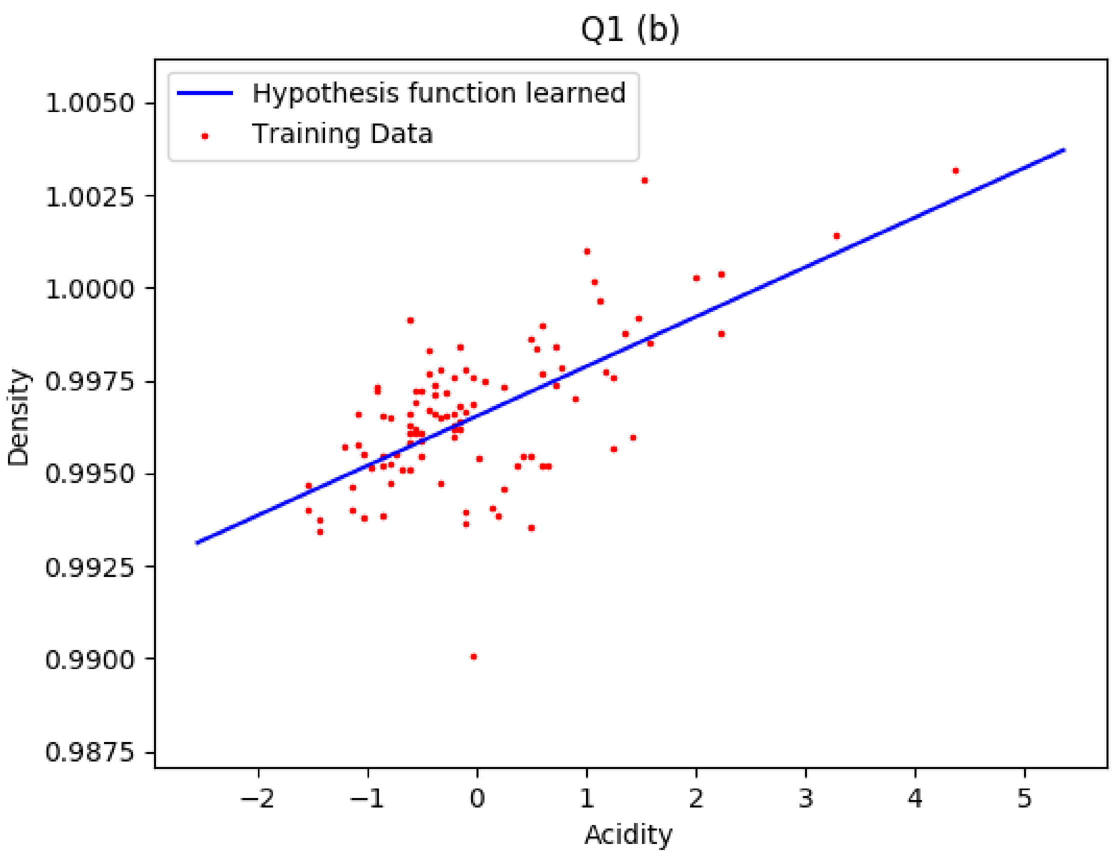
# COL774 Assignment-1

**Q1**
**Part(a)**

Learning rate: tried many values and chose η=**0.001**
Stopping criteria: If value of cost in 2 consecutive iterations changes by less than ε=**0.0000001** then stop.
I takes **89** iterations to converge at η=**0.001**

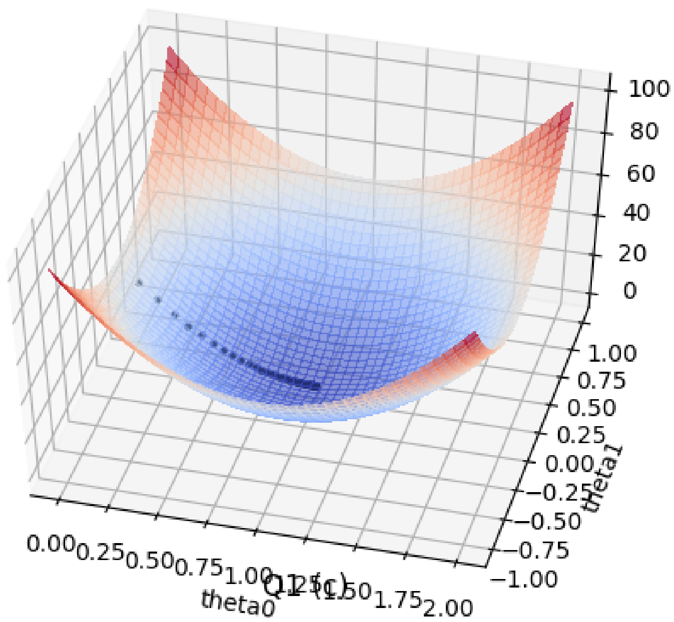
θ comes out to be [ 0.99653574 0.00134008]

**Part(b)**
Plotted training data and hypothesis function learned.

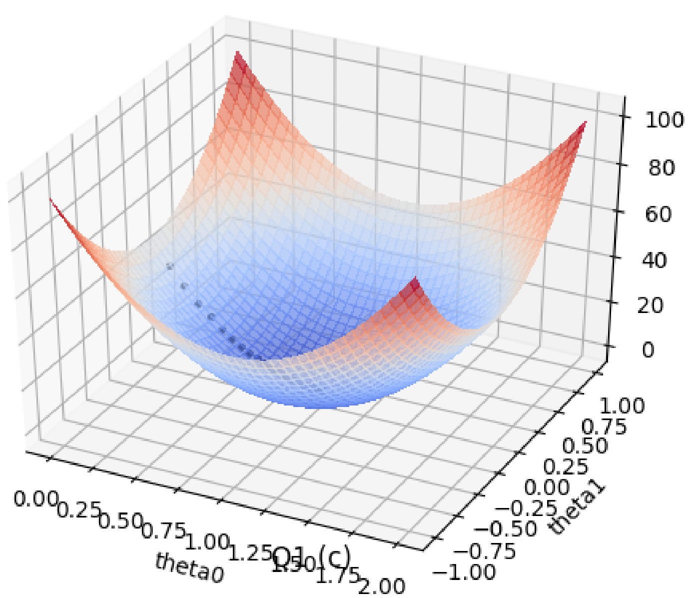


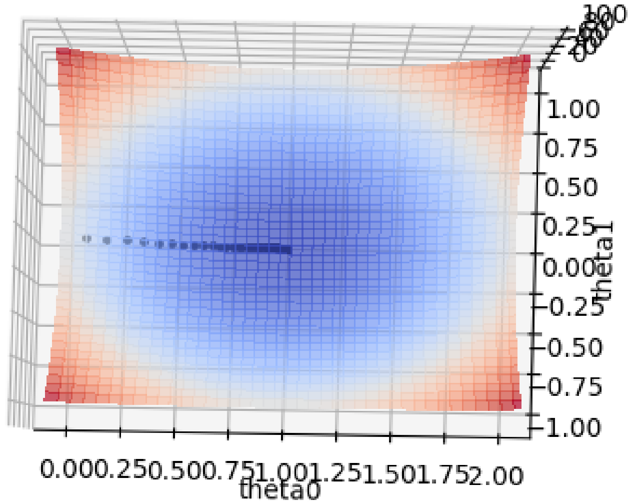

**Part(c)**

Plotted J(θ) and then plotted points at each iteration of gradient decent.



