VICTORIA-CEDAR ALLIANCE

IP 4 END OF YEAR EXAMINATION 2021

CANDIDATE NAME	**MARKERS' COMMENTS**		
CLASS		INDEX NUMBER	

INTEGRATED MATHEMATICS

Paper 2 **29 September 2021**

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

[Mark Scheme]

1 Solve the simultaneous equations

$$x+2^{y+1}+\left(\frac{8}{z}\right)^{\frac{1}{3}}=13,$$
$$x-2^{y-1}+\frac{3}{2\left(\sqrt[3]{z}\right)}=-4.5,$$
$$2^{y+3}+\left(\frac{z}{125}\right)^{-\frac{1}{3}}=4+5x.$$

Q1 is well attempted.

2 most common math errors were

(a) $\left(\frac{z}{125}\right)^{-\frac{1}{3}} \neq \frac{1}{5\sqrt[3]{z}}$

$\left(\frac{z}{125}\right)^{-\frac{1}{3}} \neq \frac{z^{-\frac{1}{3}}}{5}$

(b) $\frac{1}{\sqrt[3]{z}}=-5$

$\sqrt[3]{z}=\frac{1}{-5}$

$z \neq \sqrt[3]{\frac{1}{-5}}$

Students are reminded to arrange the terms in each equation in the same order to avoid errors when keying the coefficients into the GC.

1	$\left.\begin{array}{r} x+2^{y+1}+\left(\frac{8}{z}\right)^{\frac{1}{3}}=13 \\ x-2^{y-1}+\frac{3}{2\left(\sqrt[3]{z}\right)}=-4.5 \\ 2^{y+3}+\left(\frac{z}{125}\right)^{-\frac{1}{3}}=4+5x \end{array}\right\} \Rightarrow \left.\begin{array}{r} x+2\left(2^y\right)+2\left(\frac{1}{\sqrt[3]{z}}\right)=13 \text{ —(1)} \\ 2x-2^y+3\left(\frac{1}{\sqrt[3]{z}}\right)=-9 \text{ —(2)} \\ -5x+8\left(2^y\right)+5\left(\frac{1}{\sqrt[3]{z}}\right)=4 \text{ —(3)} \end{array}\right\}$ **[M2 - any 2 correct]** Using GC, $x=7,\quad 2^y=8,\quad \frac{1}{\sqrt[3]{z}}=-5$ $2^y=2^3,\quad \frac{1}{z}=-125$ $y=3,\quad z=-\frac{1}{125}$ **[A3]**

2 It is given that $2x^3-x^2-9x-84=Ax(x+1)(x-B)+5(x-B)(x+4)+C(x+4)$ for all real values of x. Find the value of A, B and of C. [4]

2	$2x^3-x^2-9x-84=Ax(x+1)(x-B)+5(x-B)(x+4)+C(x+4)$ Comparing coefficient of x^3, $A=2$ **[B1]** when $x=-4$, $-192=-24(4+B)$ **[M1]** $B=4$ **[A1]** when $x=4$, $-8=8C$ $C=-1$ **[A1]**

Q2 was problematic for students who did not compare coefficients to obtain A = 2 quickly.

3 Solve algebraically the simultaneous equations

$$\lg(8x+22)-\lg(y-1)=\frac{1}{\log_2 10},$$
$$e^{\ln y}=10^{2\lg x}.$$

[4]

3	$\lg\left(\frac{8x+22}{y-1}\right)=\frac{1}{\left(\frac{\lg 10}{\lg 2}\right)}$ $\lg\left(\frac{8x+22}{y-1}\right)=\lg 2$ **[M1: Quotient or Change of Base Laws]** $\left(\frac{8x+22}{y-1}\right)=2$ $y=4x+12$ ——(1) $e^{\ln y}=10^{\lg x^2}$ $y=x^2$ ——(2) **[B1]** Sub (1) into (2): $x^2=4x+12$ **[M1]** $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ (rejected $\because x>0$) $\therefore x=6,\ y=36$ **[A1]**

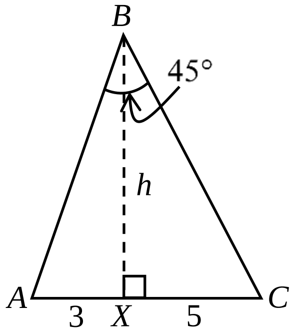
Q3 was well attempted.

The main error was oversight in not rejecting $x=-2$.

Take note that A1 was awarded only if algebraic method of solving quadratic equation was shown.

Penalty was imposed when factors do not tally with previous quadratic expression.

4 The diagram shows a triangle ABC in which angle $ABC = 45°$. Point X is the foot of the perpendicular from B to AC such that $AX = 3$ cm, $CX = 5$ cm and $BX = h$ cm.



(i) Show that $h^2 - 8h - 15 = 0$. [3]

(ii) Hence find the **exact** value of h. [2]

4(i)	$\angle ABX + \angle CBX = 45°$ $\dfrac{\tan\angle ABX + \tan\angle CBX}{1-\tan\angle ABX\tan\angle CBX} = \tan 45°$ **[M1 : correct eqn; o.e.]** $\dfrac{\frac{3}{h}+\frac{5}{h}}{1-\frac{3}{h}\left(\frac{5}{h}\right)} = 1$ **[B1 : $\tan\angle ABX = \frac{3}{h}$ OR $\tan\angle CBX = \frac{5}{h}$]** $\frac{8}{h} = 1 - \frac{15}{h^2}$ $8h = h^2 - 15$ $h^2 - 8h - 15 = 0$ **[A1 : follow through]**
4(ii)	$h = \dfrac{-(-8)\pm\sqrt{(-8)^2 - 4(1)(-15)}}{2(1)}$ **[M1]** $= \dfrac{8\pm\sqrt{124}}{2}$ $= 4+\sqrt{31}$ or $4-\sqrt{31}$ (NA) **[A1]**

Q4 was poorly done.

Many students skipped the question; surprisingly, including (ii) which only require the quadratic formula.

