Part III

Category learning with a pLoT

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Summary so far

pLoT as a picture of cognition

Some formal tools:

- Formal grammars
- Probabilistic PCFG
- Compositional interpretation
- (Approximate) Bayesian inference

Plan for session

- We'll consider some examples
- Combine the technical tools
- Build up to a pLoT category learning simulation

Part I	Introduction: On the very idea of an LoT
Part II	Technical background
Part III	**Bayesian program induction (LOTlib3)**
Part IV	Case studies
Part V	Summary & Future prospects

Example 1: Binary strings

Example 2: The visual domain

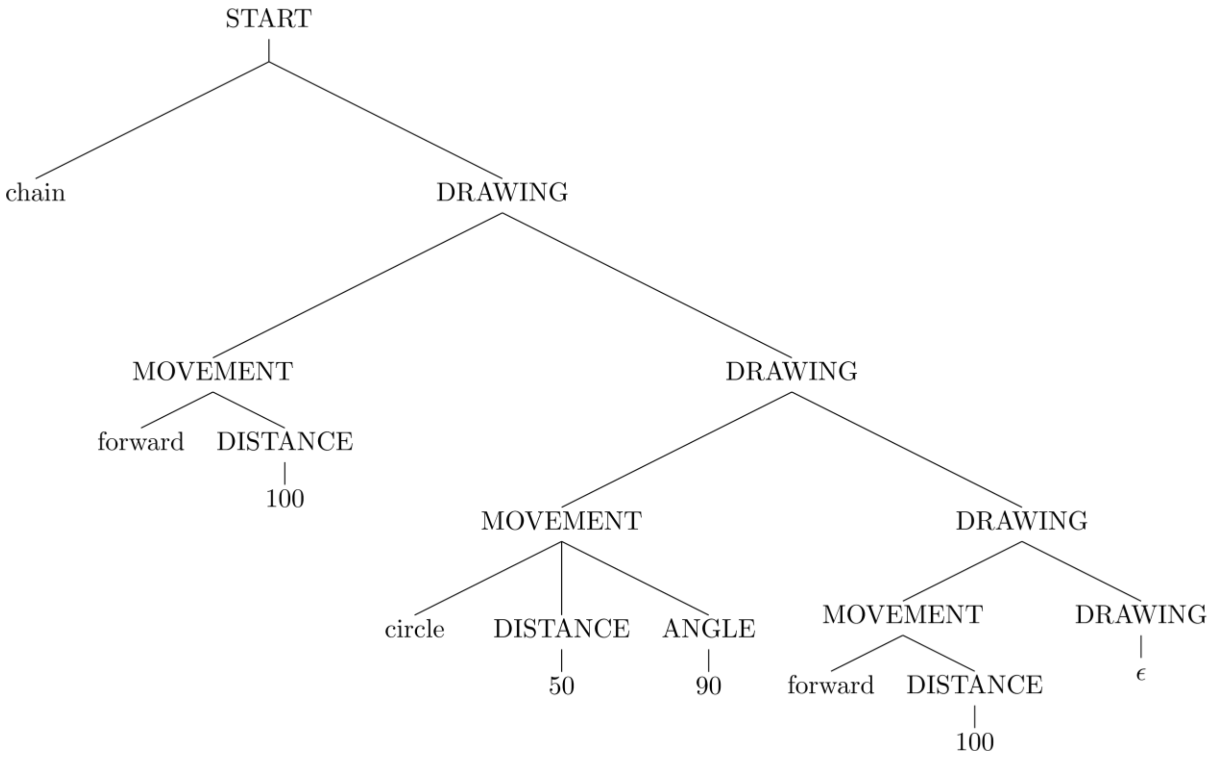
Shape grammar

What could we mean by *shape grammar*?

What would you include in a shape grammar?

