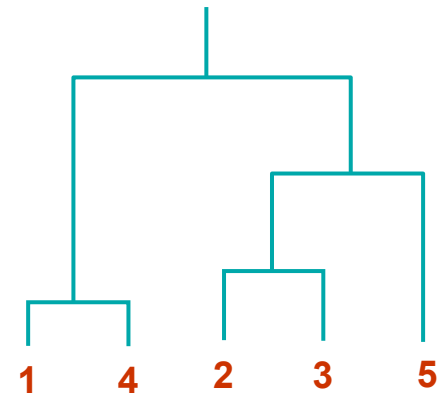
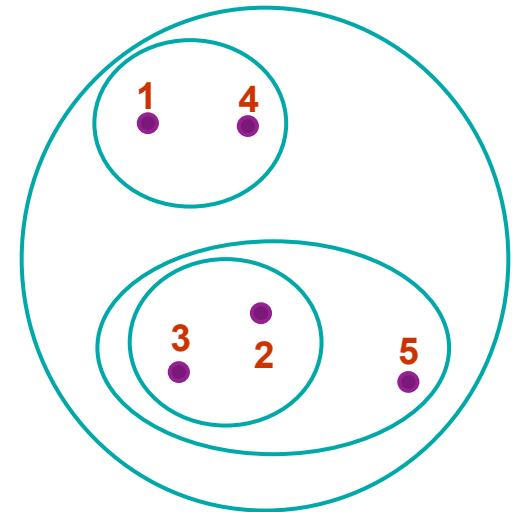




# Molecular Evolution and Phylogenetic Tree Reconstruction





# Orthology, Paralogy, Inparalogs, Outparalogs

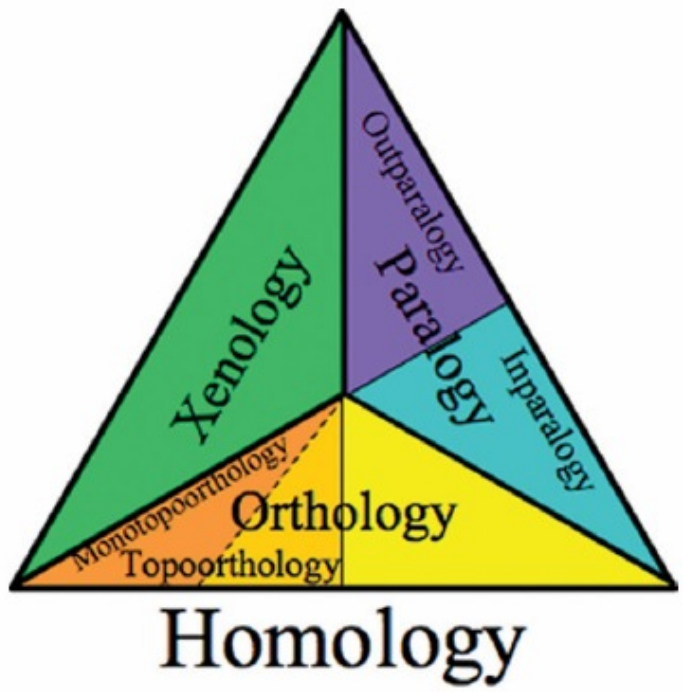


Figure 1. Refinements of homology.

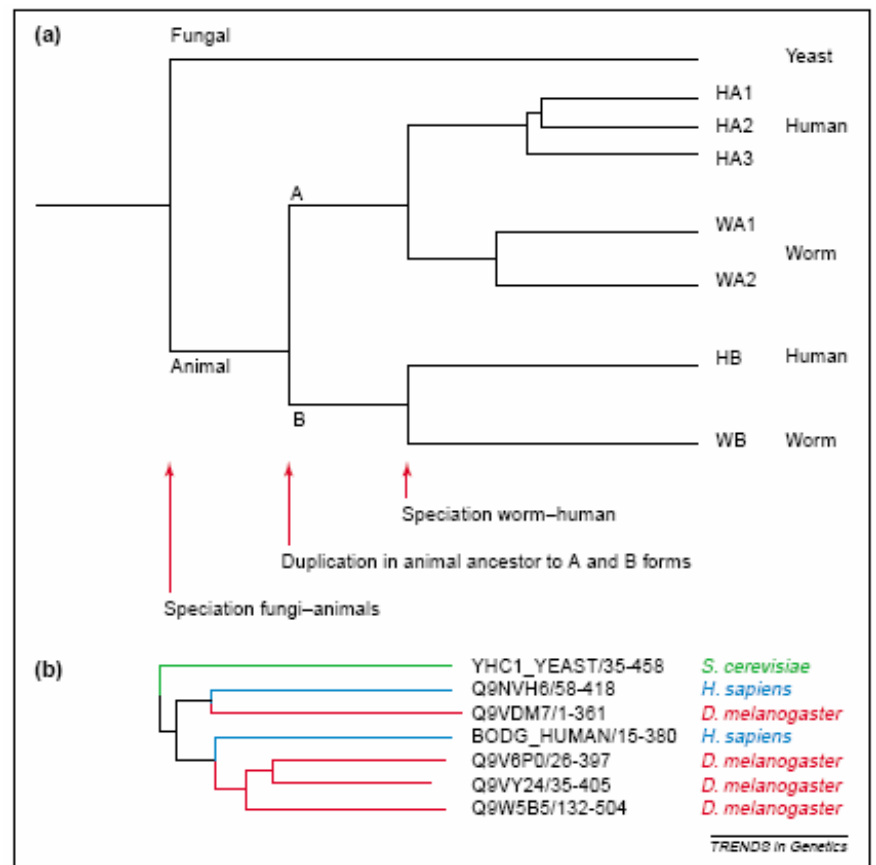
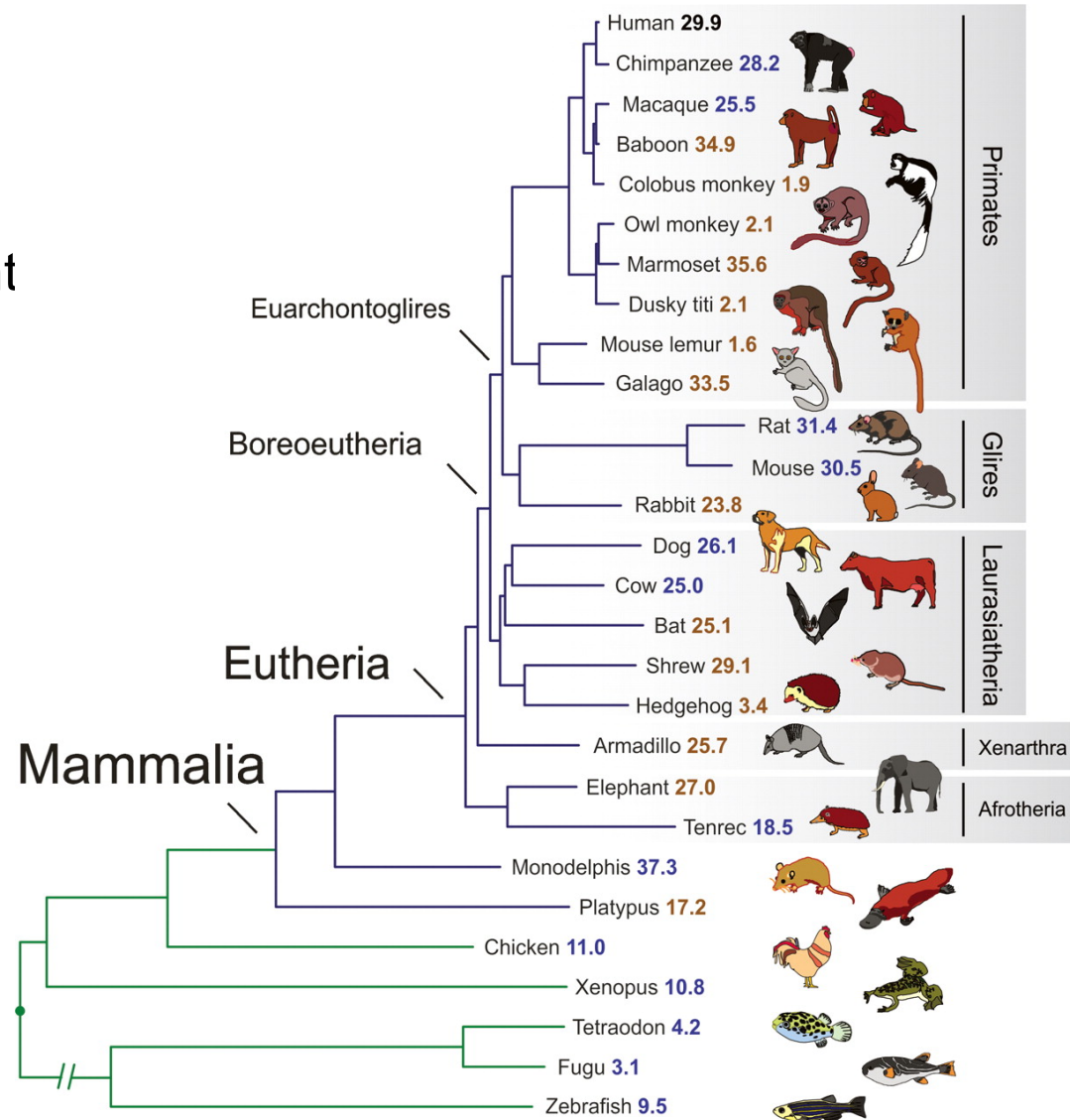


