Syllabus

APPLIED MATHEMATICS-IV
(ETMA-202)

Maximum Marks : 75

Instruction to Paper Setters:

1. *Question No. 1 should be compulsory and cover the entire syllabus. This question should have objective or short answer type questions. It should be of 25 marks.*
2. *Apart from Question No. 1, rest of the paper shall consist of four units as per the syllabus. Every unit should have two questions. However, student may be asked to attempt only 1 question from each unit. Each question should be 12.5 marks*

UNIT-I

Partial Differential Equation: linear partial differential equations with constant coefficient, homogeneous and non homogeneous linear equations. Method of separation of variables. Laplace equation, wave equation and heat flow equation in Cartesian coordinates only with initial and boundary value. [T1] [No. of Hrs. 12]

UNIT-II

Probability Theory: Definition, addition law of probability, multiplication law of probability, conditional probability, Baye's theorem, Random variable: discrete probability distribution, continuous probability distribution, expectation, moments, moment generating function, skewness, kurtosis, binomial distribution, Poisson distribution, normal distribution. [T1, T2] [No. of Hrs. 11]

UNIT-III

Curve Fitting: Principle of least square Method of least square and curve fitting for linear and parabolic curve, Correlation Coefficient, Rank correlation, line of regressions and properties of regression coefficients. Sampling distribution: Testing of hypothesis, level of significance, sampling distribution of mean and variance, Chi-square distribution, Student's T-distribution, F-distribution, Fisher's Z- distribution. [T1, T2] [No. of Hrs. 12]

UNIT-IV

Linear Programming: Introduction, formulation of problem, Graphical method, Canonical and Standard form of LPP, Simplex method, Duality concept, Dual simplex method, Transportation and Assignment problem. [T1] [No. of Hrs. 11]

FIRST TERM EXAMINATION [FEBRUARY-2015]
FOURTH SEMESTER [B. TECH]
APPLIED MATHEMATICS-IV [ETMA-202]

Time: 1 Hour **MM : 30**

Note: *Attempts Q. No. 1 which is compulsary and any two more questions from remaining. All questions carry equal marks.*

Q.1. (a) Solve $r + s - 2t = (y-1)e^x$ **(2.5)**

Ans. The given equation is

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y-1)e^x$$

$$\Rightarrow \quad (D^2 + DD' - 2D'^2)z = (y-1)e^x \text{ where } D \equiv \frac{\partial}{\partial x} \text{ and } D' \equiv \frac{\partial}{\partial y}$$

The oxiliary equation is

$$m^2 + m - 2 = 0$$

$$\Rightarrow \quad (m-1)(m+2) = 0$$

$$\Rightarrow \quad m = 1, -2$$

$$\text{C.F} = f_1(y+x) + f_2(y-2x)$$

$$\text{P.I} = \frac{1}{D^2 + DD' - 2D'^2}(y-1)e^x$$

$$= \frac{1}{(D-D')(D+2D')}(y-1)e^x$$

$$= \frac{1}{D-D'}\int (c+2x-1)e^x dx \quad \text{where } y = c+2x$$

$$= \frac{1}{D-D'} = [c-1)e^x + 2(x-1)e^x]$$

$$= \frac{1}{D-D'}[(c+2x)\, e^x - 3e^x]$$

$$= \frac{1}{D-D'}[ye^x - 3e^x] \text{ where } c = y-2x$$

$$= \int (b-x)e^x dx - 3e^x \quad \text{where } y = b-x$$

$$= be^x - (x-1)e^x - 3e^x$$

$$= (b-x-2)e^x$$

$$= (y-2)e^x, \text{ where } b = y+x.$$

Hence the complete solution is

$$z = \text{C.F} + \text{P.I}$$

$$z = f_1(y+x) + f_2(y-2x) + (y-2)e^x$$

where f_1 and f_2 are arbitrary functions.

Q.1. (*b*) $\frac{\partial u}{\partial x} = \frac{2\partial u}{\partial t} + u$; $u(x, 0) = 6e^{-3x}, x > 0$ (2.5)

Ans. The given equation is

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad \text{...(1)}$$

Let $u = X(x)\,T(t)$ ...(2)

Where X is a function of x only and T is a function of t only.

Then
$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(XT) = T\frac{\partial X}{\partial x}$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}(XT) = X\frac{\partial T}{\partial t}$$

Substituting in equation (1), we get

$$T\frac{dX}{dx} = 2x\frac{dT}{dt} + XT$$

$$\Rightarrow \quad TX' = 2\,XT' + XT$$

$$\Rightarrow \quad TX' = X(2T' + T)$$

$$\Rightarrow \quad \frac{X'}{X} = 2\frac{T'}{T} + 1 = -p^2 \text{ (say)}$$

(*i*)
$$\frac{dX}{dx} + p^2 X = 0$$

$$\Rightarrow \quad \frac{dX}{X} = -p^2 dx$$

On integration, we get

$$\log X = -p^2 x + \log C_1$$

$$X = C_1 e^{-p^2 x} \quad \text{...(3)}$$

(*ii*)
$$\frac{2T'}{T} = -(p^2+1)$$

$$\Rightarrow \quad \frac{dT}{T} = -\left(\frac{p^2+1}{2}\right)dt$$

On integration, we get

$$\log T = -\frac{(p^2+1)}{2}t + \log C_2$$

$$T = C_2 e^{-\left(\frac{p+1}{2}\right)t} \quad \text{...(4)}$$

From (2), (3) and (4) we get

$$u = XT = C_1 C_2 e^{-p^2 x - \left(\frac{p^2+1}{2}\right)t} \quad \text{...(5)}$$

$$u(x, 0) = 6e^{-3x} \text{ (given)}$$

From (5), we have

$$6e^{-3x} = C_1 C_2 e^{-p^2 x}$$

$$\Rightarrow \quad C_1 C_2 = 6 \text{ and } p^2 = 3$$

Hence the salution is

$$u(x, t) = 6e^{-3x-2t}$$

Q.1. (*c*) The odds against A solving a certain problem 5 to 7 and the odds in favour of B salving the same problem are 3 to 4. what is the probability that if both of them try the problem would be solved. (2.5)

Ans.
$$P(\bar{A}) = \frac{2}{3} \Rightarrow P(A) = 1 - P(\bar{A}) = 1\frac{-2}{3} = \frac{1}{3}$$

$$P(B) = \frac{1}{2} \Rightarrow P(\bar{B}) = 1\frac{-1}{2} = \frac{1}{2}$$

The probability that A and B can not solve the problem $= P(\bar{A}) \times P(\bar{B}) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

The probabrility that the problem can be solved $= 1 - P(\bar{A})P(\bar{B})$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

Q.1. (*d*) A random variable X has probatrility function $p(x) = 1/2^x, x = 1, 2, 3,$ .... find its moment generating function about origin. (2.5)

Ans.
$$M_0(t) = E[e^{tX}] = \sum e^{tx} p(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{1}{2^x}$$

$$= \sum_{x=0}^{\infty} \left(\frac{e^t}{2}\right)^x$$

$$= 1 + \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \left(\frac{e^t}{2}\right)^3 + \ldots$$

Q. 2. (*a*) Find the solution of partial differential equation (5)

$$(D - 3D' - 2)^2 z = 2e^{2x}\tan(y + 3x)$$

Ans. C.F $= e^{2x} f_1(y+3x) + xe^{2x} f_2(y+3x)$

P.I
$$= \frac{1}{(D - 3D' - 2)^2} 2e^{2x}\tan(y+3x)$$

$$= 2e^{2x}\frac{1}{(D + 2 - 3D' - 2)^2}\tan(y+3x)$$

$$= 2e^{2x}\frac{1}{(D-3D')^2}\tan(y+3x)$$

$$= 2e^{2x}\left[x.\frac{1}{2(D-3D')}\tan(y+3x)\right]$$

$$= 2e^{2x}x^2\frac{1}{2}\tan(y+3x)$$

Hence the complete solution is

$$z = \text{C.F}+\text{P.I} = e^{2x}f_1(y+3x)+xe^{2x}f_2(y+3x)+x^2e^{2x}\tan(y+3x)$$

Q.2. (b) A rod of lenght l with insulated sides is initially a uniform temperature u_0. Its ends are suddenly cooled to 0°C and kept at that temperature. Find the temperature function $u(x, t)$, (5)

Ans. The temperature function $u(x, t)$ satisfies the differential equation

$$\frac{\partial u}{\partial t} = C^2\frac{\partial^2 u}{\partial x^2} \quad \ldots(1)$$

let $u = XT$

where X is a function of x only and T is a function of t only be a solution of (1).

Then $\frac{\partial u}{\partial t} = XT'$ and $\frac{\partial^2 u}{\partial x^2} = X''T$

Substituting is equation (1)

$$XT' = C^2X''T$$

$$\Rightarrow \frac{X''}{X} = \frac{1}{C^2}\frac{T'}{T} = -p^2 \text{ (say)}$$

$$\frac{X''}{X} = -p^2$$

$$\Rightarrow X''+p^2X = 0$$

$$\Rightarrow X = C_1\cos px + C_2\sin px$$

and

$$\frac{1}{C^2}\frac{T'}{T} = -p^2$$

$$\Rightarrow \frac{T'}{T} = -C^2p^2$$

$$\Rightarrow \log T = -C^2p^2t+\log C_3$$

$$\Rightarrow T = e^{-C^2p^2t}$$

Thus the solution of heat eq. 4 (1) is

$$u(x,t) = (C_1\cos px + C_2\sin px)C_3e^{-c^2p^2t} \quad (2)$$

Since the ends $x = 0$ and $x = l$ are cooled to 0°C and kept at that temperature throughout, the boundary condition are $u(0,t) = u(l,t) = 0$ for all t.

Also, $u(x, 0) = u_0$ is the initial condition.

Since $u(0,t) = 0$, we have from (2)

$$0 = C_1C_3e^{-c^2p^2t} \Rightarrow c_1 = 0$$

\From (2),

$$u(x,t) = C_2C_3\sin px.\,e^{-c^2p^2t} \quad \ldots(3)$$

Since $u(l,t) = 0$, we have from (3)

$$0 = C_2C_3\sin pl\,e^{-c^2p^2t}$$

$$\Rightarrow \sin pl = 0 \Rightarrow pl = n\bar{h}$$

$$\Rightarrow p = \frac{n\pi}{l}, n \text{ being an integer}$$

Solution (3) reduces to

$$u(x,t) = b_n\sin\left(\frac{n\pi x}{l}\right)e^{-\frac{C^2n^2\pi^2t}{l^2}} \text{ on replacing}$$

C_2C_3 by b_n.

The most general solution is obtained by adding all such solution for $n = 1, 2, 3, \ldots$

$$u(x,t) = \sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{l}\right)e^{-\frac{C^2n^2\pi^2t}{l^2}} \quad \ldots(4)$$

Since $u(x, 0) = u_0$

$$\Rightarrow u_0 = \sum_{n=1}^{\infty}b_n\sin\left(\frac{n\pi x}{l}\right) \text{ which is half-range sine series for } u_0$$

$$\Rightarrow b_n = \frac{2}{l}\int_0^l u_0\sin\left(\frac{x\pi x}{l}\right)dx = \begin{cases}0, \text{ when } n \text{ is even}\\ \frac{4u_0}{x\pi}, \text{when } n \text{ is add}\end{cases}$$

Hence the temprature function is

$$u[x,t] = \frac{4u_0}{\pi}\sum_{n=1,3,5}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{l}\right)e^{\frac{-C^2n^2\pi^2t}{l^2}}$$

$$= \frac{4u_0}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)}\sin\left(\frac{(2n-1)\pi x}{l}\right)e^{\frac{-C^2n^2\pi^2t}{l^2}}$$

Q.3. (a) Solve the differential equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u = \sin t$ at $x = 0$ and $\frac{\partial u}{\partial x} = \sin t$ at $x = 0$

Ans.

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} \quad \ldots(1)$$

Let $u = XT$

Where X is a function of x only and T is a function of t only be a solution of (1).

Then $$\frac{\partial^2 u}{\partial t^2} = XT'' \text{ and } \frac{\partial^2 u}{\partial x^2} = X''T$$

Substrituting in eq. (1)

$$XT'' = 2^2 X'' T$$

$$\Rightarrow \quad \frac{1}{2^2}\frac{T''}{T} = \frac{X''}{X} = -p^2 \text{ (say)}$$

$$\frac{X''}{X} = -p^2$$

$$\Rightarrow \quad X = C_1 \cos px + C_2 \sin px$$

$$\frac{1}{2^2}\frac{T''}{T} = -p^2$$

$$\Rightarrow \quad T = C_3 \cos(2pt) + C_4 \sin(2pt)$$

Thus the solution of equetion (1) is

$$u = (C_1 \cos px + C_2 \sin px)(C_3 \cos(2pt) + C_4 \sin(2pt)) \quad \ldots (2)$$

On putting $x = 0, u = \sin t$ in (2), we get

$$\sin t = C_1(C_3 \cos(2pt) + C_4 \sin(2pt))$$

$$\Rightarrow \quad C_1C_3 = 0 \text{ and } C_1C_4 = 1, 2p = 1 \Rightarrow p = \frac{1}{2}$$

$$\Rightarrow \quad C_3 = 0 \text{ and } C_4 = \frac{1}{C_1}$$

So eqn. (2) reduce to

$$u = \left(C_1 \cos\frac{x}{2} + C_2 \sin\frac{x}{2}\right)\frac{1}{C_1}\sin t$$

$$\Rightarrow \quad u = \left(\cos\frac{x}{2} + \left(\frac{C_2}{C_1}\right)\sin\frac{x}{2}\right)\sin t$$

$$\Rightarrow \quad u = \left(\cos\frac{x}{2} + C_5 \sin\frac{x}{2}\right)\sin t \text{ where } C_5 = \frac{C_2}{C_1} \quad \ldots (3)$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = \left(\frac{-1}{2}\sin\frac{x}{2} + \frac{1}{2}C_5 \cos\frac{x}{2}\right)\sin t \quad \ldots (4)$$

On putting $x = 0, \frac{\partial u}{\partial x} = \sin t$ in (4) we get

$$\sin t = \frac{1}{2} C_5 \sin t$$

$$\Rightarrow \quad C_5 = 2$$

Hence the solution to given equation (1) is

$$u = \left(\cos\frac{x}{2} + 2\sin\frac{x}{2}\right)\sin t$$

Q.3. (b) In a toy factory, machines A, B and C manutacture respectively 25%, 35% and 40% of the total of this output 5, 4, 2 percents are respectively defective.

A toy is drawn at random from the total production. what is the probability that it was manufactured by Machine A. The toy drawn is defective? Also, find the probability that it was manufactured by machine A. (5)

Ans. Let E_1, E_2, and E_3 denote the events that a toy selected at random is manufacture by the machines A, B, and C respectively and let H denote the event of its being defective.

Then $P(E_1) = 0.25, P(E_2) = 0.35, P(E_3) = 0.40$

The probability of drawring toy manufactured by machine A is $P(H|E_1) = 0.05$

Similarly, $P(H|E_2) = 0.04$ and $P(H|E_3) = 0.02$

By Barye's theorem,

$$P(E_1|H) = \frac{P(E_1)P(H/E_1)}{\sum_{i=1}^{3} P(E_i)P(H/E_i)}$$

$$= \frac{0.25 \times 0.05}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02}$$

$$= \frac{0.0125}{0.0345} = 0.36$$

Q. 4. (a) The first four moments of a distribution about the value 4 of the variable are 1, 4, 10 and 45 respectively. Find the mean and all the four moments about the mean. also comment upon skewness and kurtosis

Ans. We have $A = 4, \mu_1' = 1, \mu_2' = 4, \mu_3' = 10, \mu_4' = 45$

we know $\mu_1' = \bar{x} - A$

$$\Rightarrow \quad \bar{x} = \mu_1' + A = 1 + 4 = 5$$

$$\Rightarrow \quad \text{mean} = 5$$

Moments about mean $\mu_1 = 0$

$$\mu_2 = \mu_2' - (m_1')^2 = 4 - (1)^2 = 3$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$
$$= 10 - 3 \times 4 \times 1 + 2(1)^3$$
$$= 10 - 12 + 2 = 0.$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$
$$= 45 - 4 \times 10 \times 1 + 6 \times 4 \times (1)^2 - 3(1)^4$$
$$= 45 - 40 + 24 - 3$$
$$= 5 + 21 = 26$$

Coefficiente of skewness $= \frac{\mu_3}{\sqrt{\mu_2^3}} = 0$

Kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{26}{9} < 3$

Since $\beta_2 < 3$ then the distribution is platy kurtic.

Q.4. (b) X is a continuous random variable with probability density function given by **(2 + 2 + 1)**

$$f(x) = \begin{cases} kx, & 0 \le x < 5 \\ k(10-x), & 5 \le x < 10 \\ 0, & \text{otherise} \end{cases}$$

Ans. (i) Since X is a continous random variable then

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^5 kx\,dx + \int_5^{10} k(10-x)dx = 1$$

$$\Rightarrow k\left|\frac{x^2}{2}\right|_0^5 + k\left|\frac{(10-x)^2}{-2}\right|_5^{10} = 1$$

$$\Rightarrow k\left(\frac{25}{2}\right)+\frac{k}{2}(25) = 1$$

$$\Rightarrow 25k = 1$$

$$\Rightarrow k = \frac{1}{25}$$

(ii) Mean of

$$X = \int_{-\infty}^{\infty} x\,f(x)\,dx$$

$$= \int_0^5 kx^2dx + \int_5^{10} k\,x(10-x)dx$$

$$= k\left[\left|\frac{x^3}{3}\right|_0^5 + 10\left|\frac{x^2}{2}\right|_5^{10} - \left|\frac{x^3}{3}\right|_5^{10}\right]$$

$$= \frac{1}{25}\left[\frac{125}{3}+5\times 50-\frac{1}{3}\times 875\right]$$

$$= \frac{5}{3}+10-\frac{1}{3}\times 35$$

$$= \frac{5+30-35}{3} = 0$$

(iii)

$$P(5 < x \le 1/2) = \int_5^{12} f(x)dx$$

$$= \int_5^{10} f(x)dx + \int_{10}^{12} f(x)dx$$

$$= \int_5^{10} k(10-x)dx + 0$$

$$= k\left|\frac{(10-x)^2}{-2}\right|_5^{10}$$

$$= k\left(\frac{25}{2}\right)$$

$$= \frac{1}{25}\times\frac{25}{2} = \frac{1}{2} = 0.5$$

SECOND TERM EXAMINATION [APRIL-2015]
FOURTH SEMESTER [B. TECH]
APPLIED MATHEMATICS [ETCS-402]

Time: 1 Hour **MM : 30**

Note: *Attempts Q. No. 1 which is compulsoary and any two more questions from remaining. All questions carry equal marks.*

Q. 1. (a) It the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 1000 individuals more than 2 will be defective. **(2.5)**

Ans. Given p = the probability of a bad reaction

$$= 0.001$$

$$n = 2000$$

$$\lambda = np = 2000 \times 0.001 = 2$$

Required Probability

$$= P(X > 2)$$

$$= 1 - P(X \le 2)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= 1 - \left[\frac{e^{-2}2^0}{0!} + \frac{e^{-2}2^1}{1!} + \frac{e^{-2}2^2}{2!}\right]$$

$$\left(\text{using poisson distribution } P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}\right)$$

$$= 1 - e^{-2}(1+2+2)$$

$$= 1 - 5e^{-2} = 1 - 5(0.1353)$$

$$= 1 - 0.6765 = 0.3235$$

Q.1. (b) It θ is the acute angle between two regression lines, show that

$$\tan\theta = \frac{1-r^2}{|r|}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$ **, where σ_x and σ_y are the S.D's of x and y- series respectivly and r is the. Correlation coefficient.**

Ans. Equations of the lines of regresion of y on x and x on y are

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \text{ and } x - \bar{x} = x\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

Their slopes are

$$m_1 = r\frac{\sigma_y}{\sigma_x} \text{ and } m_2 = \frac{\sigma_y}{r\sigma_x}$$

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_2m_1}\right| = \left|\frac{\frac{\sigma y}{r\sigma x}-\frac{r\sigma y}{\sigma x}}{1+\frac{\sigma y^2}{\sigma x^2}}\right|$$

$$= \left| \frac{1-r^2}{r} \cdot \frac{\sigma y}{\sigma x} \cdot \frac{\sigma x^2}{\sigma y^2 + \sigma y^2} \right| = \frac{1-r^2}{|r|} \cdot \frac{\sigma x\, \sigma y}{\sigma y^2 + \sigma y^2}$$

Q.1. (c) A coin was tossed 400 times and the head turned up 225 times. Test the hypothesis that the coin is unbiased at 5% level of significance? **(2.5)**

Ans. Null hypothesis H_0: the coin is unbiased i.e. $P = 0.5$ Alternative hypothesis H_1: the coin is not unbiased i.e $P \neq 0.5$

Here,
$$n = 400, X = \text{No. of success} = 225$$
$$p = \text{proportion of success in the sample}$$
$$= \frac{X}{n} = \frac{225}{400} = 0.5625$$

Population proportion $P = 0.5, Q = 1 - P = 1 - 0.5 = 0.5$

under
$$H_0, z = \frac{p-P}{\sqrt{PQ/n}} = \frac{0.5625-0.5}{\sqrt{\frac{0.5\times 0.5}{400}}} = \frac{0.0625}{\sqrt{0.000625}}$$

$$z = \frac{0.0625}{0.025} = 2.5$$

$\Rightarrow$
$$|z| = 2.5$$

Conclusion. Since $|z| = 2.5 > 1.96$ i.e $|z| > z_\alpha, z_\alpha$ is the significant value of z at 5% level of significance. *i.e.* H_0 is rejected and hence the coin is not unbiased.

(d) Calculate the rank correlation coeficient from the following data showing ranks of 5 students in two subjects; **(2.5)**

Maths:	3	2	4	1	5
Chemistry:	5	4	3	2	1

Ans.

						Total
Rank in maths (x)	3	2	4	1	5	
Rank in Chemistry (y)	5	4	3	2	1	
$d = x - y$	−2	−2	1	−1	4	0
d^2	4	4	1	1	16	26

Coefficient of rank correlation

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$
$$= 1 - \frac{6\times 26}{5(25-1)}$$
$$= 1 - \frac{156}{120}$$
$$= -0.3$$

Q.2. (a) In a normal distribution 7% of the items are under 35 and 89% are under 63. Find the mean and S.D of the distribution? given that **(5)**

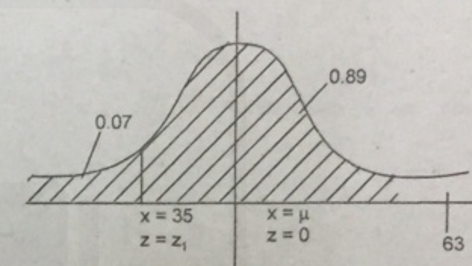
$P(0 \leq z \leq 0.18) = 0.07, P(0 < z < 1.48) = 0.43$

Ans. let μ and σ be the required mean and standard deviation.

Now 7% of the items are under 35.

It means that the area to the left of the ordinate $x = 35$ is 0.07

Also, 89% of the items are under 63. it means area to the left of the ordinate $x = 63$ is 0.89.



Let $\quad z = \frac{x-\mu}{\sigma}$ be the standard normal variate.

Now $\quad x = 35, z = \frac{35-\mu}{\sigma} = z_1$ (say)

when $\quad x = 63, z = \frac{63-\mu}{\sigma} = z_2$ (say)

When $\quad P(x < 35) = 0.07 \Rightarrow P(z < z_1) = 0.07$

$\Rightarrow \quad 1 - P(z > z_1) = 0.07$

$\Rightarrow \quad P(z > z_1) = 0.93$

$\Rightarrow \quad 0.5 - P(0 < z < z_1) = 0.93$

$\Rightarrow \quad P(0 < z < z_1) = 0.43$

$\Rightarrow \quad z_1 = 1.48$

Also, $\quad P(x < 63) = 0.89$

$\Rightarrow \quad P(z < z_2) = 0.89$

$\Rightarrow \quad 1 - P(z > z_2) = 0.89$

$\Rightarrow \quad P(z > z_2) = 0.11$

$\Rightarrow \quad 0.5 - P(0 < z < z_2) = 0.11$

$\Rightarrow \quad P(0 < z < z_2) = 0.39$

$\Rightarrow \quad z_2 = 1.23$.

Here the values of the ordinate $z = z_1$ and $z = z_2$ must be negative.

$$z_1 = -1.48 \quad z_2 = -1.23$$

When $\quad z_1 = -1.48 \Rightarrow \frac{35-\mu}{\sigma} = -1.48 \Rightarrow 35 - \mu = -1.48\,\sigma$ ... (1)

When $\quad z_2 = -1.23 \Rightarrow \frac{63-\mu}{\sigma} = -1.23 \Rightarrow 63 - \mu = -1.23\,\sigma$ ... (2)

Subtraction (1) and (2), we get

$$-28 = -0.25\,\sigma$$

$$\Rightarrow \quad \sigma = \frac{28}{0.25} = 112$$

Also from (1), $35 - \mu = -1.48 \times 112$

$$\Rightarrow \quad \mu = 200.76$$

Q.2. (*b*) By the method of least square, find the best fitted straight line from the following data: **(5)**

x :	1	2	3	4	5
y:	14	13	9	5	2

Ans. Let the straight line of best fit be

$$y = a + bx \qquad \text{... (1)}$$

Normal equations are

$$\Sigma y = na + b\Sigma x \qquad \text{... (2)}$$

and

$$\Sigma xy = a\,\Sigma x + b\,\Sigma x^2 \qquad \text{... (3)}$$

Here $n = 5$

x	y	xy	x^2
1	14	14	1
2	13	26	4
3	9	27	9
4	5	20	16
5	2	10	25
$\Sigma x = 15$	$\Sigma y = 43$	$\Sigma xy = 97$	$\Sigma x^2 = 55$

Substituting in (2) and (3), we get

$$43 = 5a + 15b \qquad \text{... (4)}$$

$$97 = 15a + 55b \qquad \text{... (5)}$$

On solving (4) and (5), we get

$$a = 18.2, b = -3.2$$

Hence the required straight line is

$$y = 18.2 - 3.2x$$

i.e., $\quad 3.2x + y = 18.2$

Q.3. (*a*) Two random variable have the regression lines with equation $3x + 2y = 26$ and $6x + y = 31$.

Find the mean value of x and y. Also, find the correlation coefficient between x and y. **(5)**

Ans. Given that

$$3x + 2y = 26 \qquad \text{... (1)}$$

$$6x + y = 31 \qquad \text{... (2)}$$

The mean volume of x and y are the values of x and y given by (1) and (2). multiplying (2) by 2 and subtracting from (1), we get

$$-9x = -36$$

$$\Rightarrow \quad x = 4 \text{ i.e. } \bar{x} = 4$$

From (1), $12 + 2y = 26$

$$\Rightarrow \quad y = 7 \text{ i.e. } \bar{y} = 4$$

Calculation of coorelation coefficient

From (1), $2y = -3x + 26$

$$y = \frac{-3}{2}x + 13 = \frac{-3}{2}\left(x - \frac{26}{3}\right)$$

i.e $\quad r\dfrac{\sigma y}{\sigma x} = 1.5 \qquad \text{... (3)}$

Also, from (2),

$$6x = -y + 31$$

$$\Rightarrow \quad x = \frac{-y}{6} + \frac{31}{6}$$

$$\Rightarrow \quad x = \frac{-1}{6}(y - 31)$$

i.e. $\quad r\dfrac{\sigma x}{\sigma y} = \dfrac{-1}{6} \qquad \text{... (4)}$

From (3) and (4), we get

$$r^2 = (-1.5)\left(-\frac{1}{6}\right) \Rightarrow r^2 = 0.25$$

$$\Rightarrow \quad r = \pm 0.5$$

Hence, $\quad \bar{x} = 4, \bar{y} = 7, r = \pm 0.5$

Q.3. (*b*) Using graphical method, solve the following L.P.P

$$\min z = x_1 + x_2$$

S.T.

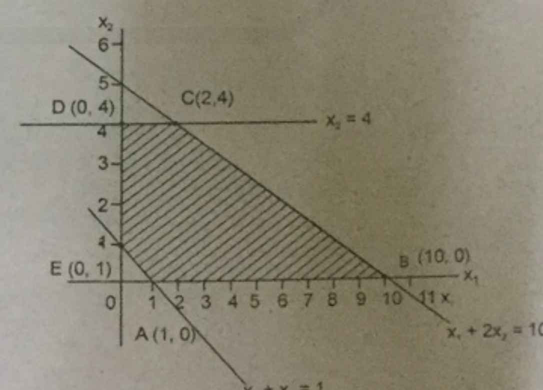
$$x_1 + 2x_2 \le 10$$

$$x_1 + x_2 \ge 1$$

$$x_2 \le 4$$

$$x_1, x_2 \le 0$$

Ans.



By corner point method	
Point	z
A (1,0)	1
B(10, 0)	10
C (2, 4)	6
D (0, 4)	4
E (0, 1)	1

Q.4. (a) The theory predicts the proportion of beans in four groups A, B C and D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans, the number in four groups were 882, 313, 287 and 118. Does the experiment result support the theory? (5)

Given that $\chi^2_{0.05}$ **for 3 d.f = 7.815**

Ans. Null Hypothesis H_0: The experimental result support the theory i.e there is no significant difference between the observed and theoretical frequency under H_0, the theoretical frequency can be calculated as

$$E(A) = \frac{1600\times 9}{16} = 300, E(B) = \frac{1600\times 3}{16} = 300$$

$$E(C) = \frac{1600\times 3}{16} = 300, E(D) = \frac{1600\times 1}{16} = 100$$

Observed frequncy O_i	882	313	287	118
Expected frequency E_i	900	300	300	100
$\frac{(O_i-E_i)^2}{E_i}$	0.36	0.5633	0.5633	3.24

$$\chi^2 = \frac{\sum(O_i-E_i)^2}{E_i} = 4.7266$$

Conclusion: Table value of χ^2 at 5% level of significance for $3d.f$ is 7.815. since the calculated volue of χ^2 is less than that of the tabulated value. Hence H_0 is accepted. i.e the experimental result support the theory.

Q.4. (b) Two sample of sizes 9 and 8 give the sum of squares of deviation from their respective means as 160 and 91 square units respectively. Test whether the samples can be regarded as drawn from two normal populations with the same variance given that $F_{0.05}(8,7) = 3.73$ (5)

Ans. We have, $n_1 = 9, n_2 = 8, \sum(x_i-\bar{x})^2 = 160, \sum(y_i-\bar{y})^2 = 91$

$$S_1^2 = \frac{\sum(x_i-\bar{x})^2}{n_1} = \frac{160}{9}, S_2^2 = \frac{\sum(y_i-\bar{y})^2}{n_2} = \frac{91}{8}$$

We test $H_0: \sigma_1^2 = \sigma_2^2$ against the right tailed alternative $H_1: \sigma_1^2 > \sigma_2^2$.

Under H_0, the statistic F is given by

$$F = \frac{S_1^2}{S_2^2} = \frac{\frac{160}{9}}{\frac{91}{8}} = \frac{160}{9}\times\frac{8}{91} = \frac{1280}{819} = 1.562$$

END TERM EXAMINATION[MAY-JUNE 2015]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA-202]

Time. 3 Hours **MM : 75**

Note: *1. Attempt any five quastions including Q. No. 1 which is compulsory. select one question from each unit.*

Q. 1. (a) Write the steady state two dimensional heat flow equation. Find its solution in cartesian coordinates. (7)

Ans. In the steady state two–dimensional heat flow equation is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0 \quad ...(1)$$

Let $u(x,y) = X(x).Y(y)$ be a solution of (1)

When X is a function of x only and y is a function of y only.

$$\frac{\partial^2 u}{\partial x^2} = X''Y \text{ and } \frac{\partial^2 u}{\partial y^2} = XY''$$

Put in equation (1)

$$X''Y = XY''$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{-Y''}{4} = K(\text{say}) \quad ...(2)$$

(*i*) When K is positive and is equal to p^2, say

$$\Rightarrow \quad X = C_1e^{px}+C_2e^{-px} \text{ and } Y = C_3\cos py + C_4 \sin py$$

(*ii*) When K is negative and is equal to $-p^2$, say

$$\Rightarrow \quad X = C_5\cos px + C_6 \sin px, Y = C_7e^{py}+C_8e^{py}$$

(*iii*) When $K = 0$

$$\Rightarrow \quad X = C_9x+C_{10} \text{ and } Y = C_{11}y+C_{12}$$

The various possible solution of (1) are

$$u = (C_1e^{px}+C_2e^{-px})(C_3\cos py + C_4\sin py) \quad ...(3)$$

$$u = (C_5\cos px + C_6\sin px)(C_7e^{py}+C_8e^{-py}) \quad ...(4)$$

$$u = (Cx_9+C_{10})(C_{11}y+C_{12}) \quad ...(5)$$

Out of these we take that solution which is consistent with the given boundary conditions.

Q. 1. (b) State Baeye's theorem. Design a suitable example and solve that using this theorem.

Ans. Baye's Theorem: Let s be the sample space and let $E_1, E_2, ... E_n$ be n mutually exclusive events associated with a random experiment. If A is any event which occurs

Example: Urn A contains 2 white, 1 black and 3 red balls, Urn B contains 3 white, 2 black and 4 red balls and Urn C contains 4 white, 3 black and 2 red balls. One Urn chosen at random and 2 balls are drawn at random from the Urn. If the chosen balls happen to be red and black what is the probability that both balls come from u_{rn} B?

Let E_1, E_2, E_3 and A denote the following events

E_1 = Urn A is chosen, E_2 = Urn B is chosen, E_3 = Urn C is chosen and A = two balls drawn at random are red and ablack.

Since one of the urns is chosen at randon, therefore

$$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$$

It E_1 has already occured, the Urn A has been chosen. Therefore the probability drawing a red and black ball from u_{rn} $A = \frac{^3C_1 \times {}^1C_1}{^6C_2}$

So, $$P(A/E_1) = \frac{^3C_1 \times {}^1C_1}{^6C_2} = \frac{3}{15} = \frac{1}{5}$$

Similarly, $$P(A/E_2) = \frac{^4C_1 \times {}^2C_1}{^9C_2} = \frac{2}{9}$$

and $$P(A/E_3) = \frac{^2C_1 \times {}^3C_1}{^9C_2} = \frac{1}{6}$$

$$\text{Required probability} = P(E_2/A)$$

$$= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)} \text{ (Using Baye's theorem)}$$

$$= \frac{\frac{1}{3}\times\frac{2}{9}}{\frac{1}{3}\times\frac{1}{5}+\frac{1}{3}\times\frac{2}{9}+\frac{1}{3}\times\frac{1}{6}} = \frac{\frac{2}{9}}{\frac{1}{5}+\frac{2}{9}+\frac{1}{6}} = \frac{20}{53}$$

Q. 1. (c) What is the difference between the problem of correlation and regression? Why are their two regression lines and where do they intersect? Find an expression for the angle of intersection between the two lines. Discuss the special cases. (7

Ans. Correlation: Whenver two variables x and y are so related that an increase in the one is accompained by an increase or decrease in the other, then the variables are said to be correlated.

we have line of regression of y on x i.e. $y = a + bx$. (Here y is dependent variable and x is independent variable).

If we have to find out unknown value x for a given value of y, then we have a line of regression of x on y i.e. $x = a + by$ (Here x is dependent variable and y is independent variable).

So, we have two lines of regression.

Yes, they can intersect with each other.

Let θ be the accute angle between the two regression lines.

Equation of lines of regression of y an x and x an y are

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \text{ and } x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$$

Their slopes are $$m_1 = r\frac{\sigma_y}{\sigma_x}\ em_2 = \frac{\sigma_y}{r\sigma_x}$$

$$\tan\theta = \pm\frac{m_2-m_1}{1+m_2m_1} = \pm\frac{\frac{\sigma_y}{r\sigma_x}-\frac{r\sigma_y}{\sigma_x}}{1+\frac{\sigma_y^2}{\sigma_x^2}}$$

$$= \pm\frac{1-r^2}{r}\cdot\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}\cdot\frac{\sigma_y}{\sigma_x}$$

$$\tan\theta = \pm\frac{1-r^2}{r}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$

Since $r^2 \le 1$ and σ_x, σ_y are positive

$\therefore$ +ve sign given the acute angle between the lines

Hence $$\boxed{\tan\theta = \frac{1-r^2}{r}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}}$$

Cases: (*I*) When $r = 0, \theta = \frac{\pi}{2}$

$\therefore$ The two lines of regression are perpendicular to each other. hence the estimated value of y is the same for all values of x and vice–versa.

(*II*) When $r = \pm 1$, $\tan\theta = 0 \Rightarrow \theta = 0$ or π.

hence the lines of regression coincide and there is perfect correlation between the two variable x and y.

Q. 1. (d) Given an example of an *LPP* with no solution at all. Write its dual. What about the solution of the dual? (5)

Ans. ***LPP* with no solution is**

Standard Primal: Introducing stack variable $s_1 \geq 0$ and surplus variables $s_2 \geq 0$, the standard from of *LPP* is

Maximize
$$Z = x_1 + x_2 + 0s_1 + 0s_2$$

Subject to constraints
$$x_1 + x_2 = s_1 + 0s_2 = 1$$
$$-3x_1 + x_2 + 0s_1 - s_2 = 3$$
$$x_1, x_2, s_1, s_2 > 0$$

Dual: Let w_1 and w_2 be the dual variables corresponding to the primal constraints. Then the dual problem will be.

Minimize
$$Z^* = w_1 + 3w_2$$

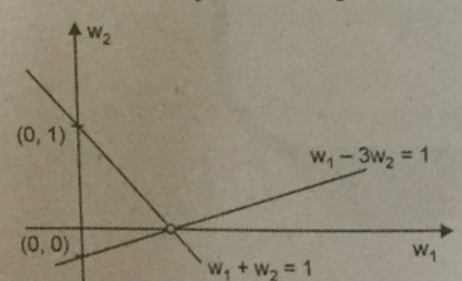
Subject to the constraints:
$$w_1 - 3w_2 \geq 1$$
$$w_1 + w_2 \leq 1$$
$$w_1 + 0w_2 \geq 0 \Rightarrow w_1 \geq 0$$
$$0w_2 - w_2 \leq 0$$

w_1 and w_2 unrestricted (redundant)

Eliminating redundant, the dual problem is

Minimize
$$Z^* = w_1 + 3w_2$$

Subject to the constraints.
$$w_1 - 3w_2 \geq 1$$
$$w_1 + w_2 \leq 1$$
$$w_1 \geq 0 \text{ and } w_2 >, 0$$



The solution of dual of on *LPP* with no solution at all is unbounded solution (by duality theorem)

UNIT – I

Q. 2. (a) Solve: $(D^2 - 4DD' + 4D'^2)\, Z = e^{2x+y}$ (6)

Ans. Auxiliary equation is
$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$

$$= x.\frac{1}{2D - 4D'}e^{2x+y}$$
$$= \frac{x^2}{2}.(e^{2x+y})$$

Hence the solution is
$$Z = f_1(y+2x) + x f_2(y+2x) + \frac{x^2}{2}(e^{2x+y})$$

Q. 2. (b) $(D^2 - D')\, z = 2y - x^2$

Ans. $D^2 - D'$ can not be resolved into linear factors in D and D'

Let $\quad Z = Ae^{hx+ky}$

$\Rightarrow \quad D^2Z = Ah^2e^{hx+ky}$

$\quad D'Z = AKe^{hx+ky}$

$\therefore \quad (D^2 - D')Z = A(h^2 - k)\, e^{hx+ky}$

Then $\quad (D^2 - D')Z = 0$

$\Rightarrow \quad A(h^2 - k) = 0$

$\Rightarrow \quad h^2 - k = 0 \Rightarrow k = h^2$

$\therefore C.F = \sum Ae^{hx+h^2y}$ where A and h are arbiitrary constants

$$P.I. = \frac{1}{D^2 - D'}(2y - x^2) = \frac{1}{D^2\left(1 - \frac{D'}{D^2}\right)}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 - \frac{D'}{D^2}\right]^{-1}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 + \frac{D'}{D^2} + \ldots\right](2y - x^2)$$
$$= \frac{1}{D^2}\left[(2y - x^2) + \frac{1}{D^2}\{D'(2y - x^2)\}\right]\ldots$$
$$= \frac{1}{D^2}\left[2y - x^2 + \frac{1}{D^2}.2\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + x^2\right] = \frac{1}{D^2}.2y = x^2y$$

Hence the required solution is
$$Z = C.F + P.I.$$

Ans. The equation of the string is

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

The solution of equation (1) is

$$u(x,t) = (C_1 \cos cpt + C_2 \sin cpt)(C_3 \cos px + C_4 \sin px) \quad ...(2)$$

Boundary conditions are

$$u(0,t) = 0 \quad ...(3)$$

$$u(l,t) = 0 \quad ...(4)$$

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \quad ...(5)$$

$$u(x,0) = u_0 \sin^3\left(\frac{\pi x}{l}\right) \quad ...(6)$$

Applying boundary condition in (2)

$$u(0,t) = 0 = (C_1 \cos cpt + C_2 \sin cpt)C_3$$

$\Rightarrow$ $C_3 = 0$

$\therefore$ From (2), $u(x,t) = (C_1 \cos cpt + C_2 \sin cpt)\, C_4 \sin px$

Again, $u(l,t) = 0 = (C_1 \cos cpt + C_2 \sin cpt)\, C_4 \sin pl$

$\Rightarrow$ $\sin pl = 0 = \sin n\pi, x \in I$ ...(7)

$\Rightarrow$ $p = \frac{n\pi}{l}$

From (7)

$$u(x,t) = \left(C_1 \cos\frac{n\pi ct}{l} + C_2 \sin\frac{n\pi ct}{l}\right) C_4 \sin\left(\frac{n\pi x}{l}\right) \quad ...(8)$$

$$\frac{\partial u}{\partial t} = \frac{n\pi c}{l}\left(-C_1 \sin\frac{n\pi ct}{l} + C_2 \cos\frac{n\pi ct}{l}\right) C_4 \text{Sin}\left(\frac{n\pi x}{l}\right)$$

At $t = 0$,

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 = \frac{n\pi c}{l} C_2 C_4 \sin\left(\frac{n\pi x}{l}\right)$$

$\Rightarrow$ $C_2 = 0$

From (8)

$$u(x,t) = C_1 C_4 \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right) = b_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right)$$

Most general solution is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right) \quad ...(9)$$

$$u(x,0) = u_0 \sin^3\frac{\pi x}{l} = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$\Rightarrow$ $$u_0 = \left\{\frac{3\sin\frac{\pi x}{l} - \sin\frac{3\pi x}{l}}{4}\right\}$$

$$= b_1 \sin\frac{\pi x}{l} + b_2 \sin\frac{2\pi x}{l} + b_3 \sin\frac{3\pi x}{l} + ...$$

On comparing, we get

$$b_1 = \frac{3u_0}{4}, b_2 = 0, b_3 = -\frac{u_0}{4}, u_4 = u_5 = \quad ...(0)$$

Hence ftrom (9),

$$u(x,t) = \frac{3u_0}{4}\sin\frac{\pi x}{l}\cos\frac{\pi ct}{l} - \frac{u_0}{4}\sin\left(\frac{3\pi x}{l}\right)\cos\left(\frac{3\pi ct}{l}\right)$$

Q. 3. (b) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u : u(0,y) = 0$

$\left(\frac{\partial u}{\partial x}\right)_{(0,y)}$ = it e^{-3y} by the method of separation of variables. (6)

Ans. let $U = XY$ ...(1)

Where X is a function of x only and Y is a function of y only.

$$\frac{\partial u}{\partial y} = X\frac{\partial Y}{\partial y} = XY' \text{ and } \frac{\partial^2 u}{\partial x^2} = YX''$$

Put in given eq^n.

$$YX'' = XY' + 2XY = X(Y' + 2Y)$$

$\Rightarrow$ $$\frac{X''}{X} = \frac{Y'}{y} + 2 = k \text{ (say)}$$

$$\frac{X''}{X} = k$$

$\Rightarrow$ $$X'' - kX = 0$$

Auxilliary equation is

$$m^2 - k = 0$$

$$m = \pm\sqrt{k}$$

$$\text{C.F.} = C_1 e^{\sqrt{kx}} + C_2 e^{-\sqrt{kx}}$$

$$\text{P.I} = 0$$

$$X = C_1 e^{\sqrt{kx}} + C_2 e^{-\sqrt{kx}}$$

and $$\frac{Y'}{y} + 2 = k$$

$$\frac{Y'}{Y} = k - 2$$

On interpation, we get

$$\log Y = (k-2)y + \log C_3$$

$$Y = C_3 e^{(k-2)y}$$

Hence from (1)

$$u(x, y) = \left(C_1 e^{\sqrt{kx}} + C_2 e^{\sqrt{kx}}\right) C_3 e^{(k-2)y} \quad ...(2)$$

Applying the condition $u(0, y) = 0$ in (2), we get

$$0 = (C_1 + C_2)\, C_3 e^{(k-2)y}$$

$$\Rightarrow \quad C_1 + C_2 = 0 \Rightarrow C_2 = -C_1$$

From (2), the most general solution is

$$u(x, y) = \sum C_1 C_3 \left(e^{\sqrt{kx}} - \bar{e}^{\sqrt{kx}}\right) e^{(k-2)y}$$

$$\frac{\partial u}{\partial x} = \sum C_1 C_3 \sqrt{k}(e^{\sqrt{kx}} + e^{-\sqrt{kx}}) e^{(k-2)y}$$

$$\left(\frac{\partial u}{\partial n}\right)_{x=0} = 1 + e^{-3y} = \sum C_1 C_3 \sqrt{k}(2) e^{(k-2)y}$$

$$= \sum b_n\, e^{(k-2)y}$$

Comparing the coefficients, we get

(i)

$$b_1 = 1, k - 2 = 0$$

$$2C_1C_3\sqrt{k} = 1, k = 2$$

$$C_1C_3 = \frac{1}{2\sqrt{2}}$$

(ii)

$$b_3 = -1, k - 2 = 3$$

$$2C_1C_3\sqrt{k} = 1, k = -1$$

$$C_1C_3 = \frac{1}{2i}$$

hence from (6). the particular solution is

$$u(x,y) = \frac{1}{2\sqrt{2}}\left(e^{\sqrt{2x}} - e^{-\sqrt{2x}}\right) + \frac{1}{2i}(e^{ix} - e^{-ix})e^{-3y}$$

$$\Rightarrow \quad \boxed{u(x,y) = \frac{1}{\sqrt{2}} \sin h\sqrt{2x} + e^{-3y} \sin x}$$

UNIT-II

Q. 4. (a) State multiplication rule of probability. From it derive the condition for two events to be independent.

For a system composed of k components in parallel, if p_i independent of others is the probability that the i^{th} component will function $i = 1, 2, ... k$, then what is the probability that system will function? (6.5)

Ans. Statement: The probability of the concurrence of two independent events is the product of their separate probabilities *i.e.*

$$P(AB) = P(A).\, P(B)$$

Proof. Suppose A and B are two independent events.

let A happen in m_1 ways and fail in n_1 ways.

$$\therefore \quad P = \frac{m_1}{m_1 + n_1}$$

Also, let B happen in m_2 ways and fail in n_2 ways.

$$\therefore \quad P(B) = \frac{m_2}{m_2 + n_2}$$

Now there are four possibilities

(i) A and B both may happen, then the number of ways $= m_1 m_2$

(ii) A may happen and B may fail, then the number of ways $= m_1 n_2$.

(iii) A may fail and B may happen, then the number of ways $= n_1 m_2$

(iv) A and B both may fail, then the number of way $= n_1 n_2$.

Thus the total number of ways $= m_1m_2 + m_1n_2 + n_1m_2 + n_1n_2. = (m_1 + n_1)(m_2 + n_2)$

Hence the probabilities of the happening of both A and B is

$$P(AB) = \frac{m_1 m_2}{(m_1 + n_1)(m_2 + n_2)} = \frac{m_1}{m_1 + n_1} \cdot \frac{m_2}{m_2 + n_2}$$

$$= P(A).\, P(B).$$

For a system composed of k components in parallel, if p_i is the probability (indendent of other) that ith component will function, $i = 1, 2, ... k$.

Probavility that system will function

= 1 – (all the k components of system is not working)

$$= 1 - [(1 - p_1)(1 - p_2)(1 - p_3) \ldots (1 - p_k)].$$

Q. 4. (b) What is the expectation of the number of failures preceding the first success in an infinite series of independent trails with constant probality 2/3 of sucess in each trials? (6)

Ans.

$$E(X) = 1.\, P(1) + 2.\, P(2) +$$

$$= 1.\left(\frac{2}{3}\right) + 2\left(1 - \frac{2}{3}\right)\left(\frac{2}{3}\right) + 3\left(1 - \frac{2}{3}\right)^2\left(\frac{2}{3}\right)$$

$$+ 4.\left(1 - \frac{2}{3}\right)^3\left(\frac{2}{3}\right) + \ldots$$

$$= \frac{2}{3}\left[1 + 2\left(\frac{1}{3}\right) + 3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right)^3 + \ldots\right]$$

$$= \frac{3}{2}\left[as \sum_{x=1}^{\infty} xp(1-p)^{x-1} = \frac{1}{p}\right]$$

$$\text{Expected value} = \frac{3}{2}$$

Q. 5. (a) Define a binomial variate. What is its mean and variance? By considering an example of your choice illustration its application. (6.5)

Ans. Binomial random variable: A specific type of discrete random variable that counts how aften a particular event occurs in a fixed number of tries or trials.

For a variable to be binomial random variable, all the following conditions must be satisfied.

(1) There are a fixed number of trails (a fixed sample size)

(2) On each trial the event of interest either occurs or does not

(3) The probability of occurrence (or not) is the same on each trial.

(4) trials are independent of one another.

The Binomial probability Distribution is

$$P(X=r) = {}^nC_r p^r q^{n-r}, p+q=1, r=0,1,2,\ldots n$$

Where p is the probability of success and X is called binomial variate.

Mean and Variance of the bnomial Distribution.

$$P(r) = {}^nC_r q^{n-r} p^r$$

$$\text{Mean}(\mu) = \sum_{r=0}^{n} r\,p(r) = \sum_{r=0}^{n} r\,{}^nC_r q^{n-r} p^r$$

$$= 0 + 1.{}^nC_1 q^{n-1}p + 2^n C_2 q^{n-2}p^2 + 3.{}^nC_3 q^{n-3}p^3 + \ldots + n.{}^nC_n p^n$$

$$= nq^{n-1}p + 2\frac{n(n-1)}{2.1}q^{n-2}p^2 + 3\frac{n(n-1)(n-2)}{3.2.1}q^{n-3}p^3 \ldots\ldots + n.p^n$$

$$= np\left[q^{n-1} + (n-1)q^{n-2}p^2 + \frac{(n-1)(n-2)}{2.1}q^{n-3}p^2 + \ldots + p^{n-1}\right]$$

$$= np\left[{}^{n-1}C_0 q^{n-1} + {}^{n-1}C_1 q^{n-2}p^2 + {}^{n-1}C_2 q^{n-3}p^2 + \ldots\ldots + {}^{(n-1)}C_{n-1}p^{n-1}\right]$$

$$= np(q+p)^{n-1} = np$$

$$\boxed{\text{Mean} = np}$$

Variance,

$$\sigma^2 = \sum_{r=0}^{n} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{n}[r + r(r-1)]P(r) - \mu^2$$

$$= \sum_{r=0}^{n} rP(r) + \sum_{r=0}^{n} r(r-1)P(r) - \mu^2$$

$$= \mu + \sum_{r=2}^{n} r(r-1)^n c_r q^{n-r}p^r - \mu^2$$

$$= \mu + \left[2.1.{}^nc_2 q^{n-2}p^2 + 3.2^n c_3 q^{n-3}p^3 + \ldots\ldots + n(n-1)^n c_n p^n\right] - \mu^2$$

$$= \mu + \left[2.1\frac{n(n-1)}{2.1}q^{n-2}p^2 + 3.2\frac{n(n-1)(n-2)}{3.2.1}q^{n-3}p^3 + \ldots\ldots + n(n-1)p^n\right] - \mu^2$$

$$= \mu + \left[n(n-1)q^{n-2}p^2 + n(n-1)(n-2)q^{n-3}p^3 + \ldots\ldots + n(n-1)p^n\right] - \mu^2$$

$$= \mu + n(n-1)p^2\left[q^{n-2} + (n-2)q^{n-3}p + \ldots\ldots + p^{n-2}\right] - \mu^2$$

$$= \mu + n(n-1)p^2\left[{}^{n-2}C_0 q^{n-2} + {}^{n-2}C_1 q^{n-3}p + \ldots\ldots + {}^{n-2}C_{n-2}p^{n-2}\right] - \mu^2$$

$$= \mu + n(n-1)p^2(q+p)^{n-2} - \mu^2$$

$$= \mu + n(n-1)p^2 - \mu^2$$

$$= np + n(n-1)p^2 - n^2p^2$$

$$= np[1 + (n-1)p - np]$$

$$= np(1-p) = npq$$

$$\boxed{Variance = npq}$$

Example: Treatment of kidney cancer: Suppose we have n = 40 patients who will be receiving an experimental therapy which is belived to be better than current treatements which historically have had a 5 year survival rate of 20% *i.e.* the probability of 5-year survival is p = 0.2.

Thus the number of patients out of 40 in our study surviving at least 5-year has a binomial distribution, i.e. X ~ Bin (40, 20)

Suppose that using the new treatment, we find that 16 out of 40 patients survive at least 5-years post diagnosis.

$$P(X=16) = {}^{40}C_{16}(0.2)^{16}(0.8)^{24} = 0.001945$$

The chance that 16 patients out of 40 surviving at least 5-years is very small 0.001945

Q. 5. (b) A manufacturer who produces medicine bottles find that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. Find that in 100 such boxes, how many boxes are expected to contain (a) no defective (b) atleast two defectives. (6)

Ans. Here N = 100, p = 0.001, n = 500

Mean (m) = np = 500 × 0.001 = 0.5

Let r be the number of defective bottles in a box.

Let $P(r)$ be the number of boxes containing r defective bottles, thes

$$P(r) = N \times \frac{e^{-m}m^r}{r!}$$

(a) P(0) = Number of boxes with no defective bottles

$$= 100 \times \frac{e^{-0.5}\times(0.5)^0}{0!} = 60.65$$

∴ Number of boxes with no defective bottles = 61

(b)

$$P(r \geq 2) = [P(2) + P(3) + \ldots\ldots\ldots + P(500)] \times 100$$

$$= [1-(P(0) + P(1)] \times 100$$

$$= \left\{1 - \left[\left(\frac{e^{-0.5}\times(0.5)^0}{0!} + \frac{e^{-0.5}\times(0.5)^1}{1!}\right)\right]\right\} \times 100$$

$$= 9.02$$

∴ Number of boxes with at least two defectives bottles = 9.

UNIT-III

Q.6. (a) Following data is for the measurement of train resistance R(lds/ton) with the velocity V(mPh). If $R = a + bV + CV^2$, find a, b, c (6)

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Ans. For convenience, let R = y and V = x

∴ the equation is

$$y = a + bx + cx^2$$

Let $$u = \frac{x-80}{h} \text{ and } v = y - 22.8$$

and Let $$h = 20.$$

$$u = \frac{x-80}{20} \text{ and } v = y - 22.8$$

Then the equation is

$$v = a + bu + cu^2$$

x	u	y	v	uv	u^2	u^2v	u^3	u^4
20	−3	5.5	−17.3	51.9	9	−155.7	−27	81
40	−2	9.1	−13.7	27.4	4	−54.8	−8	16
60	−1	14.9	−7.9	7.9	1	−7.9	−1	1
80	0	22.8	0	0	0	0	0	0
100	1	33.3	10.5	10.5	1	10.52	1	1
120	2	46.0	23.2	46.4	4	92.8	8	16
$\Sigma u = -3$			$\Sigma v = -5.2$	$\Sigma uv = 144.1$	$\Sigma u^2 = 19$	$\Sigma u^2v = -115.1$	$\Sigma u^3 = -27$	$\Sigma u^4 = 115$

Normal equations are

$$\Sigma u = na + b\Sigma u + c\,\Sigma u^2$$

$$\Rightarrow \quad -5.2 = 6a + b(-3) + 19\,c$$

$$\Rightarrow \quad -5.2 = 6a - 3b + 19c \quad \text{...(1)}$$

$$\Sigma uv = a\,\Sigma u + b\Sigma u^2 + c\Sigma u^3$$

$$\Rightarrow \quad 144.1 = -3a + 19b - 27c \quad \text{...(2)}$$

$$\Sigma u^2 v = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4$$

$$\Rightarrow \quad -115.1 = 19a - 27b + 115c \quad \text{...(3)}$$

on solving (1), (2) and (3) we get $\begin{bmatrix} a = 4.369 \\ b = -0.00175 \\ c = 0.00287 \end{bmatrix}$

a = 4.369, b = −0.00175, c = 0.00287

∴ $R = 4.369 - 0.00175V + 0.00287V^2$

Q.6. (b) From the following data

23 27 28 28 29 30 31 33 35 36

Estimate y where $x = 32$, by using suitable line of regression (6.5)

Solution:

x	y	xy	x^2
23	18	414	529
27	20	540	729
28	22	616	784
28	27	756	784
29	21	609	841
30	29	870	900
31	27	837	961
33	29	957	1089
35	28	980	1225
36	29	1044	1296
Total 300	250	7623	9138

Let y = a + bx be the equation of the line of regrssion of y on x, where a and b are given by the following equations

$$\Sigma y = na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

$$\Rightarrow \quad 250 = 10a + 300b \quad \text{...(1)}$$

$$7623 = 300a + 9138b \quad \text{...(2)}$$

on solving (1) and (2) we get (Multiplying (1) by 30)

$$7500 = \cancel{300a} + 9000b$$

$$\underline{\ \ }7623 = \underline{\ \ }\cancel{300a} + \underline{\ \ }9138b$$

$$-123 = -138b$$

$$\Rightarrow \quad b = \frac{123}{138} = 0.89$$

∴ $\boxed{b = 0.89}$

Put in (1), we get

$$\boxed{a = -1.74}$$

Hence, we get

$\boxed{y = -1.74 + 0.89x}$ is required regression of y on x.

When x = 32

$$\boxed{y = 26.74}$$

Q.7. (a) A tea company claims that its premium tea brand outsells its normal brand by 10% If it is found that 46 out of a sample of 200 tea-users prefer premium brand and 19 out of another independent sample of 100 tea-users prefer normal brand. Test the validity of the company both at 1% and 5% level of significance. (6)

Ans. Here, $n_1 = 200, n_2 = 100$

$$p_1 = \frac{x_1}{n_1} = \frac{46}{200}, n_2 = \frac{19}{100}$$

p = Proportion of premium tea brand in the population = 0.1

$$Q = 1 - P = 0.99$$

Null hypothesis: H_0: The Manufacturer claim is accepted.

Alternative hypothesis H_1: $p \neq 0.1$

Under H_0,
$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.04}{\sqrt{0.1 \times 0.99 \times \frac{3}{200}}} = 26.9$$

Conclusion: Since the calculated value of $|z| > 1.645$ and also $|z| > 2.33$. Hence H rejected 5% and 1% level of significance, i.e., the proportion of premium tea brand in population is greater than 10%.

Q.7. (b) A survey of 800 families with four children each, recorded the following data:-

No. of boys	0	1	2	3	4
No. of Girls	4	3	2	1	0
No of Families	32	178	290	236	64

Test the hypothesis that male and female births are equailly likely. (6)

Ans. Null hypothesis H_0: The data are consistent with the hypothesis of equal probability for male and female births *i.e.* $p = q = \frac{1}{2}$.

We use binomial distribution to calculate theoretical frequency given by:

$$N(r) = N \times P(X = r) = N \times {}^nC_r p^r q^{n-r}$$

Where N is the total frequency, N(r) is the number of families with r male children, p and q are possibilities of male and female births respectively, n is the no. of children.

$$N(0) = 800 \times {}^4C_0\left(\frac{1}{2}\right)^4 = 50, N(1) = 200, (2) = 300, N(3) = 200 \text{ and } N(4) = 50$$

Observed frequency O_i	32	178	290	236	64
Expected frequency E_i	50	200	300	200	50
$(O_i - E_i)^2$	324	484	100	1296	196
$\frac{(O_i - E_i)^2}{E_i}$	6.48	2.42	0.333	6.48	3.92

$$\chi^2 = \sum\left[\frac{(O_i - E_i)^2}{E_i}\right] = 19.633$$

freedom = 4

UNIT-IV

Q.8. Solve the following product mix selection LPP: (12.5)

$$\text{Max } w = 4x + 5y + 9z + 11t$$

Subject to constraints

$$x + y + z + t \le 15$$
$$7x + 5y + 3z + 2t \le 120$$
$$3x + 5y + 10z + 5t \le 100$$
$$x, y, z, t \ge 0$$

Ans. Introducing slack variables $S_1 \ge 0, S_2 \ge 0, S_3 \ge 0$ in the respective inequalities. So, the standerd LPP is

$$\text{Max w} = 4x + 5y + 9z + 11t + Os_1 + Os_2 + Os_3$$

s.t.c

$$x + y + z + t + s_1 = 15$$
$$7x + 5y + 3z + 2t + s_2 = 120$$
$$3x + 5y + 10z + 15t + s_3 = 100$$
$$x, y, z, t, s_1, s_2, s_3 \ge O$$

The set of constraints can be written in matrix form as:

$Ax = b$

$$\text{Where, } A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 7 & 5 & 3 & 2 & 0 & 1 & 0 \\ 3 & 5 & 10 & 15 & 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} x \\ y \\ z \\ t \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 15 \\ 120 \\ 100 \end{bmatrix}$$

Inittal basic feasible solution

$$x_B = B^{-1} b \text{ where } B = I_3$$

x_B = basic variable corresponding to columns of basis matrix B(=I)

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 120 \\ 100 \end{bmatrix} = \begin{bmatrix} 15 \\ 120 \\ 100 \end{bmatrix}$$

The iterative simple x tables are:

First Table.

			4	5	9	11	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
0	s_1	15	1	1	1	1	1	0	0
0	s_2	120	7	5	3	2	0	1	0

$z_4 - c_4$ is most negative, so y_4 enters into the basis.

$$\min\left\{\frac{15}{1}, \frac{120}{2}, \frac{100}{15}\right\} = \frac{100}{15}.$$

s_0, s_3 Leaves from the basis.

Second Table:

			4	5	9	11	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
0	s_1	25/3	$\boxed{4/5}$	2/3	1/3	0	1	0	–1/15
0	s_2	320/3	33/5	13/3	5/3	0	0	1	–2/15
11	y_4	100/15	1/5	1/3	2/3	1	0	0	1/15
	$z_j - c_j$	=	–9/5	–4/3	–5/3	0	0	0	11/15

Since $z_1 - c_1$ is most negative, so, y_1 enters into the basis.

$$\min\left\{\frac{25/3}{4/5}, \frac{320/3}{33/5}, \frac{100/15}{1/5}\right\} = \frac{25/3}{4/5}.$$

s_1 leaves from the basis.

IIIrd Table

			4	5	9	11	0	0	0
C_B	Y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
4	y_1	125/12	1	5/6	5/12	0	5/4	0	–1/12
0	s_2	455/12	0	–7/6	–13/12	0	–33/4	/	5/12
11	y_4	55/12	0	1/6	$\boxed{7/12}$	1	–1/4	0	1/12
	$zj - cj =$		0	1/6	–11/12	0	9/4	0	7/12

$z_3 - c_3$ is most negative, so y_3 enter into the basis

$$\text{Min}\left\{\frac{125/12}{5/12}, \frac{55/12}{7/12}\right\} = \frac{55/12}{7/12}$$

So, y_4 leaves from the basis.

IV$^{\text{th}}$ table

			4	5	9	11	0	0	0
c_B	y_B	x_B	y_1	y_2	y_3	y_4	s_1	s_2	s_3
4	y_1	50/7	1	5/7	0	–5/7	10/7	0	–1/7
0	s_2	525/7	0	–6/7	0	13/7	–61/7	1	4/7
9	y_3	55/7	0	2/7	1	12/7	–3/7	0	1/7
	$zi - cj =$		0	3/7	0	11/7	13/7	0	5/7

$$\text{Max } w = \frac{695}{7}$$

Q.9. (a) For the following transportation problem, find the initial BFS using VAM from. **(6.5)**

	1	2	3	4	5	supply
1	4	2	3	2	6	8
2	5	4	5	2	1	12
3	6	5	4	7	3	14
Demand	4	4	6	8	8	

Ans. Here the total demand is 30 and total supply is 34. Since total demand ≠ total supply. We intorduce a dummy column with is demand as (34-30) *i.e* 4 and take all the cost elements of this column as zero.

Thus the transportation table for the initial basic feasible solution of the given problem is

(I)

							Rowepenalty
	4	2	3	2	6	0	8 (2)
	5	4	5	2	1	0	12 (1)
	6	5	4	7	3	4 0	14 (3)
	4	4	6	8	8	4	
Column Penalty	(1)	(2)	(1)	(0)	(2)	(0)	

(II)

						Rewepenalty
	4	4 2	3	2	6	8 (0)
	5	4	5	2	1	12 (1)
	6	5	4	7	3	14 (1)
Column	4	4	6	8	8	
Penalty	(1)	(2)	(1)	(0)	(2)	

(III)

					Rewpenalty
	4	3	2	6	4 (1)
	5	5	2	8 1	12 (1)
	6	4	7	3	10 (1)
Column	4	6	8	8	
Penalty	(1)	(1)	(0)	(2)	

(IV)

				Row penalty
	4	3	2	4 (1)
	5	5	4 2	4 (3)
	6	4	7	10 (2)
Column penalty	4 (1)	6 (1)	8 (0)	

(V)

				Row penalty
	4	3	4 2	4 (1)
	6	4	7	10(2)
Column	4	6	4	
Penalty	(2)	(1)	(5)	

(VI)

	3 4	3	c_1 (1)
	6	4	10(2)
Column	4	6	
Penully	(2)	(1)	

(VII)

		Row Penalty
4	6	
6	4	10

$$\text{Total cost} = 4\times 0 + 4\times 2 + 8\times 1 + 4\times 2 + 4\times 2 + 4\times \varepsilon_1 + 4\times 6 + 6\times 4$$
$$= 8+8+8+8+24+24+4\varepsilon_1$$
$$= 24+24+24+8+4\varepsilon_1$$
$$= 80+4\varepsilon_1$$
$$= 80 \text{ (as } \varepsilon_1 \to 0)$$

Q. 9.(b) An engineer wants to assign 3 Jobs $J_1 J_2, J_3$ to three machines $M_1 M_2$ M_3 in such a way that each job is assigned to some machine and no machine works on more than one job. The cost matrix is given as follows (6)

	M_1	M_2	M_3
J_1	15	10	9
J_2	9	15	10
J_3	10	12	8

(i) Formulate it as LPP

(ii) Find the optimal solution using Hungarian method.

Ans. (i) Linear programing formulation of the given problem is

Minimize the total cost involved, *i.e*,

Minimize $Z = (15x_{11} + 10x_{12} + 9x_{13}) + (9x_{21} + 15x_{22} + 10x_{23}) + (10x_{31} + (2x_{32} + 8x_{33})$

Subject to the constraints:

$$x_{i1} + x_{i2} + x_{i3} = 1;\quad i = 1, 2, 3$$
$$x_{ij} + x_{2j} + x_{3j} = 1;\quad j = 1, 2, 3$$
$$x_{ij} = 0 \text{ or } 1, \text{ for all } i \text{ and } j.$$

(ii) **Step 1:** Let p_i and q_i be row i and calumn j

	M_1	M_2	M_3	Row Minimize
J_1	15	10	9	$p_1 = 9$
J_2	9	15	10	$p_2 = 9$
J_3	10	12	8	$p_3 = 8$

Step 2: We subtract the row minimum form each respective row to obtain the reduced matrix as:

	M_1	M_2	M_3
J_1	6	1	0
J_2	0	6	1
J_3	2	4	0

Minimum $q_1 = 0\ q_2 = 1\ q_3 = 0$

Step 3: We subtract the column minimum from each respective column to obtain the reduced matix as:

	M_1	M_2	M_3
J_1	6	0	0
J_2	0	5	1
J_3	0	3	0

This me...

FIRST TERM EXAMINATION [FEB. 2016]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA–202]

Time : 1: 30 hrs. **M.M. : 30**

Note: *Attempt Q. No. 1 which is compulsory and any two more questions from remaining All questions carry equal marks.*

Q.1. (a) Solve $\dfrac{\partial z}{\partial x} = 2\dfrac{\partial z}{\partial y} + y\cos x$ (2.5)

Ans.
$$\frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} = y\cos x$$
$$(D - 2D') = y\cos x$$
put $\quad D = m, D' = 1$

A.E $\quad m - 2 = 0$
$$\Rightarrow m = 2.$$
$$\text{C.F.} = f_1(y + 2x).$$
$$\text{P.I.} = \frac{1}{D - 2D'} y\cos x$$
$$= \int (c - 2x)\cos x\, dx$$
Here $\quad m = 2$, where c is replaced by $y + mx = y + 2x$.
$$= (c - 2x)\sin x - (-2)(-\cos x)$$
$$= (c - 2x)\sin x - 2\cos x$$
$$= (y + 2x - 2x)\sin x - 2\cos x$$
$$= y\sin x - 2\cos x$$
$\therefore$ Complete solution is
$$z = f_1(y + 2x) + y\sin x - 2\cos x.$$

Q.1. (b) Solve $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial z}{\partial y} = z$ (2.5)

Ans. $\quad (D^2 + DD' + D' - 1)z = 0$

A.E $\quad (D^2 - 1) + D'(D + 1) = 0$

$\Rightarrow \quad (D - 1)(D + 1) + D'(D + 1) = 0$

$\Rightarrow \quad (D + 1)(D + D' - 1) = 0$
$$D = -1$$
for $\quad (D + D' - 1), b = 1\ a = -1, c = 1$

C.F $\quad e^{-x}\phi_1(y) + e^{x}\phi_2(y - x)$

Thus, solution is
$$z = e^{-x}\phi_1(y) + e^{x}\phi_2(y - x)$$

Q.1. (c) If the probability that the man aged 60 will live 70 is 0.6, What is the probability that out of 10 men aged 60, 9 men will live upto 70. (2.5)

Ans. Let, probability of success that man will live

upto $70 = 0.6$

ie $p = 0.6$

probability of failure $q = 1 - p = 1 - 0.6 = 0.4$

Let $n = 10$

Let X be the binomial variate

$$\therefore \quad f_X(x) = p[X = x] = {}^{n}C_x p^x q^{n-x}$$
$$= {}^{10}C_x (0.6)^x (0.4)^{10-x}$$
$$p[X = 9] = {}^{10}C_9 (0.6)^9 (0.4) = 10 \times 0.4 \times (0.6)^9$$
$$= 0.0403$$

Q.1. (d) Determine the value of k, if the probability function of a random variable X is given by

$$p(x) = \begin{cases} \dfrac{kx}{20}, x = 1, 2, 3, 4 \\ 0, \text{other integers} \end{cases}$$ (2.5)

Ans. Since p_X is the probability distribution function

$$\sum p_X(x) = 1$$
$$\Rightarrow \quad \frac{k}{20} + \frac{2k}{20} + \frac{3k}{20} + \frac{4k}{20} = 1$$
$$10k = 20$$
$$k = 2$$

Q.2. (a) Find the solution of the partial differential equation $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$ (5)

Ans. $$(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$$

Replace D by m and D^1 by 1

A.E $$m^3 - 7m - 6 = 0$$
$$(m + 1)(m^2 - m - 6) = 0$$
$$\Rightarrow \quad (m + 1)(m^2 - 3m + 2m - 6) = 0$$
$$\Rightarrow \quad m = -1, (m + 2)(m - 3) = 0$$
$$\Rightarrow \quad m = -1, -2, 3$$
$$\text{C.F} = f_1(y - x) + f_2(y - 2x) + f_3(y + 3x)$$
$$\text{P.I.} = \frac{1}{D^3 - 7DD'^2 - 6D'^3} \sin(x + 2y)$$

Replace D by 1 and D' by 2

$$= \frac{1}{1 - 28 - 48} \iiint \sin u \, du \, du \, du$$

$$-1 \quad -1 \quad \ldots(x + 2y)$$

$\therefore$ complete solution is

$$z = f_1(y - x) + f_2(y - 2x) + f_3(y + 3x) \frac{-1}{75} \cos(x + 2y)$$

Q.2. (b) Use the method of separation of variable to solve the partial differential equation (5)

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \text{ given } u = 3e^{-y} - e^{-5y} \text{ when } x = 0$$

Ans. Given equation is $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$ ...(A)

Let $$u = X(x)\,Y(y) = XY$$
$$\therefore \quad \frac{\partial u}{\partial x} = \frac{\partial (XY)}{\partial x} = Y\frac{dX}{dx}$$
and $$\frac{\partial u}{\partial y} = \frac{\partial (XY)}{\partial y} = X\frac{dY}{dy}$$
$$\therefore \quad A \Rightarrow 4\frac{YdX}{dx} + X\frac{dY}{dy} = 3XY$$
$$\Rightarrow \quad 4X'Y + XY' = 3XY$$
$$\Rightarrow \quad 4X'Y = (3Y - Y')X$$
$$\Rightarrow \quad \frac{4X'}{X} = \frac{3Y - Y'}{Y}$$
$$\Rightarrow \quad \frac{4X'}{X} = 3 - \frac{Y'}{Y} = a \text{ (say)}$$

As LHS is a function of x and RHS is a function of y only

$$\therefore \quad \frac{4X'}{X} = a \Rightarrow 4\frac{dX}{dx}\cdot\frac{1}{X} = a$$

On integrating, $$\int \frac{dX}{X} = \int \frac{a}{4} dx$$
$$\Rightarrow \quad \log X = \frac{ax}{4} + \log c_1$$
$$\Rightarrow \quad X = c_1 e^{ax/4} \quad \ldots(1)$$

Now $$3 - \frac{Y'}{Y} = a \Rightarrow \frac{Y'}{Y} = 3 - a$$

On integrating, $$\int \frac{dY}{Y} = \int (3 - a) dy$$
$$\Rightarrow \quad \log Y = (3 - a)y + \log c_2$$

$$y = c_2 e^{(3-a)y}$$

$\Rightarrow$ $$U = XY = c_1 c_2 e^{ax/4} e^{(3-a)y}$$

As $$U(0,y) = 3e^{-y} - e^{-5y}$$

As given $$U(0,y) = c_1 c_2 e^{(3-a)y}$$

from (B)

comparing two, we get

$$3e^{-y} - e^{-5y} = c_1 c_2 e^{(3-a)y}$$

$$c_1c_2 = 3,\ 3-a = -1 \text{ and } c_1c_2 = -1,\ 3-a = -5$$

$\Rightarrow$ $$c_1c_2 = 3,\ a = 4 \Rightarrow c_1c_2 = -1,\ a = 8$$

Now, equation (B) becomes

$$U = 3e^{4x/4} e^{(3-4)y} - e^{8x/4} e^{-5y}$$

$$U = 3e^{x}e^{-y} - e^{2x} e^{-5y}.$$

$$U = 3e^{x-y} - e^{2x-5y}$$

Q.3. (a) Find the solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions

(i) $u \to 0$ as $y \to \infty$ for all x (ii) $u = 0$ at $x = 0$ for all y

(iii) $u = 0$ at $x = l$ for all y (iv) $u = lx - x^2$ if $y = 0$ for all $x \in (0,l)$

Ans. Given equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

The boundary conditions are

$$\left.\begin{aligned} u(0,y) &= 0 \\ u(l,y) &= 0 \end{aligned}\right\} \text{for all } y$$

$$u(x,\infty) = 0\ \forall x$$

$$u(x,0) = lx - x^2\quad 0 < x < l$$

The three possible solutions are

(i) $$u(x,y) = (c_1 e^{px} + c_2 e^{-px})(c_3 \cos py + c_4 \sin py)$$

(ii) $$u(x,y) = (c_5 \cos px + c_6 \sin px)\, c_7 e^{py} + c_8 e^{-py}$$

(iii) $$u(x,y) = (c_9 x + c_{10})(c_{11} y + c_{12})$$

from the condition that $u \to 0$ as $y \to \infty$ for all value of x, solutions (i) and (iii) le to trivial solutions and hence (ii) is the only suitable one.

$$u(x,y) = \sin px\,(C'e^{py} + D'e^{-py}) \quad ...(3)$$

Using the condition $$u(l,y) = 0$$

$$0 = \sin pl\,(C'e^{py} + D'e^{-py})$$

$\Rightarrow$ $$\sin pl = 0 \text{ or } p = \frac{n\pi}{l},\ n \text{ being an integer.}$$

Also, using $u(x,\infty) = 0$ in (3), we get $C' = 0$

By (3), we get

$$u(x,y) = \sin\frac{n\pi x}{l}.D e^{-n\pi y/l},\ n \text{ is an integer}$$

$\therefore$ General solution is of the form

$$u(x,y) = \sum_{n=1}^{\infty} D_n \sin\frac{n\pi x}{l} e^{-n y\pi/l} \quad ...(4)$$

Using, condition $u(x,0) = lx - x^2$, we get

$$lx - x^2 = \sum_{n=1}^{\infty} D_n \sin\frac{n\pi x}{l},\ 0 < x < l$$

which is half range sine series

$\therefore$ $$D_n = \frac{2}{l}\int_0^l (lx - x^2)\sin\frac{n\pi x}{l}dx$$

$$= \frac{2}{l}\left[\left[lx\left(-\cos\frac{n\pi x}{l}\right).\frac{l}{n\pi} - l\left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right]_0^l\right.$$

$$-\left(x^2\left(-\cos\frac{n\pi x}{l}\right).\frac{l}{n\pi} - 2x\left(-\sin\frac{n\pi x}{l}\right).\frac{l^2}{n^2\pi^2}\right.$$

$$\left.\left.+ 2\cos\frac{n\pi x}{l}\frac{l^3}{n^3\pi^3}\right\}_0^l\right]$$

$$= \frac{2}{l}\left[\frac{-l^3}{n\pi}\cos n\pi + \frac{l^3}{n\pi}\cos n\pi - \frac{2l^3}{n^3\pi^3}\cos n\pi + \frac{2l^3}{n^3\pi^3}\right]$$

$$= \frac{4l^2}{n^3\pi^3}[-(-1)^n + 1]$$

$$\therefore \quad u(x,y) = \frac{8l^3}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}\sin\frac{(2n-1)x\pi}{l}e^{(2n-1)\pi y/l}$$

Q.3. (b) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers.

The probability of an accident involving a scoter driver, a car driver and truck driver is 0.01, 0.03 and 0.15 respectively. One of the insured person met with an accident. What is the probability that he is a scoter driver.

Ans. Let E_1, E_2, E_3 denote the events that an insured person at random is scooter, car and truck drivers. respectively.

Let H denote the event person met with an accident.

Then $P(E_1) = \frac{2000}{12000}, P(E_2) = \frac{4000}{12000} P(E_3) = \frac{6000}{12000}$

Probability of insured person met with an accident is scooter driver $P(H/E_1) = 0.01$

similarly $\quad P(H/E_2) = 0.03$

$\quad P(H/E_3) = 0.15$

By Baye's theorem, we have

$$P(E_1/H) = \frac{P(E_1)P(H/E_1)}{P(E_1)P(H)(E_1)+P(E_2)P(H/E_2)+P(E_3)P(H/E_3)}$$

$$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15}$$

$$= \frac{1}{6}\times\frac{6}{52} = \frac{1}{52} = 0.0192$$

Q.4. (a) Find the moment generating function of the distribution $f(x) = \frac{1}{C}e^{-x/c}, 0 \le x < \infty, c > 0$ about origin. Hence find its mean and standard deviation.

Ans. $\quad f(x) = \frac{1}{C}e^{-x/c} \quad 0 \le x < \infty, c > 0$

M.G.F (about origin) $= E[e^{tx}]$

$$= \int_0^{\infty} e^{tx} \cdot f_X(x)\,dx$$

$$= \frac{1}{C}\int_0^{\infty} e^{x(t-1/c)}dx$$

$$= \frac{1}{C}\left[\frac{e^{x(t-1/c)}}{t-1/c}\right]_0^{\infty}$$

$$M_X(t) = c\frac{-1}{\frac{(tc-1)}{C}} = \frac{-1}{tc-1} = -(tc-1)^{-1}$$

Now $\quad E[X] = \frac{d}{dt}$ M.G.F.

$$= (tc-1)^{-2}.C$$

Mean $\quad = E[X] = \frac{d}{dt}Mx(t)\Big]_{t=0}$

$$= \left[(tc-1)^{-2}c\right]_{t=0} = 0$$

$$E(X^2) = \frac{d^2}{dt^2}MX(t)\Big|_{t=0}$$

$$= -2c^2(tc-1)^{-3}\Big|_{t=0} = -2C^2$$

$$\text{var}\,X = E(X^2)-[E(X)]^2 = -2c^2-c^2$$

$$= -3c^2$$

$$\text{S.D} = \sqrt{\text{var}\,X} = \sqrt{-3c^2} = c\sqrt{-3}$$

Q.4. (b) A manufacture of pins knowns that on an average 5% of his product is defative. He sells pins in boxes of 100 and gurantees that not more than 4 pins will be defective what is the probability that the box will fail to meet the guaranteed quality? ($e^{-5} = 0.0067$) (5)

Ans. Let X : no of defective pins

$X = p(\lambda)$

Let $\quad p =$ the probability that a pin is defective

$= 5\% = 0.05$

also $\quad n = 100$

$\Rightarrow \quad \lambda = np = 100 \times 0.05 = 5$

$$p.(x=r) = \frac{e^{-\lambda}\lambda^r}{r!}, r = 0,1,2,\ldots$$

The box will meet guarantee if it contains atmost 4 defective pins.

$\therefore$ Required probability $= p(X \le 4)$

$$= p[X=0] + p[X=1] + p[X=2] + p[X=3] + p[X=4]$$

$$= e^{-\lambda} + e^{-\lambda}.\lambda + e^{-\lambda}\frac{\lambda^2}{2i} + e^{-\lambda}\frac{\lambda^3}{3i} + e^{-\lambda}\frac{\lambda^4}{4i}$$

$$= e^{-5}\left[1+5+\frac{25}{2}+\frac{125}{6}+\frac{625}{24}\right]$$

$$= e^{-5}(6 + 12.5 + 20.83 + 26.04)$$

$$= e^{-5} \times 65.37 = 0.0067 \times 67.37$$

$$= 0.44$$

Box fails to meet guarantee

$$1 - 0.44 = 0.5619.$$

SECOND TERM EXAMINATION [APRIL 2016]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA–202]

Time : 1.30 hrs. M.M. : 30

Note: *Attempt Q. no. 1 which is compulsory and any two more questions from remaining. All questions carry equal marks.*

Q. 1. (a) Prove that $r = \dfrac{\text{cov}(x,y)}{\sigma_x \sigma_y}$, where σr and σr are the S.D's of x and y-series respectvely and r is the correlation coefficient. (2.5)

Ans. As

$$r = \frac{\sum (x_i - y_i)(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$= \frac{\frac{1}{n}\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2 . \frac{1}{n}\sum (y_i - \bar{y})^2}}$$

As

$$\sigma_x = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}$$

$$\sigma_y = \sqrt{\frac{1}{n}\sum (y_i - \bar{y})^2}$$

$$x = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y}$$

Q.1. (b) A random sample of 900 measurem ents from a large population ve a mean value of 64 if this sample has been drawn from a normal population h standard deviation of 20, find the 95% confidence limits for the mean in population. (2.5)

Ans. Here $n = 900, \mu = 64, \sigma = 20$

Test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

At 5% level of significance

$$|Z\alpha| = 1.96$$

95% confidence limit is

$$\bar{x} \pm 1.96\, \sigma/\sqrt{n}$$

$$= 64 \pm 1.96 \times 20/\sqrt{900}$$

$$= 64 \pm 1.96 \times \frac{20}{30} = 64 \pm 1.3066$$

$$= 62.693 < \mu < 65.3066$$

Q.1. (c) A toy company manufactures two types of toys, type A and type B. Each toy of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2000 toys per day. The supply of plastic is sufficient to produce 1500 toys per day (both A and B combined). Type B toy requires a dress of which there are only 600 per day available. If the company makes a profit of Rs 3 and Rs 5 per toy respectively on type A and B, then how many of each types of toy should be produced per day in order to maximize the total profit. Formulate this problem.

Ans. The key decision is to determine the production of toys of type A and type B respectively. Let x and y denote the number of toys of type A and type B respectively.

Since it is not possible to produce negative quantities of toys, feasible alternatives are sets of value of x and y satisfying $x \geq 0, y \geq 0$.

The constraints are

$$x + 2y \leq 2000$$
$$x + y \leq 1500$$

Since only type B requires a dress of which there are only 600 available per day

$$y \leq 600$$

profit on x toy of type A = Rs. 3x.

profit on y toy of type B = Rs 5y

Total profit on x toys of type A and y toys of type B. = Rs (3x + 5y).

So, A manufactures produce to maximize the profit.

$$z = 3x + 5y$$

mathematical formulation is

$$\text{Max } z = 3x + 5y$$

subject to constraints

$$x + 2y \leq 2000$$
$$x + y \leq 1500$$
$$y \leq 600$$
$$x, y \leq 0.$$

Q.1. (d) Write the dual of the following problem

$$\textbf{Max} z = 3x_1 + 2x_2$$

S.t. $\quad x_1 - x_2 \leq 1, x_1 + x_2 \geq 3, x_1 \geq 0, x_2$ **is unrestricted in sign.**

Ans. Standard primal

Introducing the slack variable $s_1 \leq 0$ and surplus variable $S_2 \geq 0$ and $x_2 = x_2' - x_2''$

The standard L.P.P is

$$\text{Max } z = 3x_1 + 2\left(x_2' - x_2''\right) + 0.s_1 + 0.s_2$$

s.t

$$x_1 - \left(x_2' - x_2''\right) + s_1 = 1$$
$$x_1 + \left(x_2' - x_2''\right) - s_2 = 3.$$
$$x_1 \geq 0, x_2' \geq 0, x_2'' \geq 0\ s_1 \geq 0, s_2 \geq 0.$$

Dual let w_1, w_2 be the dual variables. corresponding to the primal constraints. Then the dual problem will be

$$\text{Min } Z^* = w_1 + 3w_2.$$

subject to constraints

$$w_1 + w_2 \geq 3.$$

$$\left.\begin{aligned} -w_1 + w_2 &\geq 2. \\ w_1 - w_2 &\geq -2 \end{aligned}\right\} w_1 - w_2 = 2$$

$$w_1 + 0.w_2 \geq 0$$
$$0.w_1 - w_2 \geq 0$$
$$w_1 \geq 0, -w_2 \geq 0 \Rightarrow w_2 \leq 0.$$

it is re-written as

minimum $\quad z^* = w_1 + 3w_2$

s.t

$$w_1 + w_2 \geq 3$$
$$w_1 - w_2 = 2$$
$$w_1 \geq 0, w_2 \leq 0.$$

Q.2. (a) By the method of least squares, fit a parabola from the following data. (5)

x :	1	2	3	4	5
y :	2	6	4	5	2

Ans. To find equation of the from

$$y = a + bX + cX^2$$

By least square, normal equations are

$$\sum Y = na + b\sum X + c\sum X^2 \quad \text{...(A)}$$

$$\sum XY = a\sum X + b\sum X^2 + c\sum X^3 \quad \text{...(B)}$$

$$\sum X^2Y = a\sum X^2 + b\sum X^3 + c\sum X^4 \quad \text{...(C)}$$

x	y	x^2	xy	x^2y	x^3	x^4
1	2	1	2	2	1	1
2	6	4	12	24	8	16
3	4	9	12	36	27	81
4	5	16	20	80	64	256
5	2	25	10	50	125	625

$$\sum x = 15, \sum y = 19, \sum x^2 = 55, \sum xy = 56,$$

$$\sum x^2 y = 192, \sum x^3 = 225, \sum x^4 = 979, n = 5$$

Equation (A), (B), (C) become

$$19 = 5a + 15b + 55c$$
$$56 = 15a + 55b + 225c$$
$$192 = 55a + 225b + 979c.$$

Multiply (1) by 3. and subtract from (2)

$$57 = 15a + 45b + 165c$$
$$56 = 15a + 55b + 225c$$
$$1 = -10b - 60c.$$

$\Rightarrow$ $$10b + 60c = -1$$

Multiply (2) by 11 and (3) by 3 and subtract

$$616 = 165a + 605b + 2475c$$
$$576 = 165a + 675b + 2937c$$
$$40 = -70b - 462c$$

$\Rightarrow$ $$70b + 462c = -40$$

Multiply (4) by 7

$$70b + 420c = -7$$
$$70b + 462c = -40$$
$$-42c = 33$$

$$c = \frac{33}{42} = -0.785$$

from equation (4), $10b + 60(-0.785) = -1$

$$10b = -1 + 47.1$$

$$b = \frac{46.1}{10} = 4.61$$

By equation (1), $19 = 5a + 15 \times 4.61 + 55(-0.785)$

$$19 = 5a + 69.15 - 43.175$$
$$5a = -6.975$$

$\Rightarrow$ $$a = -1.395$$

$\Rightarrow$ $$y = -1.395 + 4.61x - 0.785x^2.$$

equation is

Q.2. (b) The equations of two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the regression coefficients byx, bxy and the correlation coefficient r. Also, find the standard deviation of y, if the variance of x is 4. (5)

Ans. Let regression equation of y on x is

$$4x + 3y = -7$$

$$y = \frac{-7}{3} - \frac{4}{3}x$$

$\Rightarrow$ Regression coefficient of y on x is $= byx = -\frac{4}{3}$

$\Rightarrow$ Regression equation of x on y is

$$3x + 4y + 8 = 0$$

$\Rightarrow$ $$x = \frac{-8}{8} - \frac{4}{3}y$$

$\Rightarrow$ Regression coefficient of x on $y = bxy = \frac{-4}{3}$

Now $$r^2 = byx.\,bxy = \frac{-4}{3} \times \frac{-4}{3} = \frac{16}{9}$$

But $$r^2 \le 1 \therefore$$

Let regression equation of y on x is

$$3x + 4y + 8 = 0 \Rightarrow y = \frac{-8}{4} - \frac{3}{4}x$$

$\therefore$ $$byx = \frac{-3}{4}$$

and regression equation of x on y is

$$4x + 3y + 7 = 0$$

$\Rightarrow$ $$x = \frac{-7}{4} - \frac{3}{4}y$$

$\therefore$ $$b_{xy} = \frac{-3}{4}$$

Now $$r^2 = b_{yx}.\,b_{xy} = \left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right) = \frac{9}{16}$$

$$r = \pm 0.75$$

As byx and bxy are negative

$\therefore$ $$r = -0.75$$

Regression coefficient of y on x

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{-3}{4}, \sigma_x = 2$$

$\Rightarrow$ $$-0.75 \times \frac{\sigma y}{2} = \frac{-3}{4}$$

$\Rightarrow$ $$\sigma_y = \frac{3}{2 \times 0.75} = 2$$

Q.3. (a) A random sample of 10 boys had the following IQ: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100.

Do these data support the assumption of population mean IQ of 100 at 5% level of significance?

(Given $t_{0.05} = 2.26$ for 9 d.f, $t_{0.05} = 2.23$ for 10 d.f, $t_{0.05} = 2.20$ for 11 d.f) **(5)**

Ans. we are testing

$$H_0 : \mu = 100 \ \sqrt{s} : H_1 : \mu_1 \ne 100$$

Under $$H_0 : \frac{\bar{X}-\mu}{s1\sqrt{n}} \sim t_{n-1}$$

$$\bar{X} = \frac{\sum x_i}{n} = \frac{972}{10} = 97.2$$

$$s = \sqrt{\frac{1}{n-1}\left(\sum x_i^2 - n\bar{x}^2\right)}$$

$$\sum x_i^2 = 4900 + 14400 + 12100 + 10201 + 7744 + 6889 + 9025 + 9604 + 11449 + 10000$$

$$= 96312$$

$$s = \sqrt{\frac{1}{9}(96312 - 10 \times (97.2)^2}$$

$$= \sqrt{\frac{1}{9}(96312 - 94478.4)} = \sqrt{\frac{1833.6}{9}}$$

$$= \sqrt{203.733} = 14.27$$

Test statistic is $$\frac{97.2-100}{14.27/\sqrt{10}} = \frac{-2.8}{4.51257}$$

$$= -0.62$$

$\therefore$ $$t = 0.62$$

Value of t at 5% level with 9 d.f = 2.26

since calculated value 0.62 < tabulated value, we accept H_0 at 5% level significance.

Q.3. (b) Solve the following L.P.P max $z = -2x_1 - x_3$ s.t. $x_1 + x_2 - x_3 \geq 5, x_1 - 2x_2 + 4x_3 \geq 8$ and $x_1, x_2, x_3 \geq 0$

Ans. $$\text{Max } z = -2x_1 - x_3.$$

s.t.
$$x_1 + x_2 - x_3 \geq 5$$
$$x_1 - 2x_2 + 4x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

Introducing slack variable x_4, x_5.

The given L.P.P can be re-written as

$$\text{Max } z = -2x_1 - x_3 + 0.x_4 + 0.x_5$$

s.t.
$$x_1 + x_2 - x_3 - x_4 = 5$$
$$x_1 - 2x_2 + 4x_3 - x_5 = 8.$$

Let us add artifical variables x_6, x_7

So, our L.PP becomes

$$\text{Max } z = -2x_1 - x_3 + 0.x_4 + 0.x_5 - Mx_6 - Mx_7$$

s.t.
$$x_1 + x_2 - x_3 - x_4 + x_6 = 5$$
$$x_1 - 2x_2 + 4x_3 - x_5 + x_7 = 8$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0.$$

Starting table

		$C_j \to$	-2	0	-1	0	0	$-M$	$-M$
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6	x_7
$-M$	x_6	5	1	1	-1	-1	0	1	0
$-M$	$\leftarrow x_7$	8	1	-2	$\boxed{4}$	0	-1	0	1
		$z_j - c_j$	$-2M+2$	M	$-3M+1$	M	M	0	0

x_3 enters the bases.

$$\min\left\{\frac{xB_i}{x_{i3}}, x_{i3} > 0\right\} = \min\left\{\frac{8}{4}\right\} = 2$$

$\Rightarrow x_7$ leaves the basis

First iteration

			-2	0	-1	0	0	$-M$
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5	x_6
$-M$	$\leftarrow x_6$	7	$\boxed{5/4}$	1/2	0	-1	$-1/4$	1
-1	x_3	2	1/4	$-1/2$	1	0	$-1/4$	0
		$z_j - c_j$	$\frac{-5}{4}M + \frac{7}{4}$	$-\frac{M}{2} + \frac{1}{2}$	0	M	$\frac{M}{4} + \frac{1}{4}$	0

x_1 enters the basis (most negative)

$$\min\left\{\frac{28}{5}, 8\right\} = \frac{28}{5}.$$

$\therefore x_6$ leave the basis

Second iteration

			-2	0	-1	0	0
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5
-2	$\leftarrow x_1$	28/5	1	$\boxed{2/5}$	0	$-4/5$	$-1/5$
-1	x_3	3/5	0	$-3/5$	1	1/5	$-1/5$
		$z_j - c_j$	0	$-\frac{1}{5}$	0	$\frac{7}{5}$	$\frac{3}{5}$

Most negative is x_2 $\therefore$ x_2 enters the basis

$$\min\{14\} = 14$$

$\therefore x_1$ leaves the basis

Third iteration

			-2	0	-1	0	0
C_B	y_B	x_B	x_1	x_2	x_3	x_4	x_5
-2	x_2	14	5/2	1	0	-2	$-1/2$
-1	x_3	9	3/2	0	1	-1	$-1/2$
		$z_j - c_j$	1/2	0	0	1	1/2

Since all $z_j - c_j \geq 0$.

$\therefore$ The solution is optimal solution.

$$x_2 = 14, x_1 = 0, x_3 = 9$$
$$\text{Max } z = -2 \times 0 - 9 = -9$$

Q.4. (a) A car hire company has one car at each of five depots a, b, c, d an A customer requires a car in each town, namel A, B, C, D and E.Distances kms) between depots (origin) as towns (destinations) are given in the follow distance matrix:

	a	*b*	*c*	*d*	*e*
A	**160**	**130**	**175**	**190**	**200**
B	**135**	**120**	**130**	**160**	**175**
C	**140**	**110**	**155**	**170**	**185**
D	**50**	**50**	**80**	**80**	**110**
E	**55**	**35**	**70**	**80**	**105**

How should cars be assigned to customers so as to minimize the dista travelled?

Ans. Here, number of tasks and number of sub-ordinates each equal 4, then problem is balanced.

Subtracting smallest element of each row from every element of corres ponding reduced matrix is

	a	b	c	d	e
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Subtracting smallest element of each column from every element of correspon column, reduced matrix is

30	$\boxed{0}$	35	30	15	√
15	⌧	$\boxed{0}$	10	⌧	
30	⌧	35	30	20	√
$\boxed{0}$	⌧	20	⌧	5	
20	⌧	25	15	15	√

number of assignments (□)is less than n (order of cost matrix), $\therefore$ optimum solution is not achieved

Mark rows having no assegned zero Mark columns that have zeros in marked rows

Draw lines through all unmarked rows and marked columns.

Revised cost matrix is

min element from reduced matrix is 15.

15	0	20	15	0
15	15	0	10	0
30	15	35	30	20
0	15	20	0	5
5	0	10	0	0

Repeat above process

Reduced matrix is

15	⌧	20	15	$\boxed{0}$
15	15	$\boxed{0}$	10	⌧
15	$\boxed{0}$	20	15	5
$\boxed{0}$	15	20	⌧	5
5	⌧	10	$\boxed{0}$	⌧

Since assignment is equal to n, therefore optimum solution is achieved.

$$A \to e, B \to c, C \to b, D \to a, E \to d$$

Now minimum assignment schedule is

$200 + 130 + 110 + 50 + 80 = 570$

Q.4. (b) Find the initial basic feasible solution the following transportation problem by VAM

From/To	W_1	W_2	W_3	W_4	Supply
F_1	11	20	7	8	50
F_2	21	16	10	12	40
F_3	8	12	18	9	70
Demand	30	25	35	40	

Ans. Here demand is 130 and supply is 160.

Since total demand ≠ total supply, we introduce a dummy column with its demand 30.

The transportation table initial *b.f.s* is given as

	W_1	W_2	W_3	W_4	W_5	
F_1	11	20	7	8	0	50 (7)
F_2	21	16	10	12	$\boxed{30}$ 0	40 (10)
F_3	8	12	18	9	0	70 (8)
	30 (3)	25 (4)	35 (3)	40 (1)	30 (0)	

Differences between the smallest and next to smallest in each row and column given in parenthesis.

Largest of these differences is (10), associated with 2nd row. of the table.

Since least cost is 0 in 2nd row, we allocate

$$x_{25} = \min(40,30) = 30$$

Exhaust 5th column. Reduced table is

	W_1	W_2	W_3	W_4	
F_1	11	20	7	8	50 (1)
F_2	21	16	10	12	10 (2)
F_3	8	$\boxed{25}$ 12	18	9	70 (1)
	30 (3)	25 (4)	35 (3)	40 (1)	

Largest difference is (4), associated with 2nd column.