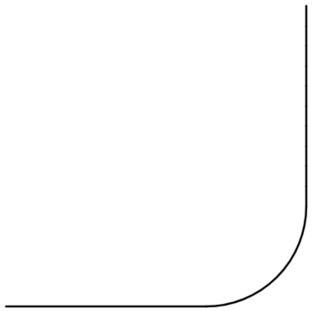
Shape grammar

$$
\begin{aligned}
\langle \text{START} \rangle &\rightarrow \text{chain}(\langle \text{DRAWING} \rangle) \\
\langle \text{DRAWING} \rangle &\rightarrow \langle \text{MOVEMENT} \rangle, \langle \text{DRAWING} \rangle \\
&\rightarrow \epsilon \\
\langle \text{MOVEMENT} \rangle &\rightarrow \text{forward}(\langle \text{DISTANCE} \rangle) \\
&\rightarrow \text{backward}(\langle \text{DISTANCE} \rangle) \\
&\rightarrow \text{right}(\langle \text{ANGLE} \rangle) \\
&\rightarrow \text{left}(\langle \text{ANGLE} \rangle) \\
&\rightarrow \text{circle}(\langle \text{DISTANCE} \rangle, \langle \text{ANGLE} \rangle) \\
&\rightarrow \text{goto}(0, 0) \\
\langle \text{DISTANCE} \rangle &\rightarrow 50 \mid 100 \mid 120 \\
\langle \text{ANGLE} \rangle &\rightarrow 30 \mid 60 \mid 90 \mid 170
\end{aligned}
$$

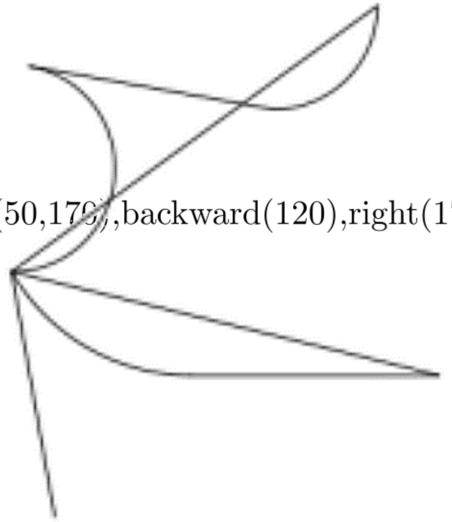


Shape semantics

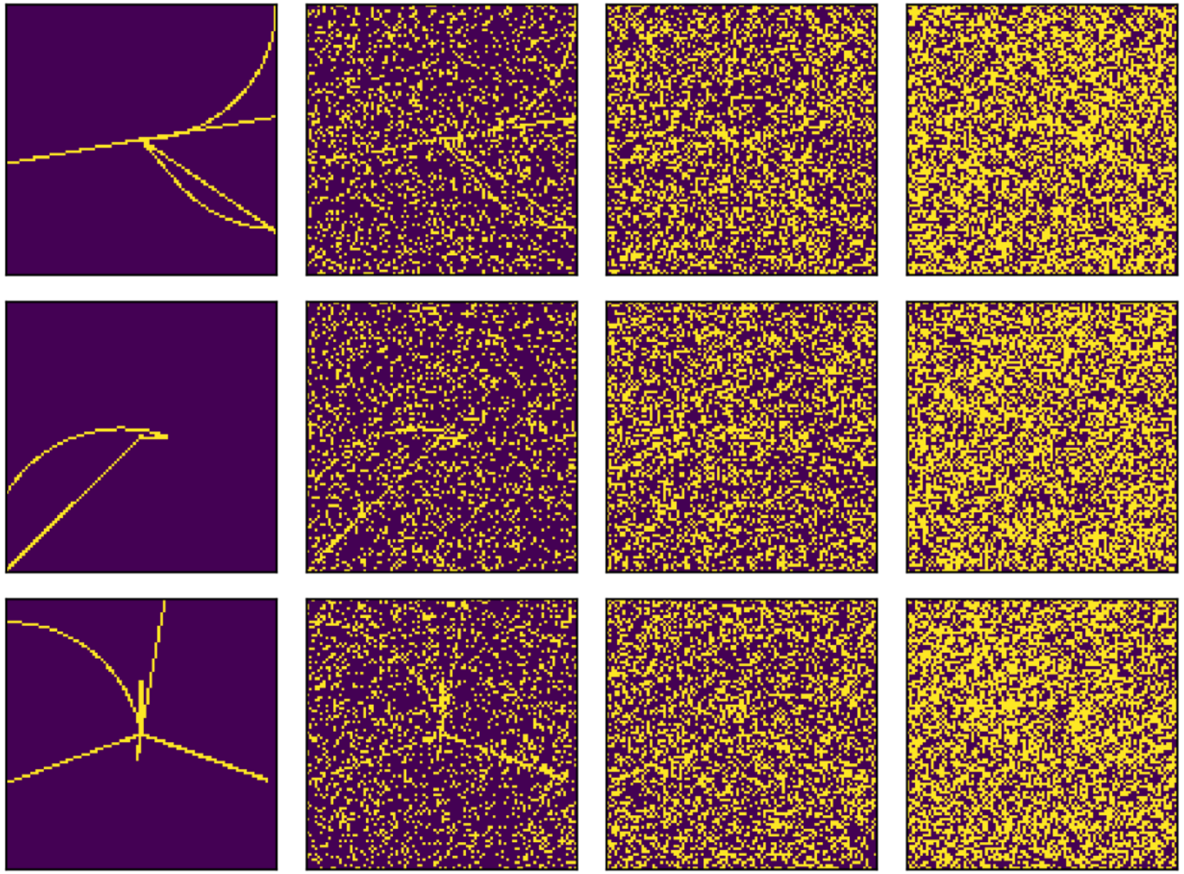
chain(forward(100), circle(50,90), forward(100),)



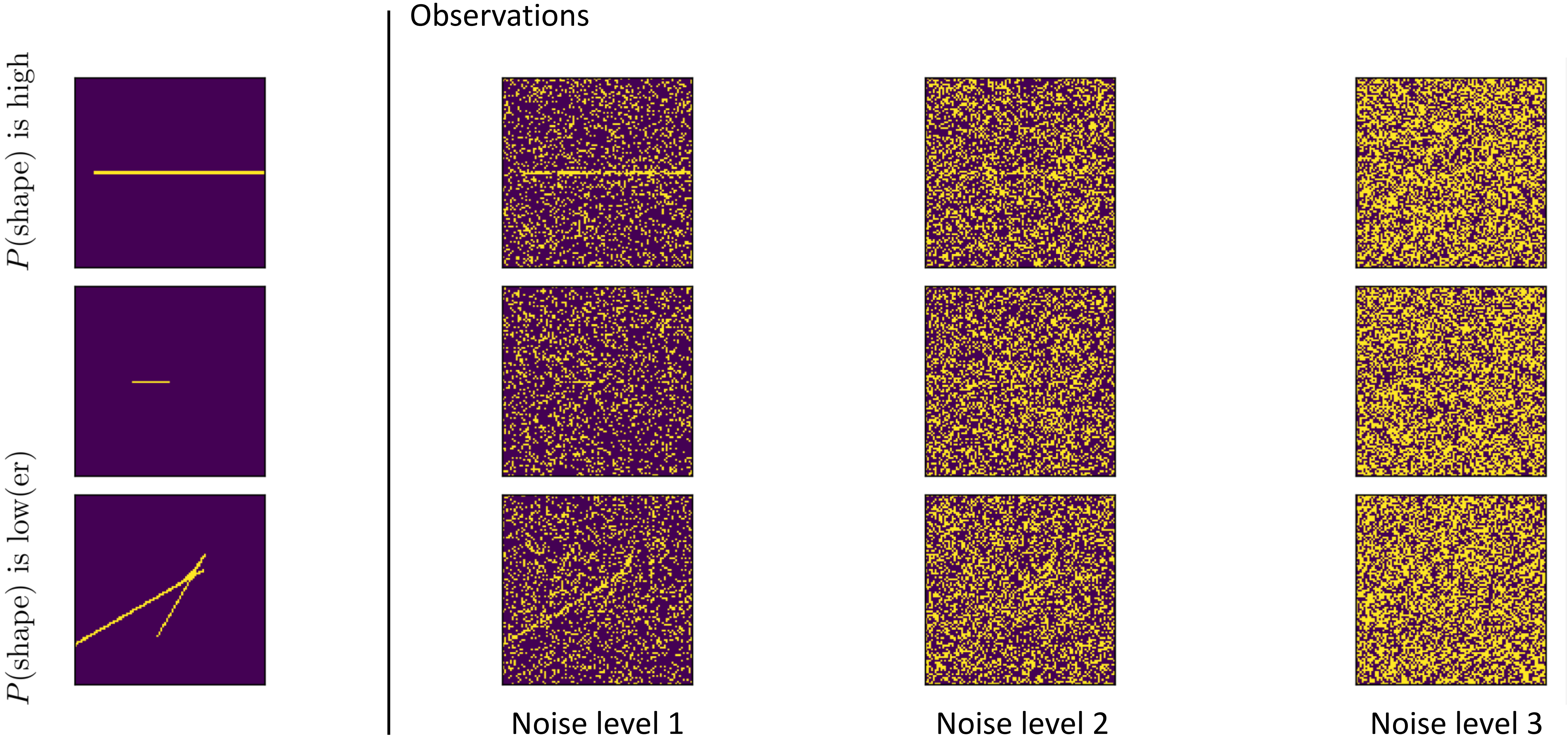
chain(right(60),circle(100,60),forward(120),goto(0,0),circle(50,170),backward(120),right(170),circle(50,90),goto(0,0),right(170),forward(120),right(90),)



Shape observations



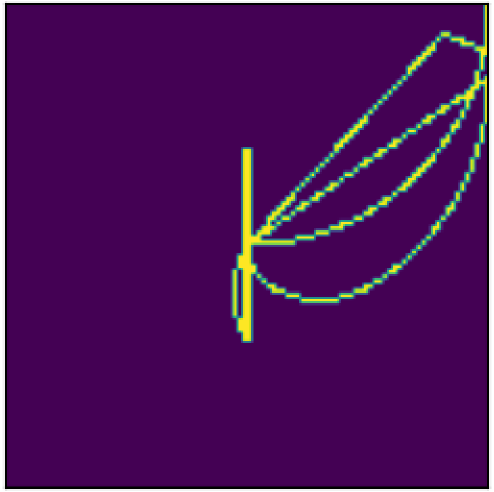
Priors and likelihoods



Bayesian inference in the shape grammar

```
chain(
circle_(50,90),
backward_(10),
right_(90),
right_(170),
right_(30),
forward_(10),
right_(90),
left_(170),
goto_(0,0),
circle_(20,60),
right_(30),
goto_(0,0),
goto_(0,0),
goto_(0,0),
backward_(20),
forward_(20),
goto_(0,0),
circle_(10,60),
circle_(20,60),
circle_(50,60),
left_(170),
goto_(0,0),
right_(60),
left_(30),
)
```

True shape



```
chain(circle_(50,90),goto_(0,0),forward_(20),goto_(0,0),backward_(20),)
```

LoT learning models: the big picture

1. Interpreted PCFG
   Defines hypotheses H
   Defines a prior over H

2. Observations O
   Generated by true H

3. Likelihood P(O | H)

4. Bayesian inference algorithm
   Approximate!

1. Shapes grammar
   Each H is a sentence…
   …which encodes a shape

2. Image (with noise)
   Multiple H produce same shape!

3. Likelihood P(image| shape)

4. MCMC w/ e.g., LOTlib3
   Gives us samples from posterior!

Example 3: The auditory domain

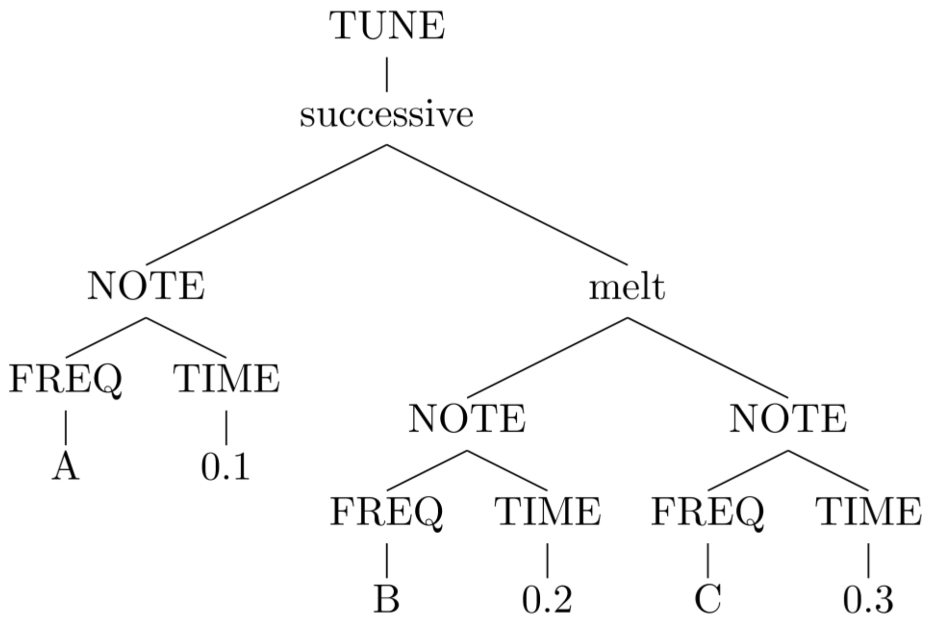
Sound grammar

What could we mean by *sound grammar*?

What would you include in a sound grammar?

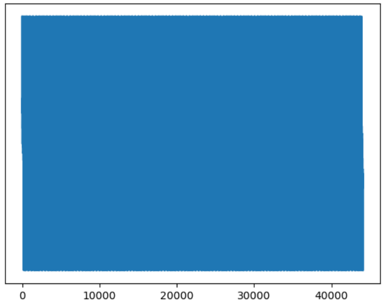
Sound grammar

$$
\begin{aligned}
\langle \text{TUNE} \rangle &\rightarrow \langle \text{NOTE} \rangle \\
&\rightarrow \text{successive}(\langle \text{TUNE} \rangle, \langle \text{TUNE} \rangle) \\
&\rightarrow \text{melt}(\langle \text{TUNE} \rangle, \langle \text{TUNE} \rangle) \\
\langle \text{NOTE} \rangle &\rightarrow \text{defineNote}(\langle \text{FREQ} \rangle, \langle \text{TIME} \rangle) \\
\langle \text{TIME} \rangle &\rightarrow 0.1 \mid 0.2 \mid 0.3 \\
\langle \text{FREQ} \rangle &\rightarrow \text{A} \mid \text{Bb} \mid \text{B} \mid \text{C} \mid \text{Db} \mid \text{D} \mid \text{Eb} \mid \text{E} \mid \text{F} \mid \text{Gb} \mid \text{G} \mid \text{Ab}
\end{aligned}
$$

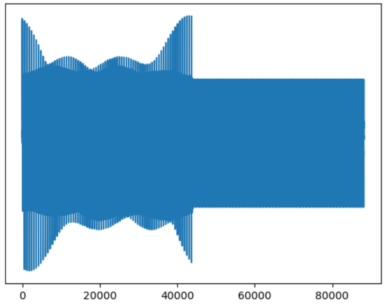


(For the sake of our ears, let’s only keep A, C, D, E, G)

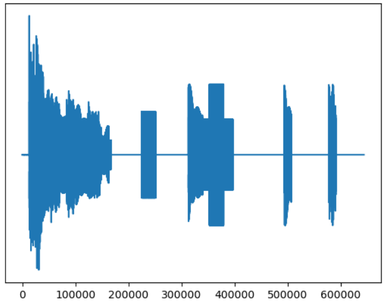
Sound semantics



A

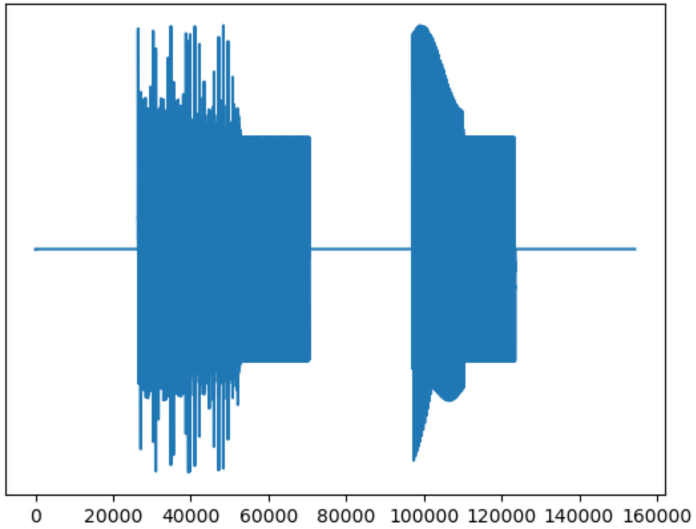


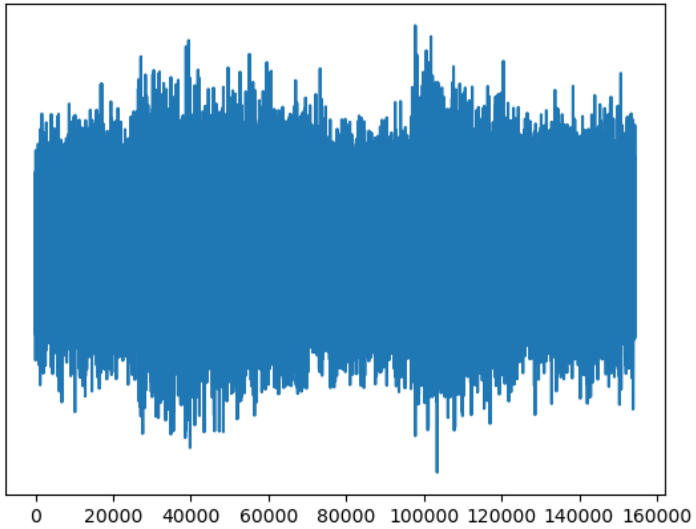
melt(defineNote(E, 1), defineNote(D, 2))



successive(defineNote('D', 0.3), melt(defineNote('A', 0.3), " "melt(melt(melt(melt(defineNote('E',
1.), " "successive(melt(successive(defineNote('D', 0.6), successive(defineNote('E',
" "0.6), defineNote('E', 0.6))), melt(successive(melt(melt(defineNote('C', " "0.3),
successive(melt(melt(successive(successive(defineNote('G', 0.3), " "melt(defineNote('E',
1.), defineNote('A', 0.6))), successive(defineNote('A', " "0.6), successive(defineNote('E',
0.6), melt(defineNote('G', 0.3), " "melt(defineNote('G', 1.), successive(defineNote('A',
0.6), defineNote('D', " "0.3))))))), melt(defineNote('D', 0.6), successive(defineNote('C',
1.), " "successive(defineNote('D', 1.), defineNote('G', 1.))))), " "successive(successive(melt(defineNote('C',
1.), " "successive(successive(melt(melt(successive(melt(successive(melt(defineNote('E',
" "0.3), defineNote('E', 0.6)), defineNote('E', 0.6)), defineNote('C', 0.3)), " "de-
fineNote('A', 0.3)), defineNote('A', 0.6)), " "successive(melt(melt(defineNote('E',
0.3), defineNote('A', 1.)), " "successive(melt(defineNote('E', 1.), successive(defineNote('G',
0.3), " "successive(melt(melt(defineNote('D', 0.3), defineNote('D', 1.)), " "de-
fineNote('A', 1.)), defineNote('A', 0.3)))), melt(defineNote('A', 0.3), " "melt(defineNote('E',
0.6), melt(successive(defineNote('A', 0.3), " "defineNote('C', 0.6)), melt(defineNote('C',
0.3), defineNote('E', " "0.3))))))), defineNote('E', 1.))), defineNote('E', 1.)),
defineNote('A', " "0.3))), melt(defineNote('G', 1.), melt(defineNote('C', 1.), "
"melt(defineNote('D', 0.6), defineNote('D', 0.6))))), defineNote('G', 1.))), " "suc-
cessive(melt(defineNote('C', 0.6), successive(defineNote('G', 0.6), " "defineNote('C',
0.3))), successive(melt(defineNote('C', 0.6), " "defineNote('C', 1.)), defineNote('E',
0.6))))), defineNote('E', 0.3)), " "defineNote('D', 1.)), defineNote('D', 0.3))),
melt(defineNote('C', 0.3), " "successive(defineNote('A', 0.6), successive(melt(successive(defineNote('E',
" "0.3), successive(defineNote('D', 1.), defineNote('E', 0.3))), " "successive(melt(successive(melt(defineNote('G',
0.3), defineNote('D', 0.3)), " "successive(successive(defineNote('E', 1.), defineNote('C',
0.6)), " "successive(melt(defineNote('D', 0.3), defineNote('E', 0.3)), defineNote('D',
" "0.6)))), defineNote('D', 0.6)), defineNote('E', 0.3))), defineNote('G', " "0.3)))))),
defineNote('C', 0.3)), successive(melt(defineNote('D', 0.3), " "melt(melt(defineNote('A',
0.6), successive(successive(defineNote('C', 0.3), " "successive(defineNote('D', 1.),
defineNote('A', 0.3))), defineNote('C', " "1.))), defineNote('G', 0.3))), defineNote('C',
1.))), " "successive(defineNote('D', 0.3), defineNote('E', 0.3)))))

Audio observations

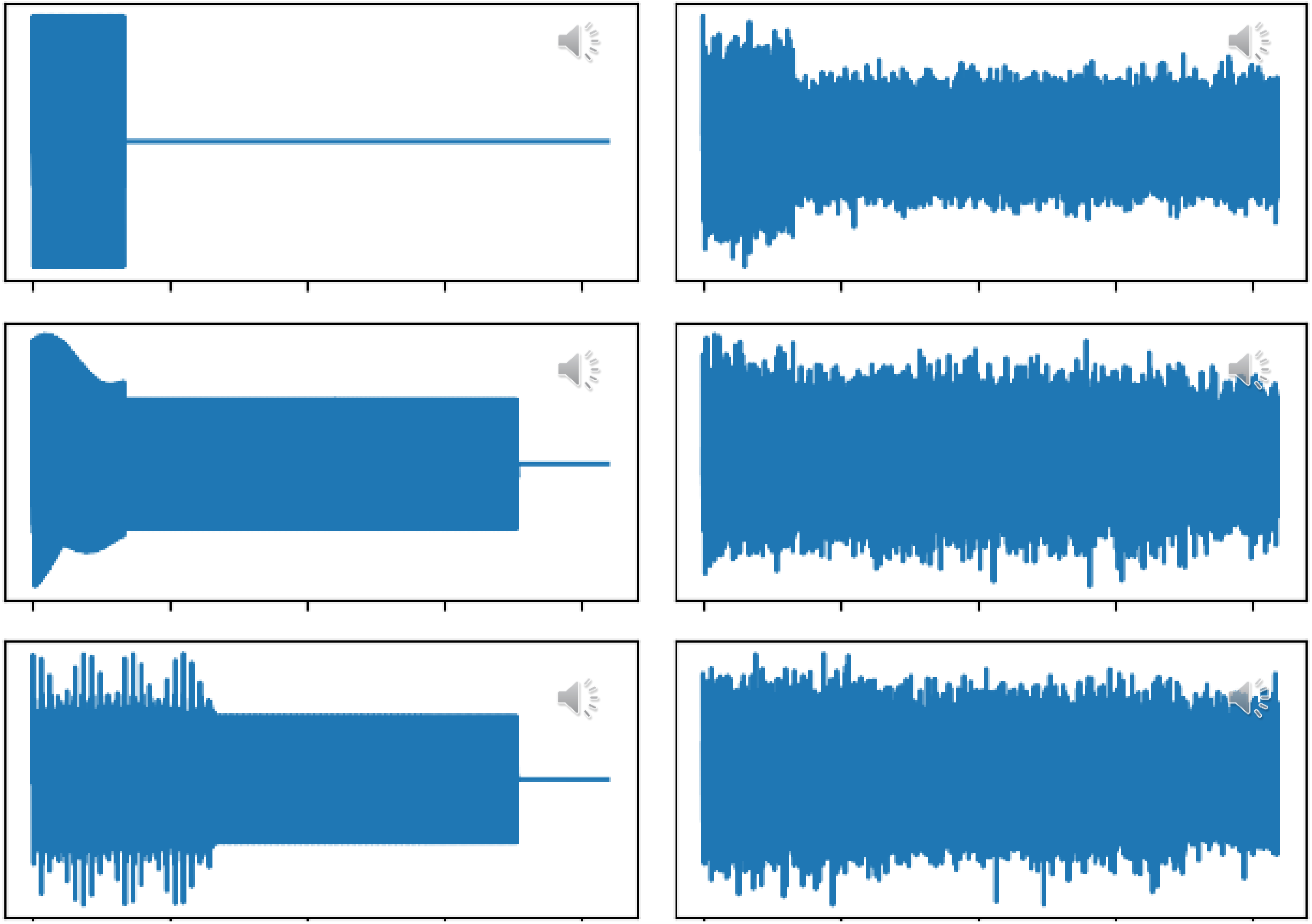




successive(defineNote('G', 0.6), successive(defineNote('D', 0.6), successive(successive(successive(successive(defineNote('G', 0.3), melt(melt(defineNote('A', 0.3), successive(defineNote('C', 1.), defineNote('D', 0.3))), defineNote('G', 0.6))), defineNote('E', 0.6)), defineNote('E', 0.6)), successive(melt(melt(defineNote('E', 0.6), successive(defineNote('C', 0.3), defineNote('D', 1.))), defineNote('G', 0.3)), defineNote('G', 0.3)))))

Priors and likelihoods

$P(\text{sound})$ is high

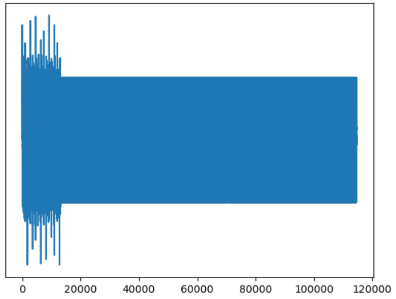


$P(\text{sound})$ is low(er)

Learning: Bayesian inference in a PCFG

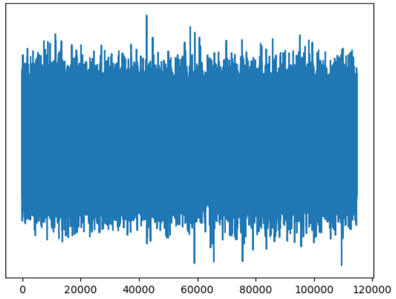
melt(defineNote('G', 0.3), successive(successive(defineNote('A', 0.6), defineNote('C', 1.)), defineNote('E', 1.)))

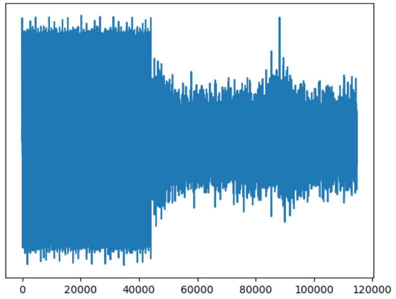
MAP



successive(successive(defineNote('A', 1.), defineNote('A', 1.)), defineNote('D', 0.6))

Posterior mean





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4. Bayesian inference
   Approximate!

1. Sound grammar
   Each H is a sentence…
   …which encodes a sound

2. Sound (with noise)
   Multiple H produce same sound!

3. Likelihood P(audio | sound)

4. MCMC w/ e.g., LOTlib3
   Gives us samples from posterior!

Summary: Building a pLoT model

1. Pick a domain, e.g., music, geometry, logical concepts
2. Write a (plausible) list of primitive concepts
3. Write a PCFG
4. Define a likelihood function
5. Produce an observation
    1. Sample from the PCFG
    2. Hand-design
    3. Use naturally occurring
6. Run inference algorithm

Important features

Simpler representations have a higher prior

- Simpler <--> More probable in the pLoT (PCFG)

In domains above, pLoT sentences denoted single objects

…but each sentence in the pLoT can denote a **function**

- Observations are (inputs, output) tuples

Categories (sets) are a special case with Boolean output

- Observations are e.g., samples from the category
- Size effect!

Some hypotheses types

Input	**Output**	**Stochastic**	**Meaning**	**Example**
$\emptyset$	Domain object	No	Single domain object	
		Yes	Graded category	
Domain object	Boolean	No	Category	
		Yes	Graded category	
Domain object	Domain object	No	Transformation	
		Yes	Stochastic transf	
...	...	...	...	

Conclusions

- We saw how to construct a model of category learning w/ a pLoT
- Two example domains:
  - Shapes
  - Sounds
  - Many more to be explored!
- Next step: Literature using this kind of model.

If there's time left…

Construct a grammar

- Grammar for categorization based on binary features
- Grammar for encoding binary strings

Improve the two discussed grammars

- Modify the shape and sound grammars to be more cognitively plausible.