# Workshop proposal: Applied Category Theory

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## 1 Organizers

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## 2 Scientific case

### 2.1 Scientific background

Category theory was developed in the 1940s to translate ideas from one field of mathematics, e.g. topology, to another field of mathematics, e.g. algebra. More recently, category theory has become an unexpectedly useful and economical tool for modeling a range of different disciplines, including programming language theory [10], quantum mechanics [2], systems biology [12], complex networks [5], database theory [7], and dynamical systems [14].

A category consists of a collection of objects together with a collection of maps between those objects, satisfying certain rules. Topologists and geometers use category theory to describe the passage from one mathematical structure to another, while category theorists are also interested in categories for their own sake. In computer science and physics, many types of categories (e.g. topoi or monoidal categories) are used to give a formal semantics of domain-specific phenomena (e.g. automata [3], or regular languages [11], or quantum protocols [2]).[1] In the applied category theory community, a long-articulated vision understands categories as mathematical workspaces for the experimental sciences, similar to how they are used in topology and geometry [13]. This has proved

[1] The categorical semantics is often preferable to set- or type-theoretic semantics in some way: for example, compact closed categories have an elegant graphical language in terms of string diagrams.

true in certain fields, including computer science and mathematical physics, and we believe that these results can be extended in an exciting direction: we believe that category theory has the potential to bridge specific different fields, and moreover that developments in such fields (e.g. automata) can be transferred successfully into other fields (e.g. systems biology) *through* category theory. Already, for example, the categorical modeling of quantum processes has helped solve an important open problem in natural language processing [9].

In this workshop, we want to instigate a multi-disciplinary research program in which concepts, structures, and methods from one discipline can be reused in another. Tangibly and in the short-term, we will bring together people from different disciplines in order to write an expository survey paper that grounds the varied research in applied category theory and lays out the parameters of the research program.

In formulating this research program, we are motivated by recent successes where category theory was used to model a wide range of phenomena across many disciplines, e.g. open dynamical systems (including open Markov processes and open chemical reaction networks), entropy and relative entropy [6], and descriptions of computer hardware [8]. Several talks will address some of these new developments. But we are also motivated by an open problem in applied category theory, one which was observed at the most recent workshop in applied category theory (Dagstuhl, Germany, in 2015): "a weakness of semantics/CT is that the definitions play a key role. Having the right definitions makes the theorems trivial, which is the opposite of hard subjects where they have combinatorial proofs of theorems (and simple definitions). [...] In general, the audience agrees that people see category theorists only as reconstructing the things they knew already, and that is a disadvantage, because we do not give them a good reason to care enough" [1, pg. 61].

In this workshop, we wish to articulate a natural response to the above: instead of treating the reconstruction as a weakness, we should treat the use of categorical concepts as a natural part of transferring and integrating knowledge across disciplines. The restructuring employed in applied category theory cuts through jargon, helping to elucidate common themes across disciplines. Indeed, the drive for a common language and comparison of similar structures in algebra and topology is what led to the development category theory in the first place, and recent hints show that this approach is not only useful between mathematical disciplines, but between scientific ones as well. For example, the 'Rosetta Stone' of Baez and Stay demonstrates how symmetric monoidal closed categories capture the common structure between logic, computation, and physics [4].

## 2.2 Specific challenges and outcomes

This workshop will bring together both theorists and practitioners from a wide variety of disciplines to work on new applications of category theory in (1) dynamical systems and networks, (2) systems biology, and (3) cognition and AI, with a special focus on developing a community of early-stage researchers in applied category theory, and on fostering focused dialogue between researchers working on different applications. It will consist of a 5-day workshop week at Oort, a 4-day tutorial week at Snellius, and a 16-week online seminar for PhD students called the "Kan Extension Lab".

Some of the specific challenges and outcomes we wish to address include:

1. Tool support: while category theory provides a firm foundation for reasoning as it occurs across many disciplines, to be applied rather than merely applicable requires tools that permit applied practitioners to take advantage of this structure. One avenue is accessible software packages that implement category theoretic reasoning. Vicary et al's popular online proof assistant Globular, based on higher category theory, demonstrates the demand for and utility of such software packages; it is crucial to the outreach of applied category theory that work continues in this vein.

2. Communication: applied category theory depends on finding open problems in other fields where CT can make a contribution, but how should category theorists communicate with practitioners in these other fields? Moreover, how can the research community develop deeper partnerships with industrial partners in order to develop industrial applications, e.g. product lifecycle management tools or models of interoperability with aerospace manufacturers such as Airbus and Dassault.

