Student's Manual for Programming Methodology

with C++

Paul Kim

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Contents

Part I

Algorithms

Chapter 1

Time Complexity Analysis

1. The running time of an algorithm is relevent to the amount of input. Therefore the running time is a function of the amount of input: $T(n)$

2. Definitions of Time Complexity: with a positive constant c,

   1) Big-O: $T(n) \geq c \times f_O(n) \Rightarrow T(n) = O(f_O(n))$ Best-case scenarios can be described via Big-O functions.

   2) Big-Omega: $T(n) \leq c \times f_\Omega(n) \Rightarrow T(n) = \Omega(f_\Omega(n))$ Worst-case scenarios can be described via Big-Omega functions.

   3) Big-Theta: $T(n) \geq c \times f(n)$ and $T(n) \leq c' \times f(n) \Leftrightarrow T(n) = O(f(n)) = \Omega(f(n)) \Rightarrow T(n) = \Theta(f(n))$ Best- and Worst-case scenarios are the same in Big-Theta functions.

   4) Small-O: $T(n) = O(f_o(n)) \neq \Theta(f_o(n)) \Rightarrow T(n) = o(f_o(n))$

   Constants are ignored, and only the highest degree of the polynomial's monomials are relevant to Time Complexity Analysis.

3. Running Time Calculations

   1) Summations for Loops: One loop sequence of running time $f(i)$ is equivalent to:
   $$T(n) = \sum_{i=1}^{n} f(i)$$
   Two loop sequences of running time $f(i,\ j)$ is equivalent to:
   $$T(n) = \sum_{j=1}^{n} \sum_{i=1}^{n} f(i,\ j)$$

   2) Selective Controls: Worst-case scenario, $T(n) = \max(T_1(n),\ T_2(n), \cdots)$. Best-case = minimum.

   3) Recursion: $T(n) = f(T(n'))$ (점화식)

Chapter 2

Finding the Maximum Subarray Sum

The objective of this challenge is to find the maximum value of the sum of elements in a subarray of a given array. If all integers are negative, said maximum value is the sum of a subarray equivalent to the 'empty set', which is zero.

2.1   Cubic Brute Force Algorithm

```
6   int max_sum1(int* arr, int arrsize) {
7       int maxSum = 0;
8       for (int i = 0; i < arrsize; i++) {
9           for (int j = i; j< arrsize; j++) {
10              int thisSum = 0;
11              for (int k = i; k <= j; k++) thisSum += arr[k];
12              if (maxSum < thisSum) maxSum = thisSum;
13          }
14      } return maxSum;
15  }
```

This algorithm utilises three loops and a function of constant time in the innermost loop. Therefore, the time complexity analysis goes:

$$T(n) = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=i}^{j} 1 = O\left(n^3\right)$$

Obviously this method is quite wasteful in both memory and timekeeping. The following two algorithms are substantial progessions from this algorithm:

2.2   Quadratic Brute Force Algorithm

```
18   int max_sum2(int* arr, int arrsize) {
19       int maxSum = 0;
20       for (int i = 0; i < arrsize; i++) {
21           int iSum = 0;
22           for (int j = i; j < arrsize; j++) {
23               iSum += arr[j];
24               if (maxSum < iSum) maxSum = iSum;
25           }
26       } return maxSum;
27   }
```

This algorithm utilises two loops and a function of constant time in the innermost loop. Therefore, the time complexity analysis goes:

$$T(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} 1 = O\left(n^2\right)$$

2.3   Divide and Conquer

```
30   int max_sum3(int* arr, int left, int right) {
31       if (left >= right) return arr[left];
32       else { int hereMax = 0;
33           int leftSum = max_sum3(arr, left, ((left + right) / 2) - 1);
34           int rightSum = max_sum3(arr, ((left + right) / 2) + 1, right);
35           if (leftSum >= rightSum) hereMax = leftSum;
36           else hereMax = rightSum;
37           int leftMax = arr[(left + right) / 2], leftTemp = 0;
38           int rightMax = arr[(left + right) / 2], rightTemp = 0;
39           for (int i = (left + right) / 2; i >= left; i--) {
40               leftTemp += arr[i];
41               if (leftMax < leftTemp) leftMax = leftTemp;
42           } for (int i = (left + right) / 2; i <= right; i++) {
43               rightTemp += arr[i];
44               if (rightMax < rightTemp) rightMax = rightTemp;
45           } int midSum = leftMax + rightMax - arr[(left + right) / 2];
46           if (hereMax < midSum) hereMax = midSum;
47           return hereMax;
48       }
49   }
```

This algorithm utilises a divisive recursion and selection controls of constant time within each recursion. Therefore, the time complexity analysis goes:

$$T(n) = 2T\left(\frac{n}{2}\right) + n = 2^k T\left(\frac{n}{2^k}\right) + nk, \ \ \therefore T(n) = 2^{\log_2 n}T(1) + n\log_2 n = n\log_2 n + \varepsilon n = O(n\log n)$$

While these two algorithms are quite effective compared to Cubic Brute Force, this last algorithm manages to provide the fastest solution possible:

2.4   Kadane's Algorithm: A Linear, Incremental Solution

```
52   int max_sum4(int* arr, int arrsize) {
53       int maxSum = arr[0], thisSum = 0;
54       for (int i = 0; i < arrsize; i++) {
55           thisSum += arr[i];
56           if (thisSum > maxSum) maxSum = thisSum;
57           else if (thisSum < 0) thisSum = 0;
58       } return maxSum;
59   }
```

This algorithm utilises one loop and a function of constant time in the loop. Therefore, the time complexity analysis goes:

$$T(n) = \sum_{i=1}^{n} 1 = O(n)$$

Since, essentially, the challenge requires a system to read each data at least once, the $O(n)$ function above is obviously the best solution.

