Chapter 1

Compiling single sections at a time

For writing purposes only.

Chapter 2

Results

2.1 Uncertainty of parameters

2.1.1 Method

For varying different parameters simultaneously a similar method was used as in section ??, where a “fudge-factor” was applied to the `Eris` best fitted parameter-values. The “fudge-factor” is distributed gaussian around 1.0 and the factor is the multiplied to the parameter-value.

Monte Carlo experiment

In order to manipulate several yield-values in `Omega` at once, a modification to `__set_yield_tables` in `Chem_Evol` were implemented. This is similar to the modification in section ??, but includes list for all isotopes and “fudge factors” applied.

Other input variables are multiplied with a similar factor before the `Omega`-simulation is executed.

```
##########################################################
### End of function as written in 'chem_evol.py' ###
"""
Change ytables(multiply yields of 'isotope' with 'factor')
This step requires
'self.loa_manip_isotope' and 'self.loa_manip_yields'!
"""
##########################################################

#AGB + massive stars, and pop3 stars
#loop over the different objects
for table_object, table_name in zip([self.ytables, self.ytables_pop3],
                                    ["agb/massive", "pop3"]):
    #get list of available metalicities
    loa_metallicities = table_object.metallicities
    for Z in loa_metallicities:
        #get list of masses for each metallicity
        loa_masses = table_object.get(Z=Z, quantity="masses")
        for M in loa_masses:
            #loop over all isotopes to manipulate
            for manip_isotope, manip_factor in zip(self.loa_manip_isotopes,self.loa_manip_yields):
                #get current yield
                try:
                    present_yield = table_object.get(M=M, Z=Z, quantity="Yields",
                                                     specie=manip_isotope)
                except IndexError: #this means that isotope doesn't exist for this table
                    continue
                #modify yield by some factor
                new_yield = present_yield*manip_factor
                #"insert" new yield back into table
                table_object.set(M=M, Z=Z, specie=manip_isotope, value=new_yield)
                #print "Fixed new yield(%s): from %1.4e to %1.4e"%(table_name, present_yield, new_yield)
```

```

# SN1a, NS-NS merger, BH-NS merger
#loop over different objects
for table_object, table_name in zip(
        [self.ytables_1a, self.ytables_nsmerger, self.ytables_bhnsmerger],
        ["sn1a", "nsm", "bhnsm"]):
    #get list of available metalicities
    loa_metallicities = table_object.metallicities
    #loop over metallicities
    for i_Z, Z in enumerate(loa_metallicities):
        #loop over all isotopes to manipulate
        for manip_isotope, manip_factor in zip(self.loa_manip_isotopes,self.loa_manip_yields):
            # get index of isotope
            index_iso = self.history.isotopes.index(manip_isotope)
            #get current yield
            try:
                present_yield = table_object.yields[i_Z][index_iso]
            except IndexError: #this means that isotope doesn't exist for this table
                continue
            #modify yield by some factor
            new_yield = present_yield*manip_factor
            #"insert" new yield back into table
            table_object.yields[i_Z][index_iso] = new_yield
            #print "Fixed new yield(%s): from %1.4e to %1.4e"%(table_name, present_yield, new_yield)
return
```

Listing 2.1: Snippet of code added to the existing function `__set_yield_tables` in `Chem_Evol` in `Omega`-framework. The code-snippet multiplies the yield of a list of isotopes, `self.loa_manip_isotope`, with a corresponding factor from a list of factors `self.loa_manip_yields` for all yield-tables where the isotopes can be found.

Postprocessing

The data-files for each simulation consists of time-arrays for a multitude of measurables from the simulation, e.g. the mass of $^{187}_{75}\mathrm{Re}$ in the interstellar medium. These measurables do not account for β^--decay of radioactive isotopes[1] Postprocessing of all the datafiles must be done in order to account for the β^--decay of $^{187}_{75}\mathrm{Re}$ to $^{187}_{76}\mathrm{Os}$. This is done, for each timestep, by calculating the amount of decayed material from parent nucleus to daughter nucleus. The amount of decayed material is calculated from the timestep and halflife of the radioactive parent nucleus, and applied to the current and all following timesteps for parent and daughter nuclei. Add reference to section of β^--decay calculations The new data is then saved to file in the same format. The function for applying the decay to parent nucleus and daughter nucleus ($^{187}_{75}\mathrm{Re}$ and $^{187}_{76}\mathrm{Os}$, respectively,

[1] This might be added in an update of `Omega`, but was not implemented during this thesis work.

in our case).

```python
def apply_decay(self, time_array, parent_array, daughter_array, halflife):
    """ Apply decay from parent to daughter with
    the corresponding time-array and nuclear halflife.
    Halflife in same units as time_array. """

    decay_constant = np.log(2)/halflife

    for i in range(len(time_array)-1):
        #calculate time
        dt = time_array[i+1] - time_array[i]
        #calculate decay
        dN = - decay_constant * parent_array[i] * dt
        #apply decay to parent forall indeces greater then i
        parent_array[i+1:] += dN
        #same for daughter, but negative decay
        daughter_array[i+1:] -= dN

    return parent_array, daughter_array
```

Listing 2.2: Snippet of code implementing β^{-}-decay in postprocessing on data calculated by `Omega`.

