Obligatorisk oppgave 1, MEK1100, Vår 2021

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Oppgave 1

$$x(t) = v_0 \cdot t \cdot \cos(\theta)$$
$$y(t) = v_0 \cdot t \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t^2$$

a)
For å finne t_m, så må vi finne nullpunktene til $y(t)$:

$$\begin{aligned} y(t) &= 0 \\ v_0 \cdot t \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t^2 &= 0 \\ t \cdot (v_0 \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t) &= 0 \end{aligned} \tag{1}$$

$$t_0 = 0 \tag{2}$$

$$\begin{aligned} v_0 \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t_m &= 0 \\ v_0 \cdot \sin(\theta) &= \frac{1}{2} \cdot g \cdot t_m \\ t_m &= \frac{2 \cdot v_0 \cdot \sin(\theta)}{g} \end{aligned} \tag{3}$$

$$\begin{aligned} x_m &= x(t_m) \\ &= v_0 \cdot \frac{2 \cdot v_0 \cdot \sin(\theta)}{g} \cdot \cos(\theta) \\ &= \frac{2 \cdot v_0^2 \cdot \sin(\theta) \cdot \cos(\theta)}{g} \end{aligned} \tag{4}$$

b)
Først så må man ha noen definisjoner:

$$T = t_m = \frac{2 \cdot v_0 \cdot \sin(\theta)}{g}$$

$$H = x_m = \frac{2 \cdot v_0^2 \cdot \sin(\theta) \cdot \cos(\theta)}{g}$$

$$V = \frac{H}{T} = \frac{\frac{2 \cdot v_0^2 \cdot \sin(\theta) \cdot \cos(\theta)}{g}}{\frac{2 \cdot v_0 \cdot \sin(\theta)}{g}} = v_0 \cdot \cos(\theta)$$

$$G = \frac{V}{T} = \frac{v_0 \cdot \cos(\theta)}{\frac{2 \cdot v_0 \cdot \sin(\theta)}{g}} = \frac{g \cdot \cos(\theta)}{2 \cdot \sin(\theta)}$$

Med dette, så kan man sette inn x^*, y^* og t^*

$$\begin{aligned}
x^* &= \frac{v_0 \cdot t \cdot \cos(\theta)}{H} \\
&= \frac{v_0 \cdot \frac{t}{T} \cdot \cos(\theta)}{\frac{H}{T}} \\
&= \frac{v_0 \cdot t^* \cdot \cos(\theta)}{V} \\
&= \frac{v_0 \cdot t^* \cdot \cos(\theta)}{v_0 \cdot \cos(\theta)} \\
&= t^*
\end{aligned} \tag{5}$$

