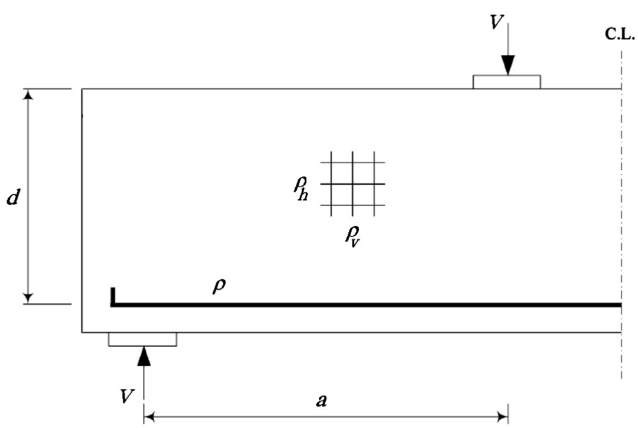

Fig. 6 – Geometrical parameters of RC deep beams.

Table 1 – Parameter settings for the GSA algorithm.

Parameter	Settings
Number of temperature levels	1000, 3000, 5000
Number of iterations per temperature level	1000
Start temperature	5
Stop temperature	0.01
Fitness function error type	Squared error
Crossover rate (%)	50, 95
Homologous crossover (%)	95
Probability of randomly generated parent in crossover (%)	99
Block mutation rate (%)	30
Instruction mutation rate (%)	30
Data mutation rate (%)	40
Offspring choice rate (%)	50
Replacement scaling factor	1
Maximum program size	128, 256
Initial program size	80
Function set	+, −, ×, /

3.2. *Experimental database*

The experimental data used in this study included test results of 214 RC deep beams collected by Park and Kuchma [37]. For the analysis, the available data sets were randomly divided into learning, validation and testing subsets. The learning data were taken for training (genetic evolution). The testing data were used to specify the generalization capability of the models on data they did not train on (model selection). Thus, both of the learning and validation data were involved in the modeling process and were categorized into one group referred to as "training data". The models with the best performance on both of the learning and validation data sets were finally selected as the outcomes of the runs. The testing data were finally employed to measure the performance of the models obtained by GSA on the data that played no role in building the models [2]. To obtain a consistent data division, several combinations of the training (learning and validation) and testing sets were considered. The selection was in a way that the maximum, minimum, mean and standard deviation of parameters were consistent in the training and testing data sets. Of the 214 data sets, 182 data vectors were taken for the training process (150 sets for learning and 32 sets for validation). The remaining 32 sets were used for the testing of the derived model. The ranges and statistics of the input and output parameters involved in the model development are given in Table 2. The range of the compressive strength (f_c) indicates that the database contains both normal and high strength concrete.

3.3. *Performance measures*

The best GSA model was chosen on the basis of a multi-objective function defined according to the following criteria [21]:

i. The simplicity of the model, although this was not a predominant factor.
ii. Providing the best fitness value of the learning set of data.
iii. Providing the best fitness value of the validation set of data.

The first objective should be controlled by the user and for other objectives the following combined objective function (Obj) was constructed as a measure of how well the model predicted output agrees with the experimentally measured output [20]:

$$\text{Minimize}: \quad Obj=\left(\frac{No._L-No._V}{No._T}\right)\frac{MAE_L}{R_L^2}+\frac{2No._V}{No._T}\frac{MAE_V}{R_V^2} \tag{3}$$

where No. $_L$, No. $_V$ and No. $_T$ are the number of learning, validation and training data, respectively. R and MAE are the correlation coefficient and mean absolute error, respectively, given as follows:

$$R=\frac{\sum_{i=1}^{n}(h_i-\bar{h}_i)(t_i-\bar{t}_i)}{\sqrt{\sum_{i=1}^{n}(h_i-\bar{h}_i)^2\sum_{i=1}^{n}(t_i-\bar{t}_i)^2}} \tag{4}$$

$$MAE=\frac{\sum_{i=1}^{n}|h_i-t_i|}{n} \tag{5}$$

in which h_i and t_i are the actual and calculated outputs for the ith output, respectively, $\bar{h}_i$ and $\bar{t}_i$ are the average of the actual

Table 2 – Descriptive statistics of variables used in model development.

	d (mm)	b (mm)	f_c (Mpa)	a/d	ρ (%)	ρ_h (%)	ρ_v (%)	V (KN)
Mean	420.8	134.2	33.4	1.220	1.813	0.314	0.542	304.6
Standard error	7.4142	3.3462	1.0887	0.0362	0.0476	0.0305	0.0453	13.564
Median	390	120	24.7	1.21	1.73	0.19	0.34	262.7
Mode	500	102	49.1	1.5	1.94	0	0	290
Standard deviation	108.46	48.950	15.926	0.530	0.696	0.447	0.662	198.42
Sample variance	11,764	2396	254	0.280	0.484	0.200	0.439	39,370
Kurtosis	1.705	1.047	−0.063	−0.484	1.037	10.011	4.696	8.884
Skewness	0.943	1.164	0.960	0.287	0.842	2.676	2.315	2.408
Range	585	229	59.8	2.43	3.56	2.45	2.65	1283.6
Minimum	216	76	13.8	0.27	0.52	0	0	73.4
Maximum	801	305	73.6	2.7	4.08	2.45	2.65	1357
Confidence level (95.0%)	14.615	6.596	2.146	0.071	0.094	0.060	0.089	26.736

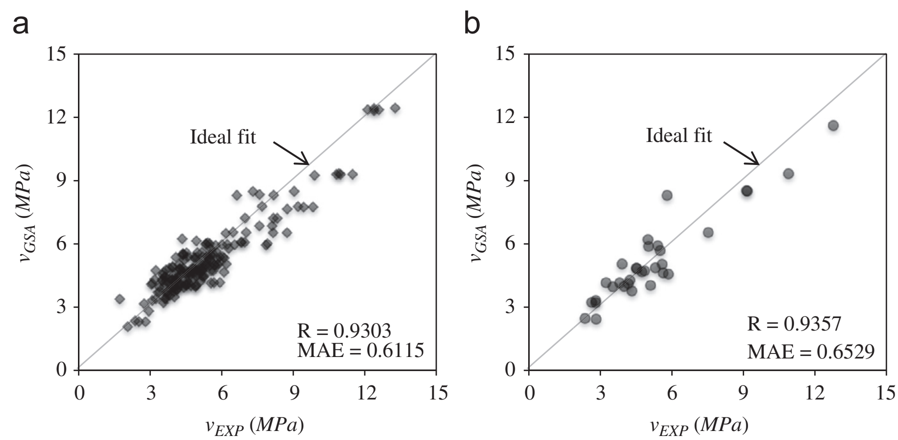

Fig. 7 – Experimental versus predicted shear strength using the GSA model: (a) training data and (b) testing data.

outputs, and n is the number of sample. It should be mentioned that R is not the only good indicator to specify prediction accuracy of a model. The reason for this opinion is that, shifting the output values of a model equally, the R value will not change. The constructed objective function takes into account the changes of R and MAE together. Higher R values and lower MAE values result in lowering the Obj value and, consequently, indicate a more precise model. In addition, the above function considers the effects of different data divisions for the learning and validation data.

Each run was observed while in progress for overfitting. In checking for overfitting, situations were checked in which the fitness of the samples for the learning of GSA was negatively correlated with the fitness on the validation data sets. To evaluate the fitness of the evolved program, the average of the squared raw errors was used. For the runs showing signs of overfitting, the GSA parameters were progressively changed so as to reduce the computational power available to the GSA algorithm until observed overfitting was minimized. The resulting run was then accepted as the production run.

3.4. *GSA-based formulation for load capacity of RC beam*

The GSA-based formulation of the load capacity of RC deep beam is as given below:

$$v_{GSA}\ (\mathrm{MPa}) = 2\frac{d}{a}\frac{\rho(\rho_h - f_c + \rho_v + \xi - 4)}{(\xi+2)} - \frac{a}{d} + 4 \tag{6a}$$

where

$$\xi = \frac{(f_c + 4a/d - 4)^2}{25} \tag{6b}$$

A comparison of the experimental and predicted load capacity is shown in Fig. 7.