2.1.2 results

There are two main experiments;

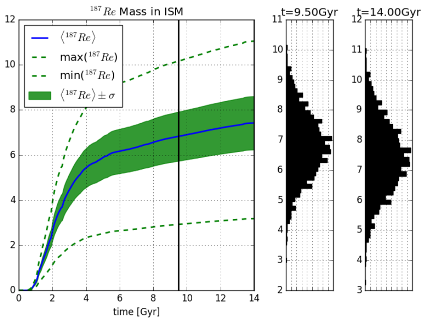
Yields The yields of isotopes are varied within their standard deviation Add reference to arnould table

Yields+IMFslope The yields of isotopes are varied *and* the high mass slope of the initial mass function, α, is varied within the uncertainty found in *(?)* , which is $\sigma_\alpha = 8.73\%$ around the mean $\langle\alpha\rangle = 2.29$.

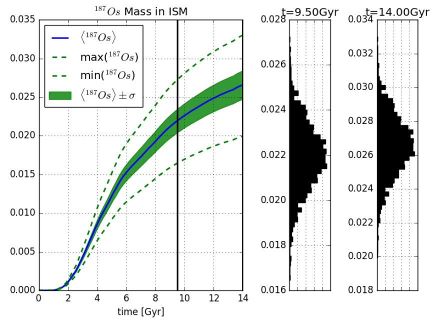
The solar system is formed from a collapse of interstellar gas. The gas is assumed to have separated from the interstellar medium at the formation of the solar system. The formation of the solar system is estimated from meteorites to be 4.5 Gyrs ago add citation and expand on meteorite articles From the semi-analytical model, `Omega`, the total mass of $^{187}_{76}\mathrm{Os}$ and $^{187}_{75}\mathrm{Re}$ in the interstellar medium is calculated. The fraction between the two isotopes, $f_{187} = \frac{^{187}_{76}\mathrm{Os}}{^{187}_{75}\mathrm{Re}}$, is also calculated. The fraction between the isotopes is relevant because it can be determined from meteorites, unlike the total mass of isotopes in the Solar system at the time of formation. In the `Eris`-simulation the galactic age is 14 Gyrs, which means the solar system formed at 9.5 Gyrs. The uncertainties of the mass of each isotope come from the uncertainty of the input parameters in each experiment, **Yields** and **Yields+IMFslope**.

In figures 2.1 the evolution and distribution of $^{187}_{75}\mathrm{Re}$ - and $^{187}_{76}\mathrm{Os}$ -mass in the inter stellar medium is plotted, as well as the ratio between them.

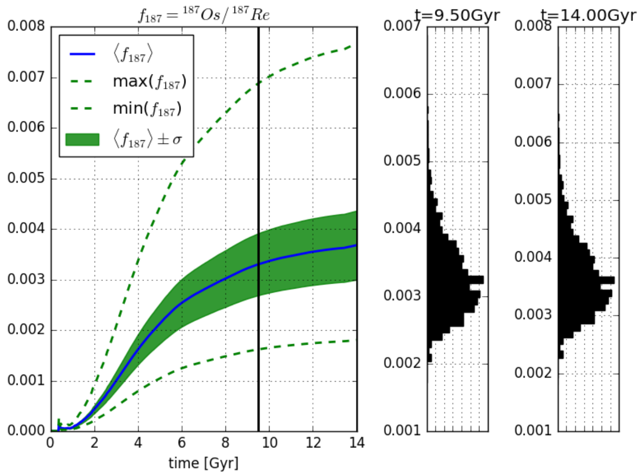
2.1.3 Without β^{-}-decay



(a) Total mass of $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.



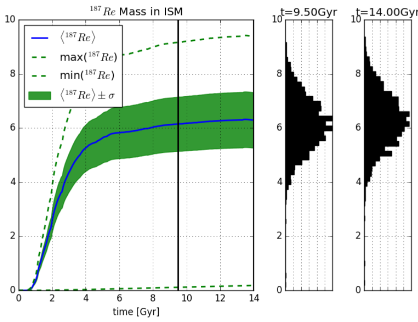
(b) Total mass of $^{187}_{76}$Os in the interstellar medium of the galaxy modelled by `Omega`.



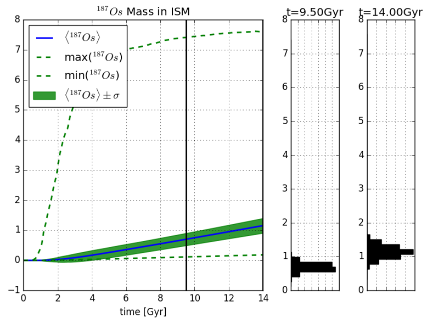
(c) Fraction of $^{187}_{76}$Os to $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.

Figure 2.1: The mass and mass fractions in the interstellar medium *before* β^--decay is applied. Only nucleosynthesis/production from stellar sources is considered.
The far left plot of all subfigures represent the timeevolution of the mass/mass-fraction in the interstellar medium, while the two right plots represent the uncertainty distribution at a given point in time. The points in time are 9.5 Gyrs (the formation of the solar system) and 14 Gyrs (current time). The points in time are also shown by black vertical lines in the far left plot.

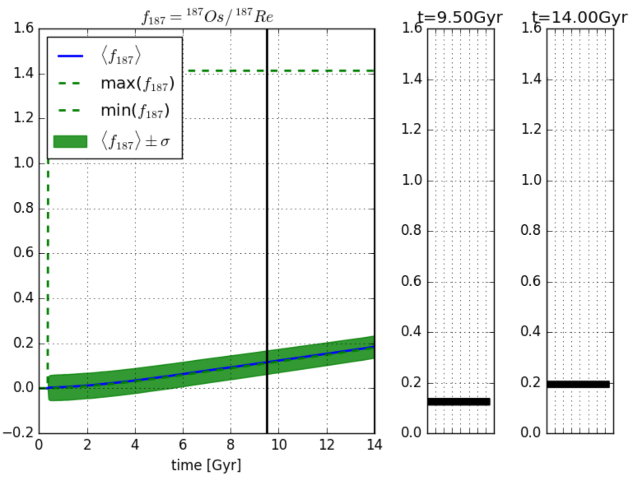
2.1.4 With β^--decay



(a) Total mass of $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.



(b) Total mass of $^{187}_{76}$Os in the interstellar medium of the galaxy modelled by `Omega`.



(c) Fraction of $^{187}_{76}$Os to $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.

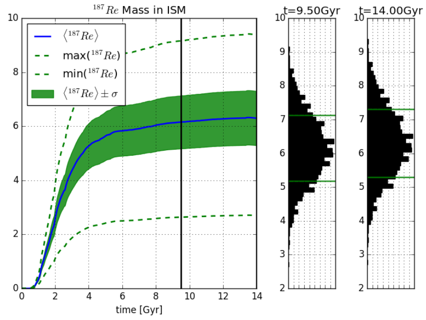
Figure 2.2: The mass and mass fractions in the interstellar medium *after* β^--decay is applied. Nucleosynthesis/production from stellar sources is considered as well as the radioactive decay from $^{187}_{75}$Re to $^{187}_{76}$Os .
The far left plot of all subfigures represent the timeevolution of the mass/mass-fraction in the interstellar medium, while the two right plots represent the uncertainty distribution at a given point in time. The points in time are 9.5 Gyrs (the formation of the solar system) and 14 Gyrs (current time). The points in time are also shown by black vertical lines in the far left plot.

2.1.5 Removing negative isotope yields

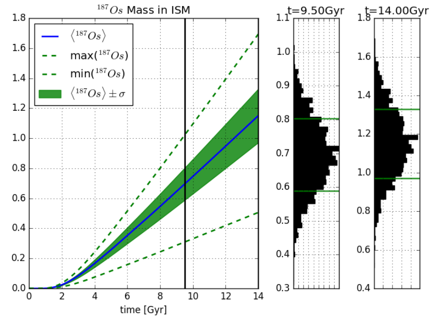
Do to the gaussian distribution of input parameters and relatively large sample size (1500 model calculations) some isotope yields will be negative. Since this is unphysical all negative yields are set to zero, since this is the closest physical interpretation of negative yields from a stellar population.

This effect leads to an overabundance of zero-yields which makes the distribution of input parameters un-gaussian. Overabundances in parameter distributions of this scale leads to outliers in the results. Such outliers also greatly affect the standard deviation of the resulting distribution, see figure 2.2c for an example. One possible solution to this is to take a gaussian distribution set it to zero below parameter-value zero and scale it to the integral (as is the norm for statistical distributions). Applying this form of distribution numerically is beyond the capabilities of the writer. An alternate method is suggested. When the statistical distribution is found from the data, all models with a parameter-value of zero or lower $\hat{Y}_{^{A}_{Z}\mathrm{X}} \leq 0$, is ignored. Signe! How do I correctly give you credit for coming up with this idea when we were discussing the distributions?

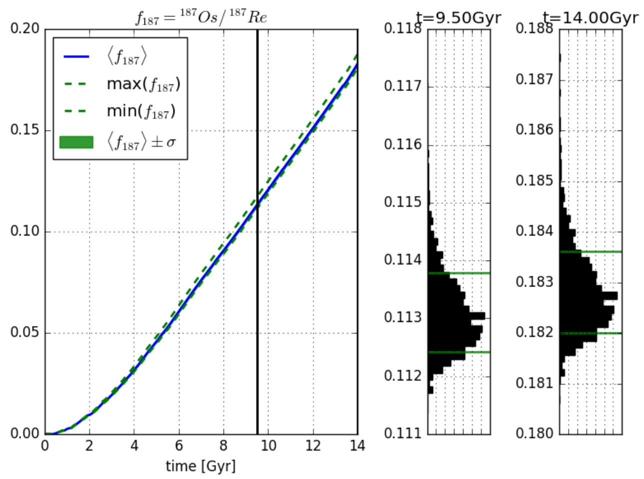
The resulting distributions can be found in figure 2.3.



(a) Mass of $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.



(b) Mass of $^{187}_{76}$Os in the interstellar medium of the galaxy modelled by `Omega`.

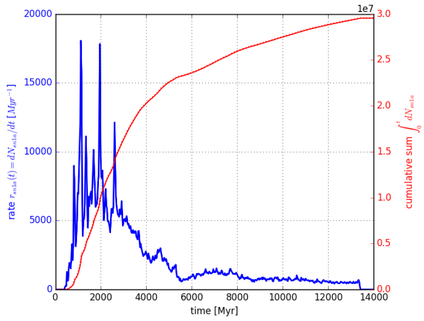


(c) Fraction of $^{187}_{76}$Os to $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.