Fig. 1. The definition of inparalogs and outparalogs. (a) Consider an ancient gene inherited in the yeast, worm and human lineages. The gene was duplicated early in the animal lineage, before the human-worm split, into genes A and B. After the human-worm split, the A form was in turn duplicated independently in the human and worm lineages. In this scenario, the yeast gene is orthologous to all worm and human genes, which are all co-orthologous to the yeast gene. When comparing the human and worm genes, all genes in the HA\* set are co-orthologous to all genes in the WA\* set. The genes HA\* are hence 'inparalogs' to each other when comparing human to worm. By contrast, the genes HB and HA\* are 'outparalogs' when comparing human with worm. However, HB and HA\*, and WB and WA\* are inparalogs when comparing with yeast, because the animal-yeast split pre-dates the HA\*-HB duplication. (b) Real-life example of inparalogs:  $\gamma$ -butyrobetaine hydroxylases. The points of speciation and duplication are easily identifiable. The alignment is a subset of Pfam:PF03322 and the tree was generated by neighbor-joining in Belvu. All nodes have a bootstrap support exceeding 95%.



# Phylogenetic Trees

- Nodes: species
- Edges: time of independent evolution
- Edge length represents evolution time
  - AKA genetic distance
  - Not necessarily chronological time



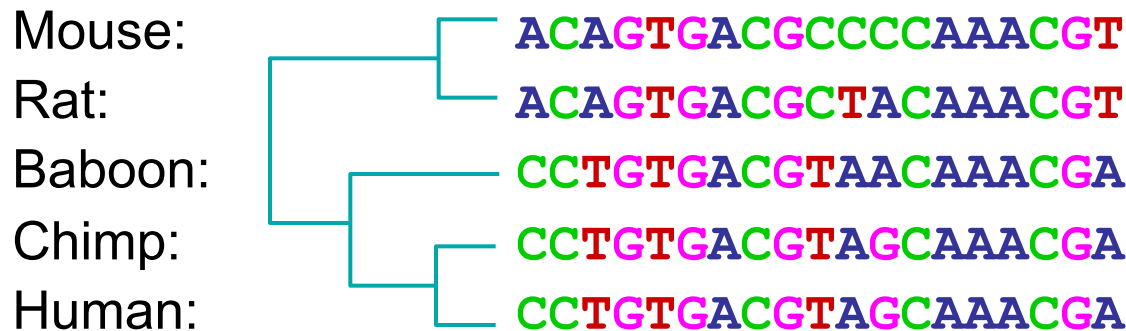


# Inferring Phylogenetic Trees

Trees can be inferred by several criteria:

- Morphology of the organisms
  - *Can lead to mistakes*
- Sequence comparison

## Example:





# Distance Between Two Sequences

## Basic principle:

- Distance proportional to degree of independent sequence evolution

Given sequences  $x^i$ ,  $x^j$ ,

$d_{ij}$  = distance between the two sequences

One possible definition:

$d_{ij}$  = fraction  $f$  of sites  $u$  where  $x^i[u] \neq x^j[u]$

Better scores are derived by modeling evolution as a continuous change process



# Molecular Evolution

## Modeling sequence substitution:

Consider what happens at a position for time  $\Delta t$ ,

- $P(t)$  = vector of probabilities of {A,C,G,T} at time  $t$
- $\mu_{AC}$  = rate of transition from A to C per unit time
- $\mu_A = \mu_{AC} + \mu_{AG} + \mu_{AT}$  rate of transition out of A
- $p_A(t+\Delta t) = p_A(t) - p_A(t) \mu_A \Delta t + p_C(t) \mu_{CA} \Delta t + p_G(t) \mu_{GA} \Delta t + p_T(t) \mu_{TA} \Delta t$



# Molecular Evolution

In matrix/vector notation, we get

$$P(t+\Delta t) = P(t) + Q P(t) \Delta t$$

where  $Q$  is the substitution rate matrix

$$Q = \begin{pmatrix} -\mu_A & \mu_{GA} & \mu_{CA} & \mu_{TA} \\ \mu_{AG} & -\mu_G & \mu_{CG} & \mu_{TG} \\ \mu_{AC} & \mu_{GC} & -\mu_C & \mu_{TC} \\ \mu_{AT} & \mu_{GT} & \mu_{CT} & -\mu_T \end{pmatrix}$$



# Molecular Evolution

- This is a differential equation:

$$P'(t) = Q P(t)$$

- $Q \Rightarrow$  prob. distribution over  $\{A, C, G, T\}$  at each position, stationary (equilibrium) frequencies  $\pi_A, \pi_C, \pi_G, \pi_T$
- Each  $Q$  is an evolutionary model
  - Some work better than others



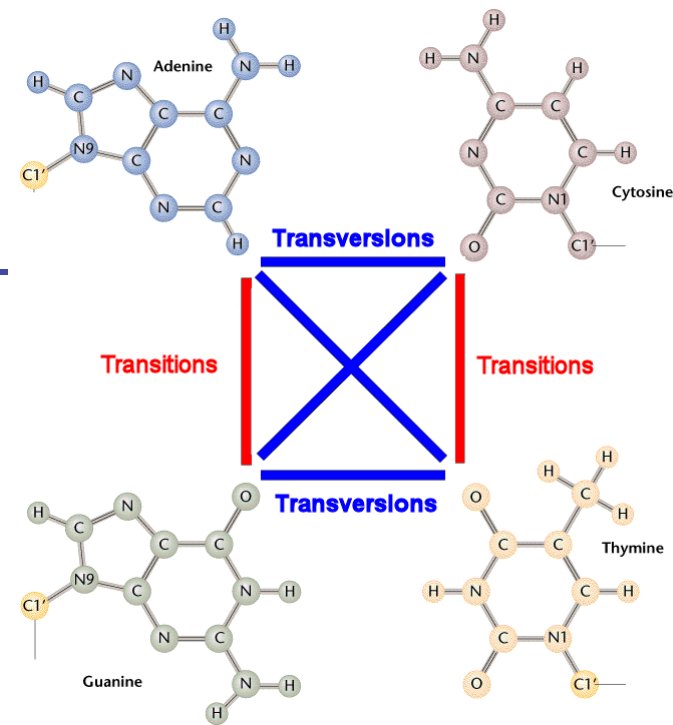
# Evolutionary Models

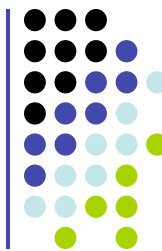
- Jukes-Cantor  $Q = \begin{pmatrix} * & \frac{\mu}{4} & \frac{\mu}{4} & \frac{\mu}{4} \\ \frac{\mu}{4} & * & \frac{\mu}{4} & \frac{\mu}{4} \\ \frac{\mu}{4} & \frac{\mu}{4} & * & \frac{\mu}{4} \\ \frac{\mu}{4} & \frac{\mu}{4} & \frac{\mu}{4} & * \end{pmatrix}$

- Kimura  $Q = \begin{pmatrix} * & \kappa & 1 & 1 \\ \kappa & * & 1 & 1 \\ 1 & 1 & * & \kappa \\ 1 & 1 & \kappa & * \end{pmatrix}$

- Felsenstein  $Q = \begin{pmatrix} * & \pi_T & \pi_T & \pi_T \\ \pi_C & * & \pi_C & \pi_C \\ \pi_A & \pi_A & * & \pi_A \\ \pi_G & \pi_G & \pi_G & * \end{pmatrix}$

- HKY  $Q = \begin{pmatrix} * & \kappa\pi_T & \pi_T & \pi_T \\ \kappa\pi_C & * & \pi_C & \pi_C \\ \pi_A & \pi_A & * & \kappa\pi_A \\ \pi_G & \pi_G & \kappa\pi_G & * \end{pmatrix}$





