# Lecture 8
# Camera Models

Professor Silvio Savarese

*Computational Vision and Geometry Lab*

# Lecture 8

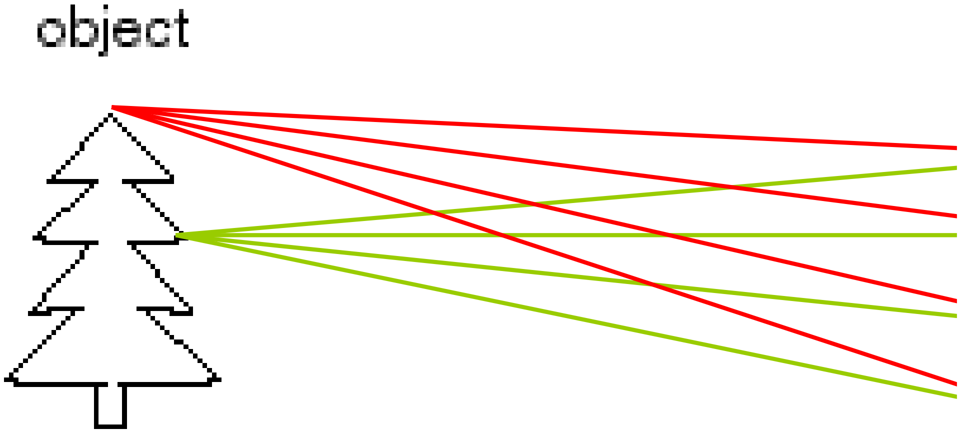
# Camera Models

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Reading: [FP] Chapter 1 “Cameras”
[FP] Chapter 2 “Geometric Camera Models”
[HZ] Chapter 6 “Camera Models”

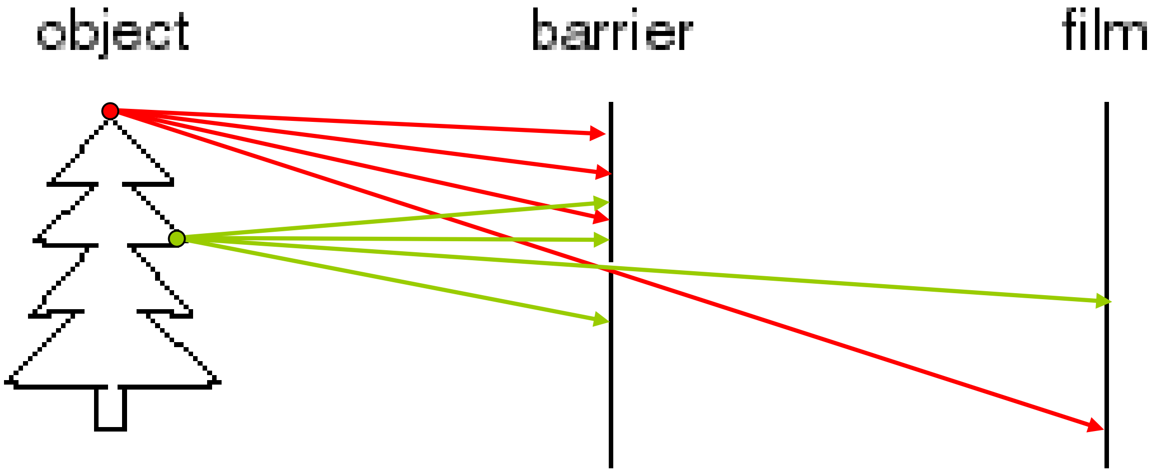
Some slides in this lecture are courtesy to Profs. J. Ponce, S. Seitz, F-F Li

# How do we see the world?




- Let’s design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

# Pinhole camera

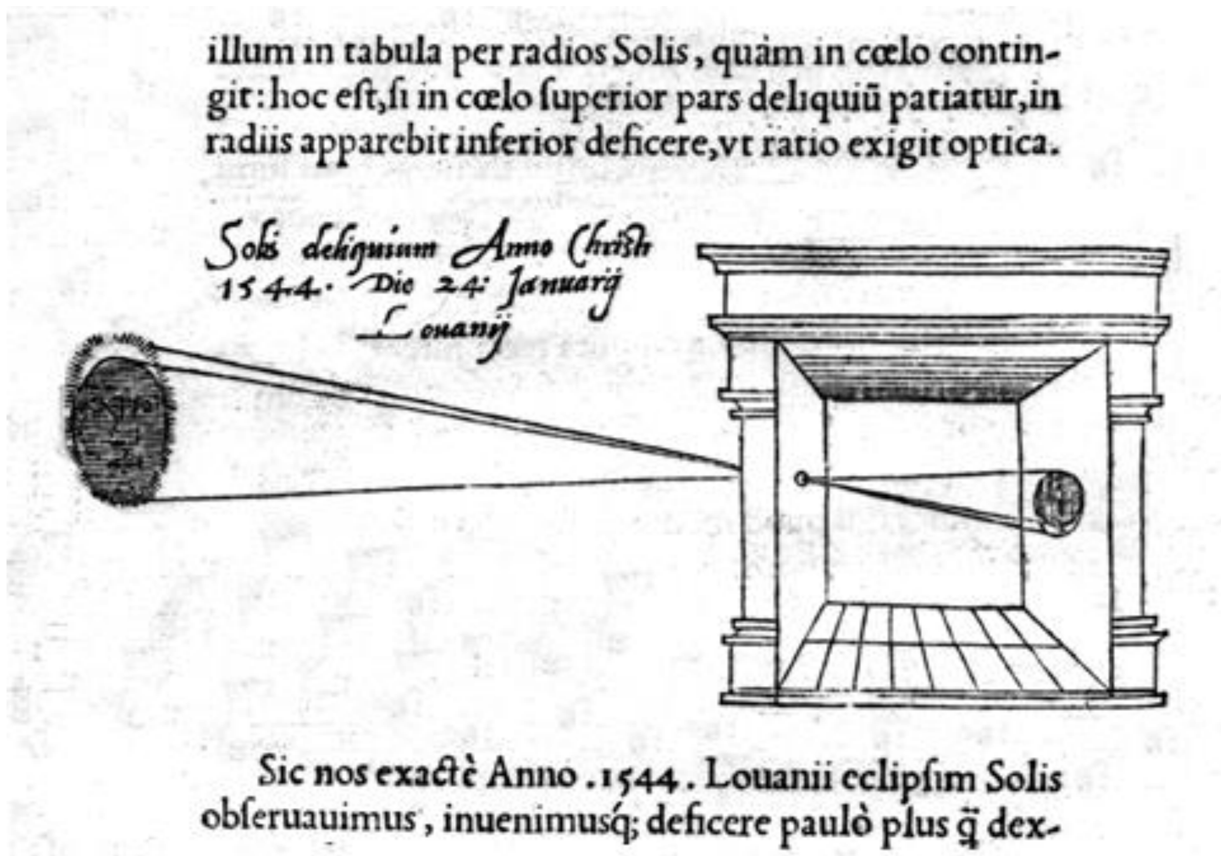



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

# Some history…

Milestones:

• Leonardo da Vinci (1452-1519):
first record of camera *obscura*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git: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in
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# Some history…

Milestones:

- Leonardo da Vinci (1452-1519): first record of camera *obscura*
- Johann Zahn (1685): first portable camera

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• Leonardo da Vinci (1452-1519): first record of camera *obscura*

• Johann Zahn (1685): first portable camera

• Joseph Nicephore Niepce (1822): first photo - birth of photography

Photography (Niepce, “La Table Servie,” 1822)

# Some history…

Milestones:
• Leonardo da Vinci (1452-1519): first record of camera *obscura*
• Johann Zahn (1685): first portable camera
• Joseph Nicephore Niepce (1822): first photo - birth of photography

• Daguerréotypes (1839)
• Photographic Film (Eastman, 1889)
• Cinema (Lumière Brothers, 1895)
• Color Photography (Lumière Brothers, 1908)

Photography (Niepce, “La Table Servie,” 1822)

# Let's also not forget…

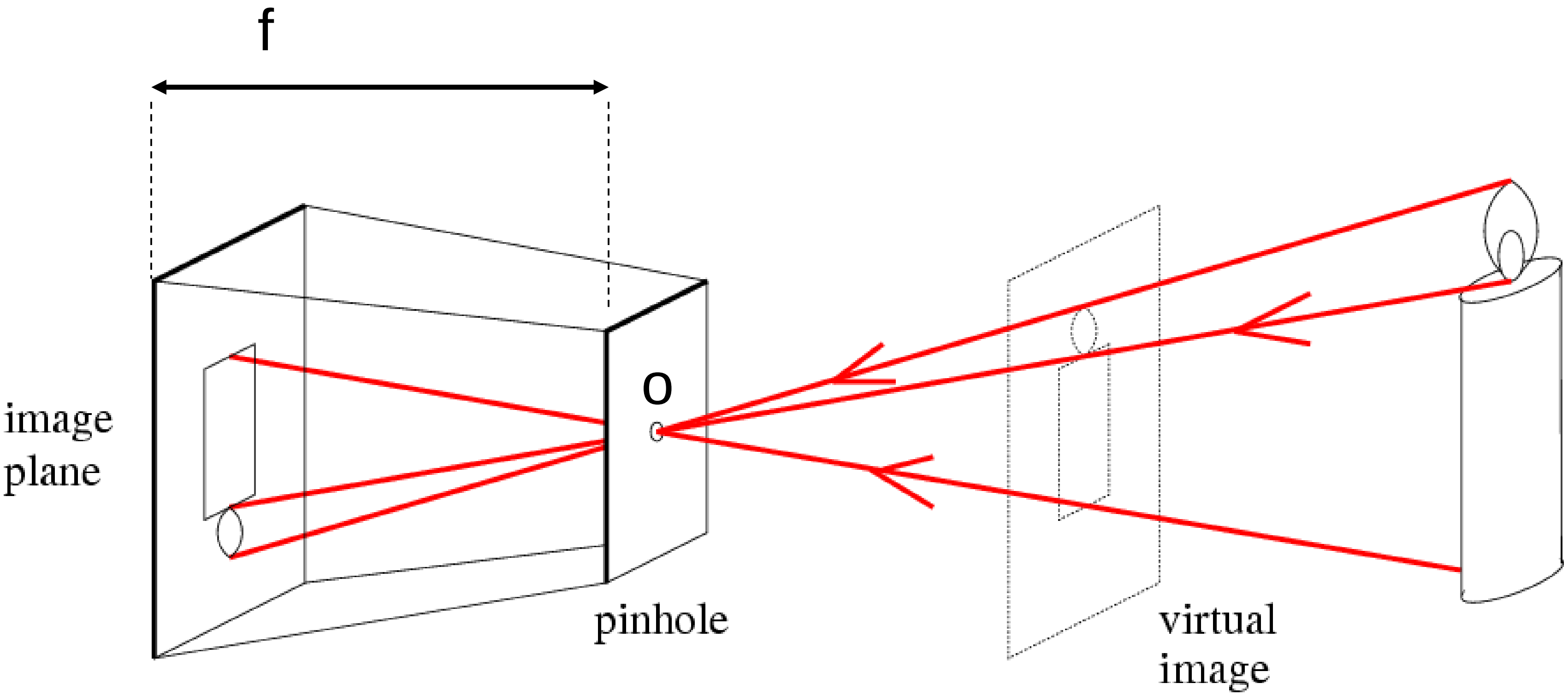
Motzu
(468-376 BC)

Oldest existent
book on geometry
in China

Aristotle
(384-322 BC)
Also: Plato, Euclid

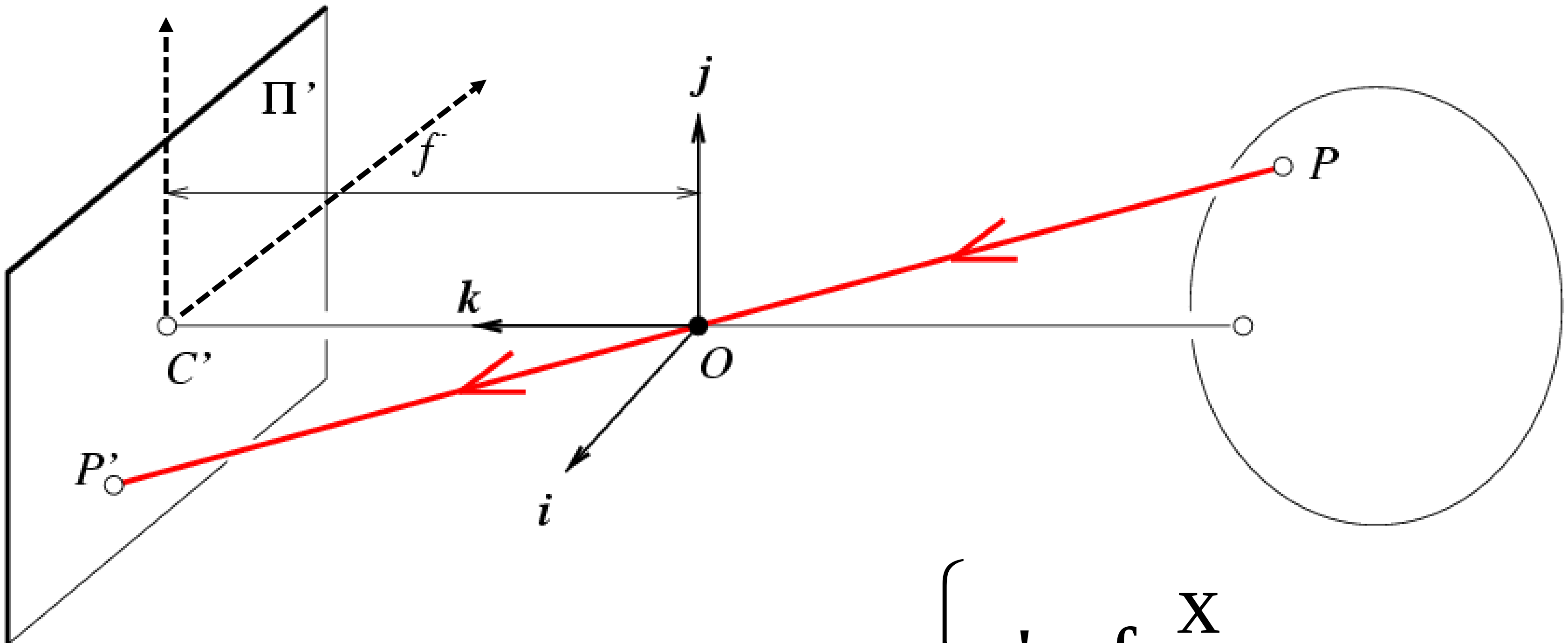
Al-Kindi (c. 801–873)
Ibn al-Haitham
(965-1040)

