

### Test 1 2016, questions and answers

Logic for Computing Science (Queen's University)

## School of Computing CISC/CMPE 204 Logic In Computer Science

Test # 1, Paper A

October 4, 2016

Please answer only in the answer boxes provided. You may use the back of the pages as scrap paper.

This is a closed-book test. No computers or calculators are allowed.

A reference page is provided at the end of the test. You may use only these rules of inference.

Should a question be unclear or ambiguous, you should make a reasonable interpretation and state what you have assumed.

To be eligible for re-marking, this tests must be answered entirely in indelible (unerasable) ink. If erasable ink or pencil is used, then the test will be marked exactly once.

#### Do not begin until instructed to do so.

#### For Marker Use Only

Question 1	/15
Question 2	/10
Question 3	/10
Question 4	/5
Total	/40

Question 1 - This question is a test of proving the validity of a simple sequent. Prove the sequent

$$\neg p \rightarrow q, \neg r \rightarrow \neg q \vdash \neg p \rightarrow r$$

Justify each step of your proof.

# Answer 1: ¬A→B, ¬C→¬B premises 2: ¬A assumption 3: B → elim 1.1,2 4: ¬C assumption 5: ¬B → elim 1.2,4 6: ⊥ ¬ elim 3,5 7: C contra (classical) 4-6 8: ¬A→C → intro 2-7

Since Jape (tool used to create answers) doesn't allow for MT, an alternative and more likely solution would be to double negate B, to get  $\neg\neg B$  and use MT to get  $\neg\neg C$  from the that and the premise, which could be double negation elimated to get to C in the conclusion:

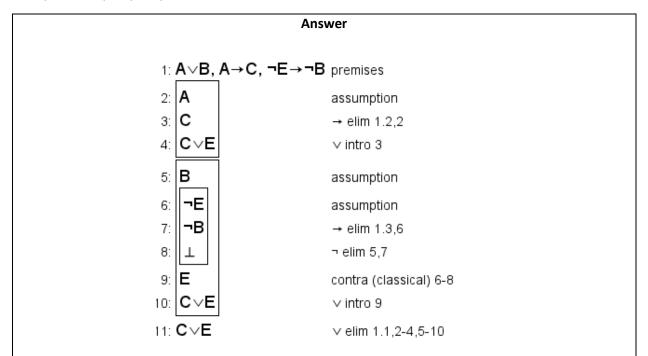
1.	~ A → B	Premise		
2.	~ C → ~ B	Premise		
3.	~ A	Assume		
4.	В	<del>→</del> e 1,3		
5.	~~ B	~~i 4		
6.	~~ C	MT 2,5		
7.	С	~~e 6		
8.	~ A → C	<del>→</del> i 2-7		

15 points

**Question 2 -** This question is a test of proving the validity of a more complicated sequent. Prove the sequent

$$p \lor q, p \rightarrow r, \neg s \rightarrow \neg q \vdash r \lor s$$

Justify each step of your proof.



Alternatively, double negation introduction can be used on line 5 to give  $\neg \neg B$  which then can be used with MT to give  $\neg \neg E$  which then using double negation elimination to reach the E on line 9.

10 points

Question 3 - This question tests your ability to establish logical equivalence. Consider the two formulas

$$p \land (q \lor r)$$
 and  $q \lor (p \land r)$ 

If these formulas are equivalent, prove the equivalence using the rules of deduction.

If they are not equivalent, provide a model of  $\{p, q, r\}$  such that one formula is true and the other is false.

If they are equivalent, justify each step of your proof; if they are not, provide a complete truth table.

#### Answer

р	q	r	$(q \lor r)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$q \lor (p \land r)$
Т	Т	Т	Т	Т	Т	Т
Т	Т	F	Т	F	Т	Т
Т	F	Т	Т	Т	Т	Т
Т	F	F	F	F	F	F
F	Т	Т	Т	F	F	Т
F	Т	F	Т	F	F	Т
F	F	Т	Т	F	F	F
F	F	F	F	F	F	F

Rows 5 or 6 (highlighted) are models of when these formulas are not equivalent.