# Estimating Distances

- Solve the differential equation and compute expected evolutionary time given sequences

$$P'(t) = Q P(t)$$

Jukes-Cantor:

$$\text{Let } P_{AA}(t) = P_{CC}(t) = P_{GG}(t) = P_{TT}(t) = r$$

$$P_{AC}(t) = \dots = P_{TG}(t) = s$$

Then,

$$r'(t) = -\frac{3}{4} r(t) \mu + \frac{3}{4} s(t) \mu$$

$$s'(t) = -\frac{1}{4} s(t) \mu + \frac{1}{4} r(t) \mu$$

Which is satisfied by

$$r(t) = \frac{1}{4} (1 + 3e^{-\mu t})$$

$$s(t) = \frac{1}{4} (1 - e^{-\mu t})$$



# Estimating Distances

- Solve the differential equation and compute expected evolutionary time given sequences

$$P'(t) = Q P(t)$$

Jukes-Cantor:

$$P = \begin{pmatrix} \frac{1}{4} + \frac{3}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} \\ \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} + \frac{3}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} \\ \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} + \frac{3}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} \\ \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} - \frac{1}{4}e^{-t\mu} & \frac{1}{4} + \frac{3}{4}e^{-t\mu} \end{pmatrix}$$



# Estimating Distances

Let  $p$  = probability a base is different between two sequences,  
Solve to find  $t$

- Jukes-Cantor  $r(t) = 1 - p = \frac{1}{4} (1 + 3e^{-\mu t})$

$$p = \frac{3}{4} - \frac{3}{4} e^{-\mu t}$$

$$\frac{3}{4} - p = \frac{3}{4} e^{-\mu t}$$

$$1 - 4p/3 = e^{-\mu t}$$

Therefore,

$$\mu t = -\ln(1 - 4p/3)$$

Letting

$d = \frac{3}{4} \mu t$ , denoting substitutions per site,

$$d = -\frac{3}{4} \ln(1 - \frac{4}{3}p)$$



# Estimating Distances

d: Branch length in terms of substitutions per site

- Jukes-Cantor

$$d = -\frac{3}{4} \ln\left(1 - \frac{4}{3}p\right)$$

- Kimura

$$d = -\frac{1}{2} \ln(1 - 2P - Q) - \frac{1}{4} \ln(1 - 2Q)$$



# Simple method for building tree: UPGMA

UPGMA (unweighted pair group method using arithmetic averages)  
Or the **Average Linkage Method**

Given two disjoint clusters  $C_i$ ,  $C_j$  of sequences,

$$d_{ij} = \frac{1}{|C_i| \times |C_j|} \sum_{\{p \in C_i, q \in C_j\}} d_{pq}$$

Claim that if  $C_k = C_i \cup C_j$ , then distance to another cluster  $C_l$  is:

$$d_{kl} = \frac{d_{il} |C_i| + d_{jl} |C_j|}{|C_i| + |C_j|}$$



# Algorithm: Average Linkage

## Initialization:

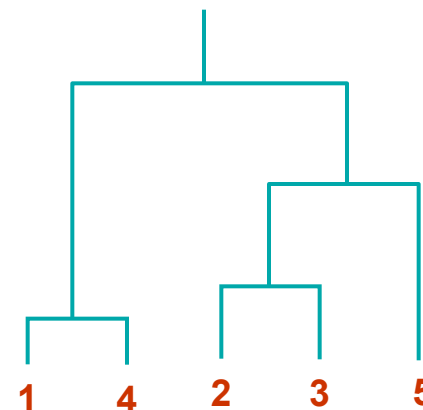
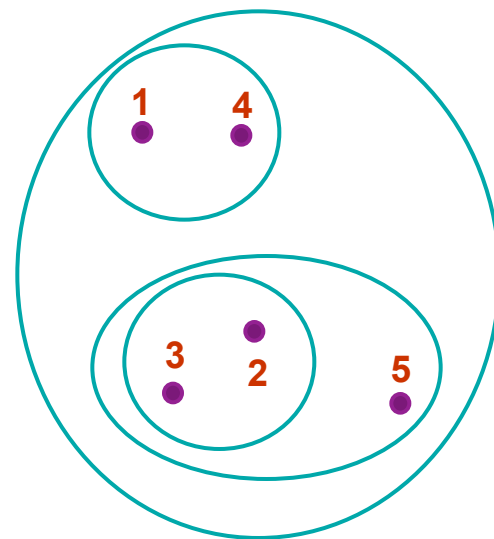
Assign each  $x_i$  into its own cluster  $C_i$   
Define one leaf per sequence, height 0

## Iteration:

Find two clusters  $C_i, C_j$  s.t.  $d_{ij}$  is min  
Let  $C_k = C_i \cup C_j$   
Define node connecting  $C_i, C_j$ , and place it at  
height  $d_{ij}/2$   
Delete  $C_i, C_j$

## Termination:

When two clusters  $i, j$  remain, place root at  
height  $d_{ij}/2$





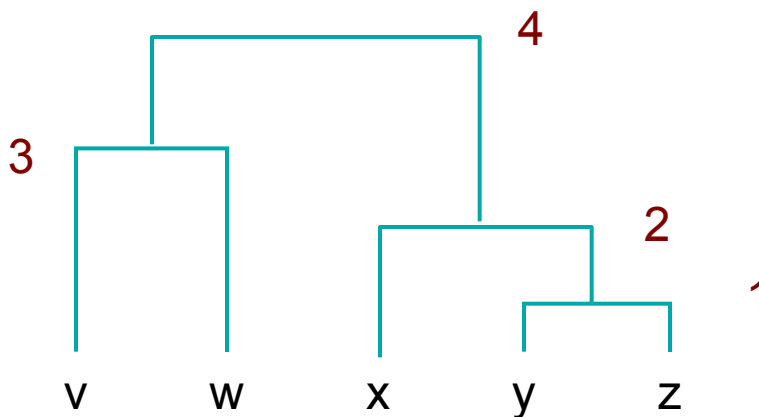
# Average Linkage Example

	v	w	x	y	z
v	0	6	8	8	8
w		0	8	8	8
x			0	4	4
y				0	2
z					0

	v	w	xyz
v	0	6	8
w		0	8
xyz			0

	vw	xyz
vw	0	8
xyz		0

	v	w	x	yz
v	0	6	8	8
w		0	8	8
x			0	4
yz				0





# Ultrametric Distances and Molecular Clock



## Definition:

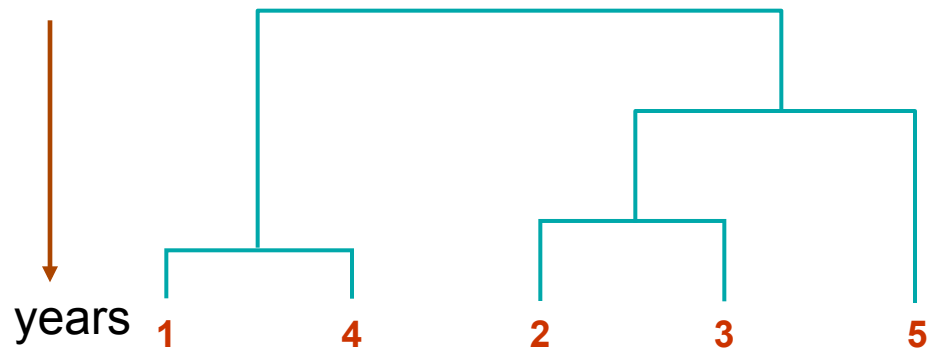
A distance function  $d(.,.)$  is ultrametric if for any three distances  $d_{ij} \leq d_{ik} \leq d_{ij}$ , it is true that

$$d_{ij} \leq d_{ik} = d_{jk}$$

## The Molecular Clock:

The evolutionary distance between species x and y is 2× the Earth time to reach the nearest common ancestor

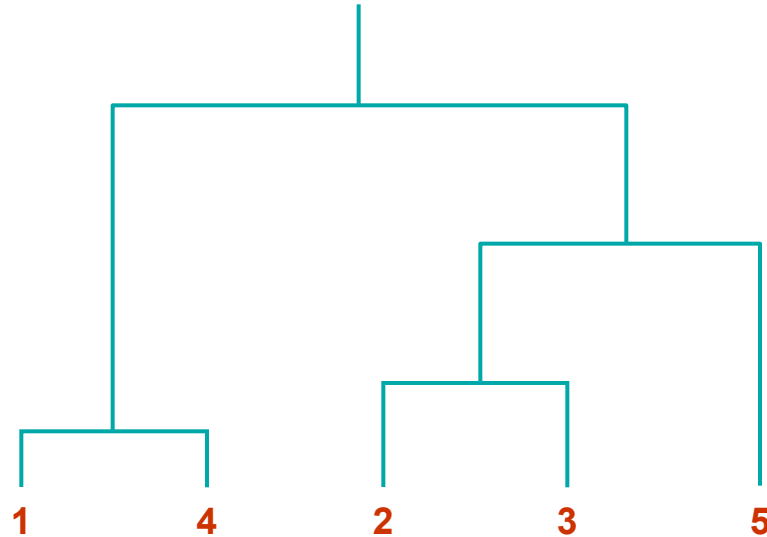
That is, the molecular clock has constant rate in all species