# Pinhole camera




f = focal length
o = aperture = pinhole = center of the camera

# Pinhole camera

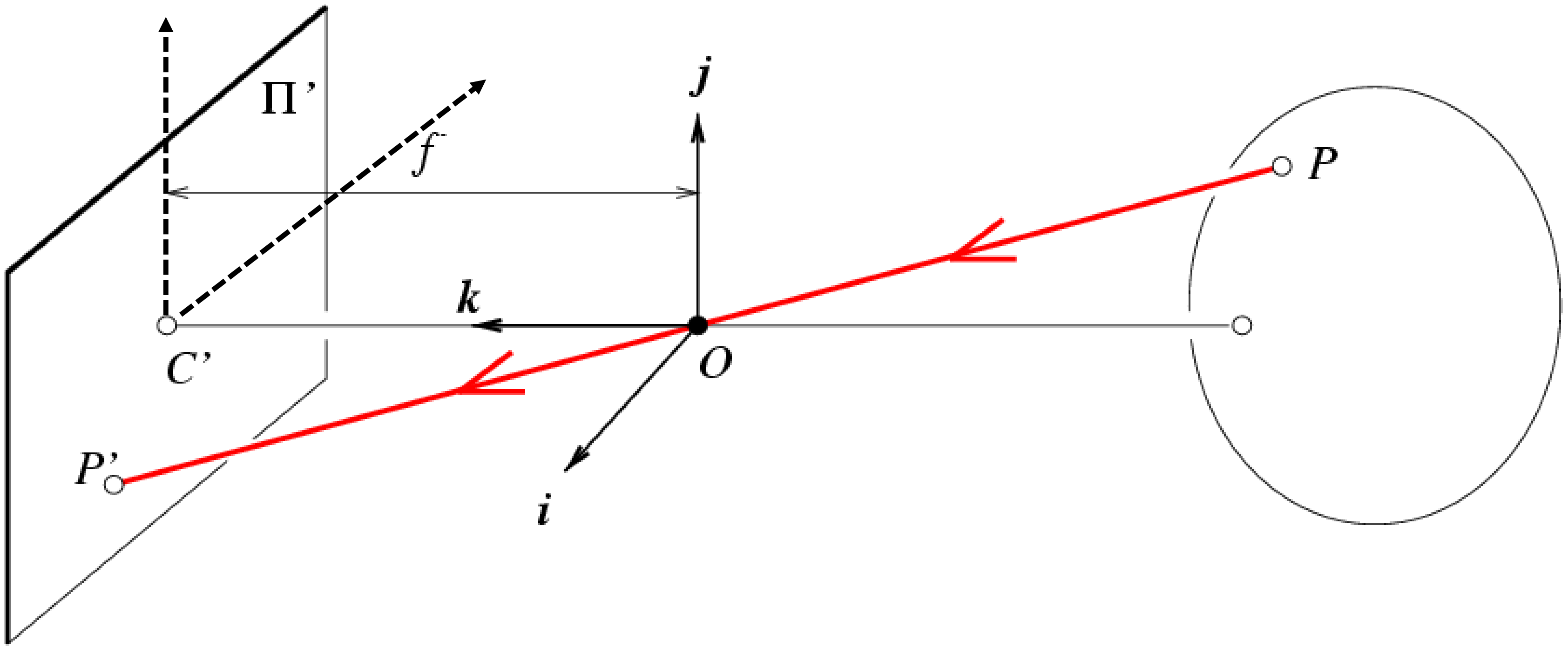


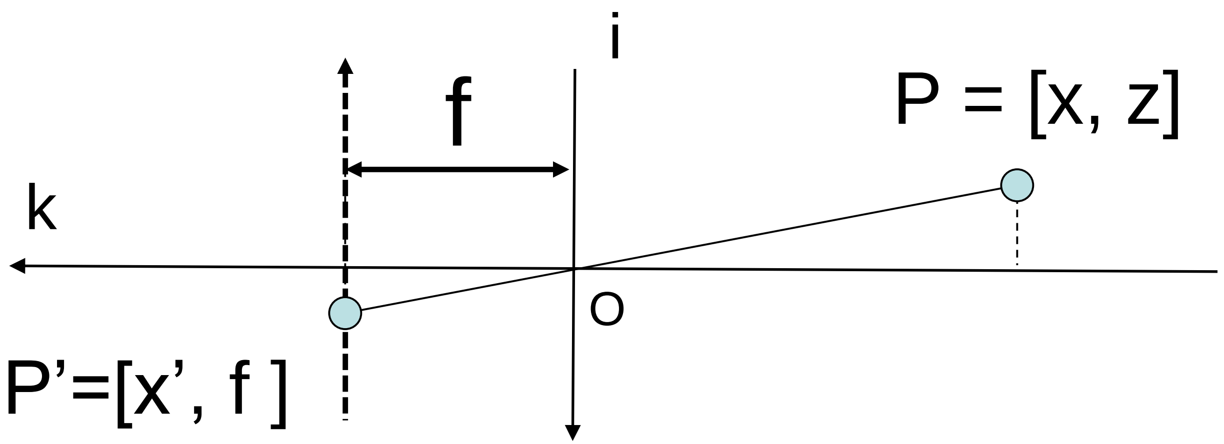

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

Derived using similar triangles

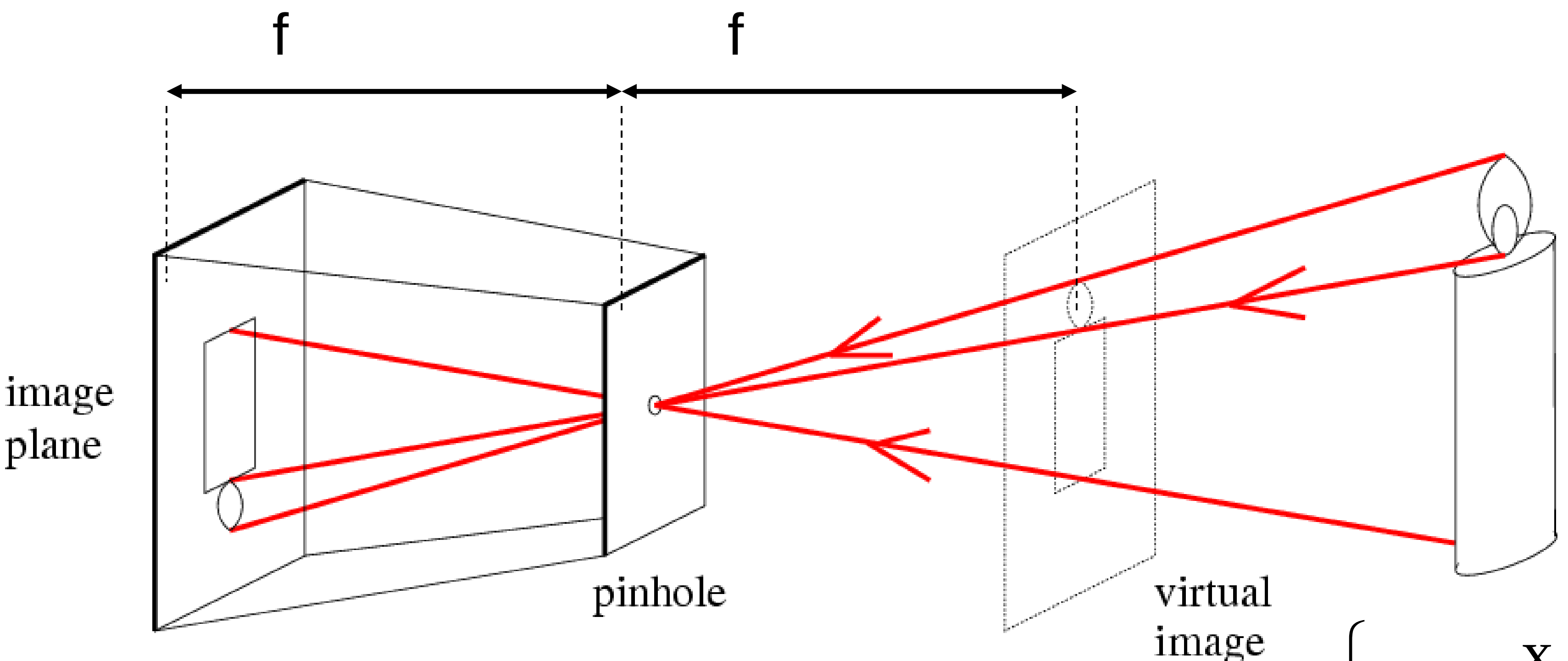
# Pinhole camera




$$\frac{x'}{f} = \frac{x}{z}$$

# Pinhole camera



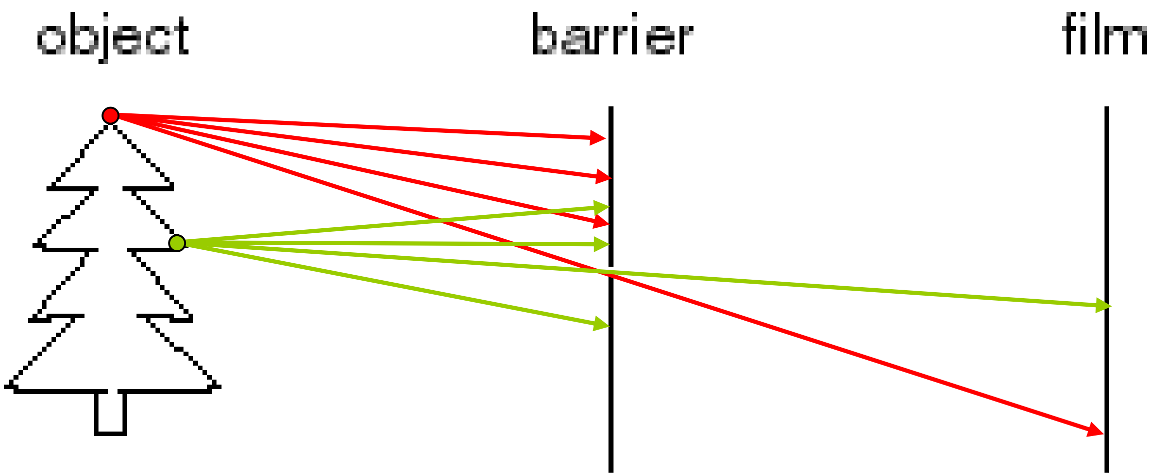

$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

Common to draw image plane *in front* of the focal point.

What's the transformation between these 2 planes?

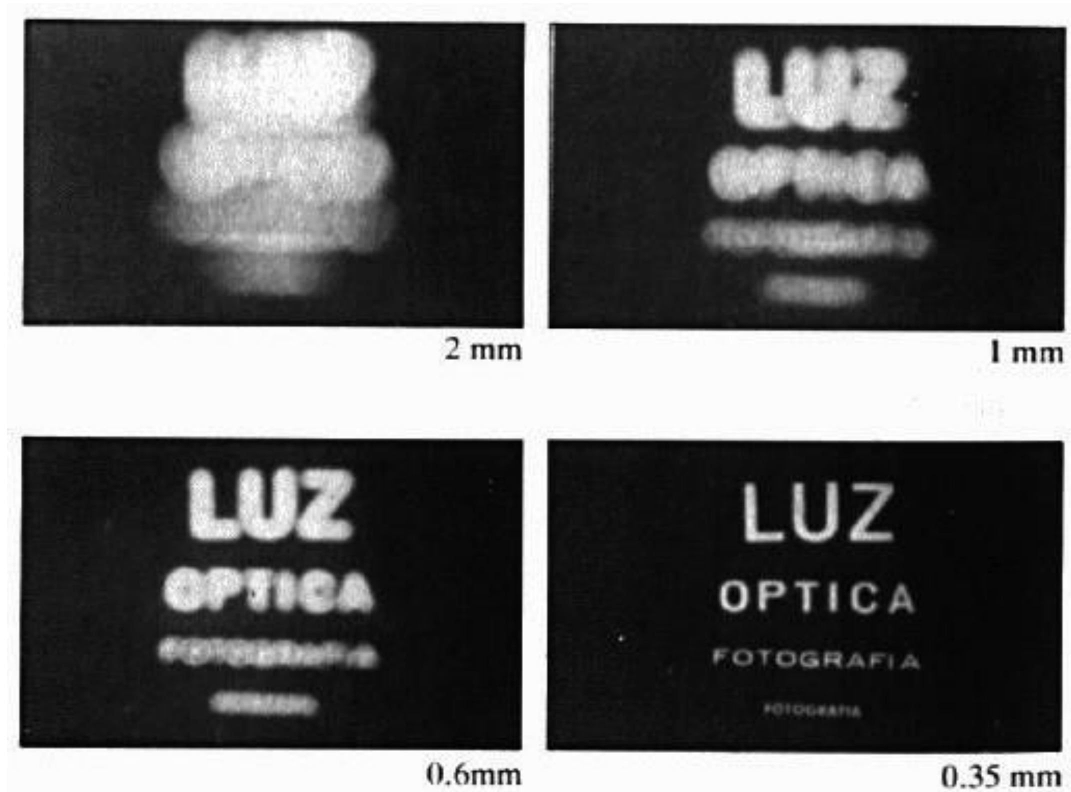
# Pinhole camera

Is the size of the aperture important?




Shrinking aperture size

- Rays are mixed up

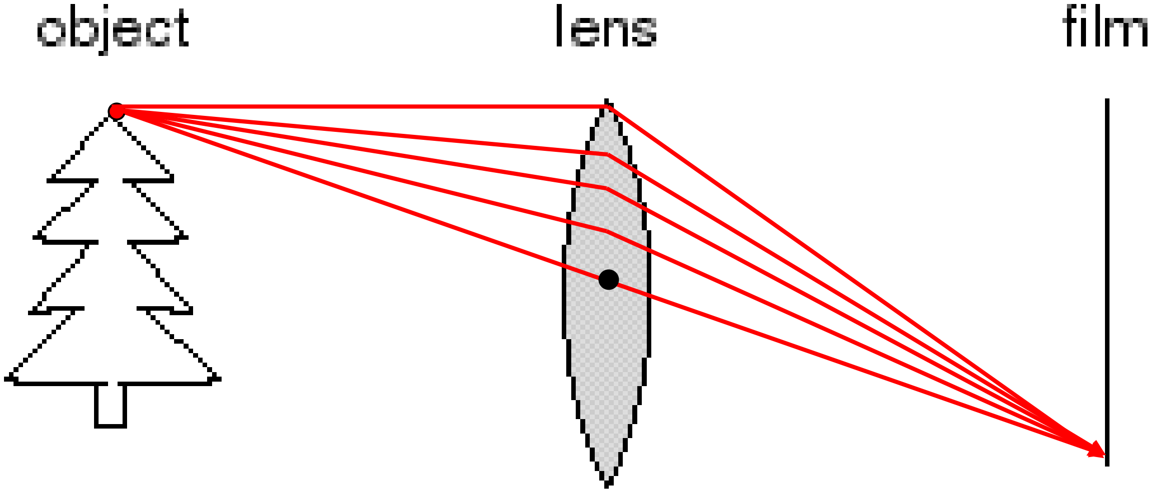



-Why the aperture cannot be too small?

-Less light passes through
-Diffraction effect

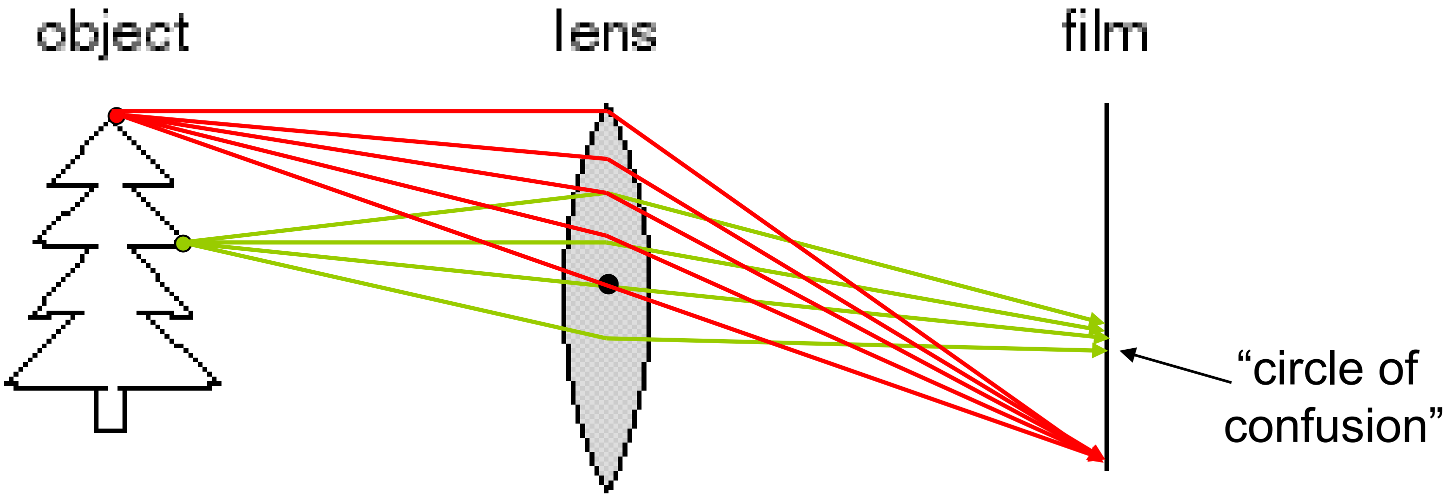
Adding lenses!

# Cameras & Lenses




- A lens focuses light onto the film

# Cameras & Lenses




- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
  - Related to the concept of depth of field

# Cameras & Lenses

- A lens focuses light onto the film
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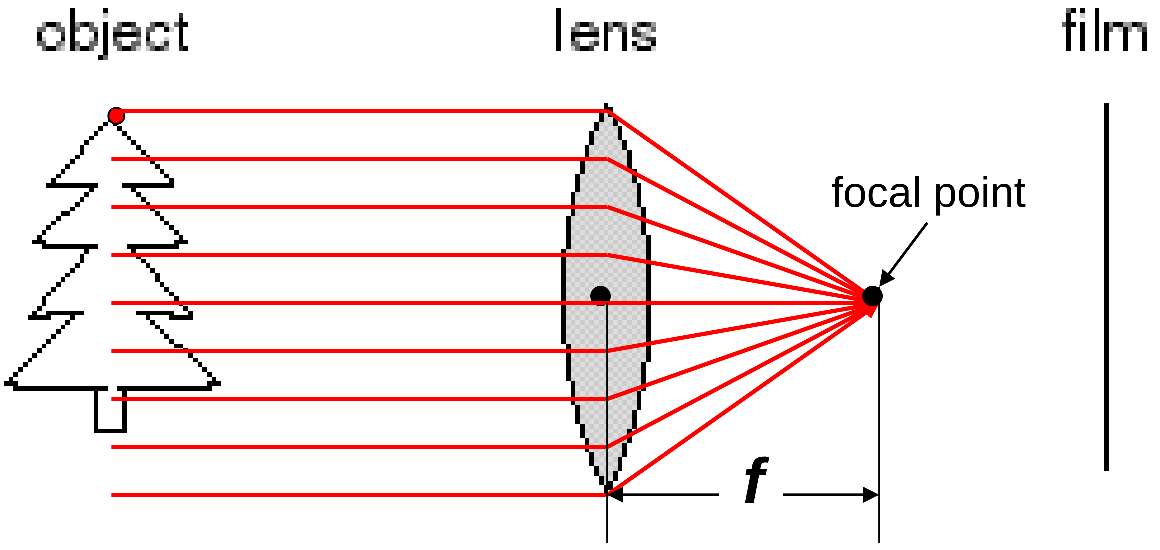
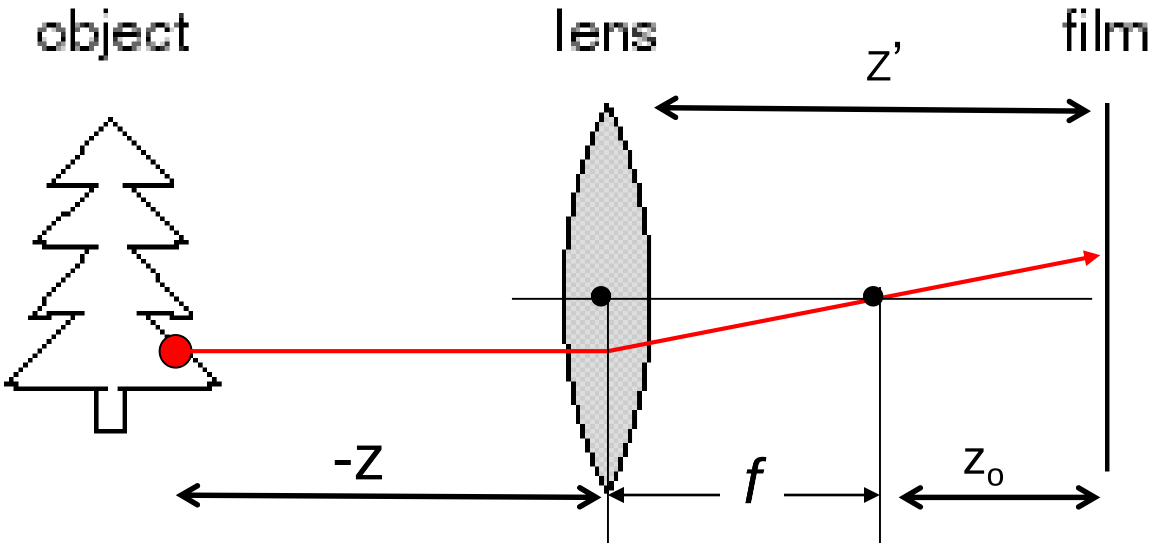
# Cameras & Lenses




- A lens focuses light onto the film
  - All parallel rays converge to one point on a plane located at the *focal length f*
  - Rays passing through the center are not deviated

# Cameras & Lenses

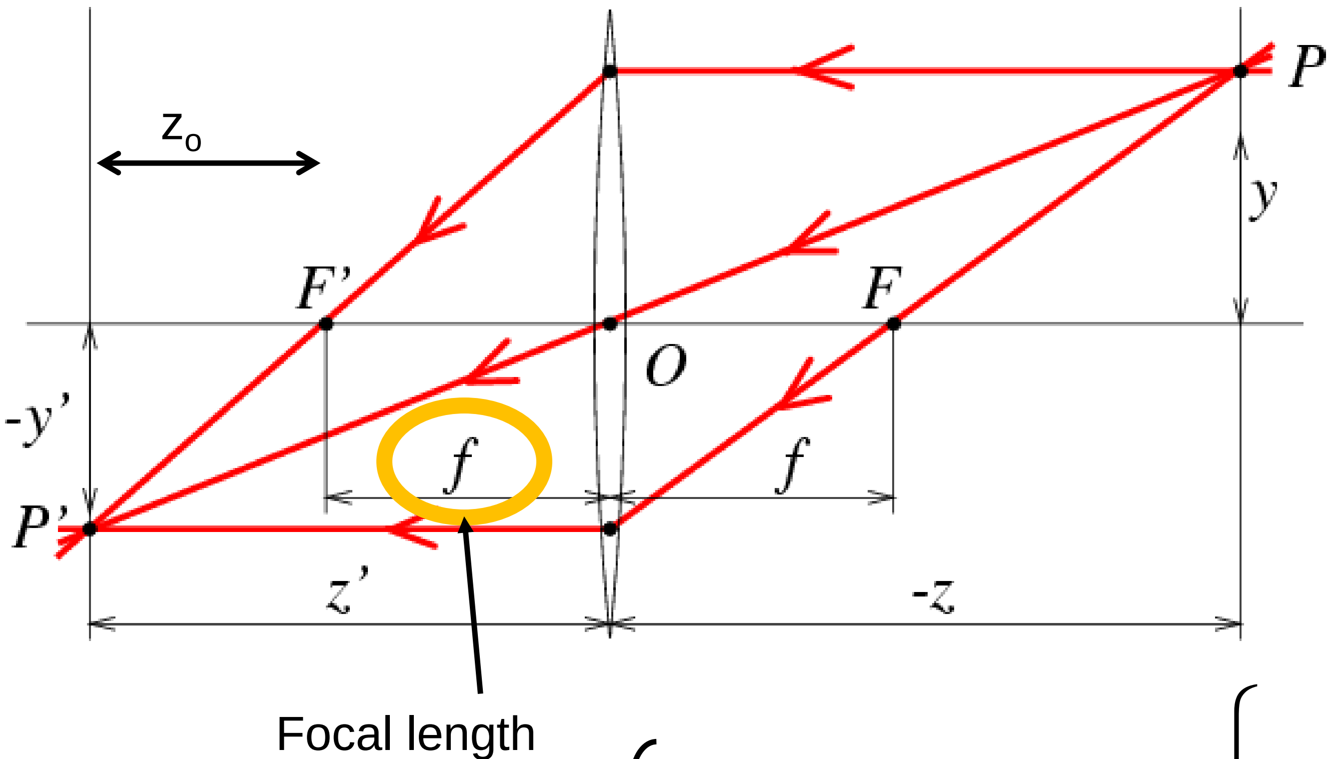



From Snell's law:

$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases}$$

$$\boxed{\begin{array}{c} z' = f + z_o \\ f = \dfrac{R}{2(n-1)} \end{array}}$$

# Thin Lenses




$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

Snell's law:

$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

Small angles:

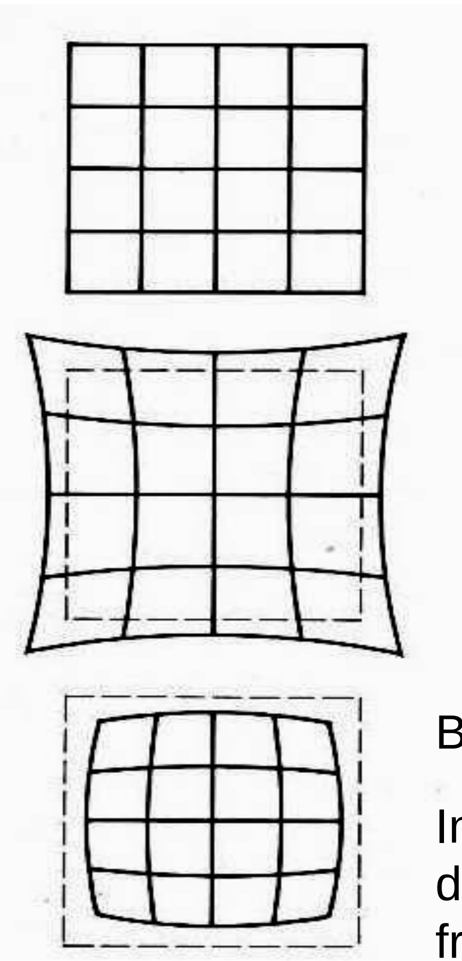
$n_1\,\alpha_1 \approx n_2\,\alpha_2$

$n_1 = \text{n}$ (lens)
$n_1 = 1$ (air)

$$\begin{cases} x' = z'\dfrac{x}{z} \\ y' = z'\dfrac{y}{z} \end{cases}$$

# Issues with lenses: Radial Distortion

– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion

Pin cushion

Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis

# Lecture 2

# Camera Models

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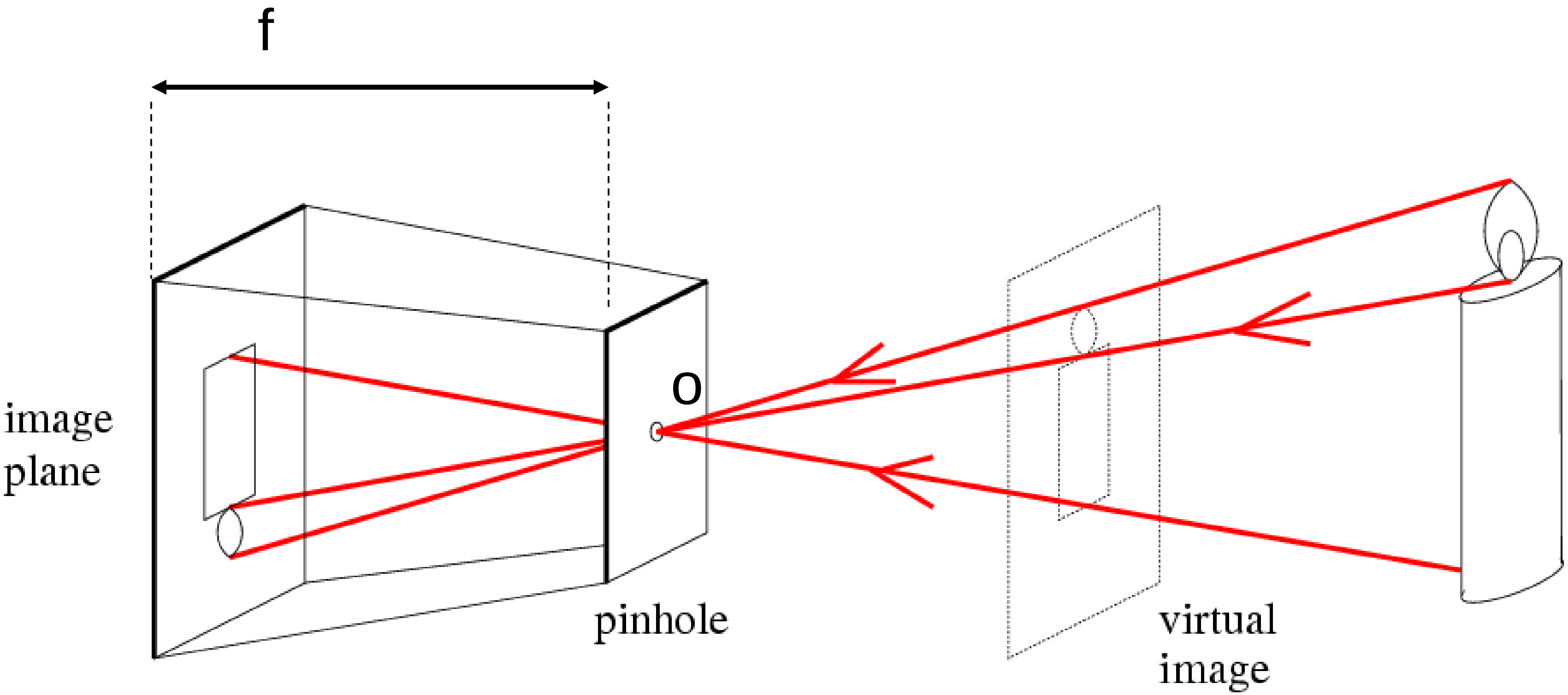
- The geometry of pinhole cameras
    - Intrinsic
    - Extrinsic
- Other camera models

# Pinhole camera

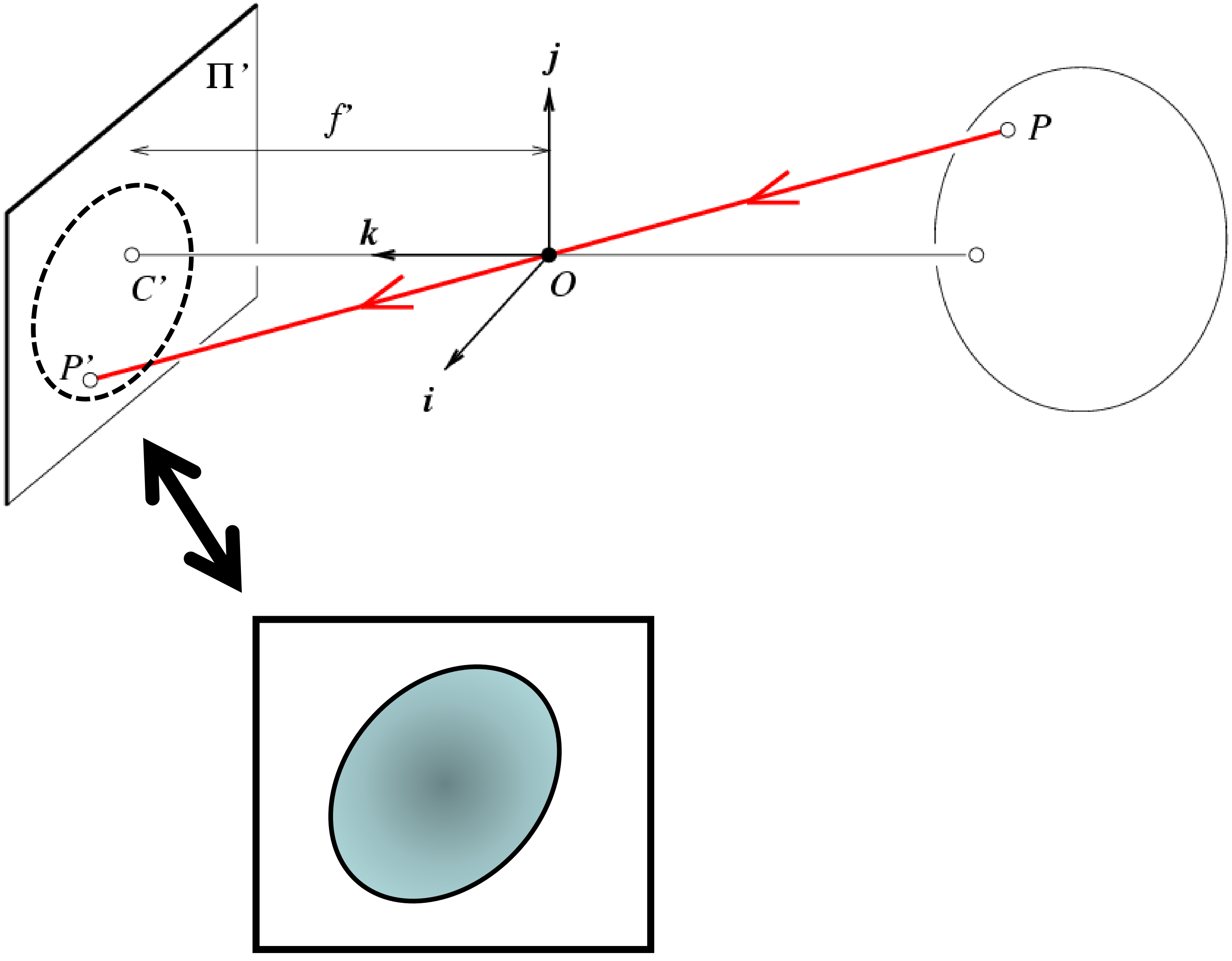



f = focal length
o = center of the camera

$$(x, y, z) \rightarrow (f\,\frac{x}{z}, f\,\frac{y}{z})$$

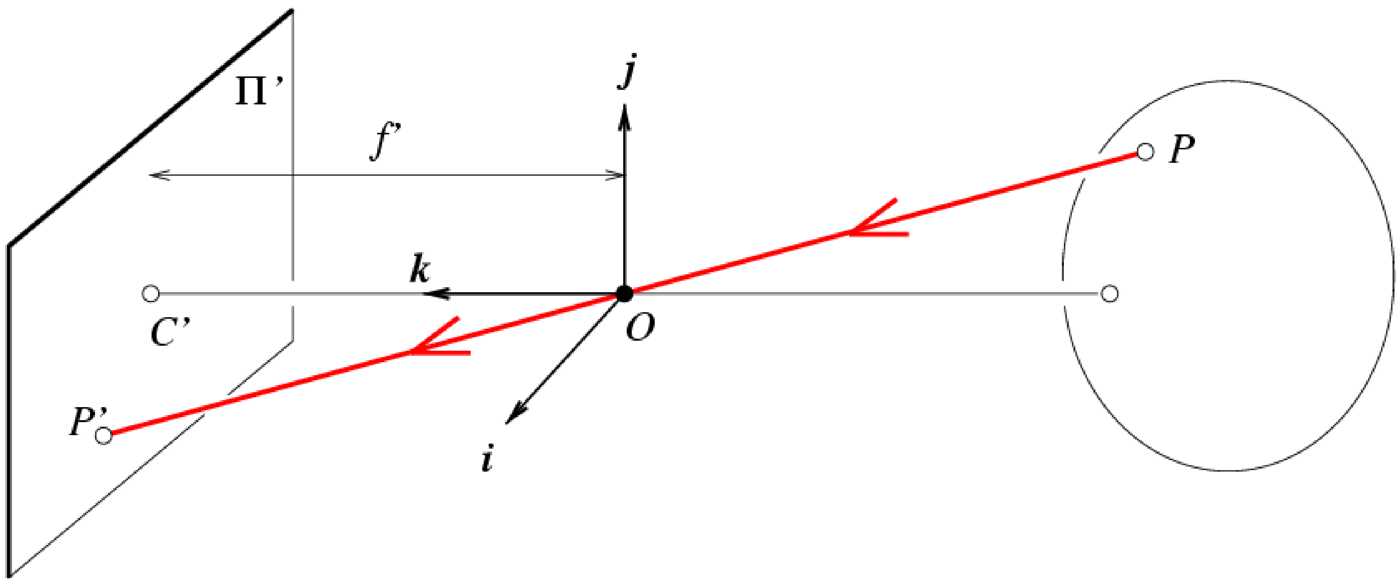
$$\Re^3 \xrightarrow{E} \Re^2$$

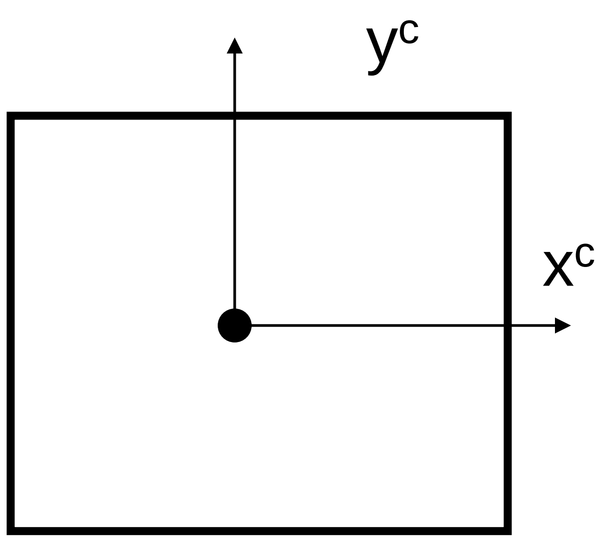
# From retina plane to images



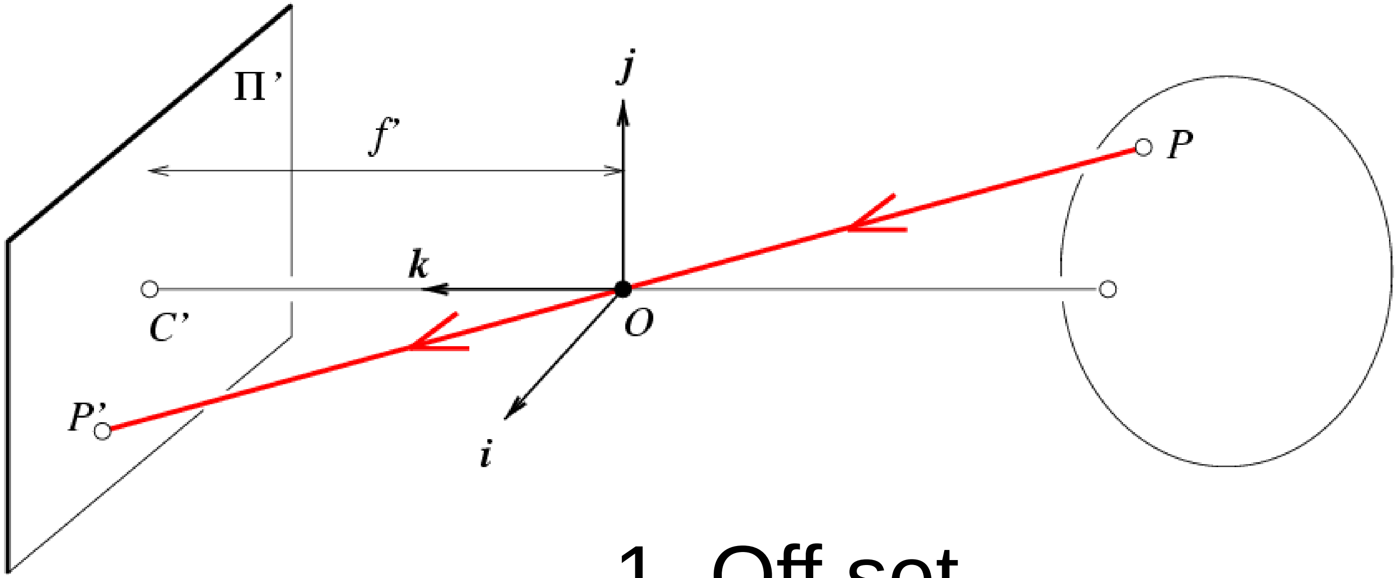

Pixels, bottom-left coordinate systems

# Coordinate systems

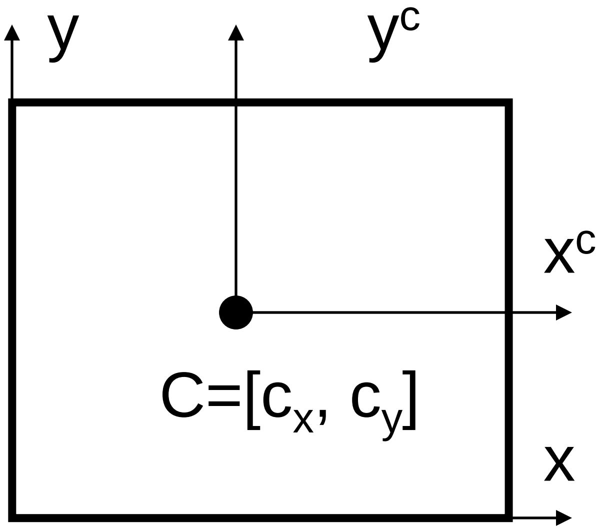


# Converting to pixels

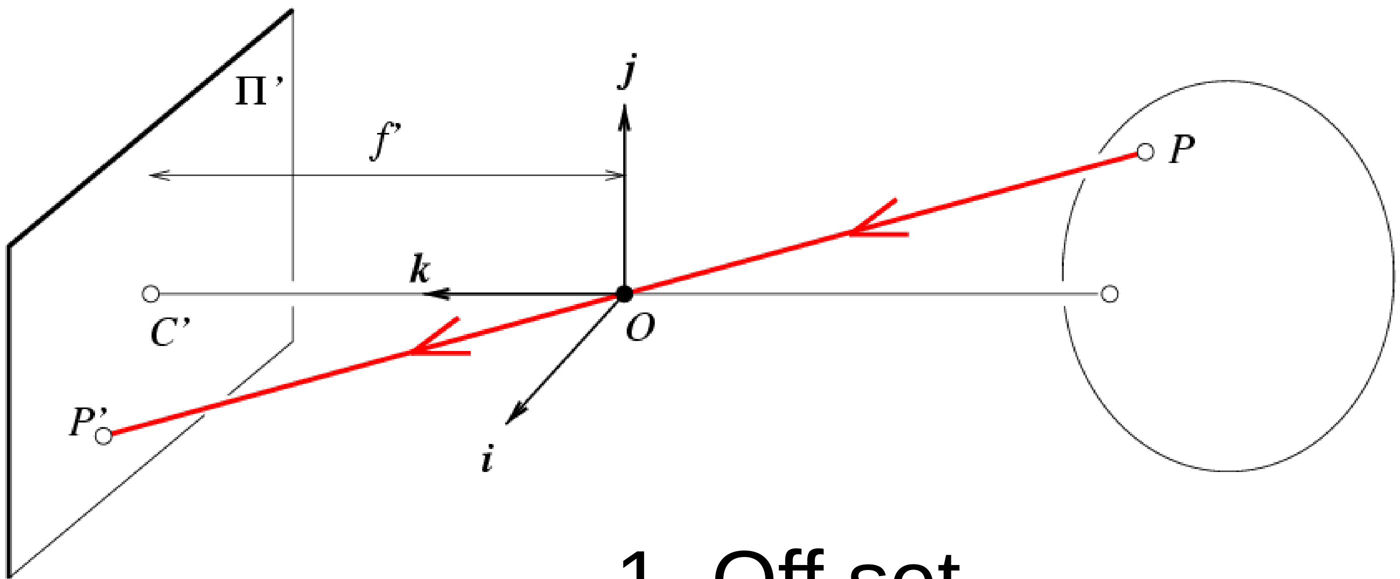


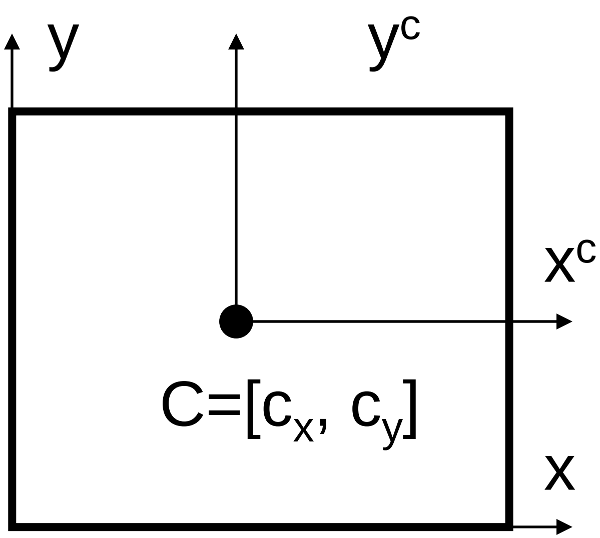

1. Off set




$$(x, y, z) \rightarrow (f\frac{x}{z} + c_x, f\frac{y}{z} + c_y)$$

# Converting to pixels




1. Off set
2. From metric to pixels

$$(x, y, z) \rightarrow (\underbrace{\boxed{f\ k}}_{\alpha}\frac{x}{z}+c_x, \underbrace{\boxed{f\ l}}_{\beta}\frac{y}{z}+c_y)$$

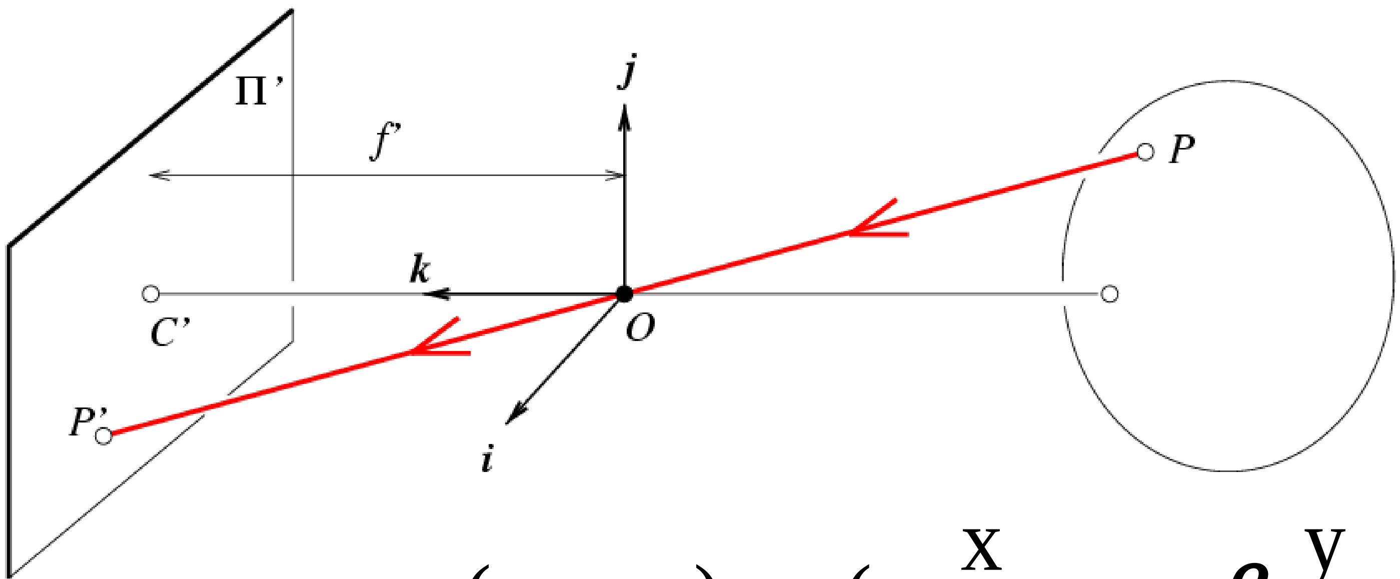
Units: k,l : pixel/m

f : m

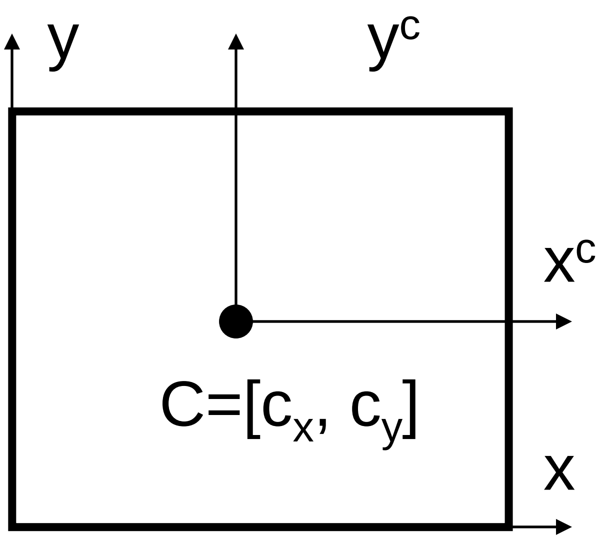
Non-square pixels

$\alpha$, $\beta$ : pixel

# Converting to pixels




$$(x, y, z) \rightarrow (\boldsymbol{\alpha} \frac{x}{z} + c_x, \boldsymbol{\beta} \frac{y}{z} + c_y)$$




- Matrix form?

A related question:

- Is this a linear transformation?

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Is this a linear transformation?

No — division by z is nonlinear

How to make it linear?

# Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

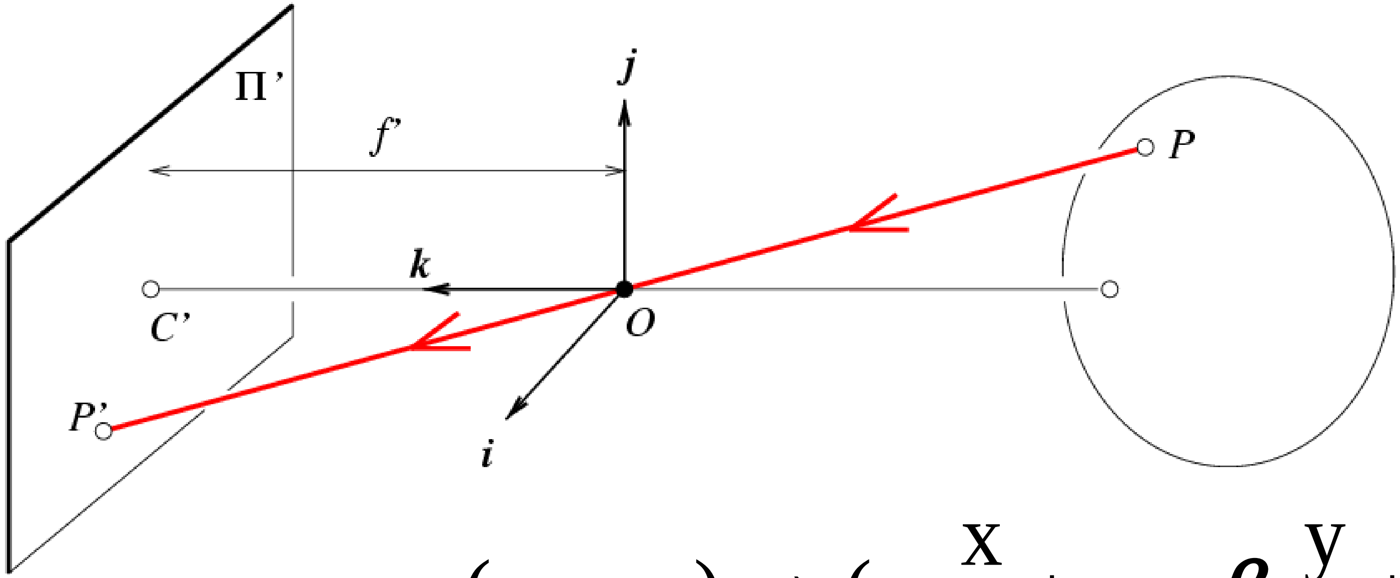
homogeneous scene coordinates

- Converting *from* homogeneous coordinates

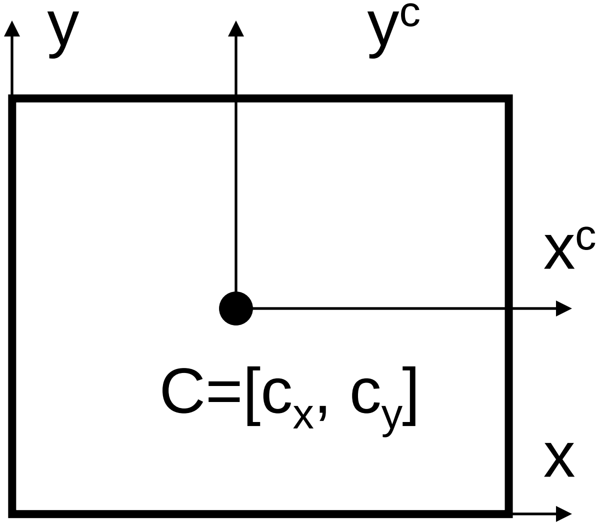
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

# Camera Matrix




$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$




$$X' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Perspective Projection Transformation

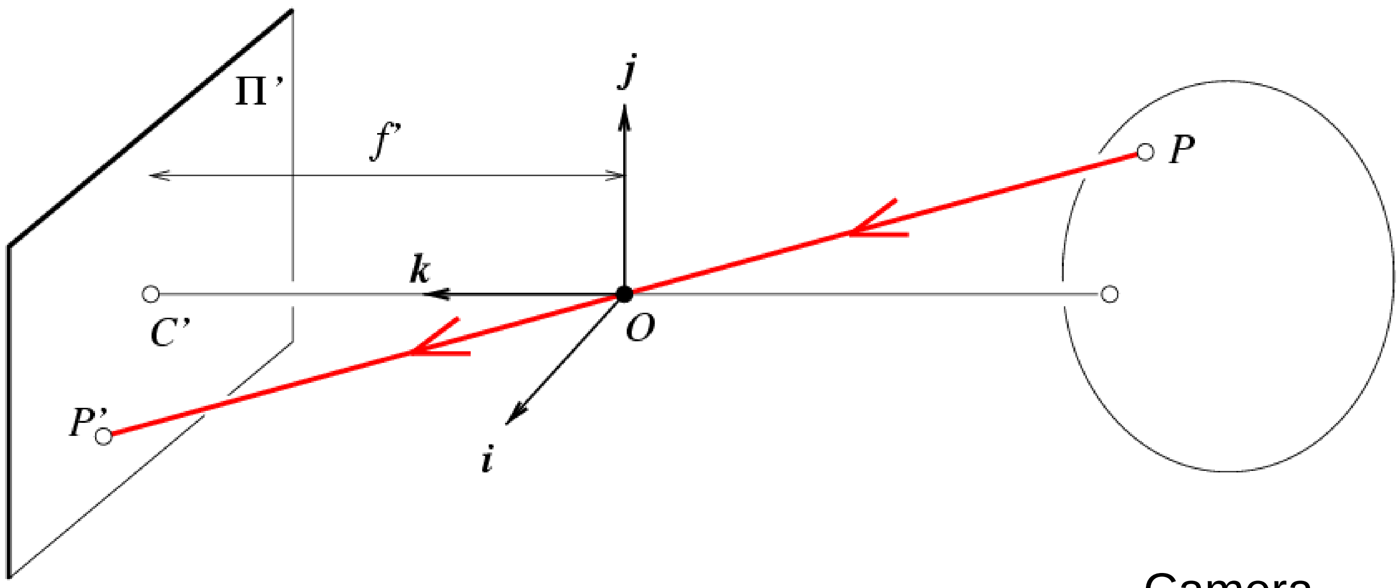
$$X' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$X' = M\,X$$

$$\Re^4 \xrightarrow{H} \Re^3$$

$$X_i' = \begin{bmatrix} f\,\dfrac{x}{z} \\ f\,\dfrac{y}{z} \end{bmatrix}$$

# Camera Matrix



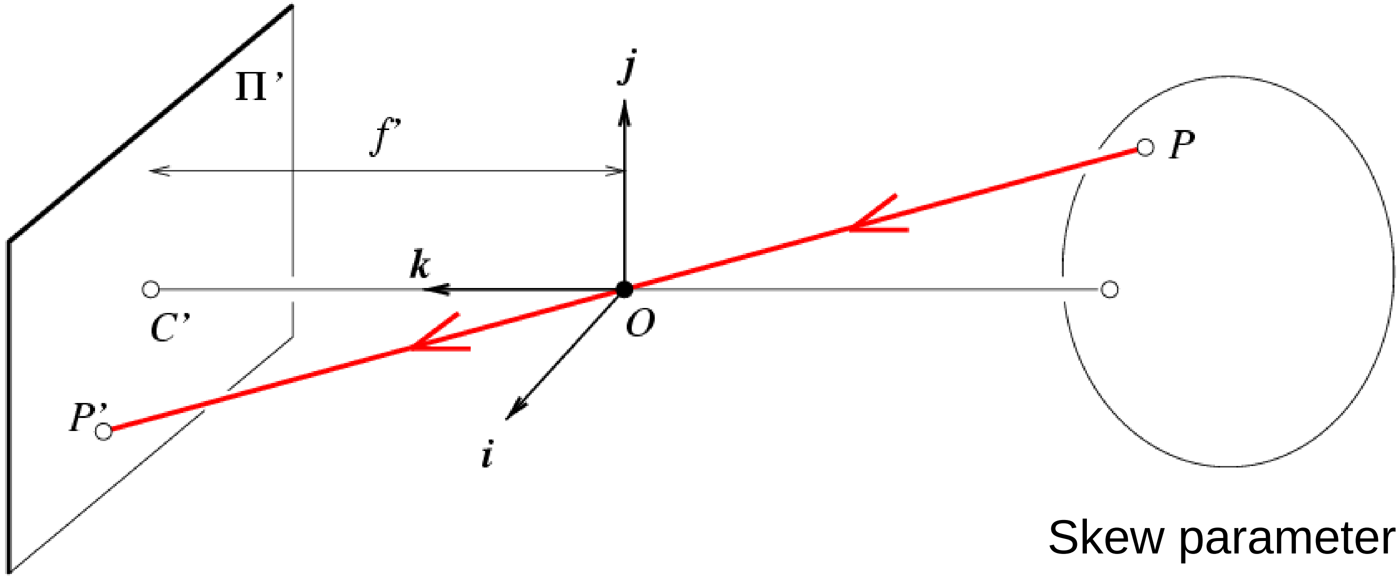

Camera matrix K

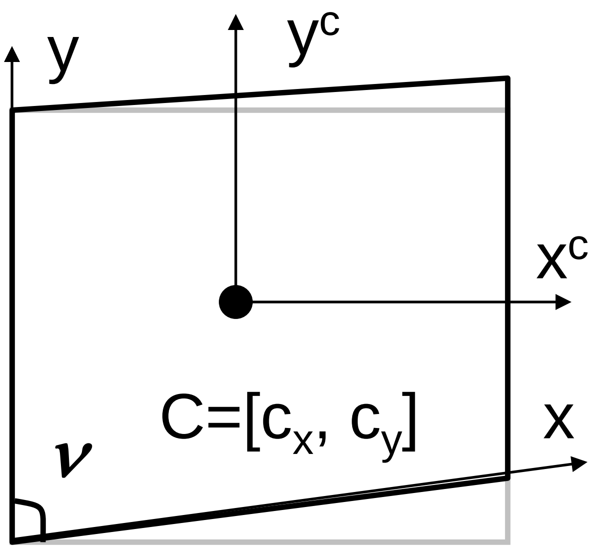
$$X' = M\,X$$

$$= K\begin{bmatrix} I & 0 \end{bmatrix} X$$

$$X' = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Finite projective cameras




Skew parameter

$$X' = \begin{bmatrix} \alpha & \boxed{s} & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

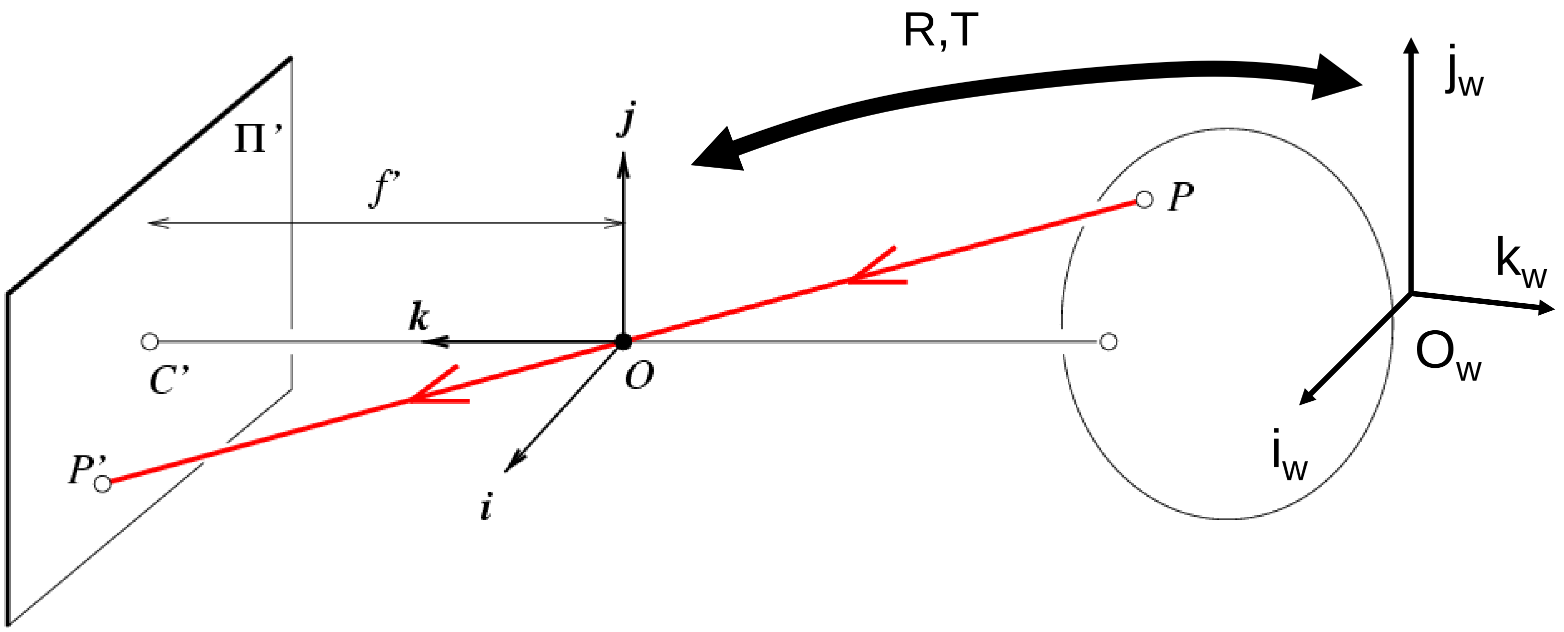
K has 5 degrees of freedom!

# Lecture 2

# Camera Models

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
  - Intrinsic
  - Extrinsic
- Other camera models

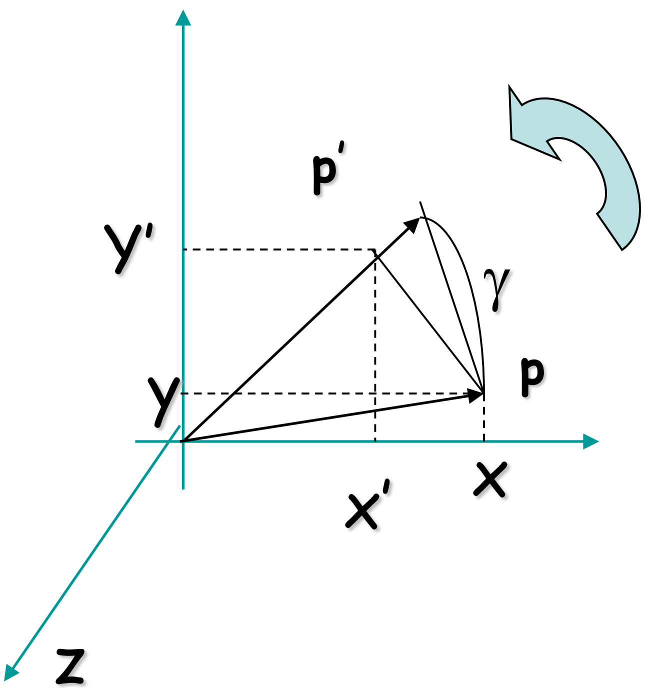
# World reference system




- The mapping so far is defined within the camera reference system
- What if an object is represented in the world reference system

# 3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

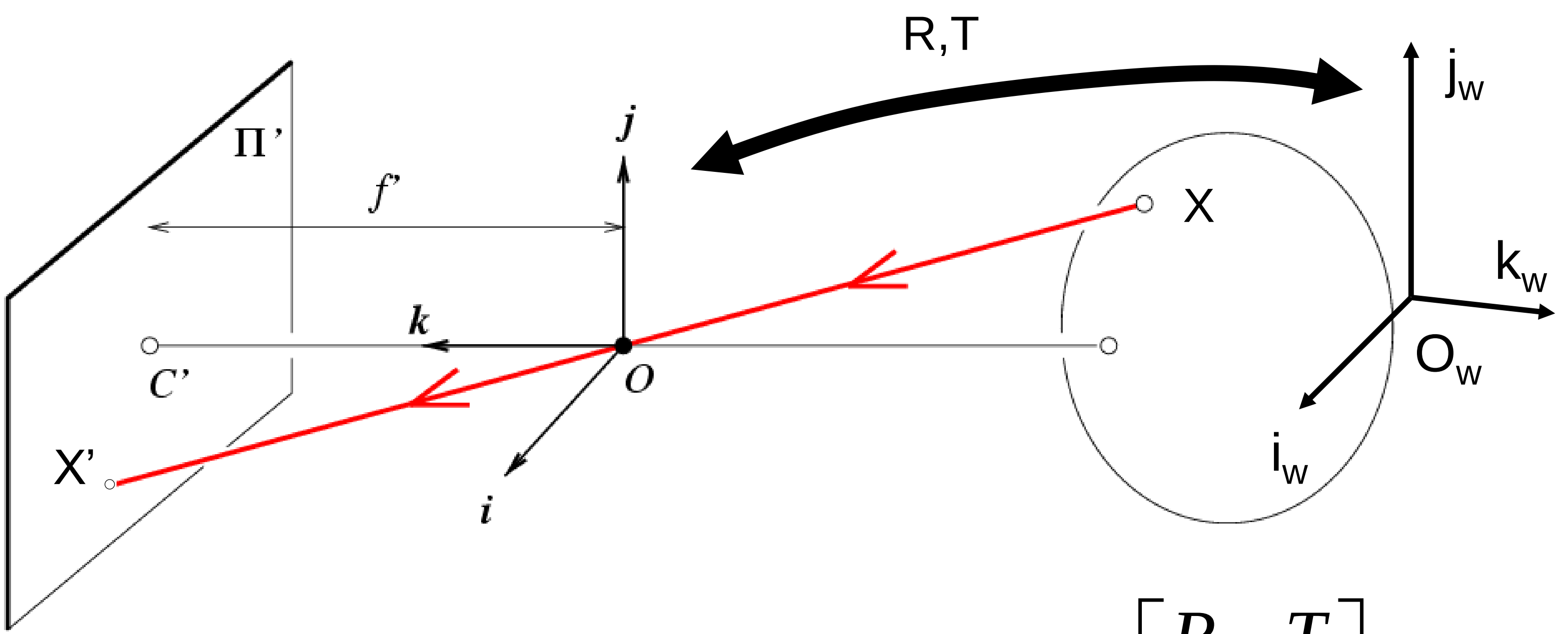



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# World reference system



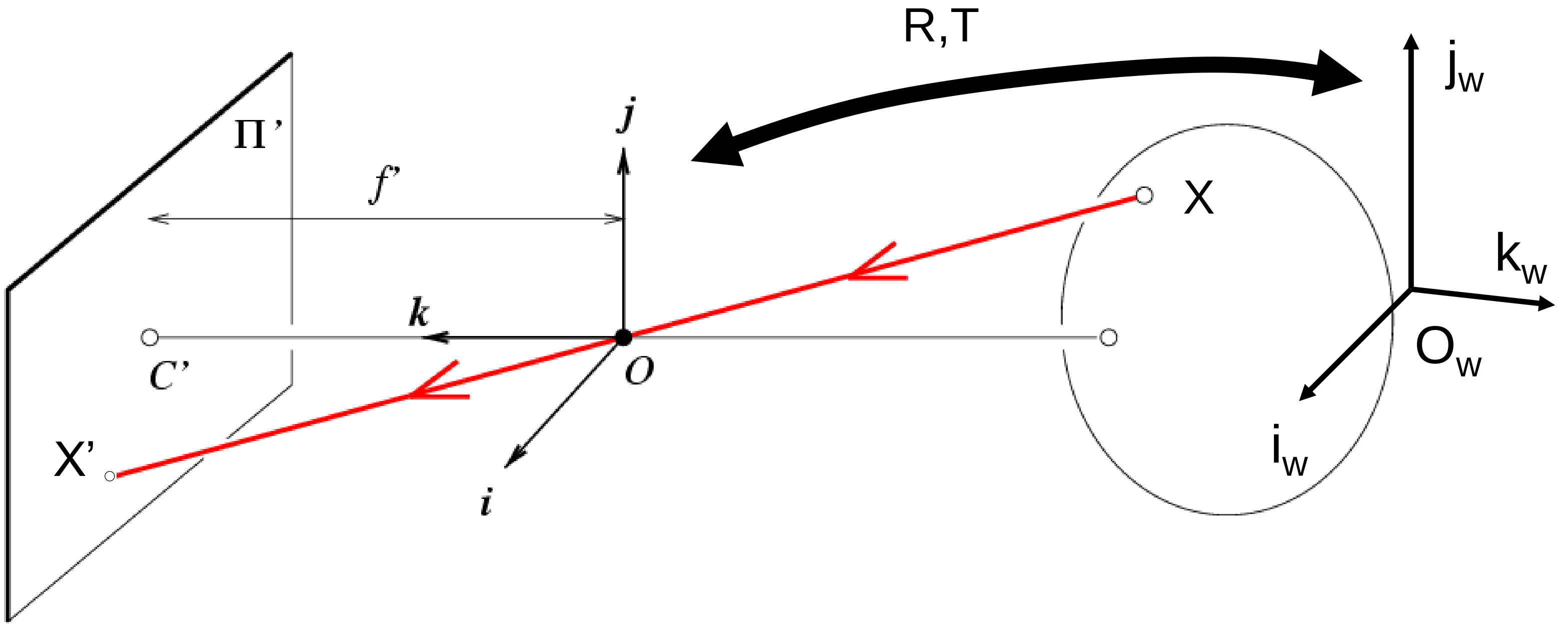

In 4D homogeneous coordinates: $X = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times 4} X_w$

Internal parameters

External parameters

$$X' = K[I \quad 0]\,X = K[I \quad 0]\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times 4} X_w = \underbrace{\boxed{K[R \quad T]}}_{M} X_w$$

# Projective cameras




$$X'_{3\times1} = M_{3x4}\ X_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} X_{w\,4\times1}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

5 + 3 + 3 =11!

# Camera calibration

More details in CS231A

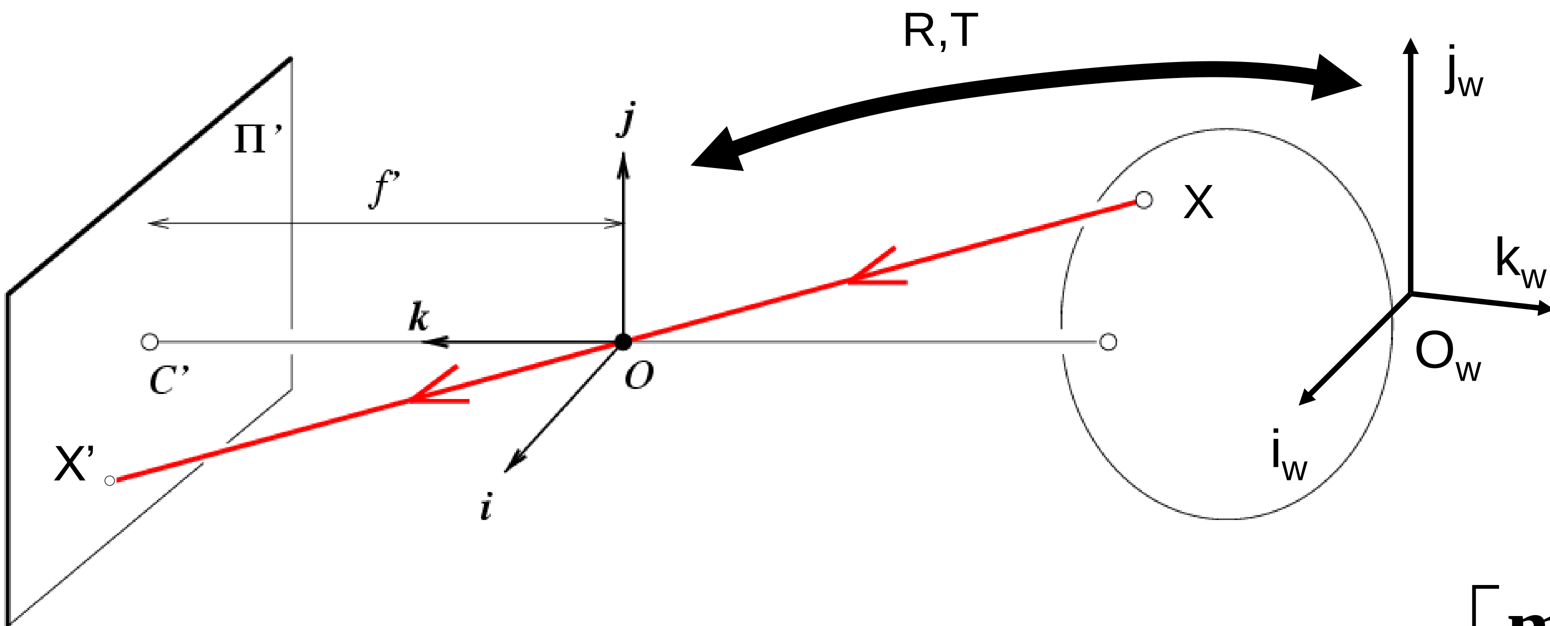
Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$X'_{3\times1} = M_{3x4}\ X_w = \mathrm{K}_{3\times3}\begin{bmatrix}\mathrm{R} & \mathrm{T}\end{bmatrix}_{3\times4}\ \mathrm{X}_{\mathrm{w}\,4\times1} \qquad \mathrm{K} = \begin{bmatrix} \alpha & \mathrm{s} & \mathrm{c_x} \\ 0 & \beta & \mathrm{c_y} \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

5 + 3 + 3 =11!

# Projective cameras




$$X'_{3\times1} = \mathrm{M}\ \mathrm{X_w} = \mathrm{K}_{3\times3}\begin{bmatrix}\mathrm{R} & \mathrm{T}\end{bmatrix}_{3\times4}\ \mathrm{X}_{\mathrm{w}\,4\times1} \qquad M = \begin{bmatrix}\mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3\end{bmatrix} X_{\mathbf{w}} = \begin{bmatrix}\mathbf{m}_1 X_{\mathbf{w}} \\ \mathbf{m}_2 X_{\mathbf{w}} \\ \mathbf{m}_3 X_{\mathbf{w}}\end{bmatrix} \quad \xrightarrow{\mathbb{E}} \left(\frac{\mathbf{m}_1 X_w}{\mathbf{m}_3 X_w}, \frac{\mathbf{m}_2 X_w}{\mathbf{m}_3 X_w}\right)$$

# Properties of Projection

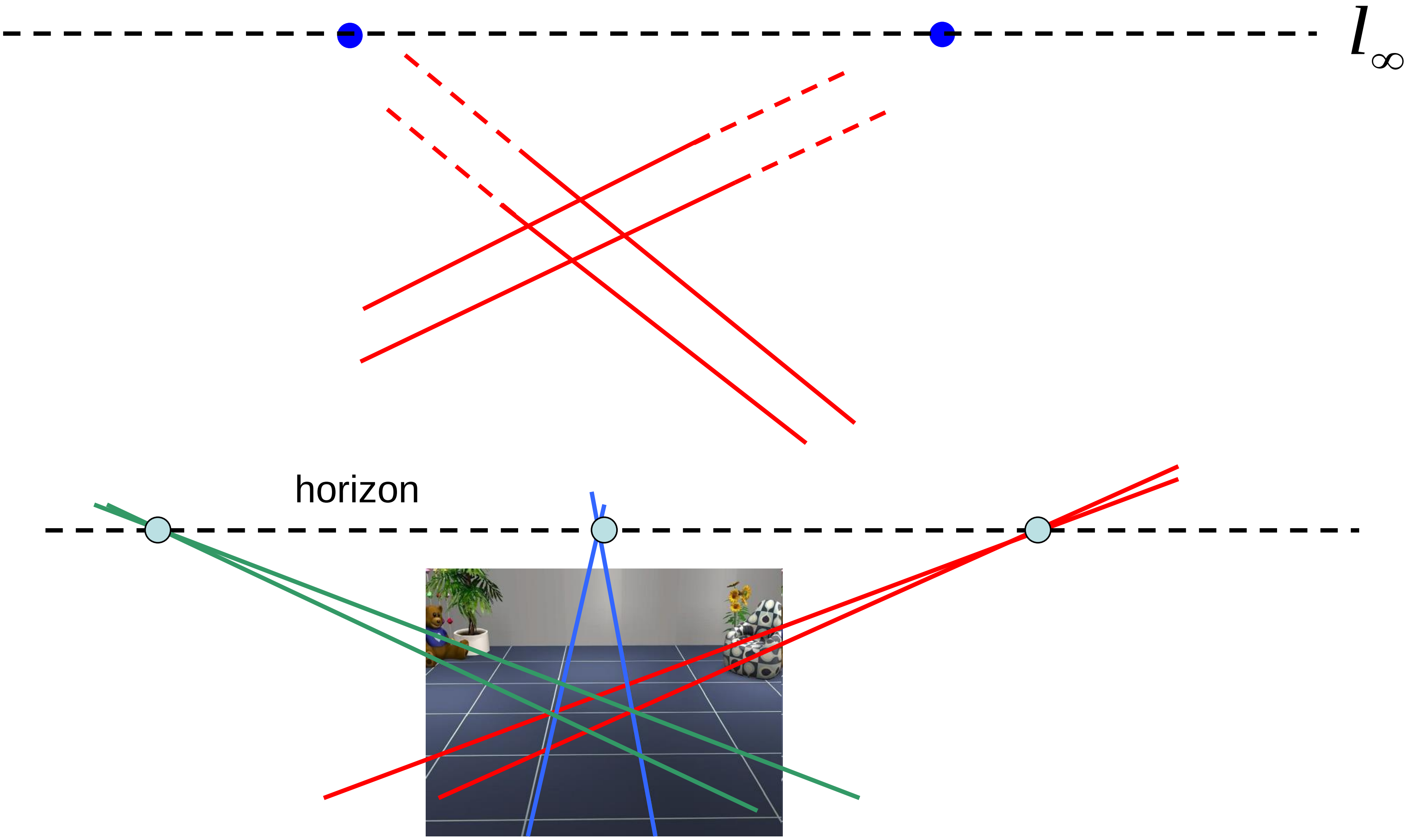
- Points project to points
- Lines project to lines
- Distant objects look smaller

# Properties of Projection

- Angles are not preserved
- Parallel lines meet!

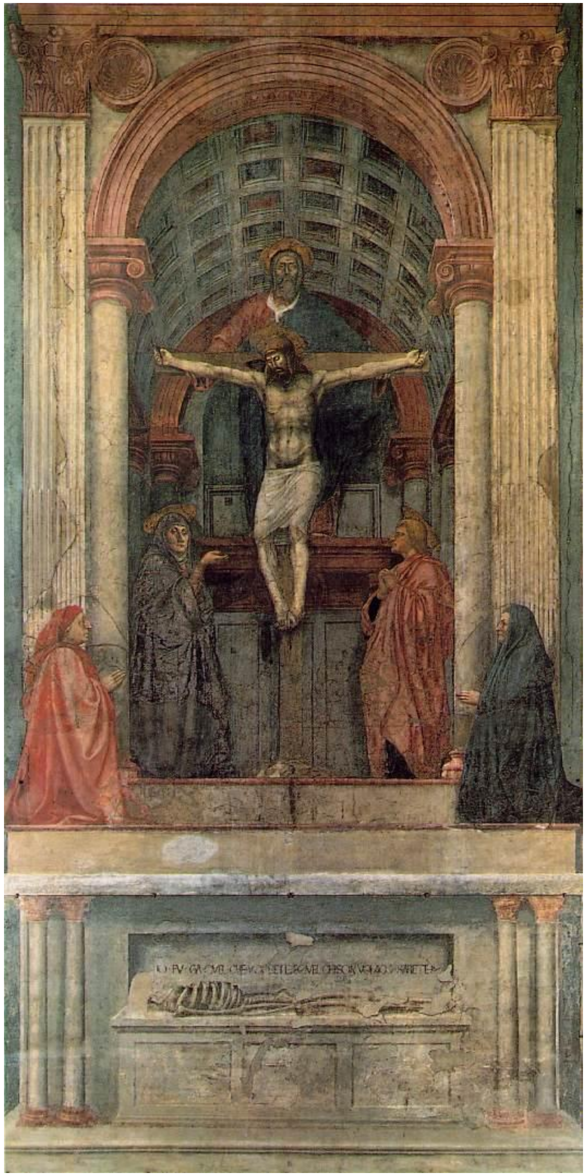
Parallel lines in the world intersect in the image at a "vanishing point"

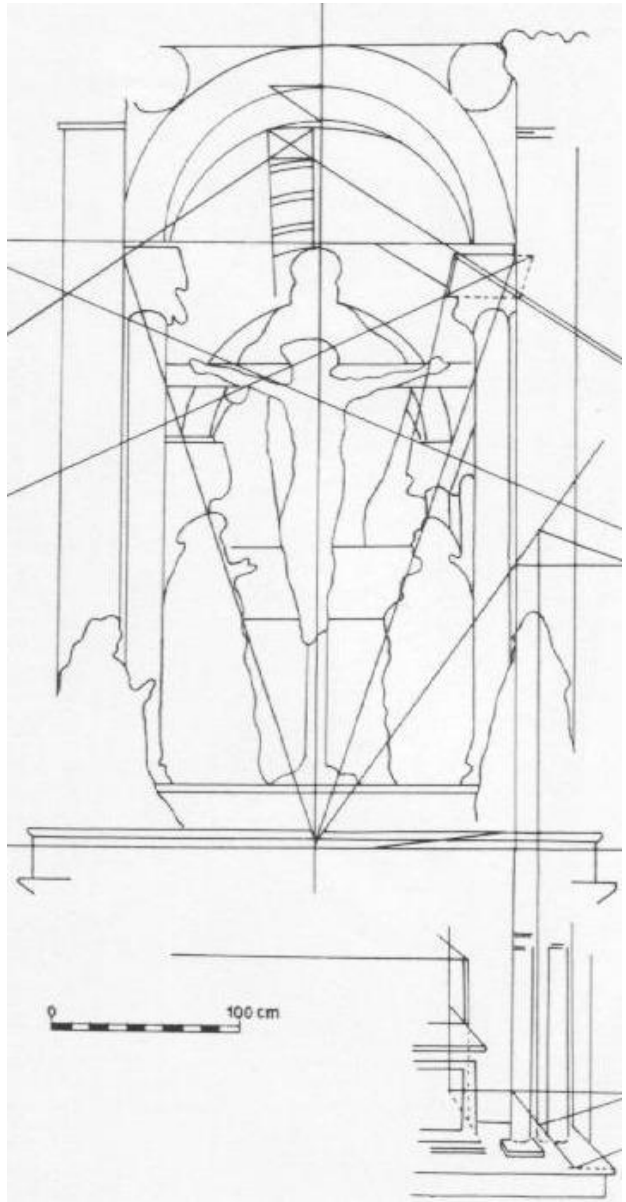
# Horizon line (vanishing line)



# Horizon line (vanishing line)

# One-point perspective






- Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

Credit slide S. Lazebnik

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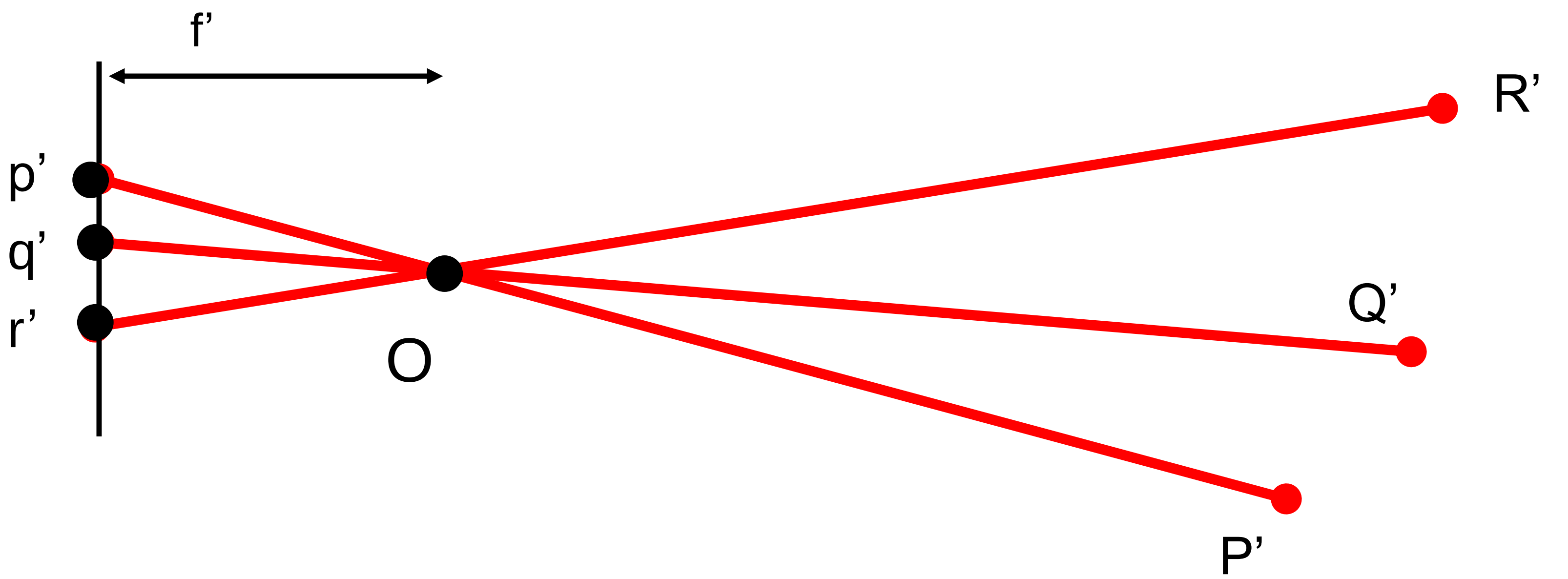
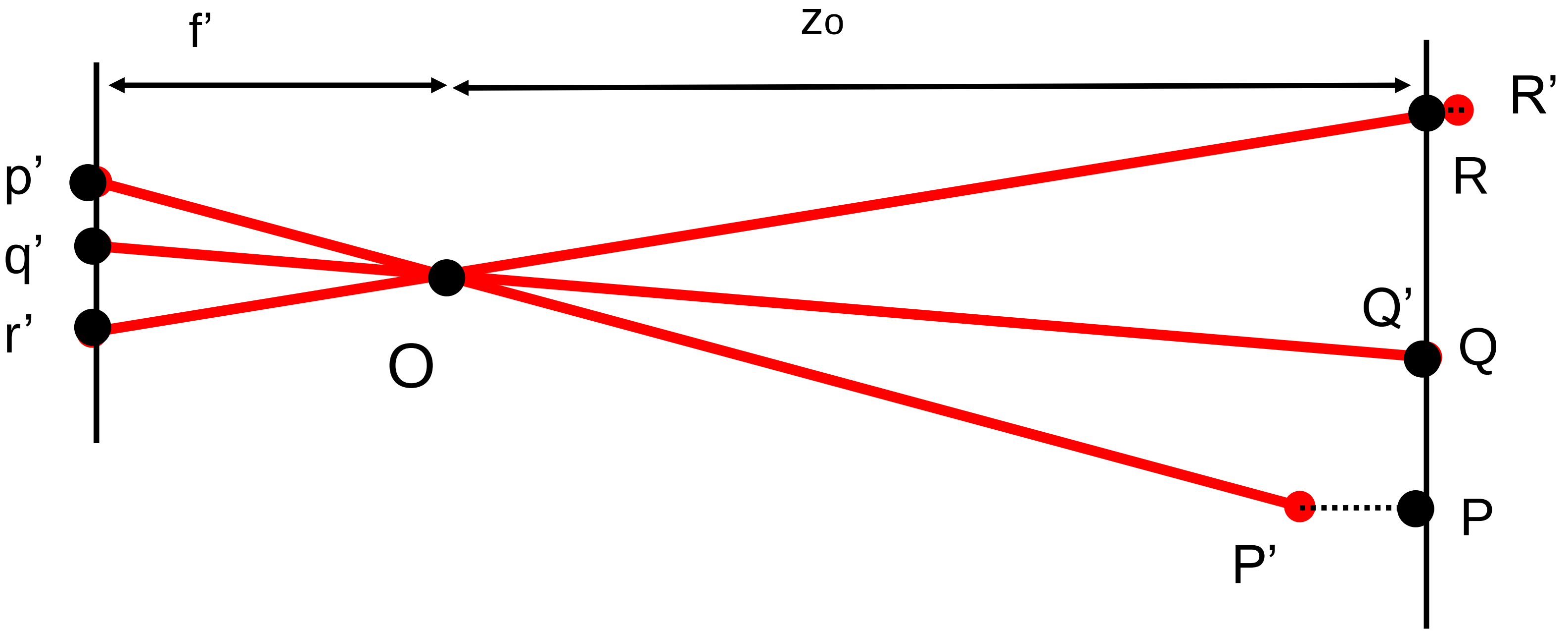
# Projective camera



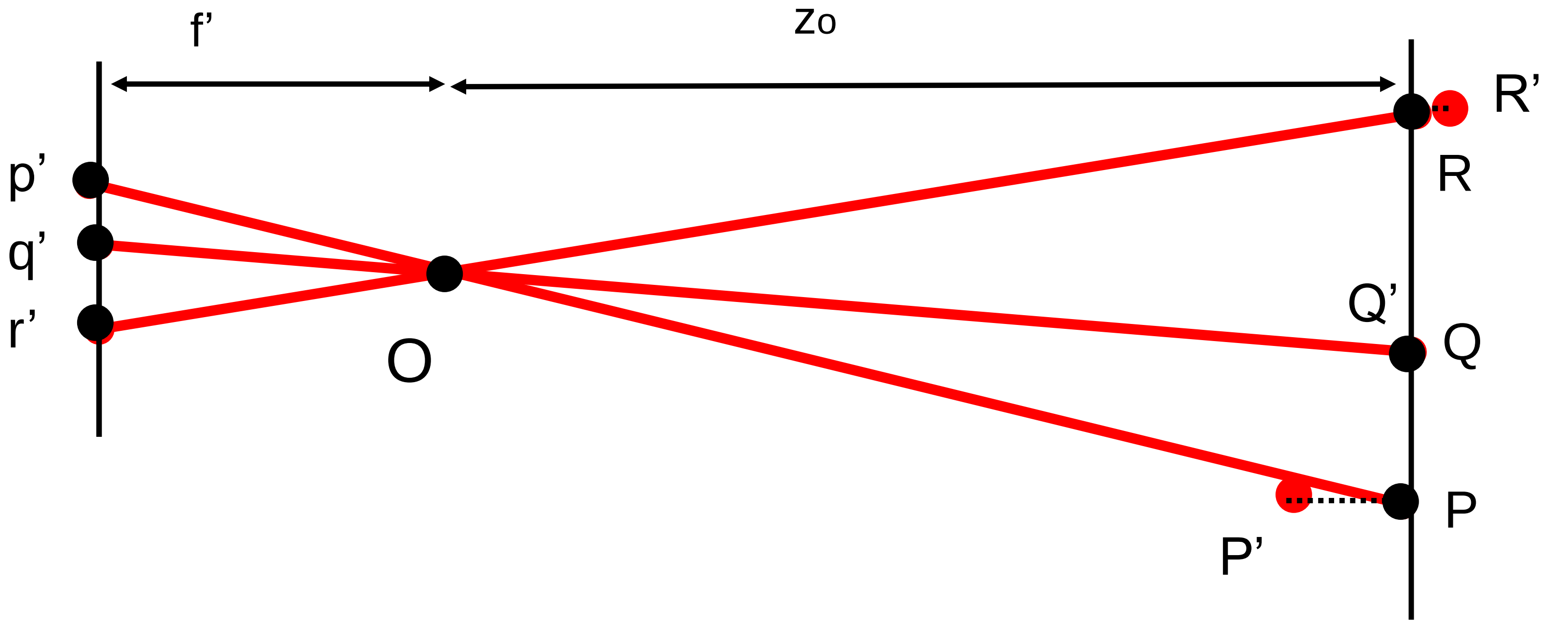
# Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



# Weak perspective projection

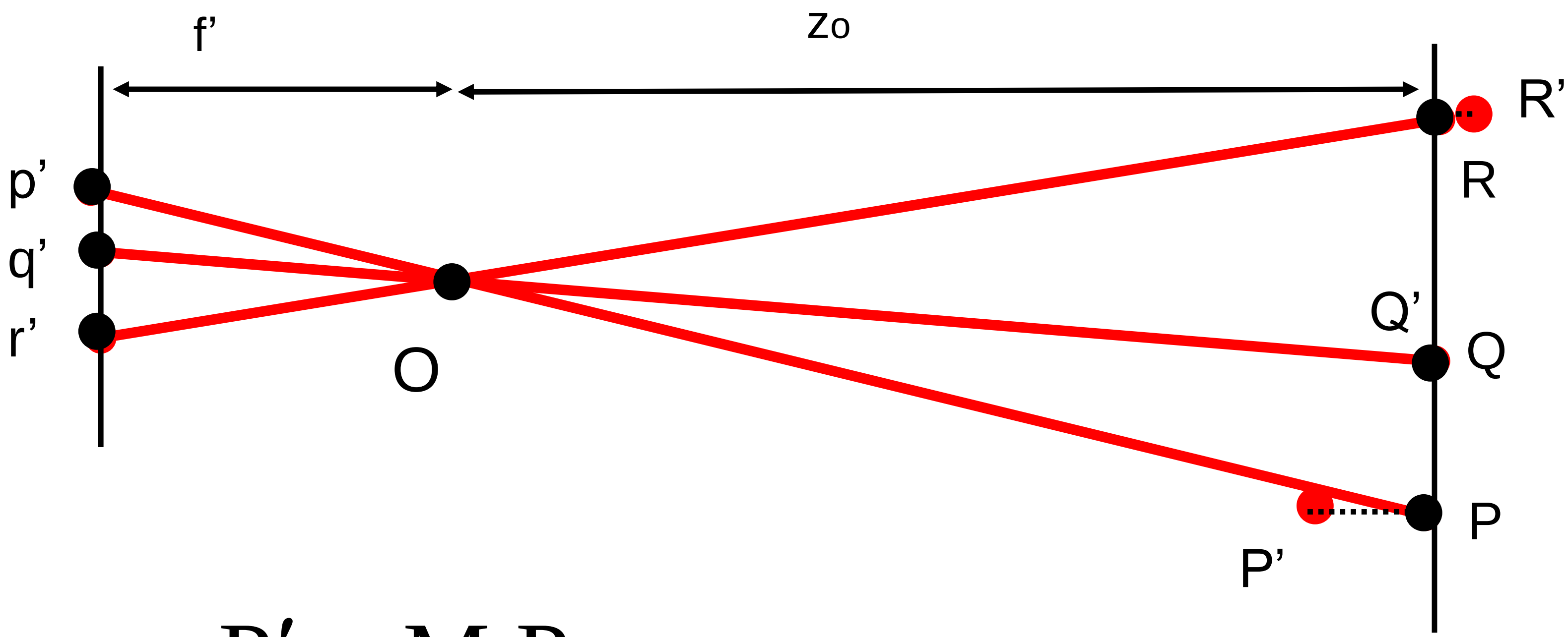
When the relative scene depth is small compared to its distance from the camera




$$\begin{cases} x' = -\dfrac{f'}{z}x \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -\boxed{\dfrac{f'}{z_0}}x \\ y' = -\dfrac{f'}{z_0}y \end{cases}$$

Magnification m

# Weak perspective projection




$$\mathrm{P}' = \mathrm{M}\ \mathrm{P}_{\mathrm{w}}$$

$$M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

Instead of

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix}$$

$$P' = M\, P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1\, P_W \\ \mathbf{m}_2\, P_W \\ \mathbf{m}_3\, P_W \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$\xrightarrow{E} \left(\frac{\mathbf{m}_1\, P_w}{\mathbf{m}_3\, P_w}, \frac{\mathbf{m}_2\, P_w}{\mathbf{m}_3\, P_w}\right)$$

Perspective

---

$$P' = M\, P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1\, P_W \\ \mathbf{m}_2\, P_W \\ 1 \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = \begin{bmatrix} & \mathbf{m}_1 & & \\ & \mathbf{m}_2 & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

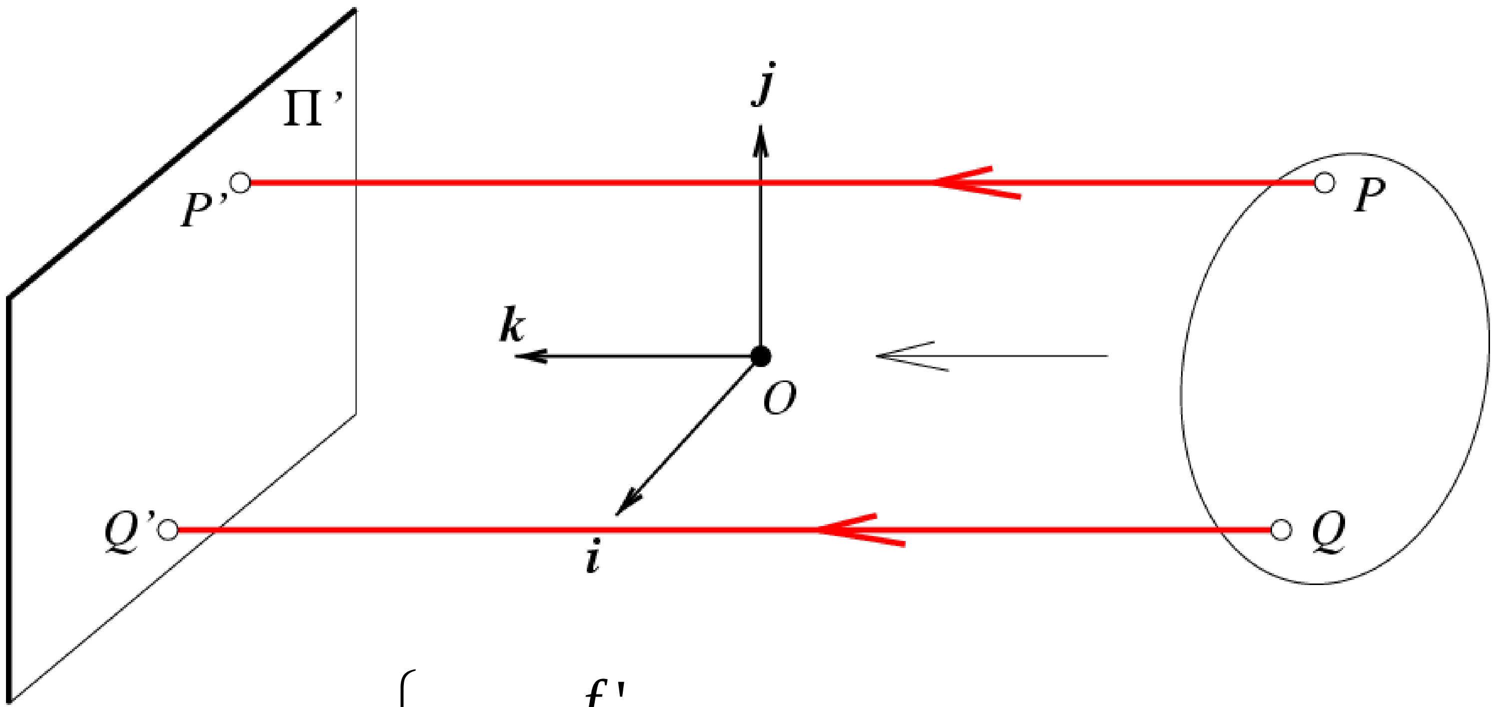
$$\xrightarrow{E} (\mathbf{m}_1\, P_w, \mathbf{m}_2\, P_w)$$

magnification

Weak perspective

# Orthographic (affine) projection

Distance from center of projection to image plane is infinite




$$\begin{cases} x' = -\dfrac{f'}{z} x \\ \\ y' = -\dfrac{f'}{z} y \end{cases} \rightarrow \begin{cases} x' = -x \\ y' = -y \end{cases}$$

# Pros and Cons of These Models

- Weak perspective much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.

- Pinhole perspective much more accurate for scenes.
  - Used in structure from motion.

# Weak perspective projection

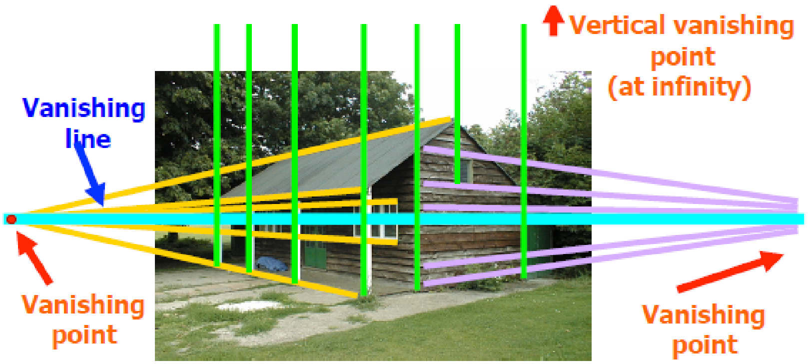
*The Kangxi Emperor's Southern Inspection Tour (1691-1698)* By Wang Hui

# Weak perspective projection

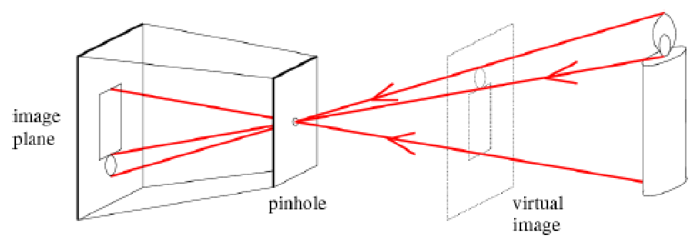
*The Kangxi Emperor's Southern Inspection Tour (1691-1698)* By Wang Hui

# Things to remember

- Vanishing points and vanishing lines




- Pinhole camera model and camera projection matrix M
    - Intrinsic parameters
    - Extrinsic parameters




$$P' = K\begin{bmatrix} R & \overline{t} \end{bmatrix} P$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Slide inspiration: J. Hayes

# What we have learned today?

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
  - Projection matrix
  - Intrinsic parameters
  - Extrinsic parameters

Reading:

[FP] Chapters 1 – 3

[HZ] Chapter 6