Obligatorisk oppgave 1, MEK1100, Vår 2021

Cory Alexander Balaton

Oppgave 2

$$\mathbf{v} = (x \cdot y, y) \tag{1}$$

a)
For å finne strømningslinjene, så må vi sette opp likningen:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{y}{x \cdot y} \\
dy &= \frac{1}{x} dx \\
y &= \ln|x| + c
\end{aligned} \tag{2}$$

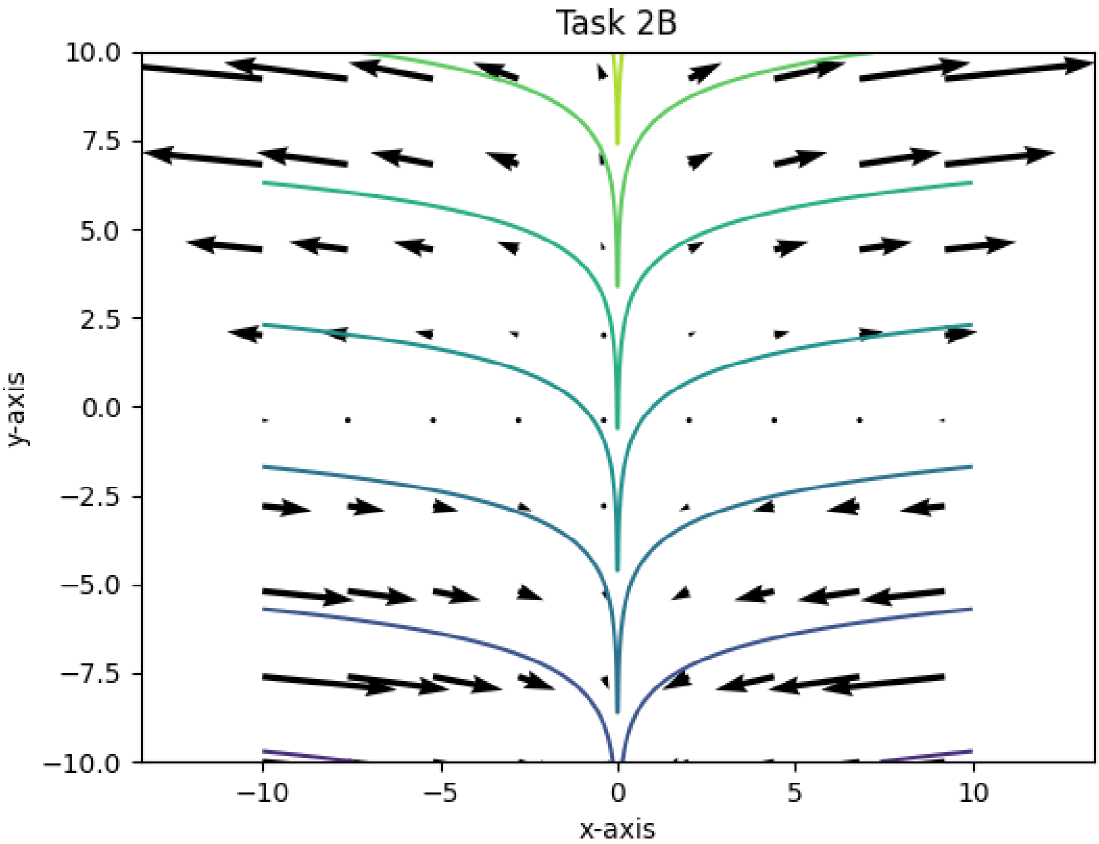
b)

Python program:

```python
import numpy as np
import matplotlib.pyplot as plt
import os

# Accessing an environment variable that points to the MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"


def mesh_grid(start, stop, dt):
    # Create a linspace
    I = np.linspace(start, stop, dt)

    # Create a meshgrid that uses the linspace dimensions
    x, y = np.meshgrid(I, I)

    return x, y


def vec_field(x, y, u, v, density):
    # Variable that tells how much to divide the number of elements in the mesh by.
    skip = (slice(None, None, density), slice(None, None, density))

    # Returns the vectorfield with the correct density.
    return u[skip], v[skip], skip


def streamlines(x, y, func):
    # Returns a meshgrid
    return func


if __name__ == "__main__":
    x, y = mesh_grid(-10, 10, 1000)

    u, v, skip = vec_field(x, y, x*y, y, 120)

    f = streamlines(x, y, y - np.log(abs(x)))

    plt.quiver(x[skip], y[skip], u, v)
    plt.contour(x, y, f, 6)
    plt.axis('equal')
    plt.title("Task 2B")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")

```

```
    plt.savefig(f"{path}/two_b.png")
```

Output:



c)
Hvis en strømfunksjon eksisterer, så må vektorfeltet være divergensfritt.

$$
\begin{aligned}
\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\
&= \frac{\partial}{\partial x} x \cdot y + \frac{\partial}{\partial y} y \\
&= y + 1 \\
&\neq 0
\end{aligned} \tag{3}
$$

Siden divergensen til v ikke er 0, så eksisterer det ingen strømfunksjon.

Oppgave 3

$$\mathbf{v} = (\cos(x) \cdot \sin(y), -\sin(x) \cdot \cos(y)) \tag{4}$$

a)
Divergensen blir lik:

$$\begin{aligned}
\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\
&= \frac{\partial}{\partial x} \cos(x) \cdot \sin(y) + \frac{\partial}{\partial y} - \sin(x) \cdot \cos(y) \\
&= -\sin(x) \cdot \sin(y) + \sin(x) \cdot \sin(y) \\
&= 0
\end{aligned} \tag{5}$$

Virvlingen blir lik:

$$\begin{aligned}
\nabla \times \mathbf{v} &= (\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}) \cdot \mathbf{k} \\
&= (\frac{\partial}{\partial x} - \sin(x) \cdot \cos(y) - \frac{\partial}{\partial y} \cos(x) \cdot \sin(y)) \cdot \mathbf{k} \\
&= (-\cos(x) \cdot \cos(y) - cos(x) \cdot \cos(y)) \cdot \mathbf{k} \\
&= (-2 \cdot cos(x) \cdot \cos(y)) \cdot \mathbf{k}
\end{aligned} \tag{6}$$

c)

$$\mathbf{F} = \mathbf{v} \tag{7}$$

Man kan først definere randa til kvadratet med 4 parametere:

$$\begin{aligned}
\mathbf{r}_1(t) &= \left(\frac{\pi}{2}, t\right) \\
\mathbf{r}_2(t) &= \left(-\frac{\pi}{2}, t\right) \\
\mathbf{r}_3(t) &= \left(t, \frac{\pi}{2}\right) \\
\mathbf{r}_4(t) &= \left(t, -\frac{\pi}{2}\right)
\end{aligned} \tag{8}$$

Deretter tar man linjeintegralet til **F** med hver parameter og adderer dem sammen:

$$
\begin{aligned}
\oint_C &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t) \cdot j \; dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_2(t)) \cdot \mathbf{r}_2'(t) \cdot -j \; dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_3(t)) \cdot \mathbf{r}_3'(t) \cdot -i \; dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_4(t)) \cdot \mathbf{r}_4'(t) \cdot i \; dt
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t) \cdot j \; dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\left(\frac{\pi}{2}\right) \sin(t) \cdot i - \sin\left(\frac{\pi}{2}\right) \cos(t) \cdot j) \cdot j \; dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\cos(t) \cdot j) \cdot j \; dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_2(t)) \cdot \mathbf{r}_2'(t) \cdot (-j) \; dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(-\frac{\pi}{2}) \sin(t) \cdot i - \sin(-\frac{\pi}{2}) \cos(t) \cdot j) \cdot (-j) \; dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \cdot j) \cdot (-j) \; dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{11}
$$

$$\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_3(t)) \cdot \mathbf{r}_3'(t) \cdot (-i)\ dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t)\sin\left(\frac{\pi}{2}\right) \cdot i - \sin(t)\cos\left(\frac{\pi}{2}\right) \cdot j) \cdot (-i)\ dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \cdot i) \cdot (-i)\ dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{12}$$

$$\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_4(t)) \cdot \mathbf{r}_4'(t) \cdot i\ dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t)\sin\left(-\frac{\pi}{2}\right) \cdot i - \sin(t)\cos\left(-\frac{\pi}{2}\right) \cdot j) \cdot i\ dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\cos(t) \cdot i) \cdot i\ dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{13}$$

Sirkulasjonen blir da:

$$\oint_C = -2 + (-2) + (-2) + (2) = -8 \tag{14}$$

d)
Siden $\nabla \cdot \mathbf{v} = 0$, så betyr det at feltet er konservativt og derfor finnes det en strømfunksjon for feltet.
For å regne ut ψ, så må man først finne $\int \frac{\partial \psi}{\partial x}$ og $\int \frac{\partial \psi}{\partial y}$

$$\begin{aligned}
\int \frac{\partial \psi}{\partial y} &= \int v_x dy \\
&= \int \cos(x) \cdot \sin(y) dx \\
&= \cos(x) \cdot \cos(y) + f(x)
\end{aligned} \tag{15}$$

$$\begin{aligned}
\int \frac{\partial \psi}{\partial x} &= \int v_y dx \\
&= \int -\sin(x) \cdot \cos(y) dx \\
&= \cos(x) \cdot \cos(y) + g(y)
\end{aligned} \tag{16}$$

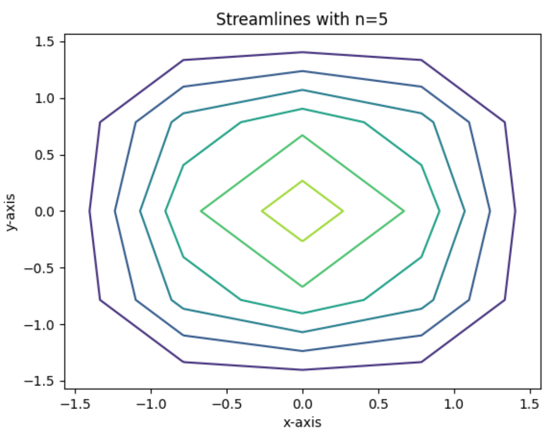
Ut i fra det vi har regnet ut, så ser vi at strømningsfunksjonen blir $\psi = \cos(x) \cdot \cos(y)$.

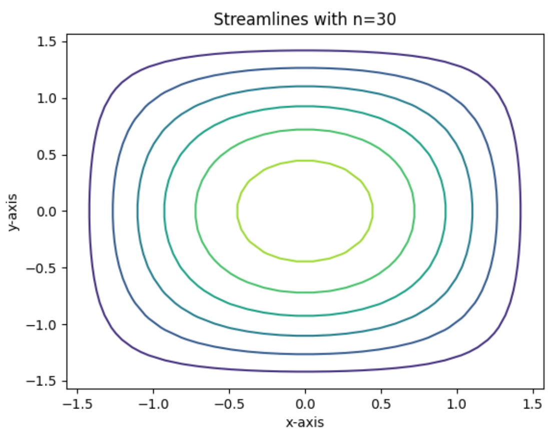
Oppgave 4

a)

```python
import matplotlib.pyplot as plt
from streamfun import streamfun
import os

# Accessing an environment variable that points to the MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"

# different values of n
n_vals = [5, 30]

# loops through different values for n and outputs .png files
for i in n_vals:
    x, y, psi = streamfun(i)
    plt.clf()
    plt.contour(x, y, psi)
    plt.title(f"Streamlines with n={i}")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/strlin_{i}.png")
```

Koden over gir to plotter:





b)

```python
import matplotlib.pyplot as plt
import os
from velfield import velfield

# Accessing an environment variable that points to the MEK1100 directory
```

```
path = f"{os.getenv('MEK1100')}/Oblig1/images"

# Chose an odd number to include the point in the middle where there is no flow.
n_val = 11

# Gets values x, y, u and v, then plots them into the vector field.
x, y, u, v = velfield(n_val)
plt.quiver(x, y, u, v)
plt.title(f"Vector field with n={n_val}")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.savefig(f"{path}/vec_{n_val}.png")
```

Koden over gir vektorfeltet under: