Article

Efficient Tensor Sensing For RF Tomographic Imaging on GPUs

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Abstract: Radio-frequency(RF)tomographic imaging is a promising technique for inferring multi-dimensional physical space by processing RF signals traversed across a region of interest. The transform-based tensor model is more considered to be more appropriate than conventional model based on vector. The Alt-Min approach proposed by Deng. demonstrate significanct improvement in recovery error and convergence sppeed compared to prior tensor-based compressed sensing. However, the running time of Tubal-Alt-Min increases exponentially with the dimension of tensors, thus not very practical for medium- or large-scale tensors. In this paper, we address this problem by exploiting massively parallel GPUs. We design, implement and optimize the Alt-Min algorithm on a GPU and evaluate the performance in terms of running time, recovery error, and convergence speed.

Keywords: keyword 1; keyword 2; keyword 3 (list three to ten pertinent keywords specific to the article, yet reasonably common within the subject discipline.)

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1. Introduction

2. Background overview

We first formulate the RF tomographic imaging task as a tensor sensing problem, then review the Alt-Min algorithm.

2.1. Notations and Problem Formulation

We use lowercase boldface letter $\mathbf{x} \in \mathbb{R}^{N_1}$ to denote a vector, uppercase boldface letter $\mathbf{X} \in \mathbb{R}^{N_1 \times N_2}$ to denote a matrix, and calligraphic letter $\mathcal{X} \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ to denote a tensor. $[k]$ denotes the set $\{1, 2, \ldots, k\}$. Let $\mathcal{A} \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ denote a thirdorder tensor. $\mathcal{A}(:, j, k)$, $\mathcal{A}(i, :, k)$, $\mathcal{A}(i, j\ :)$ denote mode-1, mode-2, mode-3 tubes of $\mathcal{A}$, and $\mathcal{A}(:, :, k)$, $\mathcal{A}(:, j, :)$ $\mathcal{A}(i, :, :)$ denote the frontal, lateral, and horizontal slices. The Frobenius norm of $\mathcal{A}$ is defined as $\|\mathcal{A}\|_F = \sqrt{\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \sum_{k=1}^{N_3} \mathcal{A}_{ijk}^2}$. The operator vec(.) transforms tensors and matrices into vectors. Let $\mathbf{X}^T$ and $\mathcal{A}^\dagger$ denote the transposes of a matrix and a tensor, respectively.

Definition 2.1. Given an invertible discrete transform $\mathcal{L} : \mathbb{R}^{1\times 1\times N_3} \to \mathbb{R}^{1\times 1\times N_3}$, the elementwise multiplication $\circ$, and $\mathbf{a}, \mathbf{b} \in \mathbb{R}^{1\times 1\times N_3}$, the tubal-scalar multiplication is defined as:

$$\mathbf{a} \bullet \mathbf{b} = \mathcal{L}^{-1}(\mathcal{L}(\mathbf{a}) \circ \mathcal{L}(\mathbf{b})) \tag{1}$$

Definition 2.2. The $\mathcal{L}$-product $\mathcal{A} = \mathcal{B} \bullet \mathcal{C}$ of $\mathcal{B} \in \mathbb{R}^{N_1\times r\times N_3}$ and $\mathcal{C} \in \mathbb{R}^{r\times \times N_2\times N_3}$ is a tensor of size $N_1 \times N_2 \times N_3$, $\mathcal{A}(i, j, :) = \sum_{s=0}^{r} \mathcal{B}(i, s, :) \bullet \mathcal{C}(s, j, :)$, for $i \in [N_1]$ and $j \in [N_2]$.

Definition 2.3. The transform domain singular value decomposition $\mathcal{L}$-SVD of $\mathcal{A} \in \mathbb{R}^{N_1\times N_2\times N_3}$ is given by $\mathcal{A} = \mathcal{U} \bullet \mathcal{S} \bullet \mathcal{V}^\dagger$, where $\mathcal{U}$ and $\mathcal{V}$ are $\mathcal{L}$-orthogonal tensors of size $N_1 \times N_1 \times N_3$ and $N_2 \times N_2 \times N_3$ respectively, and $\mathcal{S}$ is a diagonal tensor of size $N_1 \times N_2 \times N_3$. The entries of $\mathcal{S}$ are called the singular values of $\mathcal{A}$, and the number of non-zero ones is called the $\mathcal{L}$-rank of $\mathcal{A}$.

We stack the RF signal measurements $\mathbf{y}_m$ into a measurement vector $\mathbf{y} \in \mathbb{R}^M$. Given linear measurements $\mathbf{y}_m = \langle \mathcal{X}, \mathcal{A} \rangle = (\mathrm{vec}(\mathcal{A}))^T \cdot \mathrm{vec}(\mathcal{X}), 1 \le m \le M$ of a loss field tensor $\mathcal{X} \in \mathbb{R}^{N_1\times N_2\times N_3}$ with $\mathcal{L}$-rank r and the sensing tensors $\mathcal{A}_m \in \mathbb{R}^{N_1\times N_2\times N_3}$. With a linear map $\mathcal{H}(\cdot) : \mathbb{R}^{N_1\times N_2\times N_3} \to \mathbb{R}^M$. We get:

$$\mathbf{y} = \mathcal{H}(\mathcal{X}) + \mathbf{w}, \tag{2}$$

where $\mathbf{w}$ denotes the noise vector.

The goal of RF tomographic imaging is to recover the loss field tensor $\mathcal{X}$ from the measurement vector $\mathbf{y}$. We formulate this problem as a low $\mathcal{L}$-rank tensor sensing problem:

$$\widehat{\mathcal{X}} = \underset{\mathcal{X}\in\mathbb{R}^{N_1\times N_2\times N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}(\mathcal{X})\|_F^2, \text{s.t.} \mathrm{rank}(\mathcal{X}) \le r. \tag{3}$$

2.1.1. The Alt-min Algorithm

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procedure 1 Alt-min: AM($\mathcal{H}(\cdot), \mathbf{y}, r, L$)

Input: linear map $\mathcal{H}(\cdot)$, measurement vector $\mathbf{y}$, $\mathcal{L}$-rank r, iteration number L.

Initialize $\mathcal{U}^0$ randomly;
for $\ell = 1$ to L **do**
  $\mathcal{V}^\ell \leftarrow \mathrm{LS}(\mathcal{H}(\cdot), \mathcal{U}^{\ell-1}, \mathbf{y}, r)$
  $\mathcal{U}^\ell \leftarrow \mathrm{LS}(\mathcal{H}(\cdot), \mathcal{V}^\ell, \mathbf{y}, r)$
end for

Output: : Pair of tensors $(\mathcal{U}^L, \mathcal{V}^L)$.

To enable alternating minimization, we represent the loss field tensor as the $\mathcal{L}$-product of two smaller tensors, i.e., $\mathcal{X} = \mathcal{U} \bullet \mathcal{V}$, $\mathcal{X} \in \mathbb{R}^{N_1\times N_2\times N_3}$, $\mathcal{X} \in \mathbb{R}^{N_1\times r\times N_3}$ and $\mathcal{X} \in \mathbb{R}^{r\times N_2\times N_3}$. Then we reformulate equation (**??**) as the following non-convex optimization problem:

$$\widehat{\mathcal{X}} = \underset{\mathcal{U}\in\mathbb{R}^{N_1\times r\times N_3}, \mathcal{V}\in\mathbb{R}^{r\times N_2\times N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}(\mathcal{U} \bullet \mathcal{V})\|_F^2, \tag{4}$$

The main idea if Alt-min is to iteratetively estimate two low $\mathcal{L}$-rank tensors $\mathcal{U}$ and $\mathcal{V}$, each of $\mathcal{L}$ rank r. The key step is least squares(LS) minimization. The detailed implemention of LS minimization are given below.

procedure 2 Least Squares Minimization: LS($\mathcal{H}(\cdot)$,$\mathcal{U}$,r,$\mathbf{Y}$)

Input: linear map $\mathcal{H}(\cdot)$, tensor $\mathcal{U} \in \mathbb{R}^{N_1 \times r \times N_3}$, measurement vector $\mathbf{y}$, $\mathbf{L}$-rank r.

$$\widehat{\mathcal{V}} = \underset{\mathcal{V} \in \mathbb{R}^{r \times N_2 \times N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}(\mathcal{U} \bullet \mathcal{V})\|_F^2,$$

Output: tensor $\mathcal{V}$

We use the operater circ($\bullet$) to map circulants to their corresponding circular matrices which are tagged with the superscript c, i.e. $\underline{\alpha}^c$, $\underline{\mathbf{A}}^c$:

$$\underline{\alpha}^c = \text{circ}(\underline{\alpha}) = \begin{bmatrix} \alpha_1 & \alpha_{N_3} & \cdots & \alpha_2 \\ \alpha_2 & \alpha_1 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \alpha_{N_3} \\ \alpha_{N_3} & \alpha_{N_3-1} & \cdots & \alpha_1 \end{bmatrix}$$

$$\underline{\mathbf{A}}^c = \text{circ}(\underline{\mathbf{A}}) = \begin{bmatrix} \text{circ}(\underline{\mathbf{A}_{1,1}}) & \cdots & \text{circ}(\underline{\mathbf{A}}_{1,N_2})) \\ \vdots & \vdots & \vdots \\ \text{circ}(\underline{\mathbf{A}_{N_1,1}}) & \cdots & \text{circ}(\underline{\mathbf{A}}_{N_1,N_2})) \end{bmatrix}$$

For simplicity, we use $\mathcal{A}^c$ to represent the circular matrix of tensor $\mathcal{A}$.Then the $\mathcal{L}$-product $\mathcal{X} = \mathcal{U} \bullet \mathcal{V}$ has an equivalent matrix-product as:

$$\mathbf{X}^c = \mathbf{U}^c \mathbf{V}^c, \tag{5}$$

where $\mathbf{X}^c \in \mathbb{R}^{N_1 N_3 \times N_2 N_3}$, $\mathbf{U}^c \in \mathbb{R}^{N_1 N_3 \times r N_3}$, $\mathbf{V}^c \in \mathbb{R}^{r N_3 \times N_2 N_3}$. We can transform the LS minimization in Alg.2 to the corresponding circular matrix representation:

$$\widehat{\mathbf{V}}^c = \underset{\mathbf{V}^c \in \mathbb{R}^{r N_3 \times N_2 N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}(\mathbf{U}^c \mathbf{V}^c)\|_F^2, \tag{6}$$

where $\mathcal{H}^c(\bullet) : \mathbb{R}^{N_1 N_2 \times N_2 N_3} \to \mathbb{R}^M$ is the corresponding linear map inthe circular matrix representation, with $\mathbf{y} = \mathcal{H}^c(\mathbf{X}^c)$. Each sensing tensor $\mathcal{A}_m$ is transfomed into its circular matrix $\mathbf{A}_m^c \in \mathbb{R}^{N_1 N3 \times N_2 N_3}$, and $\mathbf{y}_m = \langle \mathbf{X}^c, \mathbf{A}_m^c \rangle$, $1 \leq m \leq M$. Similarly, we can estimate $\mathbf{U}^c$ in the follow way:

$$\widehat{\mathbf{U}}^c = \underset{\mathbf{U} \in \mathbb{R}^{r N_3 \times N_2 N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}(\mathbf{U}^c \mathbf{V}^c)\|_F^2. \tag{7}$$

We perform the following steps to solve this non-convex optimization problem:

Step 1). $\mathbf{U}^c$ is used to form a block diagonal matrix $\mathbf{B}_1$ of size $N_1 N_2 N_3^2 \times r N_2 N_3^2$, and the number of $\mathbf{U}^c$ is $N_2 N_3$,

$$\mathbf{B}_1 = \begin{bmatrix} \mathbf{U}^c & & & \\ & \mathbf{U}^c & & \\ & & \ddots & \\ & & & \mathbf{U}^c \end{bmatrix}$$

Step 2). Stack all the columns of $\mathbf{V}^c$, and then $\mathbf{V}^c$ is vectorized to a vector $\mathbf{b}$ of size $r N_2 N_3 N_3^2 \times 1$ as follows:

$$\mathbf{b} = \text{vec}(\mathbf{V}^c) = [\mathbf{V}^c(:,1)^T, \mathbf{V}^c(:,2)^T, \ldots, \mathbf{V}^c(:,N_2 N_3)^T]^T.$$

Step 3). Each $\mathbf{A}_m^c$, $1 \le m \le M$ is represented as a vector $\mathbf{c}_m$ of size $N_1 N_2 N_3^2 \times 1$ in the following way:

$$\mathbf{c}_m = \mathrm{vec}(\mathbf{A}_m^c) = [\mathbf{A}_m^c(:,1)^T, \mathbf{A}_m^c(:,2)^T, \ldots, \mathbf{A}_m^c(:,N_2N_3)^T]^T. \tag{8}$$

and then all the $\mathbf{c}_m$ are transfomed into a matrix $\mathbf{B}_2$ of size $M \times N_1 N_2 N_3^2$:

$$\mathbf{B}_2 = [\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M]^T$$

Therefore, the estimation of $\mathbf{V}^c$ is transformed into the following standard least squares minimization problem:

$$\widehat{\mathbf{b}} = \underset{\mathbf{b} \in \mathbb{R}^{rN_2N_3^2 \times 1}}{\arg\min} \ \|\mathbf{y} - \mathbf{B}_2\mathbf{B}_1 b)\|_F^2, \tag{9}$$

2.2. *Algorithm implemention on GPU*

procedure 3 Tensor Sensing on CPU

Input: randomly initialized $\mathbf{U}_0$, measurement vector $\mathbf{y} \in \mathbf{R}^M$, matrix $\mathbf{A} \in \mathbb{R}^{d \times N_1 N_2 N_3}$ converted from M sensing tensors $\mathcal{A}_m$ and the corresponding transpose matrix $\mathbf{A}_t$

Output: $\mathbf{X}_s \in \mathbb{R}^{N_1 N_3 \times N_2}$

for $i = 1 \to IterNum$ **do**
  $\mathbf{U}_d = \mathrm{diag}(\mathbf{U})$
  $\mathbf{W} = \mathbf{A}\mathbf{U}_d$
  $\mathbf{V}_v = \mathbf{W} \backslash \mathbf{y}$
  $\mathrm{clear}(\mathbf{W})$
  $\mathbf{V} = \mathrm{Vec2Mat}(\mathbf{V}_v)$
  $\mathbf{V}_t = \mathrm{transpose}(\mathbf{V})$
  $\mathbf{V}_{t_d} = \mathrm{diag}(\mathbf{V}_t)$
  $\mathbf{W} = \mathbf{A}_t \mathbf{V}_{t_d}$
  $\mathbf{U}_v = \mathbf{W} \backslash \mathbf{y}$
  $\mathrm{clear}(\mathbf{W})$
  $\mathbf{U} = \mathrm{transpose}(\mathrm{Vec2Mat}(\mathbf{U}_v))$
end for
return $\mathbf{X} = \mathbf{U}\mathbf{V}$

2.2.1. Optmization of Alt-min

In circular algebra, it is obvious that the first column of $\underline{\alpha}^c$ already contains all the entries of itself, and there is no need to recover the redundant information.We only need to recover the first column of each circ($\underline{\mathbf{X}}_{i,j}$). We use the Matlab function squeeze(•) to get a new definition:

$$\mathbf{X}^s = \begin{bmatrix} \mathrm{squeeze}(\mathcal{X}(1,1,:)) & \cdots & \mathrm{squeeze}(\mathcal{X}(1,N_2,:)) \\ \vdots & \ddots & \vdots \\ \mathrm{squeeze}(\mathcal{X}(N_1,1,:)) & \cdots & \mathrm{squeeze}(\mathcal{X}(N_1,N_2,:)) \end{bmatrix}$$

where squeeze($\mathcal{X}(i, j :)$) transforms the i-th tube of the j-th lateral slice of $\mathcal{X}$ into a vector of size $N_3 \times 1$.

