

# Workshop proposal: Representing Streams II

## 1 Scientific case

### 1.1 Scientific background

Streams are infinite sequences of symbols that mathematicians and computer scientists study more or less independently from one another. Researchers from these two fields are motivated by different research questions, but they study the same object. Computer scientists speak about a stream while mathematicians call it a symbolic sequence. The latter expression is perhaps a bit more precise, but the former has more appeal.

Numbers can be represented by streams, be it by decimal expansion or continued fraction. Mathematicians suspect that certain properties of a number, such as algebraicity or Diophantine approximability, can be deduced from the complexity of its stream. This is certainly the case for rationals. They correspond to the streams of lowest complexity: eventually periodic streams. The problem gets much more difficult for non-rational numbers. To mention one long-standing open problem, mathematicians suspect that numbers with bounded continued fractions are either quadratic or transcendental. Another, more recent conjecture, says that a number that can be represented by an automaton is either quadratic or transcendental. Ideas from automata theory and streams are being used by Bugeaud and his co-workers to shed new light on the classical classification of numbers due to Koksma and Mahler [8]. Mathematicians also study streams for their own sake and over the years the study of combinatorics of words has accumulated a fair amount of theory; see [6]. Streams are the simplest data types that are infinite, and that is why computer scientists study streams. One major direction of research is to find finite representations of streams, for instance by means of automata or co-algebras, and to decide when two different representations describe the same stream. One can also perform symbolic operations on streams that are similar to multiplication or division. In this set up, the eventually periodic streams correspond to the units. It turns out that in analogy with numbers, it is difficult to compute the divisors of a stream. A recent conjecture [4] says that the 2-automatic Thue-Morse stream is prime. It is well known that automatic sequences are closed under transduction, and that  $k$ -automatic and  $m$ -automatic sequences are co-prime if  $k$  and  $m$  are multiplicatively independent, but the divisibility of automatic sequences has not yet been studied. There are many other unsolved problems that arise from the study of streams. The extension of lambda calculus and co-algebras to cover infinite data types is an ongoing research project in The Netherlands, the results of which are currently being collected in an extensive monograph [10].

The research in mathematics and computer science seems to run parallel but along disjoint tracks. We think that there is enough common ground to establish a connection, and that ideas of one field can be transferred successfully into the other.

## 1.2 A brief history of streams

Some of the pioneering work on streams, from a viewpoint of dynamical systems and ergodic theory, was carried out by Michael Keane [5] and Michel Dekking [2] in the seventies. Streams that arise from substitutions have the interesting property that they are recurrent but non-periodic. This property was later used by Penrose and others to construct recurrent but non-periodic tilings of the plane. Again, this research has a Dutch flavour with important contributions of Dick de Bruyn, who had already studied the combinatorial and algebraic properties of words much earlier [1]. Research on the combinatorics of streams has also been pursued by Rob Tijdeman and his students. Recently, Michel Dekking returned to his study of streams, and extended his earlier results on the fractal structure of paperfolding curves [3].

In the Dutch computer science community, the study of streams was taken up by researchers in term-rewriting and co-algebras [7, 10, 12] in Amsterdam, Eindhoven, Utrecht and Nijmegen, who have organized a bi-monthly seminar on the topic since 2010. Computer scientists study algebraic properties of streams under equivalence relations by means of general transducers, which is a relation that combines naturally with automatic numbers. Mathematicians usually study equivalence up to conjugation of dynamical systems, which in computer science terms, is an equivalence that is defined by means of Mealy machines, a very restrictive type of transducer. Therefore, existing mathematical tools have to be adjusted, if possible, to be of any use to computer scientists.

## 1.3 Follow-up workshop with tutorial week

In December 2012, the workshop “Representing Streams” took place at Snellius. Several topics were discussed, many open problems were presented from the two communities, and a few were even solved. We have several good reasons for organising a follow-up workshop.

Firstly, the pilot workshop was considered a success by all participants, and most of them have already expressed interest in a follow-up workshop. In particular, the remaining open problems are being collected, and will be sent around to the participants. We expect that our tentative workshop will lead to more of them being solved.

Secondly, although new contacts were established during the pilot workshop, a gap remains between researchers from different areas in terms of terminology, background knowledge, and approach. In our tentative follow-up workshop, in addition to a week of specialised research talks, we plan to have a week of introductory tutorials and survey talks in which non-specialists can become familiarised with some of the areas in which streams are studied in mathematics and computer science. This tutorial week will help bridge the gap by providing an understanding of basic definitions, results and techniques. While the tutorials are particularly suited for young researchers, we expect that they will be equally appealing to senior researchers who are interested in broadening their horizon, and making connections with other areas. We plan to have tutorials on several topics related to streams such as ergodic theory (M. Keane), automatic sequences (N. Rampersad), automata and combinatorics (J.-E. Pin), Krohn-Rhodes theory (C. Nehaniv), substitutions and tilings (A. Siegel), and coalgebra (L. Moss and J. Rutten). Most of the abovementioned lecturers have already confirmed their availability.

Thirdly, the one-week pilot workshop provided limited time for presentations, and priority was given to established researchers. In the two-week follow-up workshop, we wish to allow more time for young researchers to present their work.

Our workshop will be considered a success if it results in joint research between computer

scientists and mathematicians.

## References

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- [12] J.J.M.M. Rutten. Behavioural Differential Equations: a coinductive calculus of streams, automata and power series. Theoretical Computer Science 308 (2003), 1-53.

## 2 Organizers

Jörg Endrullis (VU Amsterdam), postdoc, computer science.  
Helle Hvid Hansen (RU Nijmegen & CWI), postdoc, computer science.  
Dimitri Hendriks (VU Amsterdam), postdoc, computer science.  
Charlene Kalle (U Leiden), postdoc, mathematics.  
Evgeny Verbitskiy (U Leiden), associate professor, mathematics.

### 3 Participants

At the time of writing this proposal, the planned workshop already has 39 confirmed participants with affiliations in the Netherlands, Belgium, France, United Kingdom, Austria, Luxembourg, Czech Republic, Canada, and Japan. The workshop will thus be a highly international event.

Number of confirmed participants at the moment of proposal writing:

Tutorial week (T): 26 pp. Workshop week (W): 39 pp. Total 42 pp.

Jean-Paul Allouche (Paris XI), prof, math, WT  
Valerie Berthé (Paris Diderot), prof, math, WT  
Marcello Bonsangue (UL/CWI), UHD, cs, WT  
Wieb Bosma (RU) UHD, math, WT  
Karma Dajani (UU), UHD, math, W  
Jörg Endrullis (VUA), postdoc, cs, WT  
Robbert Fokkink (TUD), UD, math, WT  
Herman Geuvers (RU), prof, cs, WT  
Helle Hvid Hansen (RU/CWI), postdoc, cs, WT  
Dimitri Hendriks (VUA), postdoc, cs, WT  
Michael Keane (Wesleyan), prof, math, WT  
Jan Willem Klop (VUA), prof, cs, WT  
Clemens Kupke (Strathclyde), assistant prof, cs, (W)T  
Jean-Eric Pin (Paris Diderot), prof, math/cs, WT  
Michel Rigo (Liege), prof, math, WT  
Jurriaan Rot (UL/CWI), PhD, cs, WT  
Jan Rutten (CWI/RU), prof, cs, WT  
Alexandra Silva (RU/CWI), UD, cs, WT  
Bram Westerbaan (RU), PhD, cs, WT  
Wolfgang Steiner (CNRS, Diderot), math, W  
Rob Tijdeman (UL), prof, math, WT  
Joost Winter (CWI), PhD, cs, WT  
Hans Zantema (TUE/RU), prof, cs, WT  
Charlene Kalle (UL), postdoc, math, WT  
Yann Bugeaud (Strasbourg), prof, math, W  
Sara Munday (Bristol), postdoc, math, W  
Tomáš Hejda (Paris Diderot), PhD, math, T  
Emilie Charlier (Liege), UD, math, WT  
Anne Siegel (Rennes), prof, math, T  
Tom Kempton (UU), postdoc, math, WT  
Evgeny Verbitskiy (UL), UHD, math, WT  
Kan Jiang (UU), PhD, math, WT  
Peter Grabner (Graz), prof, math, W  
Shigeki Akiyama (Tsukuba), prof, math, W  
Narad Rampersad (Winnipeg, US), ass.prof., WT  
Fabien Durand (Amiens), math, W  
Joerg Thuswaldner (Leoben), math, W  
Boris Adamczewski (Lyon), math, W  
Vilmos Komornik (Strasbourg), math, W

Benoit Loridant (Leoben/Vienna), math, W  
Edita Pelantová (Prague), math/cs, W  
Minervino Milton (Graz), math, PhD, WT

## 4 Scientific Council

This follow-up workshop is (intentionally) organised by young researchers. In order to also associate the workshop with established, senior researchers, we plan to create a *scientific council*. The members of this council will be asked to give advice on scientific matters, and give future perspectives for the workshop.

The candidates for this scientific council are:

- Robbert Fokkink (math), NL, (workshop founder),
- J.P. Allouche (math), FR
- Michel Rigo (math), BE
- Valerie Berthé (math), FR
- Jean Eric Pin (cs), FR
- Jan Rutten (cs), NL
- Jan Willem Klop (cs), NL

## 5 Preliminary program

### Week 1: Tutorials

The tutorial week will consist of nine 1.5-hour timeslots divided over five to six tutorial topics, as mentioned above. Each day has scheduled Work sessions which can be used to solve exercises given by tutorial lecturers or collaboration.

In order to involve the PhD students we also plan to have a *PhD student session* where PhD students are invited to give a brief presentation (ca. 15 mins) about their work, and if possible, state open problems they have encountered in their research.

Below is a preliminary sketch of how the tutorial week will look. The exact availability of the lecturers will determine the final program. We already have confirmations from Rampersad, Keane, Pin, Siegel and Rutten. The tutorial lecturers will be asked to be present throughout the week such that participants can interact with them.

Each tutorial lecture is followed immediately by an exercise session where participants can test their understanding of the material, either individually or in collaboration with others.

Time	Monday	Tue	Wed	Thu	Fri
9 <sup>00</sup> – 9 <sup>30</sup> :	Arrival at Lorentz Center				
9 <sup>30</sup> – 10 <sup>00</sup> :	Welcome	Automatic seq. (Rampersad)	Krohn-Rhodes (Nehaniv)	Coalgebra (Moss)	Coalgebra (Rutten)
10 <sup>00</sup> – 10 <sup>30</sup> :	Automatic seq. (Rampersad)	(incl. 30 min coffee break)	(incl. 30 min coffee break)	(incl. 30 min coffee break)	(incl. 30 min coffee break)
10 <sup>30</sup> – 11 <sup>30</sup> :					
11 <sup>30</sup> – 12 <sup>15</sup> :	Exercises	Exercises	Exercises	Exercises	Exercises
12 <sup>15</sup> – 14 <sup>00</sup> :	Lunch	Lunch	Lunch	Lunch	Lunch
14 <sup>00</sup> – 14 <sup>30</sup> :	PhD session	PhD session	PhD session	PhD session	PhD session
14 <sup>30</sup> – 14 <sup>45</sup> :	Short break	Short break		Short break	Short break
14 <sup>45</sup> – 16 <sup>45</sup> :	Ergodic theory (Keane)	Ergodic theory (Keane)	(free time?)	Combinatorics (Pin)	Tilings (Siegel)
(incl. 30min coffee break)	(incl. 30 min coffee break)	(incl. 30 min coffee break)	(start time?)	(incl. 30 min coffee break)	(incl. 30 min coffee break)
16 <sup>45</sup> – 17 <sup>30</sup> :	Exercises	Exercises	Boat trip and School week dinner	Exercises	Exercises
	Wine and cheese party and Poster session				

Overview S1:

scheduled time for	per day	week total
lectures:	3.0 hrs	13 hrs
exercises:	1.5 hrs	6.75 hrs
student presentations:	.5 hrs	3.0 hrs
lunch/breaks/free time:	3.0 hrs	19 hrs

## Week 2: Workshop

The workshop week will be opened with a brief welcome by the organisers, followed by an update by a scientific council member who will summarise what has happened since the pilot workshop (Representing Streams I).

Each day will end with a Problem Session. In the first two of these sessions, the participants will present open problems, the next two essentially consist of time scheduled for working on open problems. The last problem session on the Friday will be used to report on the progress of solving them. These problem sessions are placed at the end of the day when listening to longer research talks can be a challenge.

On the Wednesday we plan a Panel Session in which scientific council members will discuss future perspectives for the workshop and research collaborations.

The research talks will be grouped into themed sessions where possible, but with mathematics and computer science sessions interleaved in order stimulate cross-fertilisation and discourage participants from going only to talks from their own area. A preliminary suggestion of the session themes is shown in the table below. In general, the aim is to have both mathematics talks and computer science talks each day.

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9 <sup>30</sup> – 10 <sup>00</sup>	Welcome				
10 <sup>00</sup> – 11 <sup>00</sup>	Keynote talk (Rigo)	Keynote talk (Endrullis)	Keynote talk (Berthé)	Keynote talk (Geuvers)	Keynote talk (Akiyama)
11 <sup>00</sup> – 11 <sup>30</sup>	break	break	break	break	break
11 <sup>30</sup> – 12 <sup>30</sup>	2 × 30min (Coalgebra)	2 × 30min (Symb.dynamics)	2 × 30min (Coalgebra)	2 × 30min (Combinatorics)	2 × 30min (Combinatorics)
12 <sup>30</sup> – 14 <sup>00</sup>	lunch	lunch	lunch	lunch	lunch
14 <sup>00</sup> – 15 <sup>45</sup> (15min break)	3 × 30min lectures (Autom. seq's)	3 × 30min lectures (Substitutions)	Problem session	3 × 30min lectures (Rewriting,logic)	2 × 30min lectures (Numbers)
15 <sup>45</sup> – 16 <sup>00</sup>	break	break		break	break
16 <sup>00</sup> – 17 <sup>00</sup>	Problem session	Problem session		Problem session	Problem session (report progress)
	(17 <sup>00</sup> – 18 <sup>30</sup> ) Wine and cheese party		(18 <sup>00</sup> ) Workshop dinner		(16 <sup>30</sup> – 17 <sup>00</sup> ) Closing

*Overview Workshop week program:*

scheduled time for	per day	week total
talks:	3.5 hrs	15.5 hrs
problem sessions:	1.0 hrs	7.0 hrs
lunch/breaks:	2.25 hrs	11.0 hrs

## 6 Factsheet

Title:	Representing Streams II
Dates 1st Preference:	tutorial week: 20-24 Jan, 2014 workshop week: 27-31 Jan, 2014
Dates 2nd Preference:	tutorial week: 13-17 Jan, 2014 workshop week: 20-24 Jan, 2014 2014
Dates 3rd Preference:	To be determined.
Finances:	funds from Lorentz center, and others
Participants:	5 key note speakers 40 participants for the tutorial week 55 participants for the workshop week
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