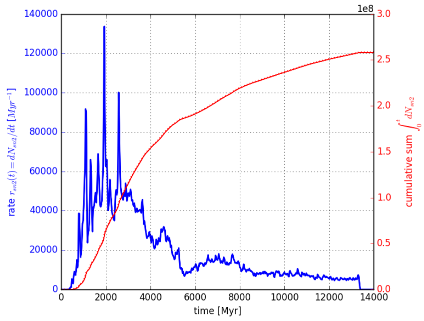
Figure 2.3

2.1.6 Rate of nucleosynthetic events

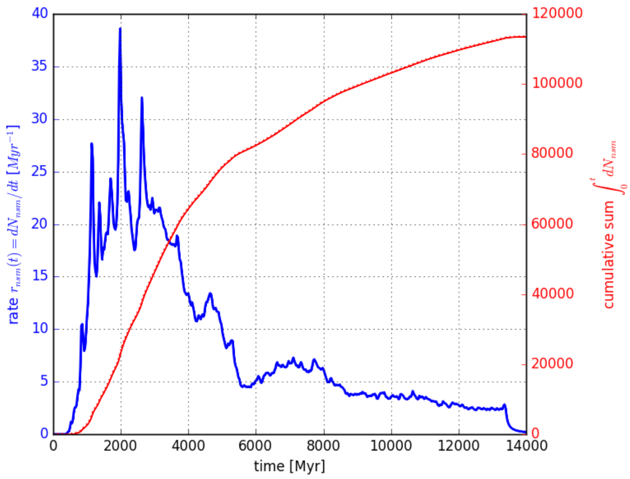
mention something about the nucleosynthetic events chosen, the times chosen and ref table and figure for result what does this result tell us?



(a) Type 1a supernovae.



(b) Type 2 supernovae.



(c) Binary neutron star mergers.

Figure 2.4: All plots show rate of nucleosynthetic events (blue), and cumulative sum of events (red) after β^--decay applied and negative isotope yields have been removed. The nucleosynthetic events are type 1a (2.4a) and type 2 (2.4b) supernovae, and binary neuron star mergers (2.4c). The rate of each event follows the star formation rate (see figure **??**) with a scale factor and delay time distribution.

Binary neutron star mergers

time	rate	ΣN
$14Gyr$	$0.201 Myr^{-1}$	114×10^3
$9.49Gyr$	$3.75 Myr^{-1}$	101×10^3

Type 1a supernovae

time	rate	ΣN
$14Gyr$	$1.1 \times 10^{-44} Myr^{-1} \simeq 0 Myr^{-1}$	29.6×10^6
$9.49Gyr$	$821 Myr^{-1}$	27.2×10^6

type 2 supernovae

time	rate	ΣN
$14Gyr$	$0 Myr^{-1}$	258×10^6
$9.49Gyr$	$8.67 \times 10^3 Myr^{-1}$	233×10^6

Table 2.1: Rates and total number of nucleosynthetic events for neuron star mergers, type 1a and 2 supernovae in `Omega`. The time is taken at $\simeq$9.5 Gyrs (the formation of the solar system, and 14 Gyrs (now). Plots of the time evolution of nucleosynthetic events are shown in figures 2.4.

2.1.7 Comparing models

add section of different models here

With a numerical model for $^{187}_{76}$Os / $^{187}_{75}$Re , the data can be compared to other, analytical models. All analytical models presented here are based on Claytons model for cosmochemical evolution of $^{187}_{76}$Os / $^{187}_{75}$Re , which assumes that the rate of events declines exponentially in time. As can be seen in some appendix the actual number of events and the amount of $^{187}_{75}$Re ejected from each is insignificant when calculating the fraction $^{187}_{76}$Os $_c$/ $^{187}_{75}$Re . $^{187}_{76}$Os $_c$ is the component of $^{187}_{76}$Os from cosmoradiogenic decay from $^{187}_{75}$Re .

Model	$^{187}_{76}\mathrm{Os}_{c}/\ ^{187}_{75}\mathrm{Re}$	$\lambda_{^{187}_{75}\mathrm{Re}}$	λ_{rncp}	Reference
Clayton	$\frac{\Lambda-\lambda}{\lambda}e^{\lambda t}\frac{1-e^{-\Lambda t}}{1-e^{-(\Lambda-\lambda)t}}-1$	$\lambda=\frac{\ln 2}{\tau_{^{187}_{75}\mathrm{Re}}}$	Λ	*(?)*
Clayton Sudden synthesis	$e^{\lambda t}-1$	$\tau_{^{187}_{75}\mathrm{Re}}=47\pm 10Gyr$	$\Lambda\rightarrow\infty$	*(?)*
Clayton Uniform synthesis	$\frac{\lambda t}{1-e^{-\lambda t}}-1$	$\tau_{^{187}_{75}\mathrm{Re}}=47\pm 10Gyr$	$\Lambda\rightarrow 0$	*(?)*
Luck	$\frac{\lambda_{\mathrm{Re}}/\beta(1-e^{-\beta t})-(1-e^{-\lambda_{\mathrm{Re}}t})}{e^{-\beta t}-e^{-\lambda_{\mathrm{Re}}t}}$	$\lambda_{\mathrm{Re}}=1.62\pm 0.08\times 10^{-11}yr^{-1}$	β	*(?)*
Luck Sudden synthesis	———"———		$\beta=10^{-6}yr^{-1}$	*(?)*
Luck Steady state synthesis	———"———		$\beta=10^{-12}yr^{-1}$	*(?)*
Shizuma	$\frac{(1-e^{-\lambda_{\beta}^{\mathrm{eff}}t})-(1-e^{-\lambda t})\lambda_{\beta}^{\mathrm{eff}}/\lambda}{e^{-\lambda_{\beta}^{\mathrm{eff}}t}-e^{-\lambda t}}$	$\lambda_{\beta}^{\mathrm{eff}}=\frac{1.2\ln 2}{\tau_{^{187}_{75}\mathrm{Re}}}$	$\lambda\in[0,2]Gyr^{-1}$	*(?)*
SciPy curvefit	———"———	TODO!	TODO!	

Table 2.2: $\lambda_{^{187}_{75}\mathrm{Re}}$ is the decay constant of radioactive $^{187}_{75}\mathrm{Re}$. λ_{rncp} is the decay constant of the rate of events for *rapid neutron capture processes.* $\lambda_{\beta}^{\mathrm{eff}}$ is the effective net β^{-}-decay constant for $^{187}_{75}\mathrm{Re}$ after thermal consitions of astration have been taken into account, equalt to 1.2 times β^{-}-decay -constant of neutral $^{187}_{75}\mathrm{Re}$. Shizuma does not give any uncertainty for the halflife of $^{187}_{75}\mathrm{Re}$, and the boundaries of λ are only found to be in good agreement with a Galctic age of 11-15 yrs. The basic model for Shizuma, Luck and Clayton are identical, even though they are written differently.

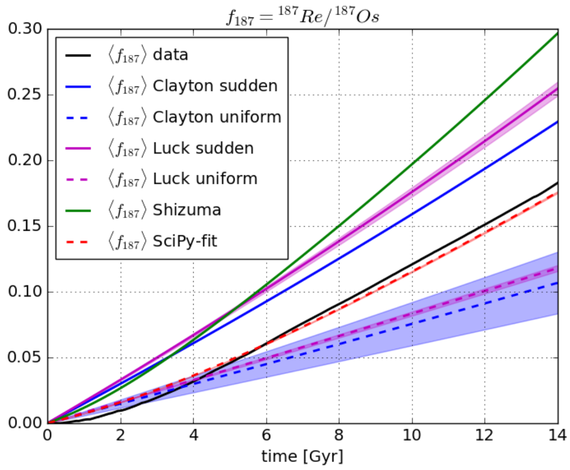
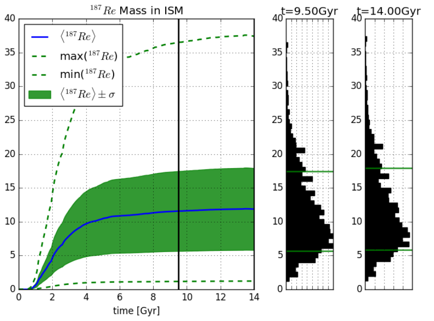


Figure 2.5

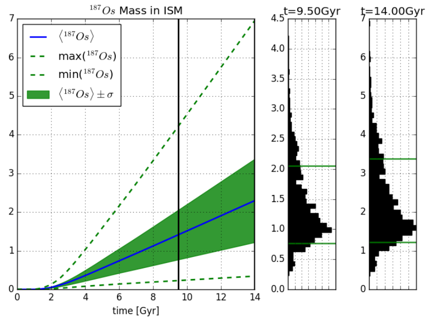
make references to calculations in appendices
add figures with uncertainty

2.1.8 Consider high mass slope of initial mass function

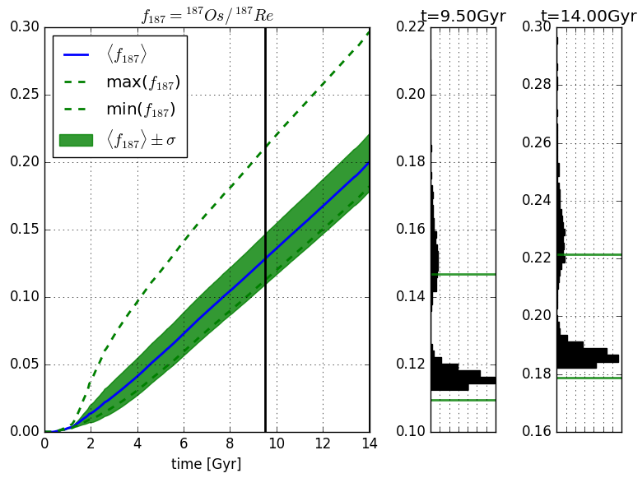
move section of IMFslope experiment here
Removed IMFslope images, trade them with similar, zero-yields removed.



(a) Mass of $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.



(b) Mass of $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.



(c) Fraction of $^{187}_{76}$Os to $^{187}_{75}$Re in the interstellar medium of the galaxy modelled by `Omega`.

Figure 2.6: The mass and mass fractions in the interstellar medium *after* β^--decay is applied and uncertainty in the high mass slope of the initial mass function. Nucleosynthesis/production from stellar sources is considered as well as the radioactive decay from $^{187}_{75}$Re to $^{187}_{76}$Os . The amount of type II supernovae are also varied because the high mass slope of the initial mass function gives more massive stars, which in turn give more type II supernovae.
The far left plot of all subfigures represent the timeevolution of the mass/mass-fraction in the interstellar medium, while the two right plots represent the uncertainty distribution at a given point in time. The points in time are 9.5 Gyrs (the formation of the solar system) and 14 Gyrs (current time). The points in time are also shown by black vertical lines in the far left plot.

2.1.9 Consider events of binary neutron star mergers

Work in progress

Add section of NSM-param experiment here
include nsm-rate plots in section

Appendix A

Calculation of cosmochronology

Following the analytical approach of *(?)* which follows the approach of *(?)* to the chemical evolution of the $^{187}_{75}$Re - $^{187}_{76}$Os -system.
The solar values and evolution of $^{187}_{75}$Re , $^{187}_{76}$Os , $^{187}_{76}$Os come, predominantly from three main sources; s-process contribution of $^{186}_{76}$Os and $^{187}_{76}$Os , r-process contribution of $^{187}_{75}$Re , and β^--decay from $^{187}_{75}$Re to $^{187}_{76}$Os . A simple exponential form is adopted for the r-process contribution to $^{187}_{75}$Re .

$$
\begin{aligned}
{}^{186}_{76}\mathrm{Os}^{\odot} &= {}^{186}_{76}\mathrm{Os}^{s} \\
{}^{187}_{76}\mathrm{Os}^{\odot} &= {}^{187}_{76}\mathrm{Os}^{s} + {}^{187}_{76}\mathrm{Os}^{\beta} \\
\frac{\mathrm{d}\,{}^{187}_{76}\mathrm{Os}^{\beta}}{\mathrm{dt}} &= \lambda_\beta\,{}^{187}_{75}\mathrm{Re} \\
\frac{\mathrm{d}\,{}^{187}_{75}\mathrm{Re}}{\mathrm{dt}} &= A(t) - \lambda_\beta\,{}^{187}_{75}\mathrm{Re} \\
&= A_0 e^{-\tau^{-1}t} - \lambda_\beta\,{}^{187}_{75}\mathrm{Re}
\end{aligned}
$$

<u>Solving for ${}^{187}_{75}\mathrm{Re}$:</u>

$$\frac{\mathrm{d}\,{}^{187}_{75}\mathrm{Re}}{\mathrm{dt}} + \lambda_\beta\,{}^{187}_{75}\mathrm{Re} = A_0 e^{-\tau^{-1}t}$$

General solution to homogenous equation: $\dot{{}^{187}_{75}\mathrm{Re}}_h + \lambda_\beta\,{}^{187}_{75}\mathrm{Re}_h = 0$

$${}^{187}_{75}\mathrm{Re}_h = Re(0)e^{-\lambda_\beta t}$$

Particular solution: $\dot{{}^{187}_{75}\mathrm{Re}}_p = A_1 e^{-\tau^{-1}t}$

$${}^{187}_{75}\mathrm{Re} = {}^{187}_{75}\mathrm{Re}_h + {}^{187}_{75}\mathrm{Re}_p = Re(0)e^{-\lambda_\beta t} + A_1 e^{-\tau^{-1}t}$$

Initial condition of ${}^{187}_{75}\mathrm{Re}$: ${}^{187}_{75}\mathrm{Re}(t=0) = 0 = Re(0) + A_1$

$$Re(0) = -A_1$$

$${}^{187}_{75}\mathrm{Re} = A_1(e^{-\tau^{-1}t} - e^{-\lambda_\beta t})$$

Where A_0 and A_1 are scaling factors for the proposed form of r-process contribution, and tau^{-1} is the "decay constant" of the proposed form of r-process contribution. $\lambda_\beta = \frac{\ln 2}{T_{1/2}}$ is the decay constant of radioactive ${}^{187}_{75}\mathrm{Re}$, and $T_{1/2}$ is the half-life of radioactive ${}^{187}_{75}\mathrm{Re}$.

<u>Solving for ${}^{187}_{76}\mathrm{Os}$:</u>

$$\frac{\mathrm{d}\,{}^{187}_{76}\mathrm{Os}^\beta}{\mathrm{dt}} = \lambda_\beta\,{}^{187}_{75}\mathrm{Re} = \lambda_\beta A_1(e^{-\tau^{-1}t} - e^{-\lambda_\beta t})$$

$${}^{187}_{76}\mathrm{Os}^\beta(\Delta t) = \lambda_\beta A_1 \int_0^{\Delta t} (e^{-\tau^{-1}t} - e^{-\lambda_\beta t})\mathrm{dt}$$

$$= \lambda_\beta A_1 \left[\frac{1}{\tau^{-1}}(1 - e^{-\tau^1\Delta t}) - \frac{1}{\lambda_\beta}(1 - e^{-\lambda_\beta\Delta t})\right]$$

$$= A_1 \left[\frac{\lambda_\beta}{\tau^{-1}}(1 - e^{-\tau^1\Delta t}) - (1 - e^{-\lambda_\beta\Delta t})\right]$$

Where the constants are the same as ${}^{187}_{75}\mathrm{Re}$.

Rename $\frac{{}^{187}_{76}\mathrm{Os}}{{}^{187}_{75}\mathrm{Re}}$ to f_{187}

<u>Calculating fraction of ${}^{187}_{76}\mathrm{Os}$ / ${}^{187}_{75}\mathrm{Re}$ (f_{187} from I</u>

$$\frac{{}^{187}_{76}\mathrm{Os}^\beta(\Delta t)}{{}^{187}_{75}\mathrm{Re}(\Delta t)} \equiv f_{187} = \frac{\frac{\lambda_\beta}{\tau^{-1}}(1 - e^{-\tau^1\Delta t}) - (1 - e^{-\lambda_\beta\Delta t})}{e^{-\tau^{-1}\Delta t} - e^{-\lambda_\beta\Delta t}} \qquad \text{(A.1)}$$

Where Δt is the time between the formation of the galaxy and the formation of the Solar system. This is, according to the model, all the time available to produce r-process isotopes outside of the Solar systembefore collapse.

Adopting meteoritic abundances for the solar values at the formation of the solar system from *(?)* , and assuming the uncertainties of ${}^{187}_{75}\mathrm{Re}$ and ${}^{187}_{76}\mathrm{Os}$ are uncorrelated:

$$\begin{aligned}
{}^{187}_{75}\mathrm{Re}^{\odot}/{}^{186}_{76}\mathrm{Os}^{\odot} &= 3.51 \pm 0.09(\pm 2.56\%) \\
{}^{187}_{76}\mathrm{Os}^{\odot}/{}^{186}_{76}\mathrm{Os}^{\odot} &= 0.793 \pm 0.001(\pm 0.126\%) \\
{}^{187}_{76}\mathrm{Os}^{\odot}/{}^{187}_{75}\mathrm{Re}^{\odot} = f_{187} &= \tfrac{0.793}{3.51} \pm \sqrt{(2.56\%)^2 + (0.126\%)^2} \\
&= 0.226 \pm 2.563\%(\pm 5.79 \times 10^{-3})
\end{aligned} \tag{A.2}$$

Since ${}^{187}_{76}\mathrm{Os}$ and ${}^{187}_{75}\mathrm{Re}$ is separated from the galaxy after the time of formation of the Solar system, the fraction ${}^{187}_{76}\mathrm{Os}$ / ${}^{187}_{75}\mathrm{Re}$ at that time can be caluclated from simple β^--decay of ${}^{187}_{75}\mathrm{Re}$. The time of solar system formation is denoted $t_{f,sos}$, and the current time is denoted t_0. Note that the normalization and integration constants used from here on are not related to the constants used previously.

$$\begin{aligned}
{}^{187}_{75}\mathrm{Re}(t) &= Ae^{-\lambda_\beta(t-t_{f,sos})} \\
\frac{\mathrm{d}\,{}^{187}_{76}\mathrm{Os}}{\mathrm{dt}} &= -\frac{\mathrm{d}\,{}^{187}_{75}\mathrm{Re}}{\mathrm{dt}} = \lambda_\beta A e^{-\lambda_\beta(t-t_{f,sos})} \\
{}^{187}_{76}\mathrm{Os} &= -Ae^{-\lambda_\beta(t-t_{f,sos})} + C
\end{aligned}$$

normalization and integration constants:

$$\begin{aligned}
C &= {}^{187}_{76}\mathrm{Os}(t = t_{f,sos}) \\
A &= {}^{187}_{75}\mathrm{Re}(t = t_{f,sos})
\end{aligned}$$

$$\begin{aligned}
\frac{{}^{187}_{76}\mathrm{Os}}{{}^{187}_{75}\mathrm{Re}} = f_{187}(t) &= \frac{C - Ae^{-\lambda_\beta(t-t_{f,sos})}}{Ae^{-\lambda_\beta(t-t_{f,sos})}} = \frac{C}{A}e^{\lambda_\beta(t-t_{f,sos})} - 1 \\
f_{187}(t_0) &= \frac{C}{A}e^{\lambda_\beta(t_0-t_{f,sos})} - 1 \\
&\Rightarrow \frac{C}{A} = \left[f_{187}(t_0) + 1\right] e^{-\lambda_\beta(t_0-t_{f,sos})} \\
f_{187}(t_{f,sos}) &= \frac{C}{A}e^{\lambda_\beta(t_0-t_{f,sos})} - 1
\end{aligned}$$

This leads to an equation for calculating the Os-Re-fraction, $f_{187}(t_{f,sos})$, at the formation of the Solar systemfrom physical parameters. The physical parameters are; the current Os-Re-fraction, $f_{187}(t_0)$, the decay-rate of ${}^{187}_{75}\mathrm{Re}$, λ_β, and the age of the solar system, $t_0 - t_{f,sos}$.

$$f_{187}(t_{f,sos}) = \left[f_{187}(t_0) + 1\right] e^{-\lambda_\beta(t_0-t_{f,sos})} - 1 \tag{A.3}$$

