Obligatorisk oppgave 1, MEK1100, Vår 2021

Cory Alexander Balaton

Oppgave 1

$$x(t) = v_0 \cdot t \cdot \cos(\theta)$$
$$y(t) = v_0 \cdot t \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t^2$$

a)

For å finne t_m, så må vi finne nullpunktene til $y(t)$:

$$y(t) = 0$$

$$v_0 \cdot t \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t^2 = 0 \tag{1}$$

$$t \cdot (v_0 \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t) = 0$$

$$t_0 = 0 \tag{2}$$

$$v_0 \cdot \sin(\theta) - \frac{1}{2} \cdot g \cdot t_m = 0$$

$$v_0 \cdot \sin(\theta) = \frac{1}{2} \cdot g \cdot t_m \tag{3}$$

$$t_m = \frac{2 \cdot v_0 \cdot \sin(\theta)}{g}$$

$$
\begin{aligned}
x_m &= x(t_m) \\
&= v_0 \cdot \frac{2 \cdot v_0 \cdot \sin(\theta)}{g} \cdot \cos(\theta) \\
&= \frac{2 \cdot v_0^2 \cdot \sin(\theta) \cdot \cos(\theta)}{g}
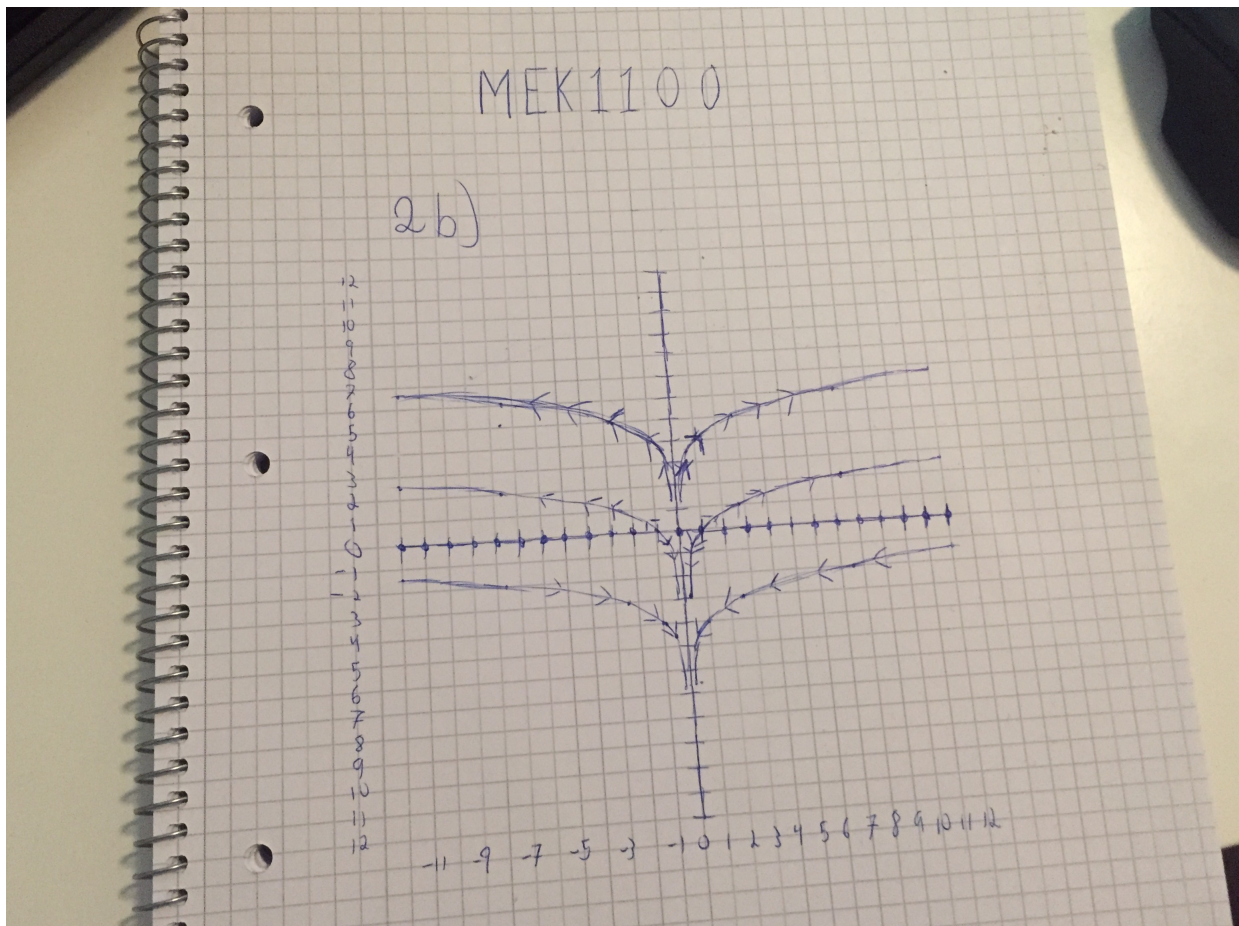\end{aligned}
\tag{4}
$$

