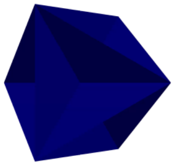
MFront User Meeting 2025

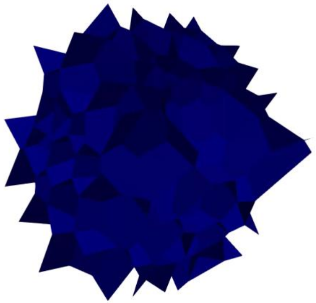
Micromechanical modelling of elasto-plastic behaviour of shell-based cellular materials by coupling FEniCS with MFront

L. Liu[1], F. Liu[1], D. Zenkert[2], M. Åkermo[2], M. Fagerström[1]

[1]Chalmers University of Technology, Sweden
[2]KTH Royal Institute of Technology, Sweden

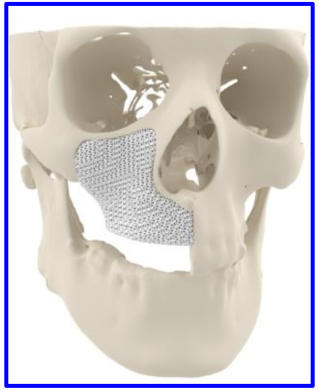
Email address: lei.liu@chalmers.se





Introduction

- Lightweight cellular materials
  - Appealing specific mechanical properties
    - High stiffness and strength
    - Energy absorption



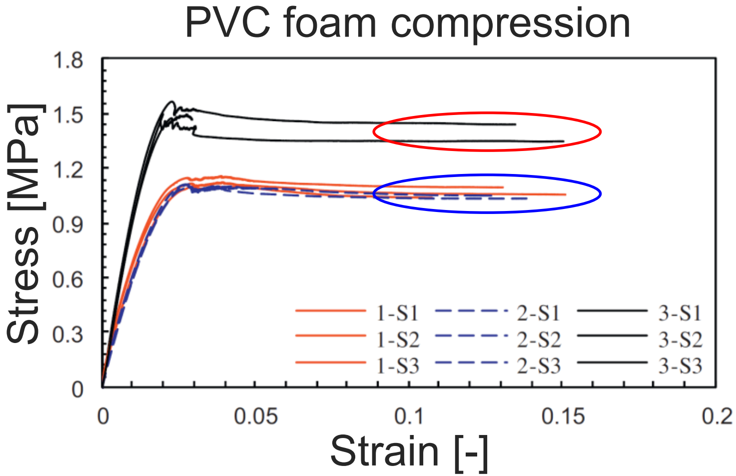
www.additivemanufacturing.media

www.sportsdirect.com

www.press.bmwgroup.com

www.amiplastics.com

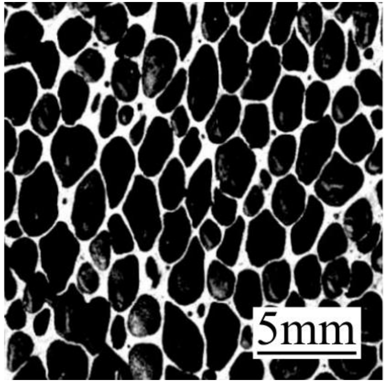
- Foams
  - Irregular and random
  - Large number of cells
  - Thin cell walls
  - Cell shape anisotropy



Shafiq et al., 2015

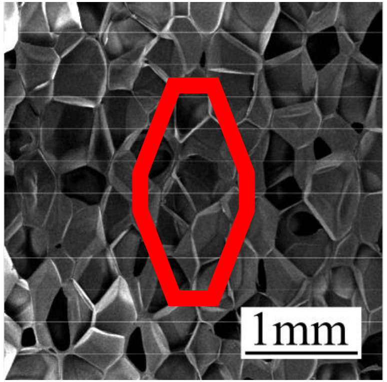
Foam rise

Aluminium



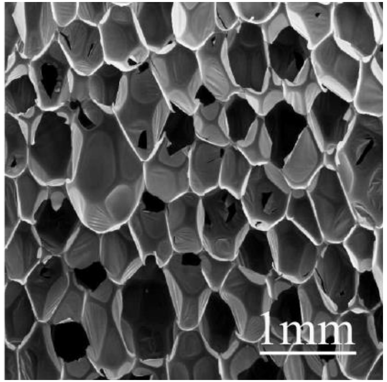
Mu et al., 2010

PVC



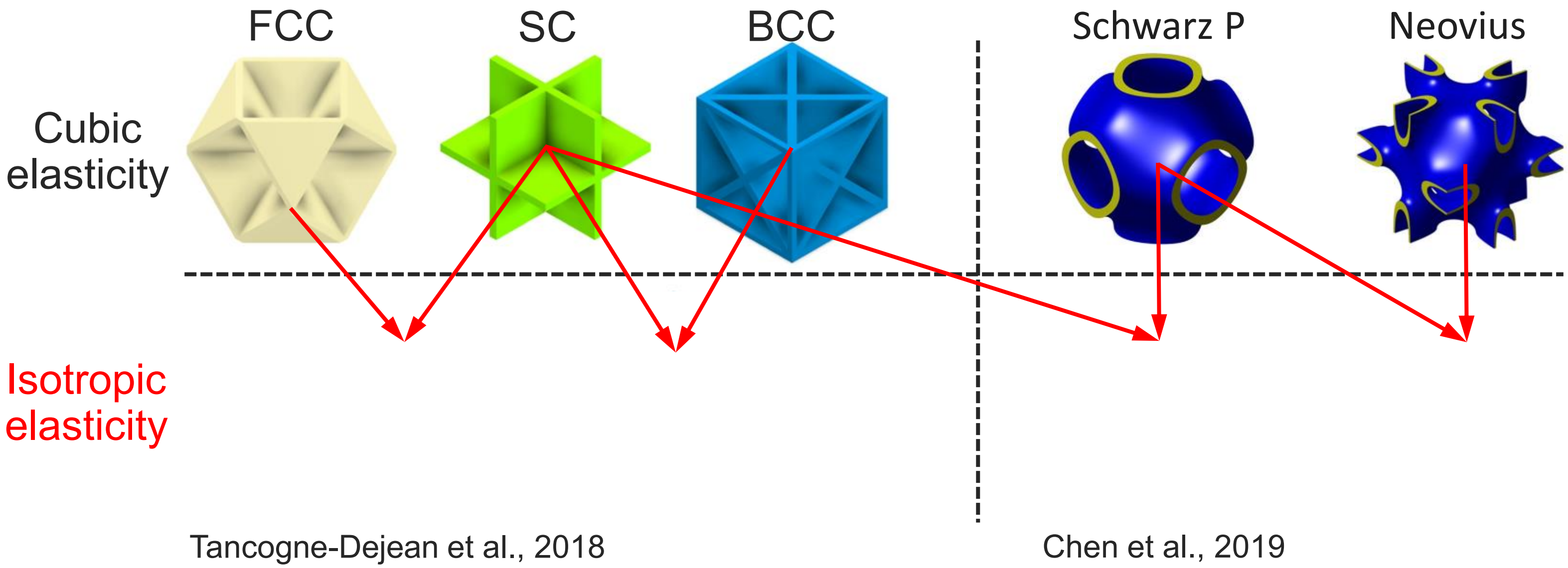
Tang et al., 2022

PIR



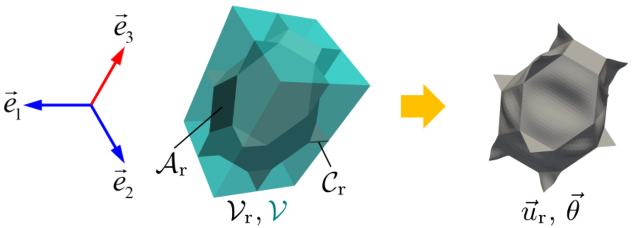
Andersons et al., 2010

- Lattice structures
  - Regular and ordered
  - Enhanced mechanical properties
  - Customization



Homogenization framework

➢Mesoscale
- Finite rotation shell



$$\tilde{\tilde{\mathbf{H}}}^\mathrm{c} = \tilde{\tilde{\mathbf{I}}} \cdot \mathbf{H}^\mathrm{c}$$

$$\vec{G}^\mathrm{c} = \vec{D} \cdot \mathbf{H}^\mathrm{c}$$

➢Macroscale
- Solid continuum $(\hat{\mathbf{F}}, \hat{\mathbf{P}})$

$$\mathbf{F} = \mathbf{L} + \eta \mathbf{K}$$

with

$$\mathbf{L} = (\tilde{\tilde{\nabla}}_0 \otimes \vec{x}_\mathrm{r})^\mathrm{T} + \mathbf{R} \cdot \vec{D} \otimes \vec{D}$$

$$\mathbf{K} = \boldsymbol{\Gamma} \cdot \left(\tilde{\tilde{\nabla}}_0 \otimes (\vec{\theta} \times \vec{D})\right)^\mathrm{T}$$

$$\mathbf{F}^\mathrm{c} = \mathbf{R}^\mathrm{T} \cdot \mathbf{F} = \mathbf{I} + \mathbf{H}^\mathrm{c} + \eta \mathbf{K}^\mathrm{c}$$

with

$$\mathbf{H}^\mathrm{c} = \mathbf{R}^\mathrm{T} \cdot (\tilde{\tilde{\nabla}}_0 \otimes \vec{x}_\mathrm{r})^\mathrm{T} - \tilde{\tilde{\mathbf{I}}}$$

$$\mathbf{K}^\mathrm{c} = \boldsymbol{\Gamma}^\mathrm{T} \cdot \left(\tilde{\tilde{\nabla}}_0 \otimes (\vec{\theta} \times \vec{D})\right)^\mathrm{T}$$

$$\mathbf{N} = \mathbf{R} \cdot (\tilde{\tilde{\mathbf{N}}}^\mathrm{c} + \vec{D} \otimes \vec{V}^\mathrm{c})$$

$$\mathbf{M} = \mathbf{R} \cdot \mathbf{M}^\mathrm{c}$$

$$\tilde{\tilde{\mathbf{N}}}^\mathrm{c} = \tilde{\tilde{\mathbf{I}}} \cdot \int_\mathcal{H} \mathbf{P}^\mathrm{c} \, \mathrm{d}\eta$$

$$\vec{V}^\mathrm{c} = \vec{D} \cdot \int_\mathcal{H} \mathbf{P}^\mathrm{c} \, \mathrm{d}\eta$$

$$\mathbf{M}^\mathrm{c} = \int_\mathcal{H} \eta \mathbf{P}^\mathrm{c} \, \mathrm{d}\eta$$

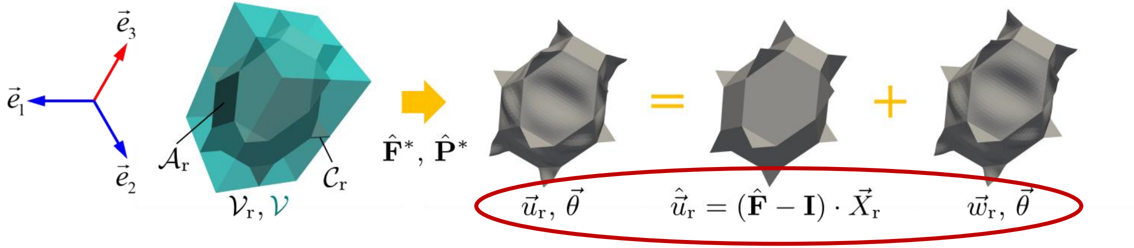
Campello et al., 2003

➢Scale transition relationships

- Classical homogenization extended for mesoscale shell problems

$$\hat{\mathbf{F}} = \frac{1}{V}\int_{\mathcal{V}} \mathbf{F}\ \mathrm{d}V \quad \Rightarrow \quad \boxed{\int_{\mathcal{C}_\mathrm{r}}\int_{\mathcal{H}} \left(\vec{w}_\mathrm{r} + \left(\mathbf{R}(\vec{\theta}) - \hat{\mathbf{R}}\right)\cdot \eta\vec{D}\right) \otimes \vec{N}_\mathrm{r}\ \mathrm{d}\eta \mathrm{d}C = \mathbf{0}}$$

$$\hat{\mathbf{P}} = \frac{1}{V}\int_{\mathcal{V}} \mathbf{P}\ \mathrm{d}V \quad \Rightarrow \quad \hat{\mathbf{P}} = \frac{1}{V}\int_{\mathcal{A}_\mathrm{r}} \mathbf{N}\ \mathrm{d}A = \frac{1}{V}\int_{\mathcal{C}_\mathrm{r}} \vec{n} \otimes \vec{X}_\mathrm{r}\ \mathrm{d}C$$



Mixed stress-strain driven formulation

- Mesoscale weak form

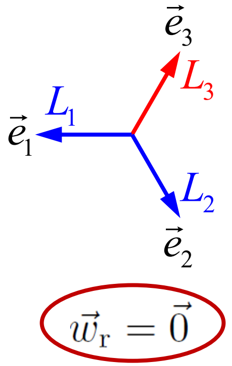
$$\int_{\mathcal{A}_\mathrm{r}} \delta\mathbf{L}^\mathrm{T}(\hat{\mathbf{F}}, \vec{w}_\mathrm{r}, \vec{\theta}) : \mathbf{N}\ \mathrm{d}A + \int_{\mathcal{A}_\mathrm{r}} \delta\mathbf{K}^\mathrm{T}(\vec{\theta}) : \mathbf{M}\ \mathrm{d}A$$
$$= V\delta\hat{\mathbf{F}}^\mathrm{T} : \hat{\mathbf{P}} + \int_{\mathcal{C}_\mathrm{r}} \delta\vec{w}_\mathrm{r} \cdot \vec{n}\,\mathrm{d}C + \int_{\mathcal{C}_\mathrm{r}} \delta\vec{\theta} \cdot \left(\boldsymbol{\Gamma}^\mathrm{T}(\vec{\theta}) \cdot \vec{m}\right)\ \mathrm{d}C, \quad \forall\{\delta\hat{\mathbf{F}}, \delta\vec{w}_\mathrm{r}, \delta\vec{\theta}\}$$

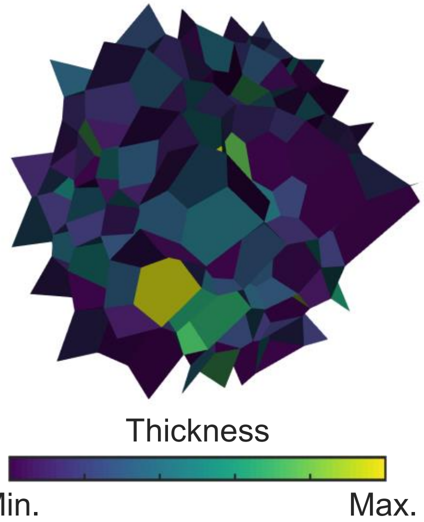
RVE simulation setup

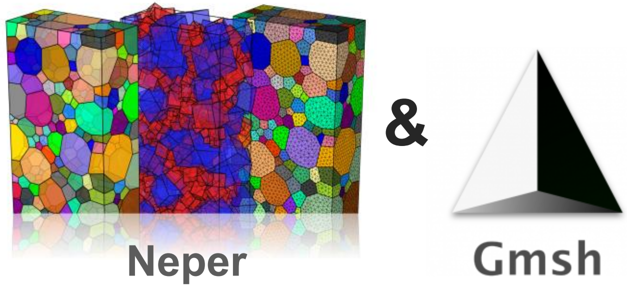
- Geometrical model configurations
  - Divinycell foam grade H100
    - Mesostructural stochastics

Zhou et al., 2023

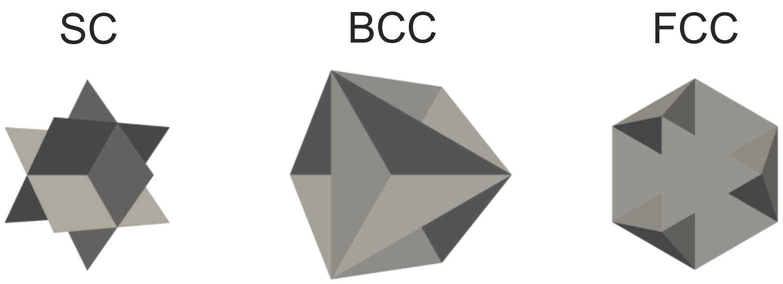
Parameter	Symbol	H100
RVE dimension 1	L_1	1.5 [mm]
RVE dimension 2	L_2	1.5 [mm]
RVE dimension 2	L_3	1.5 [mm]
Cell shape anisotropy	R	1.2 [-]
Cell equivalent diameter	(μ^d,σ^d)	(0.35,0.10) [mm]
Cell wall thickness	(μ^t,σ^t)	(0.0115,0.0059) [mm]
Nominal relative density	ρ/ρ_r	0.0714 [-]

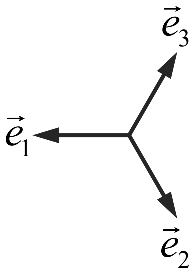






- Lattice structures
  - Constant cell wall thickness





$$\vec{w}_{\mathrm{r}}^{-} = \vec{w}_{\mathrm{r}}^{+}$$
$$\vec{\theta}^{-} = \vec{\theta}^{+}$$

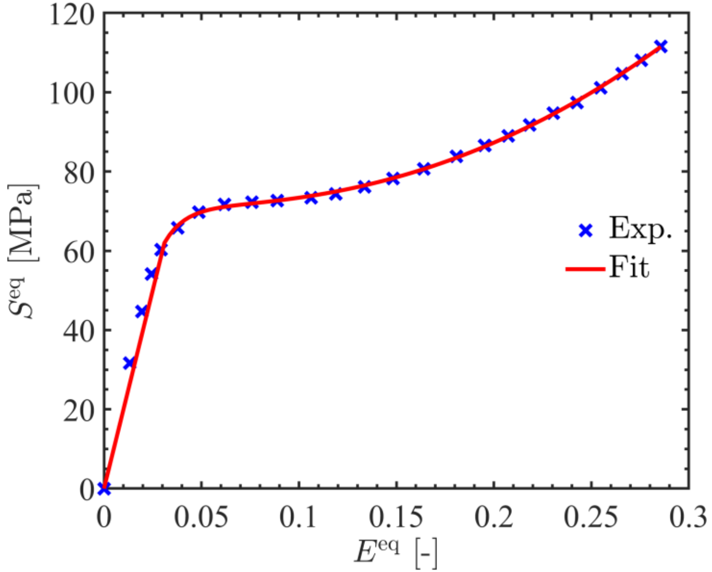
Parameter	Symbol	SC/BCC/FCC
RVE dimension 1	L_1	10 [mm]
RVE dimension 2	L_2	10 [mm]
RVE dimension 2	L_3	10 [mm]
Cell wall thickness	t	0.25/0.0885/0.1085 [mm]
Relative density	ρ/ρ_{r}	0.075 [-]

Gmsh

- Material model (MFront)
  - Isotropic elasto-plasticity
  - Exponentially saturating hardening + power law hardening

$$S_{\mathrm{y}} = S_0 + \left(S_\infty - S_0\right)\left(1 - \exp\left(-\frac{p}{p_{\mathrm{c}}}\right)\right) + K p^n$$

Kendall et al., 2014



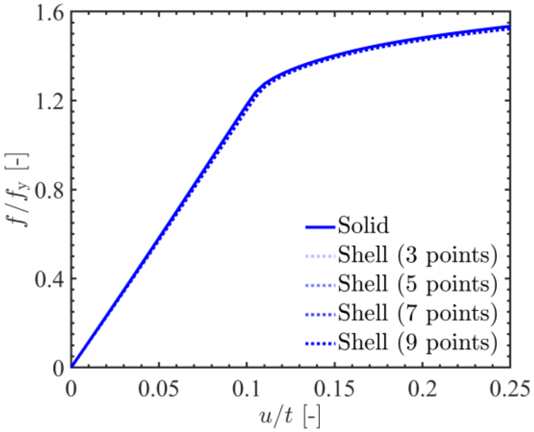
Parameter	Symbol	PVC
Young's modulus	E	2700 [MPa]
Poisson's ratio	v	0.38 [-]
Initial yield stress	S_0	62 [MPa]
Saturation yield stress	S_∞	71 [MPa]
Characteristic strain	p_{c}	0.011 [-]
Yield stress coefficient	K	870 [MPa]
Hardening exponent	n	2.2 [-]

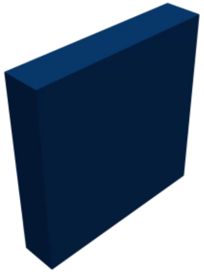
&

- Shell through-thickness integration scheme (MFront)
  - Gaussian quadrature rule

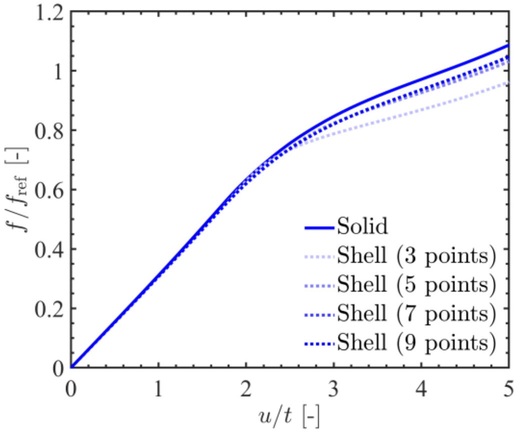
At least 5 points are required!

Uniaxial tension

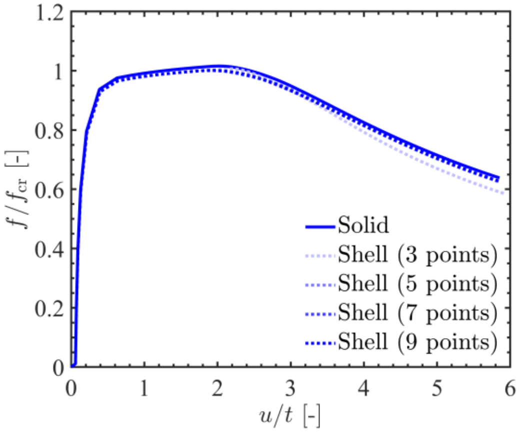




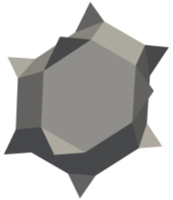
Bending

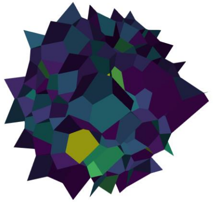


Buckling



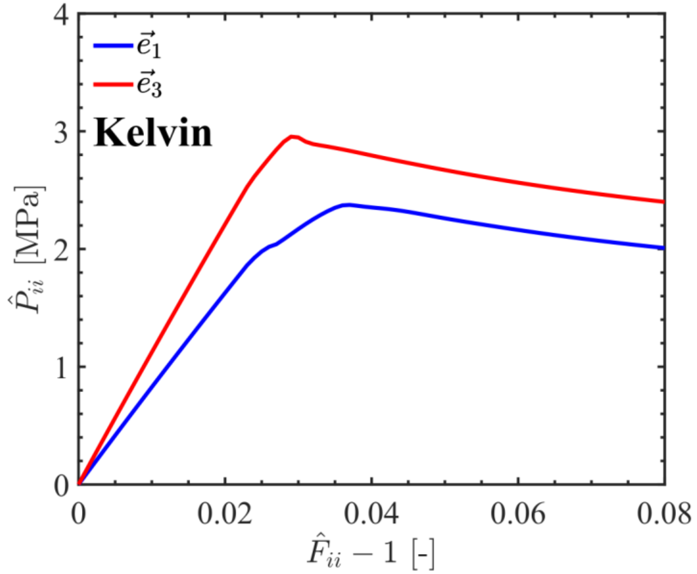
RVE simulation results

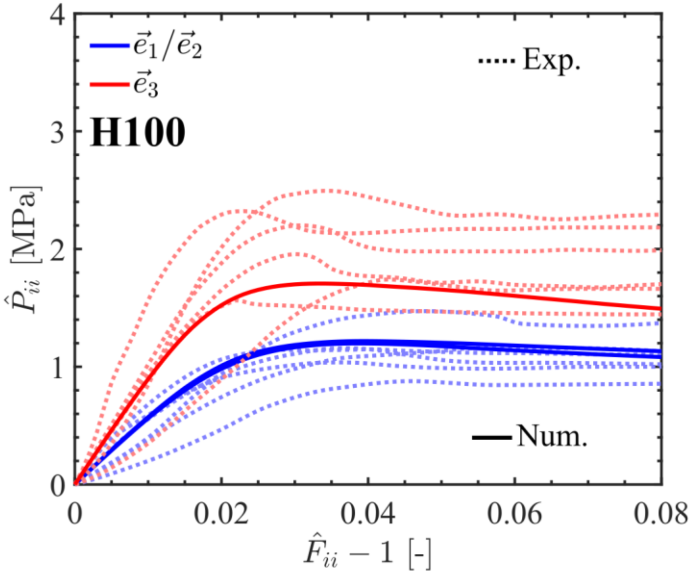


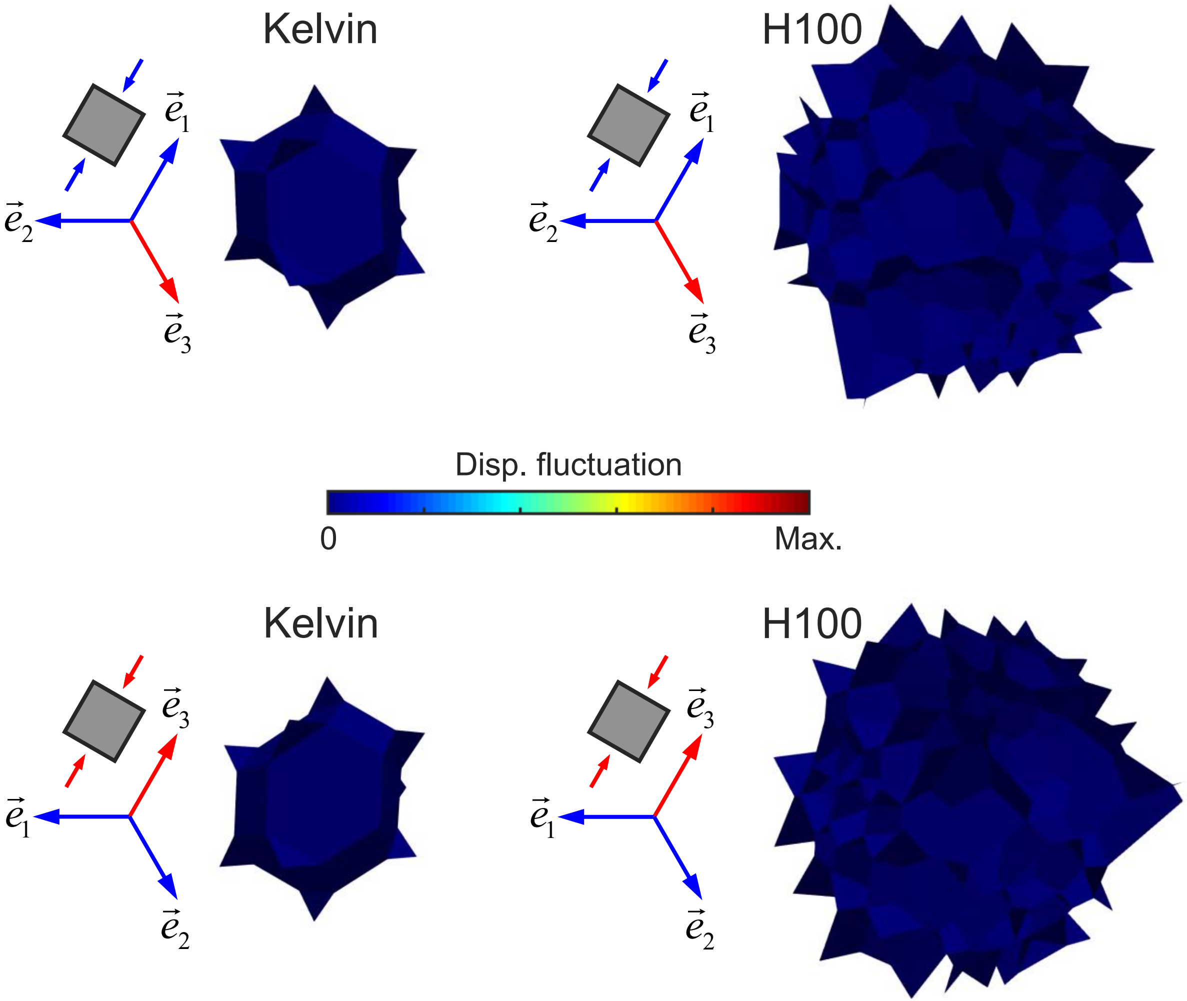


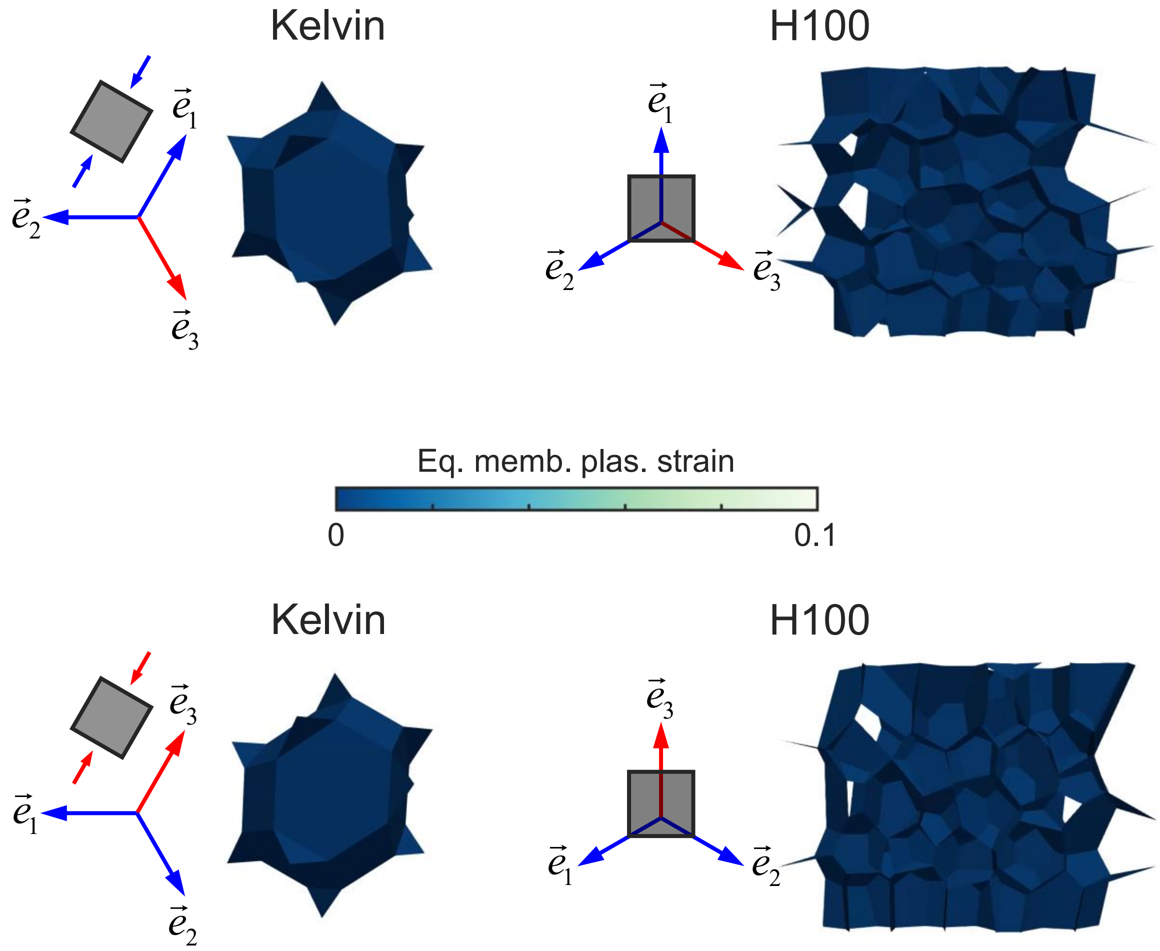
$\rho / \rho_r \approx 0.080$

- Divinycell foam grade H100
  - Uniaxial compression

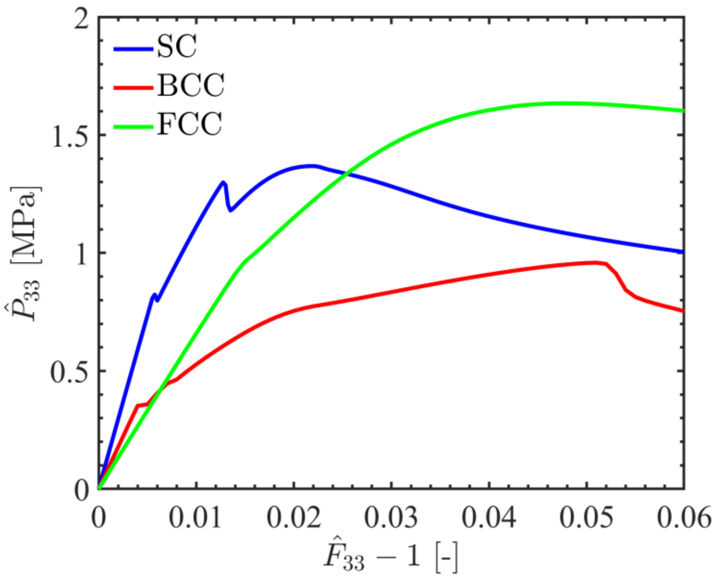


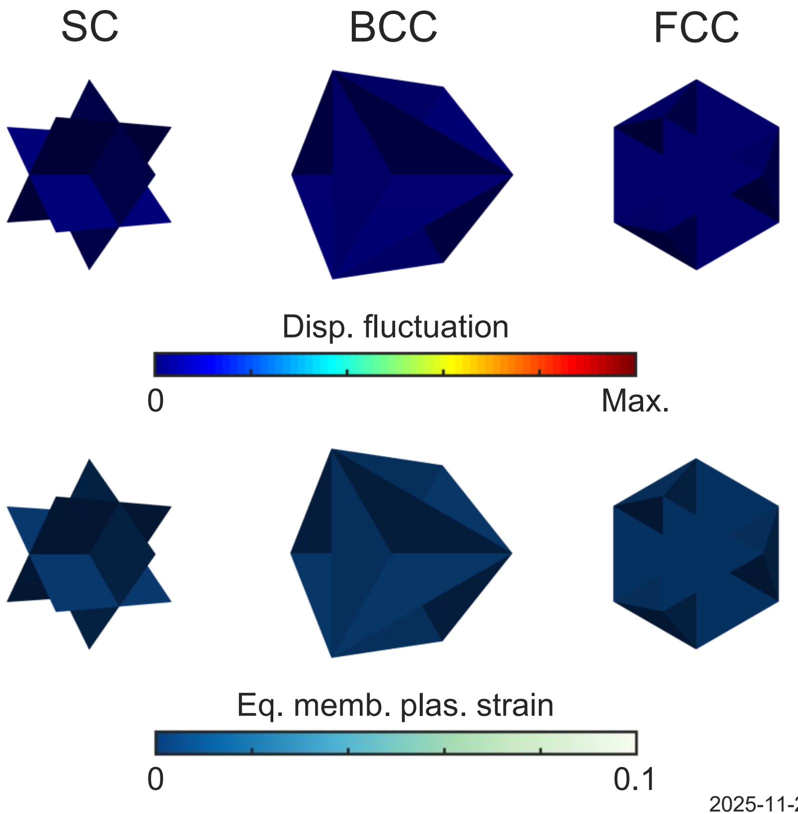



Kelvin
H100
$\vec{e}_1$
$\vec{e}_2$
$\vec{e}_3$
Disp. fluctuation
0
Max.
Kelvin
H100
$\vec{e}_3$
$\vec{e}_1$
$\vec{e}_2$


Kelvin
H100
$\vec{e}_1$
$\vec{e}_2$
$\vec{e}_3$
Eq. memb. plas. strain
0
0.1
Kelvin
H100
$\vec{e}_3$
$\vec{e}_1$
$\vec{e}_2$

- Lattice structures
    - Uniaxial compression





Conclusions and ongoing work

- Computational homogenization of shell-based cellular materials
  - Extension of classical homogenization
  - Mixed stress-strain driven formulation
  - Efficient prediction
- Ongoing work
  - Releasing codes
  - Upscaling relevant mesophysics (buckling + plasticity)
  - Data-based model description (JAX + FEniCSx)

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Thanks to MFront book draft (T. Helfer) and comet-fenics (J. Bleyer)!

Thank you!

Questions?