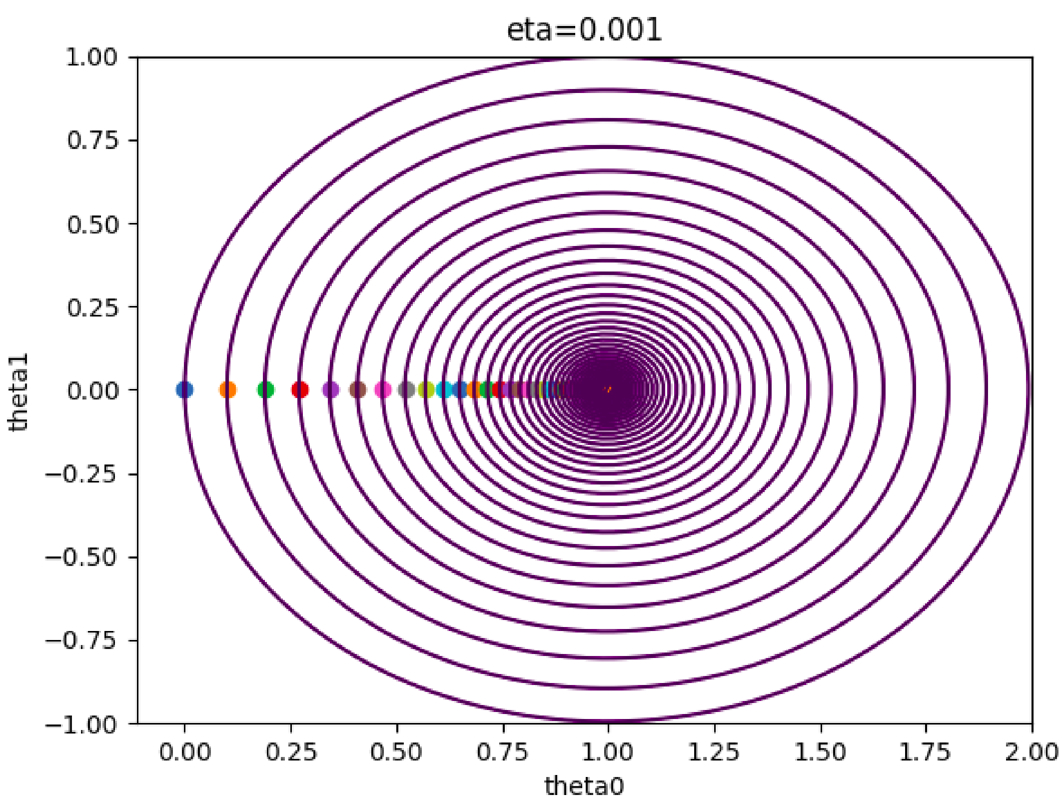

Plots of J(θ) vs θ

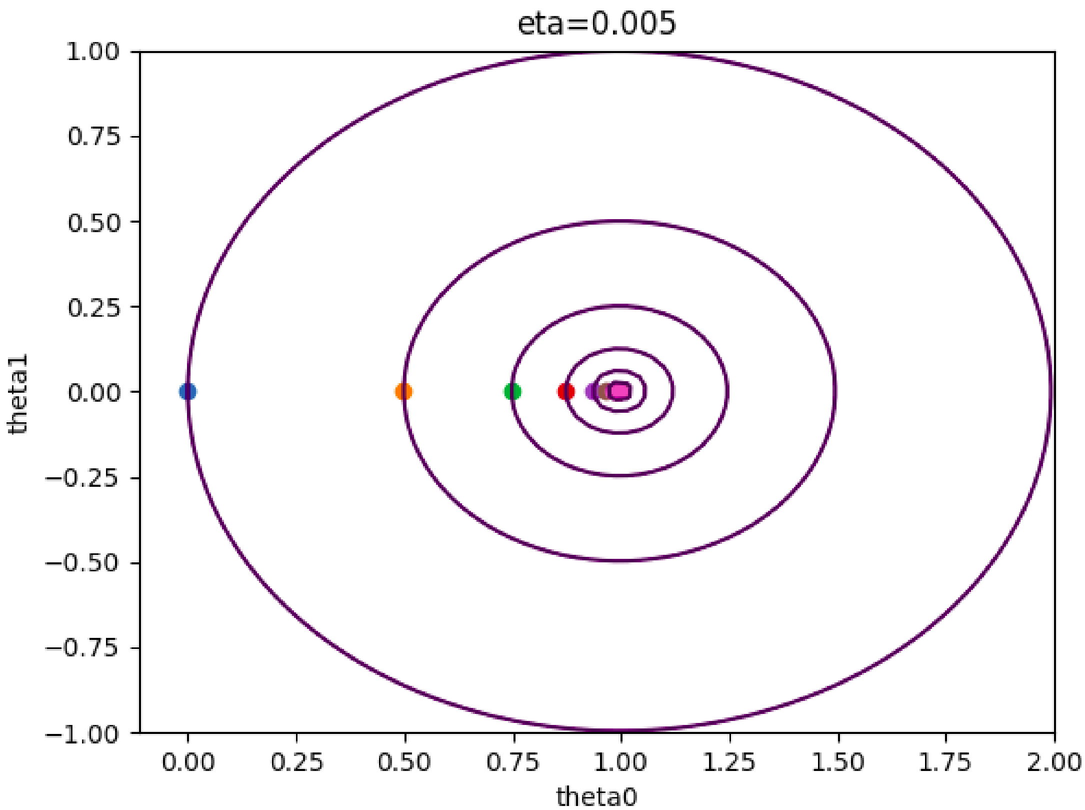


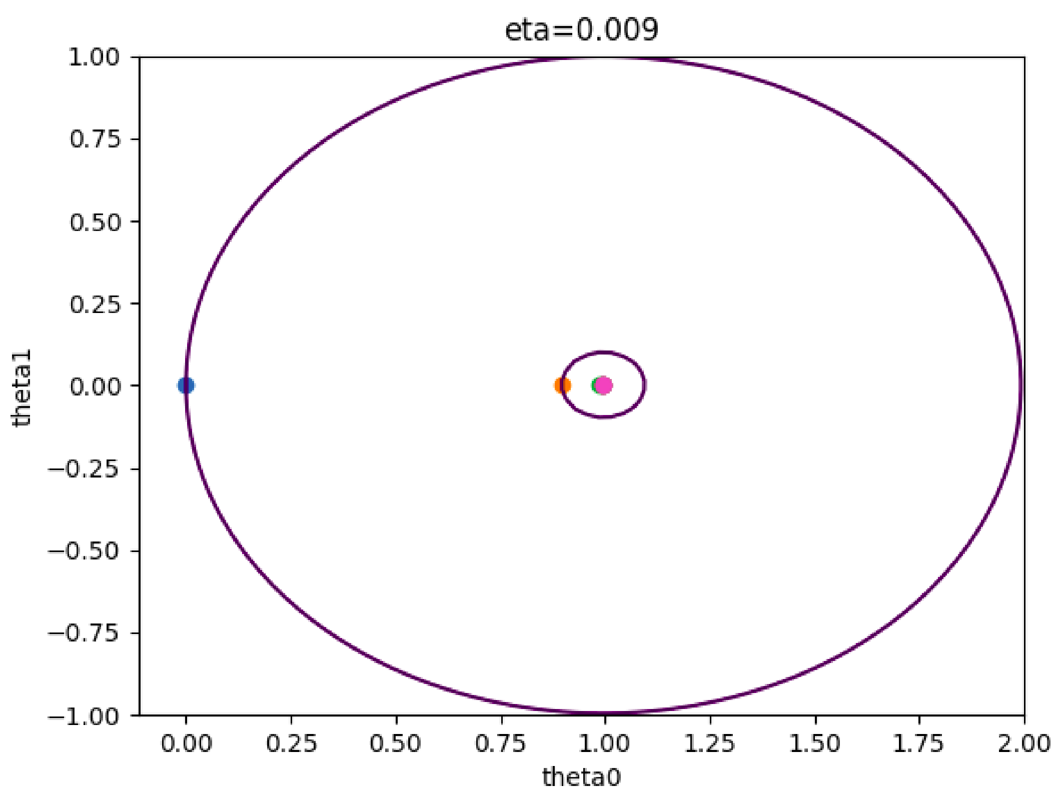


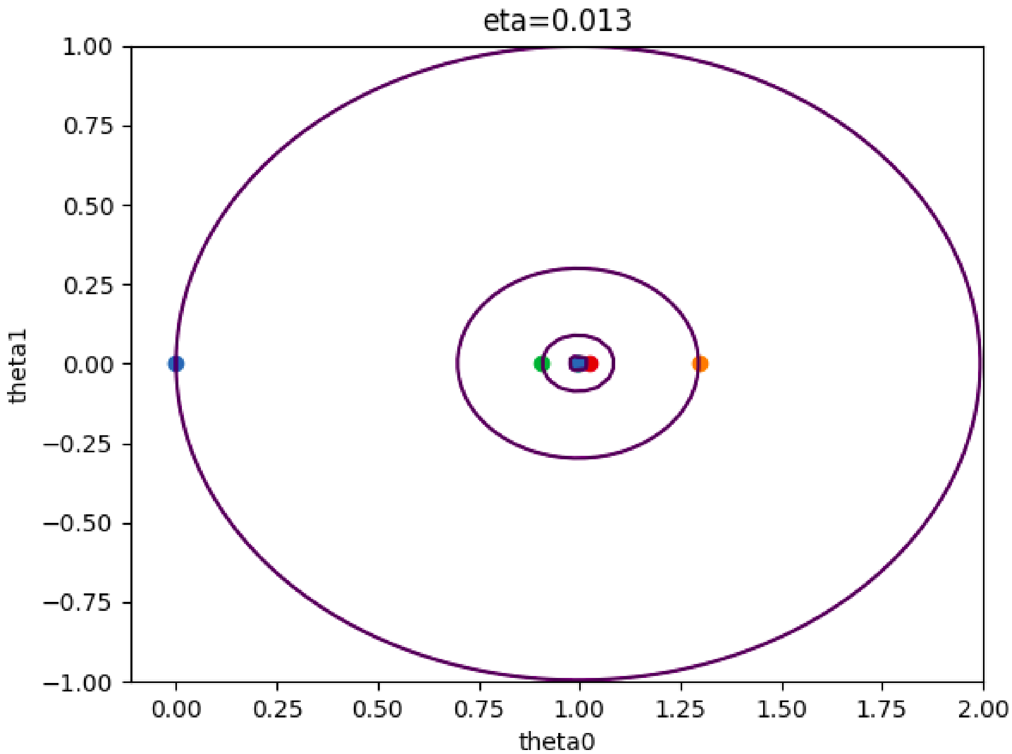

Q1 (c)

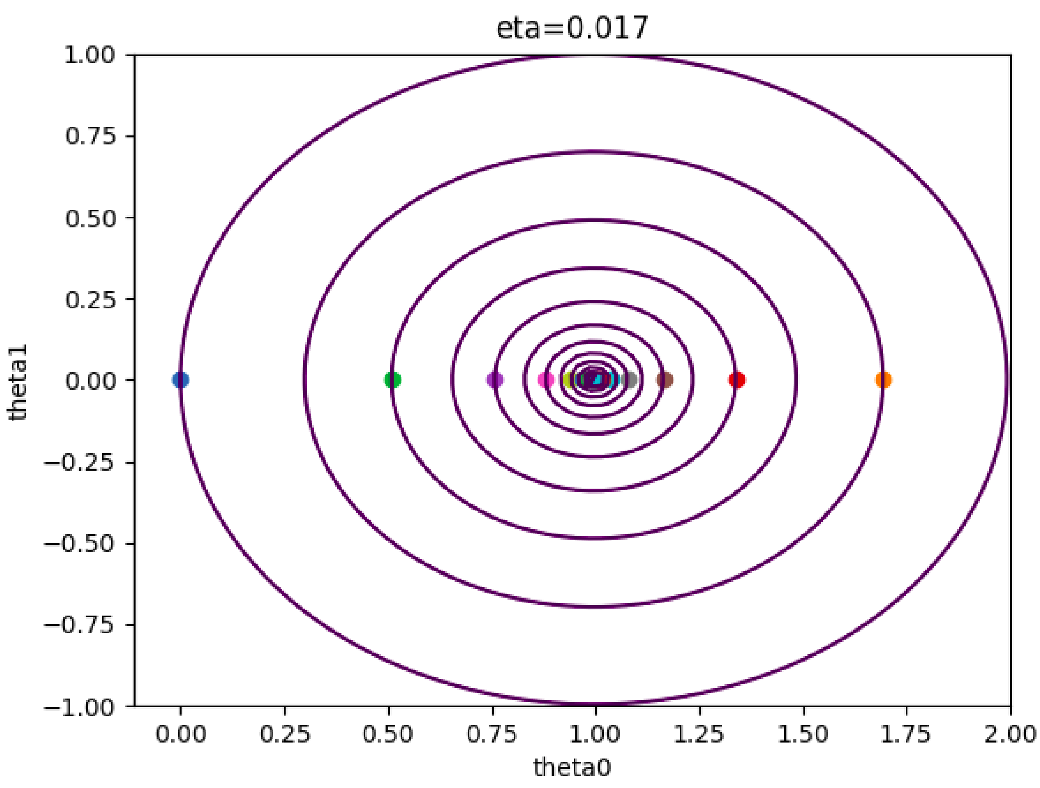
**Part(d)**
Plotted contour for cost function at each iteration.




eta=0.005
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
theta1
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
theta0


eta=0.009
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
theta1
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
theta0


eta=0.013
theta1
theta0
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


eta=0.017
theta1
theta0
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

**Part(e)**

The step size increases from η=0.001,0.005,0.009 and it take less iterations to converge.

Algorithm starts oscillating initially when η=0.013

For η=0.013 and 0.017 it oscillates initially but converges eventually.

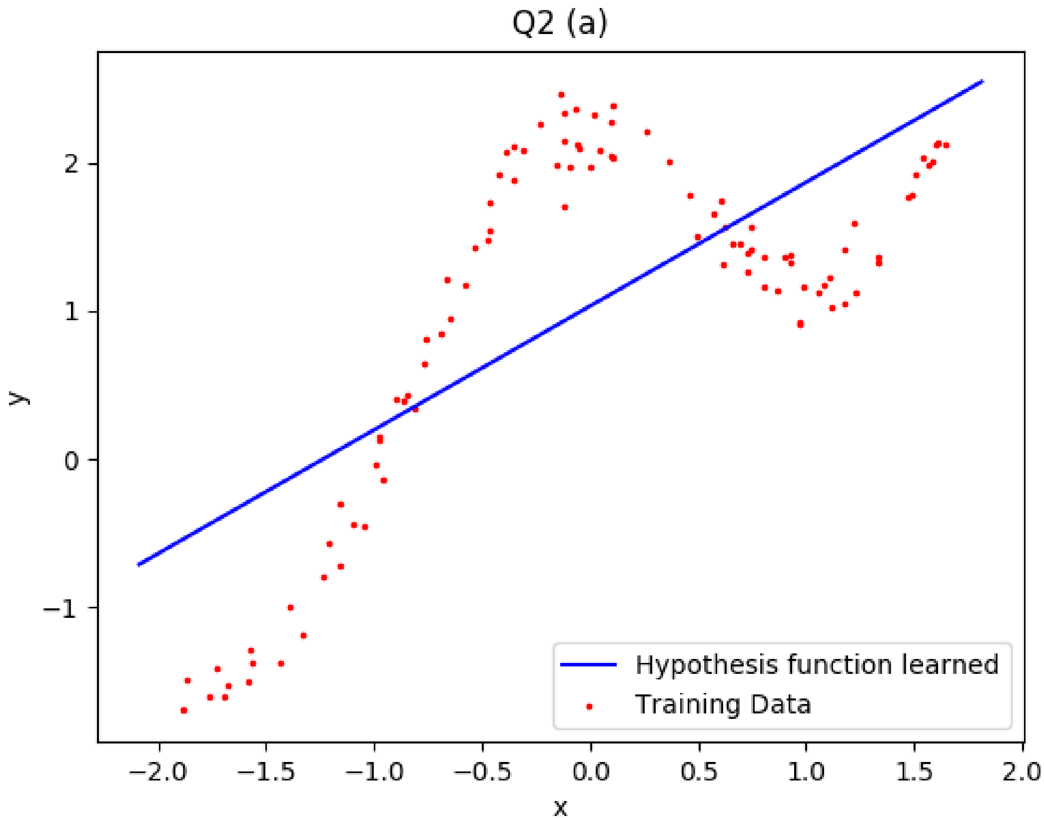
At η=0.021 and 0.025 it keeps diverging and does not converge.

