

# Supplementary lab – intro to probability

Or: Combinatorics on steroids

- *The Language of Thought: computational cognitive science approaches to category learning*
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- When: Sommer semester 2022

# Probability – finite discrete

- Suppose we flip a coin three times.
  - What is the probability of getting tails exactly once?
- Suppose we flip a coin 30 times.
  - What is the probability of getting tails exactly 5 times?
- Suppose we roll a dice twice.
  - What is the probability of the sum of the two values being 4?
- Note the ingredients of these questions:
  - We have **some situation** with a probabilistic component
  - The situation can turn out in various **ways**
  - We are interested in the **probability** of each of those ways

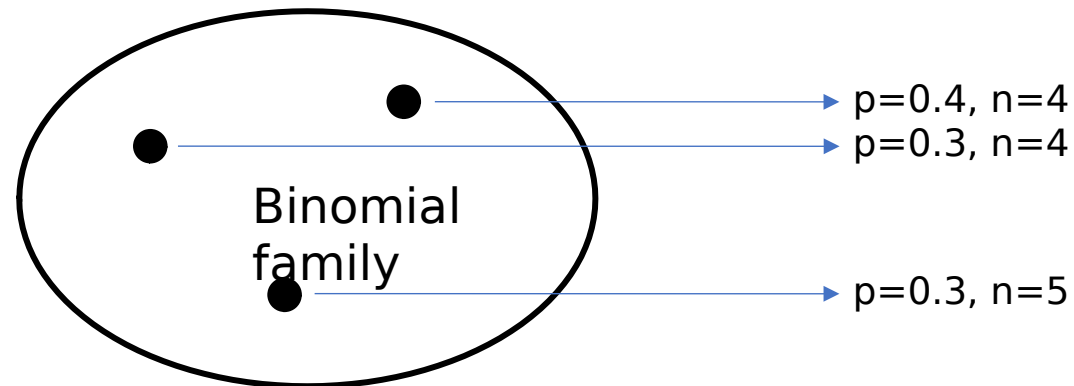
# Probability – finite discrete

Often we need one of four distributions in this context:

- Binary event  $\{0,1\}$  that happens once: **Bernoulli distribution**
  - Only has one parameter  $p$ , namely the probability of 1
- Binary event  $\{0,1\}$  that happens  $n$  times: **Binomial distribution**
  - Answers the question: What is the  $p$  that 0 happens exactly  $n$  times?
  - Has parameters  $p$  and  $n$ .
  - Bounded above by  $n$ :  $[0, n]$  is called the *support* of the distribution
- Event  $\{0,1,\dots,m\}$  that happens once: **Categorical distribution**
  - Has one parameter, which is *not a number!*, but a prob vector of length  $m$
  - Question: why is  $m$  not a parameter?
  - Can you tell me what a probability vector is?
  - What question do you think it answers?
- Event  $\{0,1,\dots,m\}$  that happens  $n$  times: **Multinomial distribution**
  - Two parameters,  $n$  and a probability vector of length  $m$
  - What question do you think it answers?

# Probability – families

- We have been talking vaguely, so let's introduce some terminology
- The various 'distributions' we have talked about are not distributions. They become distributions when a value is given for the parameters.
  - Rather, they are called *families* of distribution.
  - For instance, 'Bernoulli family', 'Binomial family', etc.
  - You can think of a family of distributions as a *parameterized set*: a set of distributions, each of which can be specified by setting some parameters.
  - Infinite sets!



# Probability – support

- The *support* of a distribution is the set of values with probability  $> 0$
- In many cases, the support is a set of numbers
  - For instance, in Bernoulli distributions the support is  $\{0, 1\}$
  - (except when  $p=0$  or  $p=1$ )
- However, the support doesn't have to be a set of numbers
  - Can you think of an example where it's not a set of numbers?
  - For instance, the support of the multinomial distribution is the set of non-negative integer vectors summing to  $n$ : Vectors, not numbers!
- Question: does the support have to be identical for all distributions in a given family?
  - Answer: No!
  - E.g. for binomial and multinomial, the support depends on  $n$

# Probability mass function

- I have talked about the *distributions* as giving us probabilities of events, but this is not quite right.
  - A probability distribution is an abstract object with various properties, like a *support* etc.
  - Think about it as an object in python.
  - What gives us the prob of an event is one of the things associated with the distribution: its *probability mass function*.
- We might want to get other information about the distribution.
  - For instance, ‘what is the probability of all elements in the support that are greater than 3?’ for a binomial distribution with  $n$ .
  - This is called the *cumulative distribution function*
  - Distributions define loads of other functions too!

# Random variables

- *Random variables* are a way of making all of this more formal.
- They're usually not taught in intro courses because they're a bit technical.
- However, not understanding them can lead to terrible confusions
- A random variable is a *function* (so it's neither a variable nor random!)
  - The support of this function is the *sample space* : a set of *events* with associated probabilities.
  - The range of the function is a set of number or vectors
  - (Range can be other things too but we'll only need those two)
  - (I'm skipping *a lot* of technical detail here)
- Random variables model quantities that have a distribution, like:
  - 'the number of heads of a coin flipped twice'
  - 'the number of times a random person has seen the moon'

# Random variable - examples

- 'whether a flipped coin shows heads'
- 'whether a flipped coin shows heads or tails'
- 'the number of heads of two flipped coins'
- 'the number of leaves in a tree in bota'
- 'whether the number of leaves in a tree in bota is odd'



# Probability – continuous support

- In order to explain the basic terminology, I have only used distributions with finite discrete support.
- But of course, there are distributions with
  - Infinite discrete support, e.g.,  $[0, 1, 2, \dots, )$
  - Bounded continuous support, e.g.,  $[0, 1]$
  - Unbounded continuous, e.g.,  $[0, )$
- It's a bit harder to deal with them mathematically, but what is important now is that you understand what's going on conceptually.
- Do you know examples of these?