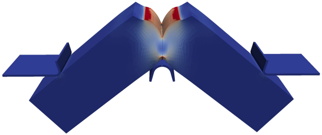
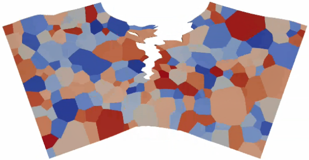
CONSTITUTIVE EQUATIONS (AND FAILURE CRITERIA) FOR POROUS DUCTILE MATERIALS IN MFRONT





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French Alternative Energies and Atomic Energy Commission

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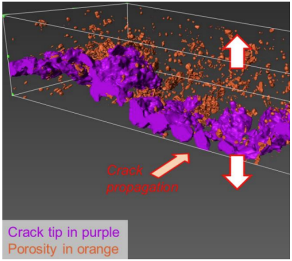
11th `MFront` User Day

Saclay, November 20, 2025

Ductile fracture by **nucleation, growth and coalescence of voids**

- **Failure mode** of many **metal alloys** used in industrial applications

From the **microstructural** scale (μm) ...



(Petit, 2019)

... to the **structural** scale (mm – m)

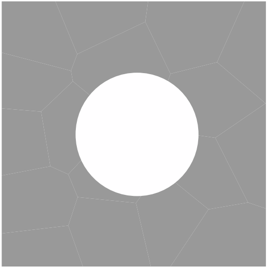
Homogenized constitutive equations for porous ductile materials needed to simulate:

- Ductile fracture **initiation**
- Ductile fracture **propagation**

Gurson-Tvergaard-Needleman (GTN) set of constitutive equations:

- (Gurson, 1977), (Tvergaard & Needleman, 1984)
- **State variable**: Porosity f
- Still the most used model to predict ductile fracture

Modelling



Equations

- **Gurson yield surface**

$$\mathscr{F} = \left(\frac{\Sigma_{vM}}{\sigma_0}\right)^2 + 2q_1 f^\star \cosh\left(\frac{3}{2} q_2 \frac{\Sigma_m}{\sigma_0}\right) - 1 - (q_1 f^\star)^2$$

- **Coalescence** $f^\star = \max\left(f_c + (f - f_c)\frac{q_1^{-1} - f_c}{f_R - f_c}, f\right)$

- **Evolution laws** $\dot{\varepsilon}_p \propto \dfrac{\partial \mathscr{F}}{\partial \Sigma}$

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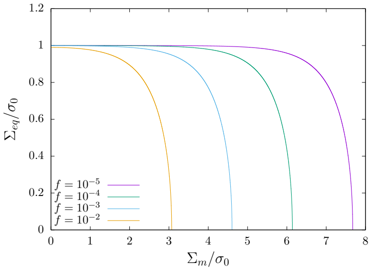

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GTN model available in `MFront` as part of the `StandardElastoViscoPlasticity` brick

Implementation choices

- Fully implicit
  - ✓ Newton-Raphson (for ε_p)
  - ✓ Fixed point (for f)
- Consistent tangent operator available

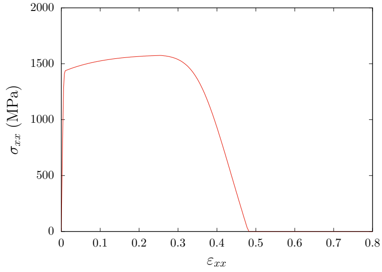
Example of `.mfront` file

```
@Brick "StandardElastoViscoPlasticity" {
  stress_potential : "Hooke" {
    young_modulus : 200000.,
    poisson_ratio : 0.3
  },
  inelastic_flow : "Plastic" {
    criterion : "GursonTvergaardNeedleman1982" {
      q1 : 1.5,
      q2 : 1.0,
      q3 : 2.25,
      fc : 0.01,
      fr : 0.1
    },
    isotropic_hardening : "Linear" {
      R0 : 150.
    }
  }
}
```

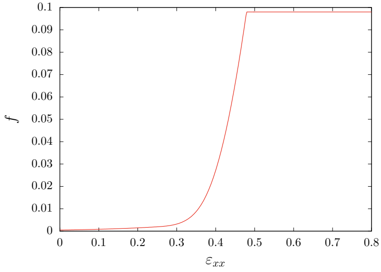
Documentation available: `https://thelfer.github.io/tfel/web/StandardElastoViscoPlasticityBrick-PorousPlasticity.html`

Typical output of the GTN model on a material point (`MTest`)

Stress-strain curve



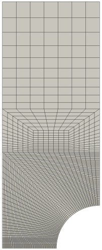
Porosity-strain curve



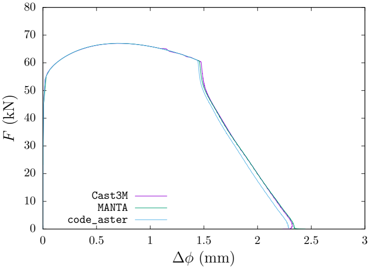
- **Competition** between **strain hardening** and **strain softening** induced by porosity
- **Fracture:** $f = f_R \Rightarrow \Sigma = 0$

Applications in FEM simulations: Benchmark between `Cast3M`, `MANTA` and `code_aster` (with Sébastien Meunier)

Mesh



Force-displacement curve



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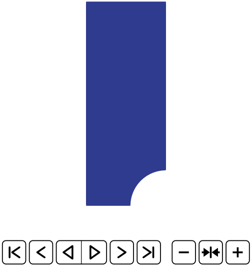

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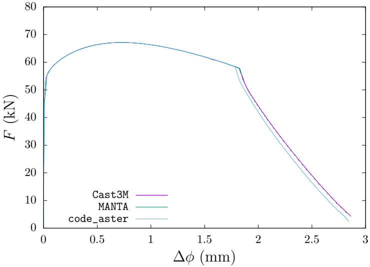

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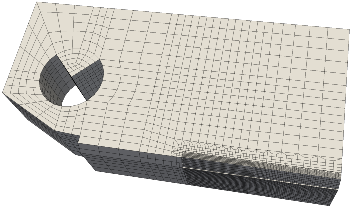

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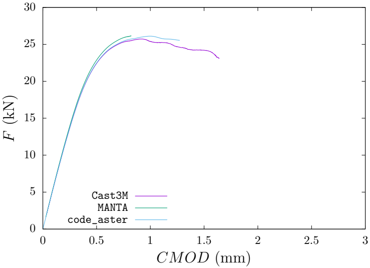

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  - ✓ **Element deletion** may also help

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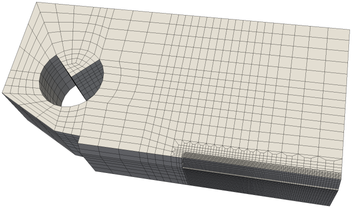

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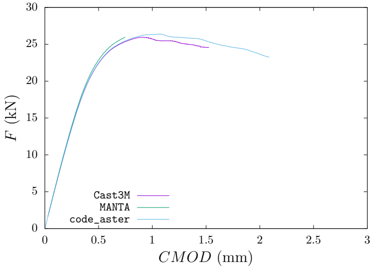

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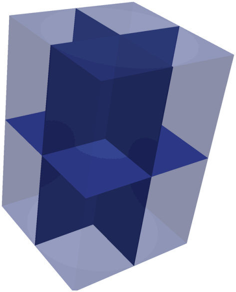
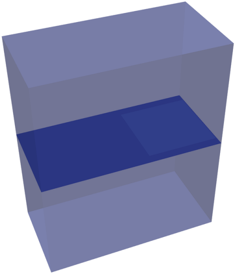
Applications in FFT simulations: `AMITEX_FFTP` (with Lionel Gélébart)





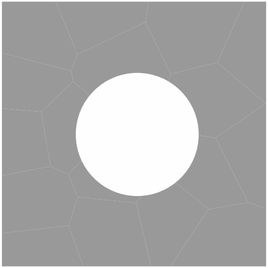
- `MFront` **implementation of the GTN model also works** in FFT solvers

But **many other models are available** in the literature for ductile fracture ! **2 examples**

Gurson-Thomason (GT) set of constitutive equations:

- (Gurson, 1977), (Thomason, 1984)
- **State variable**: Porosity f
- Physical modelling of coalescence with an additional yield surface

Modelling



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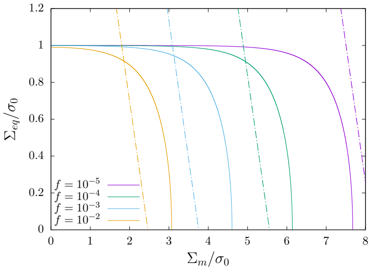

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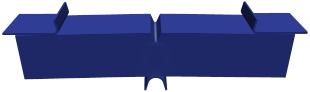

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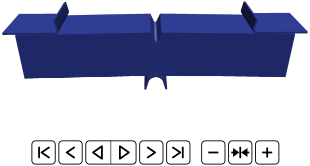

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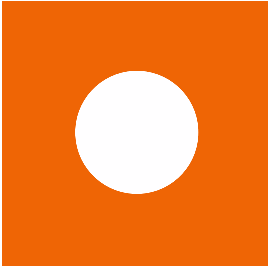

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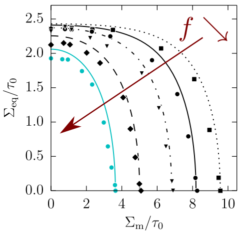
[100] loading direction



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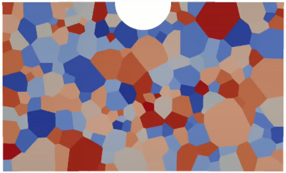

(with Cédric Sénac & Jean-Michel Scherer)

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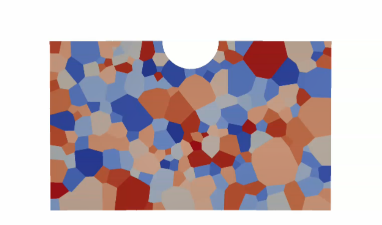

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Roadmap for the extension of the `StandardElastoViscoPlasticity` brick:

1. **Extensive set of porous constitutive equations**

   ✓ Quick / not-optimized implementation (`Numerical_Jacobian`, ...)

   ✓ JupyterLab tool to compare yield surfaces / stress-strain curves ...

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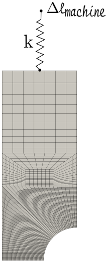
- Side project thus slow progress ...
- **Volunteers welcome !**

In practice, ductile fracture is often unstable (for uncracked samples):

- Spring effect ⇒ Elastic energy stored outside fracture zone ⇒ Snap-back event

Mesh



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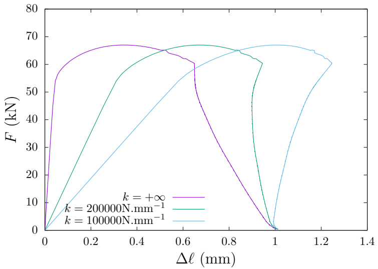


Predicting ductile fracture initiation may be sufficient for many applications:

- Uncoupled fracture modelling

`MFront` `FailureCriteria` **project:**
`https://thelfer.github.io/FailureCriteria/web/index.html`

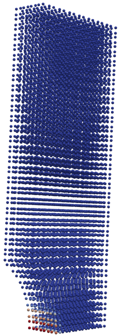
- **Implementation of ductile damage indicators**
  - ✓ **Strain based indicator** $D = \int \frac{dp}{p_c(\sigma)}$
    - ⇒ Freudenthal
    - ⇒ Cockcroft & Latham
    - ⇒ Rice & Tracey
    - ⇒ Hancock & McKenzie
    - ⇒ Johnson & Cook
    - ⇒ Huang
    - ⇒ Bai & Wierzbicki
    - ⇒ ...
  - ✓ **Localization based indicator**
    - ⇒ ...
- **Implementation of brittle fracture probabilistic models**
  - ✓ **Weakest link assumption models**
    - ⇒ Beremin
    - ⇒ Ruggieri & Dodds
    - ⇒ Forget, Marini & Vincent

to be used as **a post-processing of simulations** to predict **ductile fracture initiation**

Example of use of `MFront FailureCriteria` ductile damage indicator in python
`https://thelfer.github.io/FailureCriteria/web/analysis_python.html`

- **Requires:** vtk result files (σ, p)
- **Requires:** vtk, mgis modules

```python
from postVTK import *

model = {'LibraryPath':'src/libBehaviour.so',
         'Law':'DuctileDamageIndicator_RiceTracey1969',
         'Hypothesis':'Tridimensional'}

data = {'PrefixFiles':'FUN24',
        'Components':{'Stress':'SIG',
                      'EquivalentPlasticStrain':'P'},
        'StressNotation':'Mandel','Timestep':1}

param = {'D1':1,
         'D2':1}

output = {'OutputData':'DamageIndicator',
          'VtkOutput':'true',
          'VtkFormat':'ascii'}

postVTK(model,data,param,output)
```



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1. **Extensive set of strain based ductile damage indicator**
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   - ✓ **Main interests**: easy to use, easy to compare
2. **Implementation of localization based ductile damage indicator**
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- Side project thus slow progress ...
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- **Constitutive equations for porous ductile materials in `MFront`**
  - ✓ Few (GTN and Rousselier) are available in the `StandardElastoViscoPlasticity` brick
  - ✓ More are available as standard `MFront` implementations
    - ⇒ **Extend** the `StandardElastoViscoPlasticity` brick for porous materials
    - ⇒ **Develop** a tool to compare models
    - ⇒ **Assess** the robustness of the implementation in different FEM / FFT solvers

- **Failure criteria in `MFront`**
  - ✓ Early project / Some models are already available
  - ✓ More are available in the literature
    - ⇒ **Add** more models
    - ⇒ **Test / validate** post processing / use in FEM / FFT solvers