**Q2**

**Part(a)**

Calculated θ using :

$$\theta = (X^T X)^{-1} X^T Y$$



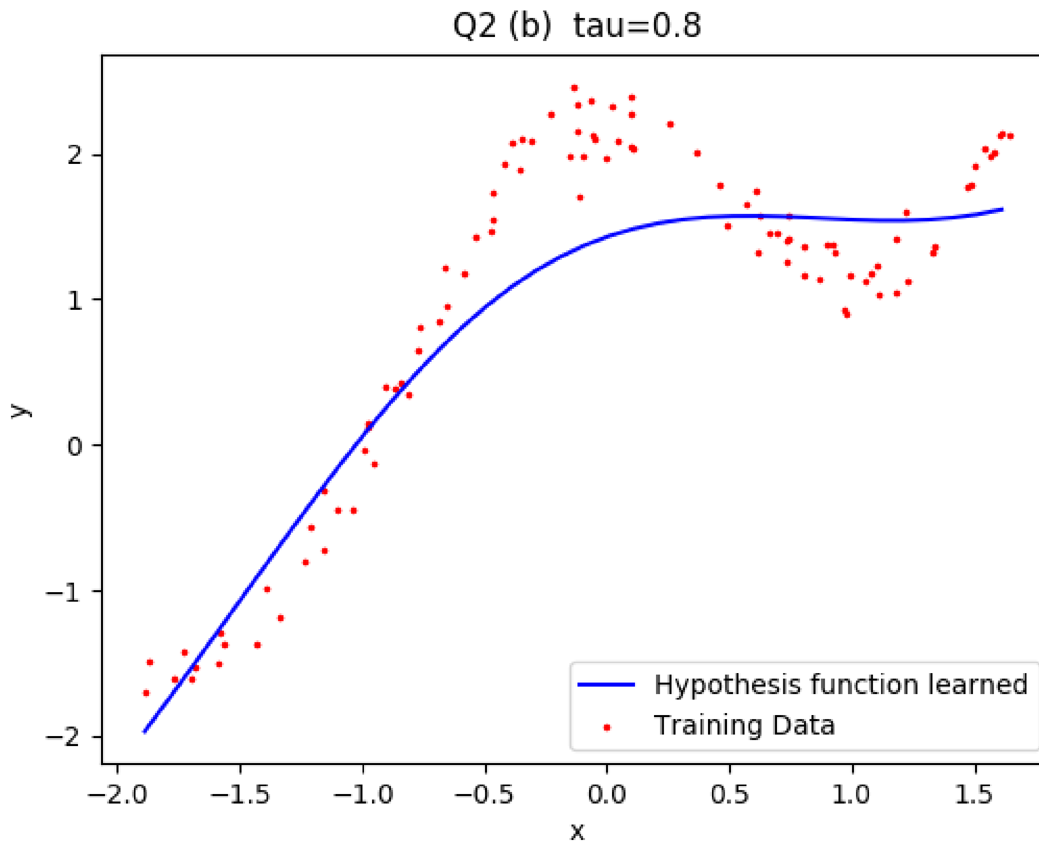

**Part(b)**

Calculated θ for each query point x where $\theta^T x$ has to be calculated using:

$$\theta = ((X^T W X)^{-1}) X^T W Y$$

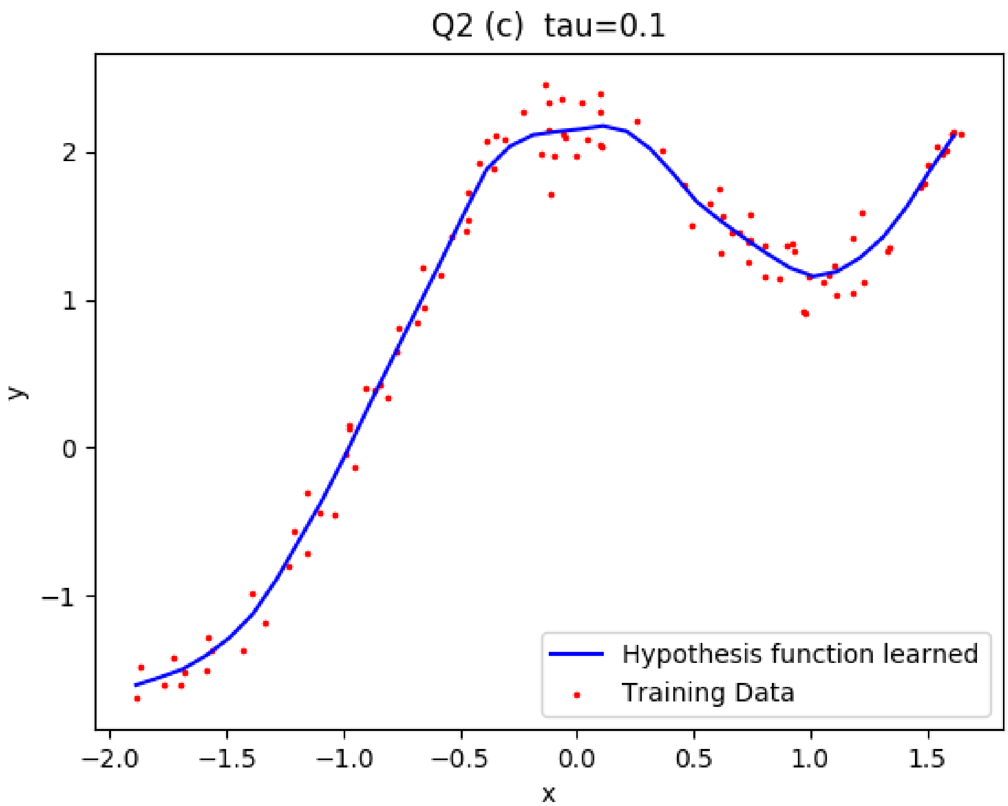
Where W is a m*m diagonal matrix, W is calculated for each query point x

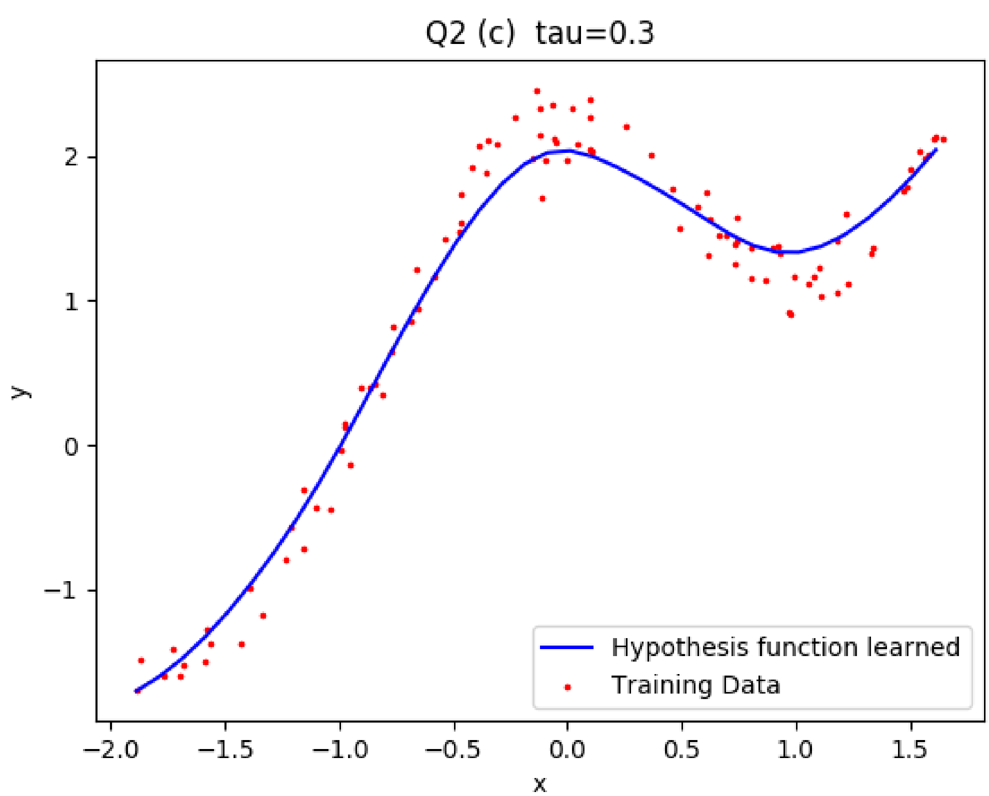
$$W_{ii}(x) = exp(-\frac{(x - x^{(i)})^2}{2\tau^2}) \qquad 0 \leq i < m$$