Estimates of the physical parameter:

$$\begin{aligned} \lambda_\beta &= \ln(2)/T_{1/2} \\ T_{1/2} &= 41.2 \pm 1.3 Gyr \quad \text{from } (?)^{1} \\ t_0 - t_{f,sos} &= 4.568^{0.2\times10^{-1}}_{0.4\times10^{-1}} Gyr \quad \text{from } (?) \\ f_{187}(t_{now}) &= 0.226 \pm 5.79\times10^{-2} \quad \text{from eq.A.2} \end{aligned} \tag{A.4}$$

The average value for $f_{187}(t_{sos})$ from eq. A.3:

$$f_{187}(t_{sos}) = [f_{187}(t_{now})+1]\, e^{-\lambda_\beta \Delta t} - 1 = 0.136 \tag{A.5}$$

Error propagation of $f_{187}(t_{sos})$ from eq. A.3: This needs to be reexamined by someone who knows what they're doing!

$$\left(\frac{\delta f(x,y,z)}{f(x_0,y_0,z_0)}\right)^2 = \left(\frac{\partial f}{\partial x}\right)^2_{(x_0,y_0,z_0)}\left(\frac{\delta x}{x_0}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2_{(x_0,y_0,z_0)}\left(\frac{\delta y}{y_0}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2_{(x_0,y_0,z_0)}\left(\frac{\delta z}{z_0}\right)^2$$

$$\left(\frac{\delta f_{187}(t_{sos})}{f_{187}(t_{sos})}\right)^2 = \begin{cases} \left(\frac{\partial f_{187}(t_{sos})}{\partial \Delta t}\right)^2_{(\Delta t, T_{1/2}, f_{187}(t_{now}))}\left(\frac{\delta \Delta t}{\Delta t}\right)^2 \\ + \left(\frac{\partial f_{187}(t_{sos})}{\partial T_{1/2}}\right)^2_{(\Delta t, T_{1/2}, f_{187}(t_{now}))}\left(\frac{\delta T_{1/2}}{T_{1/2}}\right)^2 \\ + \left(\frac{\partial f_{187}(t_{sos})}{\partial f_{187}(t_{now})}\right)^2_{(\Delta t, T_{1/2}, f_{187}(t_{now}))}\left(\frac{\delta f_{187}(t_{now})}{f_{187}(t_{now})}\right)^2 \end{cases}$$

$$\frac{\partial f_{187}(t_{sos})}{\partial \Delta t} = \frac{\partial(eq.A.3)}{\partial \Delta t} = (f_{187}(t_{now})+1)\left(\frac{-\ln 2}{T_{1/2}}\right) e^{-\ln 2\Delta t/T_{1/2}}$$

$$\frac{\partial f_{187}(t_{sos})}{\partial T_{1/2}} = \frac{\partial(eq.A.3)}{\partial T_{1/2}} = (f_{187}(t_{now})+1)\left(\frac{\ln 2\Delta t^2}{T_{1/2}}\right) e^{-\ln 2\Delta t/T_{1/2}}$$

$$\frac{\partial f_{187}(t_{sos})}{\partial f_{187}(t_{now})} = \frac{\partial(eq.A.3)}{\partial f_{187}(t_{now})} = e^{-\ln 2\Delta t/T_{1/2}}$$

$$\left(\frac{\delta f_{187}(t_{sos})}{f_{187}(t_{sos})}\right)^2 = \begin{cases} \left((f_{187}(t_{now})+1)\left(\frac{-\ln 2}{T_{1/2}}\right) e^{-\ln 2\Delta t/T_{1/2}}\right)^2 \left(\frac{\delta\Delta t}{\Delta t}\right)^2 \\ + \left((f_{187}(t_{now})+1)\left(\frac{\ln 2\Delta t^2}{T_{1/2}}\right) e^{-\ln 2\Delta t/T_{1/2}}\right)^2 \left(\frac{\delta T_{1/2}}{T_{1/2}}\right)^2 \\ + \left(e^{-\ln 2\Delta t/T_{1/2}}\right)^2 \left(\frac{\delta f_{187}(t_{now})}{f_{187}(t_{now})}\right)^2 \end{cases}$$

$$= e^{-2\ln 2\Delta t/T_{1/2}} \left[\begin{array}{l} \left(\frac{\delta f_{187}(t_{now})}{f_{187}(t_{now})}\right)^2 \\ + ((f_{187}(t_{now})+1))^2 \left(\frac{ln2}{T_{1/2}}\right)^2 \\ \qquad \times \left[\left(\frac{\Delta t}{T_{1/2}}\right)^2 \left(\frac{\delta T_{1/2}}{T_{1/2}}\right)^2 - \left(\frac{\delta\Delta t}{\Delta t}\right)^2\right] \end{array}\right]$$

Inserting values for (Δt, $T_{1/2}$, $f_{187}(t_{now})$) with uncertainties from eq.**??**
$\Delta t = 4.5682 Gyr \quad \delta\Delta t = 0.4 \times 10^{-3} Gyr$
$T_{1/2} = 41.2 Gyr \quad \delta T_{1/2} = 1.3 Gyr$
$f_{187}(t_{now}) = 0.226 \quad \delta f_{187}(t_{now}) = 57.9 \times 10^{-3}$

$$= 0.0563 \delta f_{187}(t_{sos}) \qquad = \sqrt{0.056}$$
$$= 0.0323$$

This gives us that, at the formation of the solar system: $f_{187}(t_{sos}) = 0.136 \pm 0.0323$

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