Least cost is 12, allocate

$$x_{32} = \min(70, 25) = 25$$

Exhaust 2nd column. Reduced table is

	W_1	W_3	W_4	
F_1	11	$\boxed{35}$ 7	8	50 (1)
F_2	21	10	12	10 (2)
F_3	8	18	9	45 (1)
	30 (3)	35 (3)	40 (1)	

Largest difference is (3) associated with 3 column

Least cost is 7, allocate

$$x_{13} = \min(50, 35) = 35.$$

Exhaust 3rd column. Reduced matrix is

11	8	15 (3)
~~21~~	$\boxed{10}$ ~~12~~	10 (9)
8	9	45 (1)
30 (3)	40 (1)	

Largest difference is (9), associated with 2nd row

Least cost is 12, allocate

$$x_{24} = \min(10, 40) = 10$$

Exhaust 2nd row. Reduced matrix is

~~11~~	8	15 (3)
$\boxed{30}$ ~~8~~	9	45 (1)
30 (3)	30 (1)	

Largest difference is (3), associated with 1st column Least cost is 8, allocate

$$x_{31} = \min(45, 30) = 30$$

Exhaust 1st column. Reduced matrix is

$\boxed{15}$ 8	15
$\boxed{15}$ 9	15
30	

Now the table is

	W_1	W_2	W_3	W_4	W_5
F_1	11	20	$\boxed{35}$ 7	$\boxed{15}$ 8	0
F_2	21	16	10	$\boxed{10}$ 12	$\boxed{30}$ 0
F_3	$\boxed{30}$ 8	$\boxed{25}$ 12	18	$\boxed{15}$ 9	0

No. of allocated cells is 7, which is same as required $(3 + 5 - 1) = 7$

$\therefore$ Solution is non-degenrate basic feasible

$$\text{Total cost} = 35 \times 7 + 15 \times 8 + 10 \times 12 + 30 \times 0 + 30 \times 8 + 25 \times 12 + 15 \times 9$$
$$= 1160$$

END TERM EXAMINATION [MAY 2016]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA–202]

Time : 3 hrs. **M.M. : 75**

Note : *Attempt any five questions including Q. no. 1 which is complusory. Select one question from each unit.*

Q.1. (a) Find a particular integral of $(Dx^2 + 3Dx\,Dy + Dx + Dy^2 - 2)\,z = 3x + 4y + y - 2xy$. (7)

Ans. Given quation is

$$(D + D' - 1)(D' + 2D' + 2)z = e^{3x+4y} + (y - 2xy)$$

For $(D + D' - 1)\; b = 1, a = -1, c = 1$

For $(D + 2D' + 2)\; b = 1, a = -2, c = -2$

Now, $\text{C.F} = e^x f_1(y - x) + e^{-2x} f_2(y - 2xy)$

$$\text{P.I} = \frac{1}{(D+D'-1)(D+2D'+2)} e^{3x+4y} + y - 2xy$$

Consider

$$\frac{1}{(D+D'-1)(D+2D'+2)} e^{3x+4y}$$

$$= \frac{1}{(3+4-1)(3+8+2)} e^{3x+4y} = \frac{1}{78} e^{3x+4y}$$

and

$$\frac{1}{(D+D'-1)(D+2D'+2)}(y-2xy)$$

$$= \frac{-1}{2[1-(D+D')]\left[1+\left(\frac{D}{2}+D^1\right)\right]}(y-2xy)$$

$$= \frac{-1}{2}[1-(D+D')]^{-1}\left[1+\left(\frac{D}{2}+D'\right)\right]^{-1}(y-2xy)$$

$$= \frac{-1}{2}[1+(D+D')]+2DD'\left[1-\frac{D}{2}-D'+DD'\right](y-2xy)$$

$$= \frac{-1}{2}\left[1-\frac{D}{2}-D'+DD'+D+D'+2DD'-\frac{DD'}{2}-DD'\right](y-2xy)$$

$$= \frac{-1}{2}\left[1+\frac{D}{2}+\frac{3}{2}DD'\right](y-2xy)$$

$$= \frac{-1}{2}\left[y-2xy+\frac{1}{2}(-2y)+\frac{3}{2}D(1-2x)\right]$$

$$= \frac{-1}{2}[y-2xy-y-3] = \frac{3}{2}+xy$$

$$\text{P.I} = \frac{1}{78}e^{3x+4y} + xy + \frac{3}{2}$$

$\therefore$ Complete solution is

$$Z = \text{C.F} + \text{P.I}$$

$$Z = e^x f_1(y-x) + e^{-2x} - f_x(y-2x) + \frac{1}{78}e^{3x+4y} + xy + \frac{3}{2}.$$

Q.1. (b) A die is tossed thrice Getting 2 or 4 on the die in a toss is success. Find the mean and variance of number of success. (6)

Ans. Let X denote the number of success.

i.e. $X = 0,1,2,3$

$$\text{Probability of success} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Probability of failure} = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$\text{Mean of success} = \sum f_i x_i$$

Now, distribution table is

$$P(X=0) = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}$$

$$P(X=1) = {}^3C_1 \times \frac{1}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{3!}{2!}\times\frac{4}{27}$$

$$= \frac{12}{27}$$

$$P(X=2) = {}^3C_2 \times \frac{1}{3}\times\frac{1}{3}\times\frac{2}{3} = \frac{3!}{2!}\times\frac{2}{27} = \frac{6}{27}$$

$$P(X=3) = \frac{1}{27}$$

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0	8/27	0	0	0
1	12/27	12/27	1	12/27
2	6/27	12/27	4	24/27
3	1/27	3/27	9	9/27

Mean

$$X = \sum f_i x_i = 0 + \frac{12}{27} + \frac{12}{27} + \frac{3}{27}$$

$$= 1$$

$$\text{var } X = \sum fixi^2 - \left(\sum fixi\right)^2$$

$$= \frac{12}{27} + \frac{24}{27} + \frac{9}{27} - 1 = \frac{45}{27} - 1 = \frac{18}{27} = \frac{2}{3}$$

Q.1. (c) Can $y = 5 + 2.8x$ and $x = 3 - 0.5y$ be the estimated regression equation of y on x and x or y respectively? Explain

Ans. Consider $y = 5 + 2.8x$

Here $b_{yx} = 2.8$

and $x = 3 - 0.5y \Rightarrow b_{xy} = -0.5$

Now $r^2 = -1.4$

$\Rightarrow$ $r =$ imaginary

its not feasible, r is imaginary.

$\therefore$ byx and bxy are not feasible.

Q.1. (d) Write the dual of following primal problem

$$\textbf{Max } Z = 3x_1 + 10x_2 + 2x_3$$

Subject to;

$$2x_1 + 3x_2 + 2x_3 \le 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$

Where $x_1, x_2, x_3 \ge 0.$

Prove that dual of the dual is primal.

Ans. To make it a standard linear form.

Introduce slack variable. S_1

standard form of LPP is

Max $z = 3x_1 + 10x_2 + 2x_3 + 0.S_1$ subject to constraints

$$2x_1 + 3x_2 + 2x_3 + S_1 = 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1, x_2, x_3, S_1 \ge 0$$

dual is

Min $Z^* = 7w_1 + 3w_2$. subject to constraints

$$2w_1 + 3w_2 \ge 3$$
$$3w_1 - 2w_2 \ge 10$$
$$2w_1 + 4w_2 \ge 2$$

$w_1 \ge 0, w_2$ unrestricted

w_1, w_2 are dual variables.

Introduce $S_1 \ge 0$ and $w_2 = w'_2 - w_2''$. $S_2 \ge 0, S_3 \ge 0$

standard form

Min $Z^* = 7w_1 + 3(w'_2 - w_2'') + 0.S_1 + 0.S_2 + 0.S_3$

s.t. $2w_1 + 3(w'_2 - w_2'') - S_1 = 3$

$3w_1 - 2(w'_2 - w_2'') - S_2 = 10$

$2w_1 + 4(w_2' - w_2'') - S_3 = 2$

$w_1, w_2', w_2'', S_1, S_2, S_3 \ge 0$

Dual of dual is

Max $Z^{**} = 3$...

s.t. $2x_1 + 3x_2 + 2x_3 \le 7$

$$\left.\begin{aligned} 3x_1 - 2x_2 + 4w_3 &\le 3. \\ -3x_1 + 2x_2 - 4w_3 &\le -3 \end{aligned}\right\}$$

$$-x_1 \le 0.$$
$$-x_2 \le 0.$$
$$-x_3 \le 0$$

Primal is

Max $Z = 3x_1 + 10x_2 + 2x_3$

s.t. $2x_1 + 3x_2 + 2x_3 \le 7$

$3x_1 - 2x_2 + 4x_3 = 3.$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$

Thus dual of dual is primal.

UNIT-I

Q.2. (a) Find the general solution of

$(D^3 - 4D^2 D' + 4DD'^2)\, z = \cos(2x + 3y)$ **(6)**

Ans. Consider $D^3 - 4D^2 D' + 4DD'^2 = 0$

A.E. $m^3 - 4m^2 + 4m = 0$

$\Rightarrow$ $m(m^2 - 4m + 4) = 0$

$\Rightarrow$ $m = 0, (m-2)^2 = 0$

$\Rightarrow$ $m = 0, 2, 2$

$$\text{C.F} = f_1(y) + f_2(y + 2x) + xf_3(y + 2x)$$

$$\text{P.I.} = \frac{1}{D^3 - 4D^2D' + 4DD'^2} \cos(2x + 3y)$$

Replace D by 2 and D' by 3

$$= \frac{1}{2^3 - 4\times4\times3 + 4\times2\times9} \iiint \cos u\, du\, du\, du$$

where $u = 2x + 3y$

$$= \frac{1}{8 - 48 + 72}(-\sin u) = \frac{-\sin(2x+3y)}{32}$$

General solution is

$$Z = f_1(y) + f_2(y + 2x) + xf_3(y + 2x) - \frac{\sin(2x+3y)}{32}$$

Q.2. (b) Find the complete solution of the equation

$(D^2 + D'^2 + 2DD' + 2D + 2D' + 1)\, z = e^{2x+y}$ **(6.5)**

Ans. Consider. $(D^2 + D'^2 + 2DD' + 2D + 2D' + 1) z.$

$\Rightarrow$ $[(D + D')^2 + 2(D + D') + 1] z$

A.E. $(D + D')^2 + 2(D + D') + 1 = 0$

$\Rightarrow$ $(D + D' + 1)^2 = 0.$

Here $b = 1, a = -1, c = -1$

$$C.F. = e^{-x}[\Phi_1(y-x) + x\Phi_2(y-x)]$$

$$P.I. = \frac{1}{(D+D'+1)^2}e^{2x+y}$$

Replace D by 2, D' by 1

$$\Rightarrow \quad \frac{1}{(2+1+1)^2}e^{2x+y} = \frac{1}{16}e^{2x+y}.$$

Complete solution is

$$z = e^{-x}(\Phi_1(y-x) + x\Phi_2(y-x)] + \frac{1}{16}e^{2x+y}$$

Q.3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$, where $u(0,y) = 0$ and $\frac{\partial u}{\partial x}(0,y) = e^{-3y}$ for all y using the method of separation of variables.

Ans. Given equation $\frac{\partial^2 u}{\partial x^2} - 2\cdot\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$

Let $U = X(x)Y(y)$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = X'Y, \frac{\partial u}{\partial y} = XY', \frac{\partial^2 u}{\partial x^2} = X''Y.$$

equation becomes,

$$X''Y - 2X'Y + XY' = 0.$$

$$\Rightarrow \quad XY' = Y(2X' - X'')$$

$$\Rightarrow \quad -\frac{Y'}{Y} = \frac{-2X' + X''}{X} = K \quad \left[\begin{array}{l}\text{As } x \text{ and } y \text{ are} \\ \text{independent variables}\end{array}\right.$$

$$\Rightarrow \quad Y' + KY = 0, X'' - 2X' - KX = 0$$

$$\Rightarrow \quad Y' = -KY, \text{put } \frac{d}{dx} = m$$

$$\Rightarrow \quad \frac{dY}{dY} = -KY, \Rightarrow (m^2 - 2m - K)X = 0.$$

$$, \text{ A.E } m^2 - 2m - K = 0$$

$$\Rightarrow \quad \frac{dY}{Y} = -K\,dy \Rightarrow m = \frac{2 \pm \sqrt{4+4k}}{2}$$

$$\log Y = -Ky + \log c_1, m = 1 \pm \sqrt{1+K}$$

$$\Rightarrow \quad Y = c_1e^{-Ky}, X = c_2e^{(1+\sqrt{1+K})x} + c_3e^{(\sqrt{1+K})}$$

$$\Rightarrow \quad u(x,y) = c_1e^{-Ky}\left(c_2e^{1+\sqrt{1+K}x} + c_3e^{(1-\sqrt{1+K})x}\right)$$

$$\Rightarrow \quad u(x,y) = \left(Ae^{1+\sqrt{1+kx}} + Be^{1-\sqrt{1+kx}}\right)e^{-ky} \quad ...(1)$$

for $x = 0., y = y$, we get

$$0 = (A+B)e^{-Ky}$$

$$\Rightarrow \quad A + B = 0 \quad (2)$$

Now by (1), we get

$$\frac{\partial u}{\partial x} = e^{-Ky}\left[(1+\sqrt{1+K})Ae^{1+\sqrt{1+Kx}} + (1-\sqrt{1+K})Be^{(1-\sqrt{1+K})x}\right]$$

for $x = 0, y = y$

$$e^{-3y} = [1+\sqrt{1+K}A + 1 - \sqrt{1+K}B]e^{-Ky}$$

On comparing

$$e^{-Ky} = e^{-3y}$$

$$\Rightarrow \quad K = 3$$

Now, $3A - B = 1$

and $A + B = 0$

$$\Rightarrow \quad A = -B$$

$$\Rightarrow \quad -3B - B = 1$$

$$\Rightarrow \quad B = -1/4$$

$$\Rightarrow \quad A = 1/4.$$

Now

$$u = \frac{1}{4}[e^{3x} - e^{-x}]e^{-3y}.$$

Q.3. (b) A long rectangular plate of width π cm with insulated surfaces has its temperature equal to zero on both the long sides and one of the short side so that $u(0, y) = 0$, $u(\pi, y) = 0$, $u(x, \infty) = 0$ and $u(x, 0) = kx$. find the steady state temperature within the plate. (6.5)

Ans.

Consider the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad ...(1)$$

$$\left.\begin{array}{l} u(0,y) = 0, 0 < y < \infty \\ u(\pi,y) = 0, 0 < y < \infty \end{array}\right\} B.C.$$

$$u(x,\infty) = 0, 0 < x < \pi$$

$$u(x,\infty) = kx, 0 < x < \pi$$

General solutions of (1) are

$$u(x,y) = (c_1e^{px} + c_2e^{-px})(c_3\cos py + c_4\sin py) \quad ...(2)$$

$$u(x,y) = (c_1\cos px + c_2\sin px)(c_3e^{py} + c_4e^{-py}) \quad ...(3)$$

$$u(x,y) = (c_1x + c_2)(c_3y + c_4) - (4)$$

As solution (2) does not satisfy the boundary conditions for all values of y. Also (4) oes not satisfy.

Thus only possible solution is (3), *i.e.* of the form

$$u(x,y) = (A\cos px + B\sin px)(Ce^{py} + De^{-py})$$

for $u(0,y) = 0$

$\Rightarrow$ $0 = A[Ce^{py} + De^{-py}]$

$\Rightarrow$ $A = 0$

(5) reduces to

$$u(x,y) = B\sin px\,[Ce^{py} + De^{-py}]$$

$$u(x,y) = \sin px\,[C'e^{py} + D'e^{-py}]$$

for $u(\pi,y) = 0$

$$\sin p\pi\,(C'e^{py} + D'e^{-py}) = 0.$$

$\Rightarrow$ $\sin p\pi = 0$

$\Rightarrow$ $p\pi = n\pi \Rightarrow p = n$, n being on integer

and $u(x,\infty) = 0.$

$\Rightarrow$ $c' = 0$

$\therefore$ Reduced solution (6) is,

$$u(x,y) = D'\sin nx\, e^{-ny},\ n \text{ being an integer.}$$

Adding all solution, we get

$$u = \sum D_n \sin nx e^{-ny}$$

Given $u(x,0) = kx.$

$$kx = \sum D_n \sin nx$$

This is half range sine series in interval $(0, \pi)$.

$$D_n = \frac{2}{\pi}\int_0^{\pi} kx \sin nx\, dx$$

$$= \frac{2k}{\pi}\left[\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2k}{\pi}\left[-\frac{\pi}{n}\cos n\pi\right] = \frac{-2k}{n}(-1)^n$$

$$= \frac{2k}{n}(-1)^{n+1}$$

Thus (7) reduces to

$$u(x,y) = 2k\sum \frac{(-1)^{n+1}}{n}\sin nx e^{-ny}.$$

UNIT-II

Q.4. (*a*) In a bolt factory there are four machines *A, B, C* and *D* manufacturing 20%, 15%, 25% and 40% of the total output respectively. of their outputs 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen randomly from the factory's production and is found defective What is the probability that the bolt was manufactured by machine *A* or *D*

Ans. Let

B_1 : bolt manufactured by machine A

B_2 : bolt manufactured by machine B

B_3 : bolt manufactured by machine C

B_4 : bolt manufactured by machine D.

E : bolt is defective.

Then

$P(B_1) = 0.20$, $P(E/B_1) = 0.05$

$P(B_2) = 0.15$, $P(E/B_2) = 0.04$

$P(B_3) = 0.25$, $P(E/B_3) = 0.03$

$P(B_4) = 0.4$, $P(E/B_4) = 0.02$

To find $P(B_1/E)$ and $P(B_4/E)$

By Baye's theorem

$$P(B_1/E) = \frac{P(B_1)P(E/B_1)}{\sum P(B_i)P(E/B_i)}$$

$$= \frac{0.20\times0.05}{0.20\times0.05+0.15\times0.04+0.25\times0.03+0.4\times0.02}$$

$$= \frac{0.01}{0.01+0.006+0.0075+0.008}$$

$$= \frac{0.01}{0.0315} = 0.3174$$

$$P(B_4/E) = \frac{P(B_4)P(E/B_4)}{\sum P(B_i)P(E/B_i)}$$

$$= \frac{0.4\times0.02}{0.20\times0.05+0.15\times0.04+0.25\times0.03+0.4\times0.02}$$

$$= \frac{0.008}{0.0315} = 0.253$$

Q.4. (*b*) Calculate the first four moments for the following frequency tribution about the mean and explain the skewness and kurtosis of the quency distribution (6.5)

X	:	−4	−3	−2	−1	0	1	2	3	4
f	:	3	4	5	7	12	7	5	4	3

Ans. Let $\mu_1, \mu_2, \mu_3, \mu_4$, are first four moments about the mean. Then by def.

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu'_1$$

$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu_1'^3$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4$$

Where, $\mu_1', \mu_2', \mu_3', \mu_4'$, are first four moments about any point. Consider the table.

Let $A = 0$.

x_i	fi	$fixi$	$fixi^2$	$fixi^3$	$fixi^4$
–4	3	–12	48	–192	768
–3	4	–12	36	–108	324
–2	5	–10	20	–40	80
–1	7	–7	7	–7	7
0	12	0	0	0	0
1	7	7	7	7	7
2	5	10	20	40	80
3	4	12	36	108	324
4	3	12	48	192	768

$$N_1 = \sum fi = 50,\ \sum fixi = 0,\ \sum fixi^2 = 222,$$

$$\sum fi\, xi^3 = 0,\ \sum fixi^4 = 2358,$$

Now,

$$\mu_1' = \frac{1}{N}\sum fi\, xi = \frac{1}{9}\times 0 = 0$$

$$\mu_2' = \frac{1}{N}\sum fi\, xi^2 = \frac{1}{50}\times 222 = \frac{222}{50}$$

$$\mu_3' = \frac{1}{N}\int fixi^3 = \frac{1}{50}\times 0 = 0$$

$$\mu_4' = \frac{1}{N}\sum fi\, xi^4 = \frac{1}{50}\times 2358 = 262$$

Thus, we get

$$\mu_1 = 0$$

$$\mu_2 = \frac{222}{50} - 0 = 4$$

$$\mu_3 = 0 - 3\times\frac{222}{50}\times 0 + 2\times 0 = 0$$

$$\mu_4 = 262 - 4\times 0\times 0 + 6\times\frac{222}{50}\times 0 - 3\times 0$$

$$= 262.$$

Now

$$r_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{0}{\sqrt{4^3}} = 0$$

$$r_2 = \beta_2 - 3 = \frac{\mu 4}{\mu_2^2} - 3 = \frac{262}{4^2} - 3$$

$$= \frac{214}{16} = 13.3$$

$\sigma_1 = 0$, **distribution is symmetrical**

Q.5. (*a*) Find mean, variance and moment generating function of $f(x)$, where

$$f(x) = \begin{cases} ae^{-ax}, x > 0 \\ 0, x \le 0 \end{cases}$$ **(6)**

Ans. Given

$$f(x) = \begin{cases} ae^{-ax}, x > 0 \\ 0, x \le 0 \end{cases}$$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)dx = \int_0^{\infty} xae^{-ax}dx$$

$$= a\left[x.\frac{e^{-ax}}{-a}\Big]_0^{\infty} - \int_0^{\infty} -ae^{-ax}dx\right]$$

$$= a\left[\frac{1}{a}\frac{e^{-ax}}{-a}\Big|_0^{\infty}\right] = \frac{-1}{a}(0-1) = \frac{1}{a}$$

$$\text{var}\quad X = \int_{-\infty}^{\infty}(x-\mu)^2 fX(x)dx$$

$$= \int_{-\infty}^{\infty} x^2 f_X(x)dx - [E(X)]^2$$

$$= \int_0^{\infty} x^2 e^{-ax}dx - \frac{1}{a^2}$$

$$= a\left[x^2\frac{e^{-ax}}{-a}\Big|_0^{\infty} - \int_0^{\infty}\frac{2xe^{-ax}}{-a}dx\right] - \frac{1}{a^2}$$

$$= a\left[\frac{2}{a}\int_0^{\infty} xe^{-ax}dx\right] - \frac{1}{a^2}$$

$$= a\left[\frac{2}{a}\left|x\frac{e^{-ax}}{-a} - \frac{e^{-ax}}{a^2}\right|_0^{\infty}\right] - \frac{1}{a^2}$$

$$= a\left[\frac{2}{a}\left(\frac{1}{a^2}\right)\right] - \frac{1}{a^2} = \frac{2}{a^2} - \frac{1}{a^2} = \frac{1}{a^2}$$

M.G.F

$$= \int_{-\infty}^{\infty} e^{tx} f_X(x)dx$$

$$= \int_0^{\infty} e^{tx}ae^{-ax}dx = a\int_0^{\infty} e^{(t-a)x}dx$$

$$= a\left|\frac{e^{(t-a)x}}{t-a}\right|_0^{\infty}$$

$$= a\left(0-\frac{1}{t-a}\right) = \frac{a}{a-t}.$$

Q.5. (*b*) If the probability that an individual suffers to a bad reaction an injection of a given serum is 0.001, determine the probability that out of individuals

(*i*) exactly 3 (*ii*) more than 2

individual will suffer to a bad reaction

Ans. Let '*p*' be the probability of success suffering from bad reaction = 0.001

As $n = 2000$ and X be random variable. n is very large and p is small

$\therefore$ $$X \equiv p(\lambda)$$

Here $$\lambda = np = 2000 \times 0.001 = 2$$

Now $$f_X(x) = \frac{e^{-2}(2)^x}{x!}$$

(*i*) $$P[X=3] = \frac{e^{-2}2^3}{3!} = \frac{0.1353\times 8}{6}$$

$$= 0.1804$$

(*ii*) $$P[X>2] = 1 - P[X \le 2]$$

$$= 1 - [f_X(0) + f_X(1) + f_X(2)]$$

$$= 1 - \left[e^{-2} + e^{-2}.2 + \frac{e^{-2}2^2}{2!}\right]$$

$$= 1 - e^{-2}[1+2+2] = 1 - 5e^{-2}$$

$$= 1 - 0.6765 = 0.3235$$

UNIT-III

Q.6. (*a*) The two regression equation of the variable x and y are $8x - 10y +$ and $40x - 18y = 214$ given that variance of $x = 9$. Find

(*i*) Mean of x and y

(*ii*) The standard deviation of y and

(*iii*) The coefficient of correlation between x and y.

Ans. Consider $8x - 10y + 66 = 0$., $40x - 18y - 214 = 0$

(*i*) As mean values of given series, satisfy the regression lines

$$8\bar{x} - 10\bar{y} + 66 = 0$$

$$40\bar{x} - 18\bar{y} - 214 = 0$$

$$4\bar{x} - 5\bar{y} + 33 = 0 \quad ...(1)$$

$$20\bar{x} - 9\bar{y} - 107 = 0 \quad ...(2)$$

Multiply (1) by 5 and subtract

$$\begin{array}{r} 20\bar{x} - 25\bar{y} + 165 = 0 \\ 20\bar{x} - 9\bar{y} - 107 = 0 \\ \hline -16\bar{y} + 272 = 0 \end{array}$$

$\Rightarrow$ $$16\bar{y} = 272 = 0 \quad \Rightarrow \bar{y} = 17$$

Now. using (1) $4\bar{x} - 85 + 33 = 0$

$\Rightarrow$ $$4\bar{x} = 52$$

$\Rightarrow$ $$\bar{x} = 13$$

(*iii*) Let $10y = 8x + 66$ be regression equation of y on x.

$\Rightarrow$ $$y = \frac{4}{5}x + \frac{33}{5}$$

$\therefore$ $$byx = 4/5$$

and let $40x = 18y + 214$ be regression equation of x and y.

$\Rightarrow$ $$x = \frac{9}{20}y + \frac{107}{20}.$$

$\therefore$ $$bxy = 9/20$$

As $$r^2 = bxy : byx$$

$\Rightarrow$ $$r^2 = \frac{9}{20}\times\frac{4}{5} = \frac{9}{25} = 0.36$$

$\Rightarrow$ $$r = \pm 0.6$$

As r have same sign as regression coefficients.

$\therefore$ $$r = 0.6$$

(ii) As given $$\sigma_x^2 = 9.$$

$\Rightarrow$ $$\sigma_x = 3.$$

Now $$bxy = \frac{x\sigma_x}{\sigma_y}$$

$\Rightarrow$ $$\frac{9}{20} = \frac{0.6\times 3}{\sigma_y} \Rightarrow \sigma_y = \frac{0.6\times 3\times 20}{9}$$

$\Rightarrow$ $$\sigma_y = 4.$$

Q. 6. (b) The results of measurement of electric resistance R of a copper ba[r] at various temperature t°C are listed below: (6.5)

t:	19	25	30	36	40	45	50
R:	76	77	79	80	82	83	85

if $R = a + bt$, find a and b.

Ans. $R = a + bt$

Normal equations are

$$\sum R = ma + b\sum t$$

$$\sum Rt = a\sum t + b\sum t^2$$

t	R	Rt	t^2
19	76	1444	361
25	77	1925	625
30	79	2370	900
36	80	2880	1296, m = 7
40	82	3280	1600
45	83	3735	2025
50	85	4250	2500

$\sum t = 245, \sum R = 562, \sum Rt = 19884, \sum t^2 = 9307$

substituting values in (1) and (2)

$$562 = 7a + 245b$$
$$19884 = 245a + 9307b$$

Multiply (3) by 35 and subtract

$$19670 = 245a + 8575b$$
$$19884 = 245a + 9307b$$
$$-214 = -732b$$
$$b = 0.2923.$$

By equation (3), we get

$$562 = 7a + 245 \times 0.2923$$
$$\Rightarrow \quad 7a = 490.3865$$
$$\Rightarrow \quad a = 70.05$$

Q.7. (a) Write at least three important properties of regression coeffic[ients] and prove that if two variables are uncorrelated then the regression lines [are] perpendicular to each other

Ans. Three important properties of regression coefficient are:

(1) Correlation coefficient is the geometric mean between the regression coeffici[ents]

$$r^2 = bxy.\,byx$$

(2) Correlation coefficient and both regression coefficients have same sign.

(3) Arithmatic mean of regression coefficient is greater than the correlation coeff[icient]

i.e. $\dfrac{bxy + byx}{2} > r.$

Since two variables are uncorrelated then $r = 0$.

Equation to the lines of regression of y on x and x on y, are

$$y - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x - \bar{x}) \text{ and } x - \bar{x} = \frac{r\sigma_x}{\sigma_y}(y - \bar{y})$$

Their slopes are $m_1 = \dfrac{r\sigma_y}{\sigma_x}$ and $m_2 = \dfrac{\sigma_y}{r\sigma_x}$

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| = \left|\frac{\dfrac{\sigma_y}{r\sigma_x} - \dfrac{x\sigma_y}{\sigma_x}}{1 + \dfrac{\sigma_y^2}{\sigma_x^2}}\right|$$

$$= \left|\frac{1-r^2}{r}\cdot\frac{\sigma_y}{\sigma_x}\cdot\frac{\sigma_x^2}{\sigma_y^2 + \sigma_x^2}\right| = \frac{1-r^2}{|r|}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Since $r^2 < 1$ and σ_x, σ_y are positive

$$\Rightarrow \quad \tan\theta = \frac{1-r^2}{|r|}\cdot\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2} \text{ given } r = 0 \Rightarrow \theta = \frac{\pi}{2}$$

$\therefore$ Two lines of regression are perpendicular to each other.

Q.7. (b) A sample of 10 boxes of chips is drawn in which the mean weight is 490 gm and standard deviation of weight is 9 gm. Can the sample be considered to be taken from a population having mean weight 500 gm where $t_{0.05} = 2.26$? (6.5)

Ans. Given $\quad n = 10, \bar{X} = 490, S = 9\text{gm}, \mu = 500$

$$\therefore \quad s = \sqrt{\frac{n}{n-1}S^2} = \sqrt{\frac{10}{9}\times 9^2}$$
$$= 9.486$$

Null Hypothesis H_0: The difference is not significant

i.e. $\mu = 500$

Alternative Hypothesis $H_1 : \mu \neq 500$. (Two tailed test)

Under $\quad H_0 t = \dfrac{\bar{X} - \mu}{S/\sqrt{n}} = \dfrac{490 - 500}{9.486/\sqrt{10}}$

$$= -0.333$$
$$\Rightarrow \quad |t| = 0.333$$

Also $t_{0.05} = 2.26$. for 9 d.f.

Conclusion: Since $\quad |t| = 0.333 > t_{0.05}$

$\therefore$ The hypothesis H_0 is rejected.

Thus, the sample could not have come from the population having mean 500 gm.

UNIT-IV

Q.8. (a) Write the dual of the following problem (6)

$$\text{Min } z = 2x_1 + 3x_2 + 4x_3$$
$$\text{s.t.} = 2x_1 + 3x_2 + 5x_3 = 2$$
$$3x_1 + x_2 + 7x_3 \le 3$$
$$x_1 + 4x_2 + 6x_3 = 5$$

where $x_2, x_3 \ge 0$ and x_1 is unrestricted

Ans. As x_1 is unrestricted

$\therefore \quad x_1 = x_1' - x_1''$

Introducing slack variable $S_1 \ge 0$. the primal probablem is restated as standard primal.

Min $Z \quad = 2(x_1' - x_1'') + 3x_2 + 4x_3 + 0.s_1$

S.t.

$$2(x_1' - x_1'') + 3x_2 + 5x_3 = 2$$
$$3(x_1' - x_1'') + x_2 + 7x_3 + S_1 = 3$$
$$x_1' - x_1'' + 4x_2 + 6x_3 = 5$$
$$x_1' \ge 0, x_1'' \ge 0, x_2 \ge 0, x_3 \ge 0, S_1 \ge 0$$

Let w_1, w_2, w_3 be the dual variables corresponding to the primal constraints. Dual problem is

$$\text{Max } Z^* = 2w_1 + 3w_2 + 5w_3$$

$$\left.\begin{array}{l} \text{S.t. } 2w_1 + 3w_2 + w_3 \ge 2. \\ -2w_1 - 3w_2 - w_3 \ge -2 \end{array}\right\}$$

$$3w_1 + w_2 + 4w_3 \ge 3$$
$$5w_1 + 7w_2 + 6w_3 \ge 4.$$
$$w_2 \ge 0$$

$\Rightarrow$

$$\text{Max } Z^* = 2w_1 + 3w_2 + 5w_3$$

S.t.c

$$2w_1 + 3w_2 + w_3 = 2$$
$$3w_1 + w_2 + 4w_3 \ge 3$$
$$5w_1 + 7w_2 + 6w3 \ge 4$$
$$w_2 \ge 0, w_1, w_3 \text{ unrestricted.}$$

Q.8. (b) Using dual simplex method solve the following LPP. (6.5)

$$\text{Max } Z = -3x_1 - 2x_2$$

S.t.

$$x_1 + x_2 \ge 1$$
$$x_1 + x_2 \le 7$$
$$x_1 + 2x_2 \ge 10$$
$$x_2 \le 3$$

Ans. $x_1, x_2 \ge 0$

$$x_1 + x_2 \le 7$$
$$-x_1 - 2x_2 \le 10$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Introducing slack variables $s_1 \ge 0, S_2 \ge 0, S_3 \ge 0, S_4 \ge 0$

$$\text{Max } z = -3x_1 - 2x_2 + S_1 + S_2 + S_3 + S_4.$$
$$-x_1 - x_2 + S_1 = -1$$
$$x_1 + x_2 + S_2 = 7$$
$$-s_1 - 2x_2 + S_3 = 10$$
$$x_2 + S_4 = 3$$
$$x_1, x_2, S_1, S_2, S_3, S_4 \ge 0$$

Dual simplex table is

Intial table:

			-3	-2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
$\leftarrow 0$	y_3	-1	-1	-1	1	0	0	0
0	y_4	7	1	1	0	1	0	0
0	y_5	10	-1	-2	0	0	1	0
0	y_6	3	0	1	0	0	0	1
	$zj-cj$		3	$\underset{\uparrow}{2}$	0	0	0	0

since all $(z_j - c_j) \ge 0$ and $x_{B1} (= S_1) < 0$.
min $\{-1\} = -1$ will leave the basis i.e y_3.

Since $\quad \max \left\{\frac{3}{-1}, \frac{2}{-1}\right\} = -2$

i.e y_2 enters the basis

			-3	-2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-2	y_2	1	1	1	-1	0	0	0
0	y_4	6	0	0	1	1	0	0
0	y_5	12	1	0	-2	0	1	0
0	y_6	4	-1	0	1	0	0	1
	$zj-cj$		$\underset{\uparrow}{1}$	0	2	0	0	0

since all $z_j - c_j \ge 0$. and $x_{B4} (= S_4) < 0$. *i.e.* -2.
min $\{-2\} = -2$ will leave the basis ie y_6

since max $\left\{\frac{1}{-1}\right\} = -1$

			-3	-2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-2	y_2	3	0	1	0	0	0	-1
0	y_4	6	0	0	1	1	0	0
0	y_5	16	0	0	-1	0	1	-1
← -3	y_1	-4	1	0	-1	0	0	-1
		z_j-c_j	0	0	3	0	0	1

since all $z_j - c_j^3 0$ and $x_{B4} (= -4) < 0$ *ie* y_1 leaves the basis

$$\max \left\{\frac{3}{-1}, \frac{1}{-1}\right\} = -1$$

y_6 enters.

			-3	-2	0	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-2	y_2	7	-1	1	-1	0	0	0
0	y_4	6	0	0	1	1	0	0
0	y_5	12	-1	0	0	0	1	0
0	y_6	4	-1	0	1	0	0	1
			1	0	2	0	0	1

All $\quad z_j - c_j \geq 0$ and $x_B \geq 0$.

$\therefore$ Solution is $x_1 = 0, x_2 = 7$

Q.9. Using VAM method find basic feasible solution of the following transportation problem. Check optimality and hence find the optimal solution. (12.5)

From	A	B	C	D	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	43

Ans.

Here total demand = total supply = 43.

Table for intial b.f. s is

21	16	25	[11] 13	11 (3)
17	18	14	23	13 (3)
32	27	18	41	19 (9)
6 (4)	10 (2)	12 (4)	15 (10)	

Largest of these differences is (10) associated with 4th column of the table.

M... cell (1,4)

17	18	14	[4] 23	13 (3)
32	27	18	41	19 (9)
6 (15)	10 (9)	12 (4)	4 (18)	

Largest difference is (18) in 4th column

Min cost is 23, we allocate

$$x_{24} = \min(13, 4) = 4 \text{ in cell } (2,4)$$

Exhaust 4th column. Reduced table is

[6] 17	18	14	9 (3)
32	27	18	19 (9)
6 (15)	10 (9)	12 (4)	

Largest difference is (15), associated with first column.

Min cost is 17, we allocate

$$x_{21} = \min(9, 6) = 6 \text{ in cell } (2,1). \text{ Exhaust first column}$$

[3] 18	14	3 (4)
27	18	19 (9)
10 (9)	12 (4)	

Largest difference is (9), associated with 2nd column. Min cost is 18, we allocate

$$x_{22} = \min(3, 10) = 3 \text{ on cell } (2,2)$$

Exhaust 2nd row. Reduced table is

[7] 27	[12] 18	19
7	12	

Final table is

	v_1	v_2	v_3	v_4	
u_1	21	16	25	[11] 13	11
u_2	[6] 17	[6] 18	14	[4] 23	13
u_3	32	[7] 27	[12] 18	41	19
	6	10	12	15	

Feasible solution is

$$11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + 12 \times 18$$
$$= 796$$

optimal solution is

$$u_1 + V_4 = 13$$
$$u_2 + V_1 = 17$$
$$u_2 + V_2 = 18$$
$$u_2 + V_4 = 23$$
$$u_3 + V_2 = 27$$
$$u_3 + V_3 = 18$$

Let $u_1 = 0 \Rightarrow V_4 = 13$

Thus $u_2 = 23 - 3 = 10$

Now $V_1 = 7$

and $10 + V_2 = 18 \Rightarrow V_2 = 8.$

and $U_3 + 8 = 27$

$\Rightarrow U_3 = 19$

and $19 + V_3 = 18$

$\Rightarrow V_3 = -1$

Calculating $w_{ij} = (u_i + v_j) - c_{ij}$

cells	w_{ij}
(1,1)	$(u_1 + V_1) - c_{11} = (0 + 7) - 21 = -14$
(1,2)	$(u_1 + V_2) - c_{12} = (0 + 8) - 16 = -8$
(1,3)	$(u_1 + V_3) - c_{13} = (0 - 1) - 25 = -26$
(2,3)	$(u_2 + V_3) - c_{23} = (10 - 1) - 14 = -5$
(3,1)	$(u_3 + V_1) - c_{31} = (19 + 7) - 32 = -6$
(3,4)	$(u_3 + V_4) - c_{34} = (19 + 13) - 41 = -9$

All w_{ij} are negative, thus solution is optimal

$$z = 11 \times 13 + 6 \times 17 + 3 \times 18 + 4 \times 23 + 7 \times 27 + [illegible]$$
$$= 796.$$

MID TERM EXAMINATION [FEB. 2017]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS-IV [ETMA-202]

Time : 1:30 Hrs. **M.M. : 30**

Note: *Q. no. 1 compulsory and attempt any two from remaining questions. All questions carry equal marks.*

Q.1. (a) Find Particular integral of $(4D^2 + 3DD' - D'^2 - D - D')\,z = 3e^{(x+2y)/2}$.

Ans. $$\text{P.I.} = \frac{1}{4D^2 + 3DD' - D'^2 - D - D'}3e^{\left(\frac{x}{2}+y\right)}$$

Replace D by $\frac{1}{2}$, D′ by 1.

$$\Rightarrow \quad \frac{1}{4\times\frac{1}{4}+\frac{3}{2}-1-\frac{1}{2}-1}3e^{\left(\frac{x}{2}+y\right)}$$

Case of failure

$$\Rightarrow \quad x\frac{1}{8D+3D'-1}3e^{\left(\frac{x}{2}+y\right)}$$

Replace D by $\frac{1}{2}$ and D′ by 1

$$\Rightarrow \quad x\frac{1}{4+3-1}3e^{\left(\frac{x}{2}+y\right)}$$

$$\Rightarrow \quad \frac{x}{6}.3e^{\left(\frac{x}{2}+y\right)} = \frac{x}{2}.e^{\left(\frac{x}{2}+y\right)}$$

Thus $$\text{P.I} = \frac{x}{2}.e^{\left(\frac{x}{2}+y\right)}$$

Q.1. (b) Using the method of separation of variable solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$ where $u(x, 0) = e^{-3x} - 2e^{-x}, x > 0, y > 0$

Ans. Given equation is

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u \qquad ...(1)$$

Let $$U = X(x)\,Y(y) = XY$$

$\therefore$ $$\frac{\partial u}{\partial x} = X'Y$$

and $$\frac{\partial u}{\partial y} = XY'$$

Equating (1) becomes.

$$X'Y = 2XY' + XY$$
$$X'Y = X(2Y' + Y)$$

$\Rightarrow$
$$\frac{X'}{X} = \frac{2Y'}{Y} + 1$$

$\Rightarrow$ Since x and y are independent variables $\therefore$ it is true only when each equation is equal to a constant.

Let
$$\frac{X'}{X} = \frac{2Y'}{Y} + 1 = a \text{ (say)}$$

Now
$$\frac{X'}{X} = a.$$

$\Rightarrow$
$$\frac{dX}{dx} \cdot \frac{1}{X} = a$$

$\Rightarrow$
$$\frac{dX}{X} = a\,dx$$

$\Rightarrow$
$$\log X = ax + \log c_1$$

$\Rightarrow$
$$X = C_1 e^{ax}$$

and
$$\frac{2Y'}{Y} + 1 = a$$

$\Rightarrow$
$$2\frac{dY}{dy} \cdot \frac{1}{Y} + 1 = a$$

$\Rightarrow$
$$2\frac{dY}{Y} = (a-1)dy$$

$$\frac{dY}{Y} = \left(\frac{a-1}{2}\right)dy$$

$$\log Y = \left(\frac{a-1}{2}\right)y + \log C_2$$

$$Y = C_2 e^{\left(\frac{a-1}{2}\right)y}$$

$$U = C_1 C_2 e^{ax} e^{\left(\frac{a-1}{2}\right)y}$$

As given
$$u(x, 0) = e^{-3x} - 2e^{-x}$$

$\Rightarrow$
$$e^{-3x} - 2e^{-x} = C_1 C_2 e^{ax}$$

Comparing two, we get

$$e^{-3x} = C_1 C_2 e^{ax}$$

Here
$$C_1 C_2 = 1, a = -3$$

and
$$2e^{-x} = C_1 C_2 e^{ax}$$

$\Rightarrow$
$$C_1 C_2 = 2, a = -1$$

Thus eq^n (2) becomes

$$U(x, y) = e^{-3x} e^{-2y} - 2e^{-x} e^{-y}$$

$\Rightarrow$
$$U = e^{-3x-2y} - 2e^{-x-y}$$

Q.1. (c) Find moment generating function of the following random distribution

x	-1	1
$P(x)$	1/2	1/2

Ans.
$$\text{M.G.F} = E[e^{tx}]$$
$$= \sum e^{tx} \cdot f(x)$$
$$= \frac{e^{-t}}{2} + \frac{e^{t}}{2}$$
$$= \frac{e^{-t} + e^{t}}{2} = \cosh t.$$

Q.1. (d) A can hit a target 3 times in 5 shots, B can hit a target 2 times in 5 shots and C can hit a target 3 times in 4 shots. All of them fire one shot each simultaneously at the target. What is the probability that two shots hit a target.

Ans. Let

$$P(A) = \text{Probability of A hitting target} = \frac{3}{5}$$
$$P(B) = \text{Probability of B hitting target} = \frac{2}{5}$$
$$P(C) = \text{Probability of C hitting target} = \frac{3}{4}$$

Probability two shots hit target, we have

(1) A, B hit target and C misses it

$$= \frac{3}{5} \times \frac{2}{5} \times \left(1 - \frac{3}{4}\right)$$
$$= \frac{6}{25} \times \frac{1}{4} = \frac{6}{100}$$

(2) A misses it and B, C hit target

$$= \left(1 - \frac{3}{5}\right) \times \frac{2}{5} \times \frac{3}{4}$$
$$= \frac{2}{5} \times \frac{2}{5} \times \frac{3}{4} = \frac{3}{25}$$

(3) B misses it and A, C hit target

$$= \left(1 - \frac{2}{5}\right) \times \frac{3}{5} \times \frac{3}{4}$$
$$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{4} = \frac{27}{100}$$

Since these are mutually exclusive events

$\therefore$
$$\text{Required probability} = \frac{6}{100} + \frac{3}{25} + \frac{27}{100}$$
$$= \frac{45}{100} = \frac{9}{20}$$

Q.2. (a) Find Complete solution of $(D^2 - 2DD' + D'^2)z = \cos(2x + y)$ (5)

Ans.

$$(D^2 - 2DD' + D'^2)z = \cos(2x+y)$$

A.E. $D^2 - 2DD' + D'^2 = 0$

$$m^2 - 2m + 1 = 0$$

$\Rightarrow$ $m = 1, 1$

$$\text{C.F.} = f_1(y+x) + xf_2(y+x)$$

$$\text{P.I.} = \frac{1}{D^2 - 2DD' + D'^2}\cos(2x+y)$$

Here $a = 2, b = 1$

and let $u = 2x + y$

$$\text{P.I.} = \frac{1}{2^2 - 2\times 2 + 1}\iint \cos u\, du\, du$$

$$= \int \sin u\, du = -\cos u$$

$$= -\cos(2x+y)$$

$\therefore$ Complete solution is

$$Z = f_1(y+x) + xf_2(y+x) - \cos(2x+y)$$

Q.2. (b) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{3\partial u}{\partial x} + \frac{3\partial u}{\partial y} = xy$ (5)

Ans. $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{3\partial u}{\partial x} + \frac{3\partial u}{\partial y} = xy$

$$(D^2 - D'^2 - 3D + 3D')u = xy$$

$\Rightarrow$ $[(D-D')(D+D') - 3(D-D')]u = xy$

$\Rightarrow$ $(D-D')(D+D'-3)u = xy$

For $(D-D'), b = 1, a = 1, c = 0$

For $(D+D'-3)\, b = 1, a = -1, c = 3.$

Now $\text{C.F.} = \phi_1(y+x) + e^{3x}\phi_2(y-x)$

$$\text{P.I.} = \frac{-1}{(D-D')(D+D'-3)}xy$$

$$= \frac{-1}{3\left[1-\left(\frac{D+D'}{3}\right)D\left(1-\frac{D'}{D}\right)\right]}xy$$

$$= \frac{-1}{3D}\left[1-\left(\frac{D+D'}{3}\right)\right]^{-1}\left(1-\frac{D'}{D}\right)^{-1}xy$$

$$= \frac{-1}{3D}\left[1+\frac{D+D'}{3}+\ldots\right]\left[1+\frac{D'}{D}+\ldots\right]xy$$

$$= \frac{-1}{3D}\left[1+\frac{D'}{D}+\frac{D}{3}+\frac{D'}{3}+\frac{D'}{3}+\ldots\right]xy$$

$$= \frac{-1}{3D}\left[xy+\frac{x}{D}+\frac{y}{3}+\frac{x}{3}+\frac{x}{3}\right]$$

$$= \frac{-1}{3D}\left[xy+\frac{x^2}{2}+\frac{y}{3}+\frac{2x}{3}\right]$$

$$= \frac{-1}{3}\left[\frac{1}{D}xy+\frac{1}{D}\frac{x^2}{2}+\frac{1}{D}\frac{y}{3}+\frac{1}{D}\frac{2x}{3}\right]$$

$$= \frac{-1}{3}\left[\frac{x^2y}{2}+\frac{x^3}{6}+\frac{xy}{3}+\frac{x^2}{3}\right]$$

$\therefore$ Complete Solution is

$$u = \phi_1(y+x) + e^{3x}\phi_2(y-x) - \frac{1}{3}\left[\frac{x^2y}{2}+\frac{x^3}{6}+\frac{xy}{3}+\frac{x^2}{3}\right]$$

Q.3. (a) A die is tossed twice getting '5' or '6' on a toss is taken as success. Find the probability distribution, the mean and the variance of the number of successes. (6)

Ans. Let p be the probability of success i.e. getting 5 or 6.

$\therefore$ $n = 2$

i.e. $p = \frac{1}{6}, q = 1 - \frac{1}{6} = \frac{5}{6}$

Let X be Binomial variable

$\therefore$ $f_x(x) = P[X = x] = {}^nC_x p^x q^{n-x}$

$$= {}^2C_x\left(\frac{1}{6}\right)^x\left(\frac{5}{6}\right)^{2-x}$$

Probability of obtaining success is given by,

$P[X=0] + P[X=1] + P[X=2]$

$$= {}^2C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^2 + {}^2C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) + {}^2C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^0$$

$$= \frac{2!}{2!}\times\frac{25}{36}+\frac{2!}{1!}\times\frac{5}{36}+\frac{2!}{2!}\times\frac{1}{36}$$

$$= \frac{25}{36}+\frac{10}{36}+\frac{1}{36}$$

$$= \frac{36}{36} = 1$$

Mean of Binomial variable X = np

$$= 2\times\frac{1}{6}\times\frac{1}{3}$$

Variance of Binomial variable X = npq

$$= 2\times\frac{1}{6}\times\frac{5}{6}$$

$$= \frac{5}{18}.$$

Q.3. (b) The first four moments of a distribution about the value 4 of the variable are –1.5, 17, –30 and 108. Find the moments about mean β_1 and β_2

Ans. Let $\mu'_1, \mu'_2, \mu'_3, \mu'_4$ be the first four moments of the given distribution, then $A = 4, \mu'_1 = -1.5, \mu'_2 = 17, \mu'_3 = -30, \mu'_4 = 108$

Let $\mu_1, \mu_2, \mu_3, \mu_4$ be the first four moments about the mean, then by def.

$$\mu_1 = 0$$
$$\mu_2 = \mu'_2 - \mu'^2_1 = 17 - (-1.5)^2 = 17 - 2.25 = 14.75$$
$$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1 = -30 - 3\times 17\times(-1.5) + 2\times(-1.5)^3$$
$$= -30 + 76.5 - 6.75 = 39.75$$

Also
$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1 = 108 - 4\times(-30)\times(-1.5) + 6\times 17\times(-1.5)^2 - 3\times(-1.5)^4$$
$$= 108 - 180 + 229.5 - 15.1875 = 142.3125$$

Lastly
$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(39.75)^2}{(14.75)^3} = 0.4924$$
$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{142.3125}{(14.75)^2} = 0.6541$$

Q.4. Solve the boundary value problem $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$, given that $y(0, t) = 0$, $y(5, t) = 0, y(x, 0) = 0$ and $\left(\frac{\partial y}{\partial t}\right)_{t=0} = \sin \pi x$ (10)

Ans. Wave equation is $\frac{\partial^2 y}{\partial t^2} = \frac{4\partial^2 y}{\partial x^2}$ ...(1)

Let $y = X(x)\,T(t) = XT$ ...(2)

be the solution of (1)

Then $\frac{\partial^2 y}{\partial t^2} = XT''$ and $\frac{\partial^2 y}{\partial x^2} = X''T$

By equation (1), we have

$$XT'' = 4X''T$$
$$\Rightarrow \frac{X''}{X} = \frac{1}{4}\frac{T''}{T}$$

Since LHS of (3) is a function of x only and RHS is of t only. As x and t are independent variables. Thus both sides reduce to a constant say $-a^2$.