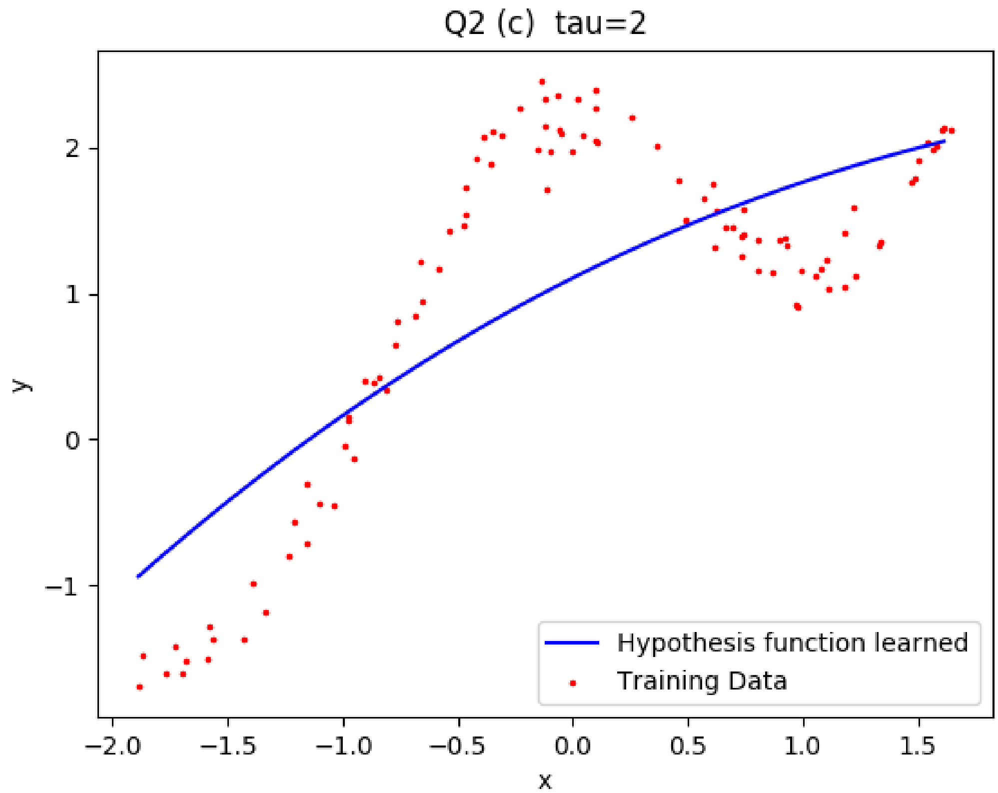


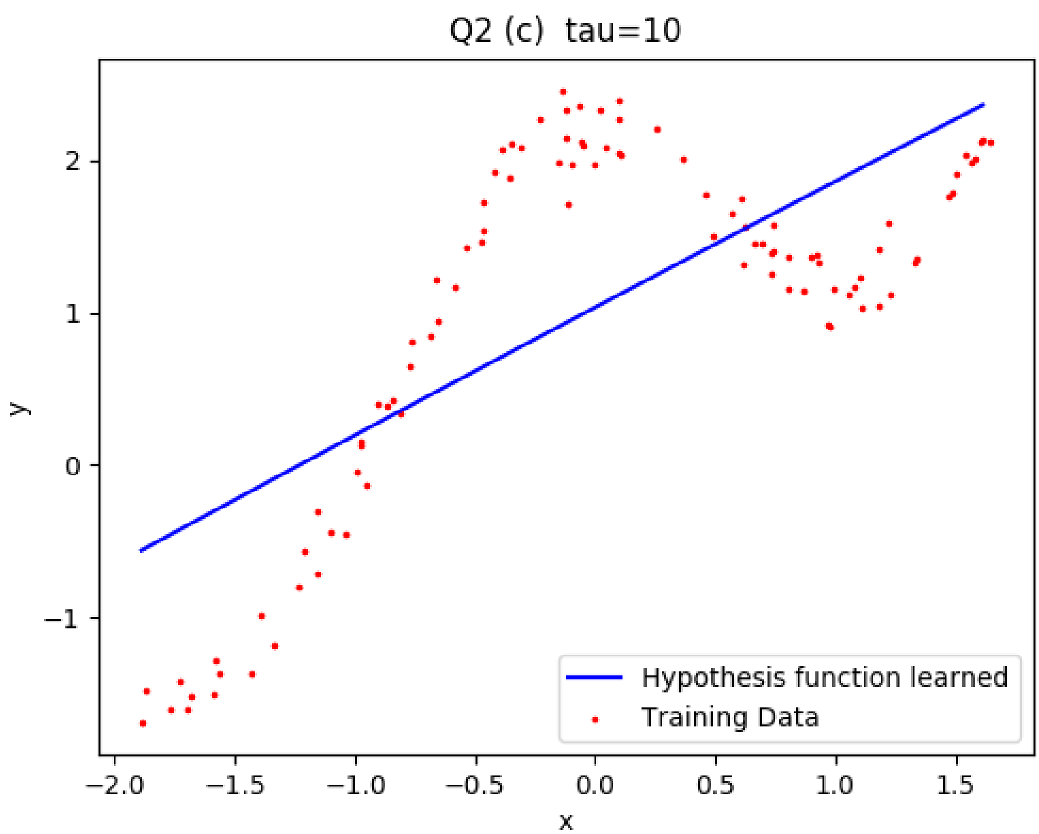

**Part(c)**

Tried tau={0.1,0.3,0.8,2,10} and plotted the hypothesis function learned.


Q2 (c) tau=0.1
Hypothesis function learned
Training Data
y
x
2
1
0
−1
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


Q2 (c) tau=0.3
Hypothesis function learned
Training Data
y
x
2
1
0
−1
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


Q2 (c) tau=2
Hypothesis function learned
Training Data
x
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
−1
0
1
2


Q2 (c) tau=10
Hypothesis function learned
Training Data
x
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
−1
0
1
2

Out of tau={0.1,0.3,0.8,2,10}, I think  tau=0.3 is best, because there is no overfitting and no underfitting.

When tau is too small, tau=0.1 then overfitting happens.
When tau is too large, tau=10 then underfitting happens and the function learned is almost a straight line.
For  tau=2,10 the curve does not represent the data at all.

**Q3**

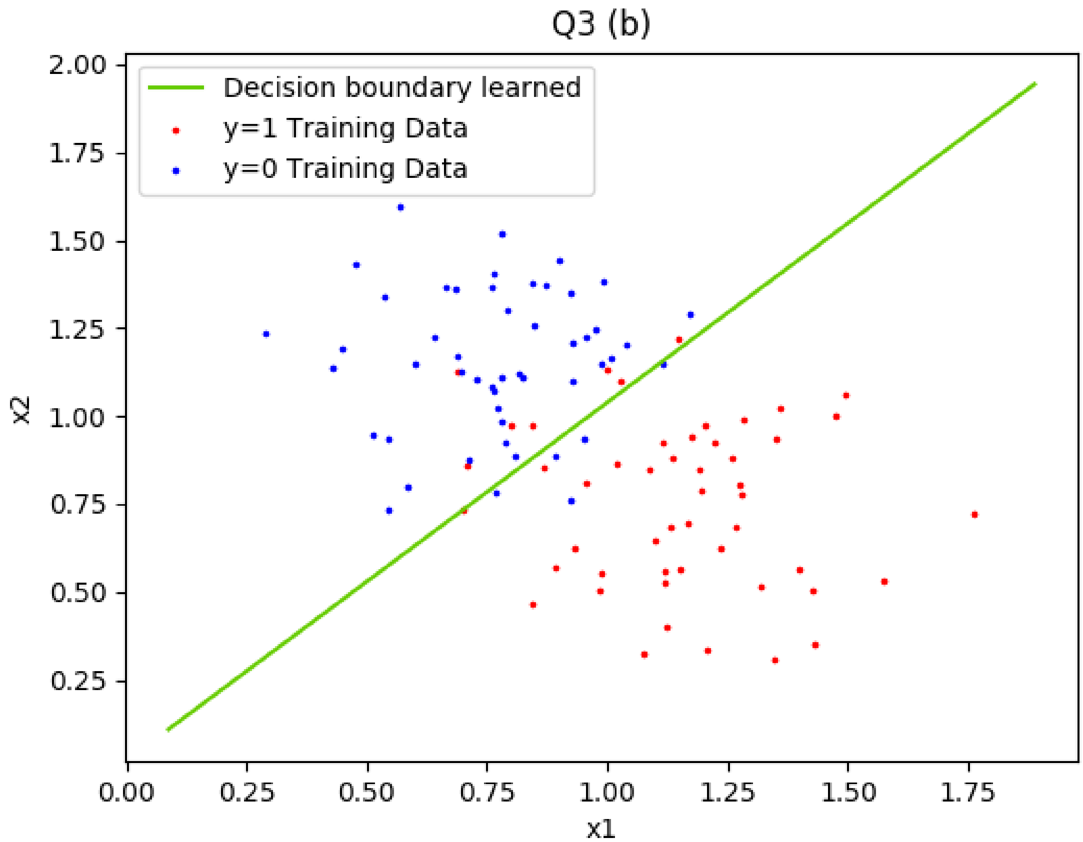
**Part(a)**
Implement Newton's method for optimizing L(θ).
Used Newton's method to find roots of L'(θ)

**Convergence condition used**: If L2 norm of difference between theta of 2 consecutive iterations is less than 0.000001 then stop

θ obtained is:
θ = [ 0.40125316  2.5885477  -2.72558849]

**Part(b)**

Plotted decision boundary learned with the training data.



**Q4**
Alaska is 0 and Canada is 1

**Part (a)**

Calculated μ0, μ1, ∑ using the formulas told in class.

μ0=[-0.75529433  0.68509431]

μ1=[ 0.75529433 -0.68509431]