We use the notation $\Leftrightarrow$ todenote a new mapping for $\mathcal{L}$-product as follows:

$$\mathcal{X} = \mathcal{U} \bullet \mathcal{V} \Leftrightarrow \mathbf{X}^s = \mathbf{U}^c \mathbf{V}^s,$$

where $\mathbf{X}^s \in \mathbf{R}^{N_1 N_3 \times N_2}$, $\mathbf{U}^c \in \mathbf{R}^{N_1 N_3 \times N_3}$, $\mathbf{V}^s \in \mathbf{R}^{r N_3 \times N_2}$. We can transform the LS minimization in Alg.2 to the following representation:

$$\widehat{\mathbf{V}}^s = \underset{\mathbf{V}^s \in \mathbb{R}^{r N_3 \times N_2 N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}^s(\mathbf{U}^c \mathbf{V}^s)\|_F^2, \tag{10}$$

where $\mathcal{H}^s(\bullet) : \mathbb{R}^{N_1 N_3 \times N_2} \to \mathbb{R}^M$ is the corresponding linear map, with $\mathbf{y} = \mathbf{H}^s(\mathbf{X}^s), \mathbf{y}_m = \langle \mathbf{X}^s, \mathbf{A}_m^s \rangle$, $1 \leq m \leq M$. Similarly, we can estimate $\mathbf{U}^c$ in the following way:

$$\widehat{\mathbf{U}}^c = \underset{\mathbf{U}^c \in \mathbb{R}^{N_1 N_3 \times r N_3}}{\arg\min} \|\mathbf{y} - \mathcal{H}^{s^T}(\mathbf{U}^{c^T} \mathbf{V}^{s^T})\|_F^2, \tag{11}$$

So we can get the pseudocode of the tensor sensing as following:

2.2.2. Data Struct

In procedure 3, after least squares minimization(line 4 and 10), we get vectorized matrices. For a matrix $\mathbf{X} \in \mathbb{R}^{m \times n}$, the corresponding vectorized $\mathbf{X}$ is $\mathbf{X}_v \in \mathbb{R}^{mn \times 1}$. And the vectorized matrices are converted back to the original matrices(line 5 and 11). In some scientific computing programming languages, such as Matlab, this conversion must be done with the appropriate conversion function. We adopt the column-first storage format to store matrices and vectors, which not only ensures read and write continuity but also avoids explicit vector-to-matrix conversions that occur in original Matlab code because in this format $\mathbf{X}_v$ and $\mathbf{X}$ are the same in memory.

2.2.3. Multiply of Block Diagonal Matrices

Using the operational properties of the block matrix we get:

$$[\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_{N_2}] \begin{bmatrix} \mathbf{U}^c & & & \\ & \mathbf{U}^c & & \\ & & \ddots & \\ & & & \mathbf{U}^c \end{bmatrix} = [\mathbf{A}_1 \mathbf{U}^c, \mathbf{A}_2 \mathbf{U}^c, \cdots, \mathbf{A}_{N_2} \mathbf{U}^c] \tag{12}$$

It shows that Multiplication of block diagonal matrices can be transformed into a batch of small matrix multiplications. As we use column-first format to store $\mathbf{A}$, the batch of $\mathbf{A}_i$ are stored in constant stride. Let p indicate the location of the first element of $\mathbf{A}_0$, then the location of the first element of $\mathbf{A}_i$ is $p + i \times N_1 N_3$. We adopt $cublas < t > gemmStridedBatchd()$ function in cuBlas Library to achieve parallelism of operations.

2.2.4. Eliminate transpose operations

We noticed that after each Least Squares method, the transpose of the target matrix is obtained. The transpose operation of the matrix needs to be performed(line 7 and 12), while the transpose operation is quite complicated and will occupy more computing resources. As the operation after transpose of the matrix is multiplication of diagonal matrices and there is a parameter to control the transpose of the input matrices in the CUDA api $cublas < t > gemmStridedBatchd$, we set the parameter to perform the transpose implicitly.