4. Results and discussion

4.1. *Comparison of GSA with Codes of practice*

Here, a comparison between the shear strength of deep beams predicted by the proposed model, ACI and CSA design codes, is drawn. Overall performance of the GSA, ACI [1] and CSA [13] models on the entire data is summarized in Table 3. The considered deep beams are unreinforced, only with vertical reinforcement, only with horizontal reinforcement, and with orthogonal shear reinforcement. Comparisons of the load capacity predictions, obtained by these models are also visualized in Fig. 8(a–f). The results clearly demonstrate that the GSA-based formula performs superior to the ACI and CSA models.

Furthermore, Figs. 9 and 10 show the ratio of experimental to the GSA predicted strength against a/d and f_c, respectively. As the

Table 3 – Shear strength results, predicted by the GSA, ACI and CSA models.

Model	Mode (No.)	$V_{Exp}/V_{Pre.}$ WO (16)	WV (83)	WH (10)	WVH (105)	Total (214)
ACI	Ave.	1.644	1.922	2.512	1.590	1.766
	S.D.	0.457	0.645	0.716	0.398	0.571
CSA	Ave.	1.213	1.807	2.081	1.524	1.637
	S.D.	0.351	0.662	0.529	0.452	0.574
GSA	Ave.	1.100	1.052	1.043	0.971	1.016
	S.D.	0.265	0.179	0.165	0.111	0.163

Note: WO, WV, WH, and WVH refer to deep beams without, with only vertical, with only horizontal, and with orthogonal shear reinforcement, respectively.

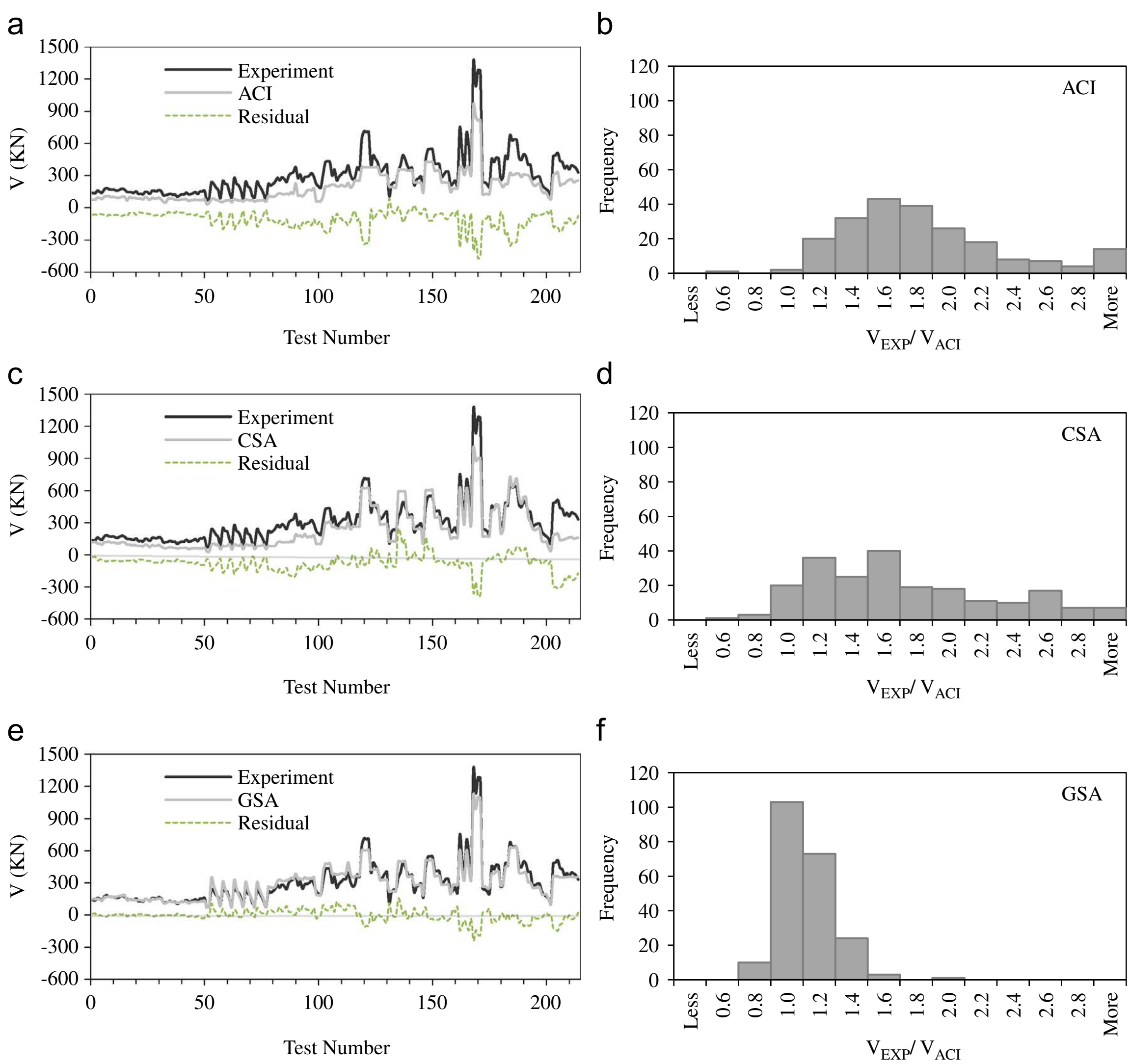

Fig. 8 – A comparison between the predicted and experimental shear strength values using different models (a–b) ACI, (c–d) CSA, and (e–f) GSA.

scattering increases, in these figures, the accuracy of the models decreases. It can be observed from these figures that the predictions obtained by the GSA model have a good accuracy with no significant trend with respect to a/d and f_c' for both normal strength concrete (NSC) and high strength concrete (HSC).

4.2. Performance analysis and the validity of the model

Based on a logical hypothesis [43], when the correlation coefficient of a model becomes more than 0.8 ($R>0.8$), and also the mean absolute error value (MAE) is minimized; a strong correlation between the predicted and measured values has been achieved. It can be observed from Fig. 7 that the GSA model with very high R and low MAE values predicts the target values to an acceptable degree of accuracy. Meanwhile, it is noteworthy that not only the MAE values are low but also as similar as possible for the training and testing sets. This implies that the proposed model has both predictive ability (low values) and generalization performance (similar values) [36].

It is known that, in most cases, the models derived using the soft computing tools have a predictive capability within the data range used for their development. Thus, the amount of data used for the modeling process is an important issue, as it bears heavily on the reliability of the final models. To cope with this limitation, Frank and Todeschini [18] argue that the minimum ratio of the number of objects over the number of selected variables for model acceptability is 3. They suggest that considering a value of 5 is safer. In the present study, this ratio is much higher and is equal to 214/5 or almost 43.

Furthermore, new criteria recommended by Golbraikh and Tropsha [26] were checked for the external verification of the GSA model on the testing data sets. It is suggested that at least one slope of regression lines (k or k') through the origin should be close to 1 [19]. Recently, Roy and Roy [41] introduced a confirmative indicator of the external predictability of models (R_m). For $R_m>0.5$, the condition is satisfied. Either the squared correlation coefficient (through the origin) between predicted and experimental values (Ro^2), or the coefficient between experimental and predicted values (Ro'^2) should be close to 1. The considered validation criteria and the relevant results obtained by the model are presented in Table 4. As it is seen, the derived model satisfies the required conditions. The validation phase

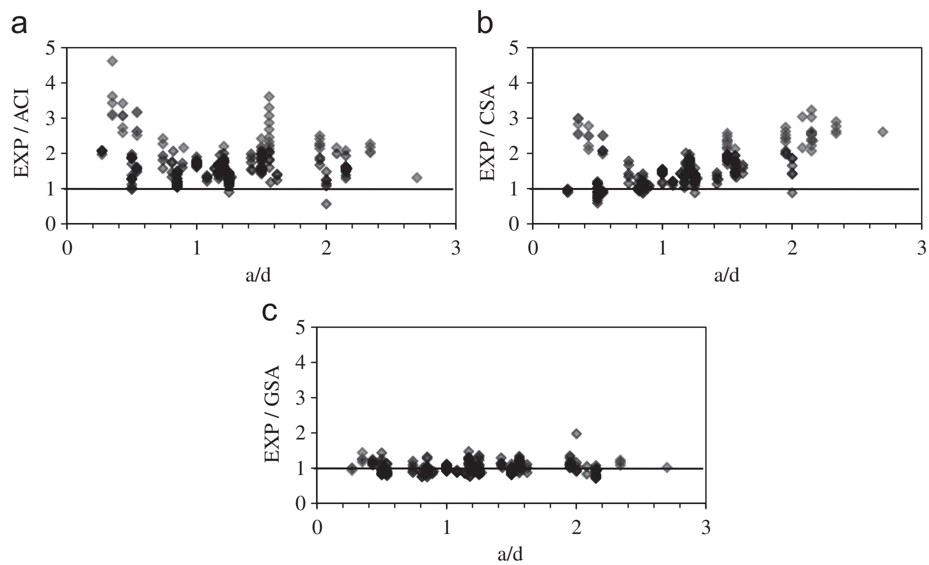

Fig. 9 – Variations of the measured-to-calculated strength ratio against *a/d*. (a) ACI, (b) CSA, and (c) GSA.

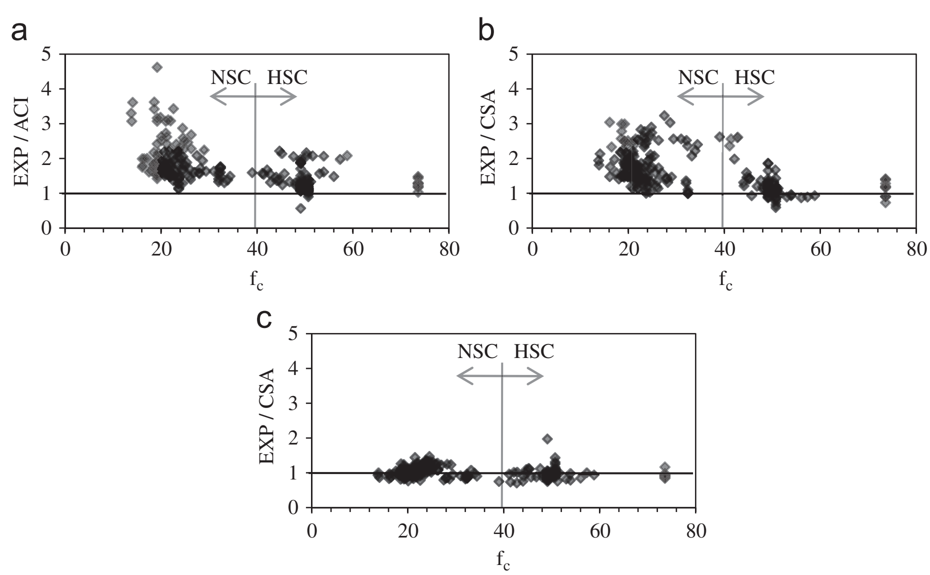

Fig. 10 – Variations of the measured-to-calculated strength ratio against f_c. (a) ACI, (b) CSA, and (c) GSA.

ensures the GSA prediction model is strongly valid, has the prediction power and is not a chance correlation.

Note that one of the major advantages of the GSA over the conventional modeling techniques is its ability to derive explicit relationships for the load capacity without assuming any predefined mathematical form. The GSA approach employed in this research is based on the data alone to determine the structure and parameters of the models. Thus, the derived constitutive model can particularly be valuable in the preliminary design stages [2]. For more reliability, the results of the GSA-based analyses are suggested to be treated as a complement to conventional computing techniques (like finite element method). In any case, the importance of engineering judgment in interpretation of the obtained results should not be underestimated.

In order to develop a sophisticated prediction tool, GSA can be combined with advanced deterministic models. Assuming that the deterministic model captures the key physical mechanisms, it needs appropriate initial conditions and carefully calibrated parameters to make accurate predictions. An idea could be to calibrate the parameters by the use of GSA which takes into account historic data sets as well as the laboratory test results. GSA provides a structured representation for the constitutive material model that can readily be incorporated into the finite element or finite difference analyses. In this case, it is possible to use a suitably trained GSA-based model instead of a conventional (analytical) constitutive model in a numerical analysis tool such as finite element code [2].

4.3. *Parametric analysis*

The influence of different parameters on the shear strength of deep beams is studied by the aid of the GSA model. Fig. 11 presents the predicted values of the shear strength of deep

Table 4 – Statistical parameters of the GSA model for external validation.

Item	Formula	Condition	GSA
1	Eq. (4)	$R>0.8$	0.9357
2	$k=\frac{\sum_{i=1}^{n}(h_i\times t_i)}{h_i^2}$	$0.85<K<1.15$	1.0076
3	$k'=\frac{\sum_{i=1}^{n}(h_i\times t_i)}{t_i^2}$	$0.85<K'<1.15$	0.9712
4	$R_m=R^2\times(1-\sqrt{\lvert R^2-Ro^2\rvert})$	$R_m>0.5$	0.5671
where	$Ro^2=1-\frac{\sum_{i=1}^{n}(t_i-h_i^o)^2}{\sum_{i=1}^{n}(t_i-\bar{t}_i)^2},\ h_i^o=k=t_i$		0.9997
	$Ro'^2=1-\frac{\sum_{i=1}^{n}(h_i-t_i^o)^2}{\sum_{i=1}^{n}(h_i-\bar{h}_i)^2},\ t_i^o=k'\times h_i$		0.9937

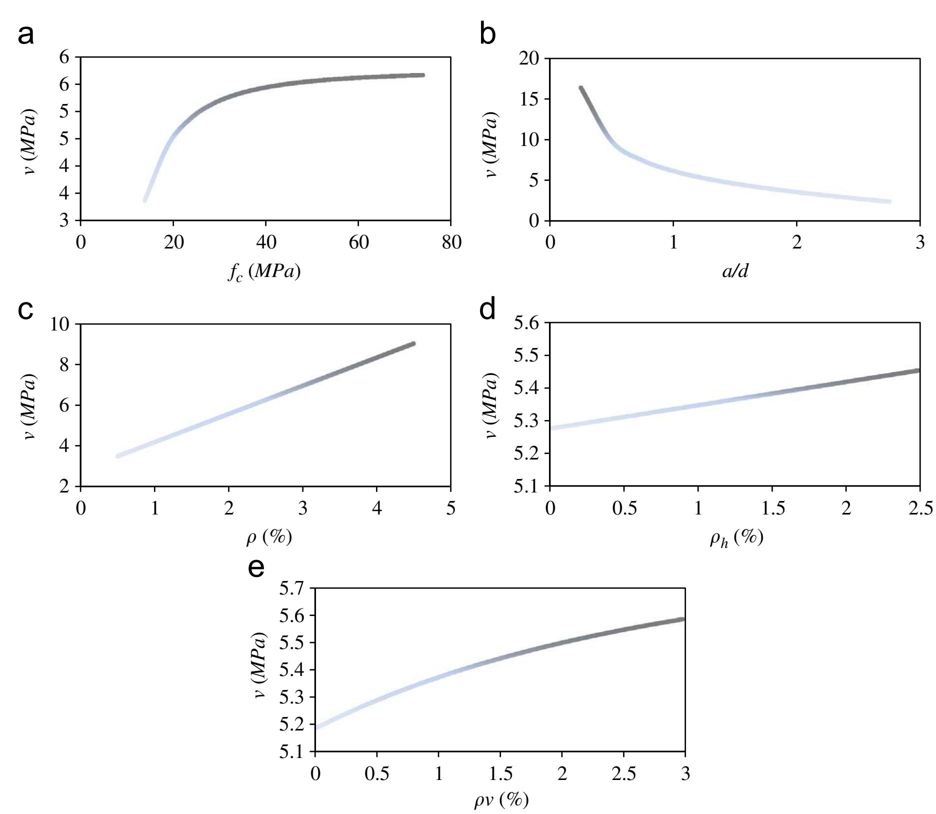

Fig. 11 – Parametric study of the load capacity in the GSA model. (a) f_c, (b) a/d, (c) ρ, (d) ρ_h, and (e) ρ_v.

Table A.1 – Experimental database and predicted values using different methods.

No.	Researcher(s)	Year	Beam ID	d (mm)	b (mm)	f_c (Mpa)	a/d	ρ (%)	ρ_h (%)	ρ_v (%)	V_{test}/bd (Mpa)	Exp./ACI	Exp./CSA	Exp./GSA
1	Smith and Vantsiotis	1982	0A0-44	305	102	20.5	1	1.94	0	0	4.48	1.9	1.19	0.94
2	Smith and Vantsiotis	1982	0A0-48	305	102	20.9	1	1.94	0	0	4.37	1.81	1.14	0.90
3	Smith and Vantsiotis	1982	1A1-10	305	102	18.7	1	1.94	0.23	0.28	5.18	1.72	1.49	1.08
4	Smith and Vantsiotis	1982	1A3-11	305	102	18	1	1.94	0.45	0.28	4.77	1.64	1.41	1.01
5	Smith and Vantsiotis	1982	1A4-12	305	102	16.1	1	1.94	0.68	0.28	4.54	1.75	1.48	1.06
6	Smith and Vantsiotis	1982	1A4-51	305	102	20.5	1	1.94	0.68	0.28	5.49	1.66	1.45	1.05
7	Smith and Vantsiotis	1982	1A6-37	305	102	21.1	1	1.94	0.91	0.28	5.92	1.74	1.53	1.09
8	Smith and Vantsiotis	1982	2A1-38	305	102	21.7	1	1.94	0.23	0.63	5.61	1.61	1.42	1.00
9	Smith and Vantsiotis	1982	2A3-39	305	102	19.8	1	1.94	0.45	0.63	5.48	1.72	1.5	1.02
10	Smith and Vantsiotis	1982	2A4-40	305	102	20.3	1	1.94	0.68	0.63	5.53	1.69	1.48	1.00
11	Smith and Vantsiotis	1982	2A6-41	305	102	19.1	1	1.94	0.91	0.63	5.20	1.69	1.46	0.96
12	Smith and Vantsiotis	1982	3A1-42	305	102	18.4	1	1.94	0.23	1.25	5.18	1.74	1.5	0.92
13	Smith and Vantsiotis	1982	3A3-43	305	102	19.2	1	1.94	0.45	1.25	5.55	1.79	1.55	0.96
14	Smith and Vantsiotis	1982	3A4-45	305	102	20.8	1	1.94	0.68	1.25	5.74	1.71	1.5	0.97
15	Smith and Vantsiotis	1982	3A6-46	305	102	19.9	1	1.94	0.91	1.25	5.40	1.68	1.47	0.91
16	Smith and Vantsiotis	1982	0B0-49	305	102	21.7	1.21	1.94	0	0	4.79	2.2	1.57	1.09
17	Smith and Vantsiotis	1982	1B1-01	305	102	22.1	1.21	1.94	0.23	0.24	4.74	1.49	1.53	1.01
18	Smith and Vantsiotis	1982	1B3-29	305	102	20.1	1.21	1.94	0.45	0.24	4.62	1.59	1.61	1.03
19	Smith and Vantsiotis	1982	1B4-30	305	102	20.8	1.21	1.94	0.68	0.24	4.51	1.5	1.53	0.98
20	Smith and Vantsiotis	1982	1B6-31	305	102	19.5	1.21	1.94	0.91	0.24	4.93	1.75	1.76	1.10
21	Smith and Vantsiotis	1982	2B1-05	305	102	19.2	1.21	1.94	0.23	0.42	4.15	1.5	1.51	0.94
22	Smith and Vantsiotis	1982	2B3-06	305	102	19	1.21	1.94	0.45	0.42	4.22	1.54	1.54	0.95
23	Smith and Vantsiotis	1982	2B4-07	305	102	17.5	1.21	1.94	0.68	0.42	4.05	1.61	1.59	0.95
24	Smith and Vantsiotis	1982	2B4-52	305	102	21.8	1.21	1.94	0.68	0.42	4.82	1.53	1.57	0.99
25	Smith and Vantsiotis	1982	2B6-32	305	102	19.8	1.21	1.94	0.91	0.42	4.67	1.64	1.65	1.00
26	Smith and Vantsiotis	1982	3B1-08	305	102	16.2	1.21	1.94	0.23	0.63	4.20	1.79	1.76	1.03
27	Smith and Vantsiotis	1982	3B1-36	305	102	20.4	1.21	1.94	0.23	0.77	5.11	1.73	1.76	1.06
28	Smith and Vantsiotis	1982	3B3-33	305	102	19	1.21	1.94	0.45	0.77	5.09	1.86	1.87	1.08
29	Smith and Vantsiotis	1982	3B4-34	305	102	19.2	1.21	1.94	0.68	0.77	4.98	1.8	1.8	1.04
30	Smith and Vantsiotis	1982	3B6-35	305	102	20.6	1.21	1.94	0.91	0.77	5.34	1.79	1.83	1.07
31	Smith and Vantsiotis	1982	4B1-09	305	102	17.1	1.21	1.94	0.23	1.25	4.93	2	1.97	1.03
32	Smith and Vantsiotis	1982	0C0-50	305	102	20.7	1.5	1.94	0	0	3.72	2.19	1.78	1.03
33	Smith and Vantsiotis	1982	1C1-14	305	102	19.2	1.5	1.94	0.23	0.18	3.83	2.42	1.95	1.07
34	Smith and Vantsiotis	1982	1C3-02	305	102	21.9	1.5	1.94	0.45	0.18	3.97	1.48	1.81	1.00
35	Smith and Vantsiotis	1982	1C4-15	305	102	22.7	1.5	1.94	0.68	0.18	4.21	1.52	1.86	1.03
36	Smith and Vantsiotis	1982	1C6-16	305	102	21.8	1.5	1.94	0.91	0.18	3.93	1.48	1.8	0.98
37	Smith and Vantsiotis	1982	2C1-17	305	102	19.9	1.5	1.94	0.23	0.31	3.99	1.64	1.97	1.06
38	Smith and Vantsiotis	1982	2C3-03	305	102	19.2	1.5	1.94	0.45	0.31	3.33	1.42	1.69	0.89
39	Smith and Vantsiotis	1982	2C3-27	305	102	19.3	1.5	1.94	0.45	0.31	3.71	1.57	1.87	0.99
40	Smith and Vantsiotis	1982	2C4-18	305	102	20.4	1.5	1.94	0.68	0.31	4.01	1.6	1.93	1.02
41	Smith and Vantsiotis	1982	2C6-19	305	102	20.8	1.5	1.94	0.91	0.31	3.99	1.57	1.91	1.00
42	Smith and Vantsiotis	1982	3C1-20	305	102	21	1.5	1.94	0.23	0.56	4.53	1.76	2.13	1.12
43	Smith and Vantsiotis	1982	3C3-21	305	102	16.5	1.5	1.94	0.45	0.56	4.02	1.99	2.33	1.14
44	Smith and Vantsiotis	1982	3C4-22	305	102	18.3	1.5	1.94	0.68	0.56	4.10	1.84	2.18	1.07
45	Smith and Vantsiotis	1982	3C6-23	305	102	19	1.5	1.94	0.91	0.56	4.41	1.9	2.26	1.11
46	Smith and Vantsiotis	1982	4C1-24	305	102	19.6	1.5	1.94	0.23	0.77	4.71	1.97	2.36	1.17

Table A.1 (*continued*)

No.	Researcher(s)	Year	Beam ID	*d* (mm)	*b* (mm)	f_c (Mpa)	*a/d*	ρ (%)	ρ_h (%)	ρ_v (%)	V_{test}/bd (Mpa)	Exp./ACI	Exp./CSA	Exp./GSA
47	Smith and Vantsiotis	1982	4C3-04	305	102	18.5	1.5	1.94	0.45	0.63	4.13	1.82	2.16	1.07
48	Smith and Vantsiotis	1982	4C3-28	305	102	19.2	1.5	1.94	0.45	0.77	4.90	2.08	2.49	1.22
49	Smith and Vantsiotis	1982	4C4-25	305	102	18.5	1.5	1.94	0.68	0.77	4.91	2.17	2.57	1.23
50	Smith and Vantsiotis	1982	4C6-26	305	102	21.2	1.5	1.94	0.91	0.77	5.13	1.98	2.39	1.20
51	Smith and Vantsiotis	1982	0D0-47	305	102	19.5	2.08	1.94	0	0	2.36	2.16	2.16	0.95
52	Smith and Vantsiotis	1982	4D1-13	305	102	16.1	2.08	1.94	0.23	0.42	2.81	1.99	3.04	1.20
53	Kong et al.	1970	1–30	724	76	21.5	0.35	0.52	0	2.45	4.34	3.11	2.56	0.69
54	Kong et al.	1970	1–25	597	76	24.6	0.43	0.62	0	2.45	4.94	2.59	2.11	0.80
55	Kong et al.	1970	1–20	470	76	21.2	0.54	0.79	0	2.45	5.32	2.62	2.06	0.89
56	Kong et al.	1970	1–15	343	76	21.2	0.74	1.09	0	2.45	6.29	2.42	1.78	1.08
57	Kong et al.	1970	1–10	216	76	21.7	1.18	1.73	0	2.45	5.48	1.64	1.7	1.03
58	Kong et al.	1970	2–30	724	76	19.2	0.35	0.52	0	0.86	4.53	4.62	2.99	0.81
59	Kong et al.	1970	2–25	597	76	18.6	0.43	0.62	0	0.86	4.94	3.42	2.78	0.92
60	Kong et al.	1970	2–20	470	76	19.9	0.54	0.79	0	0.86	6.05	3.18	2.51	1.12
61	Kong et al.	1970	2–15	343	76	22.8	0.74	1.09	0	0.86	5.37	1.93	1.42	0.99
62	Kong et al.	1970	2–10	216	76	20.1	1.18	1.73	0	0.86	6.09	1.95	2.01	1.30
63	Kong et al.	1970	3–30	724	76	22.6	0.35	0.52	2.45	0	5.02	3.43	2.83	0.85
64	Kong et al.	1970	3–25	597	76	21	0.43	0.62	2.45	0	4.98	3.07	2.49	0.89
65	Kong et al.	1970	3–20	470	76	19.2	0.54	0.79	2.45	0	5.82	3.16	2.5	1.09
66	Kong et al.	1970	3–15	343	76	21.9	0.74	1.09	2.45	0	6.10	2.27	1.67	1.14
67	Kong et al.	1970	3–10	216	76	22.6	1.18	1.73	2.45	0	5.30	1.51	1.59	1.09
68	Kong et al.	1970	4–30	724	76	22	0.35	0.52	0.86	0	4.40	3.08	2.54	0.80
69	Kong et al.	1970	4–25	597	76	21	0.43	0.62	0.86	0	4.43	2.73	2.21	0.84
70	Kong et al.	1970	4–20	470	76	20.1	0.54	0.79	0.86	0	5.07	2.63	2.08	1.00
71	Kong et al.	1970	4–15	343	76	22	0.74	1.09	0.86	0	4.22	1.57	1.15	0.84
72	Kong et al.	1970	4–10	216	76	22.6	1.18	1.73	0.86	0	5.85	1.67	1.75	1.28
73	Kong et al.	1970	5–30	724	76	18.6	0.35	0.52	0.61	0.61	4.36	3.62	2.99	0.79
74	Kong et al.	1970	5–25	597	76	19.2	0.43	0.62	0.61	0.61	4.58	3.07	2.5	0.85
75	Kong et al.	1970	5–20	470	76	20.1	0.54	0.79	0.61	0.61	4.84	2.51	1.98	0.90
76	Kong et al.	1970	5–15	343	76	21.9	0.74	1.09	0.61	0.61	4.87	1.81	1.34	0.91
77	Kong et al.	1970	5–10	216	76	22.6	1.18	1.73	0.61	0.61	4.75	1.36	1.43	0.98
78	Clark	1951	A1-1	390	203	24.6	2.34	3.1	0	0.38	2.81	2.05	2.63	0.84
79	Clark	1951	A1-2	390	203	23.6	2.34	3.1	0	0.38	2.64	2.02	2.57	0.81
80	Clark	1951	A1-3	390	203	23.4	2.34	3.1	0	0.38	2.81	2.18	2.77	0.87
81	Clark	1951	A1-4	390	203	24.8	2.34	3.1	0	0.38	3.09	2.27	2.9	0.92
82	Clark	1951	B1-1	390	203	23.4	1.95	3.1	0	0.37	3.52	2.22	2.42	0.88
83	Clark	1951	B1-2	390	203	25.4	1.95	3.1	0	0.37	3.24	1.88	2.07	0.78
84	Clark	1951	B1-3	390	203	23.7	1.95	3.1	0	0.37	3.60	2.24	2.44	0.89
85	Clark	1951	B1-4	390	203	23.3	1.95	3.1	0	0.37	3.39	2.14	2.33	0.85
86	Clark	1951	B1-5	390	203	24.6	1.95	3.1	0	0.37	3.05	1.82	2	0.74
87	Clark	1951	B2-1	390	203	23.2	1.95	3.1	0	0.73	3.80	2.41	2.62	0.91
88	Clark	1951	B2-2	390	203	26.3	1.95	3.1	0	0.73	4.07	2.28	2.52	0.93
89	Clark	1951	B2-3	390	203	24.9	1.95	3.1	0	0.73	4.23	2.5	2.74	0.98
90	Clark	1951	B6-1	390	203	42.1	1.95	3.1	0	0.37	4.79	1.67	1.98	0.99
91	Clark	1951	C1-1	390	203	25.6	1.56	2.07	0	0.34	3.51	2.46	1.59	0.82

92	Clark	1951	C1-2	390	203	26.3	1.56	2.07	0	0.34	3.93	2.68	1.73	0.91
93	Clark	1951	C1-3	390	203	24	1.56	2.07	0	0.34	3.11	2.33	1.48	0.75
94	Clark	1951	C1-4	390	203	29	1.56	2.07	0	0.34	3.61	2.24	1.47	0.81
95	Clark	1951	C2-1	390	203	23.6	1.56	2.07	0	0.69	3.66	1.83	1.77	0.86
96	Clark	1951	C2-2	390	203	25	1.56	2.07	0	0.69	3.80	1.8	1.75	0.87
97	Clark	1951	C2-3	390	203	24.1	1.56	2.07	0	0.69	4.09	2.01	1.94	0.95
98	Clark	1951	C2-4	390	203	27	1.56	2.07	0	0.69	3.64	1.6	1.58	0.82
99	Clark	1951	C3-1	390	203	14.1	1.56	2.07	0	0.34	2.83	3.61	2.13	1.15
100	Clark	1951	C3-2	390	203	13.8	1.56	2.07	0	0.34	2.53	3.3	1.94	1.08
101	Clark	1951	C3-3	390	203	13.9	1.56	2.07	0	0.34	2.38	3.07	1.81	1.00
102	Clark	1951	C4-1	390	203	24.5	1.56	3.1	0	0.34	3.91	2.87	1.73	0.77
103	Clark	1951	C6-2	390	203	45.2	1.56	3.1	0	0.34	5.35	2.13	1.41	0.89
104	Clark	1951	C6-3	390	203	44.7	1.56	3.1	0	0.34	5.49	2.21	1.46	0.91
105	Clark	1951	C6-4	390	203	47.6	1.56	3.1	0	0.34	5.41	2.05	1.37	0.89
106	Clark	1951	D1-1	390	203	26.2	1.17	1.63	0	0.46	3.80	1.4	1.14	0.78
107	Clark	1951	D1-2	390	203	26.1	1.17	1.63	0	0.46	4.51	1.67	1.36	0.93
108	Clark	1951	D1-3	390	203	24.5	1.17	1.63	0	0.46	3.24	1.28	1.03	0.68
109	Clark	1951	D2-1	390	203	24	1.17	1.63	0	0.61	3.66	1.47	1.18	0.76
110	Clark	1951	D2-2	390	203	25.9	1.17	1.63	0	0.61	3.94	1.47	1.2	0.80
111	Clark	1951	D2-3	390	203	24.8	1.17	1.63	0	0.61	4.22	1.65	1.33	0.87
112	Clark	1951	D2-4	390	203	24.5	1.17	1.63	0	0.61	4.23	1.67	1.34	0.88
113	Clark	1951	D3-1	390	203	28.2	1.17	2.44	0	0.92	4.99	1.71	1.31	0.80
114	Clark	1951	D4-1	390	203	23.1	1.17	1.63	0	1.22	3.94	1.65	1.31	0.80
115	Oh	2001	N4200	500	130	23.7	0.85	1.56	0	0	4.08	1.29	1	0.76
116	Oh	2001	N42A2	500	130	23.7	0.85	1.56	0.43	0.12	4.37	1.12	1.07	0.79
117	Oh	2001	N42B2	500	130	23.7	0.85	1.56	0.43	0.22	5.80	1.48	1.42	1.03
118	Oh	2001	N42C2	500	130	23.7	0.85	1.56	0.43	0.34	5.50	1.41	1.34	0.97
119	Oh	2001	H4100	500	130	49.1	0.5	1.56	0	0	9.88	1.7	1.03	1.07
120	Oh	2001	H41A2(1)	500	130	49.1	0.5	1.56	0.43	0.12	10.97	1.89	1.15	1.18
121	Oh	2001	H41B2	500	130	49.1	0.5	1.56	0.43	0.22	10.86	1.87	1.13	1.17
122	Oh	2001	H41C2	500	130	49.1	0.5	1.56	0.43	0.34	10.90	1.87	1.14	1.17
123	Oh	2001	H4200	500	130	49.1	0.85	1.56	0	0	6.17	1.06	0.87	0.95
124	Oh	2001	H42A2(1)	500	130	49.1	0.85	1.56	0.43	0.12	7.51	1.29	1.06	1.15
125	Oh	2001	H42B2(1)	500	130	49.1	0.85	1.56	0.43	0.22	7.02	1.21	0.99	1.07
126	Oh	2001	H42C2(1)	500	130	49.1	0.85	1.56	0.43	0.34	6.47	1.11	0.91	0.99
127	Oh	2001	H4300	500	130	49.1	1.25	1.56	0	0	5.19	1.08	1.18	1.03
128	Oh	2001	H43A2(1)	500	130	49.1	1.25	1.56	0.43	0.12	5.34	1.14	1.22	1.06
129	Oh	2001	H43B2	500	130	49.1	1.25	1.56	0.43	0.22	5.86	1.25	1.33	1.16
130	Oh	2001	H43C2	500	130	49.1	1.25	1.56	0.43	0.34	6.19	1.32	1.41	1.22
131	Oh	2001	H4500	500	130	49.1	2	1.56	0	0	1.73	0.57	0.88	0.51
132	Oh	2001	H45A2	500	130	49.1	2	1.56	0.43	0.12	3.24	1.08	1.65	0.95
133	Oh	2001	H45B2	500	130	49.1	2	1.56	0.43	0.22	3.65	1.25	1.86	1.07
134	Oh	2001	H45C2	500	130	49.1	2	1.56	0.43	0.34	3.62	1.24	1.85	1.05
135	Oh	2001	H41A0	500	120	50.7	0.5	1.29	0	0.13	5.79	0.98	0.59	0.70
136	Oh	2001	H41A1	500	120	50.7	0.5	1.29	0.23	0.13	6.63	1.12	0.67	0.80
137	Oh	2001	H41A2(2)	500	120	50.7	0.5	1.29	0.47	0.13	8.17	1.38	0.83	0.98
138	Oh	2001	H41A3	500	120	50.7	0.5	1.29	0.94	0.13	7.58	1.28	0.77	0.91
139	Oh	2001	H42A2(2)	500	120	50.7	0.85	1.29	0.47	0.13	6.54	1.13	1.12	1.09
140	Oh	2001	H42B2(2)	500	120	50.7	0.85	1.29	0.47	0.24	6.01	1.04	1.03	1.00
141	Oh	2001	H42C2(2)	500	120	50.7	0.85	1.29	0.47	0.37	6.23	1.07	1.06	1.04

Table A.1 (*continued*)

No.	Researcher(s)	Year	Beam ID	d (mm)	b (mm)	f_c (Mpa)	a/d	ρ (%)	ρ_h (%)	ρ_v (%)	V_{test}/bd (Mpa)	Exp./ACI	Exp./CSA	Exp./GSA
142	Oh	2001	H43A0	500	120	50.7	1.25	1.29	0	0.13	3.56	0.9	0.88	0.77
143	Oh	2001	H43A1	500	120	50.7	1.25	1.29	0.23	0.13	4.34	1.09	1.08	0.93
144	Oh	2001	H43A2(2)	500	120	50.7	1.25	1.29	0.47	0.13	4.61	1.19	1.14	0.99
145	Oh	2001	H43A3	500	120	50.7	1.25	1.29	0.94	0.13	4.85	1.25	1.2	1.04
146	Oh	2001	H45A2(2)	500	120	50.7	2	1.29	0.46	0.13	2.75	1.14	1.43	0.86
147	Oh	2001	U41A0	500	120	73.6	0.5	1.29	0	0.13	7.30	1.03	0.73	0.86
148	Oh	2001	U41A1	500	120	73.6	0.5	1.29	0.23	0.13	9.03	1.27	0.9	1.06
149	Oh	2001	U41A2	500	120	73.6	0.5	1.29	0.47	0.13	9.14	1.28	0.91	1.07
150	Oh	2001	U41A3	500	120	73.6	0.5	1.29	0.94	0.13	9.11	1.28	0.9	1.07
151	Oh	2001	U42A2	500	120	73.6	0.85	1.29	0.47	0.13	6.96	1.2	1.19	1.14
152	Oh	2001	U42B2	500	120	73.6	0.85	1.29	0.47	0.24	6.84	1.18	1.17	1.12
153	Oh	2001	U42C2	500	120	73.6	0.85	1.29	0.47	0.37	6.80	1.17	1.16	1.12
154	Oh	2001	U43A0	500	120	73.6	1.25	1.29	0	0.13	4.85	1.22	1.2	1.02
155	Oh	2001	U43A1	500	120	73.6	1.25	1.29	0.23	0.13	5.17	1.3	1.28	1.09
156	Oh	2001	U43A2	500	120	73.6	1.25	1.29	0.47	0.13	5.64	1.46	1.4	1.19
157	Oh	2001	U43A3	500	120	73.6	1.25	1.29	0.94	0.13	5.55	1.43	1.38	1.17
158	Oh	2001	U45A2	500	120	73.6	2	1.29	0.47	0.13	3.56	1.48	1.41	1.10
159	Oh	2001	N33A2	500	130	23.7	1.25	1.56	0.43	0.12	3.51	1.27	1.41	0.81
160	Oh	2001	N43A2	500	130	23.7	1.25	1.56	0.43	0.12	3.92	1.42	1.58	0.90
161	Oh	2001	N53A2	500	130	23.7	1.25	1.56	0.43	0.12	3.19	1.15	1.29	0.74
162	Oh	2001	H31A2	500	130	49.1	0.5	1.56	0.43	0.12	11.47	1.97	1.2	1.23
163	Oh	2001	H32A2	500	130	49.1	0.85	1.56	0.43	0.12	8.15	1.4	1.15	1.24
164	Oh	2001	H33A2	500	130	49.1	1.25	1.56	0.43	0.12	5.81	1.24	1.33	1.15
165	Oh	2001	H51A2	500	130	49.1	0.5	1.56	0.43	0.12	10.80	1.86	1.13	1.16
166	Oh	2001	H52A2	500	130	49.1	0.85	1.56	0.43	0.12	8.73	1.5	1.23	1.33
167	Oh	2001	H53A2	500	130	49.1	1.25	1.56	0.43	0.12	5.58	1.19	1.27	1.11
168	Aguilar et al.	2002	ACI-I	791	305	32	1.16	1.27	0.35	0.31	5.62	1.41	1.36	1.21
169	Aguilar et al.	2002	STM-I	718	305	32	1.27	1.4	0.13	0.31	5.18	1.33	1.29	1.15
170	Aguilar et al.	2002	STM-H	801	305	28	1.14	1.25	0.06	0.31	5.26	1.58	1.43	1.17
171	Aguilar et al.	2002	STM-M	801	305	28	1.14	1.25	0	0.1	5.23	1.56	1.42	1.17
172	Quintero-Febres	2006	A1	370	150	22	1.42	2.79	0.1	0.28	4.52	1.98	1.45	0.93
173	Quintero-Febres	2006	A2	370	150	22	1.42	2.79	0.1	0.28	4.27	1.87	1.37	0.88
174	Quintero-Febres	2006	A3	370	150	22	1.42	2.79	0	0	3.98	1.74	1.28	0.87
175	Quintero-Febres	2006	A4	370	150	22	1.42	2.79	0	0	3.53	1.54	1.13	0.77
176	Quintero-Febres	2006	B1	375	150	32.4	0.89	2.04	0.1	0.23	8.11	1.72	1.1	1.18
177	Quintero-Febres	2006	B2	375	150	32.4	0.89	2.04	0.1	0.23	7.57	1.6	1.02	1.10
178	Quintero-Febres	2006	B3	375	150	32.4	0.81	2.04	0	0	8.32	1.76	1.01	1.15
179	Quintero-Febres	2006	B4	375	150	32.4	0.81	2.04	0	0	8.16	1.73	0.99	1.13
180	Quintero-Febres	2006	HA1	380	100	50.3	1.57	4.08	0.15	0.38	6.97	1.18	1.32	0.96
181	Quintero-Febres	2006	HA3	380	100	50.3	1.43	4.08	0	0	7.68	1.52	1.26	0.99
182	Quintero-Febres	2006	HB1	380	100	50.3	0.9	4.08	0.15	0.67	12.74	2.16	1.08	1.10
183	Quintero-Febres	2006	HB3	380	100	50.3	0.82	4.08	0	0	12.11	2.06	0.93	0.98
184	Tan et al.	1995	A-0.27-2.15	463	110	58.8	0.27	1.23	0	0.48	13.25	2.08	0.93	1.07
185	Tan et al.	1995	A-0.27-3.23	463	110	51.6	0.27	1.23	0	0.48	12.37	2.07	0.99	1.00
186	Tan et al.	1995	A-0.27-4.30	463	110	53.9	0.27	1.23	0	0.48	12.57	2.06	0.96	1.02

187	Tan et al.	1995	A-0.27-5.38	463	110	57.3	0.27	1.23	0	0.48	12.37	1.97	0.89	1.00
188	Tan et al.	1995	B-0.54-2.15	463	110	56	0.54	1.23	0	0.48	9.19	1.48	0.87	1.18
189	Tan et al.	1995	B-0.54-3.23	463	110	45.7	0.54	1.23	0	0.48	8.74	1.56	0.93	1.14
190	Tan et al.	1995	B-0.54-4.30	463	110	53.9	0.54	1.23	0	0.48	9.82	1.61	0.93	1.26
191	Tan et al.	1995	B-0.54-5.38	463	110	53	0.54	1.23	0	0.48	9.43	1.56	0.89	1.22
192	Tan et al.	1995	C-0.81-2.15	463	110	51.2	0.81	1.23	0	0.48	7.91	1.33	1.14	1.31
193	Tan et al.	1995	C-0.81-3.23	463	110	44	0.81	1.23	0	0.48	7.85	1.46	1.31	1.32
194	Tan et al.	1995	D-1.08-2.15	463	110	48.2	1.08	1.23	0	0.48	5.30	1.21	1.1	1.06
195	Tan et al.	1995	D-1.08-3.23	463	110	44.1	1.08	1.23	0	0.48	5.50	1.37	1.21	1.10
196	Tan et al.	1995	D-1.08-4.30	463	110	46.8	1.08	1.23	0	0.48	5.69	1.33	1.2	1.14
197	Tan et al.	1995	D-1.08-5.38	463	110	48	1.08	1.23	0	0.48	5.69	1.3	1.18	1.13
198	Tan et al.	1995	E-1.62-3.23	463	110	50.6	1.62	1.23	0	0.48	4.32	1.41	1.67	1.14
199	Tan et al.	1995	E-1.62-4.30	463	110	44.6	1.62	1.23	0	0.48	3.73	1.38	1.59	0.99
200	Tan et al.	1995	E-1.62-5.38	463	110	45.3	1.62	1.23	0	0.48	3.40	1.24	1.43	0.90
201	Tan et al.	1995	F-2.16-4.30	463	110	41.1	2.16	1.23	0	0.48	2.95	1.57	2.36	1.04
202	Tan et al.	1995	G-2.70-5.38	463	110	42.8	2.7	1.23	0	0.48	2.06	1.31	2.61	0.98
203	Anderson and Ramirez	1989	1	425	203	39	2.15	2.67	0	2.65	5.55	1.59	2.62	1.33
204	Anderson and Ramirez	1989	2	425	203	41.4	2.15	2.67	0	2.65	5.67	1.54	2.56	1.35
205	Anderson and Ramirez	1989	3	425	203	42.7	2.15	2.67	0	2.65	5.92	1.55	2.6	1.41
206	Anderson and Ramirez	1989	4	425	203	27.5	2.15	2.67	0	2.65	5.10	2.08	3.23	1.26
207	Anderson and Ramirez	1989	5	425	203	28.7	2.15	2.67	0	2.65	4.94	1.93	3.03	1.21
208	Anderson and Ramirez	1989	6	425	203	29.4	2.15	2.67	0	2.65	4.27	1.63	2.56	1.05
209	Anderson and Ramirez	1989	7	425	203	32.1	2.15	2.67	0	2.65	4.53	1.58	2.52	1.10
210	Anderson and Ramirez	1989	8	425	203	33.9	2.15	2.67	0	2.65	4.17	1.38	2.22	1.01
211	Anderson and Ramirez	1989	9	425	203	34.4	2.15	2.67	0	2.65	4.58	1.49	2.4	1.11
212	Anderson and Ramirez	1989	10	425	203	31	2.15	2.67	0	2.65	4.48	1.63	2.56	1.09
213	Anderson and Ramirez	1989	11	425	203	32.3	2.15	2.67	0	2.65	4.27	1.49	2.36	1.04
214	Anderson and Ramirez	1989	12	425	203	33.2	2.15	2.67	0	2.65	3.83	1.3	2.07	0.93
										Average		1.77	1.64	1.01
										Standard deviation		0.57	0.57	0.15

beams with respect to involving parameters. As Fig. 11a shows, with increasing f_c, the shear strength of a deep beam increases to a certain limit and then does not change significantly. As can be observed from Fig. 11b, a significant reduction on the shear strength occurs with the increase of shear span to depth ratio, *a/d*, but the gradient of the curve decreases with the increase of *a/d* ratio. Fig. 11c–e proves that there is an almost linear relationship between the shear strength and all three main longitudinal bottom reinforcement, vertical and horizontal web reinforcement ratios.

5. Conclusions

In this study, a robust variant of GP, namely GSA, was employed to assess the shear resistance of RC deep beams. A comparison between the results of shear strength of deep beams, obtained by means of the GSA model and ACI and CSA codes, was made. The GSA-based model gives a reliable estimation for the shear strength of deep beams. It also produces better outcomes than the ACI and CSA models, considerably. Additionally, the proposed GSA-based formula for the shear strength is simple and straightforward.

```
{
 double f[8];
 double tmp = 0;


f[1]=f[2]=f[3]=f[4]=f[5]=f[6]=f[7]=0;
 f[0]=v[0];

 l0: f[0]/=f[0];
 l1: f[0]+=f[0];
 f[1]+=f[0];
 l2: f[0]*=v[4];
 l3: f[0]-=f[1];
 f[0]+=f[0];
 l4: f[0]+=v[0];
 l5: f[0]/=5;
 l6: f[0]*=f[0];
 l7: f[1]+=f[0];
 l8: f[0]-=f[1];
 f[0]+=f[1];
 l9: f[0]-=v[1];
 l10: f[0]-=3;
 l11: f[0]+=v[3];
 l12: f[0]-=1;
 l13: f[0]+=v[4];
 l14: f[0]/=f[1];
 l15: f[0]*=v[2];
 l16: f[0]+=f[0];
 l17: f[0]/=v[1];
 l18: f[0]-=v[1];
 l19: l20: f[0]-=-4;
 l21:
 l22:

 return f[0];
}
```

Fig. A.1 – The following machine learning program can be compiled in C++ environment or run in the Discipulus interactive evaluator mode. (*Note*: *v*[0], *v*[1], *v*[2], *v*[3], and *v*[4] respectively represent f_c', a/d ρ, ρ_h and ρ_v).

Appendix

See Table A.1 and Fig. A.1.

REFERENCES

[1] ACI, Committee 318. Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05), USA, 2005.

[2] A.H. Alavi, M. Ameri, A.H. Gandomi, M.R. Mirzahosseini, Formulation of flow number of asphalt mixes using a hybrid computational method, Construction and Building Materials 25 (3) (2011) 1338–1355.

[3] A.H. Alavi, A.H. Gandomi, A robust data mining approach for formulation of geotechnical engineering systems, Engineering Computations 28 (3) (2011) 242–274.

[4] A.H. Alavi, A.H. Gandomi, A.A.R. Heshmati, Discussion on soft computing approach for real-time estimation of missing wave heights, Ocean Engineering 37 (13) (2010) 1239–1240.

[5] A.H. Alavi, A.H. Gandomi, J. Bolouri, A. Mollahasani, Linear and Tree-based genetic programming for solving geotechnical engineering problems, in: X.S. Yang et al. (Ed.), Chapter 12 in Metaheuristics in Water, Geotechnical and Transportation Engineering, Elsevier, Waltham, MA, 2013, pp. 289–310.

[6] P. Aminian, H. Niroomand, A.H. Gandomi, A.H. Alavi, M. Arab Esmaeili, New design equations for assessment of load carrying capacity of castellated steel beams: a machine learning approach. Neural Computing & Applications, in press. doi:10.1007/s00521-012-1138-4.