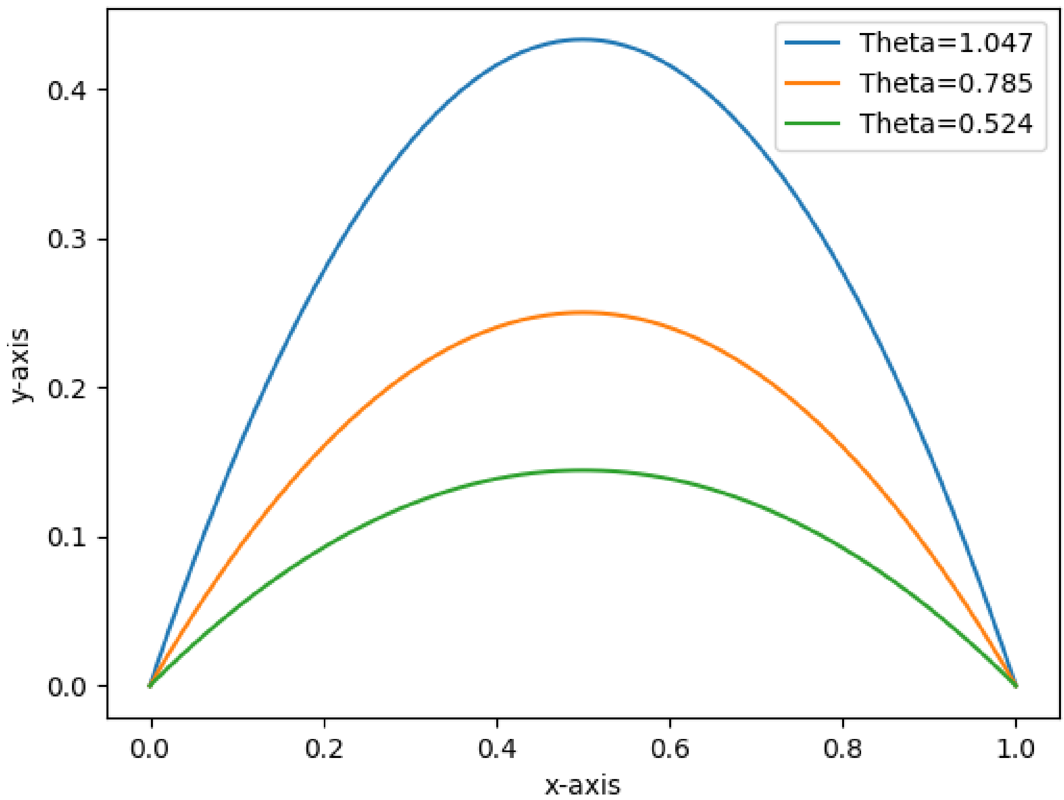
$$\begin{aligned}
y^* &= \frac{v_0 \cdot t \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t^2}{H} \\
&= \frac{v_0 \cdot \frac{t}{T} \cdot \sin(\theta)}{\frac{H}{T}} - \frac{\frac{1}{2} \cdot g \cdot \frac{t^2}{T^2}}{\frac{H}{T^2}} \\
&= \frac{v_0 \cdot t^* \cdot \sin(\theta)}{V} - \frac{\frac{1}{2} \cdot g \cdot (t^*)^2}{G} \\
&= \frac{v_0 \cdot t^* \cdot \sin(\theta)}{v_0 \cdot \cos(\theta)} - \frac{\frac{1}{2} \cdot g \cdot (t^*)^2}{\frac{g \cdot \cos(\theta)}{2 \cdot \sin(\theta)}} \\
&= t^* \cdot \frac{\sin(\theta)}{\cos(\theta)} - (t^*)^2 \cdot \frac{\sin(\theta)}{\cos(\theta)} \\
&= t^* \cdot \tan(\theta) \cdot (1 - t^*)
\end{aligned}$$

Man trenger ikke å skalere for θ ettersom den ikke har noen dimensjoner.

c)
Kode:

```
import numpy as np
import matplotlib.pyplot as plt
import os

# Accessing an environment variable that points to the MEK1100 directory
img_path = f"{os.getenv('MEK1100')}/Oblig1/images"

# function that plots x*, y* with t* and different values for theta
def plot(theta):
    t_star = np.linspace(0,1,101)

    x_star = lambda t, theta: t
    y_star = lambda t, theta: t*(np.tan(theta)) - (t**2*np.tan(theta))

    plt.plot(x_star(t_star, theta), y_star(t_star, theta), label=f"Theta={theta:.3f}")


# List of different valuesof theta
theta_list = [np.pi/3, np.pi/4, np.pi/6]

for i in theta_list:
    plot(i)
    plt.legend()
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{img_path}/one_c_revised.png")
```

Grafen som koden produserer:



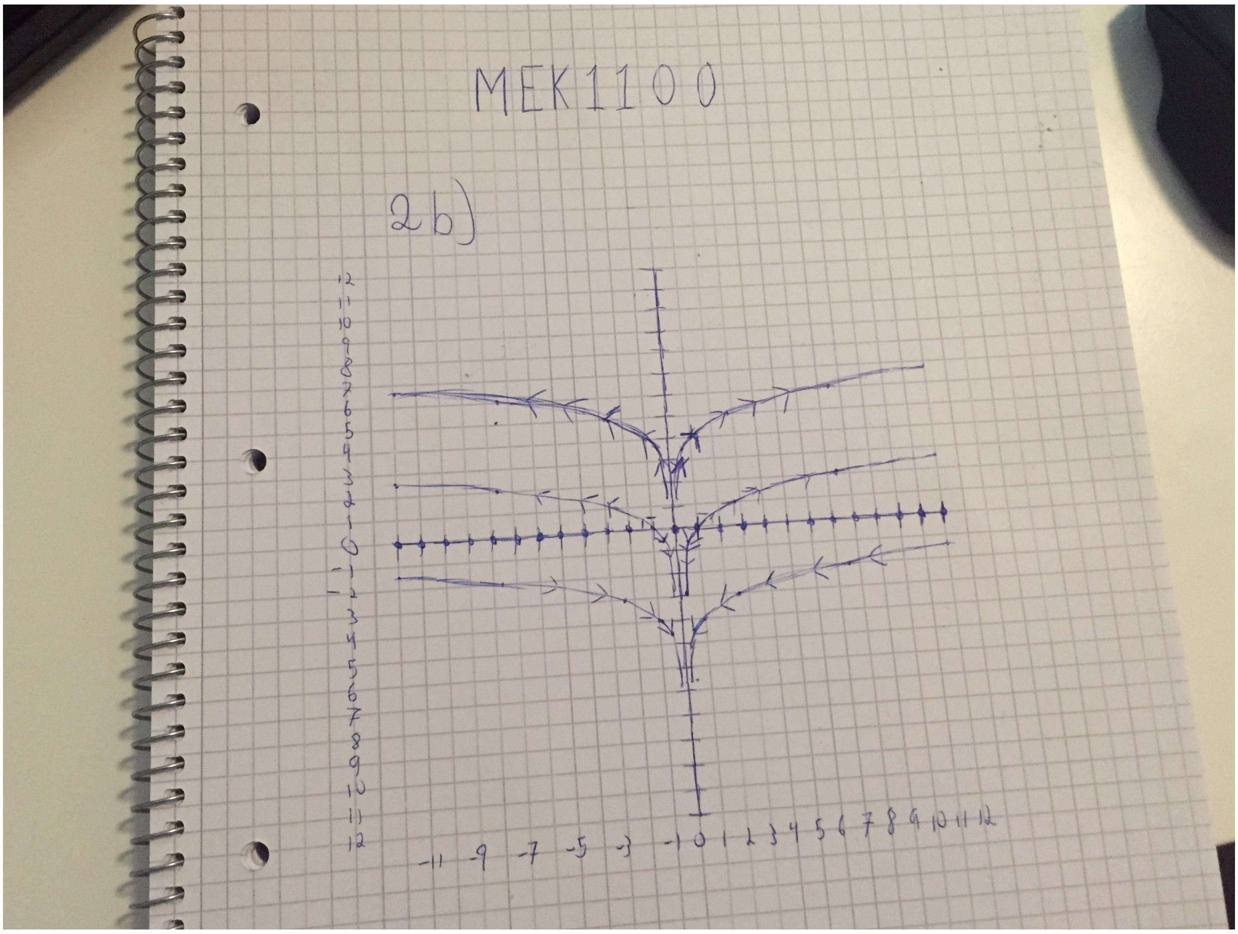
Oppgave 2

$$\mathbf{v} = (x \cdot y, y) \tag{6}$$

a)
For å finne strømningslinjene, så må vi sette opp likningen:

$$\begin{aligned} \frac{dy}{dx} &= \frac{y}{x \cdot y} \\ dy &= \frac{1}{x} dx \\ y &= \ln|x| + c \end{aligned} \tag{7}$$

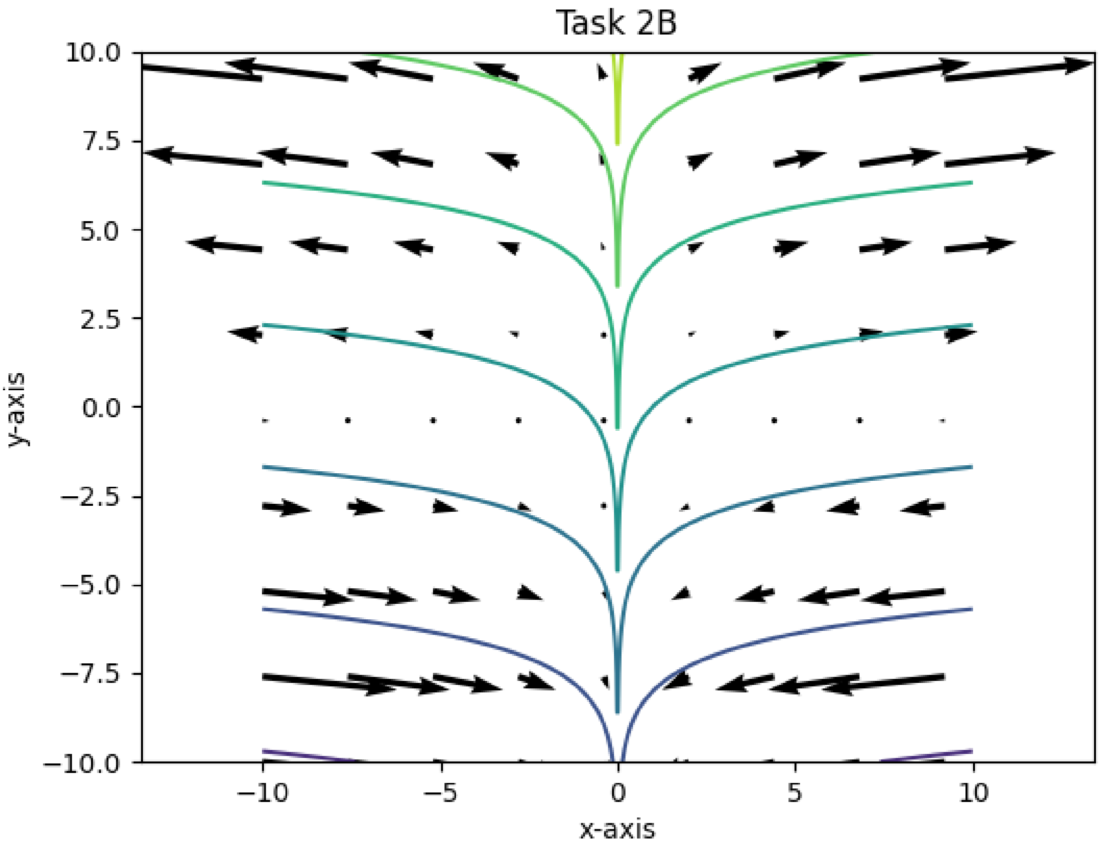
b)



Python program:

```python
import numpy as np
import matplotlib.pyplot as plt
import os

# Accessing an environment variable that points to the
    MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"


def mesh_grid(start, stop, dt):
    # Create a linspace
    I = np.linspace(start, stop, dt)

    # Create a meshgrid that uses the linspace dimensions
    x, y = np.meshgrid(I, I)

    return x, y


def vec_field(x, y, u, v, density):
    # Variable that tells how much to divide the number of
    elements in the mesh by.
    skip = (slice(None, None, density), slice(None, None,
    density))

    # Returns the vectorfield with the correct density.
    return u[skip], v[skip], skip


def streamlines(x, y, func):
    # Returns a meshgrid
    return func


if __name__ == "__main__":
    x, y = mesh_grid(-10, 10, 1000)

    u, v, skip = vec_field(x, y, x*y, y, 120)

    f = streamlines(x, y, y - np.log(abs(x)))

    plt.quiver(x[skip], y[skip], u, v)
    plt.contour(x, y, f, 6)
    plt.axis('equal')
    plt.title("Task 2B")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/two_b.png")
```

Output:



c)
Hvis en strømfunksjon eksisterer, så må vektorfeltet være divergensfritt.

$$\begin{aligned}\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\ &= \frac{\partial}{\partial x} x \cdot y + \frac{\partial}{\partial y} y \\ &= y + 1 \\ &\neq 0\end{aligned} \tag{8}$$

Siden divergensen til v ikke er 0, så eksisterer det ingen strømfunksjon.

Oppgave 3

$$\mathbf{v} = (\cos(x) \cdot \sin(y), -\sin(x) \cdot \cos(y)) \tag{9}$$

a)
Divergensen blir lik:

$$\begin{aligned}
\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\
&= \frac{\partial}{\partial x} \cos(x) \cdot \sin(y) + \frac{\partial}{\partial y} - \sin(x) \cdot \cos(y) \\
&= -\sin(x) \cdot \sin(y) + \sin(x) \cdot \sin(y) \\
&= 0
\end{aligned} \tag{10}$$

Virvlingen blir lik:

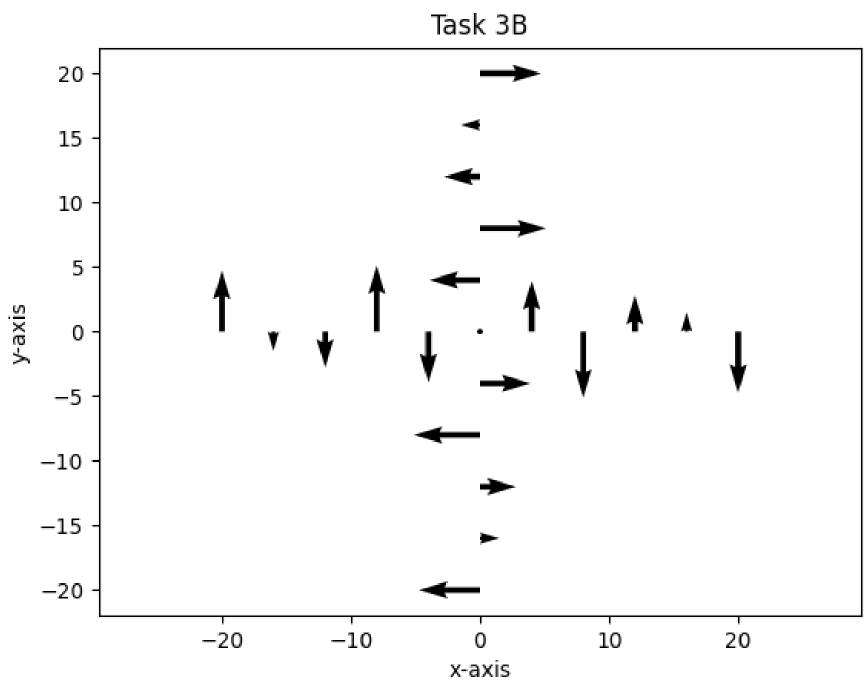
$$\begin{aligned}
\nabla \times \mathbf{v} &= \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \cdot \mathbf{k} \\
&= \left(\frac{\partial}{\partial x} - \sin(x) \cdot \cos(y) - \frac{\partial}{\partial y} \cos(x) \cdot \sin(y) \right) \cdot \mathbf{k} \\
&= (-\cos(x) \cdot \cos(y) - cos(x) \cdot \cos(y)) \cdot \mathbf{k} \\
&= (-2 \cdot cos(x) \cdot \cos(y)) \cdot \mathbf{k}
\end{aligned} \tag{11}$$

b)
Python kode:

```python
import numpy as np
import matplotlib.pyplot as plt
import os

# Accessing an environment variable that points to the MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"

if __name__ == "__main__":

    # Since we are dealing with only the x-axis and y-axis,
    # there is no need to have a mesh grid.
    # we can make an array containing only zeros, and one
    # with values and use those to plot vectors in a field.
```

```
    zeros = np.zeros(11)
    vals = [2*i for i in range(-10, 11, 2)]

    # Defining u and v
    u = lambda x,y: np.cos(x)*np.sin(y)
    v = lambda x,y: -np.sin(x)*np.cos(y)

    # Plot the vectors on the axes.
    plt.quiver(zeros, vals, u(zeros, vals), v(zeros,vals))
    plt.quiver(vals, zeros, u(vals, zeros), v(vals,zeros))
    plt.axis('equal')
    plt.title("Task 3B")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/three_b.png")
```

Feltet som blir produsert:



c)

$$\mathbf{F} = \mathbf{v} \tag{12}$$

Man kan først definere randa til kvadratet med 4 parametere:

$$\begin{aligned}
\mathbf{r}_1(t) &= \left(\frac{\pi}{2}, t\right) \\
\mathbf{r}_2(t) &= \left(-\frac{\pi}{2}, t\right) \\
\mathbf{r}_3(t) &= \left(t, \frac{\pi}{2}\right) \\
\mathbf{r}_4(t) &= \left(t, -\frac{\pi}{2}\right)
\end{aligned} \tag{13}$$

Deretter tar man linjeintegralet til $\mathbf{F}$ med hver parameter og adderer dem sammen:

$$\begin{aligned}
\oint_C &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t) \cdot j \; dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_2(t)) \cdot \mathbf{r}_2'(t) \cdot -j \; dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_3(t)) \cdot \mathbf{r}_3'(t) \cdot -i \; dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_4(t)) \cdot \mathbf{r}_4'(t) \cdot i \; dt
\end{aligned} \tag{14}$$

$$\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t) \cdot j \; dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\left(\frac{\pi}{2}\right) \sin(t) \cdot i - \sin\left(\frac{\pi}{2}\right) \cos(t) \cdot j) \cdot j \; dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\cos(t) \cdot j) \cdot j \; dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{15}$$

$$
\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_2(t)) \cdot \mathbf{r}_2'(t) \cdot (-j)\ dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(-\frac{\pi}{2}) \sin(t) \cdot i - \sin(-\frac{\pi}{2}) \cos(t) \cdot j) \cdot (-j)\ dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \cdot j) \cdot (-j)\ dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_3(t)) \cdot \mathbf{r}_3'(t) \cdot (-i)\ dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \sin\left(\frac{\pi}{2}\right) \cdot i - \sin(t) \cos\left(\frac{\pi}{2}\right) \cdot j) \cdot (-i)\ dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \cdot i) \cdot (-i)\ dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_4(t)) \cdot \mathbf{r}_4'(t) \cdot i\ dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \sin\left(-\frac{\pi}{2}\right) \cdot i - \sin(t) \cos\left(-\frac{\pi}{2}\right) \cdot j) \cdot i\ dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\cos(t) \cdot i) \cdot i\ dt \\
&= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned} \tag{18}
$$

Sirkulasjonen blir da:

$$
\oint_C = -2 + (-2) + (-2) + (2) = -8 \tag{19}
$$

d)
Siden $\nabla \cdot \mathbf{v} = 0$, så betyr det at feltet er konservativt og derfor finnes det en strømfunksjon for feltet.
For å regne ut ψ, så må man først finne $\int \frac{\partial \psi}{\partial x}$ og $\int \frac{\partial \psi}{\partial y}$

$$\begin{aligned}\int \frac{\partial \psi}{\partial y} &= \int v_x \; dy \\ &= \int \cos(x) \cdot \sin(y) \; dy \\ &= \cos(x) \cdot \cos(y) + f(x)\end{aligned} \tag{20}$$

$$\begin{aligned}\int \frac{\partial \psi}{\partial x} &= \int v_y dx \\ &= \int - \sin(x) \cdot \cos(y) \; dx \\ &= \cos(x) \cdot \cos(y) + g(y)\end{aligned} \tag{21}$$

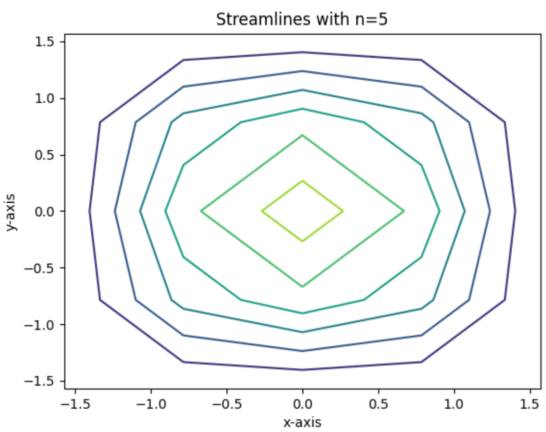
Ut i fra det vi har regnet ut, så ser vi at strømningsfunksjonen blir $\psi = \cos(x) \cdot \cos(y)$.

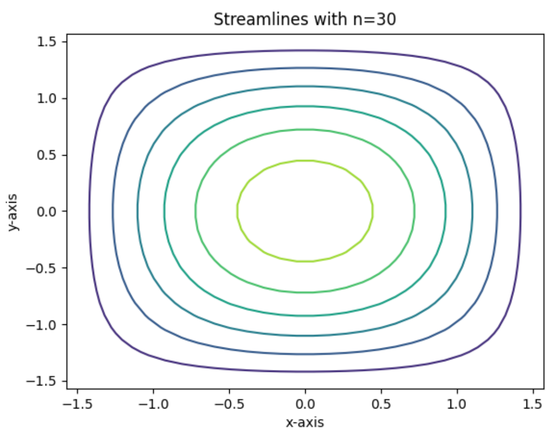
Oppgave 4

a)

```python
import matplotlib.pyplot as plt
from streamfun import streamfun
import os

# Accessing an environment variable that points to the
    MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"

# different values of n
n_vals = [5, 30]

# loops through different values for n and outputs .png files
for i in n_vals:
    x, y, psi = streamfun(i)
    plt.clf()
    plt.contour(x, y, psi)
    plt.title(f"Streamlines with n={i}")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/strlin_{i}.png")
```

Koden over gir to plotter:





b)

```python
import matplotlib.pyplot as plt
import os
from velfield import velfield

# Accessing an environment variable that points to the
    MEK1100 directory
```

```python
path = f"{os.getenv('MEK1100')}/Oblig1/images"

# Chose an odd number to include the point in the middle where there is no flow.
n_val = 11

# Gets values x, y, u and v, then plots them into the vector field.
x, y, u, v = velfield(n_val)
plt.quiver(x, y, u, v)
plt.title(f"Vector field with n={n_val}")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.savefig(f"{path}/vec_{n_val}.png")
```

Koden over gir vektorfeltet under: