MFront Cohesive Zone Models for Code_Aster
MFront User Meeting

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⋆ Failure scenarios of geotechnical structures (gravity and arch **dams**) with localized **and identified** fracture patterns at some interfaces.

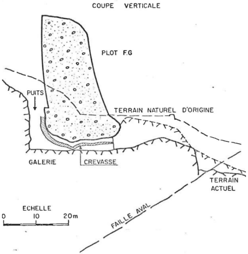
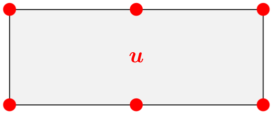


Figure: The Malpasset arch dam: failure by the sliding of the foundation (1959).

⋆ Possible **water flows** along the interface.

Primal formulation



Mixed formulation

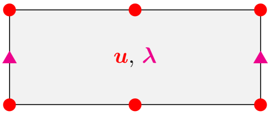
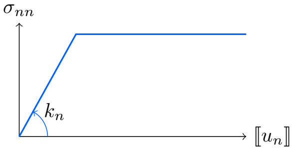
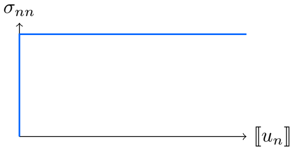


Figure: Exploded view of the joint and interface element.

$$\int_\Gamma e([\![\boldsymbol{u}]\!])d\Gamma$$

$$\int_\Gamma \left[e(\boldsymbol{\delta}) + \boldsymbol{\lambda}\cdot([\![\boldsymbol{u}]\!] - \boldsymbol{\delta}) + \frac{r}{2}\|[\![\boldsymbol{u}]\!] - \boldsymbol{\delta}\|^2\right] d\Gamma$$





- To benefit from the **mixed FE formulation** of Code_Aster with CZM constitutive equations solved by the help of MFront.

- To take into account fluid flows **along the interface**.

Outline

- A **rigid-plastic** behaviour obeying to Mohr-Coulomb plasticity with an **associated flow rule**.
- A hydromechanical coupling using the **Terzaghi** effective stress concept ($\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p\boldsymbol{I}$).

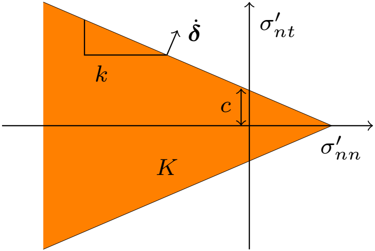
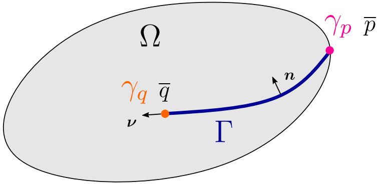


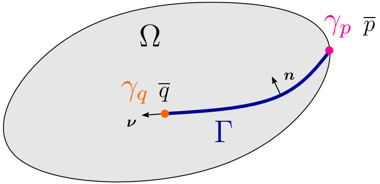
Figure: $K : f(\boldsymbol{\sigma} \cdot \boldsymbol{n}, p) = |\sigma'_{nt}| + k\sigma'_{nn} - c \leq 0.$

- A laminar flow of an incompressible fluid driven by **Darcy's law**.



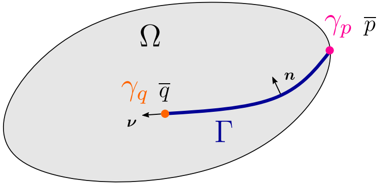
$$E_\Omega(\boldsymbol{u}, \boldsymbol{\lambda}, p, \boldsymbol{\delta}) = E_{\Omega\backslash\Gamma}(\boldsymbol{u}) +$$

$$\int_\Gamma \Big[\underbrace{\pi_K(\Delta\boldsymbol{\delta}) - p\Delta\delta_n - \frac{K}{2}\|\boldsymbol{\nabla} p\|^2\Delta t}_{e(\boldsymbol{\delta},p,\boldsymbol{\nabla} p)} + \boldsymbol{\lambda}\cdot(\llbracket\boldsymbol{u}\rrbracket - \boldsymbol{\delta}) + \frac{r}{2}\|\llbracket\boldsymbol{u}\rrbracket - \boldsymbol{\delta}\|^2\Big]d\Gamma - \int_{\gamma_q}\overline{q}p\,d\gamma$$