Oppgave 2

$$\mathbf{v} = (x \cdot y, y) \tag{5}$$

a)

For å finne strømningslinjene, så må vi sette opp likningen:

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{y}{x \cdot y} \\
dy &= \frac{1}{x}dx \\
y &= \ln|x| + c
\end{aligned}
\tag{6}
$$

b)

3

Python program:

```python
import numpy as np
import matplotlib.pyplot as plt
import os

# Accessing an environment variable that points to the
    MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"


def mesh_grid(start, stop, dt):
    # Create a linspace
    I = np.linspace(start, stop, dt)

    # Create a meshgrid that uses the linspace dimensions
    x, y = np.meshgrid(I, I)

    return x, y


def vec_field(x, y, u, v, density):
    # Variable that tells how much to divide the number of
    elements in the mesh by.
    skip = (slice(None, None, density), slice(None, None,
    density))

    # Returns the vectorfield with the correct density.
    return u[skip], v[skip], skip


def streamlines(x, y, func):
    # Returns a meshgrid
    return func


if __name__ == "__main__":
    x, y = mesh_grid(-10, 10, 1000)

    u, v, skip = vec_field(x, y, x*y, y, 120)

    f = streamlines(x, y, y - np.log(abs(x)))

    plt.quiver(x[skip], y[skip], u, v)
    plt.contour(x, y, f, 6)
    plt.axis('equal')
    plt.title("Task 2B")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/two_b.png")
```

Output:

Task 2B
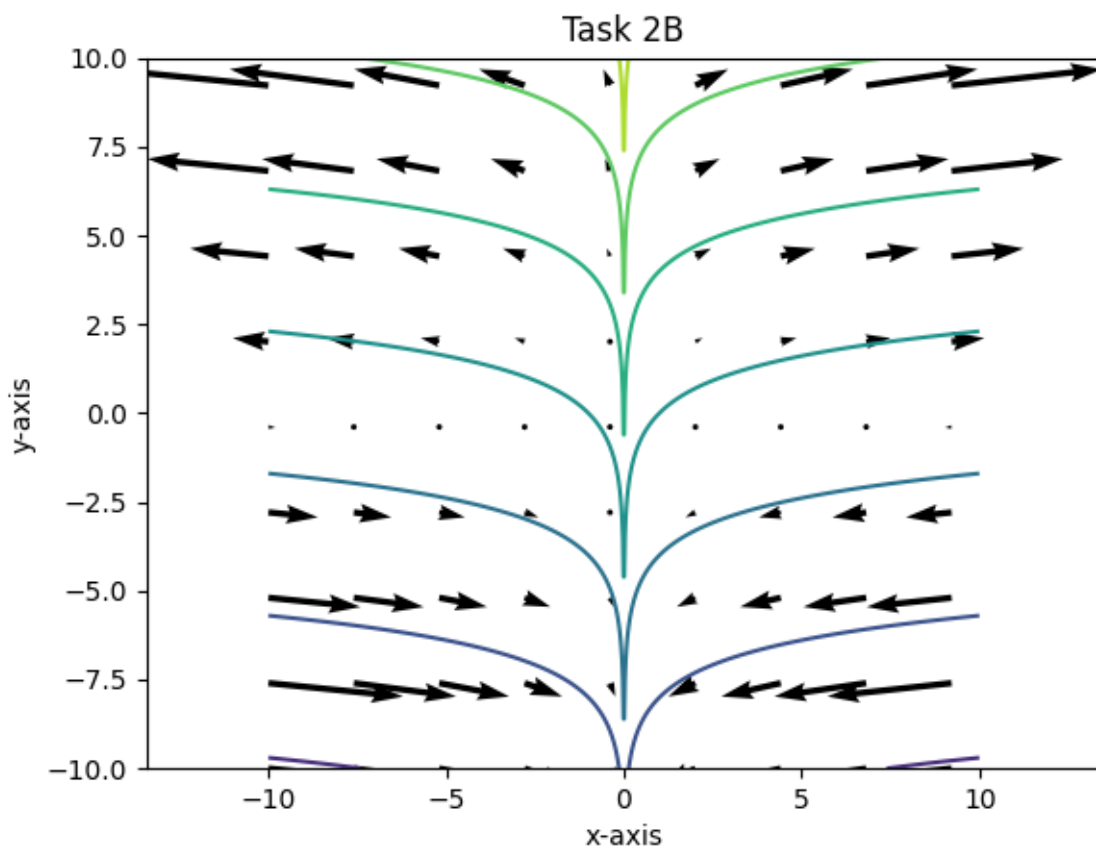
c)
Hvis en strømfunksjon eksisterer, så må vektorfeltet være divergensfritt.

$$\begin{aligned}
\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\
&= \frac{\partial}{\partial x} x \cdot y + \frac{\partial}{\partial y} y \\
&= y + 1 \\
&\neq 0
\end{aligned} \tag{7}$$

Siden divergensen til v ikke er 0, så eksisterer det ingen strømfunksjon.

Oppgave 3

$$\mathbf{v} = (\cos(x) \cdot \sin(y), -\sin(x) \cdot \cos(y)) \tag{8}$$

a)

Divergensen blir lik:

$$
\begin{aligned}
\nabla \cdot \mathbf{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \\
&= \frac{\partial}{\partial x} \cos(x) \cdot \sin(y) + \frac{\partial}{\partial y} - \sin(x) \cdot \cos(y) \\
&= -\sin(x) \cdot \sin(y) + \sin(x) \cdot \sin(y) \\
&= 0
\end{aligned}
\tag{9}
$$

Virvlingen blir lik:

$$
\begin{aligned}
\nabla \times \mathbf{v} &= \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \cdot \mathbf{k} \\
&= \left(\frac{\partial}{\partial x} - \sin(x) \cdot \cos(y) - \frac{\partial}{\partial y} \cos(x) \cdot \sin(y)\right) \cdot \mathbf{k} \\
&= (-\cos(x) \cdot \cos(y) - cos(x) \cdot \cos(y)) \cdot \mathbf{k} \\
&= (-2 \cdot cos(x) \cdot \cos(y)) \cdot \mathbf{k}
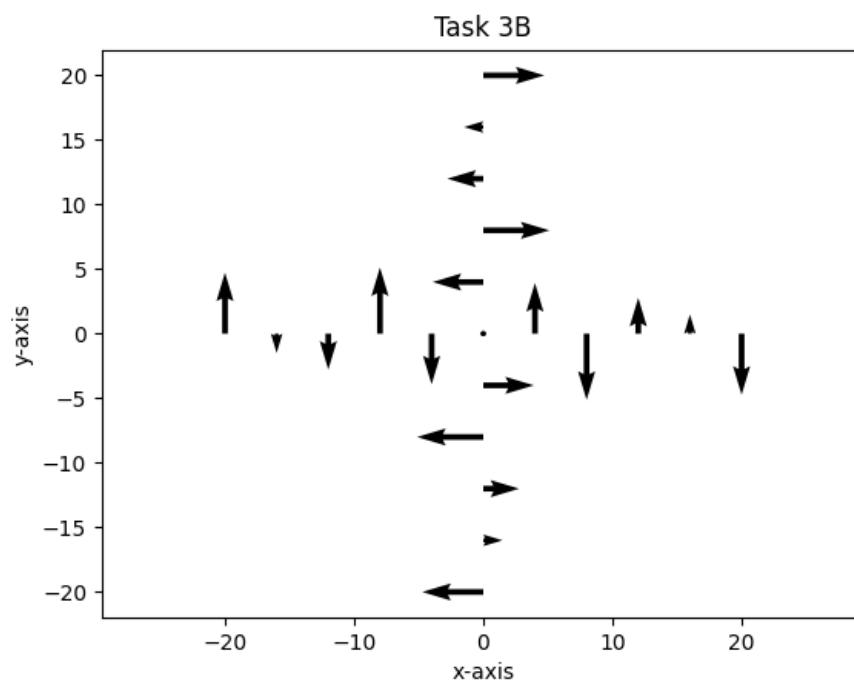\end{aligned}
\tag{10}
$$

b)

Python kode:

```python
import numpy as np
import matplotlib.pyplot as plt
import os

# Accessing an environment variable that points to the
    MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"

if __name__ == "__main__":

    # Since we are dealing with only the x-axis and y-axis,
    # there is no need to have a mesh grid.
    # we can make an array containing only zeros, and one
    # with values and use those to plot vectors in a field.
```

```
14    zeros = np.zeros(11)
15    vals = [2*i for i in range(-10, 11, 2)]
16
17    # Defining u and v
18    u = lambda x,y: np.cos(x)*np.sin(y)
19    v = lambda x,y: -np.sin(x)*np.cos(y)
20
21    # Plot the vectors on the axes.
22    plt.quiver(zeros, vals, u(zeros, vals), v(zeros,vals))
23    plt.quiver(vals, zeros, u(vals, zeros), v(vals,zeros))
24    plt.axis('equal')
25    plt.title("Task 3B")
26    plt.xlabel("x-axis")
27    plt.ylabel("y-axis")
28    plt.savefig(f"{path}/three_b.png")
```

Feltet som blir produsert:

c)

$$\mathbf{F} = \mathbf{v} \tag{11}$$

Man kan først definere randa til kvadratet med 4 parametere:

$$
\begin{aligned}
\mathbf{r}_1(t) &= \left(\frac{\pi}{2}, t\right) \\
\mathbf{r}_2(t) &= \left(-\frac{\pi}{2}, t\right) \\
\mathbf{r}_3(t) &= \left(t, \frac{\pi}{2}\right) \\
\mathbf{r}_4(t) &= \left(t, -\frac{\pi}{2}\right)
\end{aligned}
\tag{12}
$$

Deretter tar man linjeintegralet til $\mathbf{F}$ med hver parameter og adderer dem sammen:

$$
\begin{aligned}
\oint_C &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t) \cdot j \, dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_2(t)) \cdot \mathbf{r}_2'(t) \cdot -j \, dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_3(t)) \cdot \mathbf{r}_3'(t) \cdot -i \, dt \\
&+ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_4(t)) \cdot \mathbf{r}_4'(t) \cdot i \, dt
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_1(t)) \cdot \mathbf{r}_1'(t) \cdot j \, dt &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cos\left(\frac{\pi}{2}\right)\sin(t) \cdot i - \sin\left(\frac{\pi}{2}\right)\cos(t) \cdot j\right) \cdot j \, dt \\
&= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\cos(t) \cdot j) \cdot j \, dt \\
&= \left[-\sin(t)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= -1 - 1 \\
&= -2
\end{aligned}
\tag{14}
$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_2(t)) \cdot \mathbf{r}_2'(t) \cdot (-j) \, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(-\frac{\pi}{2})\sin(t) \cdot i - \sin(-\frac{\pi}{2})\cos(t) \cdot j) \cdot (-j) \, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \cdot j) \cdot (-j) \, dt$$

$$= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= -1 - 1$$

$$= -2$$

$$\tag{15}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_3(t)) \cdot \mathbf{r}_3'(t) \cdot (-i) \, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t)\sin\left(\frac{\pi}{2}\right) \cdot i - \sin(t)\cos\left(\frac{\pi}{2}\right) \cdot j) \cdot (-i) \, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t) \cdot i) \cdot (-i) \, dt$$

$$= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= -1 - 1$$

$$= -2$$

$$\tag{16}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \mathbf{F}(\mathbf{r}_4(t)) \cdot \mathbf{r}_4'(t) \cdot i \, dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos(t)\sin\left(-\frac{\pi}{2}\right) \cdot i - \sin(t)\cos\left(-\frac{\pi}{2}\right) \cdot j) \cdot i \, dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-\cos(t) \cdot i) \cdot i \, dt$$