That is to say:

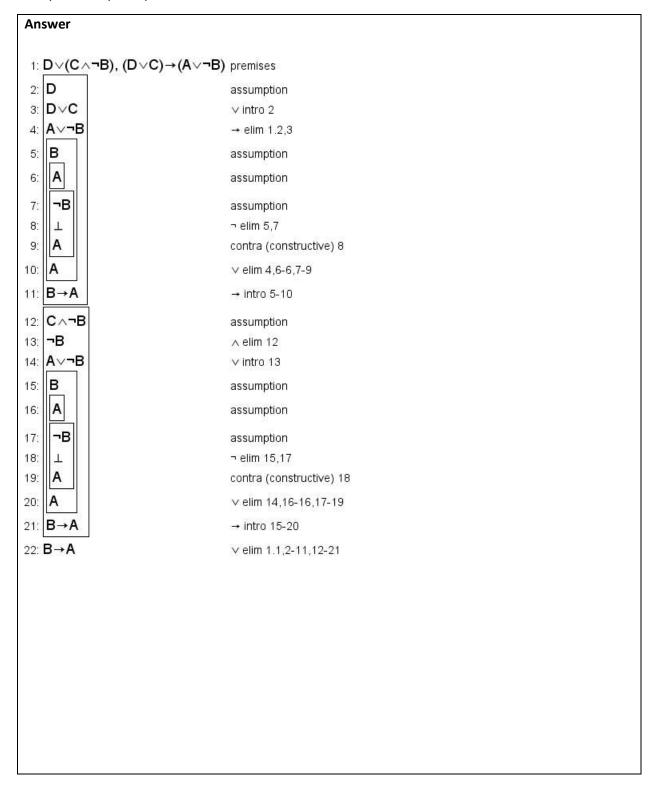
 $\begin{array}{ccc} p-false & & p-false \\ q-true & or & q-true \\ r-true & r-false \end{array}$ 

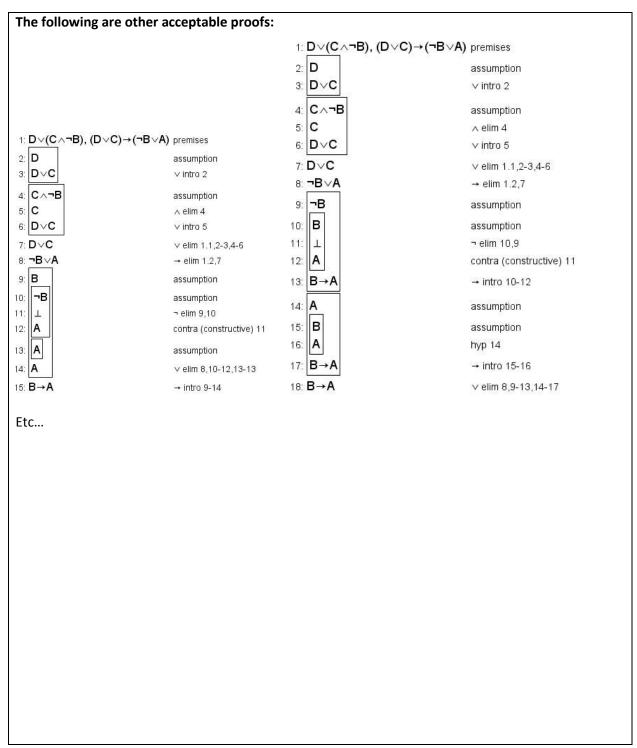
10 points

Question 4 - This question is a test of a complicated proof. Prove the sequent

$$s \lor (r \land \neg q), (s \lor r) \rightarrow (p \lor \neg q) \vdash q \rightarrow p$$

Justify each step of a proof.





5 points