[7] A. Arabzadeh, A.R. Rahaie, R. Aghayari, A simple strut-and-tie model for prediction of ultimate shear strength of rc deep beams, International Journal of Civil Engineering 7 (3) (2009) 141–153.

[8] A.F. Ashour, L.F. Alvarez, V.V. Toropov, Empirical modeling of shear strength of RC deep beams by genetic programming, Computers and Structures 81 (2003) 331–338.

[9] M. Brameier, W. Banzhaf, A comparison of linear genetic programming and neural networks in medical data mining, IEEE Transactions on Evolutionary Computation 5 (1) (2001) 17–26.

[10] M. Brameier, W. Banzhaf, Linear Genetic Programming, Springer Science +Business Media, LLC, 2007.

[11] K.K. Choi, A.G. Sherif, M.M. Reda Taha, L. Chung, Shear strength of slender reinforced concrete beams without web reinforcement: a model using fuzzy set theory, Engineering Structures 31 (2009) 768–777.

[12] M. Conrads, O. Dolezal, F.D. Francone, P. Nordin, Discipulus Lite^{TM}-fast Genetic Programming Based on AIM Learning Technology, Register Machine Learning Technologies Inc., Littleton, 2004.

[13] CSA, Committee A23.3. Design of Concrete Structures: Structures (Design)—A National Standard of Canada, Canadian Standards Association, 1994.

[14] L.M. Deschaine, F.A. Zafran, J.J. Patel, D. Amick, R. Pettit, F.D. Francone, P. Nordin, E. Dilkes, L.V. Fausett, Solving the unsolved using machine learning, Data mining and

knowledge discovery to model a complex production process, in: Proceedings of Advanced Simulation Technologies Conference, Washington DC, USA, 2000.
[15] Y.I. Elbahy, M. Nehdi, M.A. Youssef, Artificial neural network model for deflection analysis of superelastic shape memory alloy reinforced concrete beams, Canadian Journal of Civil Engineering 37 (6) (2010) 855–865.
[16] G. Folino, C. Pizzuti, G. Spezzano, Genetic programming and simulated annealing: a hybrid method to evolve decision trees, in: R. Poli, W. Banzhaf, W.B. Langdon, J.F. Miller, P. Nordin, T.C. Fogarty (Eds.), Genetic Programming, Proceedings of EuroGP'2000, vol. 1802, Springer-Verlag, Edinburgh, 2000, pp. 294–303.
[17] F.D. Francone, Discipulus Lite^{TM} Owner's Manual, Machine Learning Technologies Inc., Littleton, CO, USA, 2004.
[18] I.E. Frank, R. Todeschini, The Data Analysis Handbook, Elsevier, The Netherland, Amsterdam, 1994.
[19] A.H. Gandomi, A.H. Alavi, Expression programming techniques for formulation of structural engineering systems, in: A.H. Gandomi et al. (Ed.), Chapter 19 in Metaheuristic Applications in Structures and Infrastructures, Elsevier, Waltham, MA, USA, 2013.
[20] A.H. Gandomi, A.H. Alavi, M. Mousavi, S.M. Tabatabaei, A hybrid computational approach to derive new ground-motion attenuation models, Engineering Applications of Artificial Intelligence 24 (4) (2011) 717–732.
[21] A.H. Gandomi, A.H. Alavi, G.J. Yun, Nonlinear modeling of shear strength of sfrc beams using linear genetic programming, Structural Engineering and Mechanics 38 (1) (2011) 1–25.
[22] A.H. Gandomi, S.K. Babanajad, A.H. Alavi, Y. Farnam, A novel approach to strength modeling of concrete under triaxial compression, Journal of Materials in Civil Engineering 24 (9) (2012) 1132–1143.
[23] A.H. Gandomi, X.S. Yang, S. Talatahari, A.H. Alavi, Metaheuristics in modeling and optimization, in: A.H. Gandomi et al. (Ed.), Chapter 1 in Metaheuristic Applications in Structures and Infrastructures, Elsevier, Waltham, MA, USA, 2013.
[24] A.H. Gandomi, G.J. Yun, A.H. Alavi, An evolutionary approach for modeling of shear strength of RC deep beams, Materials and Structures, in press, http://dx.doi.org/10.1617/s11527-013-0039-z.
[25] A.H. Gandomi, A.H. Alavi, M.G. Sahab, M. Gandomi, M. Safari Gorji, Empirical models for the prediction of flexural resistance and initial stiffness of welded beam-column joints, in: Proceedings of the 11th East Asia-Pacific Conference on Structural Engineering & Construction, Taiwan, Paper no. 320, 2008.
[26] A. Golbraikh, A. Tropsha, Beware of q2, Journal of Molecular Graphics and Modeling 20 (2002) 269–276.
[27] S. Kirkpatrick, Gelatt C.D.J.R., M.P. Vecchi, Optimisation by simulated annealing, Science 220 (4598) (1983) 671–680.
[28] F.K. Kong, G.R. Sharp, S.C. Appleton, C.J. Beaumont, L.A. Kubik, Structural idealization for deep beams with web openings: further evidence, Magazine of Concrete Research 30 (103) (1978) 89–95.
[29] F.K. Kong, Reinforced Concrete Deep Beams, Taylor & Francis e-Library, Van Nostrand Reinhold New York, 2002.
[30] J.R. Koza, Genetic Programming on the Programming of Computers by Means of Natural Selection, MIT Press, Cambridge MA, USA, 1992.
[31] A.B. Matamoros, K.H. Wong, Design of simply supported deep beams using strut-and-tie models, ACI Structural Journal 100 (6) (2003) 704–712.
[32] S.T. Mau, T.T.C. Hsu, A formula for the shear strength of deep beams, ACI Structural Journal 86 (5) (1989) 516–523.
[33] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, E. Teller, Equation of state calculations by fast computing mechanics, Journal of Chemical Physics 21 (1953) 1087–1092.
[34] E. Mörsch, Concrete-Steel Construction, (Der Eisenbetonbau), English translation of the 3rd German Edition, McGraw-Hill Book Company, New York, 1909 (368 pp.).
[35] A.C. Oreta, Simulating size effect on shear strength of RC beams without stirrups using neural networks, Engineering Structures 26 (5) (2004) 681–691.
[36] Y. Pan, J. Jiang, R. Wang, H. Cao, Y. Cui, A novel QSPR model for prediction of lower flammability limits of organic compounds based on support vector machine, Journal of Hazardous Materials 168 (2009) 962–969.
[37] J.W. Park, D. Kuchma, Strut-and-Tie model analysis for strength prediction of deep beams, ACI Structural Journal 104 (6) (2007) 657–666.
[38] J.L. Pérez, A. Cladera, J.R. Rabuñal, F.M. Abella, Optimal adjustment of EC-2 shear formulation for concrete elements without web reinforcement using Genetic Programming, Engineering Structures 32 (2010) 3452–3466.
[39] N.K. Raju, Advanced Reinforced Concrete Design, CBS Publishers & Distributors, New Delhi Bangalore, 2005.
[40] W. Ritter, Die Bauweise Hennebique, Schweizerische Bauzeitung 33 (1899) 59–61.
[41] P.P. Roy, K. Roy, On some aspects of variable selection for partial least squares regression models, QSAR & Combinatorial Science 27 (2008) 302–313.
[42] A. Sanad, M.P. Saka, Prediction of ultimate shear strength of reinforced concrete deep beams using neural networks, Journal of Structural Engineering 127 (7) (2001) 818–827.
[43] G.N. Smith, Probability and Statistics in Civil Engineering, Collins, London, 1986.
[44] C.W. Tang, Using radial basis function neural networks to model torsional strength of reinforced concrete beams, Computers and Concrete 3 (5) (2006) 335–355.
[45] C.Y. Tang, K.H. Tan, Interactive mechanical model for shear strength of deep beams, Journal of Structural Engineering 30 (10) (2004) 1534–1544.
[46] R.S. Torres, A.X. Falcao, M.A. Goncalves, J.P. Papa, B. Zhang, W. Fan, E.A. Fox, A genetic programming framework for content-based image retrieval, Pattern Recognition 42 (2) (2009) 283–292.
[47] L.A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[48] N. Zhang, K.H. Tan, Direct strut-and-tie model for single span and continuous deep beams, Engineering Structures 29 (11) (2007) 2987–3001.