$$= [-\sin(t)]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= -1 - 1$$

$$= -2$$

$$\tag{17}$$

Sirkulasjonen blir da:

$$\oint_C = -2 + (-2) + (-2) + (2) = -8 \tag{18}$$

d)
Siden $\nabla \cdot \mathbf{v} = 0$, så betyr det at feltet er konservativt og derfor finnes det en strømfunksjon for feltet.
For å regne ut ψ, så må man først finne $\int \frac{\partial \psi}{\partial x}$ og $\int \frac{\partial \psi}{\partial y}$

$$\int \frac{\partial \psi}{\partial y} = \int v_x dy$$
$$= \int \cos(x) \cdot \sin(y) dx \tag{19}$$
$$= \cos(x) \cdot \cos(y) + f(x)$$

$$\int \frac{\partial \psi}{\partial x} = \int v_y dx$$
$$= \int -\sin(x) \cdot \cos(y) dx \tag{20}$$
$$= \cos(x) \cdot \cos(y) + g(y)$$

Ut i fra det vi har regnet ut, så ser vi at strømningsfunksjonen blir $\psi = \cos(x) \cdot \cos(y)$.

Oppgave 4

a)

```python
import matplotlib.pyplot as plt
from streamfun import streamfun
import os

# Accessing an environment variable that points to the
    MEK1100 directory
path = f"{os.getenv('MEK1100')}/Oblig1/images"

# different values of n
n_vals = [5, 30]

# loops through different values for n and outputs .png files
for i in n_vals:
    x, y, psi = streamfun(i)
    plt.clf()
    plt.contour(x, y, psi)
    plt.title(f"Streamlines with n={i}")
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.savefig(f"{path}/strlin_{i}.png")
```

Koden over gir to plotter:

b)
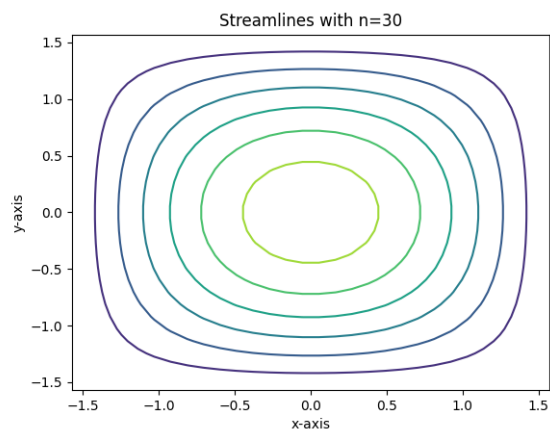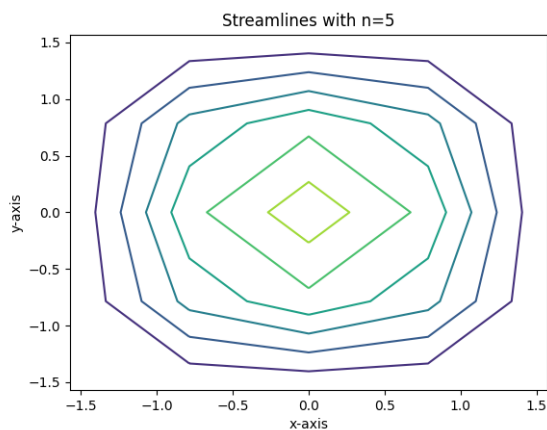
```python
import matplotlib.pyplot as plt
import os
from velfield import velfield

# Accessing an environment variable that points to the
    MEK1100 directory
```

```
6 path = f"{os.getenv('MEK1100')}/Oblig1/images"
7
8 # Chose an odd number to include the point in the middle
      where there is no flow.
9 n_val = 11
10
11 # Gets values x, y, u and v, then plots them into the vector
      field.
12 x, y, u, v = velfield(n_val)
13 plt.quiver(x, y, u, v)
14 plt.title(f"Vector field with n={n_val}")
15 plt.xlabel("x-axis")
16 plt.ylabel("y-axis")
17 plt.savefig(f"{path}/vec_{n_val}.png")
```

Koden over gir vektorfeltet under: