UNOFFICIAL SOLUTIONS BY TheLongCat

C3: CONDENSED MATTER PHYSICS

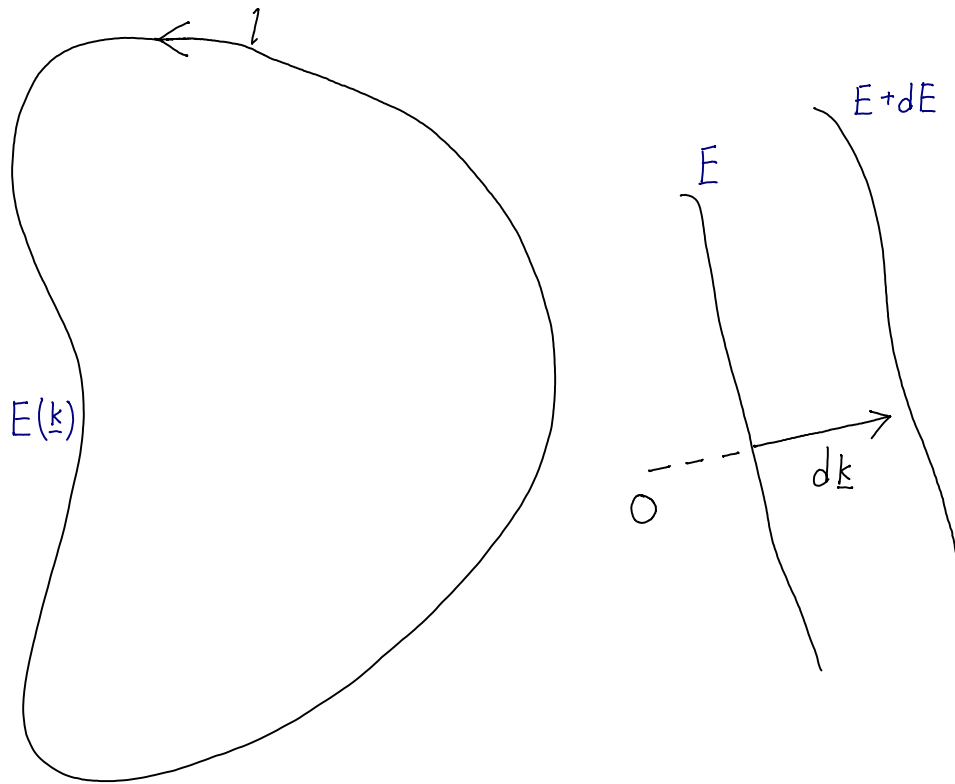**TRINITY TERM 2014**

Last updated: 30th May 2025

Turn over as you please – we are NOT under exam conditions here.

3. (DRAFT) Electronic properties under magnetic field.

(a) In 2D, the d.o.s. may be written as $g(k)\,dk = dA$ where dA is an areal loop element of equal energy.

Also we have dispersion $E(\mathbf{k}) \Rightarrow \nabla E = \nabla E \hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is the normal to the isoenergy line l.

(TO EXPAND) We know that:

$$dE = \nabla E \cdot d\mathbf{k}$$
$$= \nabla E\, dk\, (\hat{\mathbf{n}} \cdot \hat{\mathbf{n}})$$
$$\Rightarrow dk = \frac{dE}{\nabla E}$$

dA is then:

$$dA = \oint dl \cdot dk$$
$$= \oint \frac{dl}{\nabla E}\, dE$$
$$\Rightarrow g(E) = \oint \frac{dl}{\nabla E}$$

Prefactor $1/(2\pi)^n$ from continuum approx.

In 1D, we have $dE/\nabla E = g(E)\,dE$ instead.

(b)

$$\mathcal{H} = \frac{\hat{p}_x^2}{2m_1} + \frac{(\hat{p}_y + eB_z x)^2}{2m_2}$$

$$\Rightarrow \mathcal{H} = \frac{\hat{p}_x^2}{2m_1} + \frac{(\hbar k_y + eB_z x)^2}{2m_2}$$

since $[\mathcal{H}, \hat{p}_y] = 0$ allows us to write the eigenvalue in place.

Let $-\hbar k_y / eB = x_0$:

$$\mathcal{H} = \frac{\hat{p}_x^2}{2m_1} + \frac{(eB_z)^2}{2m_2}(x - x_0)^2$$

$$= \frac{1}{2m_1}\hat{p}_x^2 + \frac{1}{2}m_1\omega^2(x - x_0)^2$$

Note that the form of Hamiltonian is that of a harmonic oscillator, thus we have $\omega = eB/\sqrt{m_1 m_2}$ and energy eigenvalue $E_l = (l + 1/2)\hbar\omega$ with $l \in \mathcal{Z}$.

Gauge changing will introduce additional terms, and thus constant to the energy eigenvalue, however it has no physical meaning as only energy difference is observable.
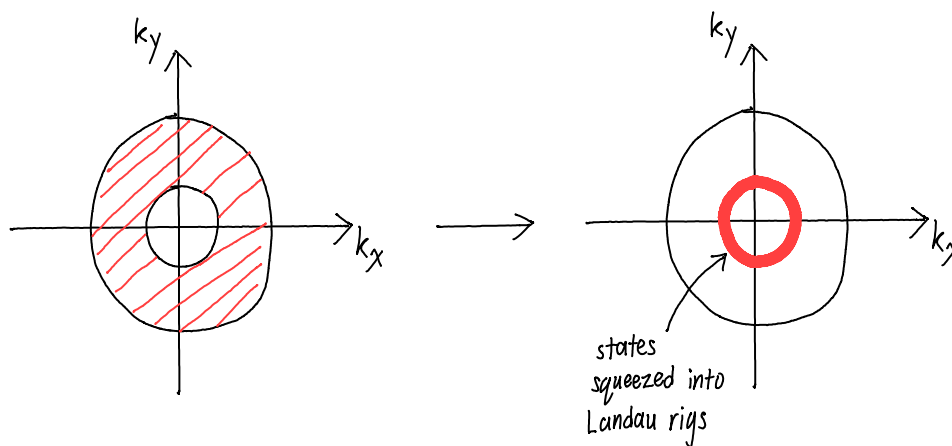
(c) Density of states in a real system: (broadening due to scattering and imperfections in 2D geometry):
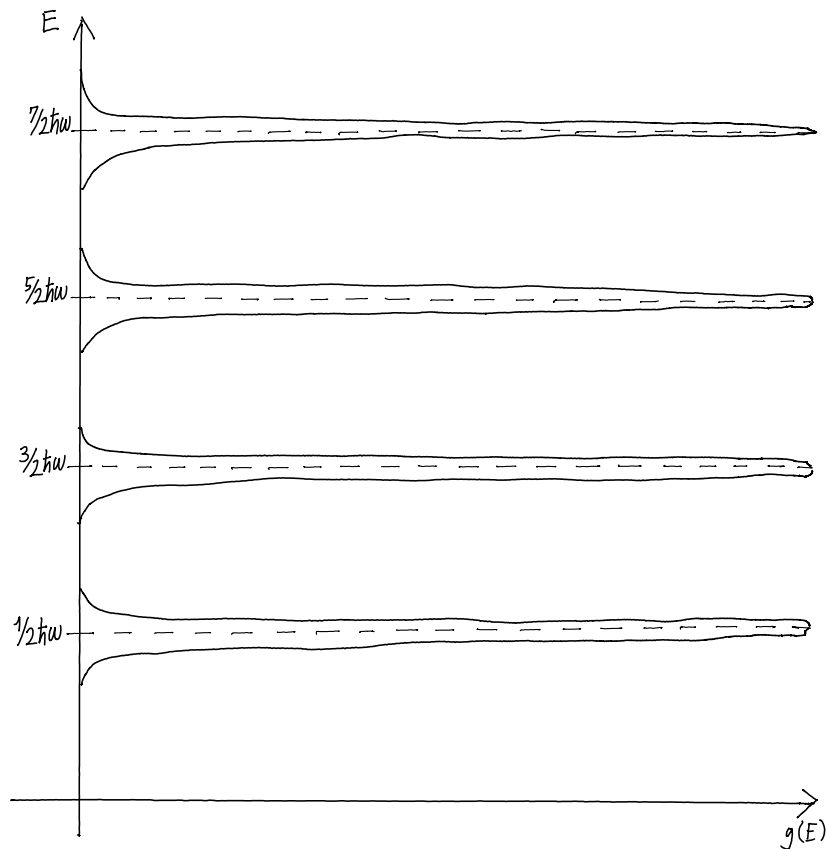
Resistivities:
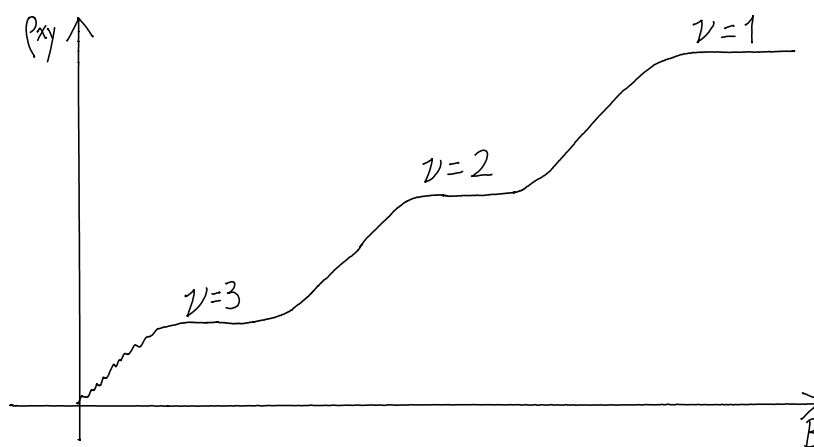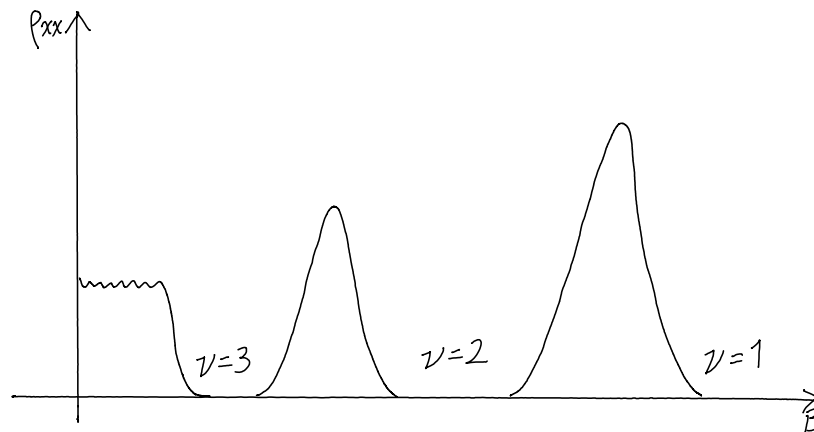
Hall resistance is quantised as it is dependent on the density of charge carrier, which is quantised due to the introduction of Landau levels:

(WRONG) The reason why this quantisation is precise is due to the density of states being partitioned precisely into the Landau levels as opposed to invoking correspondence principle which fails for small l.

(d) From Drude,

$$\frac{\mathrm{d}p}{\mathrm{d}t} = f - \underbrace{\frac{p}{\tau}}_{\substack{\to 0 \text{ as } \tau \to \infty}}$$

$$= qE$$

$$\Rightarrow m\frac{\mathrm{d}v}{\mathrm{d}t} = qE$$

$$\xrightarrow{J=nqv} \frac{m}{nq}\frac{\mathrm{d}J}{\mathrm{d}t} = qE$$

$$J = \frac{nq^2}{m}Et$$

for static voltage $V = El$ where l is sample length.

Conductance: $J = \sigma E \Rightarrow$ since n is quantised we have quantised σ too.

Unit of quantisation: ?

We expect the quantisation here to be less precise due to the lack of correspondence principle???