The molecular clock  
results in ultrametric  
distances



# Ultrametric Distances & Average Linkage



Average Linkage is guaranteed to reconstruct correctly a binary tree with ultrametric distances

Proof: Exercise

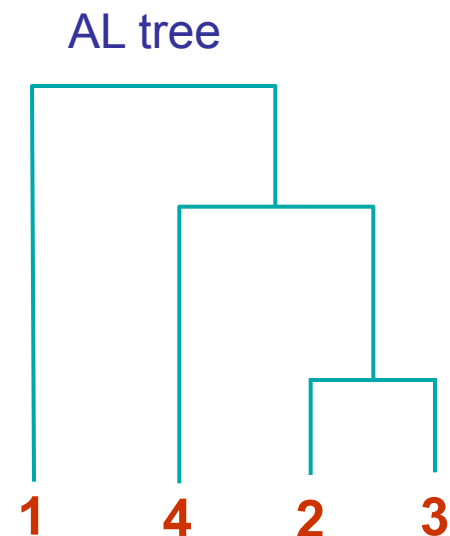
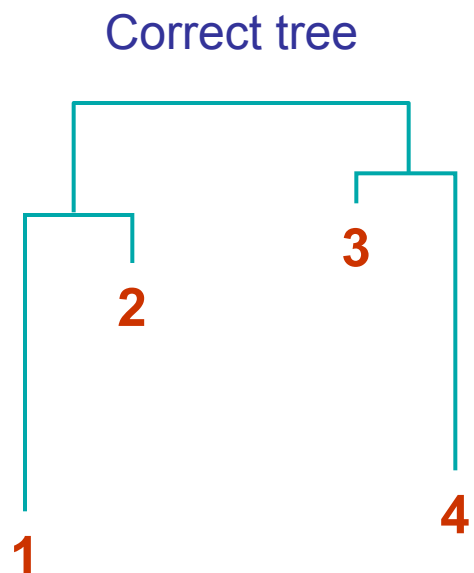


# Weakness of Average Linkage

Molecular clock: all species evolve at the same rate (Earth time)

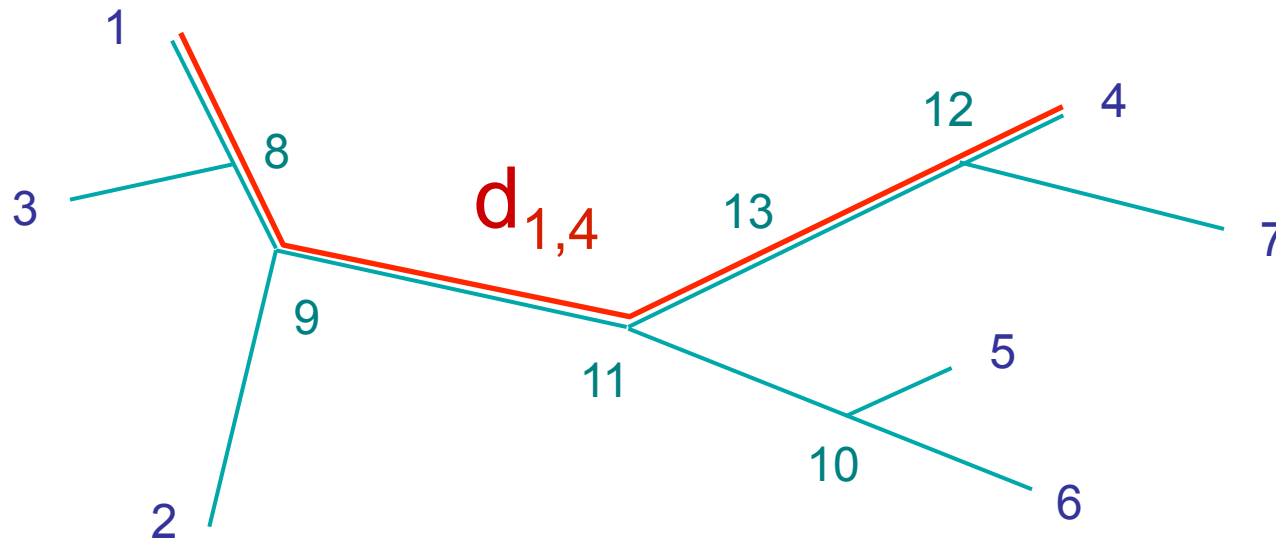
However, certain species (e.g., mouse, rat) evolve much faster

Example where UPGMA messes up:





# Additive Distances



Given a tree, a distance measure is **additive** if the distance between any pair of leaves is the sum of lengths of edges connecting them

Given a tree  $T$  & additive distances  $d_{ij}$ , can uniquely reconstruct edge lengths:

- Find two neighboring leaves  $i, j$ , with common parent  $k$
- Place parent node  $k$  at distance  $d_{km} = \frac{1}{2} (d_{im} + d_{jm} - d_{ij})$  from any node  $m \neq i, j$



# Additive Distances



For any four leaves  $x, y, z, w$ , consider the three sums

$$\begin{aligned}d(x, y) + d(z, w) \\d(x, z) + d(y, w) \\d(x, w) + d(y, z)\end{aligned}$$

One of them is smaller than the other two, which are equal

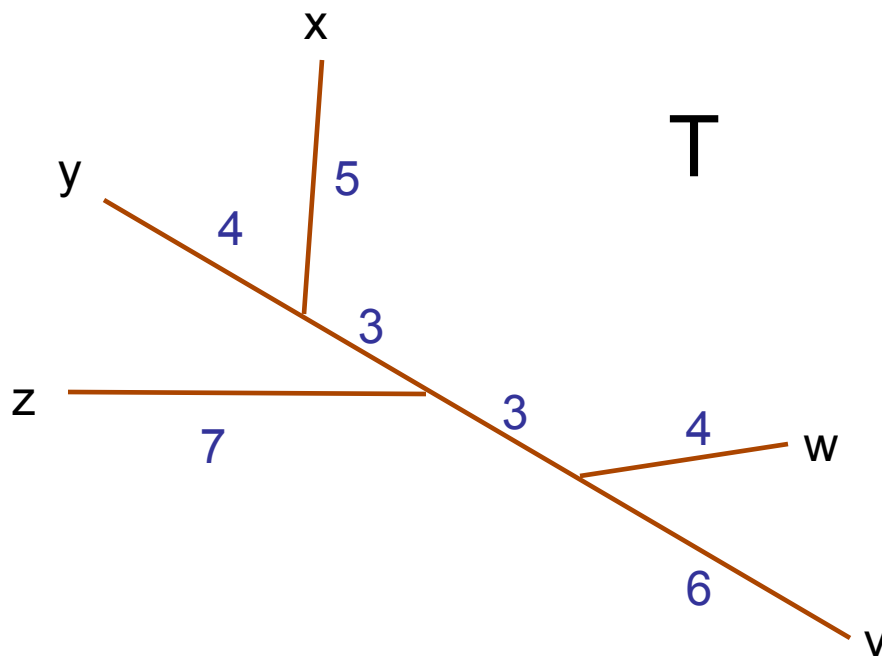
$$d(x, y) + d(z, w) < d(x, z) + d(y, w) = d(x, w) + d(y, z)$$



# Reconstructing Additive Distances Given T

D

	v	w	x	y	z
v	0	10	17	16	16
w		0	15	14	14
x			0	9	15
y				0	14
z					0



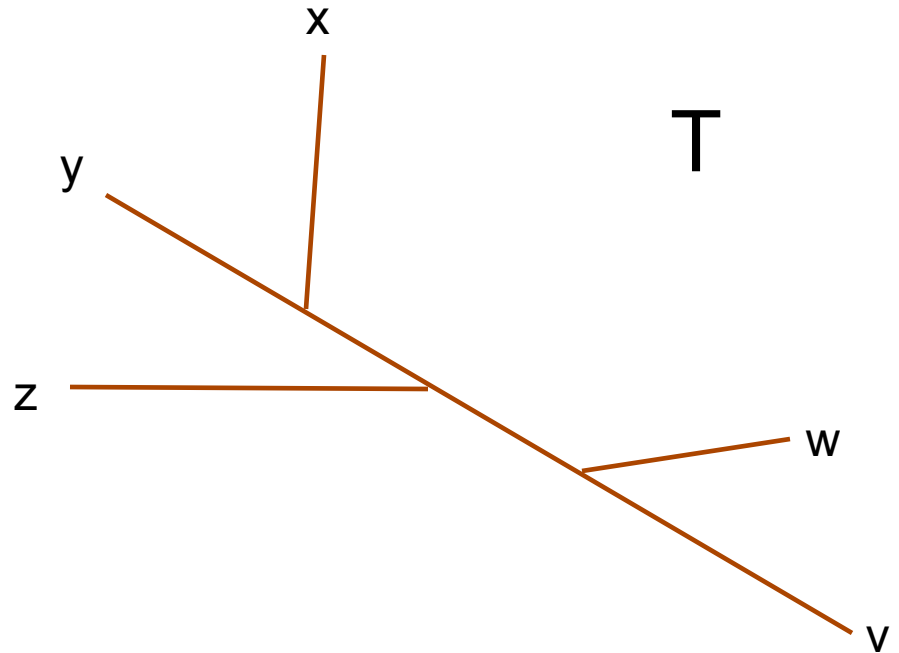
If we know T and D, but do not know the length of each leaf, we can reconstruct those lengths



# Reconstructing Additive Distances Given T

D

	v	w	x	y	z
v	0	10	17	16	16
w		0	15	14	14
x			0	9	15
y				0	14
z					0





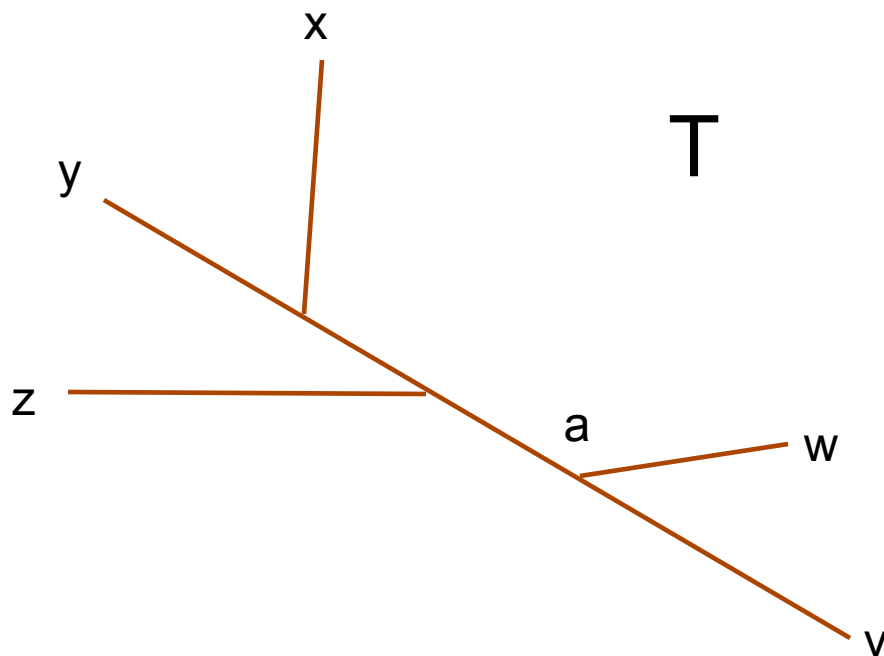
# Reconstructing Additive Distances Given T

**D**

	v	w	x	y	z
v	0	10	17	16	16
w		0	15	14	14
x			0	9	15
y				0	14
z					0

**D<sub>1</sub>**

	a	x	y	z
a	0	11	10	10
x		0	9	15
y			0	14
z				0



$$d_{ax} = \frac{1}{2} (d_{vx} + d_{wx} - d_{vw})$$

$$d_{ay} = \frac{1}{2} (d_{vy} + d_{wy} - d_{vw})$$

$$d_{az} = \frac{1}{2} (d_{vz} + d_{wz} - d_{vw})$$





# Reconstructing Additive Distances Given T

$D_1$

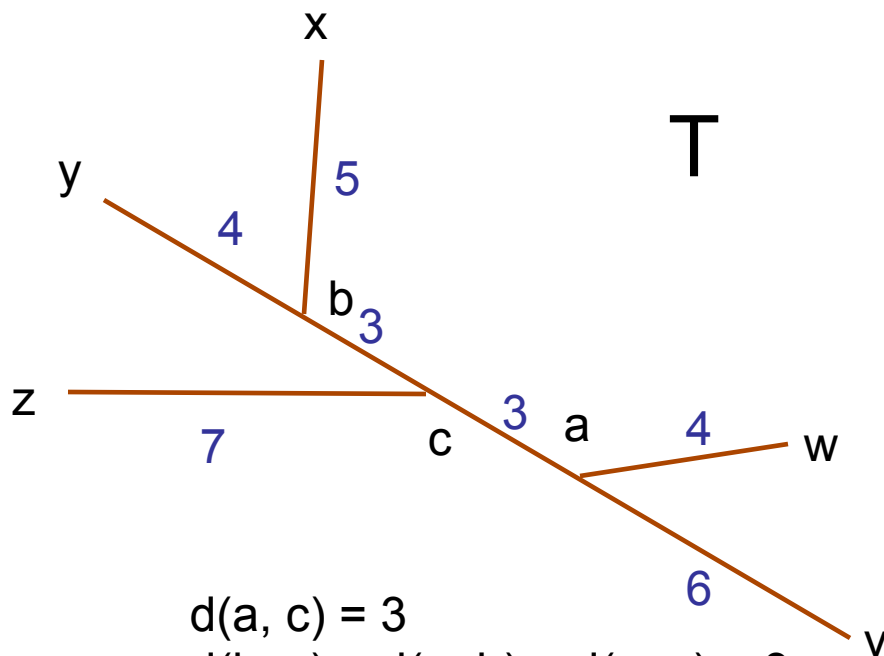
	a	x	y	z
a	0	11	10	10
x		0	9	15
y			0	14
z				0

$D_2$

	a	b	z
a	0	6	10
b		0	10
z			0

$D_3$

	a	c
a	0	3
c		0



$$d(a, c) = 3$$

$$d(b, c) = d(a, b) - d(a, c) = 3$$

$$d(c, z) = d(a, z) - d(a, c) = 7$$

$$d(b, x) = d(a, x) - d(a, b) = 5$$

$$d(b, y) = d(a, y) - d(a, b) = 4$$

$$d(a, w) = d(z, w) - d(a, z) = 4$$

$$d(a, v) = d(z, v) - d(a, z) = 6$$

**Correct!!!**



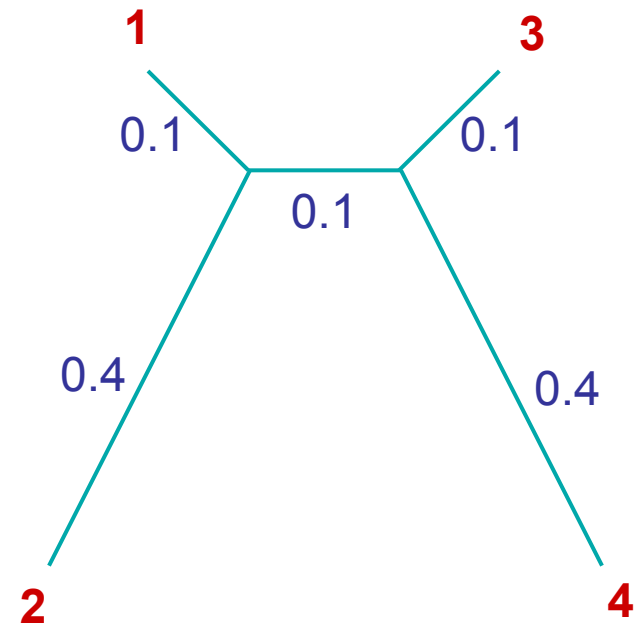
# Neighbor-Joining

- Guaranteed to produce the correct tree if distance is additive
- May produce a good tree even when distance is not additive

## **Step 1:** Finding neighboring leaves

Define

$$D_{ij} = (N - 2) d_{ij} - \sum_{k \neq i} d_{ik} - \sum_{k \neq j} d_{jk}$$

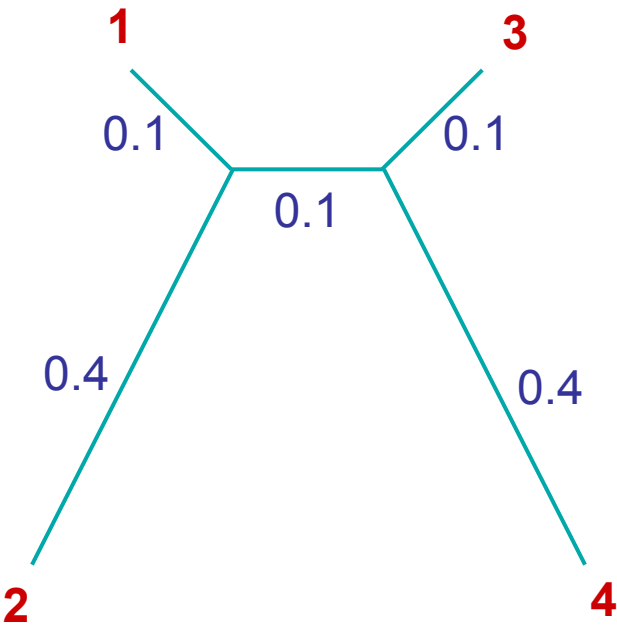
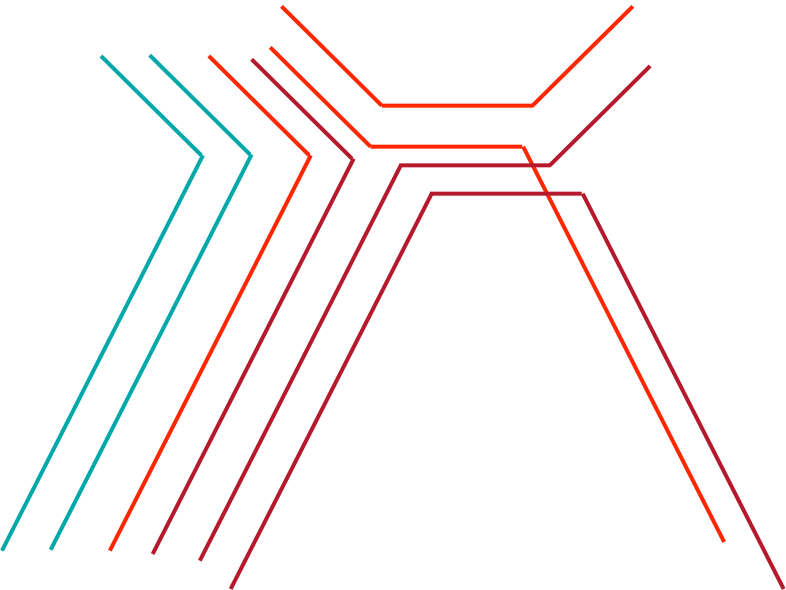


**Claim:** The above “magic trick” ensures that  $i, j$  are neighbors if  $D_{ij}$  is minimal



# Neighbor-Joining

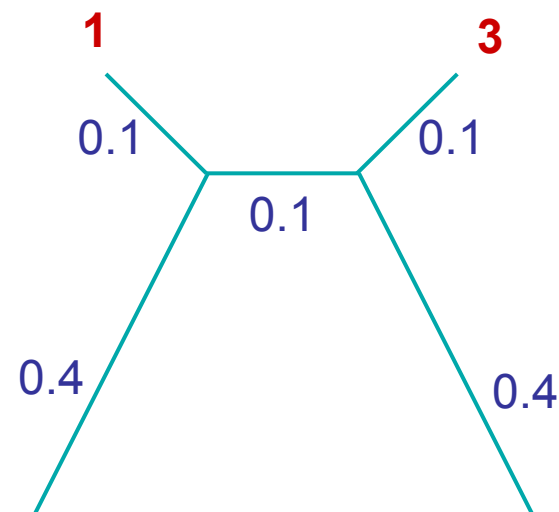
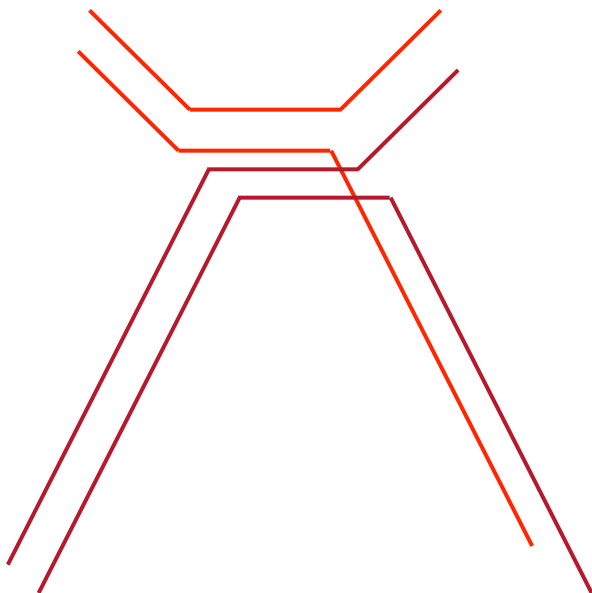
$$D_{ij} = (N - 2) d_{ij} - \sum_{k \neq i} d_{ik} - \sum_{k \neq j} d_{jk}$$





# Neighbor-Joining

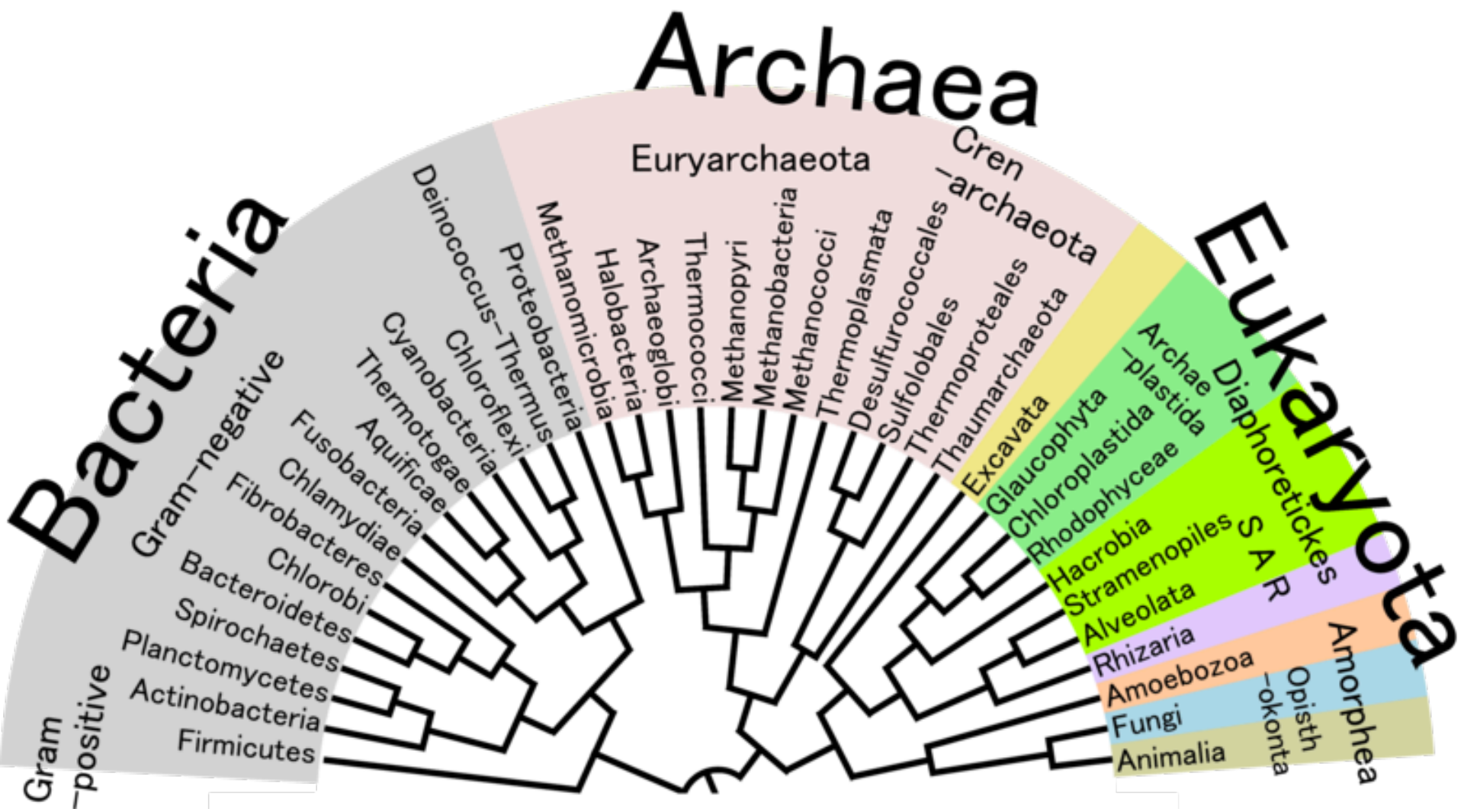
$$D_{ij} = (N - 2) d_{ij} - \sum_{k \neq i} d_{ik} - \sum_{k \neq j} d_{jk}$$



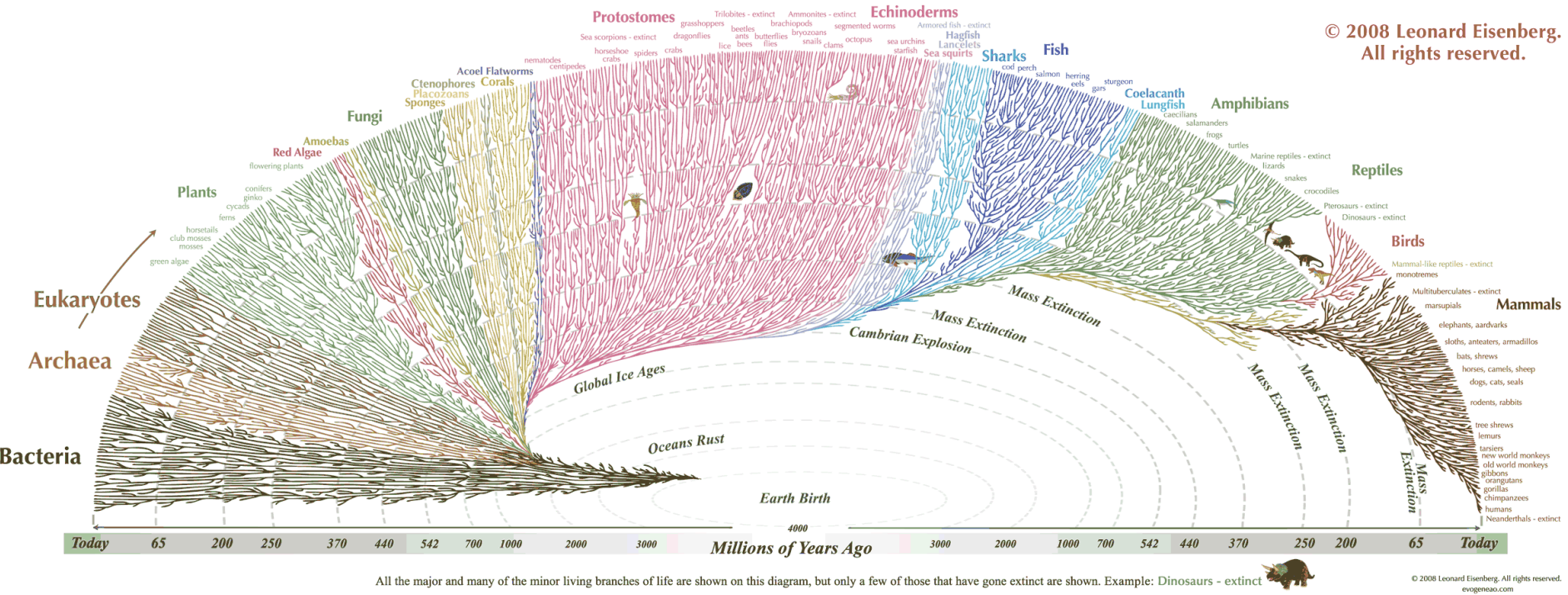
- All leaf edges appear negatively exactly twice
- All other edges appear negatively once for every path from each of the two leaves  $i, j$ , to leaves  $k \neq i, j$



# Some Trees



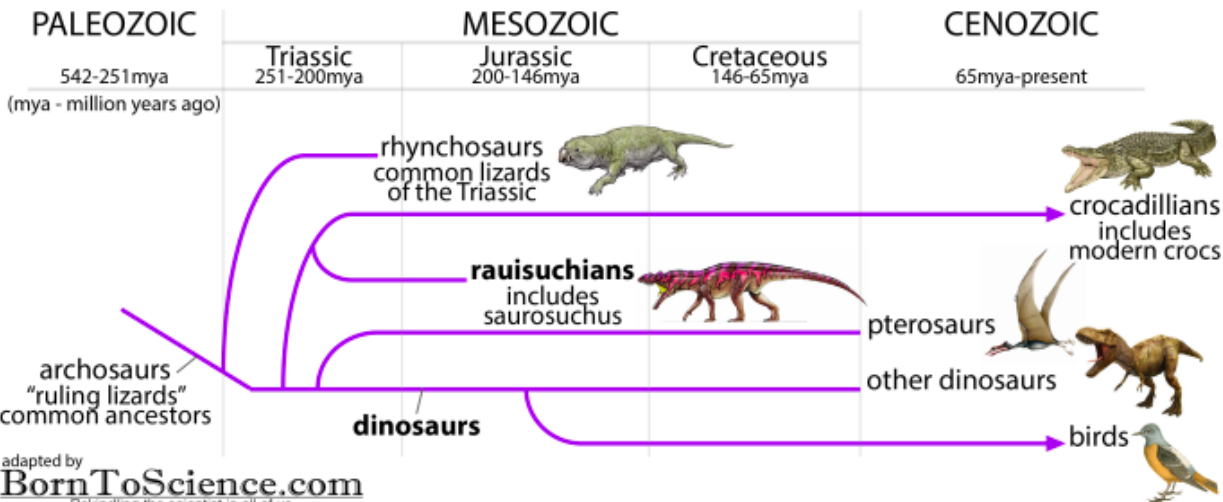
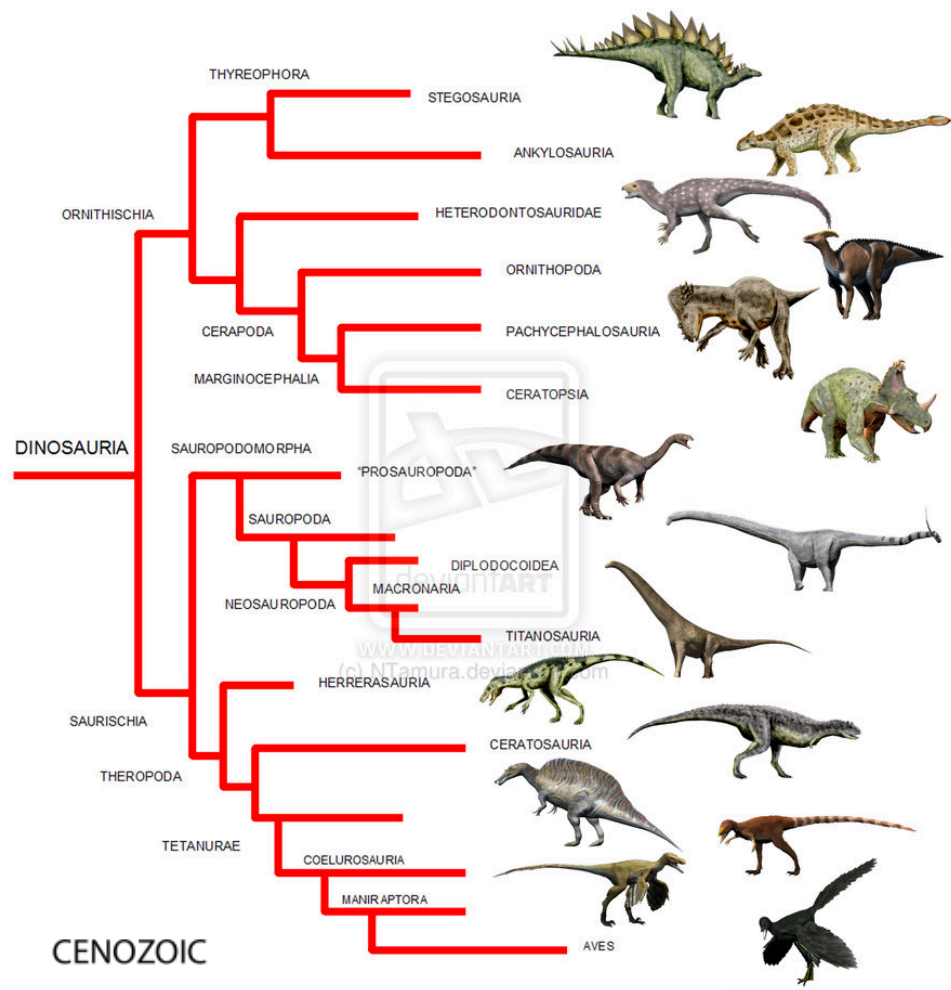
# Some Trees



All the major and many of the minor living branches of life are shown on this diagram, but only a few of those that have gone extinct are shown. Example: Dinosaurs - extinct

