

HW-6

① There are n different items.

For optimal solution, we either choose the item or we don't.

Let there be a subproblem with k items, and w capacity.

$$OPT(k, w) = \max \begin{cases} OPT(k-1, w), & \text{not select} \\ OPT(k, w - w_k) + v_k, & \text{selected} \end{cases}$$

hence add value

$$\boxed{\Theta(w_k)}$$

Initial condition: $OPT(0, 0) = 0$

② Let there be a substring from i th element to j th element.
Here, $0 \leq i \leq j$

$$\therefore OPT(j) = \max_{\text{where } S_{i+1}, j \text{ is a valid word}} OPT(i)$$

Time complexity: $\boxed{\Theta(n^2)}$

③ We select the balloon to burst at the end.
Let that balloon be i . So first we burst all balloons on its left side,
then its right side.

$$(1 \leq i \leq n)$$

$$\therefore OPT(1, n) = \max \begin{cases} OPT(1, i-1) + OPT(i+1, n) \\ + n[i] \times nums[i-1] \times nums[i+1] \end{cases}$$

$\boxed{\Theta(n^3)}$

④ We have to maximize to quality.
Let there be a substring denoted by S_{ij} for $S_i S_{i+1} S_{i+2} \dots S_j$

Here, $0 \leq i < j$

$$OPT(j) = \max (OPT(i) + \text{quality}(S_{i+1,j}))$$

$\Theta(n^2) \rightarrow \text{complexity}$

⑤ Let there be k words in first line from a set of n words; in opt. solⁿ.
Let $OPT(i)$ be sum of squares of blacks for the optimal solⁿ with words w_{i+1} to w_n .

Now suppose there are p words at present in first line.

$$OPT(i) = \begin{cases} 0 & \text{if } p \geq n - (i-1) \\ \min_{1 \leq k \leq p} \{ (S(i,k))^2 + OPT(i+k) \} & \text{if } p < n - (i-1) \end{cases}$$

Complexity \rightarrow pseudo-polynomial.

⑥ Let there be at least x ^{A-}votes in dist. 1 and y A-votes in dist. 2.
 $M[i, p, x, y] = \text{true}$.

Look for entry $M[i, p-1, x-2, y]$.

Each dist. gets $mn/2$ votes total.

$$O(n^2 m^2)$$