Most attempts to (i) involved forming equation with area of triangle, sine rule or cosine rule. Some who persevered, ended up with a degree 4 polynomial which only a few students continue to solve the equation:

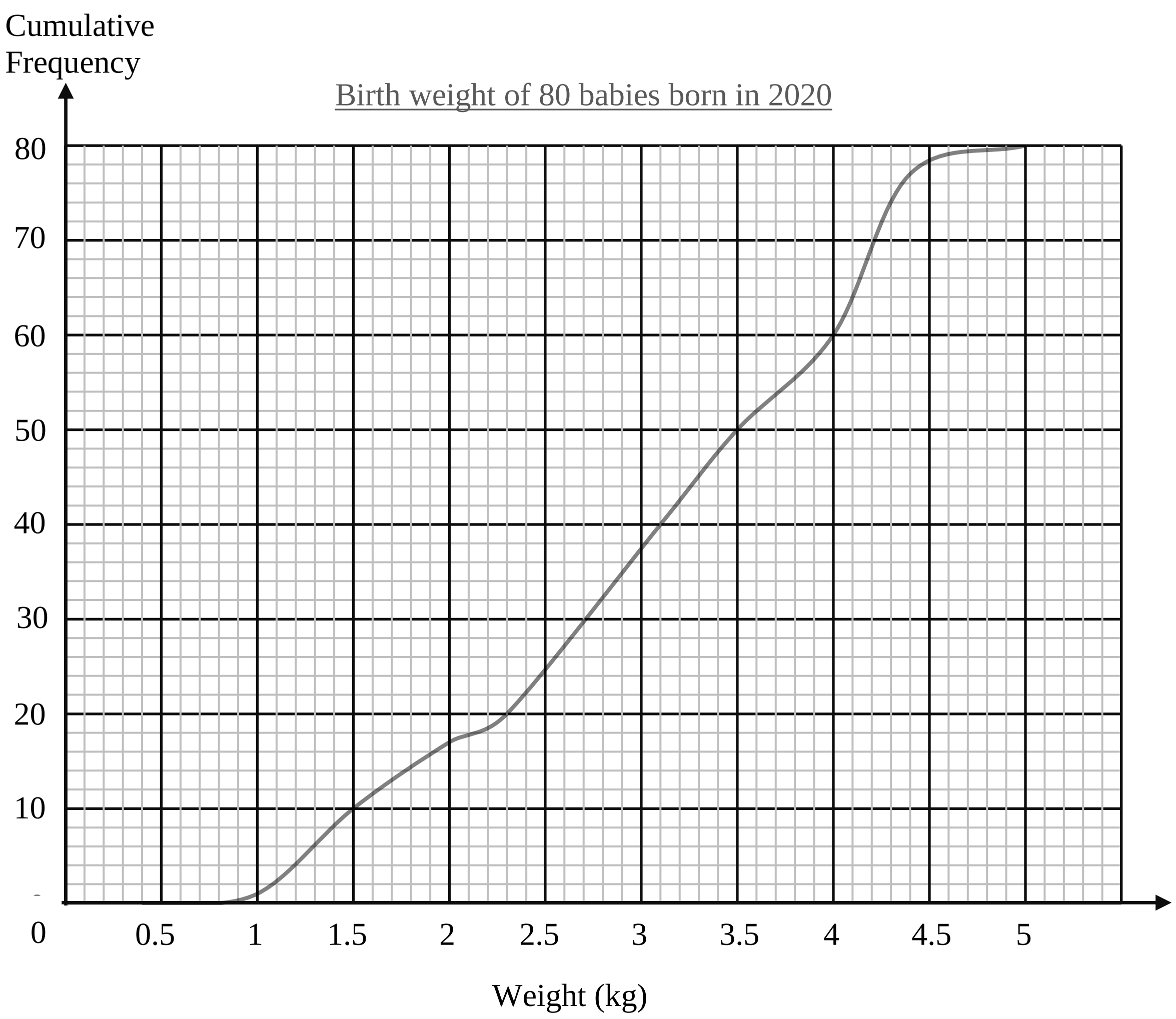
$h^4 - 94h^2 + 225 = 0$

$(h^2 - 8h - 15)(h^2 + 8h - 15) = 0$

Explain with working that $h^2 + 8h - 15 \neq 0$ as either h is negative or $h = -4 + \sqrt{31}$, which makes $\angle ABC$ obtuse. Hence $h^2 - 8h - 15 = 0$.

Take note that all exact answers in surds need to be simplified.

5 The distribution of birth weights of a sample of 80 babies born in 2020 is represented in a cumulative frequency curve as shown below.



(i) Using the cumulative frequency curve, estimate

(a) the median weight, [1]

(b) the interquartile range of the weights. [2]

(ii) It is given that babies can gain up to 0.9 kg in their first month. Using the cumulative frequency curve and the below infographics, estimate the maximum percentage of babies who may not fit into new-born size diapers at the end of the first month. [2]

(iii) It is given that the distribution of birth weights for a sample of 80 babies born in 1990 has a median weight of 3.4 kg and an interquartile range of 1.8 kg.

State whether you agree or disagree with the comments made by Rosie and Kang in their conversation, giving reason(s) to support your answer:

(a) Rosie sighed, "The new-born babies in 1990 are heavier compared to those in 2020." [1]

(b) Kang replied, "Also in 1990, the new-born babies are about the same weight, but in 2020, the new-born have very different weights." [1]

5(i)	Median = 3.1 kg **[B1]**
5(ii)	Interquartile range = 4 − 2.3 **[M1]** = 1.7 kg **[A1]**
5(iii)	Maximum % of babies $=\frac{80-64}{80}\times 100\%$ **[M1]** $= 20\%$ **[A1]**
5(iv)(a)	Agree with Rosie; median weight in 1990 is 3.4 kg which is greater than median weight in 2020 (3.1 kg), showing that the new-born babies are on average heavier in 1990 than in 2020. **[B1]**
5(iv)(b)	Disagree with Kang; interquartile range in 1990 is 1.8 kg , which is close to the interquartile range in 2020 (1.7 kg), showing similar spread in 1990 and 2020, **[B1]** **OR** which is greater than the interquartile range in 2020 (1.7 kg), showing wider/larger spread in 1990 than in 2020, **[B1]** hence not "about the same weight" in 1990 and "very different weights" in 2020.

Q5 was very well attempted. Except for a few students, the rest scored at least 5 out of 7 marks.

Most error made was with 5(iii).

6 A game of dice is played as follows:

Step 1: Roll two dice.

Step 2: Set aside the die with the larger number; if both dice show the same number, set aside one of the dice.

Step 3: Roll the other die.

Step 4: Add the two numbers on both dice.

You may assume that the dice are fair.

(i) **Copy and complete** the following possibility diagram for Step 2. The first column has been filled. [1]

		Die *A*					
	Number that is set aside	**1**	**2**	**3**	**4**	**5**	**6**
Die *B*	**1**	1	**2**	**3**	**4**	**5**	**6**
	2	2	**2**	**3**	**4**	**5**	**6**
	3	3	**3**	**3**	**4**	**5**	**6**
	4	4	**4**	**4**	**4**	**5**	**6**
	5	5	**5**	**5**	**5**	**5**	**6**
	6	6	**6**	**6**	**6**	**6**	**6**

[B1]

(ii) Hence find the probability that

(a) the number 2 is set aside, [1]

(b) the number 5 is set aside. [1]

6(ii)(a)	P(2 is set aside) $= \frac{3}{36} = \frac{1}{12}$ **[B1]**
6(ii)(b)	P(5 is set aside) $= \frac{9}{36} = \frac{1}{4}$ **[B1]**

(iii) **Copy and complete** the following possibility diagram for Step 4. The first column has been filled. [1]

		First Number					
Second Number	**Sum**	**1**	**2**	**3**	**4**	**5**	**6**
	1	2	**3**	**4**	**5**	**6**	**7**
	2	3	**4**	**5**	**6**	**7**	**8**
	3	4	**5**	**6**	**7**	**8**	**9**
	4	5	**6**	**7**	**8**	**9**	**10**
	5	6	**7**	**8**	**9**	**10**	**11**
	6	7	**8**	**9**	**10**	**11**	**12**

[B1]

(iv) Find the probability of getting a sum of 4 in a game of dice. [2]

6(iv) P(sum is 4)

$$=\frac{5}{36}\left(\frac{1}{6}\right)+\frac{3}{36}\left(\frac{1}{6}\right)+\frac{1}{36}\left(\frac{1}{6}\right)$$ **[M1 :** at least 2 correct**]**

$$=\frac{1}{24}$$ **[A1]**

(v) Victor remarked that, "In a game of dice, the chance of getting a sum less than 7 or a sum more than 7 is equal. This is because the number of occurrences for both in the possibility diagram in part **(iii)** is the same."

Is Victor correct or incorrect? Briefly explain your answer. [2]

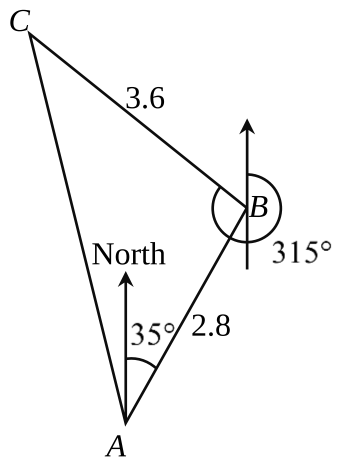
6(v) Victor is incorrect. **[B1: with correct reason]**
The probabilities of getting a 1, 2, 3, 4, 5 or 6 for the first number are not equal probabilities. Hence the probability of a particular sum is not given by counting the number of occurrences of that sum in the possibility diagram and divide by 36. **[B1]**
P(sum < 7) = 55/216
P(sum > 7) = 125/216
Okay if student calculates correctly and gives as reason.

Q6 was not well attempted.

(i)(ii)(iii) are very easy parts. But students did not read question and fill the (i) table with the *product* of 2 dice. There is nothing in the question that mention product or multiply.

(iv) Very few students answered correctly.
(v) B1B0 is awarded if answer is correct and reason is correct but lack details.

7 A yacht starts from a point A and sails on a bearing of 035° for 2.8 km to a point B. It then changes its course to a bearing of 315° and sails for 3.6 km to reach a point C.



(i) Find the distance AC. [3]

(ii) Find the bearing of C from A. [3]

(iii) When the yacht sails along the path BC, the yachtsman notices a helicopter, H, hovering at a constant height of 0.75 km directly above A. Find the smallest possible angle of elevation of the helicopter when viewed along the path BC. [2]

7(i)	$\angle ABC = 315° - 180° - 35° = 100°$ **[B1: with working]** $AC^2 = 2.8^2 + 3.6^2 - 2(2.8)(3.6)\cos 100°$ **[M1: Cosine Rule]** $AC = 4.9296$ (5 sf) $AC = 4.93$ km (3 sf) **[A1]**	
7(ii)	$\frac{\sin \angle BAC}{3.6} = \frac{\sin 100°}{4.9296}$ **[M1: Sine Rule]** $\angle BAC = 45.987°$ (3 dp) Bearing of C from A $= 360° - (45.987° - 35°)$ **[M1]** $= 349.0°$ (1 dp) **[A1]**	**[Alternate Method]** $\frac{\sin \angle BCA}{2.8} = \frac{\sin 100°}{4.9296}$ **[M1: Sine Rule]** $\angle BCA = 34.012°$ (3 dp) Bearing of C from A $= 360° - (180° - 100° - 35° - 34.012°)$ **[M1]** $= 349.0°$ (1 dp) **[A1]**
7(iii)	Let smallest angle of elevation be θ. $\tan \theta = \frac{0.75}{4.9296}$ **[M1: must use AC]** $\theta = 8.7°$ (1 dp) **[A1]** *look out for insufficient SF/DP in intermediate working.	

