Part III
Category learning with a pLoT

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13:30 – 15:00

(1h30m)

Summary so far

pLoT as a picture of cognition

Some formal tools:

- Formal grammars
- Probabilistic PCFG
- Compositional interpretation
- (Approximate) Bayesian inference

9:00-10:20	Introduction: On the very idea of an LoT
10:40-12:30	Technical background
12:30-13:30	Lunch
13:30-15:00	**Bayesian program induction (LOTlib3)**
15:20-16:30	Case studies
16:30-17:00	Summary

Plan for session

- We'll consider two examples
- Combine the technical tools
- Build up to a pLoT category learning simulation

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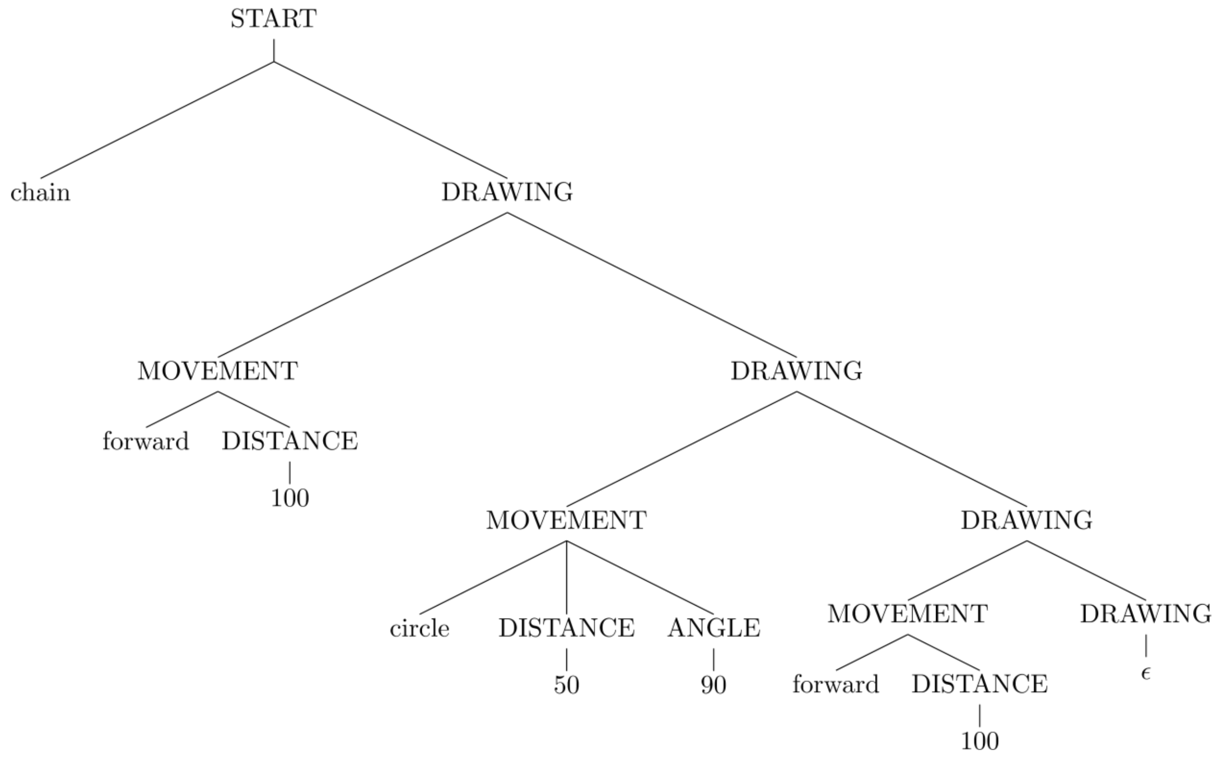
Example 1: The visual domain