$\therefore$ (3) reduces to

$$\frac{X''}{X} = -a^2 \text{ and } \frac{1}{4}\frac{T''}{T} = -a^2$$
$$\Rightarrow X'' + a^2X = 0$$
$$\Rightarrow (D^2 + a^2)X = 0 \Rightarrow D = \pm ia$$
$$\therefore X = C_1 \cos ax + C_2 \sin ax$$

and $T'' + a^2 4T = 0$
$\Rightarrow (D^2 + 4a^2)T = 0$
$\Rightarrow D = \pm i2a$
$\therefore T = C_3 \cos 2at + C_4 \sin 2at$

Solution is

$$y = (C_1 \cos ax + C_2 \sin ax)(C_3 \cos 2at + C_4 \sin 2at) \quad ...(4)$$

Now, applying boundary conditions in (4)

$$y(0, t) = 0, y(5, t) = 0$$
$$y(0, t) = C_1(C_3 \cos 2at + C_4 \sin 2at)$$
$$0 = C_1(C_3 \cos 2at + C_4 \sin 2at)$$
$$\Rightarrow C_1 = 0$$

$\therefore$ (4) reduces to

$$y(x, t) = C_2 \sin ax(C_3 \cos 2at + C_4 \sin 2at) \quad ...(5)$$

Apply $y(5, t) = 0$ in equation (5), we get

$$0 = C_2 \sin 5a\,(C_3 \cos 2at + C_4 \sin 2at)$$

This is satisfied when

$$\sin 5a = 0$$
$$\Rightarrow 5a = n\pi$$
$$\Rightarrow a = \frac{n\pi}{5} \text{ where } n = 1, 2, \ldots\ldots$$

Solution of wave equation reduces to

$$y(x, t) = C_2\left(C_3 \cos_2 \frac{n\pi t}{5} + C_4 \sin\frac{2n\pi}{5}t\right)\sin\frac{n\pi x}{5}$$
$$\Rightarrow y(x, t) = \left(a_n \cos\frac{2n\pi t}{5} + b_n \sin\frac{2n\pi}{5}t\right)\sin\frac{n\pi x}{5}$$

where $a_n = C_2C_3$ and $b_n = C_2C_4$

Adding solution for different values of n, we get

$$y(x, t) = \sum_{n=1}^{\infty}\left(a_n \cos\frac{2n\pi}{5}t + b_n \sin\frac{2n\pi}{5}t\right)\sin\frac{n\pi x}{5} \quad ...(6)$$

Now, applying initial conditions on (6),

$$y(x, 0) = 0 \text{ and } \left(\frac{\partial y}{\partial t}\right)_{t=0} = \sin \pi x$$

$\therefore$ by (6), we get

$$(y, 0) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{5}$$
$$\Rightarrow 0 = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{5}$$
$$\Rightarrow a_n = 0$$

Thus, solution reduces to

$$y(x, t) = \sum_{n=1}^{\infty} b_n \sin\frac{2n\pi}{5}t \sin\frac{n\pi x}{5} \quad ...(7)$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = \sum_{n=1}^{\infty} b_n \cos\frac{2n\pi}{5} t . \frac{2n\pi}{5} \sin\frac{n\pi x}{5}$$

As given

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = \sin \pi x$$

$$\Rightarrow \quad \sin \pi x = \frac{2\pi}{5}\sum_{n=1}^{\infty} nbn \sin\frac{n\pi x}{5}$$

This represents fourier sine series for $\sin \pi x$

$$\Rightarrow \quad \frac{2\pi}{5} nbn = \frac{2}{5}\int_0^5 \sin \pi x \sin\frac{n\pi x}{5}\, dx$$

We solve for

$$n = 1.$$

$$\frac{2\pi}{5} b_1 = \frac{2}{5}\int_0^5 \sin \pi x \sin\frac{\pi x}{5}\, dx \Rightarrow b_1 = \frac{1}{2\pi}$$

$\therefore$ Complete Solution is

$$y = \frac{1}{2\pi}\sin\frac{\pi x}{5}\sin\frac{2\pi t}{5}$$

OR

Q.4. In the normal distribution 7% of the items are under 35 and 89% are 63. Determine the mean and variance of the distribution. Given that P(0 ≤ x ≤ 0.18) = 0.07, P(0 ≤ x ≤ 1.48) = 0.43 and P(0 ≤ x ≤ 1.23) = 0.39

Ans. Let μ and σ be the required mean and standard deviation.

Now 7% of items are under 35. It means area to the left of the ordinate $x = 35$ is 0.07

Also 89% of items are under 63. It means area to the left of the ordinate $x = 63$ is 0.89

Let $z = \frac{x-\mu}{\sigma}$ be the standard normal variate

When $x = 35, z = \frac{35-\mu}{\sigma} = z_1$ (say)

When $x = 63, z = \frac{63-\mu}{\sigma} = z_2$ (say)

Now $P(x < 35) = 0.07.$

$\Rightarrow$ $P(z < z_1) = 0.07.$

$\Rightarrow$ $1 - P(z > z_1) = 0.07$

$\Rightarrow$ $P(z > z_1) = 0.93$

$\Rightarrow$ $0.5 - P(0 < z < z_1) = 0.93$

$\Rightarrow$ $P(0 < z < z_1) = -0.43.$

$\Rightarrow$ $z_1 = 1.48$

Also $P(x < 63) = 0.89$

$\Rightarrow$ $P(z < z_2) = 0.89$

$\Rightarrow$ $1 - P(z > z_2) = 0.89$

$P(z > z_2) = 0.11$

$\Rightarrow$ $0.5 - P.(0 < z < z_2) = 0.11$

$\Rightarrow$ $P(0 < z < z_2) = 0.39$

$\Rightarrow$ $z_2 = 1.23$

Here the values of the ordinate $z = z_1$ and z_2 is negative

i.e. $z_1 = -1.48, z_2 = -1.23$

When $z_1 = -1.48$, then

$$\frac{35-\mu}{\sigma} = -1.48$$

$$\Rightarrow \quad 35 - \mu = -1.48\,\sigma \quad ...(1)$$

$\Rightarrow$ When $z_2 = -1.23$, then

$$\frac{63-\mu}{\sigma} = -1.23$$

$$\Rightarrow \quad 63 - \mu = -1.23\sigma \quad ...(2)$$

Solving (1) and (2), we get

$$35 - 63 = (-1.48 + 1.23)\,\sigma$$

$\Rightarrow$ $-28 = (-0.25\,\sigma)$

$\Rightarrow$ $\sigma = 112$

$\Rightarrow$ $\text{Var} = 12544$

By (1), $35 - \mu = -1.48 \times 112$

$35 - \mu = -165.76$

$\Rightarrow$ $\mu = 200.76$

END TERM EXAMINATION [MAY-JUNE 2017]
FOURTH SEMESTER [B.TECH]
APPLIED MATHEMATICS–IV
[ETMA–202]

Time : 3 Hrs. M.M. : 75

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit.

Q.1. (a) Solve $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given $u = 3e^{-y} - e^{-5y}$ when $x = 0$, by the method of separation of variables. (3)

Ans.

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u \quad ...(1)$$

Let

$$U = X(x)\,Y(y)$$

$$\frac{\partial u}{\partial x} = X'Y, \quad \frac{\partial u}{\partial y} = XY'$$

By equation (1), we get

$$4X'Y + XY' = 3XY$$

$$\Rightarrow \quad 4X'Y = (3Y - Y')X$$

$$\Rightarrow \quad \frac{4X'}{X} = \frac{3Y - Y'}{Y}$$

Since x and y are independent variables ∴ it is true only when each equation is equal to a constant

$$\Rightarrow \quad \frac{4X'}{X} = 3 - \frac{Y'}{Y} = a$$

$$\Rightarrow \quad \frac{4X'}{X} = a \text{ and } 3 - \frac{Y'}{Y} = a$$

$$\Rightarrow \quad \frac{4X'}{X} = a \Rightarrow \frac{4dX}{dx}\cdot\frac{1}{X} = a$$

$$\Rightarrow \quad \frac{dX}{X} = \frac{a}{4}dx$$

On integrating, we get

$$\log X = \frac{ax}{4} + \log c_1$$

$$X = c_1 e^{ax/4}$$

Now

$$3 - \frac{Y'}{Y} = a \Rightarrow \frac{Y'}{Y} = 3 - a$$

$$\frac{dY}{dy}\cdot\frac{1}{Y} = 3 - a$$

$$\frac{dY}{Y} = (3 - a)\,dy$$

On integrating, we get

$$\int\frac{dY}{Y} = \int(3 - a)dy$$

$$\log Y = (3 - a)y + \log c_2$$

$$Y = c_2 e^{(3-a)y} \quad ...(3)$$

As

$$U = XY$$

$$U = c_1 e^{ax/4}\, c_2 e^{(3-a)y}$$

$$U = c_1 c_2 e^{ax/4}\, e^{(3-a)y} \quad ...(4)$$

As given

$$U(0, y) = 3e^{-y} - e^{-5y}$$

By (4)

$$3e^{-y} - e^{-5y} = c_1 c_2 e^{(3-a)y}$$

Comparing these two, we get

$$3e^{-y} = c_1 c_2 e^{(3-a)y}$$

$$c_1c_2 = 3,\ 3 - a = -1$$

$$c_1c_2 = 3,\ a = 4$$

and

$$-e^{-5y} = c_1 c_2 e^{(3-a)y}$$

$$c_1c_2 = -1,\ 3 - a = -5$$

$$c_1c_2 = -1,\ a = 8.$$

Equation (4) becomes

$$U = 3e^{4x/4}e^{-y} - e^{8x/4}e^{-5y}$$

$$U = 3e^{x}e^{-y} - e^{2x}e^{-5y}$$

$$U(x, y) = 3e^{x-y} - e^{2x-5y}.$$

Q.1. (b) Define the skewness and kurtosis. Also write their coefficients. (3)

Ans. Skewness: It refers to asymmetry or lack of symmetry in the shape of frequency distribution. **OR** A distribution is said to be 'skewed' when the mean and median fall at different points in the distribution and the balance is shifted to one side or the other. Karl Pearson coefficient of skewness is

$$S_{kp} = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}}, \text{ where } -1 \le S_{kp} \le 1.$$

Also, by using moments, we define measure of skewness

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} \text{ and } \gamma_2 = \beta_2 - 3 = \frac{\mu_4}{\mu_2^2} - 3$$

If $\gamma_1 = 0$, distribution is symmetrical

If $\gamma_1 > 0$, distribution is positively skewed

If $\gamma_2 < 0$, distribution is negatively skewed.

Kurtosis: Kurtosis is defined as degree of flatness or peakedness in the region about the mode of a frequency mode. The degree of kurtosis of a distribution is measured relative to the peakedness of normal curve.

Measure of kurtosis is value of the pearson coefficient β_2, given by $\beta_2 = \frac{\mu_4}{\mu_2^2}$

Another measure of kurtosis is $\gamma_2 = \beta_2 - 3$.

For normal curve, $\gamma_2 = 0$

Q.1. (c) Determine the Bionomial Distribution for which mean = 2 (Variance) and mean + Variance = 3. Also find $P(X \le 3)$. (4)

Ans. Let $X \equiv \beta(n, p)$.

$$P(X = x) = {}^nc_x p^x q^{n-x}$$

Given

$$\text{Mean} = np = 2 = (\text{Variance})\ npq$$

Also

$$np + npq = 3.$$

$$2 + 2q = 3$$
$$\Rightarrow q = 1/2$$
$$\Rightarrow p = 1/2$$
$$\Rightarrow$$

Now $np = 2 \Rightarrow n \times \frac{1}{2} = 2$

$\Rightarrow n = 4$

Thus, Binomial distribution

$$P(X = x) = {}^4C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x}$$

For
$$P(X \le 3) = 1 - P(X > 3)$$
$$= 1 - P(X = 4)$$
$$= 1 - {}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$$
$$= 1 - \left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16} = \frac{15}{16}.$$

Q.1. (d) The first four moments of a distribution about the value '0' are −0.20, 1.76, −2.36 and 10.88. Find the moments about the mean. **(4)**

Ans. Moments about the value '0' are

$$\mu_1' = -0.20,\ \mu_2' = 1.76,\ \mu_3' = -2.36,$$
$$\mu_4' = 10.88$$

Then moments about mean, will be

$$\mu_1 = 0$$
$$\mu_2 = \mu_2' - \mu_1'^2 = 1.76 - (-0.20)^2 = 1.72$$
$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$= -2.36 - 3\times 1.76 \times (-0.20) + 2 \times (-0.20)^3$$
$$= -2.36 + 1.056 - 0.016$$
$$= -1.32$$
$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$
$$= 10.88 - 4\times(-2.36)\times(-0.20) + 6\times 1.76 \times (-0.20)^2 - 3(-0.20)^4$$
$$= 10.88 - 1.888 + 0.4224 - 0.0048$$
$$= 9.4096$$

Q.1. (e) State Baye's theorem **(3)**

Ans. Baye's theorem states that

Let $E_1, E_2, \ldots E_n$ be n mutually exclusive and exhaustive events, with non-zero probabilities, of a random experiment. If A be any arbitrary event of the sample space of the above experiment which occurs with E_1 or E_2 or ....or E_n and $P(A) > 0$, then

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A/E_j)},\ 1 \le i \le n.$$

Q.1. (f) The standard weight of a special purpose brick is 5 kg. and it contains two basic ingredients B_1 and B_2. B_1 cost Rs. 5 per kg and B_2 cost Rs. 8 per kg. Strength considerations state that the brick contains not more than 4kg of B_1 and minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find out graphically minimum cost of the brick satisfying the above conditions. **(5)**

Ans. Let x and y be the weight for B_1 and B_2 respectively.

Cost of x bricks = Rs $5x$

Cost of y bricks = Rs 8y

Since constraints are

Not more than 4kg of B,

$\Rightarrow \quad x \le 4$

Minimum of 2kg of B_2

$\Rightarrow \quad y \ge 2$

Total weight of brick is 5 kg

$\Rightarrow \quad x + y = 5$

Min. cost of the brick is needed

$\Rightarrow \quad z = 5x + 8y$

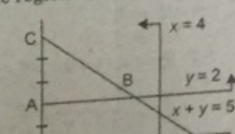
∴ Mathematical formulation of L.P.P. is

$$\text{Min } z = 5x + 8y$$

Subject to constraints

$$x \le 4$$
$$y \ge 2$$
$$x + y = 5$$
$$x \ge 0,\ y \ge 0$$

Since $x \ge 0$, $y \ge 0$ thus feasible region is in first quadrant only. Plot constraint by first treating it as a linear equation and then using the inequality condition of each constraint, mark the feasible region.



Here feasible region of the LPP is line segment AB with A = (0, 2) and B(3, 2). and C(0, 5)

We find value of objective function $z = 5x + 8y$ at each corner point

At A(0, 2), $z = 5\times 0 + 8\times 2 = 16$

At B(3, 2), $z = 5\times 3 + 8\times 2 = 31$

At C(0, 5), $z = 5\times 0 + 8\times 5 = 40$

Min. value of z is 16 at (0, 2).

Hence optimal solution of given LPP is $x = 0$, $y = 0$ and Min z = 16.

Q.1. (g) Find the dual of the following primal problem:
Min $Z = 3x_1 - 2x_2 + 4x_3$, subject to : $3x_1 + 5x_2 + 4x_3 \ge 7$, $6x_1 + x_2 + 3x_3 \ge 4$, $7x_1 - 2x_2 - x_3 \le 10$, and $x_1, x_2, x_3 \ge 0$. **(3)**

Ans. Primal problem is

$$\text{Min } z = 3x_1 - 2x_2 + 4x_3$$

S.t $\quad 3x_1 + 5x_2 + 4x_3 \ge 7$

$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Let S_1, S_2 and S_3 be the surplus and slack variables, then primal problem becomes

$$\text{Min } z = 3x_1 - 2x_2 + 4x_3 + S_1 + S_2 + S_3$$
$$3x_1 + 5x_2 + 4x_3 - S_1 = 7$$
$$6x_1 + x_2 + 3x_3 - S_2 = 4$$
$$7x_1 - 2x_2 - x_3 + S_3 = 10$$
$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

Let w_1, w_2, w_3 be the dual variables corresponding to primal constraints

Dual $\quad \text{Max } z^* = 7w_1 + 4w_2 + 10w_3$

S.t.
$$3w_1 + 6w_2 + 7w_3 \leq 3$$
$$5w_1 + w_2 - 2w_3 \leq -2$$
$$4w_1 + 3w_2 - w_3 \geq 4$$
$$-w_1 \leq 0$$
$$-w_2 \leq 0$$
$$w_3 \geq 0$$

w_1, w_2 and w_3 unrestricted (redundant)

$\Rightarrow \quad \text{Max } z^* = 7w_1 + 4w_2 + 10w_3$

S.t.
$$3w_1 + 6w_2 + 7w_3 \leq 3$$
$$5w_1 + w_2 - 2w_3 \leq -2$$
$$4w_1 + 3w_2 - w_3 \geq 4$$
$$w_1 \geq 0, w_2 \geq 0, w_3 \geq 0.$$

UNIT-I

Q.2. (a) $(D^2 - DD' - 2D'^2)z = (y-1)e^x$. (6.5)

Ans. $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

A.E. $\quad m^2 - m - 2 = 0$

$$m = 2, -1$$
$$\text{C.F.} = f_1(y+2x) + f_2(y-x)$$
$$\text{P.I.} = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$
$$= \frac{1}{D^2 - DD' - 2D'^2}ye^x - \frac{1}{D^2 - DD' - 2D'^2}e^x$$
$$= \frac{1}{(D-2D')(D+D')}ye^x - \frac{1}{1 - 1.0 - 2.0}e^x$$
$$= \frac{1}{(D-2D')(D+D')}ye^x - e^x$$
$$= \frac{1}{D-2D'}\int (c+x)e^x\,dx - e^x \qquad \text{[Here } m = -1]$$

Where C is replaced by $y + mx$ i.e., $y - x$ after integration.

$$= \frac{1}{D-2D'}\left[(c+x)e^x - \int e^x\,dx\right] - e^x$$
$$= \frac{1}{D-2D'}\left[(y-x+x)e^x - e^x\right] - e^x$$

$$= \frac{1}{D-2D'}\left(ye^x - e^x\right) - e^x$$

Now for m = 2

$$= \int (c - 2x - 1)e^x\,dx - e^x$$

where C is replaced by $y + mx = y + 2x$ after integration.

$$= (c - 2x - 1)e^x - \int -2e^x\,dx - e^x$$
$$= (y + 2x - 2x - 1)e^x + 2e^x - e^x$$
$$= ye^x - e^x + e^x$$
$$= ye^x$$

$\therefore$ complete sol^n is

$$z = f_1(y+2x) + f_2(y-x) + ye^x. \qquad (6)$$

Q.2. (b) $(D^2 - D')\,z' = 2y - x^2$.

Ans. Here $(D^2 - D')$ cannot be resolved into linear factors in D and D' ...(1)

Consider $\quad (D^2 - D')z = 0$ ...(2)

Let trial solution of (1) be $z = Ae^{hx+ky}$

So, $\quad D^2z = Ah^2e^{hx+ky}, \; D'z = Ake^{hx+ky}$

By (1)
$$A(h^2 - k)e^{hx+ky} = 0$$
$$h^2 - K = 0 \Rightarrow K = h^2$$

$\therefore$
$$\text{C.F.} = \Sigma Ae^{hx+ky} = \Sigma Ae^{hx+h^2y}$$
$$\text{P.I.} = \frac{1}{D^2 - D'}(2y - x^2)$$
$$= \frac{1}{D^2\left(1 - \frac{D'}{D^2}\right)}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 - \frac{D'}{D^2}\right]^{-1}(2y - x^2)$$
$$= \frac{1}{D^2}\left[1 + \frac{D'}{D^2} + \ldots\right](2y - x^2)$$
$$= \frac{1}{D^2}\left[(2y - x^2) + \frac{1}{D^2}D'(2y - x^2) + \ldots\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + \frac{1}{D^2}.2\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + \frac{2}{D}.x\right]$$
$$= \frac{1}{D^2}\left[2y - x^2 + x^2\right]$$
$$= \frac{1}{D^2}.2y = 2y.\frac{x^2}{2} = x^2y$$

$\therefore$ Complete sol^n is

$$z = \Sigma Ae^{h(x+hy)} + x^2y$$

A and h are arbitrary Constants.

Q.3. (a) An insulated rod of length l has its end A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t. (6.5)

Ans. The temperature function $u(x, t)$ satisfies the differential equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

Prior to temperture change at end B, when $t = 0$, the heat flow was independent of time (steady state) i.e., $\frac{\partial u}{\partial t} = 0$.

When temperature u depends upon x and not on t, (1) reduces to $c^2 \frac{\partial^2 u}{\partial x^2} = 0$

$$\Rightarrow \quad \frac{\partial^2 u}{\partial x^2} = 0$$

∴ General solⁿ is $u = ax + b$ ...(2)

Since
$$u = 0 \text{ for } x = 0$$
$$u = 100 \text{ for } x = l$$

∴ (2) gives
$$0 = b$$
$$\Rightarrow \quad u = ax$$

and
$$100 = al \Rightarrow a = \frac{100}{l}$$

∴ Initial condition is $u(x, 0) = \frac{100}{l} x$

Boundary conditions for subsequent flow are $u(0, t) = 0$, $u(l, t) = 0$ for all values of t.

Let
$$U = X(x)\,T(t) = XT$$

$$\Rightarrow \quad \frac{\partial u}{\partial t} = XT', \quad \frac{\partial^2 u}{\partial x^2} = X''T$$

Put these values in equation (1)

$$XT' = c^2 X''T$$

$$\Rightarrow \quad \frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T}$$

Now LHS is a function of x only and RHS is a function of t only. Since x and t are independent.

∴ both sides reduce to a constant say 'k'.

∴ (3) gives

$$\frac{X''}{X} = \frac{1}{c^2}\frac{T'}{T} = k.$$

Let
$$k = -p^2$$

$$\Rightarrow \quad \frac{X''}{X} = -p^2 \Rightarrow X'' + p^2 X = 0$$

Aux. eqⁿ is $m^2 + p^2 = 0$ ...

$$\therefore \quad X = c_1 \cos px + c_2 \sin px$$

Also
$$\frac{T'}{T} = -p^2c^2$$

$$\therefore \quad \log T = -p^2c^2t + \log c_3$$

$$\Rightarrow \quad T = c_3 e^{-p^2c^2t}$$

General solution is

$$u(x, t) = (c_1 \cos px + c_2 \sin px)e^{-p^2c^2t} \quad ...(3)$$

Using condition $u(0, t) = 0$ in (3), we get

$$u(0, t) = 0 = c_1 e^{-p^2c^2t}$$

$$\Rightarrow \quad c_1 = 0$$

equation (3) reduces to

$$u(x, t) = c_2 \sin px\, e^{-p^2c^2t} \quad ...(4)$$

Using
$$u(l, t) = 0 \text{ in } (4)$$
$$\Rightarrow \quad u(l, t) = 0 = c_2 \sin pl\, e^{-p^2c^2t}$$
$$\Rightarrow \quad \sin pl = 0$$
$$\Rightarrow \quad pl = n\pi$$
$$\Rightarrow \quad p = \frac{n\pi}{l}, \quad n = 1, 2, ...$$

equation (4) reduces to

$$u(x, t) = c_2 \sin\frac{n\pi x}{l} e^{\frac{-n^2\pi^2}{l^2}c^2t}$$

Adding all such solutions for different values of n, we get

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{\frac{-n^2\pi^2c^2t}{l^2}} \quad ...(5)$$

As initial condition is $u(x, 0) = \frac{100x}{l}$

$$\Rightarrow \quad \frac{100x}{l} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Which is a fourier sine series for $\frac{100x}{l}$

$$\therefore \quad b_n = \frac{2}{l}\int_0^l \frac{100x}{l}\sin\frac{n\pi x}{l}\,dx$$

$$\Rightarrow \quad b_n = \frac{200}{l^2}\left[x\left(-\cos\frac{n\pi x}{l}\right)\cdot\frac{l}{n\pi} - \left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right]_0^l$$

$$\Rightarrow \quad b_n = \frac{200}{l^2}\left[\frac{-l^2}{n\pi}\cos n\pi\right] = \frac{-200}{n\pi}(-1)^n$$

$$= \frac{200}{n\pi}(-1)^{n+1}$$

$$\therefore \quad u(x, t) = \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{n\pi x}{l} e^{-\frac{n^2\pi^2c^2t}{l^2}}$$

Q.3. (b) Solve the above problem if the change consists of raising the temperature of A to 20°C and reducing that of B to 80°C. **(6)**

Ans. Consider equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \quad ...(1)$$

As solved earlier solution is

$$u(x,0) = \frac{100x}{l} \quad ...(2)$$

Further B.c's are $\left.\begin{array}{l} u(0,t)=20 \\ u(l,t)=80 \end{array}\right\} - Ⓐ \ \forall t$

Let the required solution be

$$u(x,t) = u_s(x,t) + u_t(x,t) \quad ...(3)$$

Let u_s is steady state Solution and u_t is transient solution given by

$$u_t(x,t) = u(x,t) - u_s(x,t) \quad ...(4)$$

For steady state, solution is gives as

$$\frac{\partial^2 u}{\partial x^2} = 0$$

$$\Rightarrow \quad u_s(x,t) = c_1 x + c_2 \quad ...(5)$$

Using condition (A)

(5) $\Rightarrow$ $20 = c_2$

$\therefore$ $u_s(x,t) = c_1 x + 20$

and $80 = c_1 l + 20$

$\Rightarrow$ $c_1 = \frac{60}{l}$

$$\therefore \quad u_s(x,t) = \frac{60x}{l} + 20 \quad ...(6)$$

At $x = 0$, By (4) $\Rightarrow$

$$\begin{aligned} u_t(0,t) &= u(0,t) - u_s(0,t) \\ &= 20 - 20 \\ &= 0. \quad \text{[By (A) and (6)]} \end{aligned}$$

and

$$\begin{aligned} u_t(l,t) &= u(l,t) - u_s(l,t) \\ &= 80 - 80 = 0 \end{aligned}$$

Again by (4)

$$\begin{aligned} u_t(x,0) &= u(x,0) - u_s(x,0) \\ &= \frac{100x}{l} - \frac{60x}{l} - 20 \\ &= \frac{40x}{l} - 20 \end{aligned}$$

Now $u_t(0,t) = 0,\ u_t(l,t) = 0$

and

$$u_t(x,0) = \frac{40x}{l} - 20 \quad ...(7)$$

Then Sol^n of (1) is given by

$$u_t(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l} e^{-n^2\pi^2c^2t/l^2}$$

using (7)

$$u_t(x,0) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

This is a fourier sine series for $\left(\frac{40x}{l} - 20\right)$

where

$$b_n = \frac{2}{l}\int_0^l \left(\frac{40x}{l} - 20\right)\sin\frac{n\pi x}{l}dx$$

$$b_n = \frac{2}{l}\left[\left(\frac{40x}{l} - 20\right)\left(-\cos\frac{n\pi x}{l}\right)\frac{l}{n\pi} - \left(\frac{40}{l}\right)\left(-\sin\frac{n\pi x}{l}\right)\frac{l^2}{n^2\pi^2}\right]_0^l$$

$$b_n = \frac{2}{l}\left[-20\cos n\pi.\frac{l}{n\pi} - \frac{20l}{n\pi}\right]$$

$$b_n = \frac{-40}{n\pi}((-1)^n + 1)$$

$$b_n = \begin{cases} \frac{-80}{m\pi}, & \text{n is odd} \\ & \text{n is even} \end{cases}$$

$$b_n = \begin{cases} 0, & \text{n is odd} \\ \frac{-80}{m\pi}, & \text{n =2m (even)} \end{cases}$$

$$u_t(x,t) = \frac{-80}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{2m\pi x}{l}e^{-\frac{\pi^2c^24m^2t}{l^2}}$$

Using (3), (6) and (8), we get

$$u(x,t) = \frac{60x}{l} + 20 - \frac{80}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{2m\pi x}{l}e^{-\frac{4m^2\pi^2c^2t}{l^2}}$$

UNIT - II

Q.4. (a) Three balls are drawn successively from a box containing 6 red balls, 4 white balls, and 5 blue balls. Find the probability that they are drawn in the order red, white and blue if each ball is (*i*) replaced and (*ii*) not replaced. **(6.5)**

Ans. We have 6R, 4W, 5B

Total number of balls = 15

(*i*) We have to draw red, white and blue Probability of drawing 3 balls with replacement

First draw having 3 red balls = $\frac{^6C_3}{^{15}C_3}$

Second draw having 3 white balls = $\frac{^4C_3}{^{15}C_3}$

Third draw having 3 blue balls = $\frac{^5C_3}{^{15}C_3}$

$$\text{Required prob.} = \frac{^6C_3}{^{15}C_3} \times \frac{^4C_3}{^{15}C_3} \times \frac{^5C_3}{^{15}C_3}$$

(ii) Not replaced.