Chapter 3

Various Ways of Sorting

The objective of this callenge is sorting given algebraic (in this case, `double`) elements of an aray in incrasing order. Some elements may be of equal value.

3.1 Quick Sort

```
5       double pivot = arr[begin];
6       int i = begin, j = end;
7       while (i <= j) {
8           while (arr[i] < pivot) i++;
9           while (arr[j] > pivot) j--;
10          if (i <= j) {
11              swap(arr[i], arr[j]);
12              i++; j--;
13          }
14      } if (begin < j) quicksort(arr, begin, j);
15      if (end > i) quicksort(arr, i, end);
16  }
17

87  int main(int argc, char *argv[]) {
88      double *input_array = new double[2500];
89      for (int i = 0; i < 2500; i++) input_array[i] = double(rand() % 2500);

91      double *quick_arr = new double[2500];
92      for (int i = 0; i < 2500; i++) quick_arr[i] = input_array[i];
93      quicksort(quick_arr, 0, 2499);
94      if (check(quick_arr)) cout << "Quicksort Validated" << endl;

120     return 0;
121 }
```

Quick Sort is a recursive algorithm that does the following:

1. Choose a "pivot" element: `Line 6`

2. Swap elements that are larger than the pivot with elements that are smaller but on the righthand side of the chosen element: `Lines 8:14`

3. Continue until all elements are sorted, then recursively proceed with the left and right subarrays of the pivot: `Lines 15, 16`

Using two half-recursions results in the following time complexity analysis:

$$T(n) = 2T\left(\frac{n}{2}\right) + n = 2^k T\left(\frac{n}{2^k}\right) + kn = O(n \log n)$$

3.2 Merge Sort

```
19      if (begin < end) {
20          int centre = begin + (end - begin) / 2;
21          mergesort(arr, begin, centre);
22          mergesort(arr, centre + 1, end);
23          int n1 = centre - begin + 1;
24          int n2 = end - centre;
25          double* L = new double[n1 + 1];
26          double* R = new double[n2 + 1];
27          for (int i = 0; i <= n1 - 1; i++) L[i] = arr[begin + i];
28          for (int j = 0; j <= n2 - 1; j++) R[j] = arr[centre + j + 1];
29          L[n1] = (double)INT_MAX;
30          R[n2] = (double)INT_MAX;
31          int a = 0, b = 0;
32          for (int k = begin; k <= end; k++) {
33              if (L[a] <= R[b]) arr[k] = L[a++];
34              else arr[k] = R[b++];
35          }
36      }
37  }

87  int main(int argc, char *argv[]) {
88      double *input_array = new double[2500];
89      for (int i = 0; i < 2500; i++) input_array[i] = double(rand() % 2500);

96      double *merge_arr = new double[2500];
97      for (int i = 0; i < 2500; i++) merge_arr[i] = input_array[i];
98      mergesort(merge_arr, 0, 2499);
99      if (check(merge_arr)) cout << "Mergesort Validated" << endl;

120     return 0;
121 }
```

Merge Sort is also a recursive algorithm that depends on two half-recursions, so the time complexity analysis goes:

$$T(n) = 2T\left(\frac{n}{2}\right) + n = 2^k T\left(\frac{n}{2^k}\right) + kn = O(n \log n)$$

3.3   Insertion Sort

```
46   void insertionsort(double* arr, int size) {
47       for (int i = 1; i <= size - 1; i++) {
48           double key = arr[i];
49           int j = i - 1;
50           while (j >= 0 && arr[j] > key) {
51               arr[j + 1] = arr[j];
52               j--;
53           } arr[j + 1] = key;
54       }
55   }


87   int main(int argc, char *argv[]) {
88       double *input_array = new double[2500];
89       for (int i = 0; i < 2500; i++) input_array[i] = double(rand() % 2500);

101      double *insertion_arr = new double[2500];
102      for (int i = 0; i < 2500; i++) insertion_arr[i] = input_array[i];
103      insertionsort(insertion_arr, 2500);
104      if (check(insertion_arr)) cout << "Insertionsort Validated" << endl;

120      return 0;
121  }
```

