Supplementary Material for "Neural Network-based Knowledge Transfer for Multitask Optimization"

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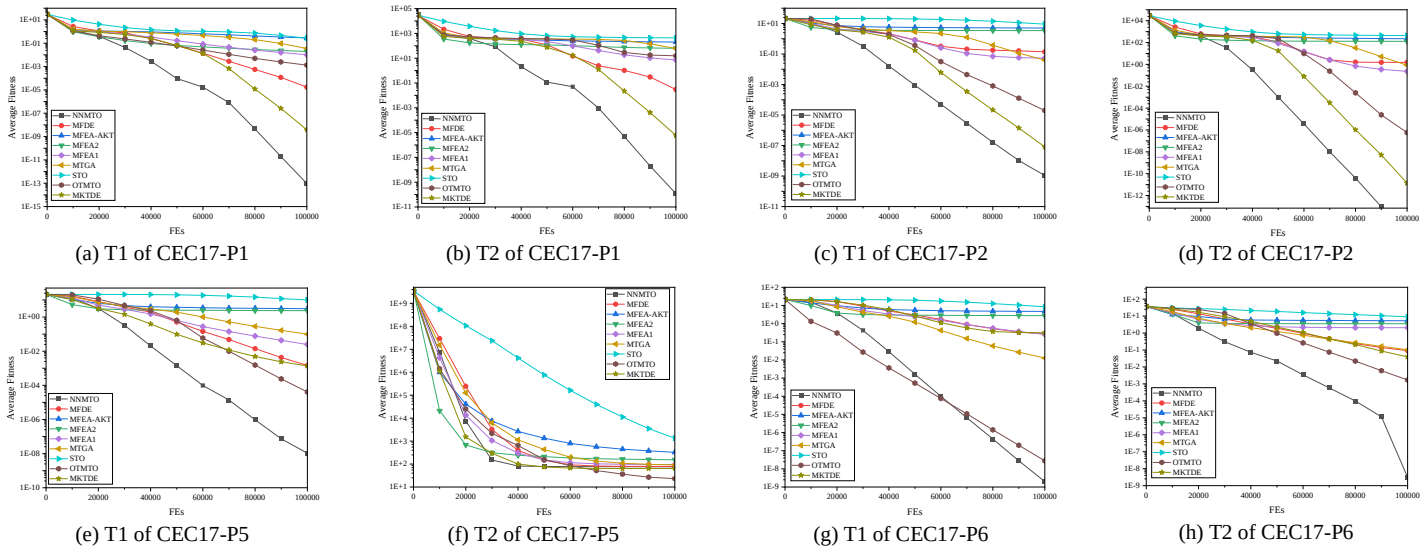
TABLE S.I
PROPERTIES OF THE CEC17 BENCHMARK PROBLEMS