Probability of drawing 3 red at first trial is

$$= \frac{^6C_3}{^{15}C_3} = \frac{6 \times 5 \times 4}{^{15}C_3}$$

Probability of drawing 3 white in second trial is

$$= \frac{^4C_3}{^{12}C_3} = \frac{4}{^{12}C_3}$$

Probability of drawing 3 blue in third trial is

$$= \frac{^5C_3}{^9C_3} = \frac{5 \times 4}{9 \times 8 \times 7 \times 6 \times 5 \times 4}$$

$$= \frac{1}{3024}$$

$\therefore$ Requid. prob. is $\frac{120}{^{15}C_3} \times \frac{4}{^{12}C_3} \times \frac{1}{3024}$

$$= \frac{120}{^{15}C_3 \times ^{12}C_3 \times 756}$$

$$= \frac{30}{^{15}C_3 \times ^{12}C_3 \times 189}$$

Q.4. (b) Find (i) E(X), (ii) E(X²), (iii) E[(X–$\bar{X}$)²] for the probability distribution shown in the following table: **(6)**

X	8	12	16	202	24
P(X)	1/8	1/6	3/8	1/4	1/12

Ans. Given

X	P(X)	xP(x)	x^2P(x)
8	1/8	1	8
12	1/6	2	24
16	3/8	6	96
202	1/4	101/2	10201
24	1/12	2	48

$$E(X) = \Sigma x.P(x) = 1 + 2 + 6 + \frac{101}{2} + 2$$

$$= 11 + \frac{101}{2} = \frac{123}{2}$$

$$E(X_2) \quad \Sigma x^2.P(x) = 8 + 24 + 96 + 10201 + 48$$

$$= 10377$$

$$E[(X - \bar{X})^2] = E(X^2) - [E(x)]^2$$

$$= 10377 - \frac{(123)^2}{4}$$

$$= 10377 - \frac{15129}{4}$$

$$= \frac{26379}{4}$$

Q.5. (a) A continuous distribution of a variable x in the range (–3, 3) is defined as

$$f(x) = \frac{1}{16}(3+x)^2, \quad -3 \le x < -1.$$

$$= \frac{1}{16}(2 - 6x^2), \quad -1 \le x < 1.$$

$$= \frac{1}{16}(3 - x)^2, \quad 1 \le x \le 3.$$

Verify that the area under the curve is unity. Show that the mean is zero. **(6)**

Ans. To show $\int_{-3}^{3} f_x(x)\, dx = 1$

Consider $\int_{-3}^{3} f_x(x)\, dx = \int_{-3}^{-1} \frac{1}{16}(3+x)^2 dx + \int_{-1}^{1} \frac{1}{16}(2-6x^2)dx + \int_{1}^{3} \frac{1}{16}(3-x)^2 dx$

$$= \frac{1}{16}\left[\int_{-3}^{-1}(9 + x^2 + 6x)dx + \int_{-1}^{1}(2 - 6x^2)dx + \int_{1}^{3}(9 + x^2 - 6x)dx\right]$$

$$= \frac{1}{16}\left[\left|9x + \frac{x^3}{3} + 3x^2\right|_{-3}^{-1} + \left|2x - 2x^3\right|_{-1}^{1} + \left|9x + \frac{x^3}{3} - 3x^2\right|_{1}^{3}\right]$$

$$= \frac{1}{16}\left[-9 - \frac{1}{3} + 3 + 27 + \frac{27}{3} - 27 + 2 - 2 + 2 - 2 + 27 + \frac{27}{3} - 27 - 9 - \frac{1}{3} + 3\right]$$

$$= \frac{1}{16}\left[\frac{16}{3}\right] = 1/3.$$

$$E(X) = \int_{-3}^{3} x.f_x(x)\,dx$$

$$= \frac{1}{16}\left[\int_{-3}^{-1} x(3+x)^2 dx + \int_{-1}^{1} x(2-6x^2)dx + \int_{1}^{3} x(3-x)^2 dx\right]$$

$$= \frac{1}{16}\left[\begin{array}{l}\int_{-3}^{-1} x(9+x^2+6x)dx + \int_{-1}^{1}(2x-6x^3)dx \\ +\int_{1}^{3}(9x+x^3-6x^2)dx\end{array}\right]$$

$$= \frac{1}{16}\left[\begin{array}{l}\left.\frac{9x^2}{2}+\frac{x^4}{4}+2x^3\right|_{-3}^{-1} + \left.x^2-\frac{6x^4}{4}\right|_{-1}^{1} \\ +\left.\frac{9x^2}{2}+\frac{x^4}{4}-2x^3\right|_{1}^{3}\end{array}\right]$$

$$= \frac{1}{16}\left[\begin{array}{l}\frac{9}{2}+\frac{1}{4}-2-\frac{81}{2}-\frac{81}{4}+54+1-\frac{6}{4}-1+\frac{6}{4} \\ +\frac{81}{2}+\frac{81}{4}-54-\frac{9}{2}-\frac{1}{4}+2\end{array}\right]$$

$$= \frac{1}{16}[0] = 0$$

Q.5. (b) If the heights of 300 students are normally distribution with mean 68.0 inch and standard deviation 3.0 inch, how many students have heights. (i) greater than 72 inch.(ii) between 65 and 71 inch. **(6.5)**

Ans. Given $n = 300$, $\mu = 68$, $\sigma = 3$

Let X denote the height of students, following the normal distribution

Let $Z = \frac{X-\mu}{\sigma}$ be the standard normal variate.

$$Z = \frac{z-68}{3}$$

When $X > 72$ inch.

$$Z = \frac{72-68}{3} = \frac{4}{3} = 1.33$$

$$\begin{aligned} P(X > 72) &= P(z > 1.33) \\ &= 0.5 - P(0 < z < 1.33) \\ &= 0.5 - 0.4082 \\ &= 0.0918 \end{aligned}$$

(ii) When $65 < X < 71$

for $X = 65$, $\quad Z = \frac{65-68}{3} = \frac{-3}{3} = -1$

for $X = 71$, $\quad Z = \frac{71-68}{3} = \frac{3}{3} = 1$

$$\begin{aligned} P(65 < X < 71) &= P(-1 < z < 1) \\ &= 2P(0 < z < 1) \\ &= 2 \times 0.3413 = 0.6826 \end{aligned}$$

UNIT - III

Q.6. (a) Fit a second degree parabola to the following data using method of least squares. **(6.5)**

X	**10**	**12**	**15**	**23**	**20**
Y	**14**	**17**	**23**	**25**	**21**

Ans. Let $y = a + bx + cx^2$ be the second degree parabola. Then normal equation are

$$\begin{aligned} \Sigma y &= na + b\Sigma x + c\Sigma x^2 \\ \Sigma xy &= a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \\ \Sigma x^2 y &= a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \end{aligned}$$

Now $\Sigma x = 80$, $\Sigma y = 100$, $\Sigma x^2 = 1398$, $n = 5$.

Let $\bar{x} = 16$, $\bar{y} = 20$

Let $A = 16$, $B = 20$

x	y	$u = x - A$	$v = y - B$	u^2	u^3	u^4	uv	u^2v
10	14	−6	−6	36	−216	1296	36	−216
12	17	−4	−3	16	−64	256	12	−48
15	23	−1	3	1	−1	1	−3	3
23	25	7	5	49	343	2401	35	245
20	21	4	1	16	64	256	4	16

$\Sigma u = 0$, $\Sigma v = 0$, $\Sigma u^2 = 118$, $\Sigma u^3 = 126$, $\Sigma u^4 = 4210$, $\Sigma uv = 84$, $\Sigma u^2 v = 0$

$$0 = 5a + 0 + 118c \quad \text{...(1)}$$
$$84 = 0 + 118b + 126c \quad \text{...(2)}$$
$$0 = 118a + 126b + 4210c \quad \text{...(3)}$$

$(1) \Rightarrow$ $\quad a = \frac{-118c}{5}$

By (3) $\Rightarrow$ $\quad 0 = 118\left(\frac{-118c}{5}\right) + 126b + 4210c$

$$0 = -\frac{13924}{5}c + 126b + 4210c$$

$\Rightarrow$ $\quad 0 = \frac{7126}{5}c + 126b$

$\Rightarrow$ $\quad 630b + 7126c = 0 \quad \text{...(4)}$

$\Rightarrow$ $\quad 118b + 126c = 84 \quad \text{...(5)}$

$\Rightarrow$ $\quad 315b + 3563c = 0 \Rightarrow c = -\frac{315}{3563}b$

$\Rightarrow$ $\quad 59b + 63c = 42$

$$\Rightarrow \quad 59b + 63\left(\frac{-315}{3563}\right)b = 42$$

$$\Rightarrow \quad 59b - \frac{19845}{3563}b = 42$$

$$\Rightarrow \quad \frac{190372}{3563}b = 42$$

$$\Rightarrow \quad b = 0.7861$$

$$\Rightarrow \quad C = \frac{-315}{3563} \times 0.7861 = -0.0695$$

$$\Rightarrow \quad a = -\frac{118}{5} \times (-0.0695)$$

$$\Rightarrow \quad a = 1.6402$$

Thus

$$\Rightarrow \quad y = 1.6402 + 0.7861x - 0.0695\,x^2$$

Q.6. (b) For 10 observations on price (x) and supply (y), the following data were obtained $\Sigma x = 130$, $\Sigma y = 220$, $\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$, $\Sigma xy = 3467$.

Obtained the two lines of regression, correlation coefficient and estimate the supply when the price is 16 units.

Ans. Given n = 10, $\Sigma x = 130$, $\Sigma y = 220$, $\Sigma x^2 = 2288$, $\Sigma y^2 = 5506$, $\Sigma xy = 3467$

$$r_{xy} = \frac{\frac{1}{n}\Sigma xy - \bar{x}\,\bar{y}}{\sqrt{\left(\frac{1}{n}\Sigma x^2 - \bar{x}^2\right)\left(\frac{1}{n}\Sigma y^2 - \bar{y}^2\right)}} \quad ...(1)$$

Now

$$\bar{x} = \frac{\Sigma x}{n} = \frac{130}{10} = 13$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{220}{10} = 22$$

Now

$$r_{xy} = \frac{\frac{3467}{10} - 13 \times 22}{\sqrt{\left(\frac{2288}{10} - 13^2\right)\left(\frac{5506}{10} - 22^2\right)}}$$

$$= \frac{346.7 - 286}{\sqrt{(228.8 - 169)(550.6 - 484)}}$$

$$= \frac{60.7}{\sqrt{59.8 \times 66.6}} = \frac{60.7}{63.1085} = 0.9618$$

Now

$$\sigma_x^2 = \frac{1}{n}(\Sigma x^2) - \bar{x}^2 = \frac{1}{10} \times 2288 - 13^2$$

$$= 228.8 - 169 = 59.8$$

$$\Rightarrow \quad \sigma_x = 7.73$$

and

$$\sigma_y^2 = \frac{1}{n}(\Sigma y^2) - \bar{y}^2 = \frac{1}{10}(5506) - 22^2$$

$$= 66.6$$

$$\Rightarrow \quad \sigma_y = 8.16$$

Two regression lines are:

Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x} = \frac{0.9618 \times 8.16}{7.73} = 1.015$$

$$\Rightarrow \quad y - 22 = 1.015\,(x - 13)$$

$$\Rightarrow \quad y = 1.015x - 13.195 + 22$$

$$\Rightarrow \quad y = 1.015x + 8.805 \quad ...(1)$$

Regression line x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$b_{xy} = \frac{r\sigma_x}{\sigma_y} = \frac{0.9618 \times 7.73}{8.16}$$

$$= 0.911$$

$$\Rightarrow \quad x - 13 = 0.911\,(y - 22)$$

$$\Rightarrow \quad x = 0.911y - 7.042 \quad ...(2)$$

To find y when $x = 16$

Using equation (1) we get

$$y = 1.015 \times 16 + 8.805 = 25.045.$$

$$\Rightarrow \quad y \approx 25$$

Q.7. (a) A group of 10 rats fed on a diet A and another group of 8 rats fed on a different diet B, recorded the following increase in weight: **(6.5)**

Diet A(gm.)	5	6	8	1	12	4	3	9	6	10
Diet B(gm.)	2	3	6	8	10	1	2	8	-	-

Does it show that superiority of diet A over that of B.

Ans. Here $n_1 = 10$, $n_2 = 8$,

Null hypothesis: $H_0 : \bar{x}_A = \bar{y}_B$

i.e., there is no significant difference between the two diets.

Alternative hypothesis: $H_1 : \bar{x}_A > \bar{y}_B$ (one tailed test)

Using t-test, we have

$$t = \frac{\bar{x}_A - \bar{y}_B}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}$$

Here

$$\bar{x}_A = \frac{\Sigma x}{n_1} = \frac{64}{10} = 6.4$$

$$\bar{y}_B = \frac{\Sigma y}{n_2} = \frac{40}{8} = 5$$

Consider the table

x_A	y_B	$(x_A - \bar{x}_A)^2$	$(y_B - \bar{y}_B)^2$
5	2	1.96	9
6	3	0.16	4
8	6	2.56	1
1	8	29.16	9
12	10	31.36	25
4	1	5.76	16
3	2	11.56	9
9	8	6.76	9
6		0.16	
10		12.96	

Now $\Sigma(x_A - \bar{x}_A)^2 = 102.4$

and $\Sigma(y_B - \bar{y}_B)^2 = 82$

Now
$$S^2 = \frac{\Sigma(x_A - \bar{x}_A)^2 + \Sigma(y_B - \bar{y}_B)^2}{n_1 + n_2 - 2}$$
$$= \frac{102.4 + 82}{16} = \frac{184.4}{16} = 11.525$$

$\Rightarrow$ $s = 3.39$

Consider
$$t = \frac{6.4 - 5}{3.39\sqrt{\frac{1}{10} + \frac{1}{8}}} = \frac{1.4}{3.39\sqrt{0.225}}$$
$$= \frac{1.4}{1.6080} = 0.8706$$

The value of t at 10% level of significance for 16 d.f is 1.75

Since $|t| = 0.8706 < 1.75$

$\therefore$ The hypotheis H_0 is accepted

i.e., there is no significant difference between the two diets.

Q.7. (b) Fit a Poisson distribution to the following data and test for its goodness of fit at 0.05 level of significance. **(6)**

X:	**0**	**1**	**2**	**3**	**4**
F:	**419**	**352**	**154**	**56**	**19**

Ans. Null hypotheis H_0: Poisson fit is good fit to data.

Mean of the given distribution,

$$\lambda = \frac{0 + 352 + 308 + 168 + 76}{1000}$$
$$\lambda = \frac{904}{1000} = 0.904$$

By Poisson distribution, the frequency of r success is

$$N(r) = N \times e^{-\lambda} \cdot \frac{\lambda^r}{r!}, \text{ N is the total frequency}$$

Now
$$N(0) = 1000 \times \frac{e^{-0.904}}{0!} = 404.94$$
$$N(1) = 1000 \times e^{-0.904} \times \frac{0.904}{1!} = 366.071$$
$$N(2) = 1000 \times e^{-0.904} \times \frac{(0.904)^2}{2!} = 165.464$$
$$N(3) = 1000 \times e^{-0.904} \times \frac{(0.904)^3}{3!} = 49.859$$
$$N(4) = 1000 \times e^{-0.904} \times \frac{(0.904)^4}{4!} = 11.27$$

X	0	1	2	3	4
0_i	419	352	154	56	19
E_i	404.94	366.071	165.464	49.859	11.27

Now $\frac{(Oi - Ei)^2}{Ei}$

for X = 0, $\frac{(419 - 404.94)^2}{404.94} = 0.4882$

for X = 1, $\frac{(352 - 366.071)^2}{366.071} = 0.5409$

for X = 2, $\frac{(154 - 165.464)^2}{165.464} = 0.7943$

for X = 3, $\frac{(56 - 49.859)^2}{49.859} = 0.7564$

for X = 4, $\frac{(19 - 11.27)^2}{11.27} = 5.3019$

$$\chi^2 = \Sigma \frac{(Oi - Ei)^2}{Ei}$$

$$= 0.4882 + 0.5409 + 0.7943 + 0.7564 + 5.3019$$
$$= 7.8817$$

The calculated value of χ^2 is 7.8817. Tabulated value of χ^2 at 5% level of significance for

$$v = 5 - 2 = 3 \text{ d.f. is } 7.815$$

Since the calculated value of χ^2 is more than that of tabulated. i.e., $7.8817 > 7.815$

$\Rightarrow H_0$ is rejected

i.e., Poisson distribution does not provide good fit to the data.

UNIT-IV

Q.8. Use penalty (or Big-M) method to solve the probelm:

Max. $z = 6x_1 + 4x_2$ subject to $2x_1 + 3x_2 \le 30$, $3x_1 + 2x_2 \le 24$, $x_1 + x_2 \ge 3$, and $x_1, x_2 \ge 0$. Is the solution is unique? If not, give two different solutions. **(12.5)**

Ans. The given L.P.P can be written as

$$\text{Max } z = 6x_1 + 4x_2 + 0.x_3 + 0.x_4 + 0.x_5$$

S.t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 = 3$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

Let us add an artifical variable x_6 to the 3rd constraint, so that our L.P.P. becomes

$$\text{Max } z = x_1 + 5x_2 + 0.x_3 + 0.x_4 + 0.x_5 - Mx_5$$

S.t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 + x_6 = 3$$

$x_1, x_2, x_3, x_4, x_5 \ge 0$ and $x_6 \ge 0$ is an artifical variable and $M > 0$ is very very large.

Starting Table

			6	4	0	0	0	−M
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_3	30	2	3	1	0	0	0
0	y_4	24	3	2	0	1	0	0
−M	y_6	3	1	$\boxed{1}$	0	0	−1	1
		$z_j - c_j$	−M−6	−M−4 ↑	0	0	M	0

y_2 enters the basis

$$\min\left\{\frac{x_{Bi}}{yi_2}, yi_2 > 0\right\} = \text{Min}\left\{\frac{30}{3}, \frac{24}{2}, \frac{3}{1}\right\} = 3$$

$\Rightarrow y_6$ leaves the basis.

First Iteration

			6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_3	21	−1	0	1	0	$\boxed{3}$
0	y_4	18	1	0	0	1	2
4	y_2	3	1	1	0	0	−1
	$z_j - c_j$		−2	0	0	0	−4 ↑

$\therefore y_5$ enters the basis

$$\text{Min}\left\{\frac{x_{Bi}}{yi_5}, yi_5 > 0\right\} = \text{Min}\left\{\frac{21}{3}, \frac{18}{2}\right\} = 7$$

$\Rightarrow y_3$ leaves the basis

Second Iteration

			6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_5	7	−1/3	0	1/3	0	1
← 0	y_4	4	$\boxed{5/3}$	0	−2/3	1	0
4	y_2	10	2/3	1	1/3	0	0
	$z_j - c_j$		−10/3 ↑	0	4/3	0	0

$\Rightarrow y_1$ enters the basis

$$\text{Min}\left\{\frac{x_{Bi}}{yi_1}, yi_1 > 0\right\} = \text{Min}\left\{\frac{4}{5/3}, \frac{10}{2/3}\right\} = \frac{12}{5}$$

$\Rightarrow y_4$ leaves the basis

Third Iteration

			6	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_5	39/5	0	0	1/5	1/5	1
6	y_1	12/5	1	0	−2/5	3/5	0
4	y_2	42/5	0	1	3/5	−2/5	0
	$z_j - c_j$		0	0	0	2	0

Since all $z_j - c_j \ge 0$.

Thus sol^n is optimal.

$$x_1 = \frac{12}{5}, \quad x_2 = \frac{42}{5}$$

$$\text{and Max } z = 6 \times \frac{12}{5} + 4 \times \frac{42}{5} = \frac{240}{5} = 80.$$

Q.9. (a) A method Engineer wants to assign four new methods to three work centers. The assignment of the new methods will increase production and they are given below. If only one method can be assigned to a work center, determine the optimum assignement. (6)

Increase in Production (unit)

Methods	Works Centers A	B	C
1	10	7	8
2	8	9	7
3	7	12	6
4	10	10	8

Ans. Since number of tasks and number of subordinates are not equal. We introduce a dummy column.

$$\begin{bmatrix} 10 & 7 & 8 & 0 \\ 8 & 9 & 7 & 0 \\ 7 & 12 & 6 & 0 \\ 10 & 10 & 8 & 0 \end{bmatrix}$$

Locate smallest element from each row and subtract from it.

$$\begin{bmatrix} 10 & 7 & 8 & 0 \\ 8 & 9 & 7 & 0 \\ 7 & 12 & 6 & 0 \\ 10 & 10 & 8 & 0 \end{bmatrix}$$

Locate smallest element from each column and subtract from it

$$\begin{bmatrix} 3 & \boxed{0} & 2 & \cancel{0} \\ 1 & 2 & 1 & \boxed{0} \\ \boxed{0} & 5 & \cancel{0} & \cancel{0} \\ 3 & 3 & 2 & \cancel{0} \end{bmatrix}$$

Since assigned zero (3) < 4 (order of matrix) optimum solution is not reached.
For optimum solution

$$\begin{bmatrix} 3 & [0] & 2 & 0 \\ 1 & 2 & 1 & \boxed{0} \\ [0] & 5 & 0 & 0 \\ 3 & 3 & 2 & 0 \end{bmatrix} \begin{matrix} \\ \checkmark \\ \\ \checkmark \end{matrix}$$

Since minimum number of lines so drawn is 3, which is less than the order of the cost matrix. To increase minimum number of lines, we generate new zeros in the modified matrix. Smallest element not covered by the lines is 1. Subtracting this element from all the uncovered elements and adding the same to all the element lying at the intersection of the lines, we obtain new reduced cost matrix as:

$$\begin{bmatrix} 3 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & 0 & 1 \\ 2 & 2 & 1 & 0 \end{bmatrix}$$

Repeating the whole procedure, we get:

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline 1 & 3 & \boxed{0} & 2 & 1 \\ 2 & \boxed{0} & 1 & \cancel{0} & \cancel{0} \\ 3 & \cancel{0} & 5 & \boxed{0} & 1 \\ 4 & 2 & 2 & 1 & \boxed{0} \end{array}$$

Since each row and each column has one and only one assignement, an optimal solution is reached.

The optimum assignement is:
$1 \to B, 2 \to A, 3 \to C, 4 \to D$
The minimum assignment given to a work center
$= 7 + 8 + 6 + 0 = 21$

Q.9. (b) Solve the following Transportation problem: (6.5)

Suppliers / Consumers	A	B	C	Available
I	6	8	4	14
II	4	9	8	12
III	1	2	6	5
Required	6	10	15	31

Ans. Since demand and availability = 31
There exists a feasible solution to the transportation problem. we solve by VAM.

6	8	4	14 (2)
4	9	8	12 (4)
1	[5] 2	6	5 (1)
6 (3)	10 (6)	15 (2)	

Largest of these differeneces is (6), associated with 2nd column of the table.
Since min. cost in 2nd column is 2, we allocate

$$x_{32} = \min(5, 10) = 5$$

Exhaust 3rd row. Reduced transportation table is

6	8	4	14 (2)
[6] 4	9	8	12 (4)
6 (2)	5 (1)	15 (4)	

Largest of these differences is (4), associated with 2nd row. Since min. cost in 2nd. row is 4, we allocate

$$x_{21} = \min(12, 6) = 6$$

Exhaust Ist column. Reduced table is

8	$\boxed{14}$ 4	14	(4)
9	8	6	(1)
5 (1)	15 (4)		

Largest of these differences is (4) in 3rd column. Since min cost in 3rd column is 4, we allocate

$$x_{13} = \min(14, 15) = 14$$

Exhaust Ist row. Reduced table is

$\boxed{5}$ 9	$\boxed{1}$ 8	6
5	1	

Basic feasible solution is

6	8	$\boxed{14}$ 4
$\boxed{6}$ 4	$\boxed{5}$ 9	$\boxed{1}$ 8
1	$\boxed{5}$ 2	6

The transportation cost is

$$\begin{aligned} &= 14 \times 4 + 6 \times 4 + 5 \times 9 + 1 \times 8 + 5 \times 2 \\ &= 56 + 24 + 45 + 8 + 10 \\ &= 143. \end{aligned}$$