

# Contracted 1-Tunnel within the Movement Union

April 15, 2016

**Theorem 1** *A Robot  $R$  consisting of two  $2 \times 2$  contracted Modules  $M_a$ ,  $M_b$  cannot perform a 1-Tunnel move within the union of the movement. Assuming that units cannot switch which module they belong to.*

1	2		
3	4		
5	6	9	10
7	8	11	12
$a_1$	$a_2$		
$a_3$	$a_4$		
$b_1$	$b_2$		
$b_3$	$b_4$		

**Definition 0.1** *An INSIDE unit space is a unit space belonging to the set  $\{2, 4, 6, 9, 10\}$*

**Definition 0.2** *An OUTSIDE unit space is a unit space belonging to the set  $\{1, 3, 5, 7, 8, 11, 12\}$*

**Lemma 0.3** *A movement of this type requires moving a unit in an INSIDE unit space to an OUTSIDE unit space or vis-versa.*

**Proof 0.4** *Before the movement,  $M_a$  has two units  $a_1, a_2 \in OUTSIDE$ , and two units,  $a_3, a_4 \in INSIDE$ .  $M_b$  has 3 units  $b_1, b_3, b_4 \in OUTSIDE$  and 1 unit,  $b_2 \in INSIDE$ .*

*It follows that to stay in the union of the movement, all units must be either INSIDE or OUTSIDE. i.e.  $UNITS = INSIDE \cup OUTSIDE$ , and  $INSIDE \cap OUTSIDE = \emptyset$*

**Lemma 0.5**