Q7 was very well attempted.

(ii) Many students left answer to only whole numbers and hence A0.
The angle was not an exact value and is in degrees, hence the bearing should be given to 1 dp.

(iii) Most errors were due to identifying the position with smallest angle of elevation wrongly.

8 The equation of a curve is $y = 2x^2 - 2px + 4 + p$, where p is a constant.

(i) Find the range of values of p for which the curve lies completely above the x-axis. [4]

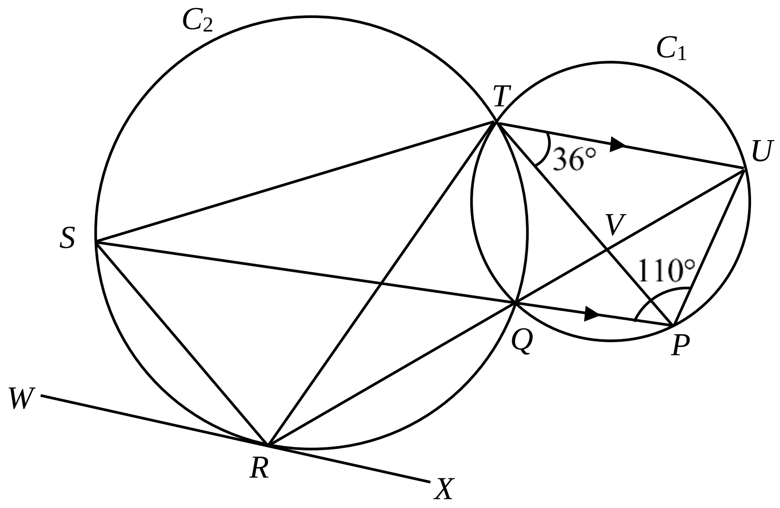
(ii) Show that the line $y = 6 - x$ will intersect the curve at two distinct points. [5]

8(i)	For $y = 2x^2 - 2px + 4 + p$ to lie completely above the x-axis, $2x^2 - 2px + 4 + p = 0$ has no real roots. Discriminant < 0 $(-2p)^2 - 4(2)(4+p) < 0$ **[B1- < 0; B1 - $b^2 - 4ac$]** $4p^2 - 8p - 32 < 0$ $p^2 - 2p - 8 < 0$ $(p+2)(p-4) < 0$ **[M1 - factorisation]** $-2 < p < 4$ **[A1]** Question was very well attempted.
8(ii)	$2x^2 - 2px + 4 + p = 6 - x$ $2x^2 + (1-2p)x + p - 2 = 0$ **[M1]** For line to intersect curve, discriminant $= (1-2p)^2 - 4(2)(p-2)$ **[M1 - $b^2 - 4ac$]** $= 4p^2 - 4p + 1 - 8p + 16$ $= 4p^2 - 12p + 17$ **[B1]** $= 4(p^2 - 3p) + 17$ $= 4\left(p - \frac{3}{2}\right)^2 - 4\left(-\frac{3}{2}\right)^2 + 17$ **[M1 - completing the square]** $= 4\left(p - \frac{3}{2}\right)^2 + 8$ Since $4\left(p - \frac{3}{2}\right)^2 \geq 0$, $4\left(p - \frac{3}{2}\right)^2 + 8 > 0$, discriminant > 0. Thus the line will intersect the curve at 2 distinct points. **[A1]**

Question was generally okay.
Some observations include:

- A few students stated discriminant greater than zero and used it to solve for *p*. The question is not about solving.
- A few students did not know that they needed to complete the square and they could not explain well why the discriminant is greater than zero.
- A few treated the expression as an equation and divided by 4 (instead of factorising)

9



In the diagram, the 2 circles C_1 and C_2 intersect at points T and Q. Point P lies on C_1 such that PQ produced cuts C_2 at point S. Point R lies on C_2 such that RQ produced cuts C_1 at point U. PT intersects QU at point V.

It is given that PQ is parallel to TU, angle $PTU = 36°$ and angle $QPU = 110°$.

(i) Determine whether triangle PQV is an isosceles triangle. [2]

(ii) WX is a tangent to the circle C_2 at R. Find

(a) angle PTQ, [2]

(b) angle SRW. [2]

(iii) Given that angle $QRS = 120°$, find angle PTR. [2]

9(i)	$\angle PQU = \angle PTU$ (angles in the same segment) $= 36°$ $\angle QPT = 36°$ (alt. $\angle$s, $PQ // TU$) $\angle PQV = \angle QPV$ (base $\angle$s of isos Δ) **[B1]** ΔPQV is an isosceles Δ **[A1]**
9(ii)(a)	$\angle QTU = 180° - 110°$ ($\angle$s in opp. segments) **[B1]** $= 70°$ $\angle PTQ = 70° - 36°$ $= 34°$ **[A1]**

Observations included:

- Wrong property. A few were confused between corresponding angles and alternate angles.
- No properties stated.
- Both will result in a loss of 1 mark due to penalty

Question is well attempted.

9(ii)(b)	$\angle RQS = \angle PQV$ (vert. opp. $\angle$s) **[B1]** $= 36°$ $\angle SRW = \angle RQS$ (Tangent Chord Theorem) $= 36°$ **[A1]** Question is well attempted.
9(iii)	$\left.\begin{cases} \angle STQ = 180° - 120° \text{ (}\angle\text{s in opp. segments)} \\ = 60° \\ \angle STR = \angle SQR \text{ (}\angle\text{s in the same segment)} \\ = 36° \end{cases}\right\}$ **[B1** - either $\angle STQ$ or $\angle STR$**]** $\angle RTQ = 60° - 36°$ $= 24°$ $\angle PTR = 24° + 34°$ $= 58°$ **[A1]** No credit will be given to students who attempted to find all the other angles except for the two intended angles that will lead to the answer.

10 A cubic expression $\mathrm{f}(x)$ has a factor (x^2-x) and leaves a remainder of -36 when divided by $(x+2)$. It is given that the coefficient of x^3 is 2.

(i) Show that $\mathrm{f}(x)=2x^3-4x^2+2x$. [3]

(ii) Factorise $\mathrm{f}(x)$ completely. [2]

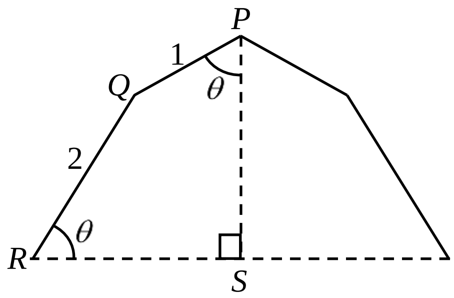
(iii) Hence express $\dfrac{7x^2-12x+1}{\mathrm{f}(x)}$ in partial fractions. [5]

10(i)	Let $\mathrm{f}(x)=2x^3+ax^2+bx$ $x^2-x=x(x-1)$ $\mathrm{f}(1)=0$ & $\mathrm{f}(-2)=-36$ $2+a+b=0$ $\quad 2(-8)+4a-2b=-36$ **[M2 - forming 2 eqns]** $a+b=-2$ —(eqn1) & $2a-b=-10$ —(eqn2) (eqn2)+(eqn1), $a=-4$ subst $a=-4$ into (eqn1), $b=2$ **[M1 - solving eqns; M0 if just use GC to solve]** $\therefore \mathrm{f}(x)=2x^3-4x^2+2x$
10(ii)	$\mathrm{f}(x)=2x^3-4x^2+2x$ $=2x(x^2-2x+1)$ **[M1]** **OR** $(x^2-x)(2x-2)$ $=2x(x-1)^2$ **[A1]** Question is well attempted.
10(iii)	$\dfrac{7x^2-12x+1}{\mathrm{f}(x)}=\dfrac{7x^2-12x+1}{2x^3-4x^2+2x}$ Let $\dfrac{7x^2-12x+1}{2x(x-1)^2}=\dfrac{A}{2x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ where A, B and C are constants $7x^2-12x+1=A(x-1)^2+2Bx(x-1)+2Cx$ **[M1]** when $x=1$, $2C=-4$ **[M1 - solving the unknowns]** $C=-2$ when $x=0$, $A=1$ **[A1]** when $x=2$, $1+4B+4(-2)=5$ $4B-7=5$ $B=3$ **[A1]** $\dfrac{7x^2-12x+1}{\mathrm{f}(x)}=\dfrac{1}{2x}+\dfrac{3}{(x-1)}-\dfrac{2}{(x-1)^2}$ **[A1]**

10(i) Students who use GC to solve for their unknowns will lose the final M1. If students were to introduce 3 unknowns in their formulation, the 3rd equation ($\mathrm{f}(0)=0$) becomes critical for the method mark to be awarded. Merely stating $\mathrm{f}(1)=0$ and $\mathrm{f}(-2)=36$ will not earn you any marks.

10(iii) well attempted.

11 The diagram below shows the vertical cross section through a tent in which $PQ = 1$ m, $QR = 2$ m, $\angle QPS = \angle QRS = \theta$ and RS is horizontal. The diagram is symmetrical about the vertical line PS.



(i) Obtain expressions for PS and RS in the form $a\cos\theta + b\sin\theta$, where a and b are integers. [2]

(ii) Hence show that A m^2, the area of quadrilateral $PQRS$ is given by

$$A = 1 + \frac{5}{4}\sin 2\theta - \cos 2\theta. \quad [3]$$

(iii) Express $\frac{5}{4}\sin 2\theta - \cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2]

(iv) State the maximum area of the vertical cross section of the tent and find the corresponding value of θ. [3]

11(i)	$PS = \cos\theta + 2\sin\theta$ **[B1]** $RS = 2\cos\theta + \sin\theta$ **[B1]** Question is well attempted.
11(ii)	$A = \frac{1}{2}(1)\left(\cos\theta + 2\sin\theta\right)\sin\theta + \frac{1}{2}(2)\left(2\cos\theta + \sin\theta\right)\sin\theta$ **[M1:** $\frac{1}{2}ab\sin C$; sum of 2Δ**]** $= \frac{1}{2}\sin\theta\cos\theta + \sin^2\theta + 2\sin\theta\cos\theta + \sin^2\theta$ $= \frac{5}{2}\sin\theta\cos\theta + 2\sin^2\theta$ **[M1:** expand & simplify**]** $= \frac{5}{4}(2\sin\theta\cos\theta) + (1 - \cos 2\theta)$ $= 1 + \frac{5}{4}\sin 2\theta - \cos 2\theta$ **[A1:** $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 2\theta = 1 - 2\sin^2\theta$**]**

11(iii)	Let $\frac{5}{4}\sin 2\theta - \cos 2\theta = R\sin(2\theta - \alpha)$ $= R\cos\alpha\sin 2\theta - R\sin\alpha\cos 2\theta$ $\frac{5}{4} = R\cos\alpha$........(1) $1 = R\sin\alpha$..........(2) Question is well attempted. $\tan\alpha = \frac{4}{5}$ and $R = \sqrt{\left(\frac{5}{4}\right)^2 + 1^2}$ **[M1: sight either]** $\alpha = 38.660°$ (3 dp) $R = \frac{\sqrt{41}}{4}$ **OR** 1.6008 (5 sf) $\therefore \frac{5}{4}\sin 2\theta - \cos 2\theta = \frac{\sqrt{41}}{4}\sin(2\theta - 38.7°)$ (1 dp) **[A1]** **OR** $1.60\sin(2\theta - 38.7°)$
11(iv)	$A = 1 + \frac{\sqrt{41}}{4}\sin(2\theta - 38.660°)$ maximum area of vertical cross section $= \left(2 + \frac{\sqrt{41}}{2}\right)$ m^2 **OR** $\frac{4+\sqrt{41}}{2}$ **OR** 5.20 (3 sf) **[B1]** when $\sin(2\theta - 38.660°) = 1$ **[M1]** $2\theta - 38.660° = 90°$ $\theta = 64.3°$ (1 dp) **[A1]**

11(iv) Most students could not find the maximum area of veritical cross section. Only a few could get it right.

12 Answer the whole of this question on a sheet of graph paper.

The table below shows a set of tabulated experimental values of two variables, x and y which are connected by an equation of the form $hx^2 = \frac{x-k}{y}$ where h and k are constants.

x	0.1	0.2	0.3	0.4	0.5
y	55.0	54.5	43.0	34.5	28.8

(i) Using a scale of 1 cm to represent 1 unit on both axes, plot xy against $\frac{1}{x}$ and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of each of the constants h and k. [4]

(iii) By drawing a suitable straight line on the same graph, find the value of x and y which satisfy the simultaneous equations

$$hx^2 = \frac{x-k}{y},$$

$$xy = \frac{1}{x} + 2.$$

[3]

12(i)

$1/x$	10	5	3.33	2.5	2
xy	5.5	10.9	12.9	13.8	14.4

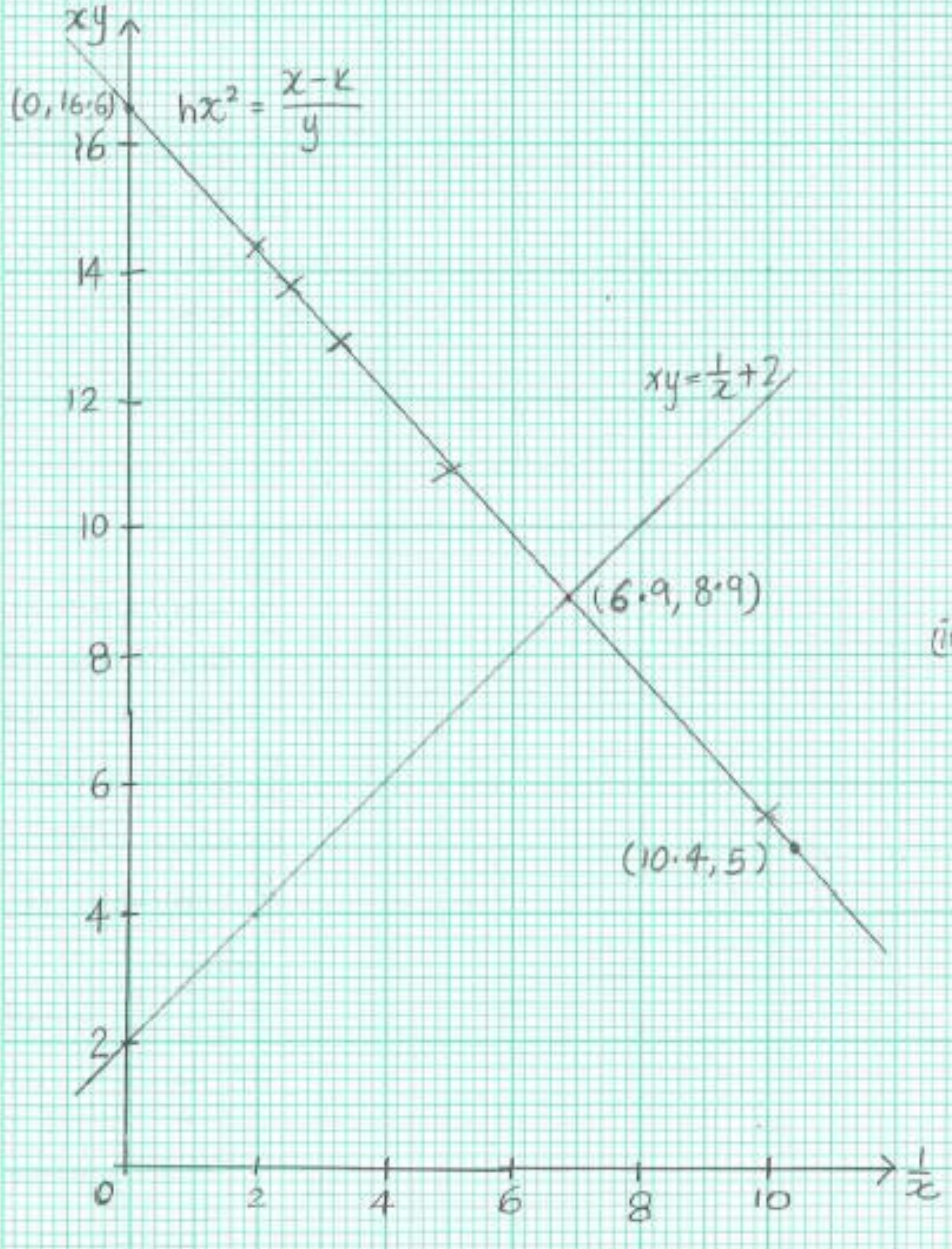
[B1]– plotting all points correctly
[B1]– straight line + labelling of axes, line
[B1] – table of values (accept to 3 or more sig fig)

12. Generally well attempted except for the following.

- A few students did not plot the correct coordinates correctly.
- Sign of k is negative of positive.
- The unknowns found should be rounded off to 3sf. Avoid using fractions.
- Many skipped (iii)

12(ii)	$hx^2 = \frac{x-k}{y}$ $hx^2 y = x - k$ $xy = \frac{x-k}{hx}$ $xy = \frac{1}{h} - \left(\frac{k}{h}\right)\frac{1}{x}$ **[B1]** Gradient $= \frac{-k}{h}$, xy − intercept $= \frac{1}{h}$ **[M1 - for relating the gradient & xy - intercept to eqn]** $\frac{1}{h} = 16.6$ (accept 16.4, 16.7) $h \approx 0.0602$ (accept 0.0610, 0.0599) **[A1]** Gradient $= \frac{-k}{h}$ $\frac{-k}{h} = \frac{16.6-5}{0-10.4}$ $k \approx 0.0672$ (accept 0.065-0.068) **[A1]**
12(iii)	Coordinates of intersection point = (6.9, 8.9) $\frac{1}{x} = 6.9$ & $xy = 8.9$ **[B1 - plotting the line $xy = \frac{1}{x} + 2$ correctly]** $x \approx 0.145$, $y \approx 61.4$ **[B2 - ecf with incorrect line from (i)]**

$\frac{1}{x}$	10	5	3.33	2.5	2
xy	5.5	10.9	12.9	13.8	14.4



(ii) $hx^2 = \frac{x-k}{y}$

$hxy = \frac{x-k}{x}$

$hxy = 1 - \frac{k}{x}$

$xy = \frac{1}{h} - \frac{k}{hx}$

xy- intercept $= \frac{1}{h}$

$16.6 = \frac{1}{h}$

$\therefore h \approx 0.0602$

Gradient $= \frac{-k}{h}$

$\frac{-k}{1/16.6} = \frac{16.6-5}{-10.4}$

$\therefore k \approx 0.0672$

(iii) $\frac{1}{x} = 6.9$

$x \approx 0.145$

$xy = 8.9$

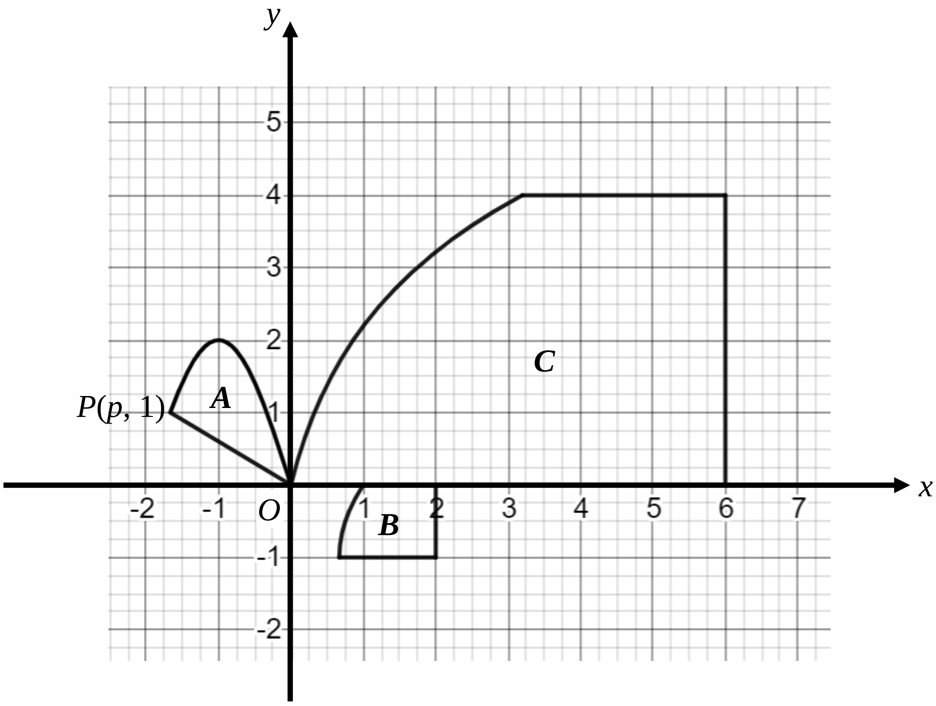
$y \approx 61.4$

13 Vincent made a design of a partial silhouette of a tortoise using a graphing software as shown below.

The head of the tortoise, region A, is an area enclosed by the curve $y=-2\sin\left(\frac{\pi}{2}x\right)$ and line OP, where P lies on $y=-2\sin\left(\frac{\pi}{2}x\right)$ and has coordinates $(p, 1)$.

The leg of the tortoise, region B, is an area enclosed by the curve $(y+1)^2=3x-2$, the lines $x=2$, $y=-1$ and the x-axis.

The shell of the tortoise, region C, is an area enclosed by the curve $y=2\ln(2x+1)$, the lines $x=6$, $y=4$ and the x-axis.



(i) Calculate the value of p. [2]

13(i)	$1=-2\sin\left(\frac{\pi}{2}p\right)$ **[M1]** $\sin\left(\frac{\pi}{2}p\right)=-\frac{1}{2}$ basic $\angle$ of $\frac{\pi}{2}p=\frac{\pi}{6}$ $\frac{\pi}{2}p=-\frac{5\pi}{6}$ $p=-\frac{5}{3}$ **[A1]** Question is well attempted.

(ii) Hence calculate the **exact** area of A. [3]

13(ii)	Area A
	$=\int_{-\frac{5}{3}}^{0}-2\sin\left(\frac{\pi}{2}x\right)\mathrm{d}x-\frac{1}{2}\left(\frac{5}{3}\right)(1)$ **[M1]**
	$=\left[\frac{2\cos\left(\frac{\pi}{2}x\right)}{\frac{\pi}{2}}\right]_{-\frac{5}{3}}^{0}-\frac{5}{6}$ **[B1:** integrate $\sin\left(\frac{\pi}{2}x\right)$ correctly**]**
	$=\frac{4}{\pi}\left[\cos 0-\cos\frac{5\pi}{6}\right]-\frac{5}{6}$
	$=\frac{4}{\pi}\left[1+\frac{\sqrt{3}}{2}\right]-\frac{5}{6}$ unit^2 **[A1:** o.e. in exact value**]**

13(i), (ii) and (iii are generally well attempted. A few students were observed to solve the parts using GC without showing any working. No credit will be given to such cases. For 13(iv), no credit is given too if students choose to integrate the logarithmic function without showing any working.

(iii) Without the use of a graphing calculator, find the area of B. [3]

13(iii)	Area B	**[Alternate Method]** Area B
	$=2(1)-\int_{-1}^{0}\frac{(y+1)^2+2}{3}\mathrm{d}y$ **[M1]**	$=\left(2-\frac{2}{3}\right)(1)-\left[-\int_{\frac{2}{3}}^{1}\sqrt{3x-2}-1\,\mathrm{d}x\right]$ **[M1]**
	$=2-\frac{1}{3}\left[\frac{(y+1)^3}{3}+2y\right]_{-1}^{0}$ **[B1:** integrate $(y+1)^2$ correctly**]**	$=\frac{4}{3}+\left[\frac{(3x-2)^{\frac{3}{2}}}{3\left(\frac{3}{2}\right)}-x\right]_{\frac{2}{3}}^{1}$ **[B1:** integrate $\sqrt{3x-2}$ correctly**]**
	$=2-\frac{1}{3}\left[\frac{1}{3}-(0-2)\right]$	$=\frac{4}{3}+\left[\frac{2}{9}-1-\left(0-\frac{2}{3}\right)\right]$
	$=\frac{11}{9}$ **OR** $1\frac{2}{9}$ unit^2 **[A1** accept 1.22 (3sf)**]**	$=\frac{11}{9}$ **OR** $1\frac{2}{9}$ unit^2 **[A1** accept 1.22 (3sf)**]**

(iv) Without the use of a graphing calculator, find the area of C. [4]

13(iv)	$y = 2\ln(2x+1)$
	$x = \dfrac{e^{\frac{y}{2}} - 1}{2}$ **[M1:** $y = \ln k \Leftrightarrow k = e^y$**]**
	Area C
	$= 4(6) - \int_0^4 \dfrac{e^{\frac{y}{2}} - 1}{2}\,dy$ **[M1]**
	$= 24 - \dfrac{1}{2}\left[\dfrac{e^{\frac{y}{2}}}{\frac{1}{2}} - y\right]_0^4$ **[B1:** integrate $e^{\frac{y}{2}}$ correctly**]**
	$= 24 - \dfrac{1}{2}\left[2e^2 - 4 - 2\right]$
	$= 27 - e^2$ unit^2 **[A1** accept 19.6 (3sf)**]**

END OF PAPER

Answer key

1	$x = 7,\ y = 3,\ z = -\dfrac{1}{125}$
2	$A = 2,\ B = 4,\ C = -1$
3	$x = 6,\ y = 36$
4(ii)	$h = 4 + \sqrt{31}$ or $4 - \sqrt{31}$ (NA)
5(i)(a)	3.1 kg
5(i)(b)	1.7 kg
5(ii)	20%
5(iii)(a)	Agree with Rosie; median weight in 1990 is 3.4 kg which is greater than median weight in 2020 (3.1 kg), showing that the new-born babies are on average heavier in 1990 than in 2020.
5(iii)(b)	Disagree with Kang; interquartile range in 1990 is 4.1 − 2.3 = 1.8 kg, which is close to the interquartile range in 2020 (1.7 kg), showing similar spread in 1990 and 2020, **OR** which is greater than the interquartile range in 2020 (1.7 kg), showing greater spread in 1990 than in 2020, hence not “about the same weight” in 1990 and “very different weights” in 2020.
6(i)	1,2,3,4,5,6 2,2,3,4,5,6 3,3,3,4,5,6 4,4,4,4,5,6 5,5,5,5,5,6 6,6,6,6,6,6
6(ii)(a)	$\dfrac{1}{12}$

7(i)	$AC = 4.93$ km (3 sf)
7(ii)	349.0° (1 dp)
7(iii)	8.7° (1 dp)
8(i)	$-2 < p < 4$
8(ii)	discriminant = $4\left(p - \dfrac{3}{2}\right)^2 + 8$ $4\left(p - \dfrac{3}{2}\right)^2 \geq 0,$ $4\left(p - \dfrac{3}{2}\right)^2 + 8 > 0.$
9(i)	$\angle PQV = \angle QPV$ (base $\angle$s of isos Δ) [B1] ΔPQV is an isosceles Δ
9(ii)(a)	34°, 36°
(b)	58°
10(i)	$\text{f}(x) = 2x^3 - 4x^2 + 2x$
10(ii)	$\text{f}(x) = 2x(x-1)^2$
10(iii)	$\dfrac{x^4 - 4}{\text{f}(x)} = \dfrac{x}{2} + 1 - \dfrac{2}{x} + \dfrac{7}{2(x-1)} - \dfrac{3}{2(x-1)^2}$
11(i)	$PS = \cos\theta + 2\sin\theta$ $RS = 2\cos\theta + \sin\theta$
11(iii)	$\dfrac{5}{4}\sin 2\theta - \cos 2\theta = \dfrac{\sqrt{41}}{4}\sin(2\theta - 38.7°)$ **OR** $1.60\sin(2\theta - 38.7°)$
11(iv)	$\left(2 + \dfrac{\sqrt{41}}{2}\right)$ m^2 **OR** $\dfrac{4+\sqrt{41}}{2}$ when $\theta = 64.3°$
12(ii)	$h \approx 0.0602;\ k \approx 0.0672$
12(iii)	Coordinates of intersection point = (6.9, 8.9) $x \approx 0.145,\ y \approx 61.4$

6(ii)(b)	$\frac{1}{4}$	**13(i)**	$p = -1\frac{2}{3}$
6(iii)	2,3,4,5,6,7 3,4,5,6,7,8 4,5,6,7,8,9 5,6,7,8,9,10 6,7,8,9,10,11 7,8,9,10,11,12	**13(ii)**	$\frac{4}{\pi}\left[1+\frac{\sqrt{3}}{2}\right]-\frac{5}{6}$ unit^2
		13(iii)	$1\frac{2}{9}$ unit^2
6(iv)	$\frac{1}{24}$	**13(iv)**	$27-e^2$ unit^2
6(v)	Victor is incorrect. The probabilities of getting a 1, 2, 3, 4, 5 or 6 for the first number are not equal probabilities. Hence the probability of a particular sum is not given by counting the number of occurrences of that sum in the possibility table and divide by 36.		