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HW-11

① Yes it is a decision problem in NP.
for every "yes" instance (when the no is composite), a factor of the number is certificate — verified by dividing the number with its factor (giving remainder 0 if yes) — done in polynomial time.

② Sum of degrees: $2|E|$ — even number.
$\langle G, k \rangle$

Now suppose $\langle \bar{G}, k+2 \rangle$ is a new instance of vertex cover. create new triangle with the new two vertices and old one.
Degree is even in both.
Hence, $\bar{G}$ has a vertex cover of size $k+2$ iff G has a vertex cover of size k.

③ → certificate - verified in polynomial time.
Hence, NP.

→ $\langle G_1, m)$: Clique problem is to decide if the graph has a clique of size m.
Its complement G_1' has same set of vertices but incident edge is inverted.
> Independent set $\leq_p$ Clique.
> $\langle G_2 = (V_2, E_2), m \rangle$
reduce it to half-clique: $m = \left\lceil \frac{V_2}{2} \right)$

Clique $\leq_p$ Half-Clique.
From transitivity: Independent Set $\leq_p$ Half-Clique

④ Yes it is a decision problem in NP.
⇒ simple path of length k → certificate → verified in polynomial time.
⇒ Hamiltonian Path $\leq_p$ K-Path
and Hamiltonian Cycle $\leq_p$ Ham. Path.
∴ Transitivity : HC $\leq_p$ K-path
Thus K-Path is NP-complete, and P ≠ NP,
so K-Path is not ~~NP~~ in P.

⑤ → certifier : BHC instance.
> certificate: sequence of edges.
> verifies ~~in~~ sequence of edges form HC & the total wt. is atleast half the total wt. of edges in graph.
> Hence NP-complete.
→ HAM Cycle $\leq_p$ BIG HAM Cycle.
→ When the weighted graph is feed to BHC decider black box, it returns "yes" iff. G has a HC containing e.

⑥ → verify in NP
→ HAM Path $\leq_p$ Pebble.
→ use black box to solve pebble more than once.

⑦ $G = (V, E)$ $A(G) = 1$
Let $A(\bar{G}) = 1'$ [$\overset{G}{\text{an}}$ edge removed from e of G]
If $A(\bar{G}) = 0$, then every HC in G contains e.
> Run BFS to enumerate the edges of HC in order.