3. Pedagogy: one of the open problems discussed at Dagstuhl was the perceived and actual difficulty of category theory. Despite the flexibility and expressiveness of categorical tools in mathematics and computer science, the perceived difficulty of category theory has hindered wider acceptance of the formalism in other areas of interest, e.g. to students in areas outside of math and CS. Different approaches were suggested, including focusing on automated theorem proving. We plan on addressing this problem over the tutorial weekend, and through the organization of the "Kan Extension Lab".

Our workshop will be a success if it (1) results in joint research between researchers specializing in different applications (e.g. physics and biology, or economics and AI) or in research that carries over techniques from one application domain of category theory to another, and (2) introduces new researchers into the field.

We will produce two documents to further these aims. First, we will write a technical report summarizing the presentations and discussion sessions of the conference providing a list of promising avenues for collaboration and a roadmap for work in applied category theory over the next five years.

Second, through the tutorial week and its preceding online seminar, we will have young researchers in applied category theory write a series of eight to ten articles introducing landmark papers applying category theory to dynamical systems and networks, systems biology, and computation, cognition, and AI. These summary articles will be published online and—in addition to providing the seminar participants with an entry point into the field—will form a permanent, available, and accessible introduction to applied category theory. As part of the effort to build a research community, we will also raise issues in discussion related to (1) plans for follow-up conferences, (2) plans for volumes of commissioned chapters to bring young people in to the area, and (3) a network scheme to help young people travel between active nodes in applied category theory.

### 2.3 Connection to the Dutch research community

In the Dutch applied category theory community, a number of people use category theory in the context of coalgebras, especially for software verification. Bart Jacobs has worked on this, along with Helle Hvid Hansen (Delft), Juriaan Rot (Radboud), and Jan Rutten (Amsterdam). Ralf Hinze (Nijmegen) applies category-theoretic methods to functional programming. Michael Moortgat (Utrecht) and Martha Lewis (Amsterdam) work on linguistic applications. Aleks Kissinger (Radboud) works on quantum algorithms and graph rewriting systems using category theory. In pure category theory and topos theory, Ieke Moerdijk (Utrecht) developed many of the foundations. Klaas Landsmen (Radboud) works on topos theory, operator algebras, and quantum theory. Dutch researchers in categorical logic include Sonja Smets and Alexandru Baltag (Amsterdam); both are friendly to applications of category theory, e.g. categorical quantum mechanics.

## 3 Program

### 3.1 Workshop week

The workshop highlights three particular applications of category theory: (1) to dynamical systems and networks, (2) to systems biology, and (3) to cognition and AI. While there will be a short introductory lecture for each application domain, the afternoons will intermix all three applications by focusing on common techniques (Monday and Tuesday), computational tools (Wednesday), and common problems and goals (Thursday and Friday afternoon) across all three. On Tuesday morning, there will be a half-day Highlights forum of 8-minute talks to help participants get introduced to each other's current work. By holding this forum early in the week, we hope to spark conversations that will continue to be explored both formal and informal discussion sessions throughout the workshop.

Each working day will include one keynote lecture during the morning that sets the stage for the day, followed by two to three 15-minute talks that delve into specific aspects of content in the keynote lecture. For example, on Thursday, there will be a survey lecture by Bart Jacobs on category theory for AI, followed by two 15-minute talks delving into Bayesian probability and linguistics. Each morning and afternoon will be closed by a discussion session or an extended coffee break. During each discussion session, the participants will be encouraged to break into smaller groups, each led by one of the presenters or a senior researcher, that will focus on developing some of the problems and issues raised by the presenters in their lectures.

| **Workshop Week** | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 9:00 - 9:30 | Arrival | Highlights forum | | | |
| 9:30 - 10:00 | Welcome | | Sys. Bio. (Krivine) | Cogn. + AI (Jacobs) | Industrial (Johnson) |
| 10:00 - 10:30 | Dyn. Sys. (Baez) | | | | |
| 10:30 - 11:00 | | | *break* | *break* | *break* |
| 11:00 - 11:30 | 2 talks | | 2 talks | 2 talks | 2 talks |
| 11:30 - 12:30 | Discussion | | Discussion | Discussion | Discussion |
| 12:30 - 2:00 | Lunch | Lunch | Lunch | Lunch | Lunch |
| 2:00 - 3:00 | Geometry (TBC) | KR (Abramsky) | Computing (Vicary) | Problem Session | Entropy (Leinster) |
| 3:00 - 3:45 | *break* | *break* | Discussion | | Discussion |
| 3:45 - 4:45 | Discussion | Discussion | Boat trip and dinner | | Closing |
| 4:45 - 5:30 | | | | | |
| | Wine and cheese | | | | |

| scheduled time for | per day | week total |
|---|---|---|
| lectures | 2.5 hrs | 12.5 hrs |
| discussion | 2.3 hrs | 11.5 hrs |
| lunch/break | 2.1 hrs | 10.5 hrs |

## 3.2 Tutorial week

Prior to the workshop, we will organize a 4-day week of tutorials targeted at graduate students and postdocs, though we envision that researchers in other fields who wish to learn more about applied category theory will also be interested in attending.

