UNOFFICIAL SOLUTIONS BY TheLongCat

B4. NUCLEAR AND PARTICLE PHYSICS

TRINITY TERM 2022

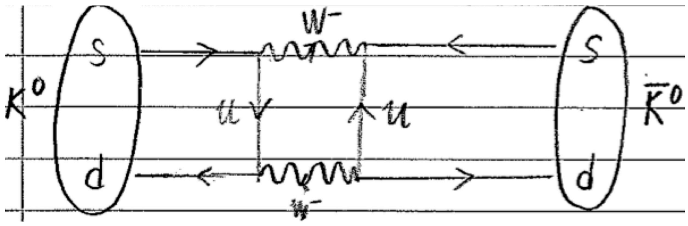
Last updated: 30th May 2025

Disclaimer: due to its unofficial nature, the author does not warrant the accuracy of the presented solutions in any form. However, the author is happy to discuss the typos and errors should one arises.

Turn over as you please – we are NOT under exam conditions here.

1. (DRAFT)

(a) $\mathrm{K}^0 \to \bar{\mathrm{K}}^0$ ($\mathrm{s\bar{d}} \to \mathrm{\bar{s}d}$):



Weak charged current allows generation crossing via Cabibbo mixing.

(b)

$$\hat{C}\hat{P}\left|+\right\rangle = \left|+\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle + \left|\bar{\mathrm{K}}^0\right\rangle\right)$$
$$\hat{C}\hat{P}\left|-\right\rangle = -\left|-\right\rangle = -\frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle - \left|\bar{\mathrm{K}}^0\right\rangle\right)$$

$$\hat{P}\left|+\right\rangle = \hat{P}\left[\frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle + \left|\bar{\mathrm{K}}^0\right\rangle\right)\right] = -\frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle + \left|\bar{\mathrm{K}}^0\right\rangle\right)$$

since kaons are fermions.

Similarly,

$$\hat{P}\left|-\right\rangle = -\frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle - \left|\bar{\mathrm{K}}^0\right\rangle\right)$$

So $\hat{C}$ should introduce a negative parity to the overall wavefunction while exchanging K^0 with $\bar{\mathrm{K}}^0$:

$$\hat{C}\hat{P}\left|+\right\rangle = \hat{C}\left[-\frac{1}{\sqrt{2}}\left(\left|\mathrm{K}^0\right\rangle + \left|\bar{\mathrm{K}}^0\right\rangle\right)\right] = \frac{1}{\sqrt{2}}\left(\left|\bar{\mathrm{K}}^0\right\rangle + \left|\mathrm{K}^0\right\rangle\right) = \left|+\right\rangle$$

$$\hat{C}\hat{P}\left|-\right\rangle = \hat{C}\left[-\frac{1}{\sqrt{2}}\left(\left|\bar{\mathrm{K}}^0\right\rangle - \left|\mathrm{K}^0\right\rangle\right)\right] = \frac{1}{\sqrt{2}}\left(\left|\bar{\mathrm{K}}^0\right\rangle - \left|\mathrm{K}^0\right\rangle\right) = -\left|-\right\rangle$$

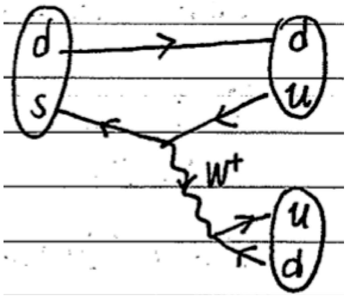
CP symmetry is obeyed under strong interaction, so the final state must have the same CP parity.

Each pion carries intrinsic parity -1, so $\left|+\right\rangle$ should decay into 2 pions, $\left|-\right\rangle$ into 3 pions (1 pion would be kinematically unfavourable due to the mass difference). Also no way to conserve momentum in CoM frame.

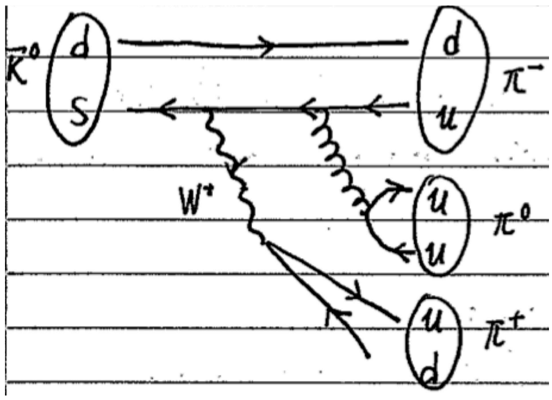
$$\hat{C}\hat{P}\,|\pi\rangle = -\,|\pi\rangle$$

So $|+\rangle \to$ even π's, $|-\rangle \to$ odd π's.

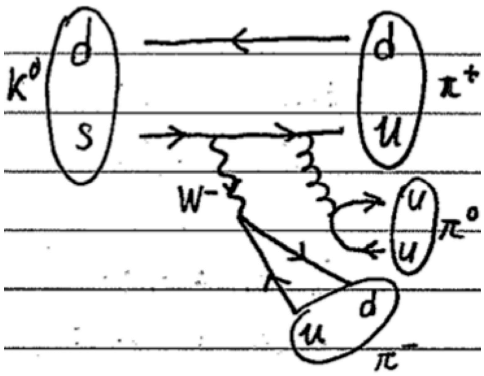
Feynman diagram of $|+\rangle \to \pi^+\pi^-$:



$|-\rangle \to \pi^+\pi^0\pi^-$:

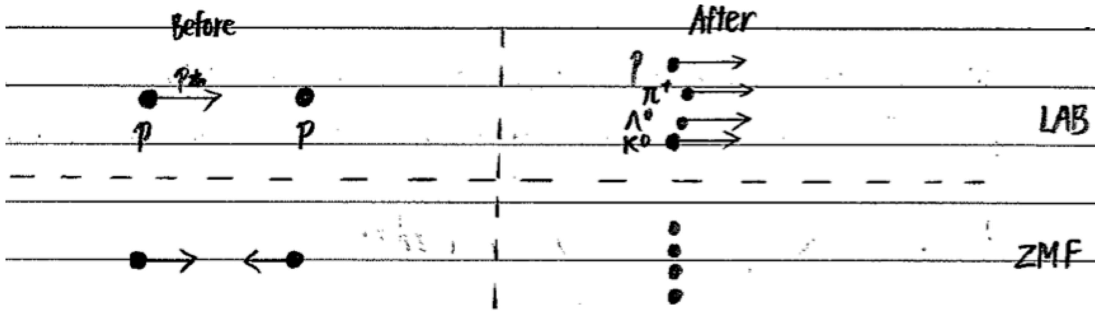


or



While both decays are OZI-suppressed, the kinematic factor from having 2 daughters is lower than having 3, so the $|+\rangle$ is short-lived compared to $|-\rangle$.

$|+\rangle$ and $|-\rangle$ aren't actually states containing both K^0 and $\bar{K}^0$, it's a single particle but a mix of two possible states. Need to show either K^0 or $\bar{K}^0$ decaying via both channels. $|-\rangle$ decay involve only 1 virtual particle, so larger $M_{if} \to$ lower τ



(c) In ZMF, the daughters possess no momentum at threshold.

Lorentz invariance:

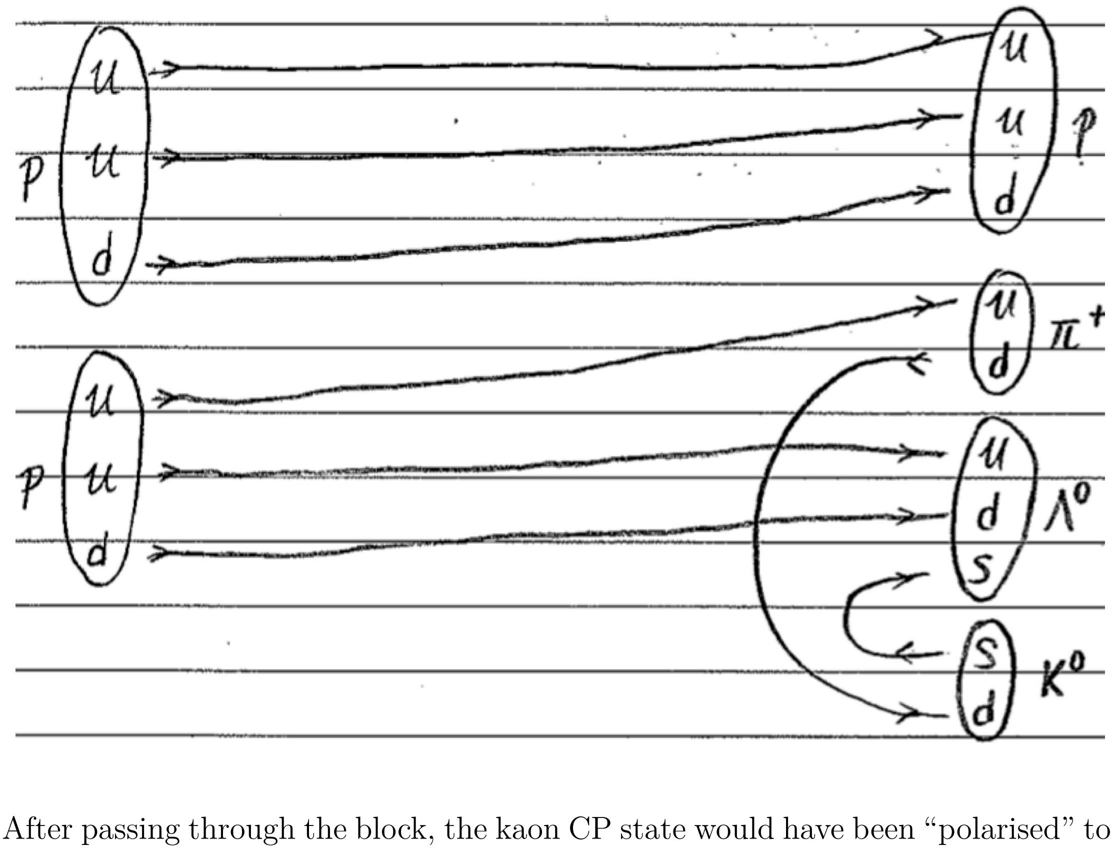
$$\mathsf{P}_\mu \mathsf{P}^\mu = -\left(\frac{E_{\text{th}}}{c} + m_{\text{p}}c\right)^2 + p_{\text{th}}^2 = -(m_{\text{p}} + m_{\pi^+} + m_{\Lambda^0} + m_{\text{K}^0})^2c^2$$

$$\Rightarrow -\cancel{\frac{E_{\text{th}}^2}{c^2}} - 2E_{\text{th}}m_{\text{p}} - m_{\text{p}}^2c^2 + \cancel{\frac{E_{\text{th}}^2}{c^2}} - m_{\text{p}}^2c^2 = -(m_{\text{p}} + m_{\pi^+} + m_{\Lambda^0} + m_{\text{K}^0})^2c^2$$

$$E_{\text{th}} = \frac{(m_{\text{p}} + m_{\pi^+} + m_{\Lambda^0} + m_{\text{K}^0})^2c^2 - 2m_{\text{p}}^2c^2}{2m_{\text{p}}}$$

$$= 2922\,\text{MeV}$$

Quark flow diagram:



After passing through the block, the kaon CP state would have been “polarised” to a specific state which depends on the material, similar to photons.

A mix of K_L^0 and K_S^0 states will be regenerated.

Thin – no interaction with the material. (except resetting composition)

The material acts as a measurement on the beam so $|\psi\rangle \to |\mathrm{K}^0\rangle$ OR $|\bar{\mathrm{K}^0}\rangle$, so K_L^0 and K_S^0 revert to mixing state.

(d) $A_S(0)$ is the amplitude of the wavefunction.

$\exp(-im_s t)$ is the plane wave (e^{iHt}) from Schrödinger's equation.

$\exp(-\dfrac{t}{2\tau_s})$ is the decay factor, $2\tau_s$ so that $I \propto e^{-\frac{t}{\tau_s}}$.

$$|\mathrm{K}^0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) \qquad |\bar{\mathrm{K}^0}\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$\begin{aligned}\langle \mathbf{r}|\,|\mathrm{K}^0\rangle &= \frac{1}{\sqrt{2}}\left(\langle \mathbf{r}|\,|+\rangle + \langle \mathbf{r}|\,|-\rangle\right)\\ &= \frac{1}{\sqrt{2}}\left[A_S(t) + A_L(t)\right]\end{aligned}$$

$$\langle \mathbf{r}|\,|\bar{\mathrm{K}^0}\rangle = \frac{1}{\sqrt{2}}\left[A_S(t) - A_L(t)\right] \quad \text{similarly}$$

(e)

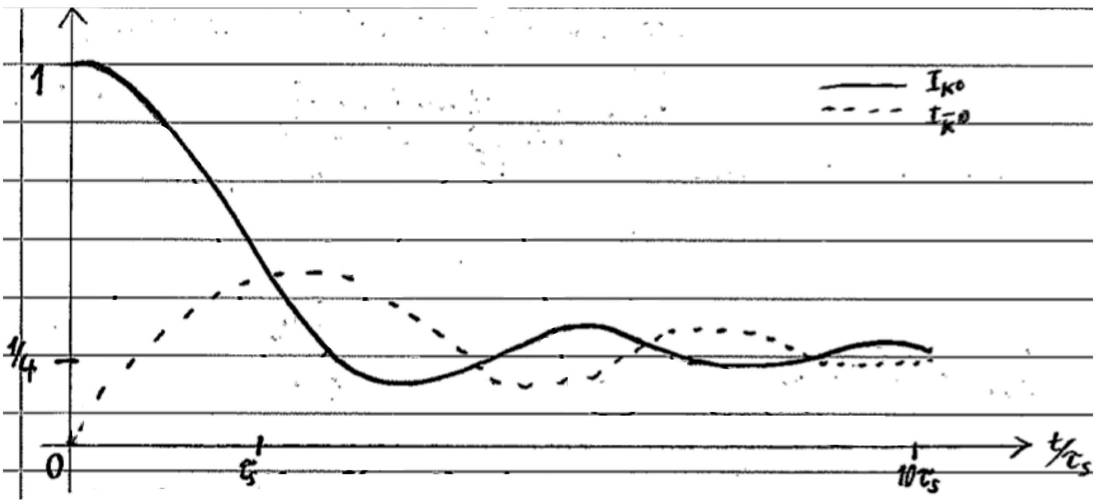
$$\begin{aligned}I &= \left|\langle \mathbf{r}|\,|\mathrm{K}^0\rangle\right|^2\\ &= \frac{1}{2}\left[|A_S(t)|^2 + A_S^*(t)A_L(t) + A_S(t)A_L^*(t) + |A_L(t)|^2\right]\\ &= \frac{1}{2}\left[\begin{aligned}&(A_S(0))^2\, e^{-\frac{t}{\tau_s}} + A_S^*(0)A_L(0)e^{i(m_S-m_L)t}e^{-\frac{t}{2\tau_s}}e^{-\frac{t}{2\tau_L}}\\ &+ A_S(0)A_L^*(0)e^{-i(m_S-m_L)t}e^{-\frac{t}{2\tau_s}}e^{-\frac{t}{2\tau_L}} + (A_L(0))^2\, e^{-\frac{t}{\tau_L}}\end{aligned}\right]\\ &= \underbrace{\frac{A_S(0)^2}{2}}_{A} e^{-\frac{t}{\tau_s}} + \underbrace{A_S(0)A_L(0)e^{-\frac{t}{2\tau_s}}e^{-\frac{t}{2\tau_L}}}_{C} \cos\underbrace{(m_S - m_L)t}_{\phi} + \underbrace{\frac{A_L(0)^2}{2}}_{B} e^{-\frac{t}{\tau_L}}\end{aligned}$$

For $\bar{\mathrm{K}^0}$, $C \to -C$.

$$\begin{aligned}I_{\bar{\mathrm{K}^0}}(t=0) &= 0\\ \Rightarrow A_S(0)^2 + A_L(0)^2 - 2A_S(0)A_L(0) &= 0\\ \Rightarrow A_S(0) = A_L(0) &= A\end{aligned}$$

So $I_{\mathrm{K}^0}(t=0) = 4A^2$.

