UNOFFICIAL SOLUTIONS BY TheLongCat

C3: CONDENSED MATTER PHYSICS

TRINITY TERM 2021

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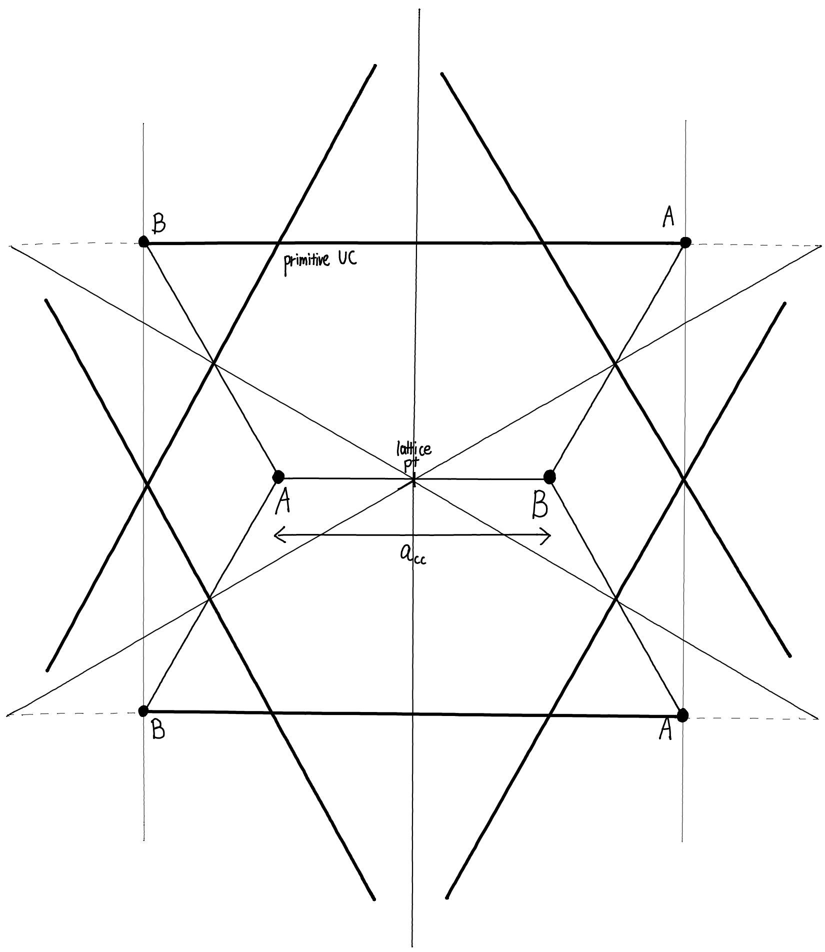
Disclaimer: due to its unofficial nature, the author does not warrant the accuracy of the presented solutions in any form. However, the author is happy to discuss the typos and errors should one arises.

Turn over as you please – we are NOT under exam conditions here.

1. **(DRAFT)** Scattering structure factor.

Sketch of the direct lattice:

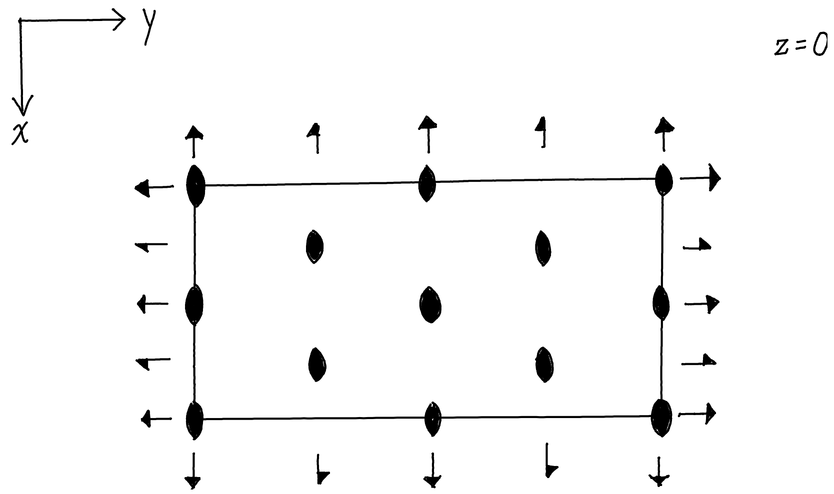


(a) Scattering amplitude for reciprocal lattice vector $\mathbf{G} = h\mathbf{a}^* + k\mathbf{b}^* + \mathrm{l}\mathbf{c}^*$ and C-centred lattice with centering translation $(1/2, 1/2, 0)$:

$$\begin{aligned} S_{hkl} &= f_x \left[\mathrm{e}^{i0} + \mathrm{e}^{2\pi i(h\cdot 1/2 + k\cdot 1/2 + 0)} \right] \\ &= f_x \left(1 + \mathrm{e}^{\pi i(h+k)} \right) \end{aligned}$$

where f_x is the atomic factor for atom x (assumed to be common throughout the lattice points).

So extinction condition is when $S_{hkl} = 0 \Rightarrow h + k = \text{odd}$.



(b) As seen above, there is no point of inversion. Nor are there any mirror planes. However, there exists screw axis of 2_1 as pointed by the half-pointed arrows in the diagram above.

(c) Generally,

$$\begin{aligned}
S_{hkl} &= S_{\text{basis}} \times S_{\text{lattice}} \\
&= \left(1 + \mathrm{e}^{i\pi(h+k)}\right)\left[f_{\mathrm{Y}}\mathrm{e}^{i0} + f_{\mathrm{Sb}}\left(\mathrm{e}^{2\pi i(h/4+k/4+l\eta)} + \mathrm{e}^{2\pi i(3h/4+k/4-l\eta)}\right)\right] \\
&= \left(1 + \mathrm{e}^{i\pi(h+k)}\right)\left[f_{\mathrm{Y}} + f_{\mathrm{Sb}}\left(\mathrm{e}^{2\pi i(h/4+k/4+l\eta)} + \mathrm{e}^{2\pi i(-h/4+k/4-l\eta)}\right)\right] \\
&= \ldots\left[f_{\mathrm{Y}} + f_{\mathrm{Sb}}\mathrm{e}^{i\pi k/2} \cdot 2\cos\left(h/2 + 2l\eta\right)\pi\right] \\
&= 2 \times \begin{cases} f_{\mathrm{Y}} + f_{\mathrm{Sb}}\left(-1\right)^{k/2} \cdot 2\cos\left(h/2 + 2l\eta\right)\pi & h,\ k \text{ even} \\ f_{\mathrm{Y}} + f_{\mathrm{Sb}} i^{k/2} \cdot 2\cos\left(h/2 + 2l\eta\right)\pi & h,\ k \text{ odd} \end{cases}
\end{aligned}$$

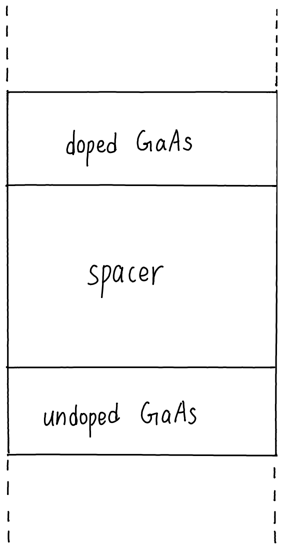
(d) Note that enantiomorphs have opposite signs for η, so the extinction condition for either would have a phase difference of $4l\eta$. By comparing between the calculations for $\pm\eta$ and the scattering angle, we may distinguish between the 2 forms.

(e) Note that shearing has destroyed the 2-fold rotational symmetry along x and y. However the 2 along z and C centering is preserved, so we have a monoclinic C121 lattice.

2. **(DRAFT)** Quantum Hall effect in a semiconductor. Naturally the question is a lot more difficult due to its open-book nature.