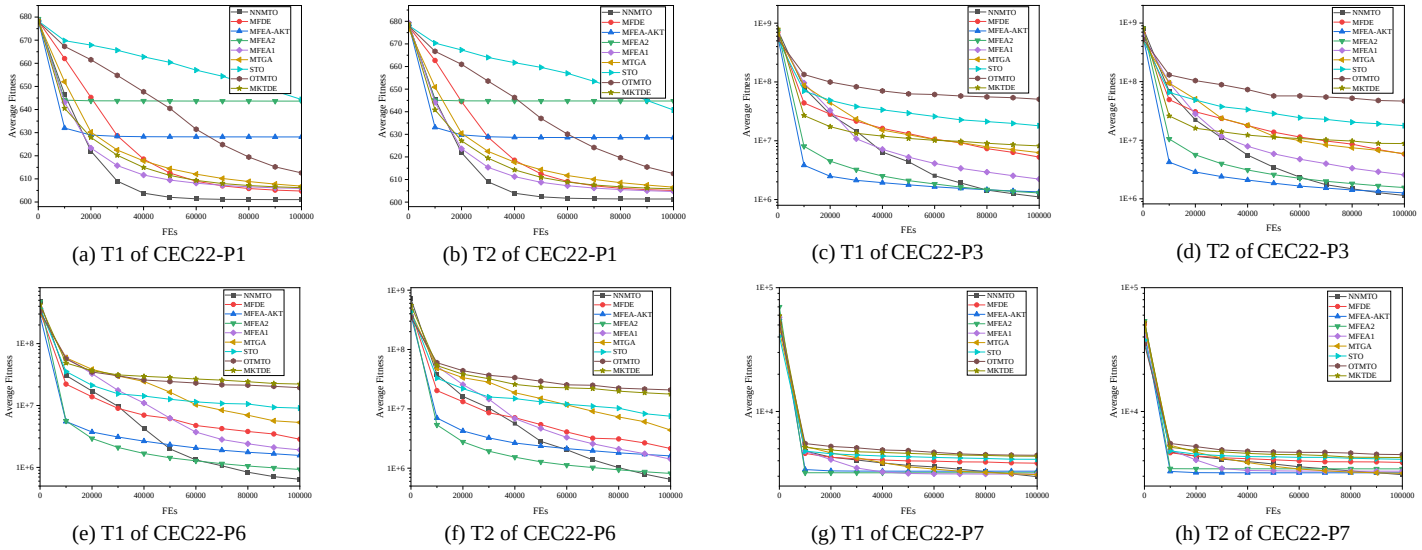
Problem	Task	Dimensionality	Degree of Intersection	Inter-task similarity
CI+HS	Griewank (T_1)	50	Complete Intersection	1.0000
	Rastrigin (T_2)	50		
CI+MS	Ackley (T_1)	50	Complete Intersection	0.2261
	Rastrigin (T_2)	50		
CI+LS	Ackley (T_1)	50	Complete Intersection	0.0002
	Schwefel (T_2)	50		
PI+HS	Rastrigin (T_1)	50	Partial Intersection	0.8670
	Sphere (T_2)	50		
PI+MS	Ackley (T_1)	50	Partial Intersection	0.2152
	Rosenbrock (T_2)	50		
PI+LS	Ackley (T_1)	50	Partial Intersection	0.0725
	Weierstrass (T_2)	25		
NI+HS	Rosenbrock (T_1)	50	No Intersection	0.9434
	Rastrigin (T_2)	50		
NI+MS	Griewank (T_1)	50	No Intersection	0.3669
	Weierstrass (T_2)	50		
NI+LS	Rastrigin (T_1)	50	No Intersection	0.0016
	Schwefel (T_2)	50		



(a) T1 of CEC17-P1 (b) T2 of CEC17-P1 (c) T1 of CEC17-P2 (d) T2 of CEC17-P2

(e) T1 of CEC17-P5 (f) T2 of CEC17-P5 (g) T1 of CEC17-P6 (h) T2 of CEC17-P6

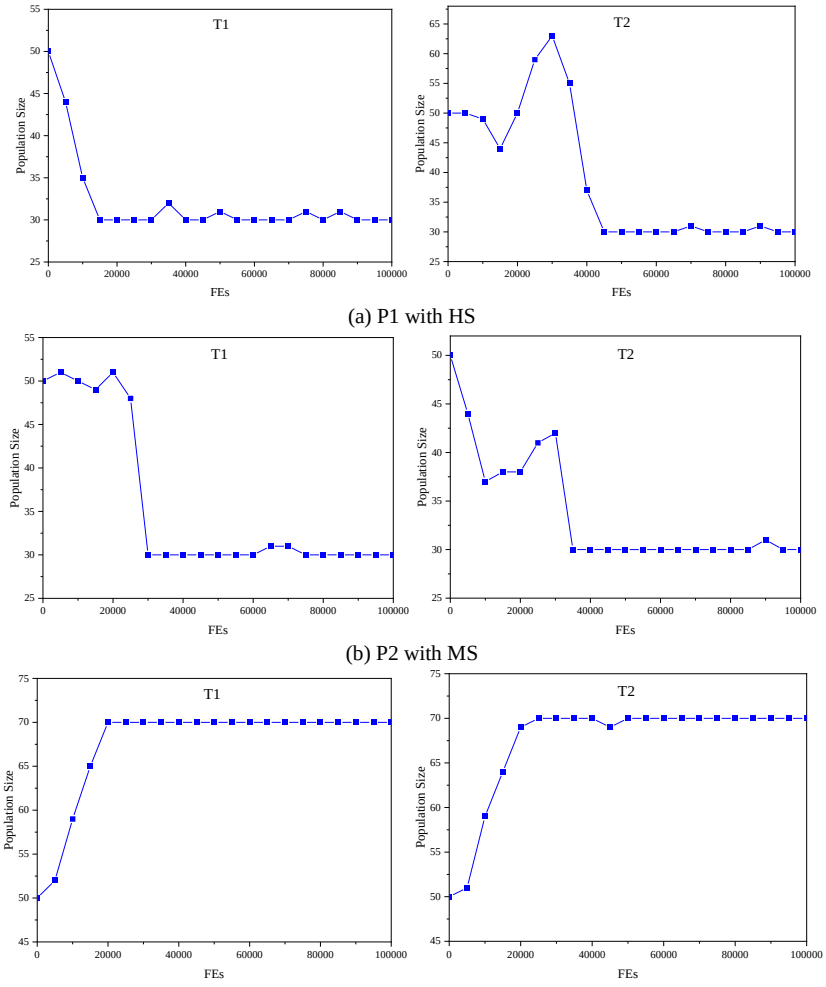
Fig. S1. Convergence curves of the average fitness on (a) T1 of CEC17-P1; (b) T2 of CEC17-P1; (c) T1 of CEC17-P2; (d) T2 of CEC17-P2; (e) T1 of CEC17-P5; (f) T2 of CEC17-P5; (g) T1 of CEC17-P6; (h) T2 of CEC17-P6.



