UNOFFICIAL SOLUTIONS BY TheLongCat

C3: CONDENSED MATTER PHYSICS


TRINITY TERM 2022

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Disclaimer: due to its unofficial nature, the author does not warrant the accuracy of the presented solutions in any form. However, the author is happy to discuss the typos and errors should one arises.
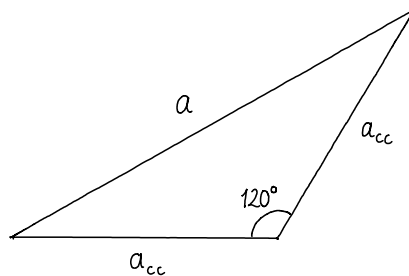
Turn over as you please – we are NOT under exam conditions here.

1. Classic tight-binding question with graphene.

    (a) Sketch of the direct lattice:
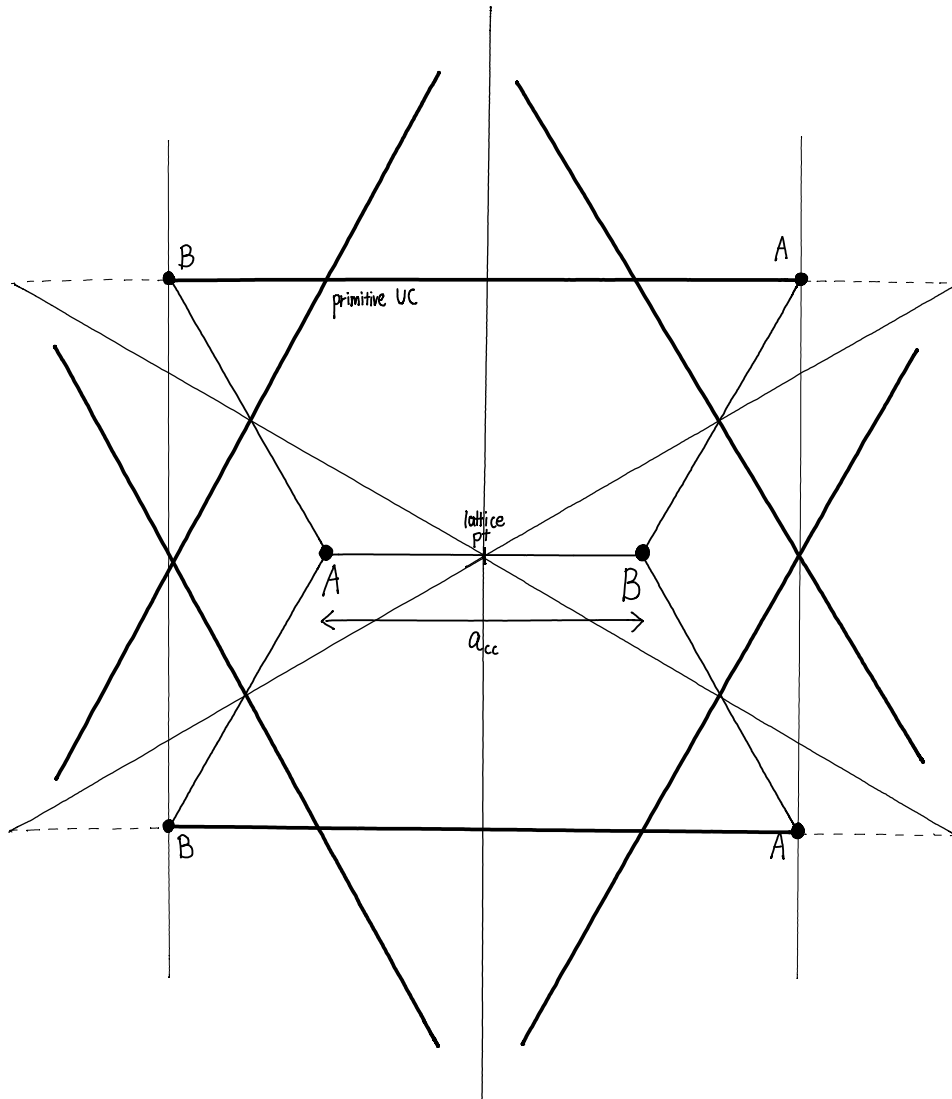
First we note the relation between a and a_{CC}:

From the diagram, it is clear that $\dfrac{a}{\sin 120°} = \dfrac{a_{\mathrm{CC}}}{\sin 30°} \Rightarrow a_{\mathrm{CC}} = \dfrac{a}{\sqrt{3}}.$

For each type, the nearest neighbours have relative positions:

$$(a_{\mathrm{CC}}, 0) = \left(\frac{a}{\sqrt{3}}, 0\right)$$

$$\left(-\frac{a_{\mathrm{CC}}}{2}, \frac{a}{2}\right) = \left(-\frac{a}{2\sqrt{3}}, \frac{a}{2}\right)$$

$$\left(-\frac{a_{\mathrm{CC}}}{2}, -\frac{a}{2}\right) = \left(-\frac{a}{2\sqrt{3}}, -\frac{a}{2}\right)$$

(b) In the tight-binding model, we begin with the Hamiltonian for the ith atom:

$$\mathcal{H}_i = H_{\mathrm{at}} + \Delta V$$

where $H_{\mathrm{at}} = p_i^2/2m + V_i(\mathbf{r})$ is the atomic Hamiltonian, $\Delta V = \sum_{j \neq i} V_j(\mathbf{r})$ is the potential from other atoms.

Next we consider the LCAO as the trial wavefunction:

$$\Psi = \sum_{\substack{i=1 \\ \text{sum over} \\ \text{lattice}}}^{N} \sum_{\substack{l \in \nu \\ \text{sum over} \\ \text{orbitals}}} \sum_{\substack{d=1 \\ \text{sum over} \\ \text{basis}}}^{M} \underbrace{\alpha_{ild}}_{\substack{\text{normalisation} \\ \text{constant}}} \underbrace{e^{i\mathbf{k}\cdot(\mathbf{R}_i + \mathbf{d}_d)}}_{\text{Bloch wavevector}} \underbrace{\phi_{ld}(\mathbf{r} - \mathbf{R}_i)}_{\substack{\text{standalone} \\ \text{atomic wavefunction}}}$$

For nearest neighbours, we have:

$$\Psi = \alpha_{\mathrm{A}} \Phi_{\mathrm{A}} + \alpha_{\mathrm{B}} \Phi_{\mathrm{B}}$$

where $\alpha_i = 1/\sqrt{N}$, $\Phi_\alpha = \sum_{i=1}^{N} e^{i\mathbf{k}\cdot(\mathbf{R}_i + \mathbf{d}_\alpha)} \Phi_\alpha(\mathbf{r} - \mathbf{R}_i)$.

(c) Substituting such wavefunction into TISE then gives:

$$\langle \Psi | \mathcal{H} | \Psi \rangle = \langle \Psi | E | \Psi \rangle$$

$$\Rightarrow \langle \Psi | H_{\mathrm{at}} | \Psi \rangle + \langle \Psi | \Delta V | \Psi \rangle = E \quad \text{assuming } \Phi_{\mathrm{A}}, \Phi_{\mathrm{B}} \text{ are orthonormal}$$

$$= \sum_{a=1}^{N} \sum_{b=1}^{N} \int d\mathbf{r} \phi_i^*(\mathbf{r} - \mathbf{R}_a + \mathbf{d}_i) \Delta V \phi_j(\mathbf{r} - \mathbf{R}_b + \mathbf{d}_j) \cdot e^{i\mathbf{k}\cdot(-\mathbf{R}_a - \mathbf{d}_i + \mathbf{R}_b + \mathbf{d}_j)}$$

$$= \sum_{\substack{\text{nearest} \\ \text{neighbour}}} e^{i\mathbf{k}\cdot(\mathbf{R}_{\mathrm{nn}} + \mathbf{d}_{ij})} \underbrace{\int d\mathbf{r}' \phi_i^*(\mathbf{r}') \Delta V \phi_j(\mathbf{r}' - \mathbf{R}_{\mathrm{nn}} + \mathbf{d}_{ij})}_{\tilde{t}_{ij}}$$

For nearest neighbours we then have:

$$t_{ij} = \left[e^{i\mathbf{k}\cdot(a/\sqrt{3}, 0)} + e^{i\mathbf{k}\cdot(-a/2\sqrt{3}, a/2)} + e^{i\mathbf{k}\cdot(-a/2\sqrt{3}, -a/2)} \right] \qquad \underbrace{\tilde{t}_{ij}}$$

$$\begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix}$$

where t is the hopping integral

Hence:

$$t_{ij} = \underbrace{\left[e^{ik_x \cdot a/\sqrt{3}} + e^{ik_x\left(-a/2\sqrt{3}\right) \cdot 2\cos(k_y \cdot a/2)} \right]}_{\gamma} \tilde{t}_{ij}$$

