HW10

(1) False.

If $B \in P$, then $A \leq_p B$ implies that $A \in P$.
Given that $A \in NPC$, so any NP problem can be polynomial time reducible to A.
In this case $P = NP$.

(2) False.

(3) True.

→ $B \in NP$, means it has polynomial time verifier V_B, constant k_B such that $x \in B$ iff $\exists C_x \Rightarrow \|C_x\| \leq \|x\|^k$ & $V_B(x, C_x) = 1$

(4) Yes it is in NP.

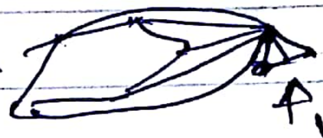
→ For every composite number, a factor of the num. is a certificate. Divide the num. by factor → remainder 0 then verified.
So the factor - at most n bits.
verification - polynomial in n.
Hence NP.

(5) True.

(6) Let there be a graph $G = (V, E)$ [Vertex Cover]
Since each edge will make at least one as a count to the vertices connected to it.
So the total will be $2|E|$ → even number.
∴ There is an even number of vertices in G that have odd degrees.

Now let there be a subset V' of vertices with odd degrees in G. Now let there be $\langle \bar{G}, k+2 \rangle$ as new instance of vertex cover. We add a new triangle with 3 new vertices & then connect one of them to all $\bar{G}$ vertices.

Now the degree will be even for every vertex



↑ VC of Δ is min. size 2.

$\therefore$ $\bar{G}$ has a vertex cover of size $k+2$ iff G has a vertex cover of size k.

(7) $Ax \leq b$, $\therefore$ 0-1 integer programming is NP-

→ Now, 3-CNF-SAT $\leq_p$ 0-1 Integer Programming.

n var. ⟵ a var. x & its negation: $(1-x)$

(a var. x)

$$(x \vee (1-y) \vee z) \wedge ((1-x) \vee y \vee (1-z)) \wedge \ldots$$

$$\therefore \quad x + (1-y) + z \geq 1$$

$$\Rightarrow -x + y - z \leq 0$$

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