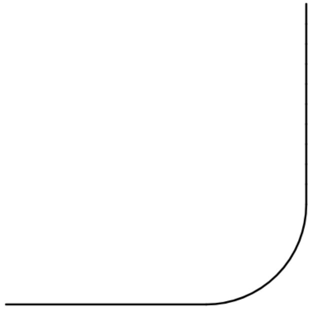
Example 1: Shape grammar

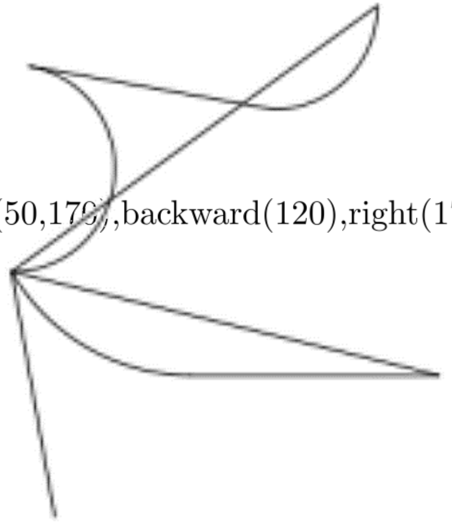
$$
\begin{aligned}
\langle \text{START} \rangle &\to \text{chain}(\langle \text{DRAWING} \rangle) \\
\langle \text{DRAWING} \rangle &\to \langle \text{MOVEMENT} \rangle, \langle \text{DRAWING} \rangle \\
&\to \epsilon \\
\langle \text{MOVEMENT} \rangle &\to \text{forward}(\langle \text{DISTANCE} \rangle) \\
&\to \text{backward}(\langle \text{DISTANCE} \rangle) \\
&\to \text{right}(\langle \text{ANGLE} \rangle) \\
&\to \text{left}(\langle \text{ANGLE} \rangle) \\
&\to \text{circle}(\langle \text{DISTANCE} \rangle, \langle \text{ANGLE} \rangle) \\
&\to \text{goto}(0, 0) \\
\langle \text{DISTANCE} \rangle &\to 50 \mid 100 \mid 120 \\
\langle \text{ANGLE} \rangle &\to 30 \mid 60 \mid 90 \mid 170
\end{aligned}
$$



Example 1: Shape semantics

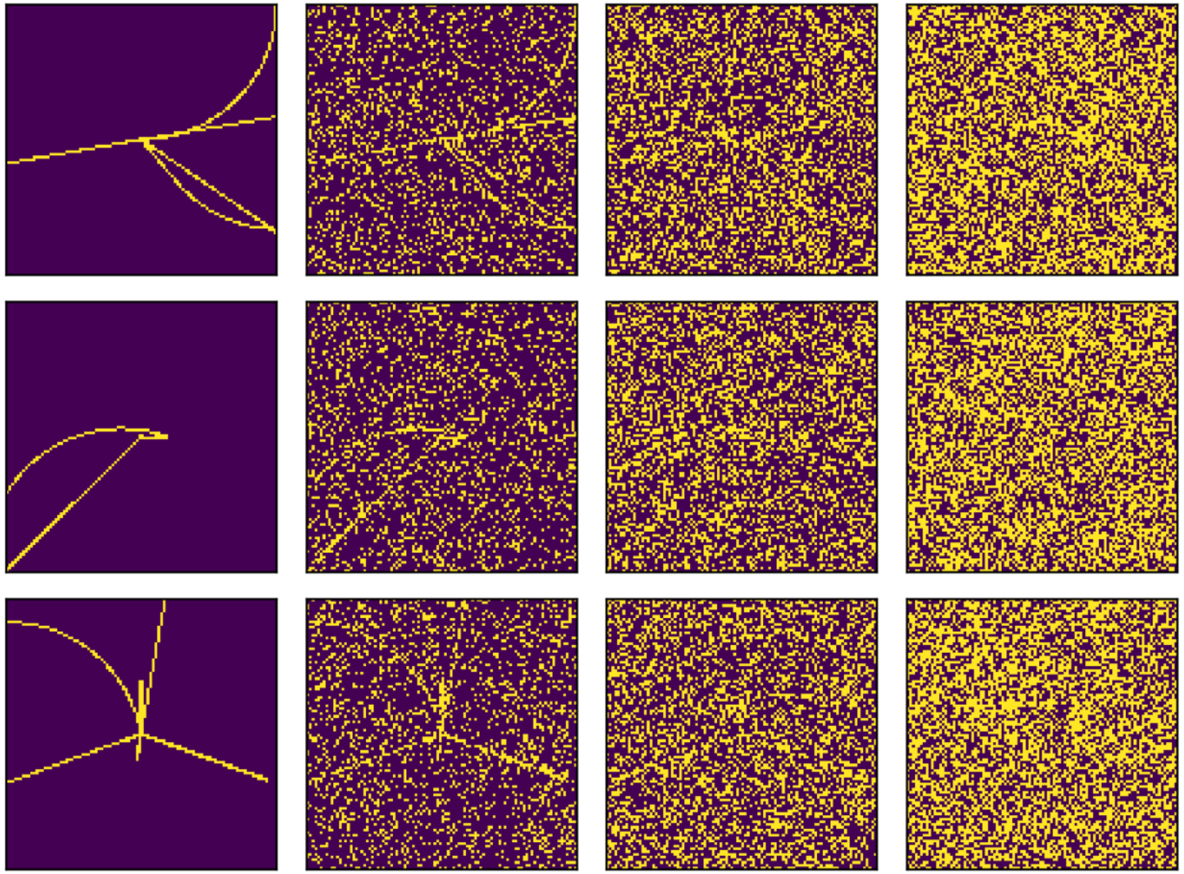
chain(forward(100), circle(50,90), forward(100),)



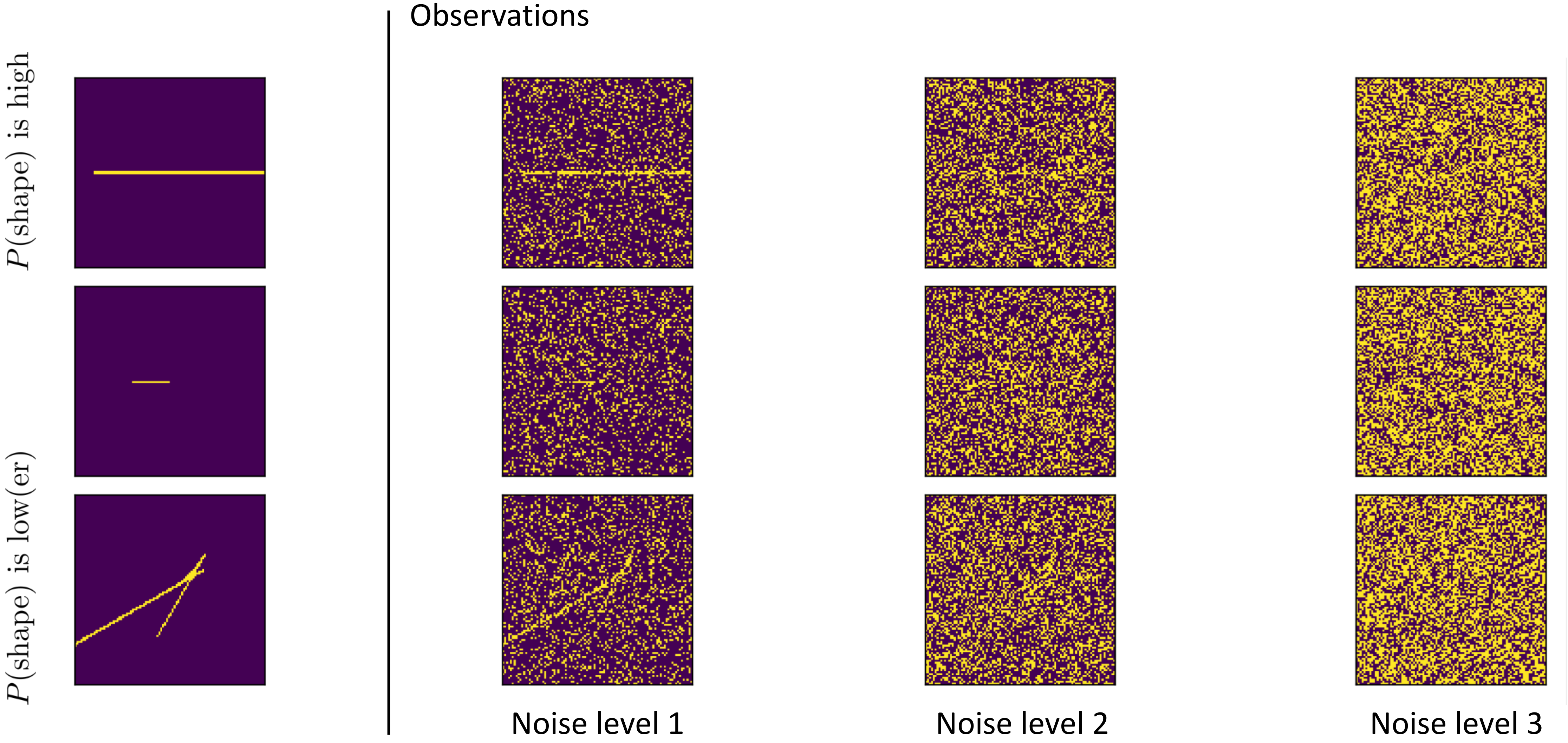


chain(right(60),circle(100,60),forward(120),goto(0,0),circle(50,170),backward(120),right(170),circle(50,90),goto(0,0),right(170),forward(120),right(90),)

Example 1: Shape observations



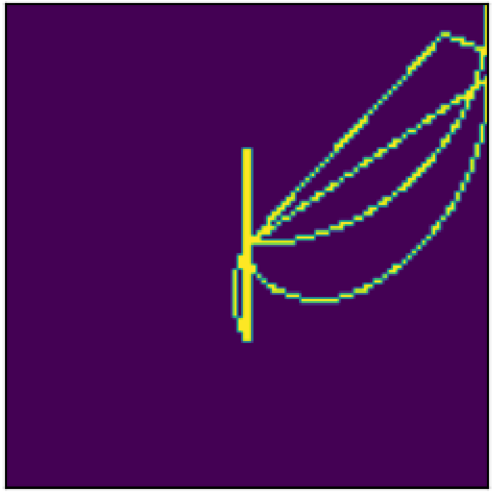
Example 1: Priors and likelihoods



Bayesian inference in the shape grammar

chain(
circle_(50,90),
backward_(10),
right_(90),
right_(170),
right_(30),
forward_(10),
right_(90),
left_(170),
goto_(0,0),
circle_(20,60),
right_(30),
goto_(0,0),
goto_(0,0),
goto_(0,0),
backward_(20),
forward_(20),
goto_(0,0),
circle_(10,60),
circle_(20,60),
circle_(50,60),
left_(170),
goto_(0,0),
right_(60),
left_(30),
)

True shape



chain(circle_(50,90),goto_(0,0),forward_(20),goto_(0,0),backward_(20),)

LoT learning models: the big picture

1. Interpreted PCFG
   Defines hypotheses H
   Defines a prior over H

2. Observations O
   Generated by true H

3. Likelihood P(O | H)

4. Bayesian inference algorithm
   Approximate!

1. Shapes grammar
   Each H is a sentence…
   …which encodes a shape

2. Image (with noise)
   Multiple H produce same shape!

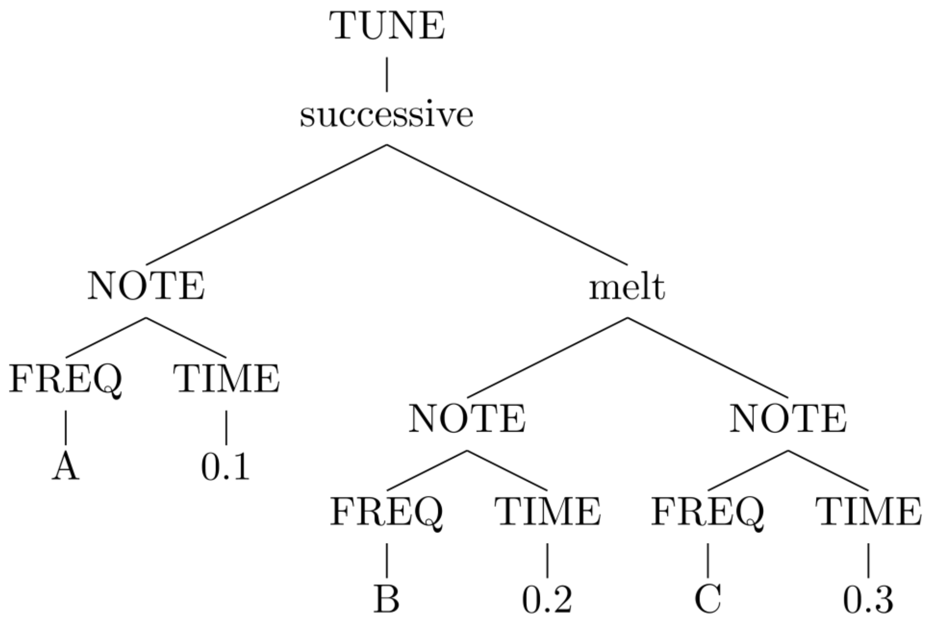
3. Likelihood P(image| shape)