Sketch of I_{K^0} and $I_{\bar{\mathrm{K}^0}}$ against t:



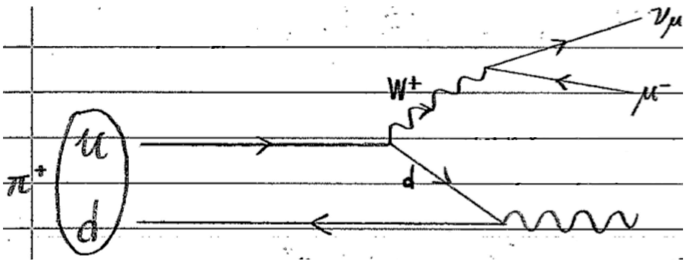
$I_{K^0} + I_{\bar{K}^0} = \frac{A^2}{2} e^{-\frac{t}{\tau_s}} + \frac{A^2}{2} e^{-\frac{t}{\tau_L}}$ so approaches τ_L as $t \to \infty$.

2. (DRAFT)

(a) Fermi's Golden Rule: reaction rate $\Gamma = \frac{2\pi}{\hbar}|M_{if}|^2\rho(E)$, where $|M_{if}|$ is the matrix element of the interaction ($\langle f|\,\hat{V}\,|i\rangle$), which may be determined with the aid of Feynman diagram. What are $\langle f|$, $\hat{V}$, $|i\rangle$?

$\rho(E)$ is the density of states of each particle in the interaction.

(b) $\pi^+ \to \mu^+\nu_\mu\gamma$:



Assuming the neutrino is massless, then the decay is similar to a Fermi 4-point interaction and thus Sargent's Rule applies: $\Gamma \propto Q^5$ where Q is the energy available after decay. Derive it.

Since ν_μ and γ are massless,

$$\begin{aligned} Q^2 &= m_\mu^2 c^4 + p_\nu c + p_\gamma c - m_{\pi^+}^2 c^4 \\ &\simeq (m_\mu^2 - m_{\pi^+}^2)c^4 \end{aligned}$$

at low energy scale so $m_\mu c \gg p_\nu c/p_\gamma c$.

What happened to Q^5? $\Gamma_{\pi\to\mu} \propto Q^5 = (m_\pi - m_\mu)^5 \propto BR$, $BR = \frac{\Gamma_{\pi\to\mu}}{\sum_i \Gamma_i}$

(c) τ decay channels:

$$\tau \to \begin{cases} \mu \\ \mathrm{e}^- \\ \pi \times 3 \text{ colours} \end{cases}$$

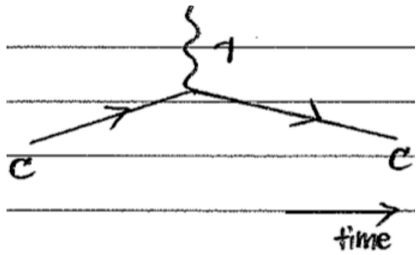
Sargent's rule gives a scaling of Q^5, so:

$$\tau_\tau \propto \frac{\tau_\mu}{5}\left(\frac{m_\mu}{m_\tau}\right)^5$$
$$\sim 3 \times 10^{-13}\,\text{s}$$

(d) As described above, Feynman diagram provides aid in calculating the matrix element of the interaction M_{if}.

In a Feynman diagram, there are vertex which couples different particles, and propagator that transmits 4-momentum between vertices.

e.g. EM interaction vertex, a charged particle $\mathcal{C}$ emits a photon.

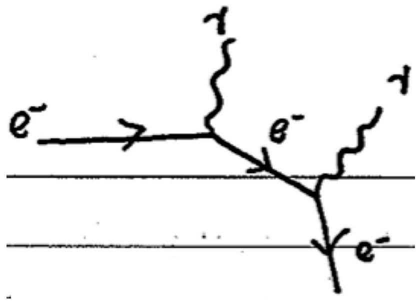


The matrix element is then:

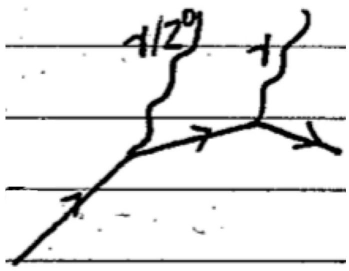
$$M_{if} = \prod_i \text{vertex factor} \times \prod_i \text{propagator}$$

where vertex factor encodes the strength of interaction, propagator $= \dfrac{1}{\mathsf{P}_\mu \mathsf{P}^\mu - m^2c^2}$ where m is the on-shell rest mass of the propagating boson, P^μ is the 4-momentum transfer of the interaction. Derive everything.

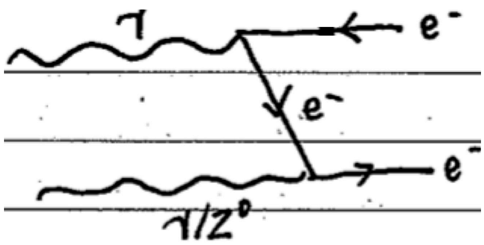
(e) Compton scattering:



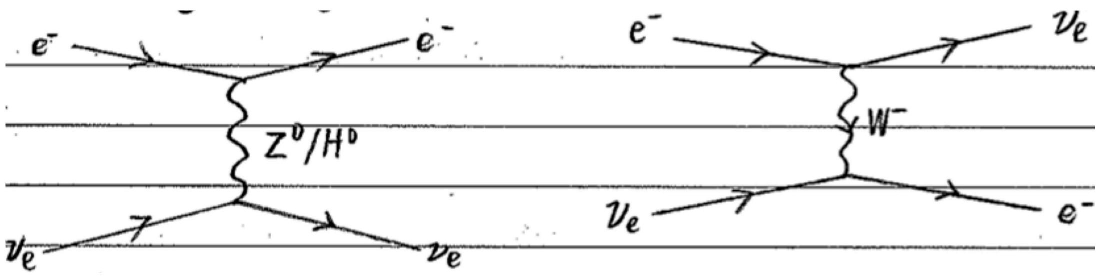
Bremsstrahlung:



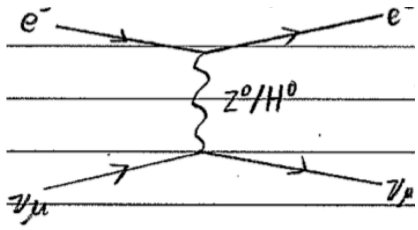
Pair production:



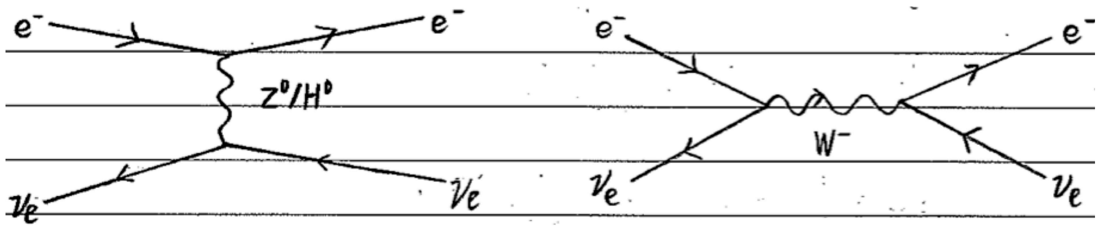
(f) i. $e^- \nu_e \to e^- \nu_e$:



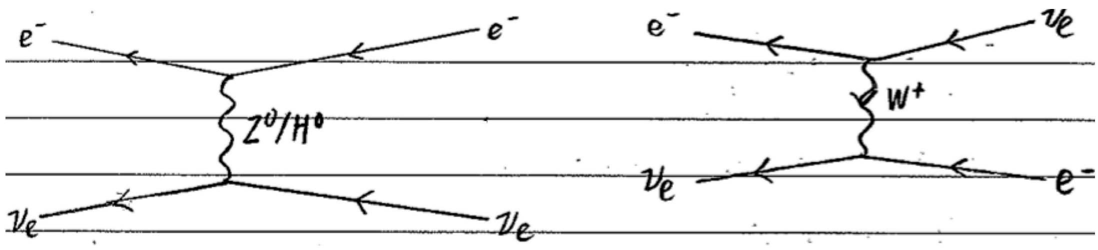
ii. $e^- \nu_\mu \to e^- \nu_\mu$:



iii. $e^- \bar{\nu}_e \to e^- \bar{\nu}_e$:

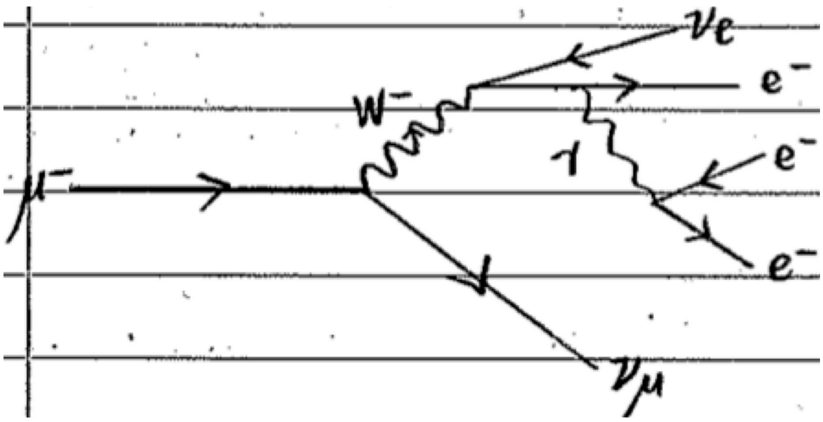


iv. $e^+ \bar{\nu}_e \to e^+ \bar{\nu}_e$:



All but $e^- \nu_\mu \to e^- \nu_\mu$ have the same cross section since W boson exchange is forbidden between different lepton flavours. $e^- \nu_\mu \to e^- \nu_\mu$ should have half of the cross section due to it having only half of the available channels.

(g) $\mu^- \to e^- e^- e^+ \nu_\mu \bar{\nu}_e$:



$$\begin{array}{rll} \mu \text{ num:} & 1 & \quad 1 \\ \text{e num:} & 0 & \to 1+1-1-1=0 \\ \text{charge:} & -1 & \quad -1-1+1=-1 \end{array}$$

Since lepton numbers and charge are all conserved, the process is allowed in the Standard Model.

3. (DRAFT)

(a) β^- decay: ${}^{\mathrm{A}}{}_{\mathrm{Z}}P \to {}^{\mathrm{A}}{}_{\mathrm{Z+1}}D^+ + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$

β^+ decay: ${}^{\mathrm{A}}{}_{\mathrm{Z}}P \to {}^{\mathrm{A}}{}_{\mathrm{Z\text{-}1}}D^- + \mathrm{e}^+ + \nu_\mathrm{e}$

EC: ${}^{\mathrm{A}}{}_{\mathrm{Z}}P + \mathrm{e}^- \to {}^{\mathrm{A}}{}_{\mathrm{Z\text{-}1}}D^- + \nu_\mathrm{e}$

Assuming neutrino to be massless, then Q value of β^- decay is:

$$Q = (m_P - m_D)c^2$$

since m_e is included in the atomic mass of D.

For β^+ decay,

$$Q = (m_P - m_D)c^2 - 2m_\mathrm{e}c^2$$

to account for the extra e^- and e^+.

For EC,

$$Q = (m_P - m_D)c^2$$

for the extra electrons cancel each other out across the LHS and RHS.

(b) α decay: ${}^{\mathrm{A}}{}_{\mathrm{Z}}P \to {}^{\mathrm{A\text{-}4}}{}_{\mathrm{Z\text{-}2}}D + {}^{4}{}_{2}\alpha$

$$Q = (m_D + m_\alpha)c^2 - m_P c^2$$

Also:

$$\begin{aligned} Q &= m_\alpha c^2 + T \quad \text{where } T \text{ is kinetic energy of } \alpha \text{ particle} \\ \Rightarrow T &= Q - m_\alpha c^2 \end{aligned}$$

In ZMF, some of Q goes to the kinetic energy of the recoiling nucleus, so T_α is “ratio-ed” by the masses:

$$T_\alpha = Q\frac{(A-4)}{A}$$

(c) Assuming that the entire α particle tunnels out of the nucleus (i.e. probability of formation independent of atomic number), potential square well in nucleus and Coulomb outside, which may be approximated as:

$$\frac{(Z-2)e62}{4\pi\epsilon_0 r} \simeq \frac{Ze^2}{4\pi\epsilon_0 r}$$

for large Z and r is the radius of the nucleus (assumed to be constant).

The tunneling probability, assuming plane wave, is given by e^{-2G} with G being the Gamow factor,

$$\begin{aligned} G &\propto \frac{V}{pc} \\ &\propto \frac{Ze^2}{\epsilon_0 \hbar \underbrace{kc}_{\omega = \frac{v}{r}} r} = \frac{Ze^2}{\epsilon_0 \hbar v} \end{aligned}$$

(d) i. $N_b(0) = \alpha + \beta = N_{b0}$

Also:

$$\begin{aligned}\frac{\mathrm{d}N_a}{\mathrm{d}t} &= -\lambda_a N_a \\ \Rightarrow N_a &= N_{a0}e^{-\lambda_a t}\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}N_b}{\mathrm{d}t} &= \lambda_a N_a - \lambda_b N_b \\ \Rightarrow -\lambda_a \alpha e^{-\lambda_a t} - \lambda_b \beta e^{-\lambda_b t} &= \lambda_a N_a - \lambda_b N_b\end{aligned}$$

Combining the expressions gives:

$$\begin{aligned}-\lambda_a \alpha e^{-\lambda_a t} - \lambda_b \beta e^{\lambda_b t} &= \lambda_a N_{a0} e^{-\lambda_a t} - \lambda_b \alpha e^{-\lambda_a t} - \lambda_b \beta e^{-\lambda_b t} \\ \Rightarrow \left(\lambda_b \alpha - \lambda_a \alpha - \lambda_a N_{a0}\right) e^{-\lambda_a t} &= 0 \\ \Rightarrow \alpha &= \frac{\lambda_a N_{a0}}{\lambda_b - \lambda_a} \\ \Rightarrow \beta &= N_{b0} - \frac{\lambda_a N_{a0}}{\lambda_b - \lambda_a}\end{aligned}$$

ii. $N_{b0} = 0 \Rightarrow \beta = -\alpha$, so:

$$\begin{aligned}N_b(t) &= \alpha\left(e^{-\lambda_a t} - e^{-\lambda_b t}\right) \\ \frac{\mathrm{d}N_b}{\mathrm{d}t} &= \alpha\left(\lambda_b e^{-\lambda_b t} - \lambda_a e^{-\lambda_a t}\right)\end{aligned}$$

At max N_b, $\frac{\mathrm{d}N_b}{\mathrm{d}t}$:

$$\begin{aligned}\Rightarrow e^{(\lambda_a - \lambda_b)t_{\max}} &= \frac{\lambda_a}{\lambda_b} \\ t_{\max} &= \frac{\ln(\lambda_a) - \ln(\lambda_b)}{\lambda_a - \lambda_b}\end{aligned}$$

iii. Assuming no other decay channels, $N_c = N_{a0} - N_a - N_b$ or:

$$\begin{aligned}\frac{\mathrm{d}N_c}{\mathrm{d}t} &= \lambda_b N_b \\ \Rightarrow \frac{\mathrm{d}N_c}{\mathrm{d}t} &= \lambda_b \alpha\left(e^{-\lambda_a t} - e^{-\lambda_b t}\right) \\ N_c(t) &= \lambda_b \alpha \left[\frac{e^{-\lambda_b t}}{\lambda_b} - \frac{e^{-\lambda_a t}}{\lambda_a}\right]_{t=0}^{t} \\ &= \lambda_b \alpha \left[\frac{e^{-\lambda_b t} - 1}{\lambda_b} - \frac{e^{-\lambda_a t} - 1}{\lambda_a}\right]\end{aligned}$$

4. (DRAFT)

(a) In the quark model, the baryon number is defined such that a baryon has $B = 1$, an antibaryon has $B = -1$, i.e. a quark has $B = +1/3$, an antiquark has $B = -1/3$. This number arises from the fact that strong interaction conserves the quark number (gluons couple quark-antiquark so $\Delta B = 0$).

Strangeness is a label that mark the number of strange quarks a baryon contains, this is added to resolve the mystery of some long lived barons. $S- = 1$[1] upon adding a strange quark.

And I_z is added in a similar fashion as angular momentum J to mark the fact that strong interaction is invariant of quark flavour (originally marking neutron and proton as 2 states with $I = 1/2$, $I_z = \pm 1/2$).

Draw 3-quark diagram of baryon octet.

(b) Recall that baron and quarks are fermions, thereby obeying Pauli's Exclusion Principle.

For $J = 1/2$ barons, the quarks should occupy the lowest energy shell, akin to the Rydberg model of hydrogen. However there only exists 2 possible "slots" for the same quark to lay in a shell since $I_z = \pm 1/2$, the third quark must be of different flavour to satisfy Pauli's Exclusion Principle.

An OK argument, but key point's antisymmetry of composite wavefunction under particle exchange.

ddu

$\uparrow\uparrow\downarrow$ not possible

$\left.\begin{matrix}\uparrow\downarrow\uparrow \\ \downarrow\uparrow\uparrow\end{matrix}\right\}$ singlet so one particle state only

ddd

$\left.\begin{matrix}\uparrow\uparrow\downarrow \\ \uparrow\downarrow\uparrow \\ \downarrow\uparrow\uparrow\end{matrix}\right\}$ all impossible!

Antisymmetry w.r.t. exchange.

(c) Again, since baryons are fermions, the parity of its wavefunction must be antisymmetric under particle exchange.

The centre of the Eightfold Way contains u, d and s. There exists 2 possible ways of arranging the flavour wavefunction for it to be antisymmetric:

uds + dsu	−sud − dus	−usd − sdu	about	$(12) \leftrightarrow 3$
uds + dsu	−sud − dus	−usd − sdu	about	$1 \leftrightarrow (23)$

And those wavefunctions correspond to the particles Σ^0 and Λ^0. Check `https://physics.stackexchange.com/a/321439`

Spin wavefunction:

	dus
Σ^0	$\uparrow\uparrow\downarrow$
	$\uparrow\downarrow\uparrow$
Λ^0	$\downarrow\uparrow\uparrow$

where Λ^0 has the isospin degeneracy (approximately symmetric) so identical $\Psi \Rightarrow$ some mass difference but very small so indistinguishable.

[1] Or $S--$ if C++ is your cup of tea.

(d) The mass and charge of the particles should indicate that the combination of 3 quarks / antiquarks would explain these properties, e.g. to make up a +1 charge, there can be u$\bar{d}$ or uud, but the large mass of proton would suggest that uud would be its composition instead. Also the fact that $J = \frac{1}{2}$ would indicate that the number of quarks must be odd.

(e) For Σ^0/Λ^0:

$$S + B = 0, \qquad I_z = 0 \tag{1}$$

For Ξ^0:

$$S + B = -1, \qquad I_z = +\frac{1}{2} \tag{2}$$

So try:

$$\begin{aligned}(S + B) &= mI_z + c \\ \Rightarrow c &= 0 \quad \text{from (1)}\end{aligned}$$

From (2),

$$\begin{aligned}-1 &= \frac{1}{2}m \\ \Rightarrow m &= -2\end{aligned}$$

Gell-Mann Nishijima relation for $Q = 0$: $(S + B) = -2I_z$

Similarly for $Q = +1$, $Y(p)$:

$$S + B = +1, \qquad I_z = +\frac{1}{2}$$

Σ^+:

$$S + B = 0, \qquad I_z = 1$$

So $(S + B) = -2I_z + 2$.

Hence the full relation is:

$$\begin{aligned}(S + B) &= -2I_z + 2Q \\ \Rightarrow Q &= \frac{S + B}{2} + I_z\end{aligned}$$

For u, $I_z = +\frac{1}{2}$, $B = \frac{1}{3}$, $S = 0$:

$$Q = \frac{1}{6} + \frac{1}{2} = +\frac{2}{3}$$

For d, $I_z = -\frac{1}{2}$, $B = \frac{1}{3}$, $S = 0$:

$$Q = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$$

For s, $I_z = 0$, $B = \frac{1}{3}$, $S = 1$:

$$Q = -\frac{2}{6} = -\frac{1}{3}$$

Can I get this directly directly from assuming quark properties?

Better use ansatz $Q = \alpha(I_z) + \beta(S + B)$

$Q = +1$: p=uud

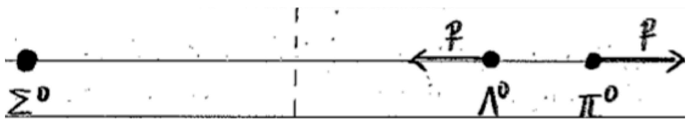
$Q = 0$: n=dud

So $u - d = 1 \Rightarrow$ repeat and solve

(f) Strong interaction conserves quark number $\Rightarrow$ new particle must be meson. How do I know if it's strong?

Also conservation of charge gives $Q = 0$ for the new particle.

Thus the particle must be π^0.



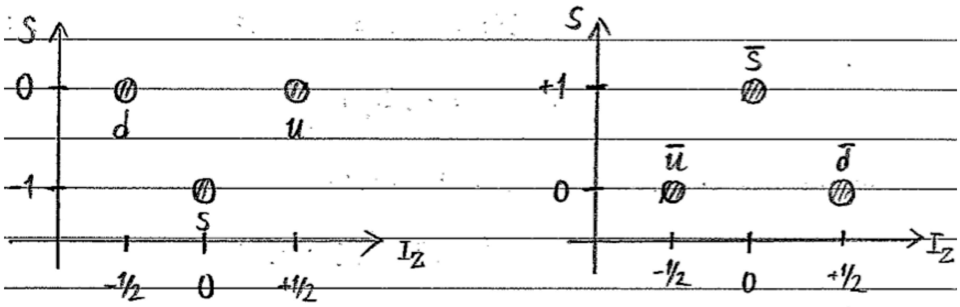
Conservation of energy gives:

$$\begin{aligned} m_{\Sigma^0}c^2 &= m_{\Lambda^0}c^2 + E_{\pi^0} \quad \text{since } m_{\Lambda^0} \gg m_{\pi^0} \\ E_{\pi^0} &= (m_{\Sigma^0} - m_{\Lambda^0})\, c^2 \\ &= 77\,\text{MeV} \end{aligned}$$

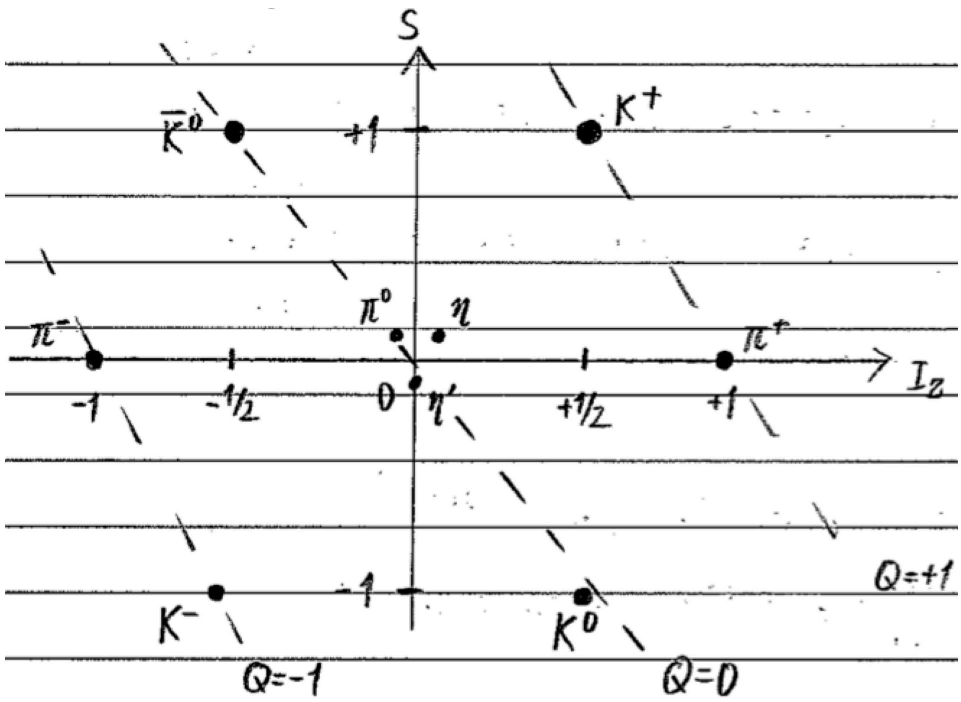
Not the energy, but a good estimate of rest mass.

But $m_{\pi^0}c^2 > E_{\pi^0}$! So the resulting particle has to be photon instead (preserves quark content too). $\Sigma^0 \to \mathrm{n}\pi^0$ and $\Sigma^0 \to \mathrm{p}\pi^-$ not observed since quark content is not conserved.

(g) Propose a basis representation as follows:



Then combining the quarks to form a $J = 0$ meson nonet:



Again, mesons are also fermions therefore there are three ways of arranging antisymmetric flavour wavefunctions:

$$\mathrm{u\bar{u} - \bar{u}u}, \qquad \mathrm{d\bar{d} - \bar{d}d}, \qquad \mathrm{s\bar{s} - \bar{s}s}$$

Mass difference: $\mathrm{u\bar{u}}$, $\mathrm{d\bar{d}}$ have approximate symmetry so they occupy the same E level. But $\mathrm{s\bar{s}}$ has significantly larger mass and difference from u and d.