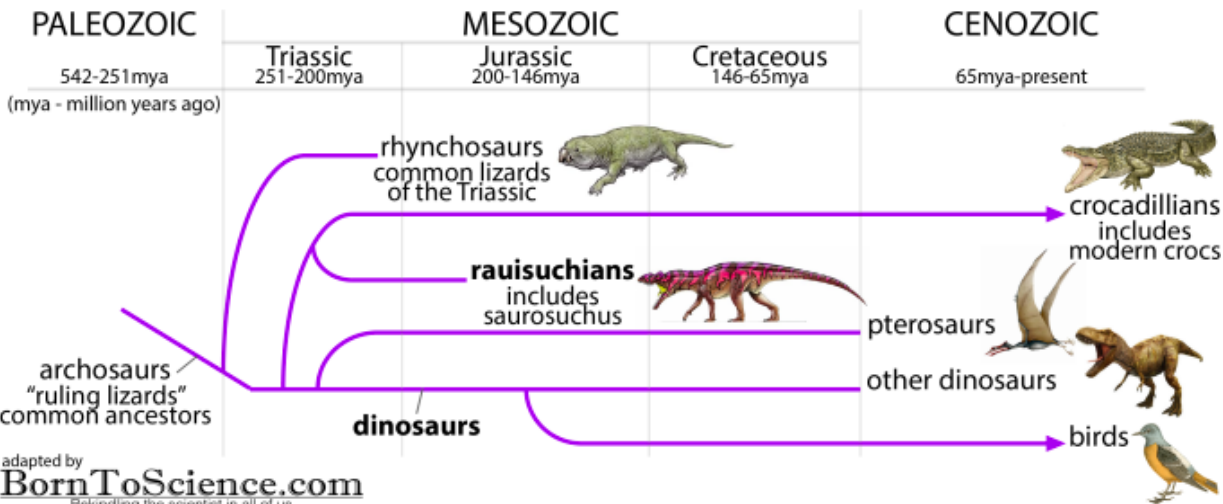
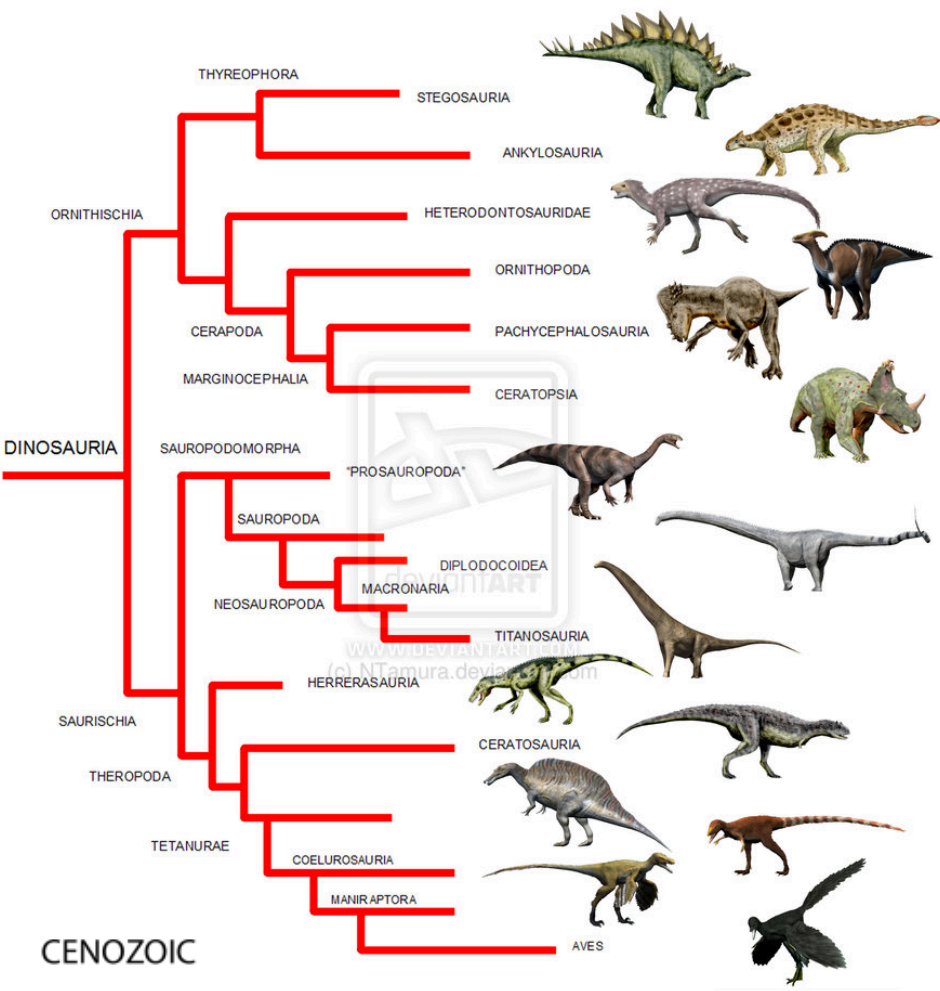
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# Some Trees





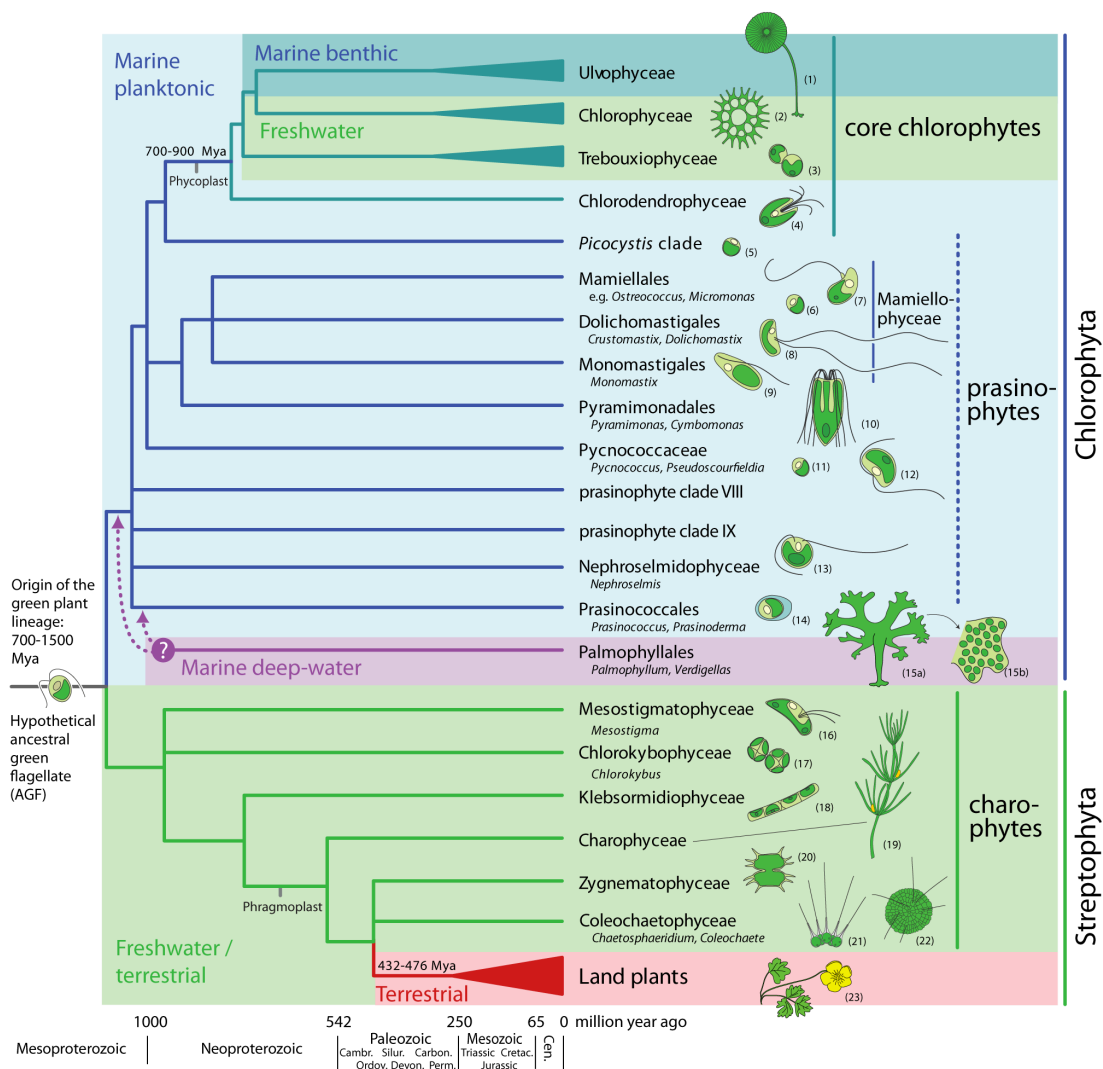
# Some Trees







# Some Trees



# Some Trees

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