4. MCMC w/ e.g., LOTlib3
   Gives us samples from posterior!

Example 2: The auditory domain

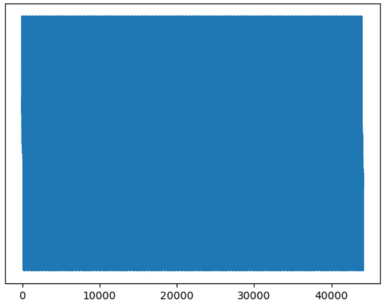
Example 2: Sound grammar

$$
\begin{aligned}
\langle \mathrm{TUNE} \rangle &\to \langle \mathrm{NOTE} \rangle \\
&\to \mathrm{successive}(\langle \mathrm{TUNE} \rangle, \langle \mathrm{TUNE} \rangle) \\
&\to \mathrm{melt}(\langle \mathrm{TUNE} \rangle, \langle \mathrm{TUNE} \rangle) \\
\langle \mathrm{NOTE} \rangle &\to \mathrm{defineNote}(\langle \mathrm{FREQ} \rangle, \langle \mathrm{TIME} \rangle) \\
\langle \mathrm{TIME} \rangle &\to 0.1 \mid 0.2 \mid 0.3 \\
\langle \mathrm{FREQ} \rangle &\to \mathrm{A} \mid \mathrm{Bb} \mid \mathrm{B} \mid \mathrm{C} \mid \mathrm{Db} \mid \mathrm{D} \mid \mathrm{Eb} \mid \mathrm{E} \mid \mathrm{F} \mid \mathrm{Gb} \mid \mathrm{G} \mid \mathrm{Ab}
\end{aligned}
$$

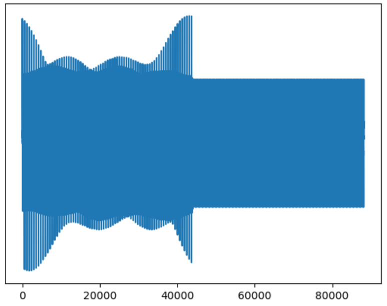


(For the sake of our ears, let’s only keep A, C, D, E, G)

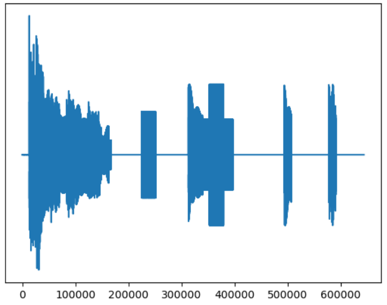
Example 1: Sound semantics



A

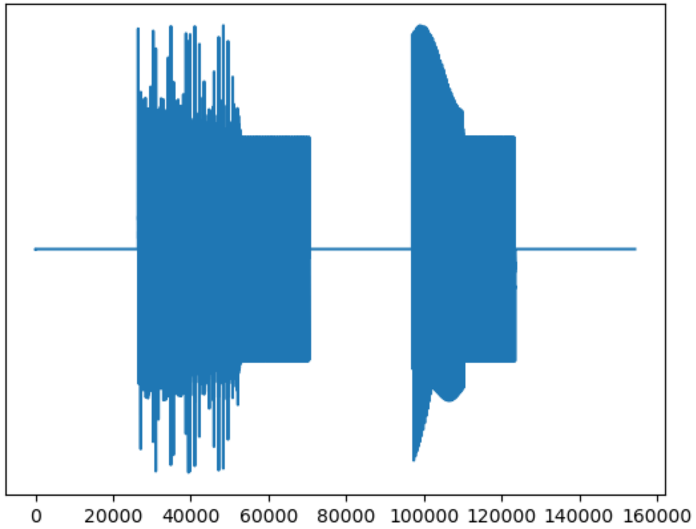


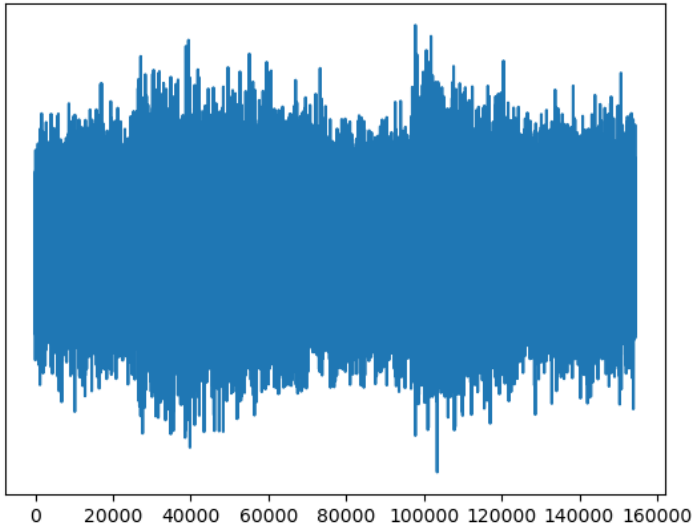
melt(defineNote(E, 1), defineNote(D, 2))



successive(defineNote('D', 0.3), melt(defineNote('A', 0.3), " "melt(melt(melt(melt(defineNote('E', 1.), " "successive(melt(successive(defineNote('D', 0.6), successive(defineNote('E', " "0.6), defineNote('E', 0.6))), melt(successive(melt(melt(defineNote('C', " "0.3), successive(melt(melt(successive(successive(defineNote('G', 0.3), " "melt(defineNote('E', 1.), defineNote('A', 0.6))), successive(defineNote('A', " "0.6), successive(defineNote('E', 0.6), melt(defineNote('G', 0.3), " "melt(defineNote('G', 1.), successive(defineNote('A', 0.6), defineNote('D', " "0.3))))))), melt(defineNote('D', 0.6), successive(defineNote('C', 1.), " "successive(defineNote('D', 1.), defineNote('G', 1.))))), " "successive(successive(melt(defineNote('C', 1.), " "successive(successive(melt(melt(successive(melt(successive(melt(defineNote('E', " "0.3), defineNote('E', 0.6)), defineNote('E', 0.6)), defineNote('C', 0.3)), " "defineNote('A', 0.3)), defineNote('A', 0.6)), " "successive(melt(melt(defineNote('E', 0.3), defineNote('A', 1.)), " "successive(melt(defineNote('E', 1.), successive(defineNote('G', 0.3), " "successive(melt(melt(defineNote('D', 0.3), defineNote('D', 1.)), " "defineNote('A', 1.)), defineNote('A', 0.3)))), melt(defineNote('A', 0.3), " "melt(defineNote('E', 0.6), melt(successive(defineNote('A', 0.3), " "defineNote('C', 0.6)), melt(defineNote('C', 0.3), defineNote('E', " "0.3))))))), defineNote('E', 1.))), defineNote('E', 1.)), defineNote('A', " "0.3))), melt(defineNote('G', 1.), melt(defineNote('C', 1.), " "melt(defineNote('D', 0.6), defineNote('D', 0.6))))), defineNote('G', 1.))), " "successive(melt(defineNote('C', 0.6), successive(defineNote('G', 0.6), " "defineNote('C', 0.3))), successive(melt(defineNote('C', 0.6), " "defineNote('C', 1.)), defineNote('E', 0.6))))), defineNote('E', 0.3)), " "defineNote('D', 1.)), defineNote('D', 0.3))), melt(defineNote('C', 0.3), " "successive(defineNote('A', 0.6), successive(melt(successive(defineNote('E', " "0.3), successive(defineNote('D', 1.), defineNote('E', 0.3))), " "successive(melt(successive(melt(defineNote('G', 0.3), defineNote('D', 0.3)), " "successive(successive(defineNote('E', 1.), defineNote('C', 0.6)), " "successive(melt(defineNote('D', 0.3), defineNote('E', 0.3)), defineNote('D', " "0.6)))), defineNote('D', 0.6)), defineNote('E', 0.3))), defineNote('G', " "0.3)))))), defineNote('C', 0.3)), successive(melt(defineNote('D', 0.3), " "melt(melt(defineNote('A', 0.6), successive(successive(defineNote('C', 0.3), " "successive(defineNote('D', 1.), defineNote('A', 0.3))), defineNote('C', " "1.))), defineNote('G', 0.3))), defineNote('C', 1.))), " "successive(defineNote('D', 0.3), defineNote('E', 0.3)))))

Example 1: Audio observations

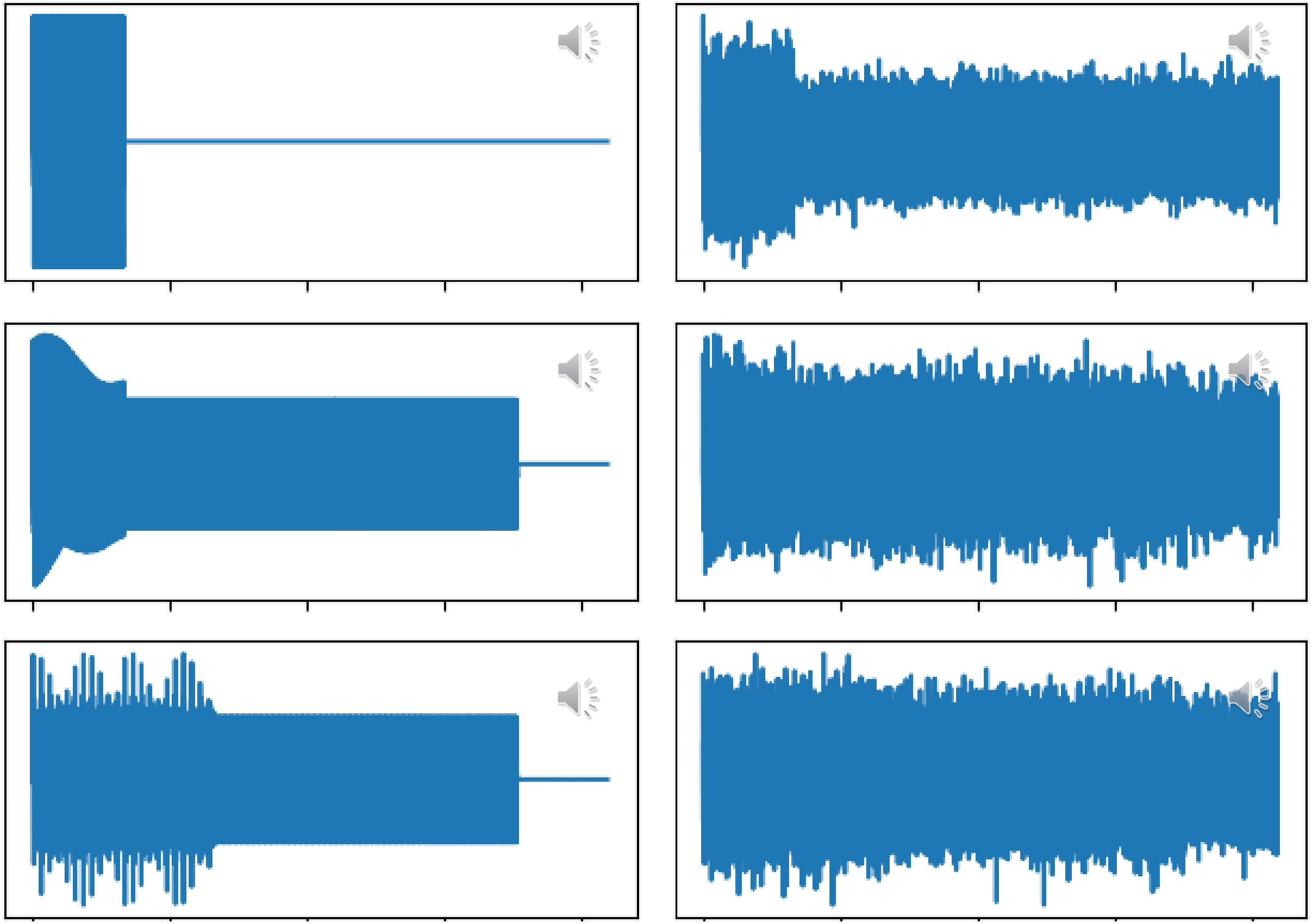




successive(defineNote(’G’, 0.6), successive(defineNote(’D’, 0.6), successive(successive(successive(successive(defineNote(’G’, 0.3), melt(melt(defineNote(’A’, 0.3), successive(defineNote(’C’, 1.), defineNote(’D’, 0.3))), defineNote(’G’, 0.6))), defineNote(’E’, 0.6)), defineNote(’E’, 0.6)), successive(melt(melt(defineNote(’E’, 0.6), successive(defineNote(’C’, 0.3), defineNote(’D’, 1.))), defineNote(’G’, 0.3)), defineNote(’G’, 0.3)))))

Example 1: Priors and likelihoods

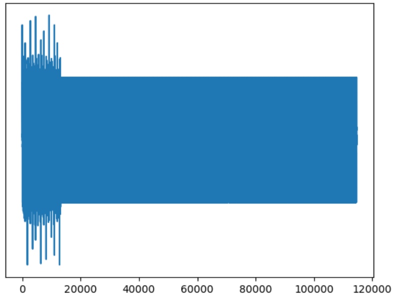
$P(\text{sound})$ is high



$P(\text{sound})$ is low(er)

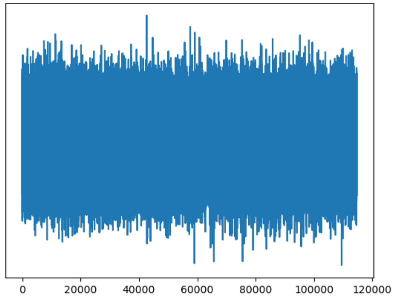
Learning: Bayesian inference in a PCFG

melt(defineNote('G', 0.3), successive(successive(defineNote('A', 0.6), defineNote('C', 1.)), defineNote('E', 1.)))

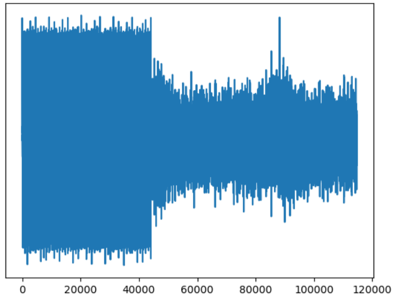


MAP

successive(successive(defineNote('A', 1.), defineNote('A', 1.)), defineNote('D', 0.6))



Posterior mean



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   Approximate!

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2. Sound (with noise)
   Multiple H produce same sound!

3. Likelihood P(audio | sound)

4. MCMC w/ e.g., LOTlib3
   Gives us samples from posterior!

Summary: Building a pLoT model

1. Pick a domain, e.g., music, geometry, logical concepts
2. Write a (plausible) list of primitive concepts
3. Write a PCFG
4. Define a likelihood function
5. Produce an observation
   1. Sample from the PCFG
   2. Hand-design
   3. Use naturally occurring
6. Run inference algorithm

Important features

Simpler representations have a higher prior

- Simpler <--> More probable in the pLoT (PCFG)

In domains above, pLoT sentences denoted single objects

…but each sentence in the pLoT can denote a **function**

- Observations are (inputs, output) tuples

Categories (sets) are a special case with Boolean output

- Observations are e.g., samples from the category
- Size effect!

Some hypotheses types

Input	**Output**	**Stochastic**	**Meaning**	**Example**
$\emptyset$	Domain object	No	Single domain object	
		Yes	Graded category	
Domain object	Boolean	No	Category	
		Yes	Graded category	
Domain object	Domain object	No	Transformation	
		Yes	Stochastic transf	
…	…	…	…	

Conclusions

- We saw how to construct a model of category learning w/ a pLoT
- Two example domains:
  - Shapes
  - Sounds
  - Many more to be explored!
- Next step: Literature using this kind of model.

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If there's time left…

Construct a grammar

- Grammar for categorization based on binary features
- Grammar for encoding binary strings

Improve the two discussed grammars

- Modify the shape and sound grammars to be more cognitively plausible.