ADFT CA3

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Question 1

Moving average (MA) models of order q consider the past q error terms. The formula for a MA(q) model is:

$y_t = \mu + \epsilon_t + \theta_1 \epsilon_t - 1 + ... + \theta_q \epsilon_{t-q}$ [1]

In order to determine the best value of q for the Amazon stock price time series, we can plot the autocorrelation function (ACF) or consider the RMSE.

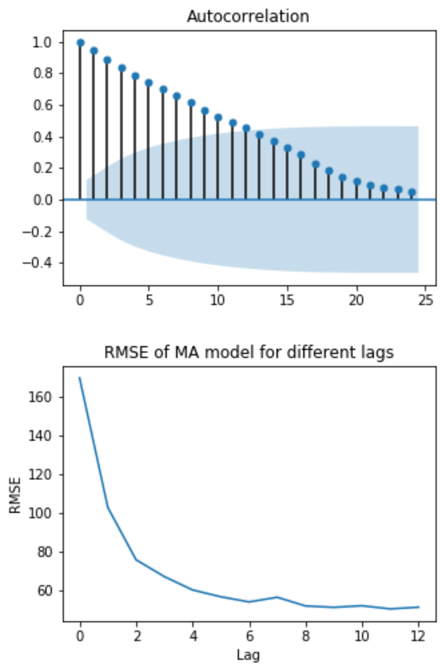

Fig. 1. ACF and RMSE

From Figure 1 it seems a lag of $q = 12$ is a sensible choice. So, we train an MA(12) model on 70% of the data, and reserve the rest for testing. The forecast of the trained model is compared to the test data in Figure 2.

As you can see, the predicted prices are fairly accurate. They also move in the same direction as the actual prices, showing that the model has taken into account the trend in the series. This model could potentially be improved by introducing an autoregressive component, or including a signal to help with predictions.

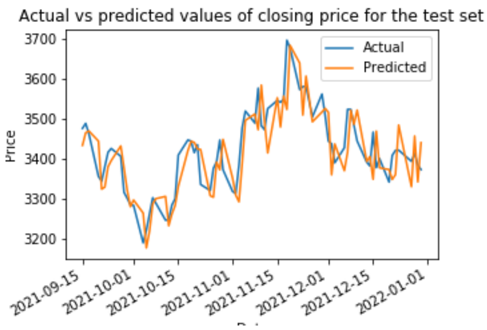

Fig. 2. MA(12) forecast

Question 2

The Augmented Dickey-Fuller (ADF) test is a statistical hypothesis test for testing whether the process generating a time series is stationary. The null hypothesis is that there is a unit root; i.e. 1 is a solution to the characteristic equation of the process. If the resulting test statistic is large, there is significant evidence that the process does not have a unit root; so we reject the null hypothesis and conclude the process is stationary. However, if the test statistic is small, there is likely to be a unit root and so we accept the null hypothesis; concluding the series is non-stationary. The deicision of 'largeness' of the test statistic is based on the pre-defined significance level. [1]

When applying the ADF test to the original time series of stock prices, we get a test statistic of $T = -2.623$ and p-value of 0.088. At a 5% significance level there is not enough evidence to reject the null hypothesis; so the series is non-stationary. This confirms our results from CA2, where we suggested the series was non-stationary due to the presence of a trend.

We can apply first order differencing to transform the series into a stationary series. Applying the ADF to the differenced series, we get a test statistic of $T = -14.869$ and p-value of 1.67×10^{-27}. We reject the null hypothesis and conclude that the differenced series is stationary.

Question 3

An autoregressive (AR) model of order p models a time series as a function of the previous p values, plus an error term. It is of the form:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... + \phi_p y_{t-p} + \epsilon_t$$

[1]

A problem with AR models is that they don't consider the past error terms $\epsilon_{t-i} fori > 0$. As we've seen, MA models are similar to AR models, except they model past error terms instead of past values. An MA model of order q considers the past q error terms, and is of the form:

$$y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + ... + \theta_q \epsilon_{t-p}$$

[1]

The error terms that are included in the MA model are assumed to be i.i.d. with mean 0, meaning MA models are inherently weakly stationary.

We can approximate the order of AR and MA models (p and q respectively) using the PACF and ACF. If the PACF drops off quickly after a lag p, we can use this knowledge to approximate the order of an AR model. If the ACF drops off quickly after a lag q, we can use this knowledge to approximate the order of an MA model. This is because MA models can only be correlated with the past q terms, since the error terms in the model don't incorporate past data.

Wold's theorem - that any stationary time series can be written as the sum of a deterministic series and a stochastic series - implies a combination of both AR and MA models is necessary for modelling stationary time series. Hence, we introduce the combined ARMA(p,q) model, which has AR order p and MA order q. The formula is:

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_{t-i} y_{t-i} + \epsilon_t + \sum_{i=1}^{q} \theta_{t-i} \epsilon_{t-i}$$

The ARIMA(p,d,q) model introduces d, the order of differencing required to make the series stationary. Stationarity of the time series is necessary for the model to be valid and reliable.

One problem with ARIMA models is that, despite their simplicity, they are prone to overfitting [1]. To avoid this problem, it's important to keep the parameters p, d and q as small as possible.

Another problem with ARIMA models is that they are backwards-looking, and rely on the assumption that we can use the past to predict the future. This often leads to lagged predictions; for example see Figure **??**. To counteract this, the model can be enhanced with a signal, which indicates when something is about to change. Implementing

Fig. 3. Amazon daily returns

signals usually requires an external data source. For example, changing interest rates could be an effective signal in financial forecasting.

In general, ARIMA models are good at short term forecasting. However, for long term forecasting ARIMA models tend not to work so well, and in the long run tend towards predicting the mean of the underlying process. [1]

Question 4

For financial time series, the assumption of constant variance of the error terms often doesn't hold. ARCH models introduce a parameter q for modelling the series as a function of the past q error terms. GARCH stands for generalised ARCH, which introduces a parameter p for modelling the series as a function of the past p variance terms in addition to the past q error terms. The formula for a GARCH(1,1) model is as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

This can be generalised to p past variance terms and q past error terms.

Since we're predicting the volatility, we first perform a transformation to convert the Amazon price data into a series of daily returns, which can be seen in Figure 3.

We want to predict the volatility in December 2021, so we train a GARCH model on the months January-November. Plotting the ACF and PACF for the variance should help us choose the parameters for our GARCH model.

Both the ACF and PACF drop off quickly after a lag of 0, and aren't particularly helpful in choosing p and q. However, we can see a slight drop off in correlation at a lag of 2, and significant correlation at a lag of 18. We fit a GARCH(2,2) and a GARCH(18,18) model, and compare the resulting forecasts for December in 5.

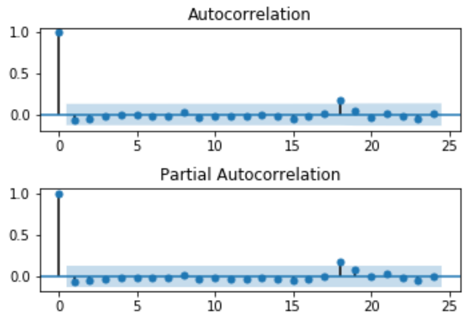

Fig. 4. ACF and PACF of Amazon daily returns

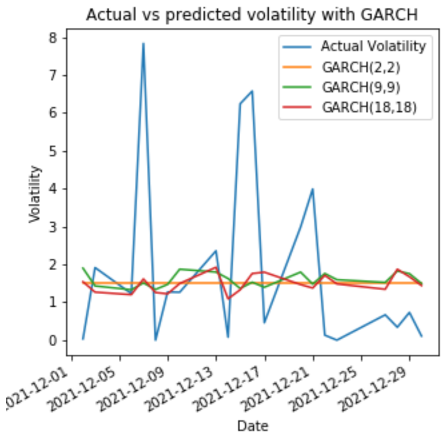

Fig. 5. Predicted volatility of GARCH models

The GARCH(18,18) model gives a better forecast of the volatility, because it considers a greater range of lags. The GARCH(2,2) model gives a constant forecast of volatility, because it is overly simple. Having said this, it's good practice to simplify GARCH models where possible. The GARCH(9,9) model nicely balances the trade-off between complexity and forecast accuracy.

When looking back at 3, the volatility doesn't appear to fluctuate much. Perhaps this explains why the GARCH(2,2) model predicts constant variance. This could indicate that perhaps variance is constant, and a GARCH model isn't necessary.

References

[1] Nielson, Aileen, *Practical Time Series Analysis : Prediction with Statistics and Machine Learning* 2019, First ed. Sebastopol, CA: O'Reilly Media.