Test 1 2016, questions and answers

Logic for Computing Science (Queen's University)

School of Computing
CISC/CMPE 204
Logic In Computer Science

Test # 1, Paper A

October 4, 2016

Please answer only in the answer boxes provided. You may use the back of the pages as scrap paper.

This is a closed-book test. No computers or calculators are allowed.

A reference page is provided at the end of the test. You may use only these rules of inference.

Should a question be unclear or ambiguous, you should make a reasonable interpretation and state what you have assumed.

To be eligible for re-marking, this tests must be answered entirely in indelible (unerasable) ink. If erasable ink or pencil is used, then the test will be marked exactly once.

Do not begin until instructed to do so.

For Marker Use Only

Question 1	/15
Question 2	/10
Question 3	/10
Question 4	/5
Total	**/40**

Question 1 - This question is a test of proving the validity of a simple sequent. Prove the sequent

$$\neg p \rightarrow q, \neg r \rightarrow \neg q \vdash \neg p \rightarrow r$$

Justify each step of your proof.

Answer

1:	$\neg A \rightarrow B, \neg C \rightarrow \neg B$	premises
2:	$\neg A$	assumption
3:	B	→ elim 1.1,2
4:	$\neg C$	assumption
5:	$\neg B$	→ elim 1.2,4
6:	$\perp$	¬ elim 3,5
7:	C	contra (classical) 4-6
8:	$\neg A \rightarrow C$	→ intro 2-7

Since Jape (tool used to create answers) doesn't allow for MT, an alternative and more likely solution would be to double negate B, to get $\neg\neg B$ and use MT to get $\neg\neg C$ from the that and the premise, which could be double negation elimated to get to C in the conclusion:

1. **~ A → B** **Premise**
2. **~ C → ~ B** **Premise**
3. **~ A** **Assume**
4. **B** **→e 1,3**
5. **~~ B** **~~i 4**
6. **~~ C** **MT 2,5**
7. **C** **~~e 6**
8. **~ A → C** **→i 2-7**

15 points

Question 2 - This question is a test of proving the validity of a more complicated sequent. Prove the sequent

$$p \vee q, p \ \rightarrow r, \neg s \ \rightarrow \ \neg q \vdash \ r \vee s$$

Justify each step of your proof.

Answer

1:	$A \vee B, A \rightarrow C, \neg E \rightarrow \neg B$	premises
2:	A	assumption
3:	C	$\rightarrow$ elim 1.2,2
4:	$C \vee E$	$\vee$ intro 3
5:	B	assumption
6:	$\neg E$	assumption
7:	$\neg B$	$\rightarrow$ elim 1.3,6
8:	$\bot$	$\neg$ elim 5,7
9:	E	contra (classical) 6-8
10:	$C \vee E$	$\vee$ intro 9
11:	$C \vee E$	$\vee$ elim 1.1,2-4,5-10

Alternatively, double negation introduction can be used on line 5 to give $\neg\neg B$ which then can be used with MT to give $\neg\neg E$ which then using double negation elimination to reach the E on line 9.

10 points

Question 3 - This question tests your ability to establish logical equivalence. Consider the two formulas

$$p \wedge (q \vee r) \ \ and\ q \vee (p \wedge r)$$

If these formulas are equivalent, prove the equivalence using the rules of deduction.

If they are not equivalent, provide a model of $\{p, q, r\}$ such that one formula is true and the other is false.

If they are equivalent, justify each step of your proof; if they are not, provide a complete truth table.

Answer

p	q	r	$(q \vee r)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$q \vee (p \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	T	T	T	T
T	F	F	F	F	F	F
F	T	T	T	F	F	T
F	T	F	T	F	F	T
F	F	T	T	F	F	F
F	F	F	F	F	F	F

Rows 5 or 6 (highlighted) are models of when these formulas are not equivalent.

That is to say:

p – false		**p – false**
q – true	**or**	**q – true**
r – true		**r – false**

10 points

Question 4 - This question is a test of a complicated proof. Prove the sequent

$$s \vee (r \wedge \neg q), (s \vee r) \rightarrow (p \vee \neg q) \vdash q \rightarrow p$$

Justify each step of a proof.

Answer

1:	$D \vee (C \wedge \neg B), (D \vee C) \rightarrow (A \vee \neg B)$	premises
2:	D	assumption
3:	$D \vee C$	$\vee$ intro 2
4:	$A \vee \neg B$	$\rightarrow$ elim 1.2,3
5:	B	assumption
6:	A	assumption
7:	$\neg B$	assumption
8:	$\perp$	$\neg$ elim 5,7
9:	A	contra (constructive) 8
10:	A	$\vee$ elim 4,6-6,7-9
11:	$B \rightarrow A$	$\rightarrow$ intro 5-10
12:	$C \wedge \neg B$	assumption
13:	$\neg B$	$\wedge$ elim 12
14:	$A \vee \neg B$	$\vee$ intro 13
15:	B	assumption
16:	A	assumption
17:	$\neg B$	assumption
18:	$\perp$	$\neg$ elim 15,17
19:	A	contra (constructive) 18
20:	A	$\vee$ elim 14,16-16,17-19
21:	$B \rightarrow A$	$\rightarrow$ intro 15-20
22:	$B \rightarrow A$	$\vee$ elim 1.1,2-11,12-21

The following are other acceptable proofs:

1:	$D \vee (C \wedge \neg B), (D \vee C) \rightarrow (\neg B \vee A)$	premises
2:	D	assumption
3:	$D \vee C$	$\vee$ intro 2
4:	$C \wedge \neg B$	assumption
5:	C	$\wedge$ elim 4
6:	$D \vee C$	$\vee$ intro 5
7:	$D \vee C$	$\vee$ elim 1.1,2-3,4-6
8:	$\neg B \vee A$	$\rightarrow$ elim 1.2,7
9:	B	assumption
10:	$\neg B$	assumption
11:	$\perp$	$\neg$ elim 9,10
12:	A	contra (constructive) 11
13:	A	assumption
14:	A	$\vee$ elim 8,10-12,13-13
15:	$B \rightarrow A$	$\rightarrow$ intro 9-14

1:	$D \vee (C \wedge \neg B), (D \vee C) \rightarrow (\neg B \vee A)$	premises
2:	D	assumption
3:	$D \vee C$	$\vee$ intro 2
4:	$C \wedge \neg B$	assumption
5:	C	$\wedge$ elim 4
6:	$D \vee C$	$\vee$ intro 5
7:	$D \vee C$	$\vee$ elim 1.1,2-3,4-6
8:	$\neg B \vee A$	$\rightarrow$ elim 1.2,7
9:	$\neg B$	assumption
10:	B	assumption
11:	$\perp$	$\neg$ elim 10,9
12:	A	contra (constructive) 11
13:	$B \rightarrow A$	$\rightarrow$ intro 10-12
14:	A	assumption
15:	B	assumption
16:	A	hyp 14
17:	$B \rightarrow A$	$\rightarrow$ intro 15-16
18:	$B \rightarrow A$	$\vee$ elim 8,9-13,14-17

Etc…

5 points