(a) T1 of CEC22-P1 (b) T2 of CEC22-P1 (c) T1 of CEC22-P3 (d) T2 of CEC22-P3

(e) T1 of CEC22-P6 (f) T2 of CEC22-P6 (g) T1 of CEC22-P7 (h) T2 of CEC22-P7

Fig. S2. Convergence curves of the average fitness on (a) T1 of CEC22-P1; (b) T2 of CEC22-P1; (c) T1 of CEC22-P3; (d) T2 of CEC22-P3; (e) T1 of CEC22-P6; (f) T2 of CEC22-P6; (g) T1 of CEC22-P7; (h) T2 of CEC22-P7.



(a) P1 with HS

(b) P2 with MS

(c) P3 with LS

Fig. S3. Population size fluctuation of NNMTO on the CEC17 problems.

TABLE S.II

THE CEC17 EXPERIMENTAL RESULTS OF STO AND NNMTO VARIANTS WITH OR WITHOUT NNKT OR FAMP

Problem		NNMTO	NNMTO -w/o-NNKT	NNMTO -w/o-FAMP	STO
P1	T1	**9.41E-14**	3.20E-04(≈)	4.57E-06(+)	2.68E-01(+)
	T2	**1.18E-10**	4.97E-02(≈)	1.53E+02(+)	4.40E+02(+)
P2	T1	1.08E-09	**3.43E-10(-)**	3.59E-05(+)	9.04E+00(+)
	T2	**0.00E+00**	**0.00E+00(≈)**	3.06E+02(+)	4.36E+02(+)
P3	T1	**2.12E+01**	2.12E+01(≈)	2.12E+01(≈)	2.12E+01(≈)
	T2	1.05E+04	1.13E+04(+)	**8.51E+03(-)**	1.40E+04(+)
P4	T1	**2.57E+02**	3.90E+02(+)	2.94E+02(≈)	4.53E+02(+)
	T2	**2.40E-11**	3.62E+00(+)	5.29E-09(+)	4.32E+00(+)
P5	T1	**1.06E-08**	6.53E-02(≈)	5.30E-05(+)	1.01E+01(+)
	T2	7.62E+01	8.24E+01(+)	**7.41E+01(≈)**	1.34E+03(+)
P6	T1	**1.99E-09**	2.11E+00(+)	3.18E-05(+)	8.48E+00(+)
	T2	**3.11E-09**	2.04E-01(+)	3.21E-01(+)	9.10E+00(+)
P7	T1	**6.16E+01**	9.26E+01(+)	6.17E+01(≈)	1.23E+03(+)
	T2	**1.42E+02**	2.29E+02(+)	2.77E+02(+)	4.39E+02(+)
P8	T1	**2.81E-08**	3.93E-03(+)	3.83E-06(+)	2.84E-01(+)
	T2	1.02E+00	**2.71E-01(-)**	9.14E-01(≈)	4.23E+01(+)
P9	T1	3.28E+02	3.98E+02(+)	**9.89E+01(-)**	4.44E+02(+)
	T2	9.12E+03	1.12E+04(+)	**8.42E+03(≈)**	1.38E+04(+)
Number of +/≈/−			11/5/2	10/6/2	17/1/0

TABLE S.III

THE CEC17 EXPERIMENTAL RESULTS OF NNMTO VARIANTS WITH THE PARAMETER OF DIFFERENT UPDATE FREQUENCY OF NN

Problem		NNMTO(*G*-50)	*G*-25	*G*-100	*G*-200
P1	T1	9.41E-14	**2.85E-15(≈)**	3.50E-13(+)	6.16E-04(+)
	T2	1.18E-10	**3.66E-12(≈)**	3.18E-10(+)	1.54E+01(+)
P2	T1	**1.08E-09**	1.09E-09(≈)	1.80E-09(≈)	1.03E-01(+)
	T2	**0.00E+00**	**0.00E+00(≈)**	**0.00E+00(≈)**	2.98E-01(≈)
P3	T1	**2.12E+01**	2.12E+01(≈)	2.12E+01(≈)	2.12E+01(≈)
	T2	1.05E+04	**1.02E+04(≈)**	1.08E+04(≈)	1.12E+04(+)
P4	T1	2.57E+02	**1.25E+02(-)**	3.75E+02(+)	3.68E+02(+)
	T2	**2.40E-11**	6.65E-02(-)	1.55E-10(+)	3.75E-09(+)
P5	T1	1.06E-08	1.46E-07(≈)	**2.06E-09(≈)**	4.40E-02(+)
	T2	7.62E+01	**6.97E+01(≈)**	8.02E+01(≈)	7.23E+01(≈)
P6	T1	**1.99E-09**	1.76E-01(≈)	4.40E-02(+)	1.74E-01(+)
	T2	**3.11E-09**	4.40E-02(≈)	1.75E-02(≈)	7.65E-04(≈)
P7	T1	**6.16E+01**	8.39E+01(+)	7.48E+01(≈)	6.16E+01(≈)
	T2	1.42E+02	**6.98E+01(≈)**	2.59E+02(+)	1.45E+02(≈)
P8	T1	**2.81E-08**	1.14E-03(+)	3.70E-04(≈)	3.70E-04(+)
	T2	1.02E+00	2.00E+00(+)	**5.69E-01(-)**	7.55E-01(≈)
P9	T1	3.28E+02	**2.49E+02(≈)**	3.24E+02(≈)	3.67E+02(≈)
	T2	9.12E+03	**8.87E+03(≈)**	9.74E+03(≈)	9.77E+03(+)
Number of +/≈/−		~	3/13/2	6/11/1	10/8/0

TABLE S.IV

THE CEC17 EXPERIMENTAL RESULTS OF NNMTO VARIANTS WITH THE PARAMETER OF DIFFERENT FREQUENCY OF KNOWLEDGE REUSE

Problem		NNMTO(*g*-5)	*g*-1	*g*-10	*g*-15	*g*-20
P1	T1	9.41E-14	5.02E-05(≈)	**1.37E-15(≈)**	2.19E-13(≈)	9.29E-04(≈)
	T2	1.18E-10	**3.03E-14(≈)**	1.62E-12(≈)	1.69E-10(≈)	1.77E+01(≈)
P2	T1	1.08E-09	**2.238E-10(-)**	5.50E-10(≈)	3.30E-10(-)	2.60E-10(-)
	T2	**0.00E+00**	**0.00E+00(≈)**	**0.00E+00(≈)**	**0.00E+00(≈)**	**0.00E+00(≈)**
P3	T1	**2.12E+01**	2.12E+01(≈)	2.12E+01(≈)	2.12E+01(≈)	2.12E+01(≈)
	T2	**1.05E+04**	1.11E+04(+)	1.09E+04(≈)	1.09E+04(+)	1.12E+04(+)
P4	T1	**2.57E+02**	3.90E+02(+)	3.21E+02(+)	3.43E+02(+)	3.79E+02(+)
	T2	2.40E-11	1.05E+00(+)	**1.82E-11(≈)**	4.73E-05(≈)	6.86E-11(≈)
P5	T1	**1.06E-08**	4.54E-02(+)	4.09E-08(≈)	2.97E-07(-)	6.87E-02(-)
	T2	**7.62E+01**	8.53E+01(+)	8.15E+01(+)	8.28E+01(+)	7.85E+01(+)
P6	T1	**1.99E-09**	1.95E+00(+)	1.92E-01(≈)	5.78E-02(≈)	1.72E-01(+)
	T2	**3.11E-09**	1.54E-01(+)	1.04E-02(≈)	1.83E-03(≈)	2.96E-02(≈)
P7	T1	**6.16E+01**	1.62E+02(+)	7.00E+01(+)	1.01E+02(≈)	8.79E+01(+)
	T2	1.42E+02	2.31E+02(+)	**1.01E+02(≈)**	2.09E+02(≈)	2.43E+02(+)
P8	T1	**2.81E-08**	1.64E-03(+)	3.11E-08(≈)	2.10E-03(+)(≈)	1.79E-05(+)
	T2	1.02E+00	7.82E-01(≈)	1.70E+00(+)	**7.49E-01(≈)**	1.13E+00(≈)
P9	T1	**3.28E+02**	3.95E+02(+)	3.50E+02(≈)	3.74E+02(+)	3.78E+02(+)
	T2	**9.12E+03**	1.11E+04(+)	9.98E+03(+)	1.04E+04(+)	1.08E+04(+)
Number of +/≈/−		~	12/5/1	5/13/0	6/10/2	9/7/2

TABLE S.V

THE CEC17 EXPERIMENTAL RESULTS OF NNMTO VARIANTS WITH THE PARAMETER OF DIFFERENT NUMBER OF TRANSFER SOLUTIONS

Problem		NNMTO(*S*-10)	*S*-1	*S*-5	*S*-15	*S*-20
P1	T1	9.41E-14	**0.00E+00(-)**	1.25E-15(≈)	3.70E-04(+)	1.36E-03(+)
	T2	1.18E-10	**9.06E-15(-)**	1.52E-12(≈)	3.98E+00(+)	4.53E+00(+)
P2	T1	1.08E-09	**2.24E-10(-)**	4.89E-10(-)	4.45E-09(+)	8.23E-02(+)
	T2	**0.00E+00**	**0.00E+00(≈)**	**0.00E+00(≈)**	1.15E-14(≈)	2.79E+00(≈)
P3	T1	**2.12E+01**	2.12E+01(≈)	2.12E+01(≈)	2.12E+01(≈)	2.12E+01(≈)
	T2	1.05E+04	1.11E+04(+)	1.11E+04(+)	1.03E+04(≈)	**9.75E+03(-)**
P4	T1	2.57E+02	3.73E+02(+)	3.18E+02(+)	2.38E+02(≈)	**1.59E+02(-)**
	T2	2.40E-11	4.12E-11(≈)	**4.68E-12(≈)**	1.28E-10(≈)	3.06E+00(≈)
P5	T1	1.06E-08	2.54E-06(≈)	**9.81E-10(-)**	4.40E-02(+)	1.13E-01(+)
	T2	7.62E+01	8.13E+01(+)	8.08E+01(+)	**7.40E+01(≈)**	8.27E+01(+)
P6	T1	**1.99E-09**	5.78E-02(+)	1.18E-01(≈)	6.46E-09(+)	1.51E-01(+)
	T2	**3.11E-09**	2.11E-03(≈)	7.25E-04(≈)	5.70E-07(+)	1.80E-03(≈)
P7	T1	**6.16E+01**	8.94E+01(+)	7.88E+01(≈)	6.65E+01(≈)	7.66E+01(+)
	T2	1.42E+02	2.51E+02(+)	2.82E+02(+)	**8.23E+01(≈)**	1.11E+02(≈)
P8	T1	2.81E-08	5.30E-04(+)	6.46E-08(≈)	**5.60E-09(≈)**	3.70E-04(+)
	T2	1.02E+00	**8.27E-01(≈)**	1.15E+00(≈)	1.32E+00(≈)	1.22E+00(≈)
P9	T1	3.28E+02	3.97E+02(+)	3.73E+02(+)	1.69E+02(-)	**1.61E+02(-)**
	T2	9.12E+03	1.07E+04(+)	1.01E+04(+)	8.51E+03(≈)	**7.98E+03(-)**
Number of +/≈/−		~	9/6/3	6/10/2	6/11/1	8/6/4