⋆ First order optimality conditions:

$$
\left\{
\begin{array}{llll}
\boldsymbol{u}: & \boldsymbol{\lambda} + r([\![\boldsymbol{u}]\!] - \boldsymbol{\delta}) & = \boldsymbol{\sigma} \cdot \boldsymbol{n} & (\Gamma) \\
\boldsymbol{\delta}: & \boldsymbol{\lambda} + r([\![\boldsymbol{u}]\!] - \boldsymbol{\delta}) + p\boldsymbol{n} & \in \partial \pi_K(\Delta \boldsymbol{\delta}) & (\Gamma) \\
\boldsymbol{\lambda}: & [\![\boldsymbol{u}]\!] - \boldsymbol{\delta} & = 0 & (\Gamma) \\
p: & \Delta \delta_n + div(-K \boldsymbol{\nabla} p)\Delta t & = 0 & (\Gamma), \quad (-K\boldsymbol{\nabla} p)\Delta t \cdot \boldsymbol{\nu} = \overline{q} \quad (\gamma_q)
\end{array}
\right.
$$



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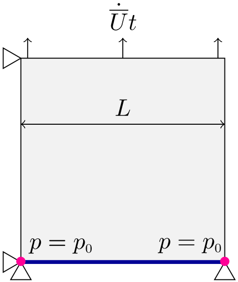
$$\left\{\begin{array}{llll} \boldsymbol{u}: & \boldsymbol{\lambda}+r([\![\boldsymbol{u}]\!]-\boldsymbol{\delta}) & =\boldsymbol{\sigma}\cdot\boldsymbol{n} & (\Gamma) \\ \boldsymbol{\delta}: & \boldsymbol{\lambda}+r([\![\boldsymbol{u}]\!]-\boldsymbol{\delta})+p\boldsymbol{n} & \in\partial\pi_K(\Delta\boldsymbol{\delta}) & (\Gamma) \quad \text{solved with MFront} \\ \boldsymbol{\lambda}: & [\![\boldsymbol{u}]\!]-\boldsymbol{\delta} & =0 & (\Gamma) \\ p: & \Delta\delta_n+div(-K\boldsymbol{\nabla}p)\Delta t & =0 & (\Gamma), \quad (-K\boldsymbol{\nabla}p)\Delta t\cdot\boldsymbol{\nu}=\overline{q} \quad (\gamma_q) \end{array}\right.$$

- A FE with a P2-P1-P2 interpolation for the **DOFs** $(\boldsymbol{u}, \boldsymbol{\lambda}, p)$.
- A monolithic Newton scheme on the DOFs at the global scale.
- An **analytic resolution** of the constitutive law on $\boldsymbol{\delta}$ at the integration points, using MFront with the **@DSLGenericBehaviour**.

Outline


$\dot{\bar{U}}t$
L
$p = p_0$
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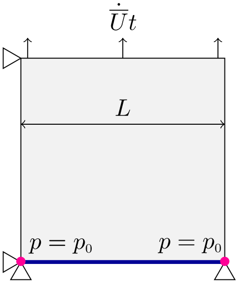
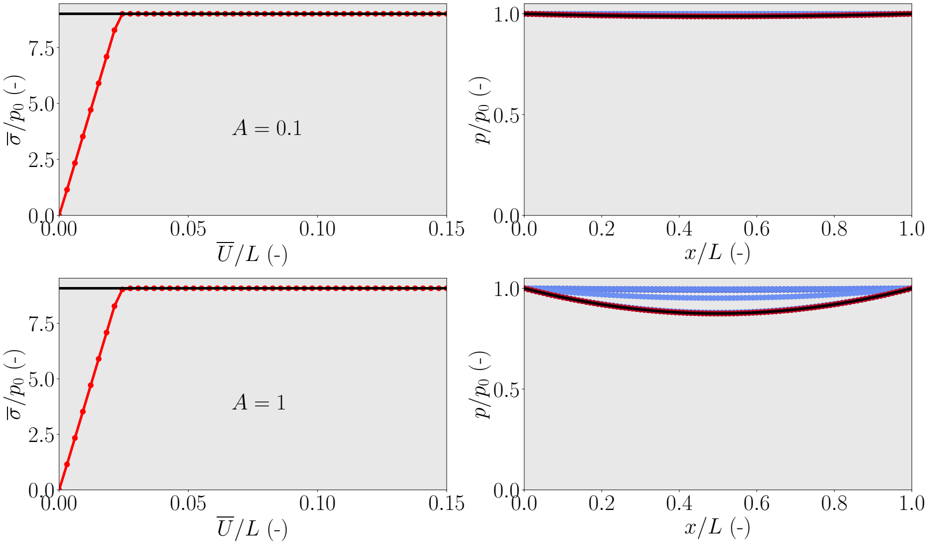


- Solution for $t \gg L^3(1-\nu^2)/(\pi^2 K E)$:

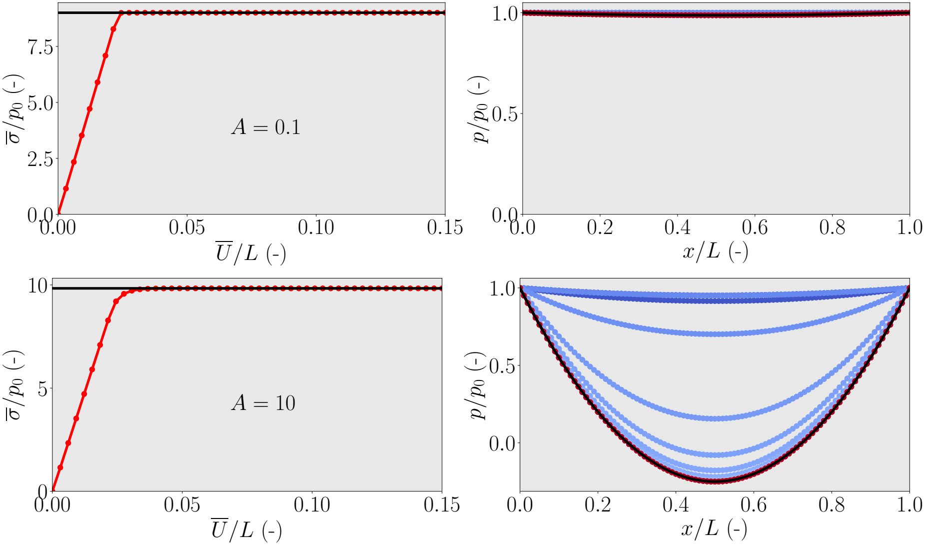
$$\sigma = \frac{c}{k} - p_0 + \frac{\dot{\overline{U}}}{12K} L^2$$
$$p(x) = p_0 \qquad + \frac{\dot{\overline{U}}}{2K} x(x-L)$$

- Dimensionless number $A = \dot{\overline{U}} L^2 / (p_0 K)$ to weight the 2 contributions.



- More "hardening" and parabolic pressure profile for higher A.

Illustration of the opening rate of the interface



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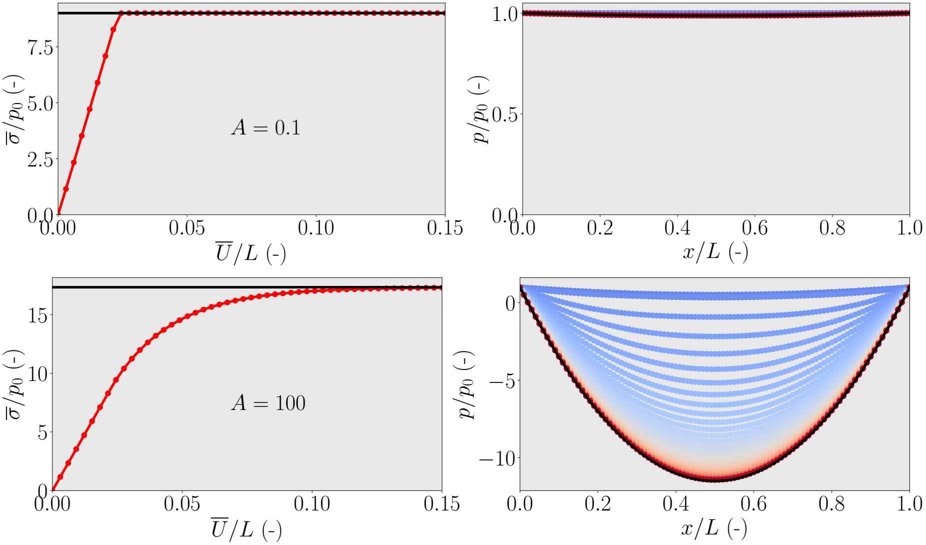

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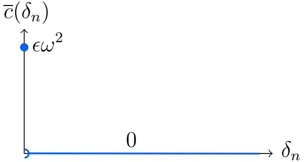
Outline

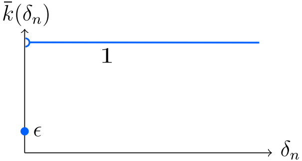
Main idea

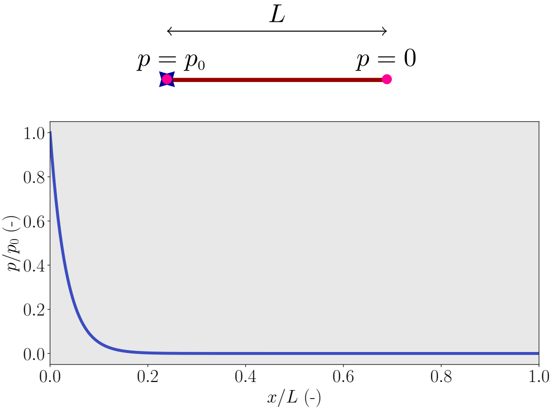
- Currently, fluid pressure profile is linear in a still-closed interface (solution of $div(-K\boldsymbol{\nabla} p) = 0$).
- Not very realistic.

- ⋆ Currently, fluid pressure profile is linear in a still-closed interface (solution of $div(-K\boldsymbol{\nabla} p) = 0$).
- ⋆ Not very realistic.
- ⋆ Main idea: change the fluid mass conservation equation for a still-closed interface:

$$\Delta\delta_n + K\left[\bar{c}(\delta_n)p + div(-\bar{k}(\delta_n)\boldsymbol{\nabla} p)\right]\Delta t = 0 \qquad \begin{cases} \bar{c}(\delta_n) & = \epsilon\omega^2 1_{\{0\}}(\delta_n) \\ \bar{k}(\delta_n) & = 1 - (1-\epsilon)1_{\{0\}}(\delta_n) \end{cases}$$

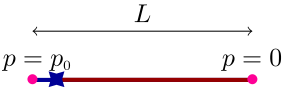


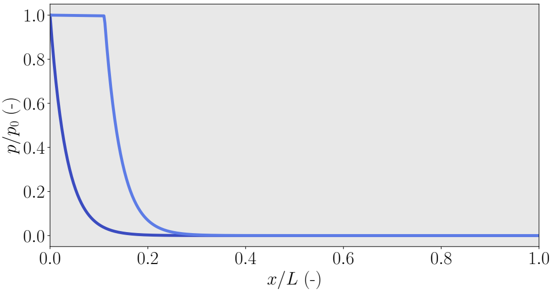




- Exponential decay in the still-closed region mainly controled by ω.

Graphical illustration (K "large")





- Exponential decay in the still-closed region mainly controled by ω.
- Slope in the open region controled by ϵ.

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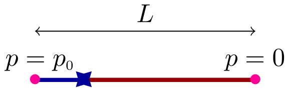

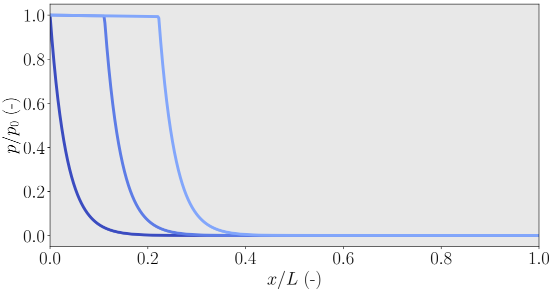

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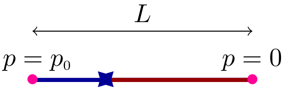

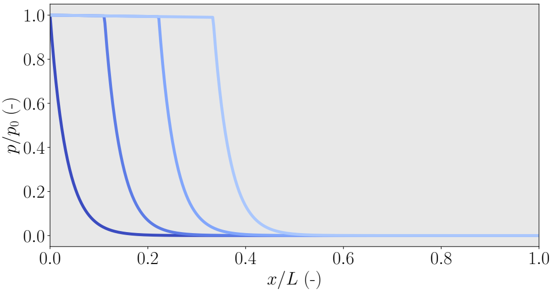

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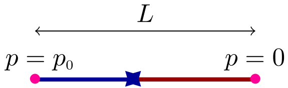

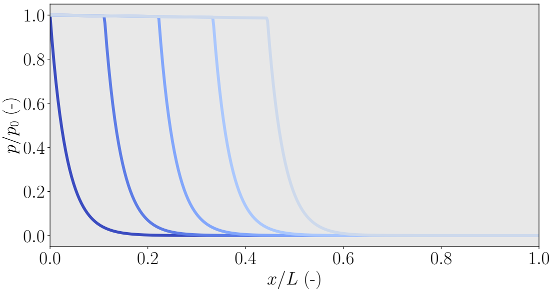

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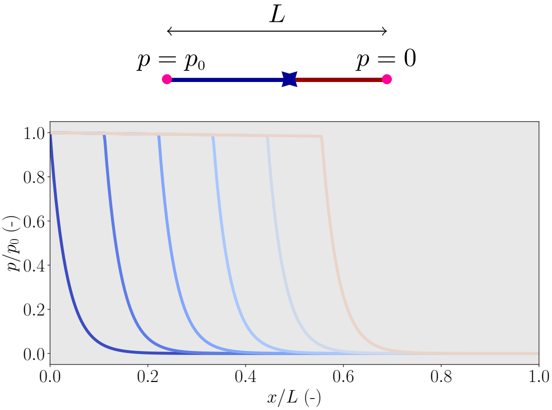

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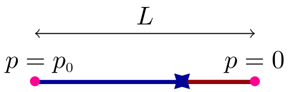

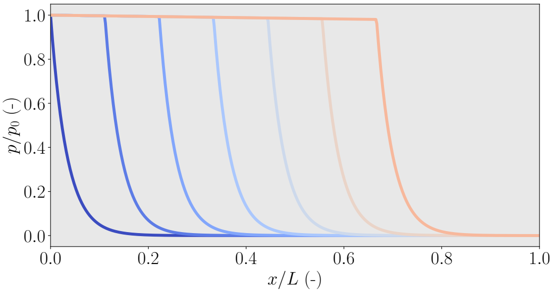

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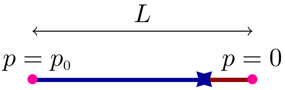
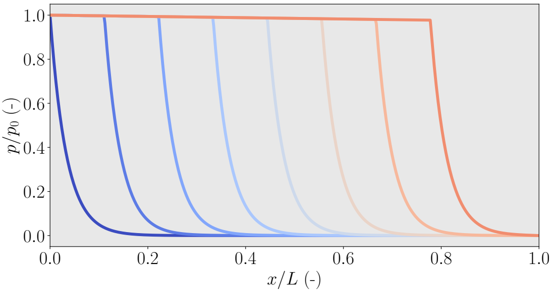
⋆ Slope in the open region controled by ϵ.





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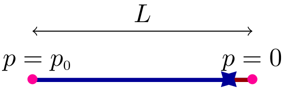

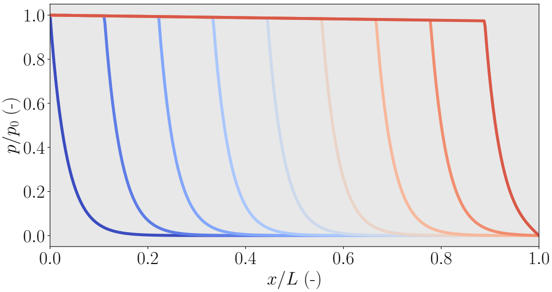

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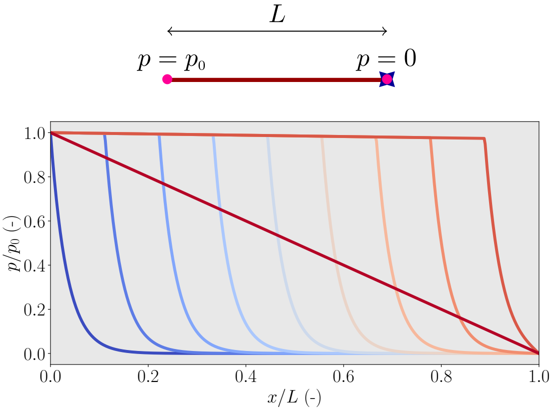


- ⋆ Exponential decay in the still-closed region mainly controled by ω.
- ⋆ Slope in the open region controled by ϵ.
- ⋆ Once fully open, the linear fluid pressure profile is recovered.

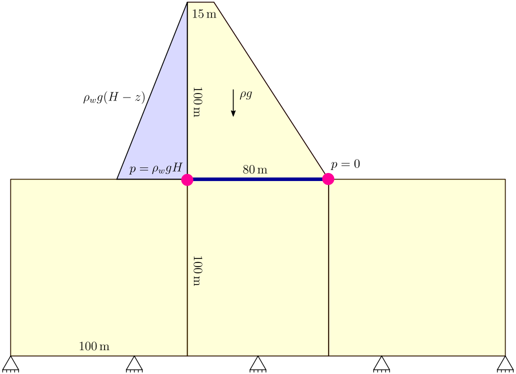
Outline

⋆ Behaviour:

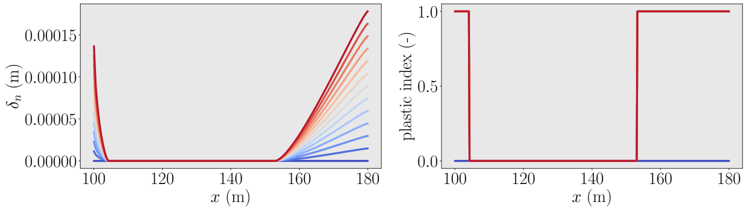
- Dam / Foundation: $E = 15\,\text{GPa}, \nu = 0.3, \rho = 2400\,\text{kg/m}^3$.
- Interface: $c = 0, k = 0.3$, K "large", $\omega = 0.1\,\text{m}^{-1}, \epsilon = 10^{-5}$.

⋆ Two steps loading:

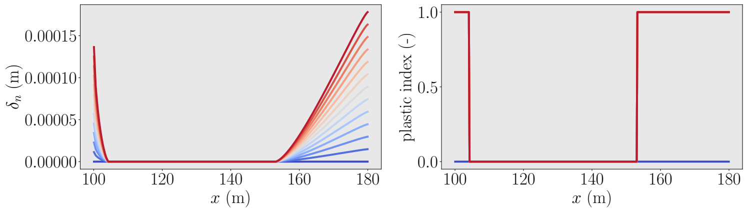
1) Dam weight applied in 5 years.
2) Water filling during 1 month.



⋆ Comparison between the simple and the more realistic CZM.

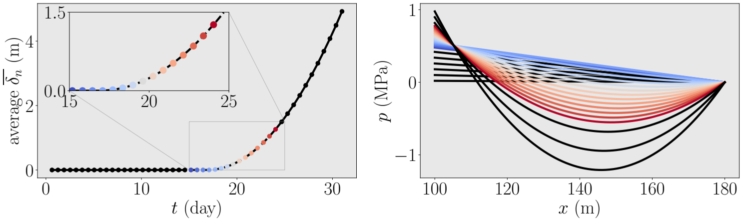


(a) Simple model.



(b) More realistic model.

- Very small opening of the interface near both extremities of the dam.



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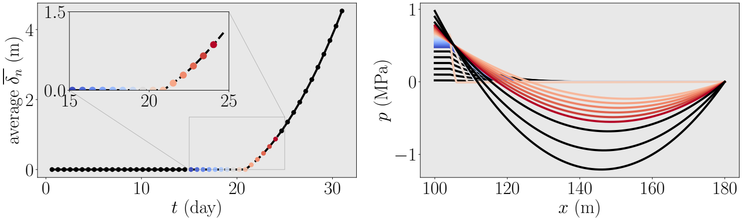
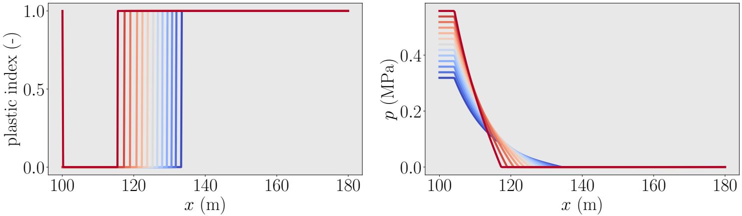
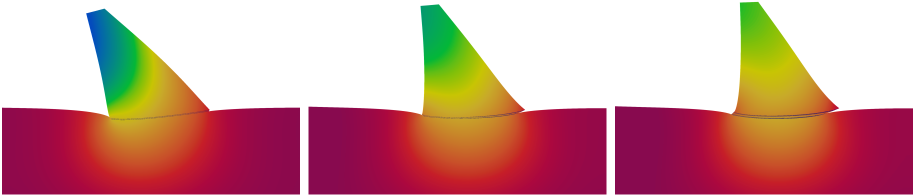


(b) More realistic model.

- ⋆ Different pressure distribution $\rightarrow$ different time for which a sudden large opening happens.



- ⋆ Opening propagates from the right dam extremity to the left, which explains the pressure profile evolution.



x1500

- A FE mixed formulation + Mohr-Coulomb plasticity + hydromechanical full coupling.
- CZM solved in MFront (@DSLGenericBehaviour).
- Description of the fluid flow for opening interfaces.
- *Numerical performances much better than penalized formulations.*

Outlooks

- Integration of a less trivial mechanical behaviour (i.e. with hardening).
- Account of a shear-driven fluid flow.
- Industrial (3D) applications.