**UNOFFICIAL SOLUTIONS BY TheLongCat**

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# C2: LASER SCIENCE AND QUANTUM INFORMATION PROCESSING

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**TRINITY TERM 2009**

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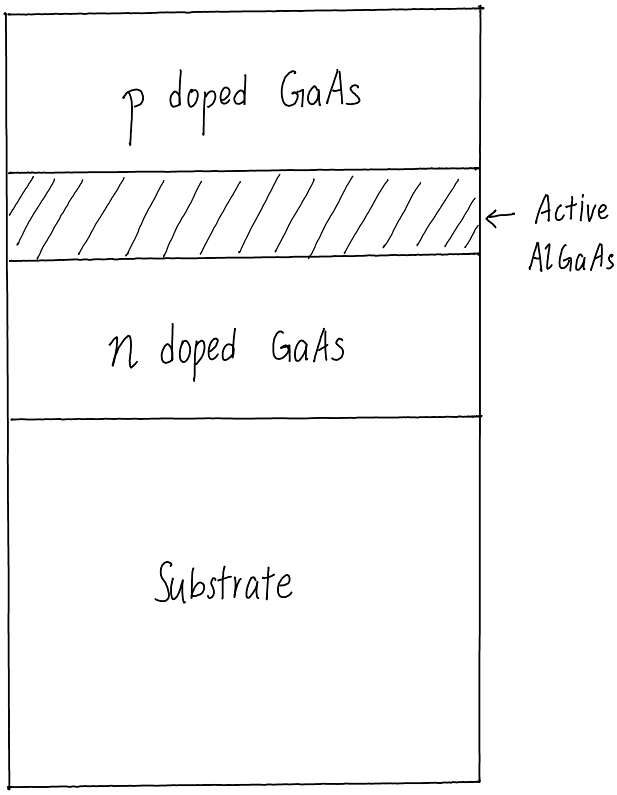
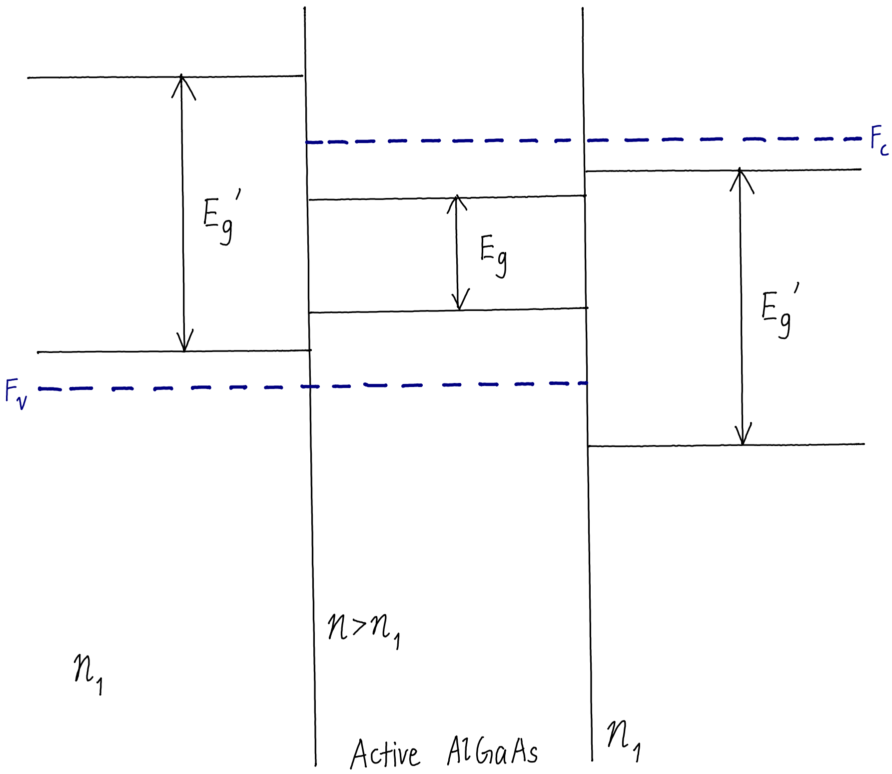
*Disclaimer: due to its unofficial nature, the author does not warrant the accuracy of the presented solutions in any form. However, the author is happy to discuss the typos and errors should one arises.*

**Turn over as you please – we are NOT under exam conditions here.**

**1.** Semiconductor laser – knowledge from Condensed Matter Physics can be very useful here.




(a) A double heterostructure is a sandwich of p and n doped substance with much higher bandgap than the active region. This is useful in low threshold laser due to the good confinement of charge carriers, photon, and low leakage.



Note the separation of bandgap $\rightarrow$ charge carrier confined and little absorption for escaped beam. Also the refractive indices are such that the active region forms a waveguide.

(b) Quasi-Fermi energies are chemical potentials of the conduction/valence band in a local equilibrium. This is useful for lasing as pumping lasing means that the band population will be altered to a new and dynamic equilibrium than the static case.

(c) The gain coefficient is dependent on the joint density of states of transitions, together with the probability of net emission:

$$\begin{aligned} E_{\text{transition}} &= E_1 - E_2 \\ &= E_g + \frac{\hbar^2 k^2}{2\mu} \\ &= \hbar\omega \\ \frac{\hbar^2 k^2}{2\mu} &= \hbar\omega - E_g \end{aligned}$$

where $\mu^{-1} = m_e^{-1} + m_h^{-1}$ is the reduced mass of an electron-hole pair.

Assuming isotropy, we have the following density of states:

$$\begin{aligned} g(k)\,\mathrm{d}k &\propto k^2\,\mathrm{d}k \\ \Rightarrow g(E)\,\mathrm{d}E &\propto (\hbar\omega - E_g)^{1/2}\,\mathrm{d}E \quad \text{since } k\,\mathrm{d}k \propto \mathrm{d}E \end{aligned}$$

Probability of emission: $f_c(E_1)\,[1 - f_v(E_2)]$

Probability of absorption: $f_v(E_2)\,[1 - f_c(E_1)]$

Hence probability of net emission:

$$P(\text{emission}) - P(\text{absorption}) = f_c(E_1) - f_v(E_2)$$

As $\omega$ increases, gain increases as the d.o.s. of allowed transitions grows. Note that for temperature, the dependence has an optimal window as both $f_c$ and $f_v$ have the same limit as $T \rightarrow 0$ or $T \rightarrow \infty$, this is due to the fact that transitions in a semiconductor is a dynamic equilibrium.

(d) We want gain $> 0$:

$$\begin{aligned} \Rightarrow f_c &> f_v \\ \mathrm{e}^{(E_2 - F_v)/k_\mathrm{B}T} + 1 &> \mathrm{e}^{(E_1 - F_c)/k_\mathrm{B}T} + 1 \\ \Rightarrow E_2 - F_v &> E_1 - F_c \\ E_1 - E_2 &< F_c - F_v \end{aligned}$$

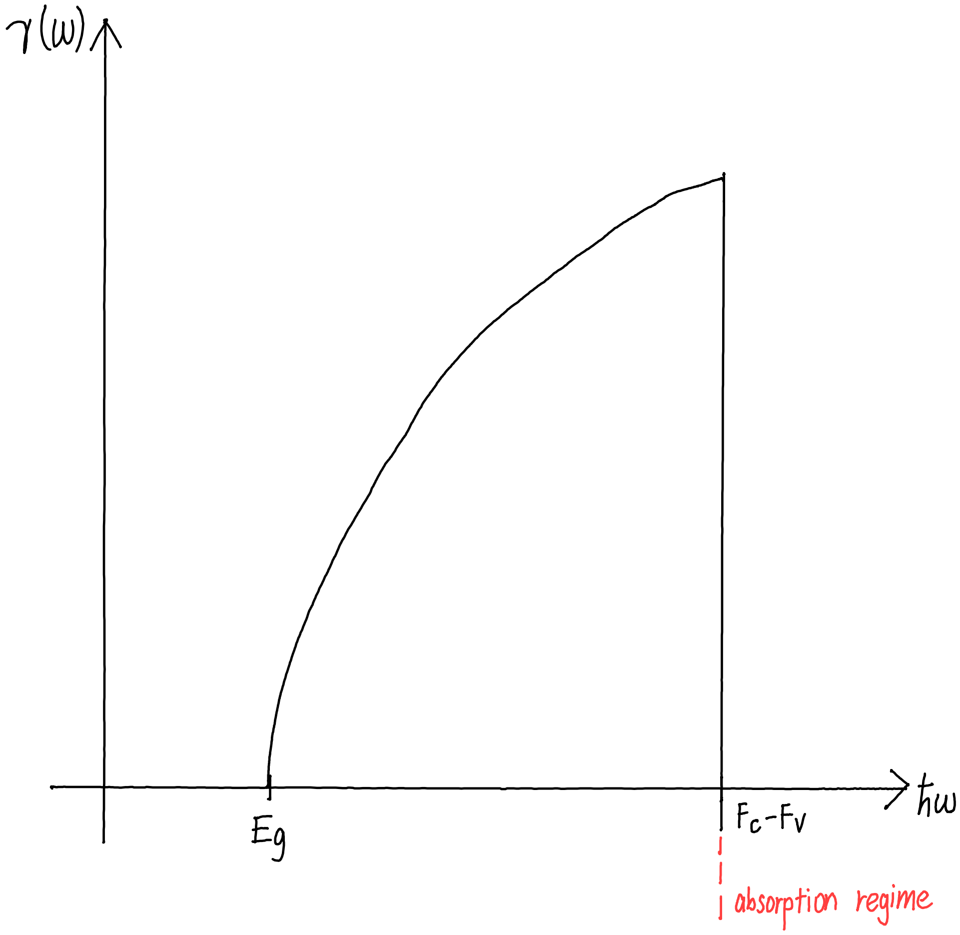
From above we also have $E_1 - E_2 = E_g + \hbar^2 k^2/2\mu > E_g$ hence the inequality.

At $T = 0\,\mathrm{K}$, we have:

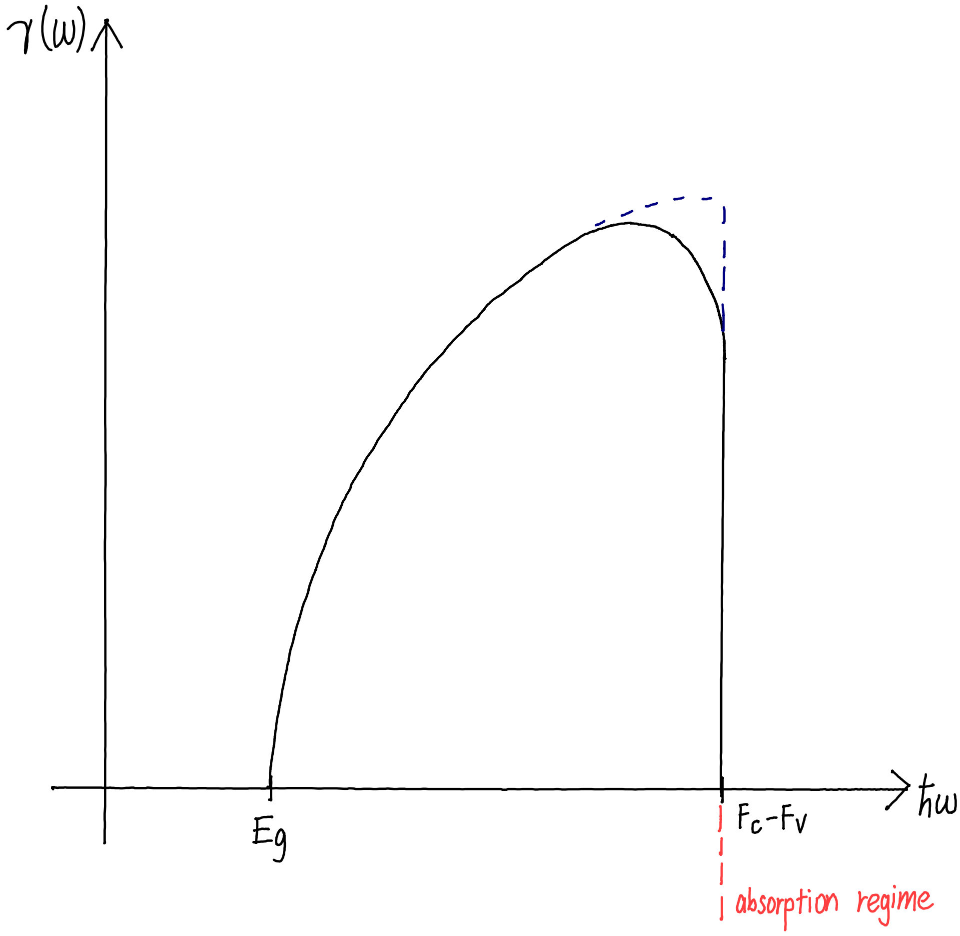
$$f_c \rightarrow \begin{cases} 1 & E_1 < F_c \\ 0 & E_1 > F_c \end{cases}$$

$$f_v \rightarrow \begin{cases} 1 & E_2 < F_v \\ 0 & E_2 > F_v \end{cases}$$

But since $E_1 - E_2 < F_c - F_v$, we won't have $f_c = 1$ and $f_v = 0$ for all $\omega$, which leads to zero gain.




At $T > 0\,\mathrm{K}$, provided $E_1 - E_2 < F_c - F_v$, we have $f_c - f_v > 0$ and the limiting factor is the d.o.s. – the gain begins at $\hbar\omega = E_g$ and grows before dropping to 0 as $f_c - f_v \to 0$.




(e) From the given electron density, we find:

$$F_c = \frac{\hbar^2}{2m_e}\left(N \cdot 3\pi^2\right)^{2/3} + E_g = 1.78\,\mathrm{eV}$$

Similarly for holes:

$$N_{\text{hole}} = \frac{1}{3\pi^2}\left(\frac{2m_h F_v}{\hbar^2}\right)^{3/2}$$
$$\Rightarrow F_v = \frac{\hbar^2}{2m_h}\left(N_{\text{hole}} \cdot 3\pi^2\right)^{2/3} = 0.03\,\mathrm{eV}$$

So $F_c - F_v = 1.75\,\mathrm{eV} > E_g = 1.54\,\mathrm{eV}$ is verified.

Peak gain should occur when $E_1 - E_2$ is maximised, i.e. $F_c - F_v = \hbar kc = hc/\lambda$:

$$\Rightarrow \lambda = \frac{hc}{F_c - F_v} = 781\,\mathrm{nm}$$