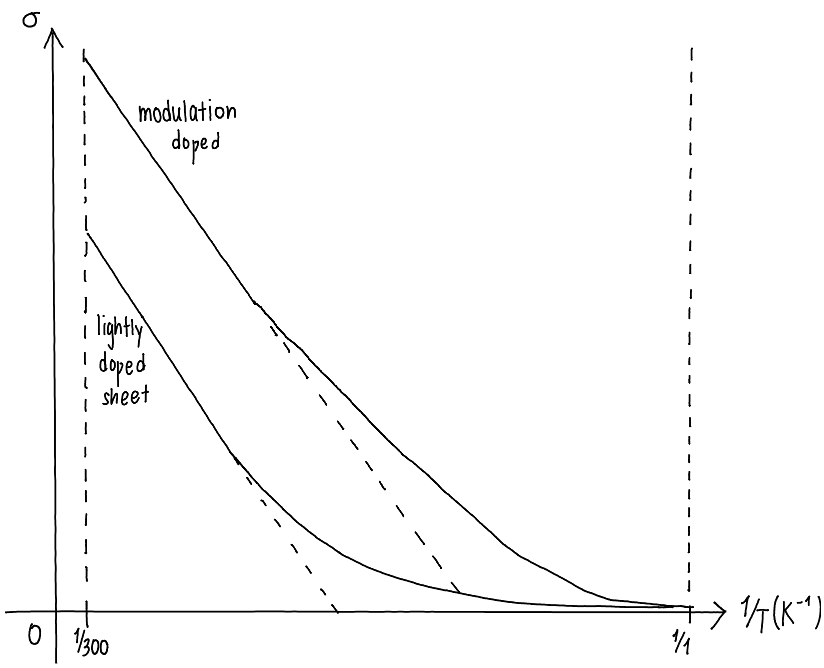
(a) Heterostructure: by forming a sandwich of lower band gap between 2 p and n doped layers, a 2D electron gas is formed due to the confinement along layers, this can be further realised by populating only the lowest sub-band with low enough carrier density.

Modulation doping:



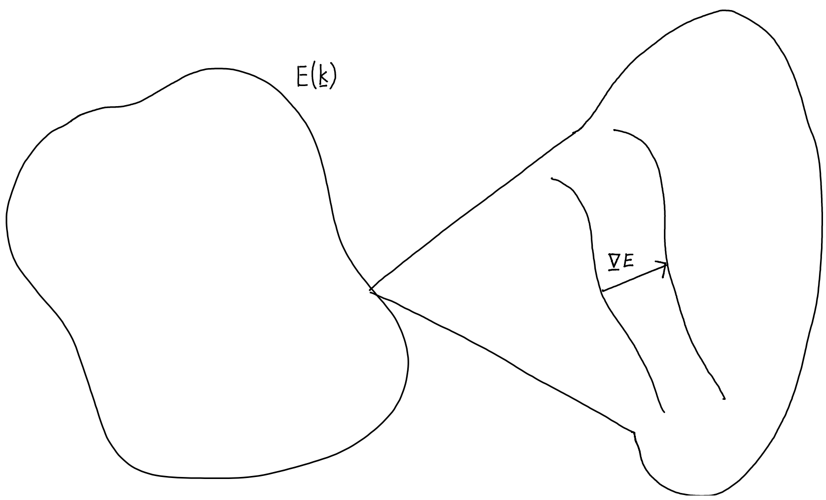
By repeatedly applying the layers shown above, we may modulate the doping of GaAs. This is important as the spacer separates the ionised donors from the electrons, thus reducing the scattering rate $\Rightarrow$ larger carrier mobilities.

Sketch of conductivity σ against temperature T (silicon doping so n-type $\rightarrow$ no cusp due to change in dominant carrier):



The temperature dependence arises from the freezing of thermally excited charge carriers.

(b) Sketch of an arbitrary energy surface:



In 2D, where g_e is the e$^-$ spin degeneracy:

$$g(k)\,\mathrm{d}k \to \frac{g_e}{(2\pi)^2}\,\delta A \quad \text{where } (2\pi)^2 \text{ comes from continuum approximation}$$

$$\delta A = \oint_l \mathrm{d}\mathbf{l} \times \hat{\mathbf{n}} \quad \text{where } \hat{\mathbf{n}} = \frac{\boldsymbol{\nabla} E}{|\nabla E|} \text{ is the normal unit vector to the iso-energy line}$$

$$= \oint_l \frac{\mathrm{d}l}{|\nabla E|}$$

For the conduction s-band, $E = \dfrac{\hbar^2 k^2}{2m^*} \Rightarrow \nabla E = \dfrac{\hbar^2 k}{m^*}$:

$$\delta A = 2\pi k \cdot \frac{m^*}{\hbar^2 k}$$

$$= \frac{2\pi m^*}{\hbar^2}$$

$$\Rightarrow g(E) = \frac{2\pi g_e}{(2\pi)^2 \hbar^2} m^* = \underbrace{\frac{g_e}{2\pi\hbar^2}}_{\alpha} m^*$$

(c) i. We know that # of states per Landau level = # of states between orbits at $B = 0$:

$$n_s = \frac{g_e \pi}{(2\pi)^2}\left(k_{n+1}^2 - k_n^2\right)$$

where $\frac{\hbar^2 k_n^2}{2m^*} = (n + 1/2)\hbar\omega_c = (n + 1/2)\hbar eB/m^* \Rightarrow k_n^2 = (n + 1/2)2eB/\hbar$:

$$\Rightarrow n_s = \frac{g_e \pi}{(2\pi)^2}\frac{2eB}{\hbar}$$

$$= g_e \frac{2\pi eB}{\hbar} \cdot \frac{1}{(2\pi)^2}$$

$$= g_e \frac{eB}{h} \quad \text{where the factor of } (2\pi)^2 \text{ comes from continuum approximation}$$

ii. Total number of areal density:

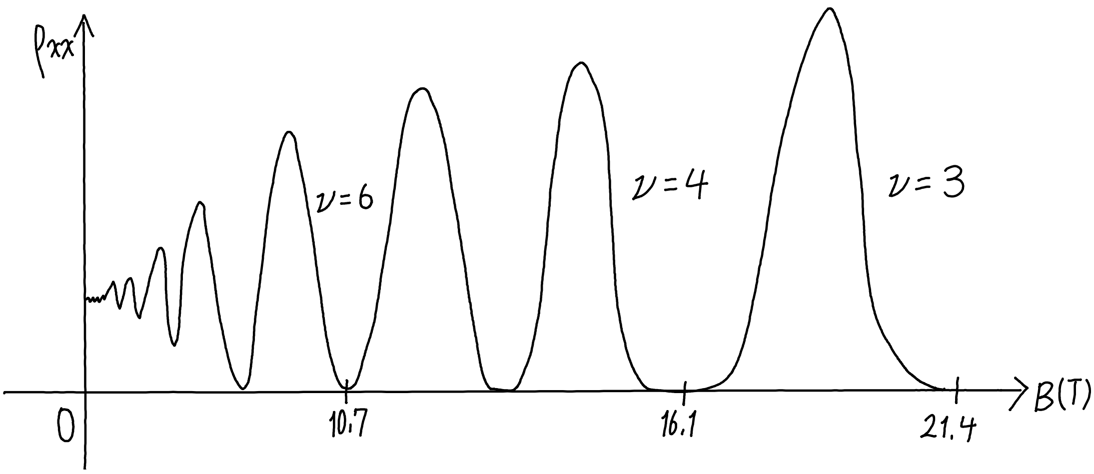
$$\begin{aligned} n_s \nu &= N \\ \frac{g_e e B}{h} &= \frac{N}{\nu} \\ \Rightarrow \Delta\left(\frac{1}{B}\right) &= \frac{g_e e}{hN} \\ \Delta\left(\frac{1}{B}\right) \cdot B &= \frac{1}{\nu} \\ B &= \frac{1}{\nu \cdot \Delta\left(1/B\right)} \end{aligned}$$

From the graph, we have:

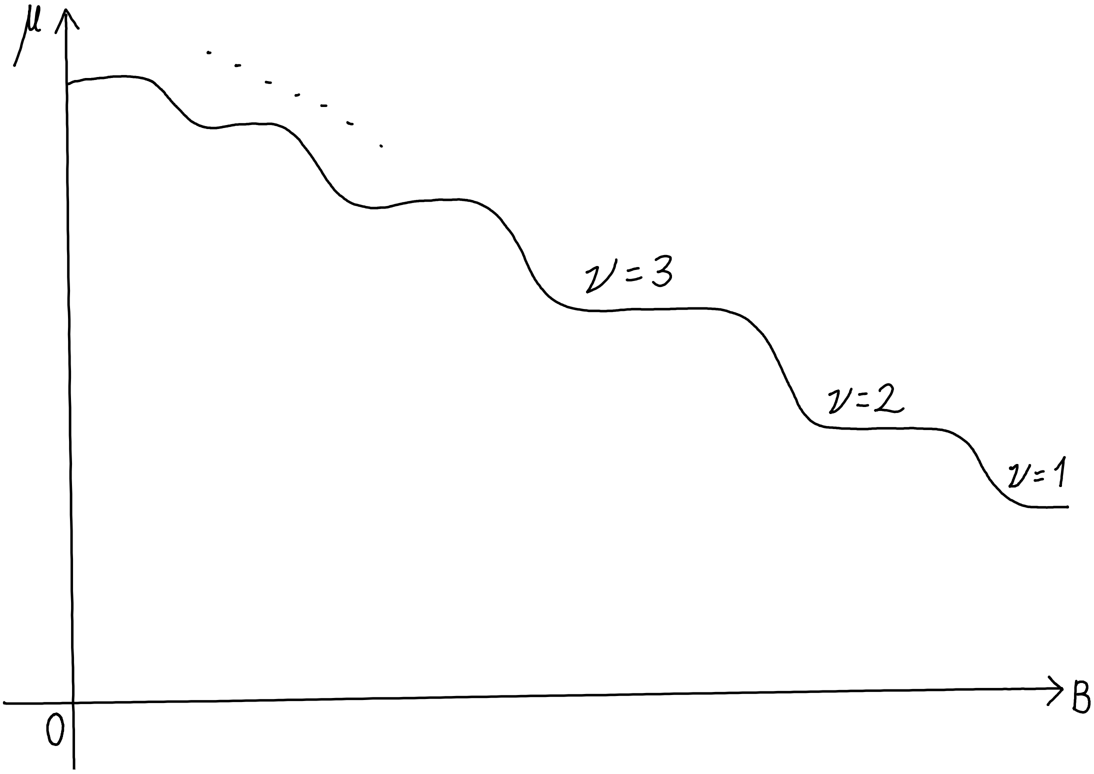
$$\begin{aligned} \Delta\left(\frac{1}{B}\right) &= \frac{1}{18} - \frac{1}{25} \\ &= 0.0156(\mathrm{T}^{-1}) \end{aligned}$$

So for $\nu = 3$, $B = 21.4\,\mathrm{T}$. For $\nu = 4$, $B = 16.1\,\mathrm{T}$. For $\nu = 6$, $B = 10.7\,\mathrm{T}$.

Sketch of ρ_{xx} against B:



iii. Sketch of μ against B:



Ideally μ should be a series of Heaviside step function. However due to disorders, there are broadening near the boundaries of Landau levels, μ decreases as each Landau level is able to hold more states.

iv. From the graph, we have:

$$\begin{aligned}
\Delta\left(\frac{1}{B}\right) &= \frac{g_e e}{hN} = 0.0156\,\mathrm{T}^{-1} \\
\Rightarrow N &= \frac{g_e e}{h\Delta(1/B)} \\
&= 3.11\times 10^{16}\,\mathrm{m}^{-2} \\
&= 3.11\times 10^{12}\,\mathrm{cm}^{-2} = n_{\mathrm{dopant}} \quad \text{assuming all charge carriers come from dopant}
\end{aligned}$$