Now we seek the eigenvalues of the matrix above:

$$\det\left[(E_0 - E)\,\mathbb{1} + t_{ij}\right] = 0$$

$$\begin{vmatrix} E_0 - E & \gamma t \\ \gamma^* t^* & E_0 - E \end{vmatrix} = 0$$
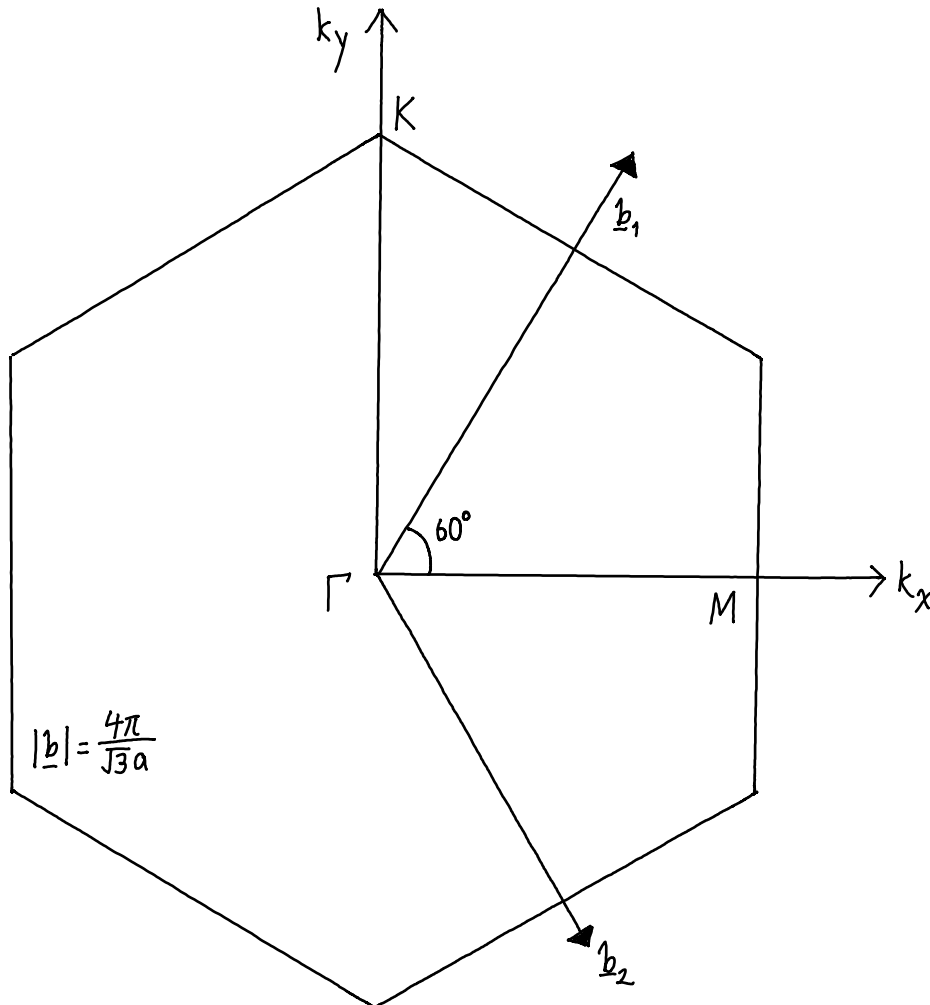
$$\Rightarrow E = E_0 \pm |\gamma|\, t \quad \text{where } E_0 \text{ is the unperturbed energy per atomic site}$$

Now:

$$|\gamma|^2 = \left(e^{ik_x \cdot a/\sqrt{3}} + e^{-ik_x a/2\sqrt{3}} \cdot 2\cos\left(k_y \cdot a/2\right)\right)\left(e^{-ik_x \cdot a/\sqrt{3}} + e^{ik_x a/2\sqrt{3}} \cdot 2\cos\left(k_y \cdot a/2\right)\right)$$

$$= 1 + 4\cos^2\left(\frac{k_y a}{2}\right) + 2\cos\left(\frac{k_y a}{2}\right)\underbrace{\left[e^{ik_x 3a/2\sqrt{3}} + e^{-ik_x 3a/2\sqrt{3}}\right]}_{2\cos\left(\sqrt{3}k_x a/2\right)}$$

$$= 1 + 4\cos\left(\frac{\sqrt{3}k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right) + 4\cos^2\left(\frac{k_y a}{2}\right)$$

Thus recovering the shown dispersion in the question.

We then have the following reciprocal lattice:

With κ being a path parameter, we have the following dispersions:

$M \to \Gamma$: choose path $k_x = \kappa$, $k_y = 0$, $\kappa \in \left[2\pi/\sqrt{3}a, 0\right]$:

$$E \simeq E_1 \pm t\sqrt{5 + 4\cos\left(\frac{\sqrt{3}\kappa a}{2}\right)}$$

$$\Rightarrow E(M) = E_1 \pm t \qquad E(\Gamma) = E_1 \pm 3t$$

$\Gamma \to K$: choose path $k_x = 0$, $k_y = \kappa$, $\kappa \in [0, 4\pi/3a]$:

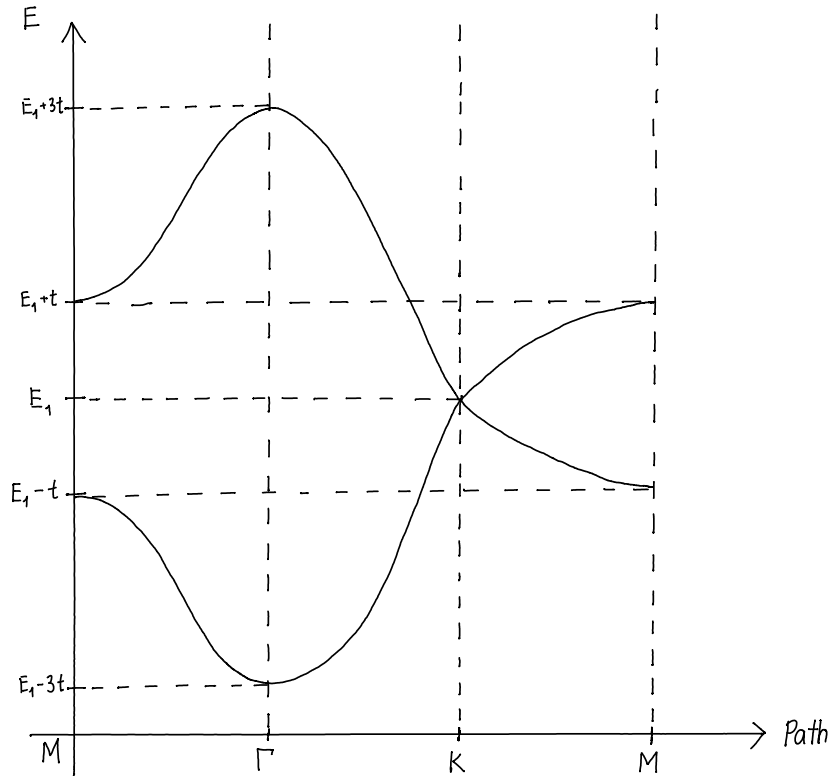$$E \simeq E_1 \pm t\sqrt{\underbrace{1 + 4\cos\left(\frac{\kappa a}{2}\right) + 4\cos^2\left(\frac{\kappa a}{2}\right)}_{2\cos(\kappa a/2)+1}}$$

$$\Rightarrow E(K) = E_1$$

$K \to M$: choose path $k_x = 2\pi/\sqrt{3}a$, $k_y = \kappa$, $\kappa \in [2\pi/3a, 0]$:

$$E \simeq E_1 \pm t\sqrt{\underbrace{1 - 4\cos\left(\frac{\kappa a}{2}\right) + 4\cos^2\left(\frac{\kappa a}{2}\right)}_{2\cos(\kappa a/2)-1}}$$

Sketch of graphene dispersion:

As the Fermi energy of graphene is near the Dirac point, its properties will be determined by the dispersion around it – linear dispersion, conductance...

(d) Carbon nanotubes are rolled up along the chiral vector, $\mathbf{n} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$ where n_i are the chiral indices.

Due to the perfect von-Karman boundary conditions, the wavevector along $\mathbf{n}$ is quantised, leaving the perpendicular wavevector quasi-continuous.
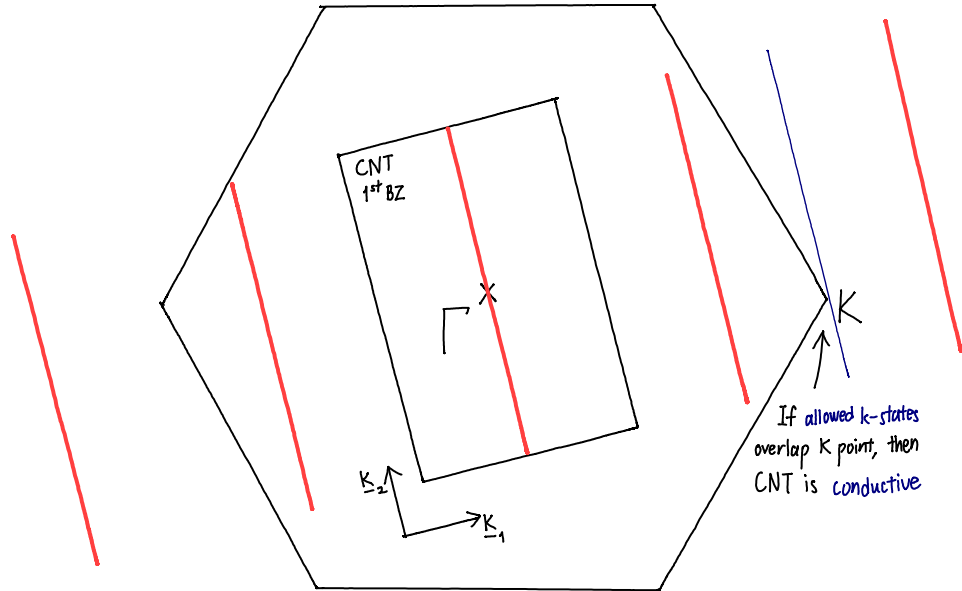
As illustrated in the sketch, the allowed energy is then restricted by the locus of allowed k states. In particular if such locus overlaps the K point, the nanotube will be conductive.

2. Absorption spectrum of 3D and 2D materials.

(a) The series of peaks is due to excitons – a bound state of e$^-$-h$^+$ pair in a material. In its ground state (i.e. stationary centre-of-mass), its energy level resembles that of a hydrogen atom, hence the $-1/n^2$ dependence of ΔE in the data.

(b) From the graph:

$$\Delta E_1 = 1.515 - 1.5188 (\text{eV}) = -\frac{R_X}{1^2} \Rightarrow R_X = 3.8\,\text{meV}$$

$$\Delta E_2 = 1.518 - 1.5188 (\text{eV}) = -\frac{R_X}{2^2} \Rightarrow R_X = 3.2\,\text{meV}$$

Thus we have average R_X of $3.5\,\text{meV}$.

(c)   i. Larger excitonic peak in QW: due to lower dimensionality, the size of exciton decreases $\Rightarrow$ higher binding energy

   ii. Step-like structure in QW: in 2D, the d.o.s. is constant $\Rightarrow$ flat structure. The step increase is due to the quantised harmonic level in QW, which also causes it to have a later onset than bulk. 3D case has the usual $\sqrt{E}$ dependence in d.o.s.
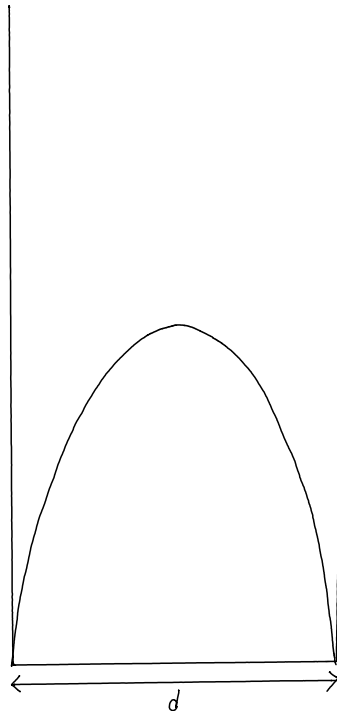

(d) From TISE:

$$\frac{\hat{p}^2}{2m}\psi = E\psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi = E\psi$$

$$\Rightarrow \text{Allowed energy } E_n = \frac{\hbar^2 k_n^2}{2m} \quad \text{where } k_n = \frac{n\pi}{L}$$

$$= \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

Note that $m = m_e^*$ in conduction band, $m = m_h^*$ in valence band so we have shifted the bands by $E_n + E'_n$.

Selection rules for optical transitions – from FGL, we know the matrix element is non-zero when:

- ψ and ψ' have the same parity $\Rightarrow$ transitions within the same oscillator level n.

- ψ and ψ' must have sufficient overlap.

In addition to the dipole selection rules for bulk material.
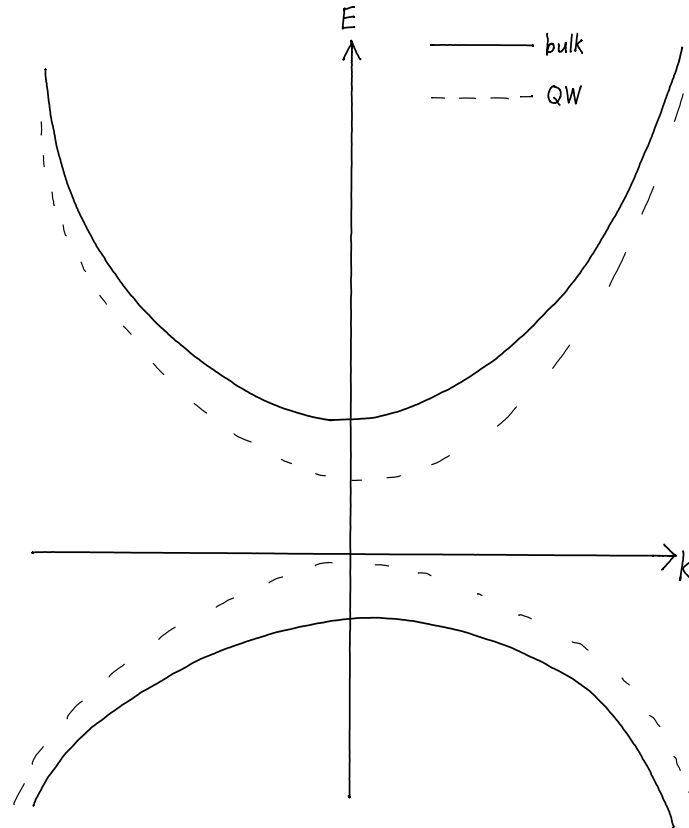
(e) From bulk we have $E_g \simeq 1.54\,\text{eV}$.

From QW we have $E_g + E_n + E'_n \simeq 1.62\,\text{eV}$ where $n = 1$.

$$\Rightarrow E_1 + E'_1 = 0.08\,\text{eV}$$
$$= \frac{\hbar^2 \pi^2}{2\mu L^2} \quad \text{where } \mu^{-1} = m_e^{*-1} + m_{lh}^{*-1}$$

m_{lh}^* is chosen since $m_{lh}^* < m_{hh}^*$ so has lower binding energy. Then:

$$\Rightarrow L^2 = \frac{\hbar^2 \pi^2}{2\mu\,(0.08\,\text{eV})}$$
$$= 7.76 \times 10^{-17}\,\text{m}^2$$
$$\Rightarrow L = 8.81\,\text{nm}$$

(f) Optical transitions require us to have:

$$\Delta E = E_{\text{conduction}} - E_{\text{valence}}$$
$$= \left(\frac{\hbar^2 k^2}{2m_e^*} + E_g \right) - \left(-\frac{\hbar^2 k^2}{2m_h^*} \right)$$

where E_g is a general band gap, m_h^* may be either m_{hh}^* or m_{lh}^*.

$$\Rightarrow \mathrm{d}E = \frac{\hbar^2 k \, \mathrm{d}k}{\mu} \quad \text{where } \mu^{-1} = m_h^{*^{-1}} + m_e^{*^{-1}}$$
$$k \, \mathrm{d}k = \frac{\mu}{\hbar^2} \, \mathrm{d}E$$

Thus in k space, the joint density of transition states N is:

$$N = \int g(k) \, \mathrm{d}k$$
$$\Rightarrow g(k) \, \mathrm{d}k = 2\pi k \, \mathrm{d}k \quad \text{in 2D QW}$$
$$= \frac{2\pi \mu}{\hbar^2} \, \mathrm{d}E = g(E) \, \mathrm{d}E$$

3. Usual SC with a brief exploration of thin film SC.

(a) From the BCS theory, we know that due to the attractive phonon interaction, electrons pair up to form Cooper pairs via the Pauli exclusion principle.

Since this interaction is attractive, there is an energy saving by the electrons at Fermi surface to form Cooper pairs, and this leads to the opening of an energy gap as those electrons are now in the SC phase and form a condensate.
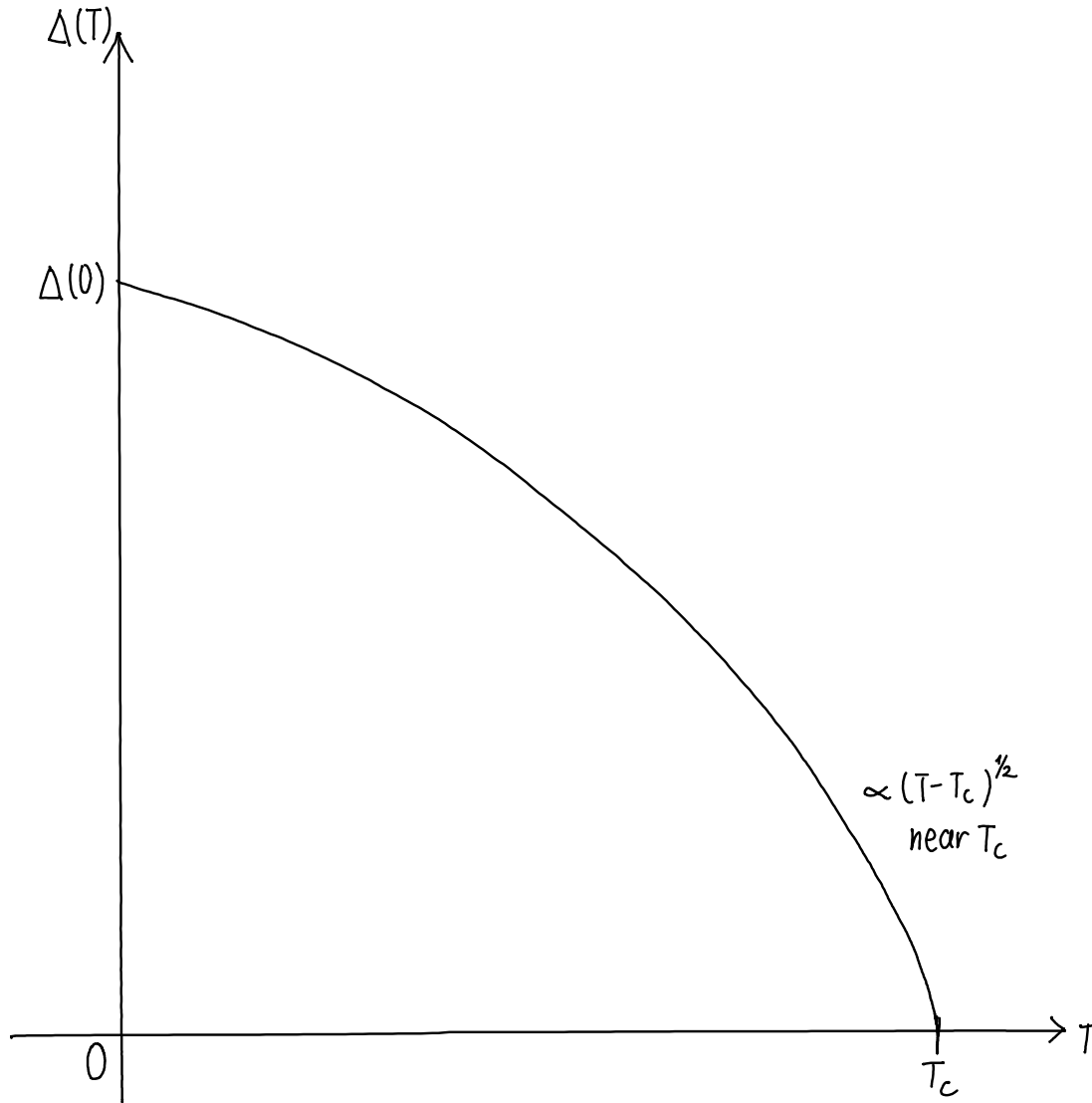
Since Δ relates to the SC carrier density n_s, it follows that near T_c it exhibits the same T dependence as $\psi = \sqrt{n_s}e^{i\theta}$.

To explore the momentum dependence of Δ, we may perform a measurement of tunnelling current through the SC. The bias voltage simply adjusts the explored energy level.

(b) Sketch of the setup:
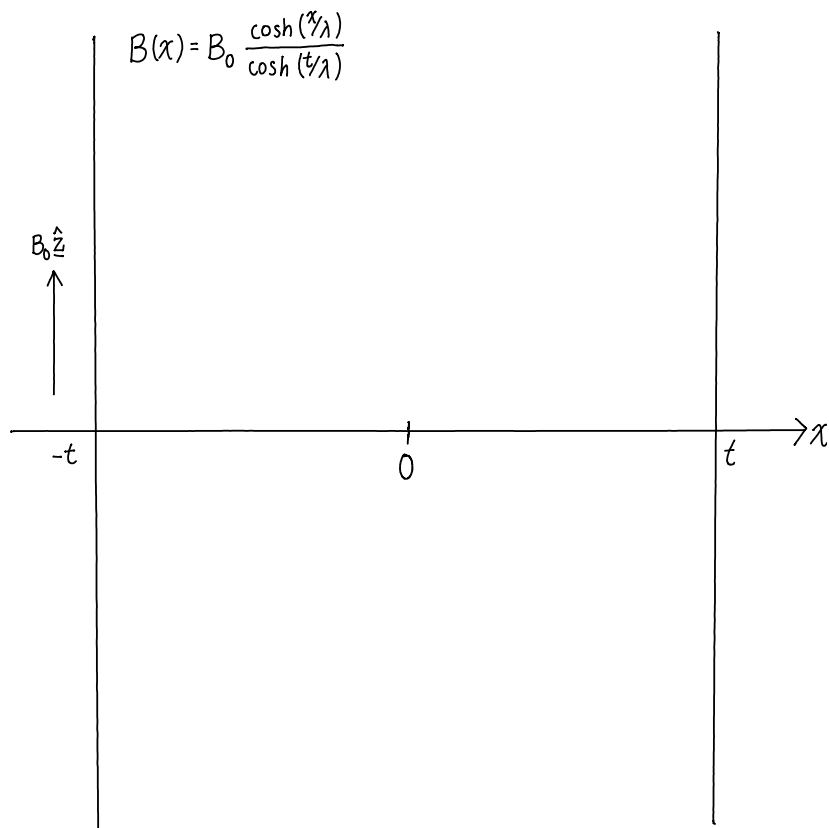
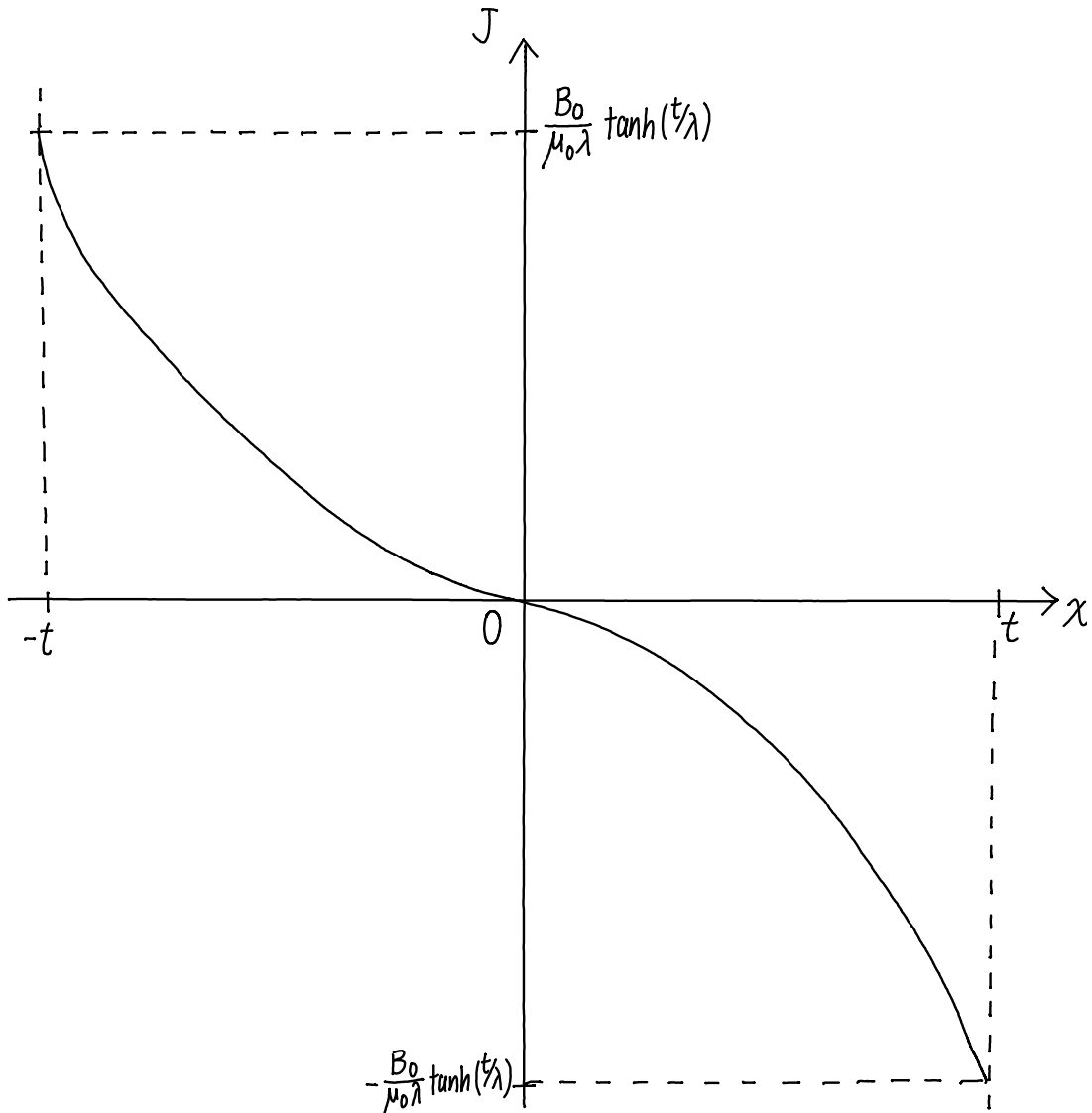$$B(x) = B_0 \frac{\cosh(x/\lambda)}{\cosh(t/\lambda)}$$

From Maxwell,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & B \end{vmatrix} = \mu_0 \mathbf{J}$$

$$\Rightarrow \mu_0 \mathbf{J} = -\frac{\partial}{\partial x} \left(B_0 \frac{\cosh(x/\lambda)}{\cosh(t/\lambda)} \right) \hat{\mathbf{y}}$$

$$\mathbf{J} = -\frac{B_0}{\mu_0 \lambda} \frac{\sinh(x/\lambda)}{\cosh(t/\lambda)} \hat{\mathbf{y}}$$

Sketch of J along x:

For $\lambda \gg t$, we see that $B(x) \to B_0 \left[1 + \mathcal{O}\left(x^2/\lambda^2\right)\right]$ so we have the B field virtually penetrating the sample.

Critical current density wise, it follows from the graph that it must also shrink to 0.

(c) $\kappa = \lambda/\xi = 100 > 1/\sqrt{2} \Rightarrow$ type-II so vortex formation favourable.

Extrapolating from the graph gives us $B_{c2} \simeq 100\,\text{T}$.

Also for type-II SC, upper field corresponds to when $\left(2\pi\xi^2\right) B_c \Phi_0$ where ξ is coherence length:

$$\left(2\pi\xi^2\right) B_c \Phi_0 = 100\,\text{T}$$
$$\Rightarrow \xi = 1.81\,\text{nm}$$

Hence:

$$\lambda = \xi\kappa = 181\,\text{nm}$$

Now we have:

$$\xi = \frac{\hbar v_{\mathrm{F}}}{\pi \Delta}$$

$$\Rightarrow \Delta = \frac{\hbar v_{\mathrm{F}}}{\pi \xi} = 0.18 \, \mathrm{eV}$$

From BCS theory we have $\Delta(0) = 2\hbar\omega_D \mathrm{e}^{-1/\lambda}$ and $k_{\mathrm{B}}T_{\mathrm{c}} = 1.13\hbar\omega_D \mathrm{e}^{-1/\lambda}$:

$$\Rightarrow \Delta(0) \simeq 1.77 k_{\mathrm{B}} T_{\mathrm{c}}$$

$$\simeq 5.34 \times 10^{-3} \, \mathrm{eV} \quad \text{for } T_{\mathrm{c}} \simeq 35 \, \mathrm{K}$$

Since there is a huge difference between the 2 predictions, this SC may not be explained by BCS theory (unless strong phonon interaction is accounted for).

4. Question on point group symmetries and their consequences.

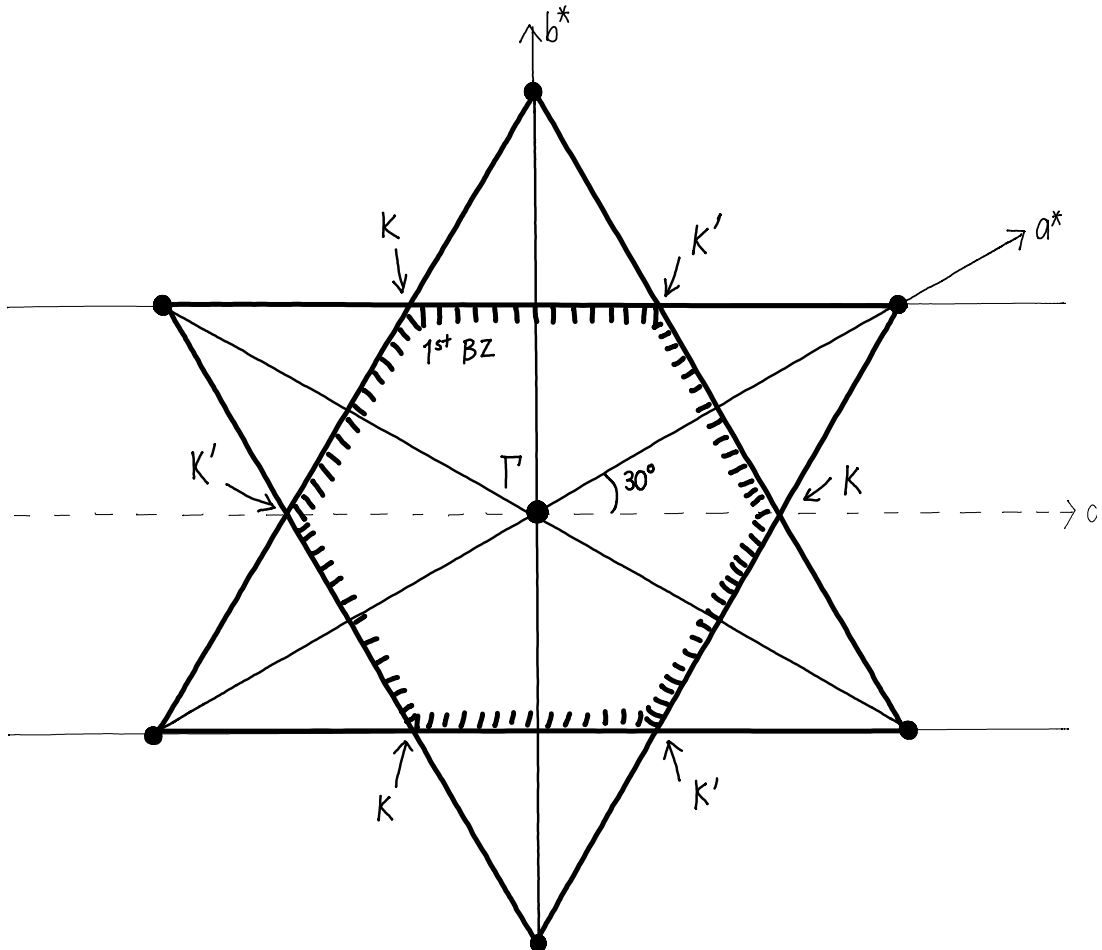(a) Sketch of the direct lattice and its symmetry elements:
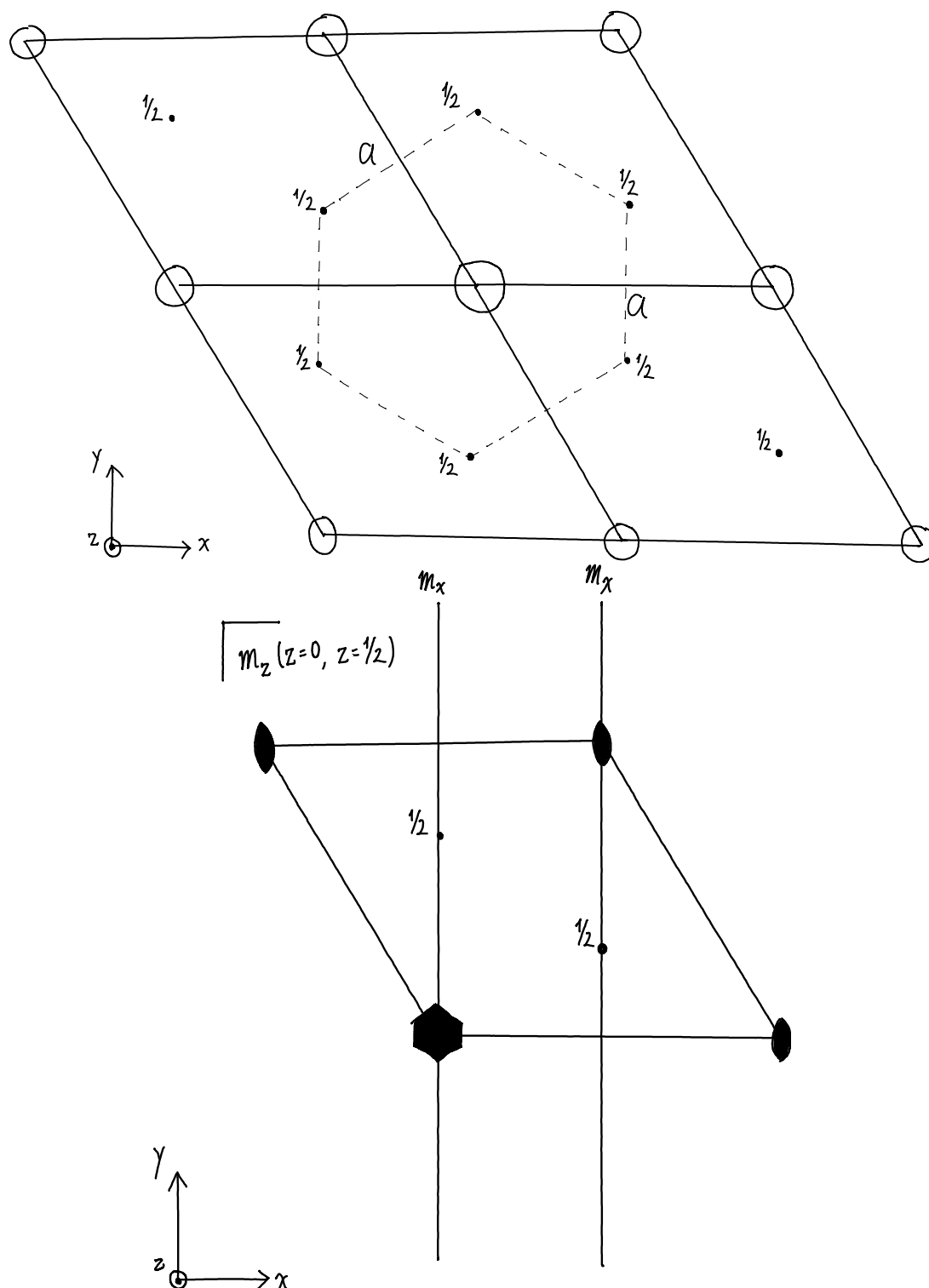
Noting the ◯ structure due to $a = b$, we have the highest rotational symmetry of 6_z. 2_z through the Mg atoms. m_z at $z = 0$ and $z = 1/2$. m_x through the B atoms. We also have $\bar{1}$ present at the centre $(1/2, 1/2, 1/2)$.

(b) We find the reciprocal lattice vector by definition:

$$\mathbf{a}^* = 2\pi \frac{\mathbf{b} \times \mathbf{c}}{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})} \Rightarrow \mathbf{a}^* \; 30° \text{ from } \mathbf{a}$$

$$\mathbf{b}^* \propto \mathbf{c} \times \mathbf{a} \Rightarrow \mathbf{b}^* \; -30° \text{ from } \mathbf{b} \Rightarrow \hat{\mathbf{y}}$$
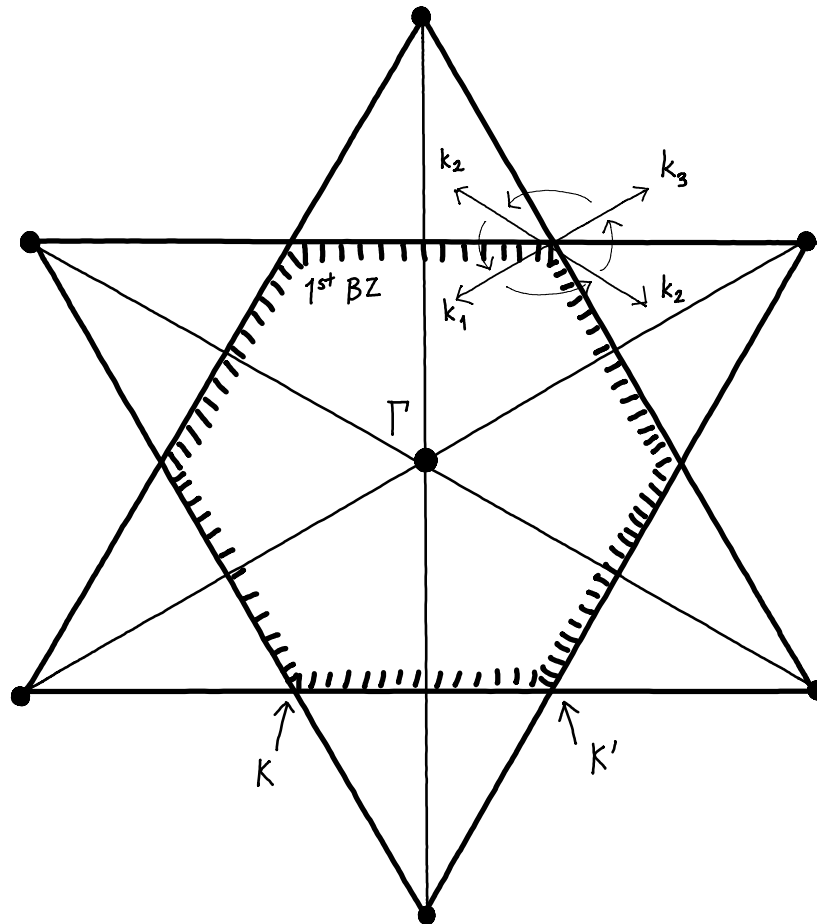
Sketch of the reciprocal lattice:

(c) Character table (p_i is polarisation along i):

|	A	B	p_x	p_y	p_z
$\bar{1}$	-1	-1	-1	-1	-1
2_z	+1	-1	-1	-1	+1
m_z	-1	-1	+1	+1	-1
m_x	+1	-1	-1	+1	+1

So only mode A is IR active with polarisation along z.

(d) Sketch of the reciprocal lattice points again:

Note that within Γ point, all wavevectors within the 1st BZ remain inside after 3_z about Γ. However, for K there is a mixing between 1st and 2nd BZ (see the sketch above), which carries a phase of $\pi \Rightarrow$ thus A and B are degenerate about K by symmetry.

5. Classic magnetism question.
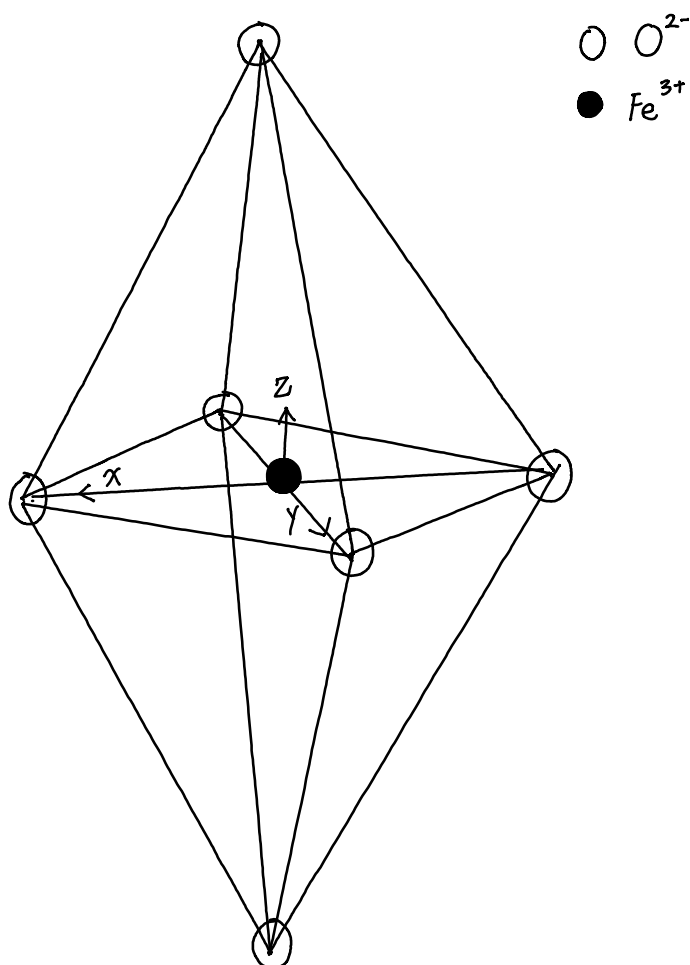
(a) As shown in the diagram above, as the O^{2-} ions are located along the principal axes, the $3d$ orbitals of the central Fe ion along x, y, z have greater overlap with the O^{2-} orbitals, thereby experiencing greater Coulomb repulsion $\Rightarrow$ higher energy, this splitting also causes the effective orbital angular momentum to vanish, this is called *orbital quenching*. Strength of the field can alter the magnetic ground state as above.

At high pressure when the crystal is compressed, the crystal field effectively increases and lead to the transition above, the slight mismatch between the low field μ_{eff} is due to the strength of crystal field not being strong enough to fully quench L.
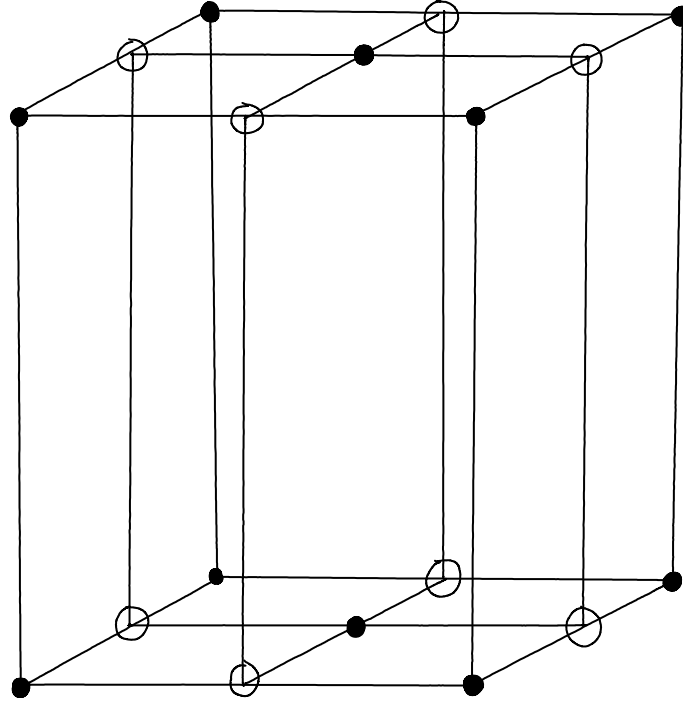
(b) Note that the magnetic UC has dimensions $2a \times 2a \times c$ with c-centering. The sublattices are located $a\hat{\mathbf{x}}$ from one another.

Thus the structure factor is:

$$S = f\left[1 + e^{2\pi i(ha/2a + ka/2a + 0)}\right] - f e^{2\pi i(ha/2a)}\left[1 + e^{2\pi i(ha/2a + ka/2a + 0)}\right]$$

So for S to vanish, $h + k$ must be odd.

Miller indices with the smallest k's: (1,0,0) and (0,1,0).

(c) Nearest-neighbour Heisenberg: $\mathcal{H} = \sum_{i,j} -J_1/2\,\mathbf{S}_i \cdot \mathbf{S}_j + J_2/2\,(\mathbf{S}_i)^2$.

B_{mf} is the molecular mean field – this is the effective field that one spin would encounter on average in the crystal.

Mean field energy per spin:

$$E = g\mu_{\mathrm{B}} B_{\mathrm{mf}} S_i = \left[-\frac{J_1}{2}\mathbf{S}_j + \frac{J_2}{2}\mathbf{S}_i\right] \cdot \mathbf{S}_i$$

$$\Rightarrow B_{\mathrm{mf}} = -\left(\frac{J_1}{2}\right)\left(\frac{1}{g\mu_{\mathrm{B}}}\right)^2 \underbrace{(g\mu_{\mathrm{B}} S_j)}_{M_{\mathrm{B}}} + \left(\frac{J_2}{2}\right)\left(\frac{1}{g\mu_{\mathrm{B}}}\right)^2 \underbrace{(g\mu_{\mathrm{B}} S_i)}_{M_{\mathrm{A}}}$$

$$\Rightarrow \begin{cases} \lambda_{\mathrm{AA}} = -\left(\dfrac{J_2}{2}\right) \cdot \left(\dfrac{1}{g\mu_{\mathrm{B}}}\right)^2 \\[3mm] \lambda_{\mathrm{AB}} = \left(\dfrac{J_1}{2}\right) \cdot \left(\dfrac{1}{g\mu_{\mathrm{B}}}\right)^2 \end{cases}$$

At T_N, $M_A = M_s B_s(y)$:

$$\Rightarrow B_s(y) = \frac{M_A}{M_s}$$

$$\Rightarrow \frac{S+1}{3S} \cdot \frac{g\mu_B S}{k_B T} \cdot \left(-\lambda_{AA} M_A \underbrace{-\lambda_{AB} M_B}_{+\lambda_{AB} M_A}\right) = \frac{M_A}{M_s}$$

$$\Rightarrow T_N = \underbrace{M_s}_{g\mu_B S} \cdot \frac{S+1}{3S} \cdot \underbrace{(-\lambda_{AA} + \lambda_{AB})}_{(1/g\mu_B)^2 [J_1+J_2]/2}$$

$$= \frac{S+1}{3(g\mu_B)} \left[\frac{J_1 + J_2}{2}\right]$$

Since the mean field ignored other fluctuations in the crystal, the prediction would be higher than the actual value.

(d) To measure M_A, apply magnetic field and measure the response field ($B = \mu_0 H + M$) via a magnetometer like SQUID.
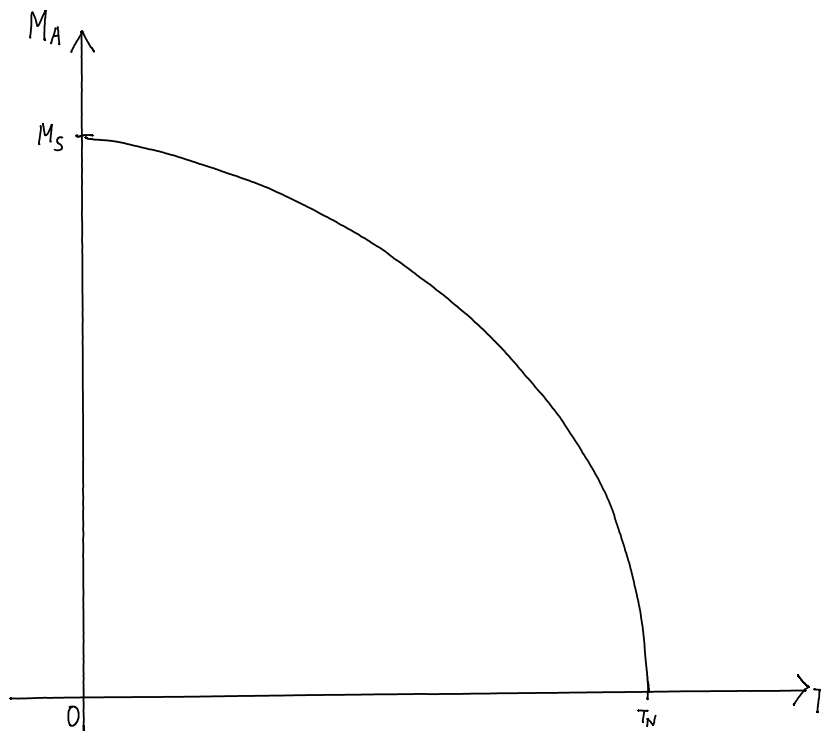
6. (**DRAFT**) Energy levels of an anisotropic paramagnetic ion.
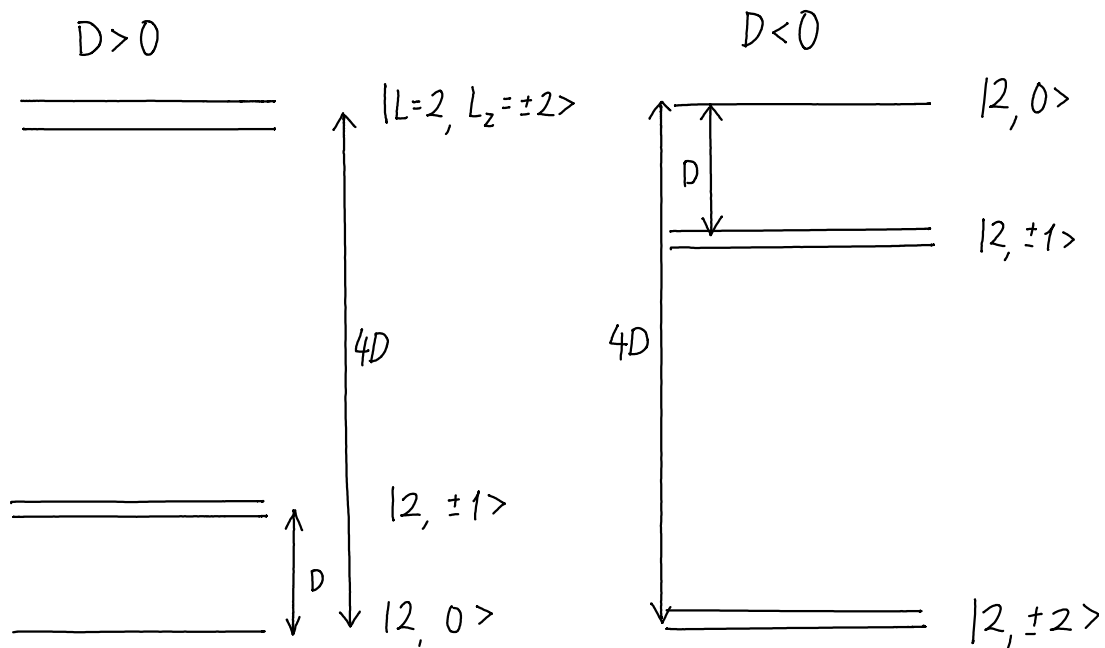
(a) From the given quantity:

$$\mu_{\text{sat}} = \frac{\mu_{\text{B}}}{2} \begin{pmatrix} g_\perp n_x \\ g_\perp n_y \\ g_\parallel n_z \end{pmatrix}$$

$$\Rightarrow \mu_{\text{sat}} = \frac{\mu_{\text{B}}}{2} \sqrt{g_\perp^2 \underbrace{\left(n_x^2 + n_y^2\right)}_{r^2 \text{ in } xy \text{ plane}}}$$

So μ_{sat} is independent of the orientation in xy plane.
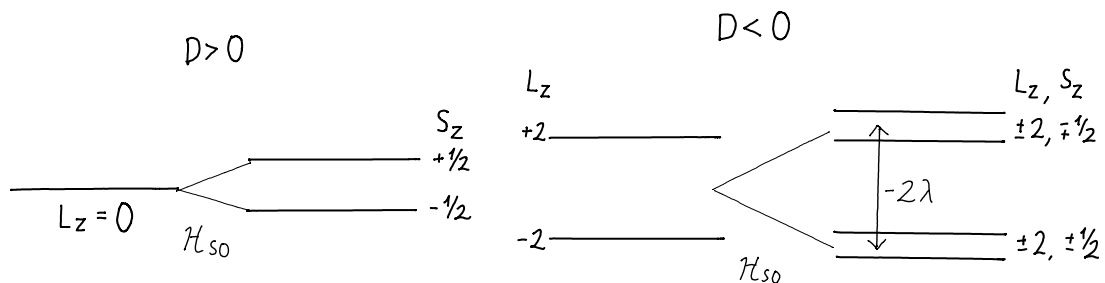
(b) Crystal field Hamiltonian: $\mathcal{H}_{\text{CF}} = D L_z^2$.

For $L = 2$ we then have degeneracies in $L_z = \pm 1$, $L_z = \pm 2$:

(c) Spin-orbit Hamiltonian: $\mathcal{H}_{\text{SO}} = \lambda \mathbf{L} \cdot \mathbf{S}$, $\lambda < 0$ and $|\lambda| \ll |D|$ so perturb from the crystal field.

For $L = 2$, $S = \frac{1}{2}$, Wigner-Eckart gives $\langle \mathbf{L} \cdot \mathbf{S} \rangle \propto L_z S_z$ so for the lowest orbital term:

(d) Zeeman Hamiltonian:

$$\mathcal{H}_{\text{Z}} = -\mu \cdot \mathbf{B}$$
$$= \mu_{\text{B}} \left(\mathbf{L} \cdot \mathbf{B} + 2 \mathbf{S} \cdot \mathbf{B} \right)$$

Since we are further perturbing from the previous interactions, we have:

$$\mu_\text{B}\left(\mathbf{L}\cdot\mathbf{B}\right)+2\mu_\text{B}\left(\mathbf{S}\cdot\mathbf{B}\right)=\begin{cases}\mu_\text{B}\left(L_z B+2S_z B\right) & \mathbf{B}\parallel\hat{z}\\[4mm]\mu_\text{B}\left(\underbrace{L_x}_{\frac{L_++L_-}{2}}B+2\underbrace{S_x}_{\frac{S_++S_-}{2}}B\right) & \mathbf{B}\perp\hat{z}\end{cases}$$

choosing x axis by isotropy.

So for the $\parallel$ case, we have a Zeeman shift of:

- <u>$D>0$</u>

$$\Delta E_z=\pm\mu_\text{B}B\quad\text{for}\quad|L_z=0,S_z=\pm\tfrac{1}{2}\rangle$$

- <u>$D<0$</u>

$$\Delta E_z=\pm\mu_\text{B}B\left(2+1\right)=\pm 3\mu_\text{B}B\quad\text{for}\quad|L_z=\pm2,S_z=\pm\tfrac{1}{2}\rangle$$

For the $\perp$ case, we have:

- <u>$D>0$</u>

$$\Delta E_z=\frac{1}{2}\left(\langle L_z=0,S_z=+\tfrac{1}{2}|+\langle L_z=0,S_z-\tfrac{1}{2}|\right)$$
$$\mathcal{H}_\text{Z}$$
$$\left(|L_z=0,S_z=+\tfrac{1}{2}\rangle+|L_z=0,S_z=-\tfrac{1}{2}\rangle\right)$$
$$=\frac{1}{2}\left(\langle\dots|+\langle\dots|\right)\left[\mu_\text{B}B\cdot2\cdot\frac{1}{2}\sqrt{\tfrac{1}{2}(\tfrac{3}{2})-\tfrac{1}{2}(\tfrac{1}{2})}\,|0,-\tfrac{1}{2}\rangle\right.$$
$$\left.+\mu_\text{B}B\cdot2\cdot\frac{1}{2}\sqrt{\tfrac{1}{2}(\tfrac{3}{2})-(-\tfrac{1}{2})(\tfrac{1}{2})}\,|0,+\tfrac{1}{2}\rangle\right]$$
$$=\frac{1}{2}\left[B\mu_\text{B}+\mu_\text{B}B\right]=\mu_\text{B}B$$

- <u>$D<0$</u>

$$\Delta E_z=\frac{1}{2}\left(\dots\right)\left[\mu_\text{B}B\cdot\frac{1}{2}\sqrt{2\cdot3-(-2)+\dots}\,|\dots\rangle+\dots\right]$$
$$=0\quad\text{since we have no bras with }L_z=\pm1$$

At saturation, we have all moments pointing in the same direction. Picking the new ground states then gives:

$$D<0\Rightarrow\begin{cases}\parallel:\ \mu_\text{sat}=-\mu_\text{B}\hat{z}\\[2mm]\perp:\ \mu_\text{sat}=+\mu_\text{B}\hat{z}\end{cases}\Rightarrow g_\parallel=-\frac{1}{2}\qquad g_\perp=+\frac{1}{2}$$

$$D>0\Rightarrow\begin{cases}\parallel:\ \mu_\text{sat}=-3\mu_\text{B}\hat{z}\\[2mm]\perp:\ \mu_\text{sat}=0\end{cases}\Rightarrow g_\parallel=-\frac{3}{2}\qquad g_\perp=0$$

(e) An EPR experiment applies an RF pulse to the ion to look for the absorption spectra. At transition frequencies, there would be absorption peaks and they may be used to infer the energy levels. (Hence the Zeeman splitting too.)

For $D<0$, since $\mu_\text{sat}=0\perp\hat{z}$, the splitting at such direction will be unobservable.