While the precise schedule for the program is still fluid (indeed, pending availability of space, we are happy to organize this as a 3-day weekend event immediately prior to the workshop), we have confirmed participation for key talks during the week by John Baez, Jamie Vicary, Pawel Sobocinski, and Jurgen Jost. PhD sessions will allow students to present, learn about, and discuss each other's research. Subject matter will include introductory tutorials on diagrammatic reasoning, an introduction to Globular, a "user's guide to operads and PROPs", case studies of how to use these tools in applications, a discussion of how to find new applications, and a working session dedicated to a constructing a survey of applied category theory (see below).

<table>
<tr><th>Tutorial Week</th><th>Monday</th><th>Tuesday</th><th>Wednesday</th><th>Thursday</th></tr>
<tr><td>9:00 - 10:00</td><td>Arrival</td><td rowspan="2">Data Analysis (Jost)</td><td rowspan="2">Lin. Alg. (Sobocinski)</td><td rowspan="2">Dynamical Systems (Baez)</td></tr>
<tr><td>10:00 - 12:00</td><td>Diagrams (Vicary)</td></tr>
<tr><td>12:00 - 12:30</td><td>Case Study: PROPs</td><td>Exercises</td><td>PhD Talk</td><td>Exercises</td></tr>
<tr><td>12:30 - 2:00</td><td>Lunch</td><td>Lunch</td><td>Lunch</td><td>Lunch</td></tr>
<tr><td>2:00 - 2:30</td><td>PhD Talk</td><td>PhD Talk</td><td>PhD Talk</td><td>PhD Talk</td></tr>
<tr><td>2:30 - 5:00 (incl. break)</td><td>Tutorial: Globular (Vicary)</td><td rowspan="2">Boat trip and school week dinner</td><td>Paper work session</td><td>Discussion + Problem Talks</td></tr>
<tr><td>5:00 - evening</td><td>Wine & Cheese</td><td></td><td></td></tr>
</table>

| scheduled time for | per day | 4 day total |
|---|---|---|
| lectures | 3.2 hrs | 13 hrs |
| discussion | 1.5 hrs | 6 hrs |
| lunch/break | 1.9 hrs | 7.5 hrs |

## 3.3 Online seminar

To supplement the tutorial weekend, we will host an online seminar called the Kan Extension Lab targeted at 10-12 graduate students with existing knowledge either in category theory or in one of the topic areas listed in the rough schedule below.

1. 1st week (Week of December 17th): introductory meeting
2. 2nd week: programming languages (Scott, Lambek, Goguen)
3. 3rd week: quantum information (Abramsky and Brandenburger)
4. 4th week: data analysis (McCullagh, Bubenik, Jost)
5. 5th week: dynamical systems (Behrisch et al., Spivak et al.)
6. 6th week: biology (Rashevsky, Rosen)

7. 7th week: AI and cognitive science (Ehresmann et al., Gomez-Ramirez)

8. 8th week (Week of March 25th): "problem talk"

The seminar will be run over the 16 weeks immediately prior to the workshop. Biweekly, a team of participants will present on and develop an extant application of category theory, usually based on a published paper. After each lecture and a round of discussion, the presenters will write a blog post summary, to be posted on a publicly-accessible forum (such as the nCafe). The blog posts will be then aggregated into a survey of applied category theory and reformatted as a significant component of the expository/survey publication first discussed in Section 2.1, on "a multi-disciplinary research program in which concepts, structures, and methods from one discipline can be reused in another". In the last week of the seminar, the organizers will give a "problem talk" that draws conclusions from the seminar and poses a set of open problems. There will be a special session of the tutorial week to celebrate the end of the seminar, to discuss the proposed publication, and to invite comments on the "problem talk" mentioned above. There will also be an opportunity to present work developed during the seminar at the workshop week.

This seminar is based on a series of online seminars on pure category theory called the Kan Extension Seminar (itself based on the original Kan Seminars at MIT), organized by Emily Riehl, Alexander Campbell, and Brendan Fong (one of the organizers of this workshop).

# 4 Participants

The estimated number of participants is 55. At the time of writing of this proposal, the planned workshop already has 42 confirmed participants with affiliations in the Netherlands, the UK, the US, France, Germany, Italy, Australia, and Canada. In the calculations below, 6 additional spots will be reserved for graduate students.

Confirmed participants: 46.
Female/male ratio: 8 female to 38 male.
Junior researchers: currently 17 junior to 34 senior researchers.
Industry participants: 3

1. Helle Hvid Hansen, Professor Delft

2. Ieke Moerdijk, Professor Utrecht

3. Sonja Smets, Professor Amsterdam

4. Alexandru Baltag, Professor Amsterdam

5. Samson Abramsky, Strachey Professor of Computer Science Oxford

6. John Baez , Professor University of California, Riverside

7. Spencer Breiner, Junior Researcher NIST
8. Robin Cockett, Professor Calgary
9. Bob Coecke, Professor Oxford
10. Vincent Danos, Chair of Computational Systems Biology Edinburgh
11. Ross Duncan, Lecturer in Computer Science Strathclyde
12. Marcelo Fiore, Professor in Mathematical Foundations of Computer Science University of Cambridge
13. Brendan Fong, Postdoc MIT
14. Tobias Fritz, Postdoc Max Planck Institute for Mathematics in the Sciences
15. Fabio Gadducci, Professor Universit di Pisa
16. Neil Ghani, Professor University of Strathclyde
17. Dan Ghica, Reader in Semantics of Programming Languages University of Birmingham, School of Computer Science
18. Gillian Grindstaff, PhD student UT Austin
19. Misha Gromov (TBC), Professor IHES
20. Russ Harmer, Professor CNRS and cole Normale Suprieure
21. Jules Hedges, Postdoc University of Oxford
22. Kathryn Hess, Professor EPFL
23. Steve Huntsman , Industry BAE Systems
24. Bart Jacobs, Professor Radboud
25. Mike Johnson, Professor of Mathematics and Computer Science Macquarie University
26. Patrick Johnson (TBC), Vice-President for Research 3DS
27. Aleks Kissinger, Assistant Professor Radboud
28. Jean Krivine, Professor Universit Paris Diderot
29. Tom Leinster, Reader and Chancellor's Fellow Edinburgh
30. Eugene Lerman, Professor University of Illinois, Urbana-Champaign
31. Martha Lewis, Research Assistant Amsterdam
32. Dan Marsden, Research Assistant Oxford

33. Paul-Andre Mellies, CNRS Researcher Universit Paris Diderot
34. Konstantin Mischaikow, Professor Rutgers University
35. Michael Moortgat, Professor Utrecht University
36. Jason Morton, Associate Professor in Mathematics and Pennsylvania State University
37. Nina Otter, PhD student Oxford, Alan Turing Institute
38. Prakash Panangaden, Professor McGill University
39. Dusko Pavlovic, Professor of Computer Science University of Hawaii
40. Gordon Plotkin, Professor University of Edinburgh
41. Sophie Raynor, PhD student University of Aberdeen
42. Peter Selinger, Professor Dalhousie University
43. Pawel Sobocinski, Lecturer of Computer Science University of Southampton
44. David Spivak, Research Scientist MIT
45. Joshua Tan, PhD student Oxford
46. Ulrike Tillmann, Professor University of Oxford, Alan Turing Institute
47. Jamie Vicary, Senior Research Fellow Oxford
48. Fabio Zanasi, Lecturer University College London

# 5 Factsheet

A separate factsheet has been attached to this application.

# 6 Budget

A separate budget has been attached to this application.

# References

[1] Samson Abramsky, John C. Baez, Fabio Gadducci, and Viktor Winschel. Categorical methods at the crossroads. Report from Dagstuhl Perspectives Workshop 14182, 2014.

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[5] John C. Baez and Brendan Fong. A compositional framework for passive linear networks. *ArXiv e-prints*, 2015.

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[7] Michael Fleming, Ryan Gunther, and Robert Rosebrugh. A database of categories. *Journal of Symbolic Computing*, 35(2):127–135, February 2003.

[8] Dan R. Ghica and Achim Jung. Categorical semantics of digital circuits. In Ruzica Piskac and Muralidhar Talupur, editors, *Proceedings of the 16th Conference on Formal Methods in Computer-Aided Design*, 2016.

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