Operational indicator frameworks

An <u>operational indicator framework</u> is just a <u>list</u>. % Can we 'enrich' these lists?

- net income
- sales
- hack attempts

- approval rating
- taxes avoided
- retweets

- reliability
- trustworthiness
- system downtime

% often: time-series,
regularly updated / added-to,
something we do low-level statistical
analysis to, e.g. correlation

They are used for:

1. optimization of processes
2. strategic decision-making
3. public (or internal) communication
4. ontology / simplification

<u>Problem:</u> ~~the~~ how do we take data from one indicator framework and relate it to data from another framework?

~~i.e. data integration~~:

<u>Eg.</u> how do I illustrate the benefits / extended impact of my project to another aspect of my organization / domain?

Abstract indicator frameworks

Defn. a $\boxed{\text{"formula"}}$ from a $\boxed{\text{causal model}}$ to a $\boxed{\text{data model}}$ (measurement/observation).

The philosophy: toward a type system for measuring and modeling complex systems

math artifact, e.g.	math artifact, e.g.
- (relational) table of data	☆ - graphs / DAGs
- List of random variables	- Bayesian networks
☆ - Hilbert space of random var.	- dynamical systems
- observation tables of a regular language	- finite automata

To start: fix one of each!

Ex. bus data from Nashville, TN

	actual travel time AT	scheduled travel time ST	jam factor JF	traffic speed TS
AT	1	.81	.15 ?	~~-.04~~
ST		1	~~.05~~	~~-.02~~
JF			1	-.48
TS				1

Correlation

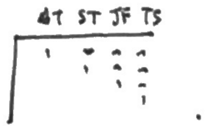
Why .15? ~~Both~~ First step toward analysis: build a (basic, maybe false) model!

$$\begin{tikzcd} & & AT & \\ ST & TS \arrow[ur] & & JF \arrow[ul] \arrow[ll] \end{tikzcd}$$

Causation

Dumb question: how do you "wrap" the two pieces ~~of~~ / perspectives on the system {AT, ST, JF, TS} into one, consistent data structure?

Bus example, continued

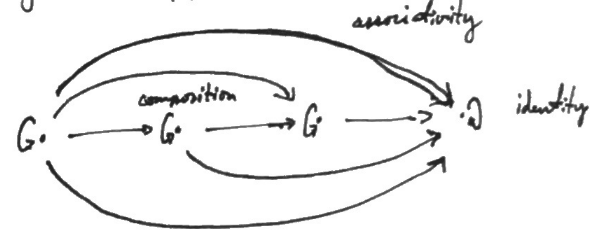
Turning
$$\begin{tikzcd} & & AT \\ ST & TS \arrow[ur] & JF \arrow[l] \arrow[u] \end{tikzcd}$$
into 

.

Step 1. Use
$$\begin{tikzcd} & & AT \\ ST & TS \arrow[ur] & JF \arrow[l] \arrow[u] \end{tikzcd}$$
to construct a category.

% what is a category? The nat'l language of systems is data.
The nat'l languages of people are models (which are math. struc.)
★ The nat'l language of math. structures is <u>category theory</u>.

Has objects and morphisms (arrows) that characterize the (mathematical) structure of those objects, satisfying



★ warning: need to justify this statement! You can represent all the usual models in <u>set theory</u> too!

~~The~~ Each causal variable ~~[illegible]~~ becomes an <u>object</u> of the category, C, so

$\text{Ob } C = \{ AT, ST, JF, TS$

add all the tensor products

$AT \otimes ST,\ AT \otimes JF,\ AT \otimes AT,\ \ldots$

and a unit object 1 s.t. $\cancel{AT \otimes 1}$ $X \otimes 1 = X$

$1 \}$

$\text{Mor } C = \{ TS \to AT,\ TS \leftarrow JF,\ AT \leftarrow JF,$

$$\begin{tikzcd} & \text{"} AT \text{"} \\ TS \arrow[ur] & JF \arrow[l] \end{tikzcd}, \quad \begin{tikzcd} & \text{"} AT \text{"} \\ TS & JF \arrow[l] \arrow[u] \end{tikzcd}, \quad \begin{tikzcd} & \text{"} AT \text{"} \\ TS \arrow[ur] & JF \arrow[u] \end{tikzcd},$$

$$\begin{tikzcd} & \text{"} AT \text{"} \\ TS \arrow[ur] & JF \arrow[l] \arrow[u] \end{tikzcd}, \quad \emptyset \to TS,\ \emptyset \to JF,\ \emptyset \to AT,$$

$AT \to 1,\ JF \to 1,\ TS \to 1,$ and all the identity morphisms ξ, and the co-multiplication (i.e. duplication) morphisms. $\}$

where
$$\begin{tikzcd} & \text{"} AT \text{"} \\ TS \arrow[ur] & JF \arrow[u] \end{tikzcd}$$
is really a map

$TS \otimes JF \to AT$

Bus example, part 3

Step 2. Define the category Rand of random variables. % There are other choices for a "measurement model"! And maybe Rand isn't the best choice...

Defn. The category Rand is given by:

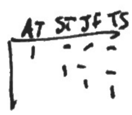
Ob Rand = { finite-dim Hilbert spaces $\mathcal{X} = L^2(\Omega_\mathcal{X}, \Sigma_\mathcal{X}, \mathbb{P}_\mathcal{X})$ endowed with basis

Mor Rand = { bounded linear operators $\mathcal{X} \to \mathcal{Y}$ }

Id. $1 : \mathcal{X} \to \mathcal{X}$.

Composition is just the usual composition of bounded linear operators

Tensor $\mathcal{X} \otimes \mathcal{Y}$ is the pushout of $\mathcal{X}, \mathcal{Y}$ over their joint support in $\Omega_X \times \Omega_Y$

Step 3. Use 

to specify a functor from C to Rand

Defn. An abstract indicator framework is... a $\boxed{\text{functor}}$ from a $\boxed{\text{causal theory C}}$ to $\boxed{\text{Rand}}$.