CGS616 Assignment 2: Dynamic Computational Model of Latent Cognitive States

Markov Chain / HMM-Based Behavioral Modeling with Reinforcement Learning

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1. Introduction

This assignment presents a dynamic computational model explaining how **latent cognitive states** evolve over time and generate **observable behaviors** during crisis events. Building on Assignment 1's ABC framework applied to crisis-related social media data, we construct a probabilistic state transition model using Hidden Markov Models (HMM) with sigmoid-based transitions and a Q-Learning reinforcement learning extension.

Dataset: CrisisNLP Combined Dataset (41,149 tweets; 11 disasters; 10 countries). Tweets carry timestamps enabling temporal analysis.

Model Variables: S_t = latent cognitive state (0 = passive/impact, 1 = active/engaged); X_t = stimulus antecedent (Global North = 1, South = 0); B_t = observable behavior (Action, Impact, Info); $\epsilon_t \sim \mathcal{N}(0, \tau^2)$.

2. Latent State Model

We model two conditional distributions:

$$P(S_t \mid S_{t-1}, X_t) \quad \text{(State Transition)} \tag{1}$$

$$P(B_t \mid S_t) \quad \text{(Emission)} \tag{2}$$

State Mapping: Action (B_t=0) and Info (B_t=2) map to Active state (S_t=1); Impact (B_t=1) maps to Passive (S_t=0). Tweets are sorted by `tweet_time_utc` within each disaster to preserve the Markov property.

2.1 State Transition & Emission

$$P(S_{t+1}{=}1 \mid S_t, X_t) = \sigma(\beta X_t - \delta S_t + \epsilon_t), \quad \sigma(z) = \frac{1}{1+e^{-}} \tag{3}$$

$$P(B_t{=}k \mid S_t{=}s) = \frac{e^{\phi_{s,k}}}{\sum_{j=0}^{2} e^{\phi_{s,j}}} \tag{4}$$

Parameters: β = stimulus sensitivity; δ = decay/recovery rate; τ = noise s.d. Long-run equilibrium: $S^* = \frac{\beta}{\delta} \cdot X$.

3. Empirical Transition Analysis

Table 1: Empirical transition matrices $P(S_{t+1} \mid S_t)$

	North (n=24,147)		**South** (n=17,002)	
	→Pass	→Act	→Pass	→Act
Passive	0.4294	0.5706	0.3370	0.6630
Active	0.3286	0.6714	0.2592	0.7408

Both regions tend to remain in or transition to the active state; Global South shows slightly higher active-state persistence.

4. Parameter Estimation (MLE)

Objective: Minimize negative log-likelihood:

$$-\log \mathcal{L}(\theta) = -\sum_t \log P(B_t|S_t) - \sum_{t>0} \log P(S_t|S_{t-1}, X_t) \tag{5}$$

Optimized via L-BFGS-B (`scipy.optimize.minimize`), with $\beta \in [-5, 5]$, $\delta \in [0.01, 5]$, $\tau \in [0.01, 5]$, emission logits $\in [-5, 5]$.

Table 2: MLE-estimated parameters

Parameter	**Global North**	**Global South**
β (Sensitivity)	0.5583	0.5000
δ (Decay Rate)	0.0100	0.0100
τ (Noise)	1.0152	1.0081
Log-Likelihood	−25,854.02	−19,315.08
AIC	51,722.04	38,644.15
BIC	51,778.69	38,698.34

Table 3: Fitted emission probabilities $P(B_t \mid S_t)$

	Global North			**Global South**		
	Act	Imp	Info	Act	Imp	Info
Passive (S=0)	0.0000	0.9933	0.0067	0.0000	0.9933	0.0067
Active (S=1)	0.6523	0.0023	0.3454	0.7021	0.0020	0.2959

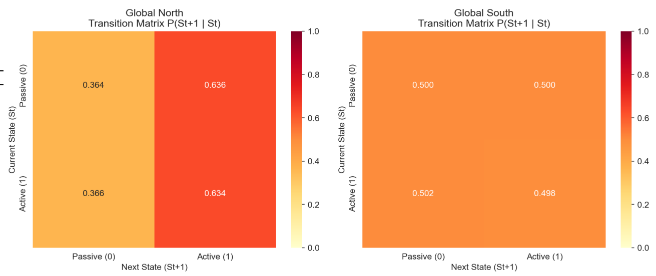

Figure 1: Fitted transition matrix heatmaps

5. Q-Learning RL Extension

Update rule: $Q_{t+1} = Q_t + \alpha(r_t - Q_t)$, where rewards are: Action → +1, Impact → −1, Info → 0.

Softmax behavior: $P(B_t{=}k) = e^{Q_{t,k}/\tau} / \sum_j e^{Q_{t,j}/\tau}$.

Grid search over $\alpha \in \{0.01, 0.05, 0.1, 0.2, 0.3, 0.5\}$ and $\tau \in \{0.1, 0.5, 1.0, 2.0, 5.0\}$.

Table 4: Q-Learning results

Parameter	Global North	Global South
α (Learning Rate)	0.5000	0.0100
τ (Temperature)	5.0000	2.0000
Log-Likelihood	−26,616.67	−18,109.33
AIC	53,237.34	36,222.67
BIC	53,253.52	36,238.15

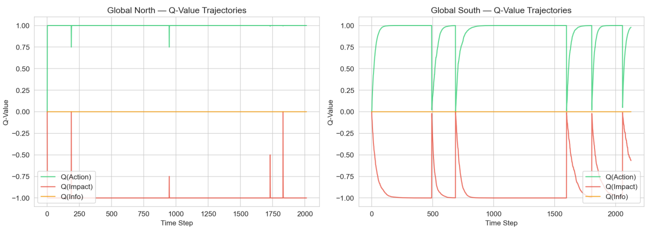

Figure 2: Q-value trajectories over time

6. Model Evaluation

Table 5: AIC/BIC comparison (lower is better)

	AIC		BIC	
Model	North	South	North	South
HMM	51,722	38,644	51,779	38,698
Q-Learning	53,237	36,223	53,254	36,238

HMM achieves better fit for Global North; Q-Learning performs marginally better for Global South.

5-Fold Cross-Validation:

Table 6: CV accuracy across folds

	F1	F2	F3	F4	F5	Mean±Std
North	.763	.790	.782	.776	.785	$\mathbf{0.779 \pm 0.009}$
South	.777	.789	.789	.784	.790	$\mathbf{0.786 \pm 0.005}$

Both regions achieve ≈78% accuracy with low variance across folds, confirming stable generalization.

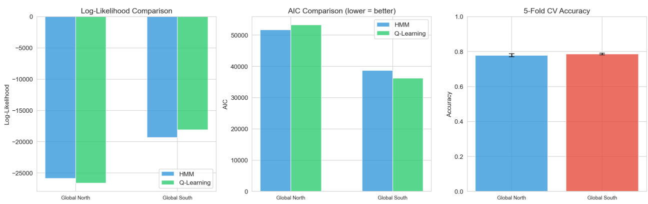

Figure 3: Model evaluation: log-likelihood, AIC, and CV accuracy

7. Behavioral Trajectory Simulation

50 Monte Carlo trajectories of 200 steps each. Initialize $S_0 \sim \text{Bern}(0.5)$; at each step emit $B_t \sim P(B_t|S_t)$, then transition $S_{t+1} \sim \text{Bern}(\sigma(\beta X_t - \delta S_t + \epsilon_t))$.

Table 7: Simulated vs. real behavior distributions

	North		South	
Behavior	Sim	Real	Sim	Real
Action	40.5%	41.4%	34.6%	50.5%
Impact	38.0%	36.5%	50.8%	28.1%
Info	21.6%	22.1%	14.7%	21.4%

Global North simulations closely match real data. Global South discrepancy suggests the binary antecedent does not fully capture developing-region crisis dynamics.

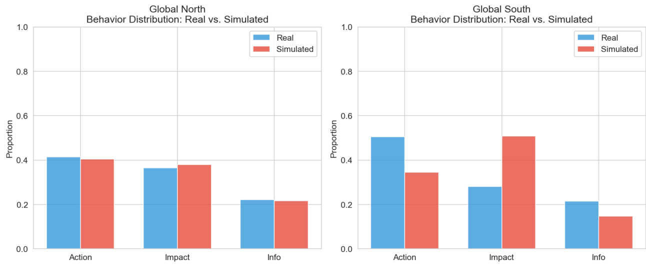

Figure 4: Real vs. simulated behavior distributions

8. Psychological Interpretation

β **(Stimulus Sensitivity):** Both regions show moderate sensitivity (≈ 0.5); North's slightly higher value suggests marginally greater responsiveness, possibly due to better communication infrastructure.

δ **(Decay/Recovery Rate):** Both regions: $\delta = 0.01$ (very low). Once activated, cognitive states are **highly persistent** — consistent with the sustained attentional demands of natural disasters.

τ **(Noise):** ≈ 1.0 for both regions, reflecting moderate stochastic variability in behavioral responses.

α **(RL Learning Rate):** Most striking regional difference. North ($\alpha = 0.5$): **rapid habit formation**, driven by fast feedback loops (real-time media). South ($\alpha = 0.01$): **gradual reinforcement**, slower behavioral updating.

Softmax Temperature $\tau_{\mathbf{RL}}$: Both regions exploratory (North: 5.0, South: 2.0), indicating no strong behavioral fixation.

Equilibrium Analysis: For X_t=1 (North stimulus): S^*=55.83, strongly driving active engagement. For X_t=0 (South stimulus): S^*=0, tending toward passive/impact states. This reflects structural differences in crisis response capacity between regions.

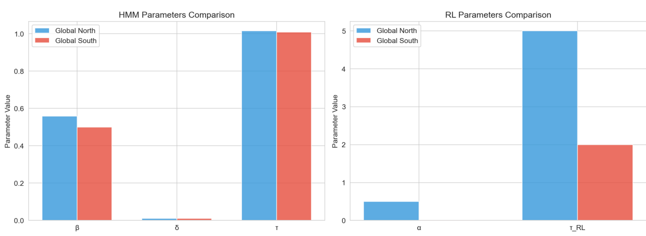

Figure 5: Parameter comparison: Global North vs. South

9. Conclusion

A sigmoid-based HMM effectively captures crisis-related cognitive state dynamics.

Key findings:

- Moderate $\beta \approx 0.5$ shows economic context measurably influences behavior
- $\delta \approx 0.01$ confirms highly persistent crisis schemas
- Divergent RL learning rates (α_{N}=0.5 vs. α_{S}=0.01) reflect infrastructure and feedback loop differences
- Equilibrium analysis reveals Global North structurally drives active engagement while Global South trends toward impact-reporting
- ≈78% CV accuracy confirms robust generalization