UNOFFICIAL SOLUTIONS BY TheLongCat

C2: LASER SCIENCE AND QUANTUM INFORMATION PROCESSING

TRINITY TERM 2011

Last updated: 30th May 2025

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Turn over as you please – we are NOT under exam conditions here.

4. **(DRAFT)** Electro-optic effect and its applications.

(a) Pockels effect is a 1st order effect. By Neumann's Principle, upon inversion the polarisation $\mathbf{P}^{(1)} \to -\mathbf{P}^{(1)}$ must be invariant in a centrosymmetric material, which implies that $\mathbf{P}^{(1)} = -\mathbf{P}^{(1)} = 0$. So a crystal with Pockel effects must be non-centrosymmetric.

(b) For $\mathbf{E} = E\hat{\mathbf{z}}$, we have:

$$\begin{pmatrix} \Delta(1/n^2)_1 \\ \dots \\ \Delta(1/n^2)_6 \end{pmatrix} = \begin{pmatrix} 0 & & \\ 0 & & \\ 0 & & \\ r_{41} & & \\ & r_{52} & \\ & & r_{63} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix}$$

$$\Rightarrow \Delta(1/n^2)_6 = r_{63}E \qquad \Delta(1/n^2)_{1,2\dots5} = 0$$

And since ADP has a tetragonal symmetry, we only have 1 unique optic axis for $E = 0$. The indicatrix should then take the following form:

$$\frac{x^2+y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

$$\xRightarrow{\text{Pockel}} \frac{x^2+y^2}{n_o^2} + 2xyr_{63}E + \frac{z^2}{n_e^2} = 1 \tag{1}$$

Writing (1) with $z = 0$ in quadratic form and diagonalising it:

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1/n_o^2 & r_{63}E \\ r_{63}E & 1/n_o^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1$$

$$\Rightarrow \begin{vmatrix} 1/n_o^2 - \lambda & r_{63}E \\ r_{63}E & 1/n_o^2 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda + \frac{1}{n_o^2} = \pm r_{63}E$$

$$\lambda = \frac{1}{n_o^2} \pm r_{63}E$$

So we have eigenvectors $(x+y)/\sqrt{2}$ and $(x-y)/\sqrt{2}$.

Inverse transforming (1) with $x = (x'+y')/\sqrt{2}$ and $y = (x'-y')/\sqrt{2}$ then gives:

$$\frac{1}{2}\frac{(x'+y')^2 + (x'-y')^2}{n_o^2} + 2\frac{(x')^2 - (y')^2}{2}r_{63}E + \frac{z^2}{n_e^2} = 1$$

$$\frac{(x')^2 + (y')^2}{n_o^2} + \frac{(x')^2 - (y')^2}{1}r_{63}E = 1$$

$$\Rightarrow \frac{(x')^2}{n_{x'}^2} + \frac{(y')^2}{n_{y'}^2} + \frac{z^2}{n_e^2} = 1$$

where the new effective refractive indices take the forms:

$$(n_{x'})^{-2} = \frac{1}{n_o^2} + r_{63}E \Rightarrow n_{x'}^2 = \frac{n_o^2}{1 + n_o^2 r_{63}E}$$

$$(n_{y'})^{-2} = \frac{1}{n_o^2} - r_{63}E \Rightarrow n_{y'}^2 = \frac{n_o^2}{1 - n_o^2 r_{63}E}$$

For such propagation, we have $x' = E'/\sqrt{2}$, $y' = E'/\sqrt{2}$.

For 90° polarisation rotation we need a phase lag of π

$$|k_0(n_{x'} - n_{y'}) \cdot l| = \pi \quad \text{where } l \text{ is the length of crystal along z,}$$
$$k_0 \text{ is wavenumber in vacuum}$$
$$|n_{x'} - n_{y'}| = \frac{\lambda_0}{2l}$$
$$\Rightarrow \left| \frac{n_0}{\sqrt{1 + n_0^2 r_{63} E}} - \frac{n_0}{\sqrt{1 - n_0^2 r_{63} E}} \right| = \frac{\lambda_0}{2l}$$
$$\xRightarrow{\text{Pockel is small so expand}} n_o \left| 1 - \frac{1}{2} n_o^2 r_{63} E - \left(1 + \frac{1}{2} n_o^2 r_{63} E \right) \right| = \frac{\lambda_0}{2l}$$
$$n_o^3 r_{63} E = \frac{\lambda_0}{2l}$$
$$El = V_0 = \frac{\lambda_0}{2 n_o^3 r_{63}} \quad \text{assuming uniform field}$$

(c) **(TO EXPAND)** Snell's Law

(d) **(TO EXPAND)** 2nd prism cancels out constant birefringence.