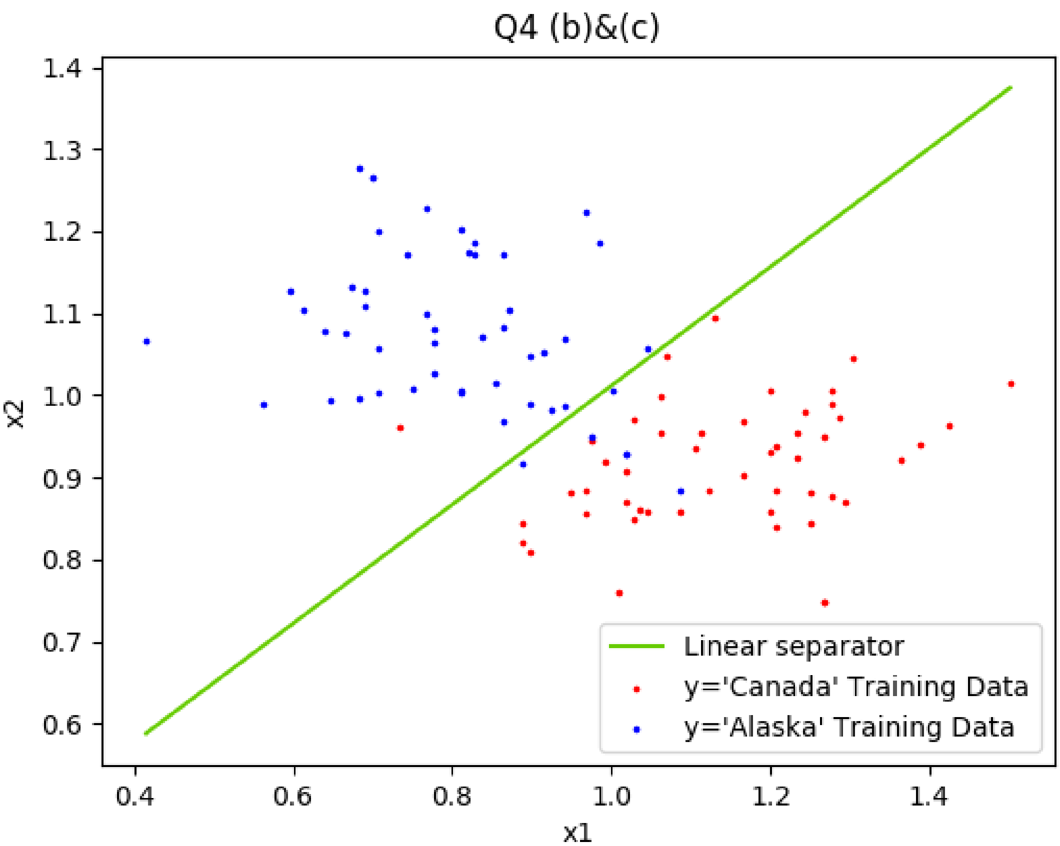
∑=[[ 0.42953048 -0.02247228]
    [-0.02247228  0.53064579]]

**Part (b)&(c)**

When ∑0=∑1 the equation of separator is:

$$X^T\Sigma^{-1}(\mu_0-\mu_1)+(\mu_0-\mu_1)^T\Sigma^{-1}X+(\mu_1^T\Sigma^{-1}\mu_1-\mu_0^T\Sigma^{-1}\mu_0)=2log_e(\frac{\phi}{1-\phi})$$

This is the equation of a straight line, plotted the above line.



**Part(d)**

μ0 and μ1 are same as before.

Calculated ∑0 and ∑1 using the formula given in assignment pdf.

∑0=
[[ 0.38158978 -0.15486516]
 [-0.15486516  0.64773717]]

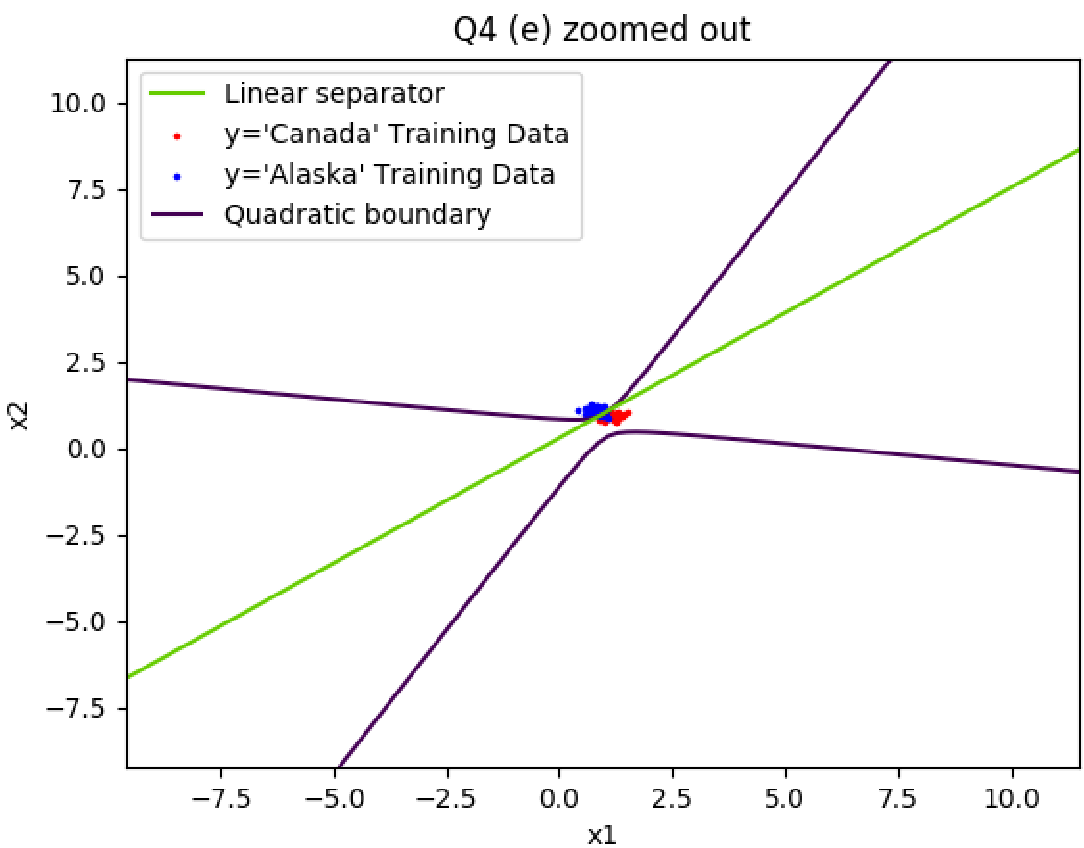
∑1=
[[ 0.47747117  0.1099206 ]
 [ 0.1099206   0.41355441]]

**Part(e)**

When ∑0≠∑1 the equation of separator is:

$$(X-\mu_1)^T\Sigma_1^{-1}(X-\mu_1)-(X-\mu_0)^T\Sigma_0^{-1}(X-\mu_0)-2log_e(\frac{\phi}{1-\phi})+log_e(\frac{|\Sigma_1|}{|\Sigma_0|})=0$$

Plotted quadratic separator as a contour of above equation at Z=0



**Part(f)**

When $\Sigma 0 = \Sigma 1$ the equation of separator is a straight line because the quadratic terms cancel.
When $\Sigma 0 \neq \Sigma 1$ the equation of separator is quadratic in x1 and x2.

The quadratic separator is actually a hyperbola, and it intersects linear separator at two points.

The linear separator classifies few ‘Alaska’ points as ‘Canada’ , but quadratic separator bends and correctly classifies those points as ‘Alaska’.