UNOFFICIAL SOLUTIONS BY TheLongCat

C3: CONDENSED MATTER PHYSICS

TRINITY TERM 2018

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Disclaimer: due to its unofficial nature, the author does not warrant the accuracy of the presented solutions in any form. However, the author is happy to discuss the typos and errors should one arises.

Turn over as you please – we are NOT under exam conditions here.

1. (DRAFT) Phonon dispersion.

(a) A phonon is a quantisation of lattice vibration due to the discrete translational symmetry in a crystal. Generally there are 2 branches of phonon modes, optic mode is where the neighbouring atoms are moving out of phase to one another, and acoustic mode where they oscillate in phase.

(b) i. 3D, N lattice points, p atoms.

ii. Space group (no longer in syllabus)

iii. Close to the BZ centre, due to the 4-fold rotational symmetry, the spectra should have degeneracy about its principal axes.

(c) i.

$$E_i = 30\,\mathrm{meV} \Rightarrow k_i = \frac{1}{\hbar}\sqrt{2mE_i} = 3.80 \times 10^{10}\,\mathrm{m}^{-1}$$
$$E_{\mathrm{ph}} = 20\,\mathrm{meV} \Rightarrow E_f = 10\,\mathrm{meV} \Rightarrow k_f = \frac{1}{\hbar}\sqrt{2mE_f} = 2.20 \times 10^{10}\,\mathrm{m}^{-1}$$

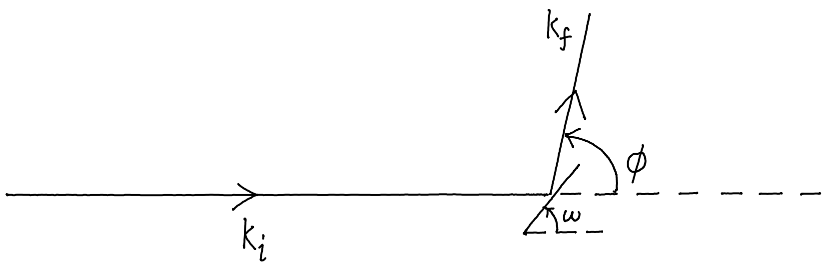
So $Q = k_i - k_f$ may range from $|k_i - k_f| = 1.61 \times 10^{10}\,\mathrm{m}^{-1} = 16.1\,\mathrm{nm}^{-1}$ to $|k_i + k_f| = 6.00 \times 10^{10}\,\mathrm{m}^{-1} = 60.0\,\mathrm{nm}^{-1}$.

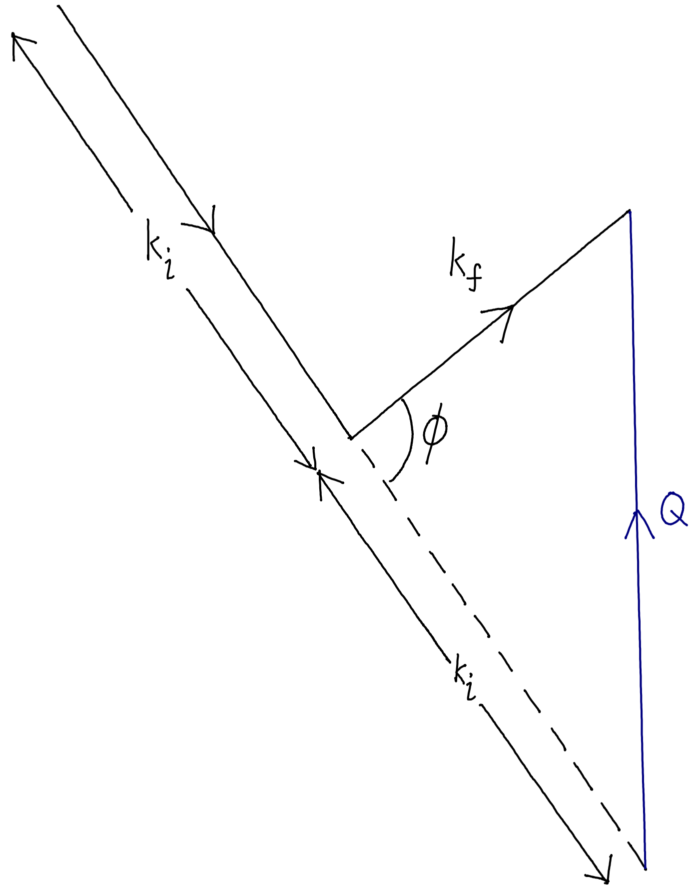
$$|Q| = \frac{2\pi l}{c}$$

So for $c = 0$, we have:

$$|Q| = \begin{cases} 14.0\,\mathrm{nm}^{-1} & (l = 1,\ \text{forbidden}) \\ 27.9\,\mathrm{nm}^{-1} & (l = 2,\ \text{allowed}) \\ 41.9\,\mathrm{nm}^{-1} & (l = 3,\ \text{allowed}) \\ 55.9\,\mathrm{nm}^{-1} & (l = 4,\ \text{allowed}) \\ 69.8\,\mathrm{nm}^{-1} & (l = 5,\ \text{forbidden}) \end{cases}$$

ii. **(TO EXPAND)**

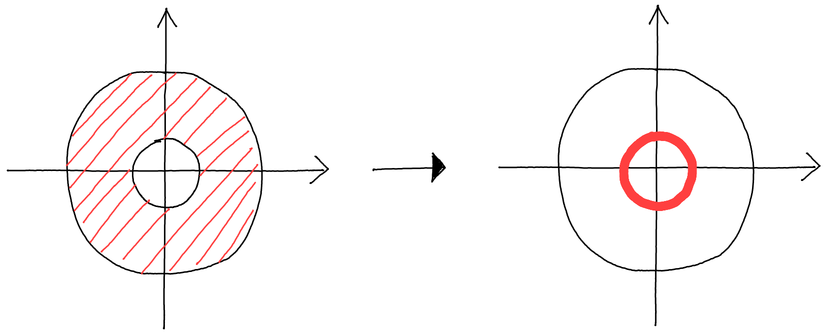




k_i
k_f
ϕ
Q
k_i

6. **(DRAFT)** Classic quantum Hall effect question.

(a) The quantum Hall effect is a phenomenon where the Hall resistivity ρ_{xy} is quantised to take $1/n$ where n is an integer. This is attributed to the formation of Landau levels on large field limit where $\omega_c \tau \gg 1$ ($\omega_c = eB/m_{CR}$ is the cyclotron frequency, τ is the scattering time).



Introduction of Landau levels squeezes the free electron into each ring, so for each Landau ring we have areal density:

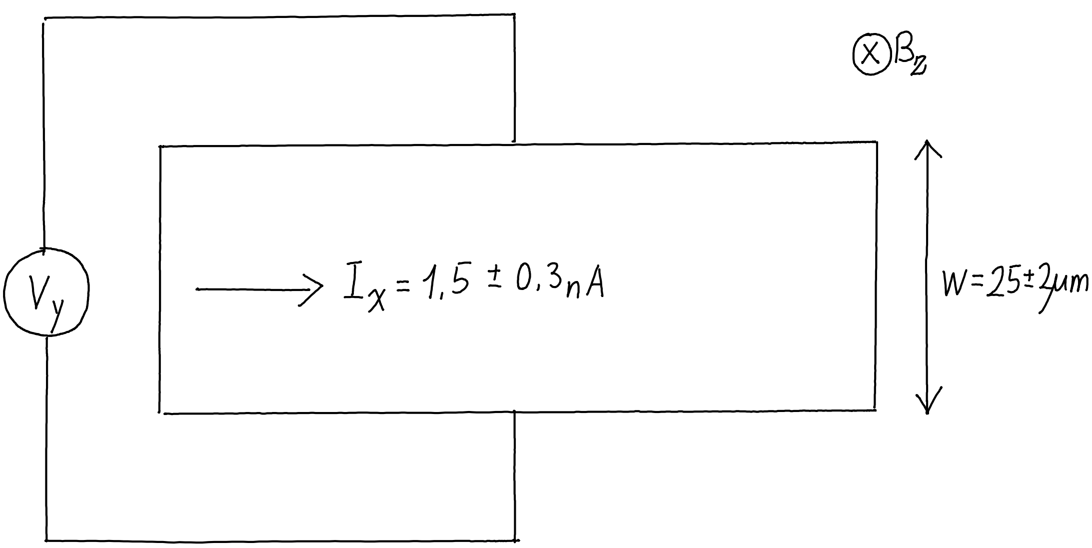
$$\begin{aligned} n_l &= \frac{1}{(2\pi)^2} \cdot \pi \left(k_{l+1}^2 - k_l^2\right) \\ &= \frac{1}{4\pi} \cdot \frac{2m}{\hbar} \cdot \frac{eB}{m} \\ &= \frac{eB}{h} \end{aligned}$$

Hence in total we have total areal density:

$$\begin{aligned} N_s &= n_l \cdot \nu \\ &= \frac{eB\nu}{h} \quad \text{where } \nu \in \mathcal{Z} \text{ is the filling factor} \end{aligned}$$

Note that for a changing B we have a period of $\Delta(1/B) = e/hN_s$. And the reason why Hall resistivity may be used as a resistance standard is due to it purely being the ratio of fundamental constants h and e.

(b) Sketch of setup:



From Drude:

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) - \frac{\mathbf{p}}{\tau}$$
$$\xRightarrow{\text{Steady state}} d\mathbf{p}/dt = 0$$

$$\begin{cases} v_x = \dfrac{q\tau}{m}\left(E_x + v_y B_z\right) & (1) \\ v_y = \dfrac{q\tau}{m}\left(E_y - v_x B_z\right) & (2) \end{cases}$$

Enforcing the condition of no Hall current gives $v_y = 0$:

$$\begin{aligned} E_y &= v_x B_z \\ \xRightarrow{(1)} E_y &= \frac{qB_z}{m}\tau E_x \\ \frac{E_y}{E_x} &= \omega_c \tau \end{aligned}$$

Also resistivity in the limit $\omega_c \tau \gg 1$:

$$\begin{aligned} \rho_{yx} &= R_H B \\ &= \frac{E_y}{J_x} \\ &= \frac{E_y}{N_s q v_x} \\ &= \frac{B_z}{N_s q} \\ &= \frac{1}{\nu}\frac{h}{eB_z} \cdot \frac{B_z}{e} \\ &= \frac{1}{\nu} \cdot \frac{h}{e^2} \end{aligned}$$

We also know that $V_y = E_y w$, $I_x = J_x w$:

$$\begin{aligned} \frac{V_y}{I_x} &= \rho_{yx} \\ V_y &= I_x \rho_{yx} \\ &= \frac{h}{\mu e^2} I_x \\ &= \begin{cases} 3.87 \times 10^{-4}\,\mathrm{V} & \nu = 1 \\ 1.94 \times 10^{-4}\,\mathrm{V} & \nu = 2 \\ 1.29 \times 10^{-4}\,\mathrm{V} & \nu = 3 \\ 9.68 \times 10^{-5}\,\mathrm{V} & \nu = 4 \end{cases} \end{aligned}$$

Error analysis:

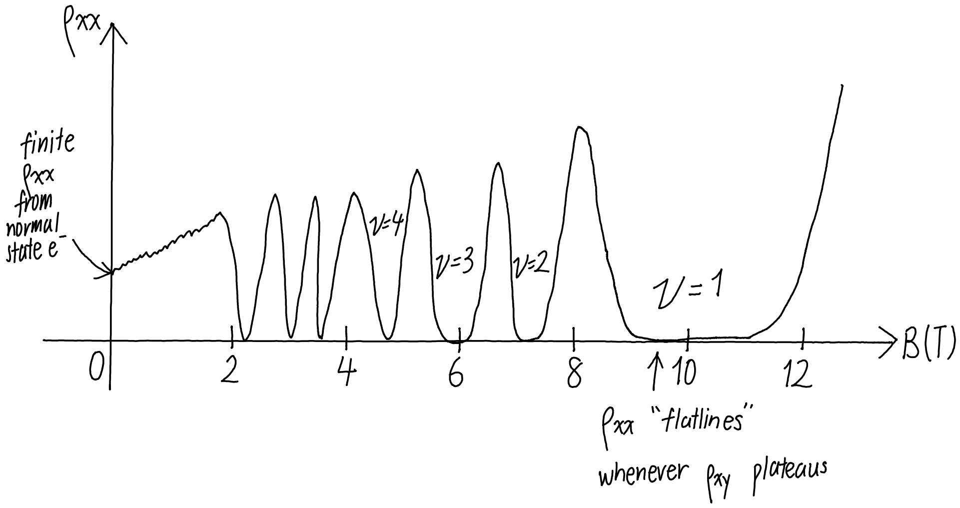
$$\Delta\left(\frac{V_y}{I_x}\right) = \text{sum of \% errors} = 2\% \text{ from } I_x \ (w \text{ cancels})$$

Assuming the top peak corresponds to $\nu = 1$, we then have:

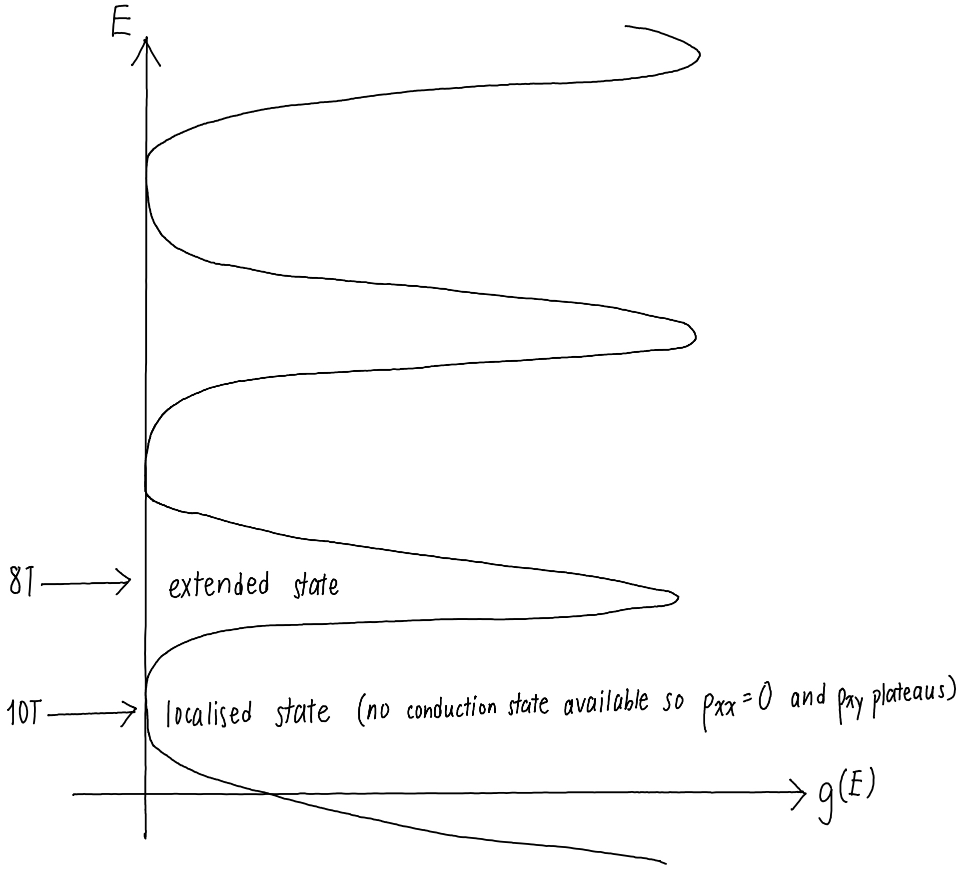
$$\begin{aligned} 19x &= 3.87 \times 10^{-4}\,\mathrm{V} \\ 12x &= 1.94 \times 10^{-4}\,\mathrm{V} \\ 9x &= 1.29 \times 10^{-4}\,\mathrm{V} \\ 6x &= 9.68 \times 10^{-5}\,\mathrm{V} \end{aligned} \quad \Rightarrow \quad x = \begin{cases} 2.04 \times 10^{-5}\,\mathrm{V/unit} \\ 1.62 \times 10^{-5}\,\mathrm{V/unit} \\ 1.43 \times 10^{-5}\,\mathrm{V/unit} \\ 1.61 \times 10^{-5}\,\mathrm{V/unit} \end{cases}$$

$$\Rightarrow \lambda \equiv \bar{x} = 1.68 \times 10^{-5}\,\mathrm{V/unit} \pm 2\%$$

(c) Sketch of ρ_{xx} against B:

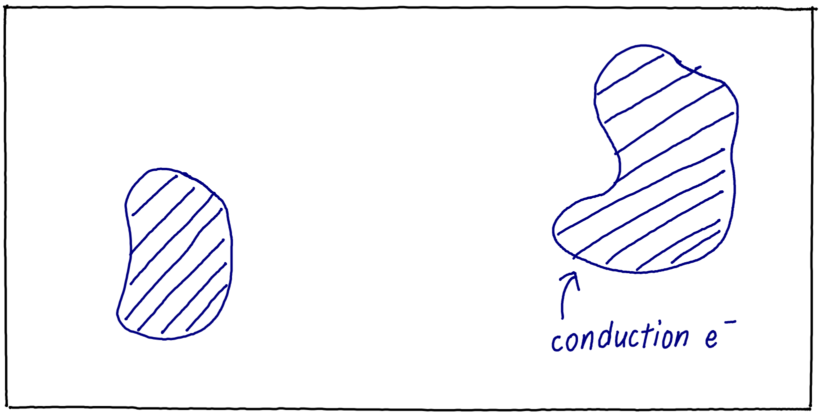


Sketch of d.o.s.:



(d) As B_z increases, the filling factor decreases as each Landau level may accommodate more states. When ν is an integer, we have a fully filled Landau level which can contribute no longitudinal current $\Rightarrow$ fixed Hall voltage.

The reason why we have a jump over some range of B is due to the variation of Landau level throughout the sample due to impurities. This gives rise to localised states where electrons available for conduction are not globally connected, rendering $\rho_{xx} = 0$. And extended states where the opposite happens.



localised state with e⁻ puddles