Table 1. $cublas < t > gemmStridedBatchd$ parameter settings

Parameters	meaning	value
transA	operation op($\mathbf{A}$) that is non- or transpose	non-transpose
transU	operation op($\mathbf{U}$) that is non- or transpose	transpose
A	pointer to the **A** matrix corresponding to the first instance of the batch	d_A
U	pointer to the **U** matrix	d_y
W	pointer to the **W** matrix	d_W
strideA	the address offset between $\mathbf{A}_i$ and $\mathbf{A}_{i+1}$	$d \times N_1 N_3$
strideU	the address offset between $\mathbf{U}_i$ and $\mathbf{U}_{i+1}$	0
strideW	the address offset between $\mathbf{W}_i$ and $\mathbf{W}_{i+1}$	$d \times N_3$
batchNum	number of *gemm* to perform in the batch	N_2

2.2.5. Data reuse and reduce data exchange

The two main operations in the loop are completed on the device side. Frequent data exchange will seriously affect the efficiency of the program. We use data reuse strategy to reduce data exchange and reduce storage resources occupied on the device side as much as possible. In the whole loop, we only do two data exchanges. One is that the data is transferred from the CPU to the GPU at the beginning of the loop, and the final result matrix is sent back from the GPU to the CPU. All intermediate results are covered with new results.One noteworthy thing is that since we use QR decomposition to solve the least squares problem, the input vector y is covered by the result vector (U, V), so we need to re-assign the vector y each time we perform the least squares method. However, it is obviously too time-consuming to read the original data of the vector y from the CPU every time. In order to solve this problem, we pre-apply a dyL that stores the original data of the vector y. Every time we need to reassign the dy, we use the api. . In this way, since the device-to-device assignment is much faster than the data exchange from the CPU to the GPU, at the expense of a small amount of storage space (the space of the vector y is small compared to the data such as A).) This is a very good acceleration we have achieved, which will show up in the experimental results in next section.

Bulleted lists look like this:

- First bullet
- Second bullet
- Third bullet

Numbered lists can be added as follows:

1. First item
2. Second item
3. Third item

The text continues here.

2.3. Figures, Tables and Schemes

All figures and tables should be cited in the main text as Figure 1, Table 1, etc.

Figure 1. This is a figure, Schemes follow the same formatting. If there are multiple panels, they should be listed as: (**a**) Description of what is contained in the first panel. (**b**) Description of what is contained in the second panel. Figures should be placed in the main text near to the first time they are cited. A caption on a single line should be centered.

Text

procedure 4 Tensor Sensing on GPU

Input: data on CPU memory: randomly initialized $\mathbf{U}^0$, measurement vector $\mathbf{y} \in \mathbf{R}^M$, matrix $\mathbf{A} \in \mathbb{R}^{d \times N_1 N_2 N_3}$ converted from M sensing tensors $\mathcal{A}_m$ and the corresponding transpose matrix $\mathbf{A}_t$

Output: $\mathbf{X}_s \in \mathbb{R}^{N_1 N_3 \times N_2}$

apply for memory on GPU device: dy, dA, dAt, dW, dyL

data transfer: $\mathcal{A}, \mathcal{A}_t, \mathbf{y}, \mathbf{U}^0 \xrightarrow{cudaMemcpyHostToDevice} dA(\mathcal{A}), dAt(\mathcal{A}_t), dy(\mathbf{U}^0), dyL(\mathbf{y})$ ($dA(\mathcal{A})$ means that the content in dA is $\mathcal{A}$, the same below)

for $i = 0 \to IterNum$ **do**

    diagonal matrix multiplication:$dW(\text{uninitialized}, dA(\mathcal{A}), dy(\mathbf{U}^i)) \xrightarrow{cublas<t>gemmStridedBatchd} dW(\mathcal{A}\,\text{diag}(\mathbf{U}^i)), dA(\mathcal{A}), dy(\mathbf{U}^i)$

    rewrite dy with $\mathbf{y}$: $dy(\mathbf{U}^i), dyL(\mathbf{y}) \xrightarrow{cudaMemcpyDeviceToDevice} dy(\mathbf{y}), dyL(\mathbf{y})$

    least squares minimization:$dW(\mathcal{A}\,\text{diag}(\mathbf{U}^i)), dy(\mathbf{y}) \xrightarrow{cusolver<t>qr} dy(\mathbf{V}), dW(\text{uninitialized})$

    diagonal matrix multiplication:$dW(\text{uninitialized}, dAt(\mathcal{A}_t), dy(\mathbf{V}^i)) \xrightarrow{cublas<t>gemmStridedBatchd} dW(\mathcal{A}_t\,\text{diag}(\mathbf{V}^i)), dAt(\mathcal{A}_t), dy(\mathbf{V}^i)$

    rewrite dy with $\mathbf{y}$: $dy(\mathbf{V}^i), dyL(\mathbf{y}) \xrightarrow{cudaMemcpyDeviceToDevice} dy(\mathbf{y}), dyL(\mathbf{y})$

    least squares minimization:$dW(\mathcal{A}_t\,\text{diag}(\mathbf{V}^i)), dy(\mathbf{y}) \xrightarrow{cusolver<t>qr} dy(\mathbf{U}^i), dW(\text{uninitialized})$

end for

return $\mathbf{X}^s = \mathbf{UV}$

Text

Table 2. This is a table caption. Tables should be placed in the main text near to the first time they are cited.

Title 1	**Title 2**	**Title 3**
entry 1	data	data
entry 2	data	data

Text

Text

2.4. Formatting of Mathematical Components

This is an example of an equation:

$$a + b = c \tag{13}$$

Please punctuate equations as regular text. Theorem-type environments (including propositions, lemmas, corollaries etc.) can be formatted as follows:

Theorem 1. *Example text of a theorem.*

The text continues here. Proofs must be formatted as follows:

Proof of Theorem 1. Text of the proof. Note that the phrase 'of Theorem 1' is optional if it is clear which theorem is being referred to. □

The text continues here.

3. Discussion

Authors should discuss the results and how they can be interpreted in perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

4. Materials and Methods

Materials and Methods should be described with sufficient details to allow others to replicate and build on published results. Please note that publication of your manuscript implicates that you must make all materials, data, computer code, and protocols associated with the publication available to readers. Please disclose at the submission stage any restrictions on the availability of materials or information. New methods and protocols should be described in detail while well-established methods can be briefly described and appropriately cited.

Research manuscripts reporting large datasets that are deposited in a publicly available database should specify where the data have been deposited and provide the relevant accession numbers. If the accession numbers have not yet been obtained at the time of submission, please state that they will be provided during review. They must be provided prior to publication.

Interventionary studies involving animals or humans, and other studies require ethical approval must list the authority that provided approval and the corresponding ethical approval code.

5. Conclusions

This section is not mandatory, but can be added to the manuscript if the discussion is unusually long or complex.

6. Patents

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	linear dichroism

Appendix A

Appendix A.1

The appendix is an optional section that can contain details and data supplemental to the main text. For example, explanations of experimental details that would disrupt the flow of the main text, but nonetheless remain crucial to understanding and reproducing the research shown; figures of replicates for experiments of which representative data is shown in the main text can be added here if brief, or as Supplementary data. Mathematical proofs of results not central to the paper can be added as an appendix.

Appendix B

All appendix sections must be cited in the main text. In the appendixes, Figures, Tables, etc. should be labeled starting with 'A', e.g., Figure A1, Figure A2, etc.

References

1. Author1, T. The title of the cited article. *Journal Abbreviation* **2008**, *10*, 142-149, doi:xxxxx.
2. Author2, L. The title of the cited contribution. In *The Book Title*; Editor1, F., Editor2, A., Eds.; Publishing House: City, Country, 2007; pp. 32-58, ISBN.

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