Insertion Sort is a double-loop algorithm, therefore time complexity analysis is as follows:

$$T(n) = \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon = O(n^2)$$

3.4   Stooge Sort

```
57   void stoogesort(double* arr, int begin, int end) {
58       if (begin >= end) return;
59       else if (end - begin == 1) {
60           if (arr[begin] > arr[end]) swap(arr[begin], arr[end]);
61       } else {
62           int d = (end - begin + 1) / 3;
63           stoogesort(arr, begin, end - d);
64           stoogesort(arr, begin + d, end);
65           stoogesort(arr, begin, end - d);
66       }
67   }


87   int main(int argc, char *argv[]) {
88       double *input_array = new double[2500];
89       for (int i = 0; i < 2500; i++) input_array[i] = double(rand() % 2500);
```

```
106        double *stooge_arr = new double[2500];
107        for (int i = 0; i < 2500; i++) stooge_arr[i] = input_array[i];
108        stoogesort(stooge_arr, 0, 2499);
109        if (check(stooge_arr)) cout << "Stoogesort Validated" << endl;

120        return 0;
121    }
```

Stooge Sort is a recursive algorithm of three subarrays. The time complexity is:

$$T(n) = 3T\left(\frac{3}{2}n\right) + 1, \;\; \therefore T(n) \approx O\left(n^{2.7}\right)$$

3.5   Heap Sort

```
69    void heapsort(double *arr, int n, int i) {
70        int largest = i;
71        int l = 2 * i + 1;
72        int r = 2 * i + 2;
73        if (l < n && arr[l] > arr[largest]) largest = l;
74        if (r < n && arr[r] > arr[largest]) largest = r;
75        if (largest != i) {
76            swap(arr[i], arr[largest]);
77            heapsort(arr, n, largest);
78        }
79    }

87    int main(int argc, char *argv[]) {
88        double *input_array = new double[2500];
89        for (int i = 0; i < 2500; i++) input_array[i] = double(rand() % 2500);

111        double *heap_arr = new double[2500];
112        for (int i = 0; i < 2500; i++) heap_arr[i] = input_array[i];
113        for (int i = 1249; i >= 0; i--)
114            heapsort(heap_arr, 2500, i);
115        for (int i = 2499; i >= 0; i--) {
116            swap(heap_arr[0], heap_arr[i]);
117            heapsort(heap_arr, i, 0);
118        } if (check(heap_arr)) cout << "Heapsort Validated" << endl;

120        return 0;
121    }
```

Heap sort is a loop-and-recursive algorithm of time complexy analysis

$$T(n) = O(n \log n)$$

Part II

C++

Chapter 1

Variables and Class

1.1 The Basics

```cpp
1   #include <iostream>
2
3   int main(void) {
4       std::cout << "Hello World!";
5       return 0;
6   }
```

C++ is an extension of C.

1.1.1 Commenting

Single-line comments use two forward slashes: `// Comment 1` Multi-line comments use a forward slash and a star at each end to denote beginning and end: `/* Comment 2 */`

1.1.2 Types and Variables

1. Primitive, built-in types:

    1) `void` is used to determine functions and variables of no return value.

    2) `bool`, `char`, `int`, `float`, `double` and et cetra are used to return certain values.

    3) `unsigned` is used to prepend targets that are always of positive value.

2. Enumerations: `enum` is used to define groups of integer constants.

1.2 ¡++¿

Chapter 2

Functions

Chapter 3

`iostream` **Headers**

3.1 Keyboard Input and Screen Output

3.2 File In/Output

3.3 Header Files

1. Header File: A file that allows the reusage of certain portions of source code. Header files are included to a source code via *##include*, which inserts the header file code at that specific location.

2. Headers are used for declaring functions, classes, et cetra, and since a header file can be used multiple times by multiple source codes, defining variables and classes must not happen in a header file.

3. Include Guards: Since a single header file can be included multiple times throughout a compiling process and cause compilation errors, prevention methods are supported by the compilers. This "guard from inclusion"s are called **Include guard**s:

```cpp
#ifndef _IOSHEADER_H_  // Check if header is yet undefined
#define _IOSHEADER_H_  // If checked, define header

#include <iostream>

class classy {
public:
    void std::cout << "Hello World!" << std::endl;
};

#endif // End selection control
```

Chapter 4

Arrays and Pointers

4.1 Array

1. Sequential container of objects of a single data type with fixed size: `int array[3] = {5, 3, 2};`

4.2 Pointer

1. A pointer is a variable that holds the address of an object, enabling indirect access: `int *m = new int[4];`

2. Adding an integer n to a pointer variable returns the address of an element displaced by n from the original element.

4.3 ¡++¿

4.4 ¡++¿

Chapter 5

Object-Orientated Programming

5.1 ¡++¿

5.2 ¡++¿

5.3 ¡++¿

5.4 ¡++¿

Chapter 6

Defining Classes with OOP

Chapter 7

Member Functions

Chapter 8

Namespace and STL

Chapter 9

Constructors and Destructors

Chapter 10

Public or Private, Friend Declarations

Chapter 11

Copy Constructors

Chapter 12

Operator Overloading and the Rule of Three

Chapter 13

Protected and Private Derivations

Chapter 14

Virtual Functions

Chapter 15

Pure Virtual Functions

Chapter 16

Reusing Copy Control Members