Obligatorisk oppgave 1, MEK1100, Vår 2021

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Oppgave 2

$$\mathbf{v} = (x \cdot y, y) \tag{1}$$

a)
For å finne strømningslinjene, så må vi sette opp likningen:

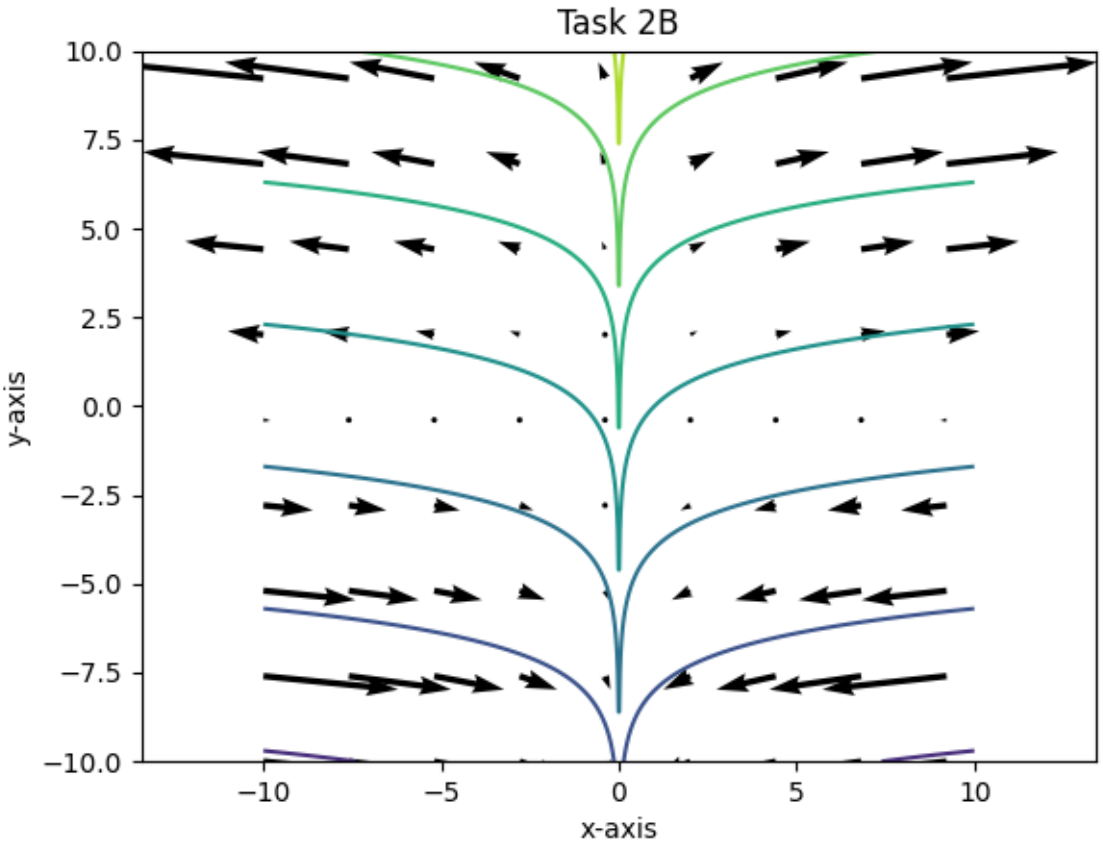
$$\begin{aligned} \frac{dy}{dx} &= \frac{y}{x \cdot y} \\ dy &= \frac{1}{x} dx \\ y &= \ln|x| + c \end{aligned} \tag{2}$$

b)

Python program:

```python
import numpy as np
import matplotlib.pyplot as plt
import abspath

# Local pypackage that I created to easily access the
    absolute path of some selected directories.
path = abspath.get_path("MEK1100") + "/Oblig1/images"

def mesh_grid(start, stop, dt):
    # Create a linspace
    I = np.linspace(start, stop, dt)

    # Create a meshgrid that uses the linspace dimensions
    x, y = np.meshgrid(I, I)

    return x, y

def vec_field(x, y, u, v, density):
    # Variable that tells how much to divide the number of
    elements in the mesh by.
    skip=(slice(None,None,density),slice(None,None,density))

    # Returns the vectorfield with the correct density.
    return u[skip], v[skip], skip

def streamlines(x, y, func):
    # Returns a meshgrid
    return func


if __name__ == "__main__":
    x,y = mesh_grid(-10, 10, 1000)

    u, v, skip = vec_field(x,y,x*y,y, 120)

    f = streamlines(x,y,y - np.log(abs(x)))

    plt.quiver(x[skip],y[skip],u,v)
    plt.contour(x,y,f,6)
    plt.axis('equal')
    plt.title("Task 2B")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")

    plt.savefig(f"{path}/two_b.png")
```

Output:



c)
Hvis en strømfunksjon eksisterer, så må vektorfeltet være divergensfritt.

$$\begin{aligned} \nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\ &= \frac{\partial}{\partial x} x \cdot y + \frac{\partial}{\partial y} y \\ &= y + 1 \\ &\neq 0 \end{aligned} \tag{3}$$

Siden divergensen til v ikke er 0, så eksisterer det ingen strømfunksjon.

Oppgave 3

$$\mathbf{v} = (\cos(x) \cdot \sin(y), -\sin(x) \cdot \cos(y)) \tag{4}$$

a)
Divergensen blir lik:

$$\begin{aligned} \nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\ &= \frac{\partial}{\partial x} \cos(x) \cdot \sin(y) + \frac{\partial}{\partial y} - \sin(x) \cdot \cos(y) \\ &= -\sin(x) \cdot \sin(y) + \sin(x) \cdot \sin(y) \\ &= 0 \end{aligned} \tag{5}$$

Virvlingen blir lik:

$$\begin{aligned} \nabla \times \mathbf{v} &= (\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}) \cdot \mathbf{k} \\ &= (\frac{\partial}{\partial x} - \sin(x) \cdot \cos(y) - \frac{\partial}{\partial y} \cos(x) \cdot \sin(y)) \cdot \mathbf{k} \\ &= (-\cos(x) \cdot \cos(y) - cos(x) \cdot \cos(y)) \cdot \mathbf{k} \\ &= (-2 \cdot cos(x) \cdot \cos(y)) \cdot \mathbf{k} \end{aligned} \tag{6}$$

d)
Siden $\nabla \cdot \mathbf{v} = 0$, så betyr det at feltet er konservativt og derfor finnes det en strømfunksjon for feltet.
For å regne ut ψ, så må man først finne $\int \frac{\partial \psi}{\partial x}$ og $\int \frac{\partial \psi}{\partial y}$

$$\begin{aligned} \int \frac{\partial \psi}{\partial y} &= \int v_x dy \\ &= \int \cos(x) \cdot \sin(y) dx \\ &= \cos(x) \cdot \cos(y) + f(x) \end{aligned} \tag{7}$$

$$\begin{aligned} \int \frac{\partial \psi}{\partial x} &= \int v_y dx \\ &= \int -\sin(x) \cdot \cos(y) dx \\ &= \cos(x) \cdot \cos(y) + g(y) \end{aligned} \tag{8}$$

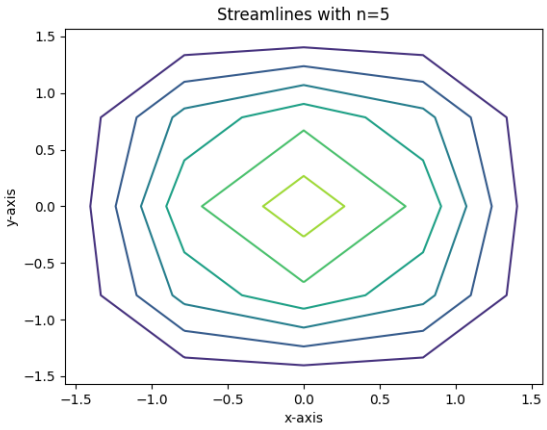
Ut i fra det vi har regnet ut, så ser vi at strømningsfunksjonen blir $\psi = \cos(x) \cdot \cos(y)$.

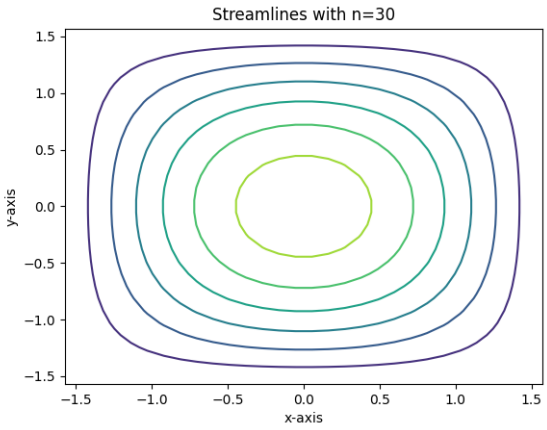
Oppgave 4

a)

```python
import numpy as np
import matplotlib.pyplot as plt
import abspath
from streamfun import streamfun

# Local pypackage that I created to easily access the
    absolute path of some selected directories.
path = abspath.get_path("MEK1100") + "/Oblig1/images"

# different values of n
n_vals = [5, 30]

# loops through different values for n and outputs .png files
for i in n_vals:
    x, y, psi = streamfun(i)
    plt.clf()
    plt.contour(x, y, psi)
    plt.title(f"Streamlines with n={i}")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/strlin_{i}.png")
```

Koden over gir to plotter:





b)

```python
import numpy as np
import matplotlib.pyplot as plt
import os
from velfield import velfield

```

```
# Local pypackage that I created to easily access the
    absolute path of some selected directories.
path = f"{os.getenv('MEK1100')}/Oblig1/images"

# Chose an odd number to include the point in the middle
    where there is no flow.
n_val = 11

# Gets values x, y, u and v, then plots them into the vector
    field.
x, y, u, v = velfield(n_val)
plt.quiver(x, y, u, v)
plt.title(f"Vector field with n={n_val}")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.savefig(f"{path}/vec_{n_val}.png")
```

Koden over gir vektorfeltet under: