UNOFFICIAL SOLUTIONS BY TheLongCat

C3: CONDENSED MATTER PHYSICS

TRINITY TERM 2016

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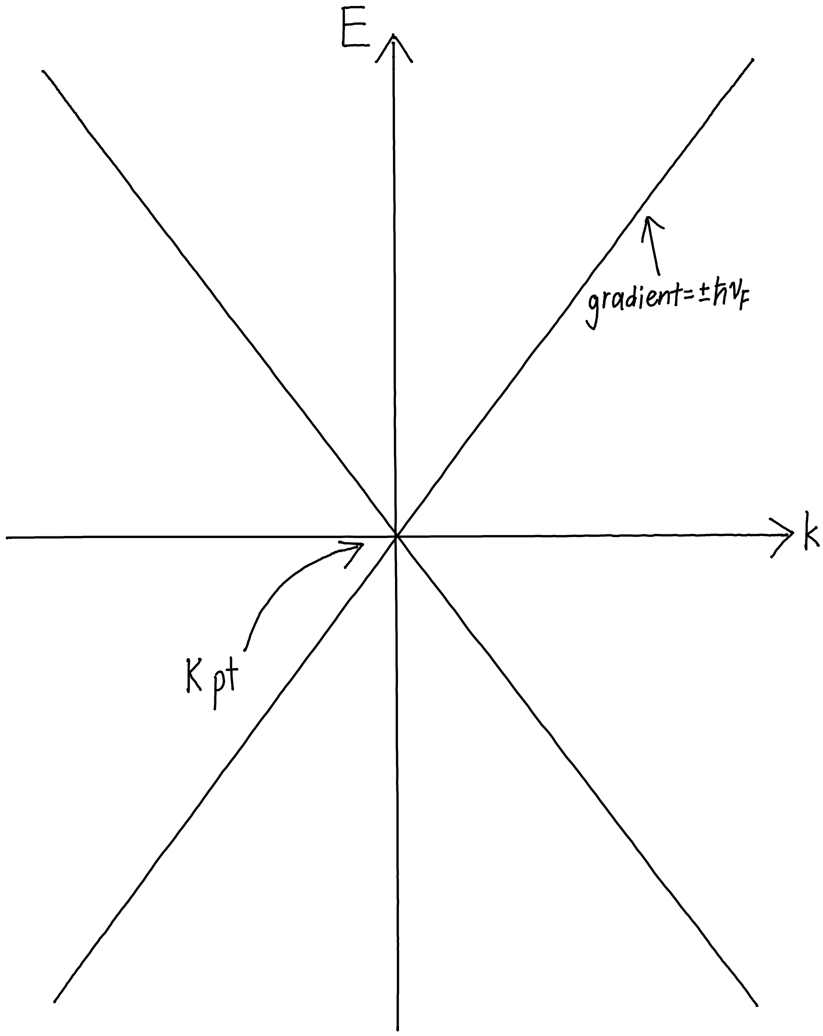
Disclaimer: due to its unofficial nature, the author does not warrant the accuracy of the presented solutions in any form. However, the author is happy to discuss the typos and errors should one arises.

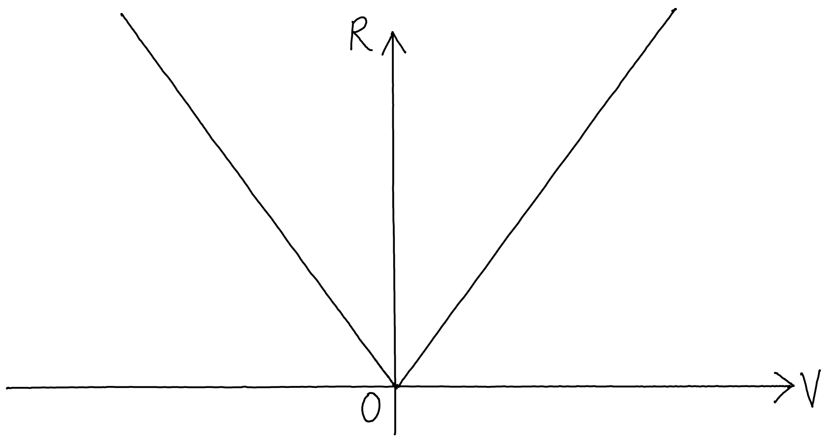
Turn over as you please – we are NOT under exam conditions here.

6. **(DRAFT)** Dirac cone of graphene.

(a) Close to the K point we have the Dirac cone as follows:



Resistance $R = V/I \propto E/J$ and $J = nqv \propto n$ so $R \propto 1/n$ the inverse of carrier density. Also note that around the K point, contour of constant energy is approximately a circle so the circumference determines the d.o.s. $\Rightarrow$ carrier density. Assuming $\mu = 0$ when the gate voltage is 0, we have the following sketch:

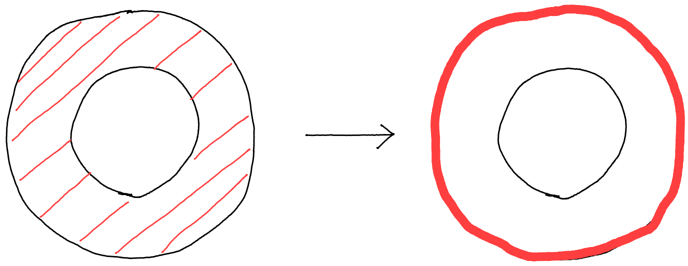


Landau levels have form:

$$\begin{aligned} E_l &= \left(l + \frac{1}{2}\right)\hbar\omega_c \quad \text{where } \omega_c = eB/m_{CR} \text{ is the cyclotron frequency} \\ &= \frac{\hbar^2 k^2}{2m_{CR}} \\ \Rightarrow k_l^2 &= \frac{2m_{CR}}{\hbar^2}\left(l + \frac{1}{2}\right)\hbar\frac{eB}{m_{CR}} \\ &\propto \frac{2eB}{\hbar} \end{aligned}$$

Hence k_j has the form $\sqrt{|2eB/\hbar|\,j}$. Plugging it into $E(k)$ then gives $E_j = v_{\mathrm{F}}\sqrt{|2eB\hbar j|}$ as required.

# of states in a Landau level = Δ(# states between adjacent orbits)



$$\begin{aligned} N_j &= \underbrace{2}_{\text{spin}} \cdot \underbrace{2}_{\text{basis}} \cdot \frac{A}{(2\pi)^2} \cdot \pi \underbrace{\left(k_{j+1}^2 - k_j^2\right)}_{2eB/\hbar} \\ \Rightarrow n_j &= \frac{2eB}{\pi\hbar} \quad \text{in a Landau level} \end{aligned}$$

The areal density is then:

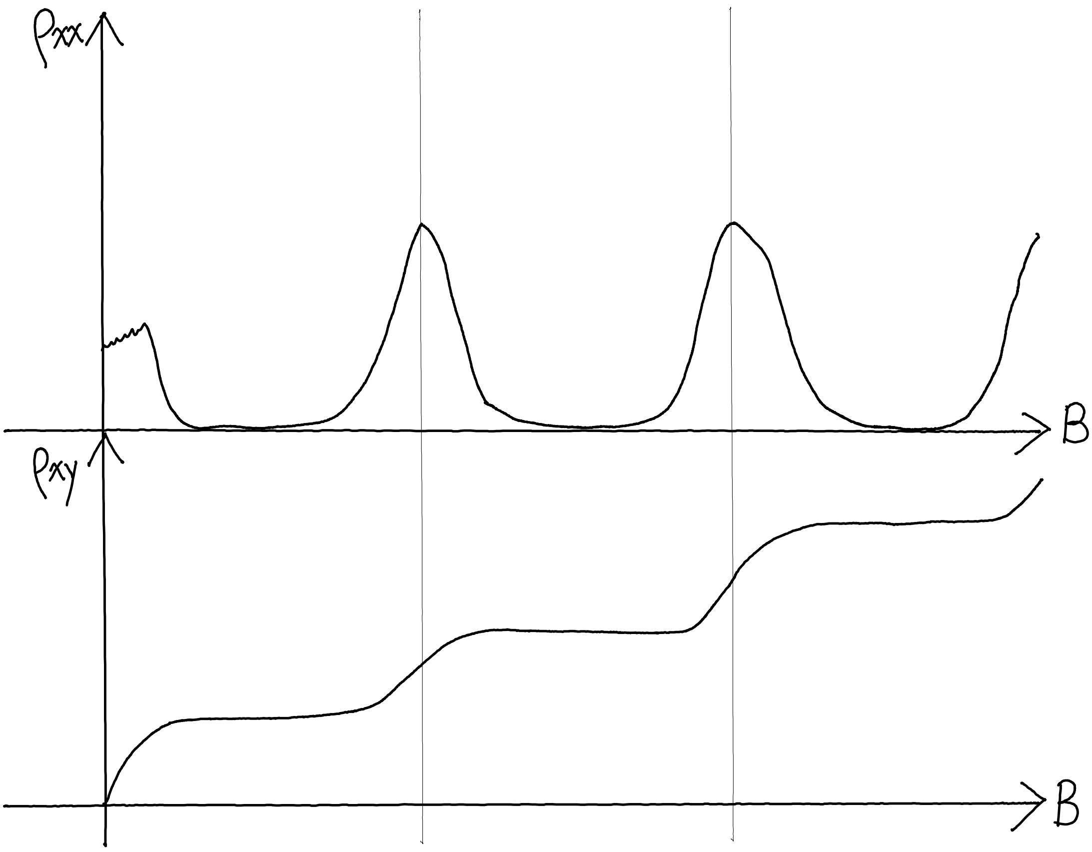
$$\begin{aligned} n &= n_j \cdot j \\ &= \frac{2eB}{h/2} j \\ &= \frac{4eB}{h} j \end{aligned}$$

Oscillatory period in $\Delta(1/B)$ for $n = 1 \times 10^{16}\,\mathrm{m}^{-2}$:

$$\begin{aligned} \Delta\left(\frac{1}{B}\right) &= \frac{4e}{hn} \\ &= 0.097\,\mathrm{T}^{-1} \end{aligned}$$

When a Landau level is filled, there are no available holes for the electron to flow, hence $\rho_{xx} = 0$. At this stage, we are in the localised states where puddles of isolated conduction electrons are separated from one another, thereby making ρ_{xy} constant.

Sketch of ρ_{xx} and ρ_{xy} on the same axis:



From Drude,

$$
\begin{aligned}
\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} &= \mathbf{f} - \frac{\mathbf{p}}{\tau} \\
\mathbf{p} &= \mathbf{f}\tau \quad \text{at stationary state} \\
m^*\mathbf{v} &= \left(q\mathbf{v} \times \mathbf{B} + qE\right)\tau \\
\frac{m^*}{nq}\hat{\mathbf{j}} &= q\tau \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} + q\mathbf{E} \\
&= q\tau\left[v_y B_z \hat{\mathbf{x}} - v_x B_z \hat{\mathbf{y}}\right] + q\mathbf{E} \\
&\Rightarrow \begin{cases} \frac{m^*}{nq} j_x = q\tau \cdot \frac{1}{nq} j_y B + qE_x \\ \frac{m^*}{nq} j_y = -q\tau \cdot \frac{1}{nq} j_x B + qE_y \Rightarrow \frac{\tau}{nq} j_x B = E_y \quad \text{since } j_y = 0 \end{cases}
\end{aligned}
$$

Group velocity:

$$
\begin{aligned}
v_g &= \frac{\partial \omega}{\partial k} \\
&= v = \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k}
\end{aligned}
$$

So we have $p = m^* v = \hbar k$:

$$\begin{aligned}
\hbar k &= \frac{m^*}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k} \\
\Rightarrow m^* &= \hbar^2 k \cdot \frac{\mathrm{d}k}{\mathrm{d}E} \\
&= \frac{\hbar k}{v_\mathrm{F}}
\end{aligned}$$

For quantum Hall effect to be well observed, we need:

$$\begin{aligned}
\omega_c \tau = \frac{eB}{m^*}\tau = \frac{eBv_\mathrm{F}}{\hbar k}\tau &\gg 1 \\
\Rightarrow eBv_\mathrm{F}\tau &\gg \hbar k \\
\tau &\gg \frac{\hbar(1\times 10^9\,\mathrm{m}^{-1})}{eBv_\mathrm{F}} = 1.01\times 10^{-13}\,\mathrm{s}
\end{aligned}$$

Phonon broadening has the form:

$$\begin{aligned}
\hbar \underbrace{\omega_\mathrm{ph}}_{2\pi/\tau} &\sim k_\mathrm{B} T \\
\Rightarrow T_\mathrm{C} &\simeq \frac{h}{k_\mathrm{B}\tau} = 474\,\mathrm{K}
\end{aligned}$$