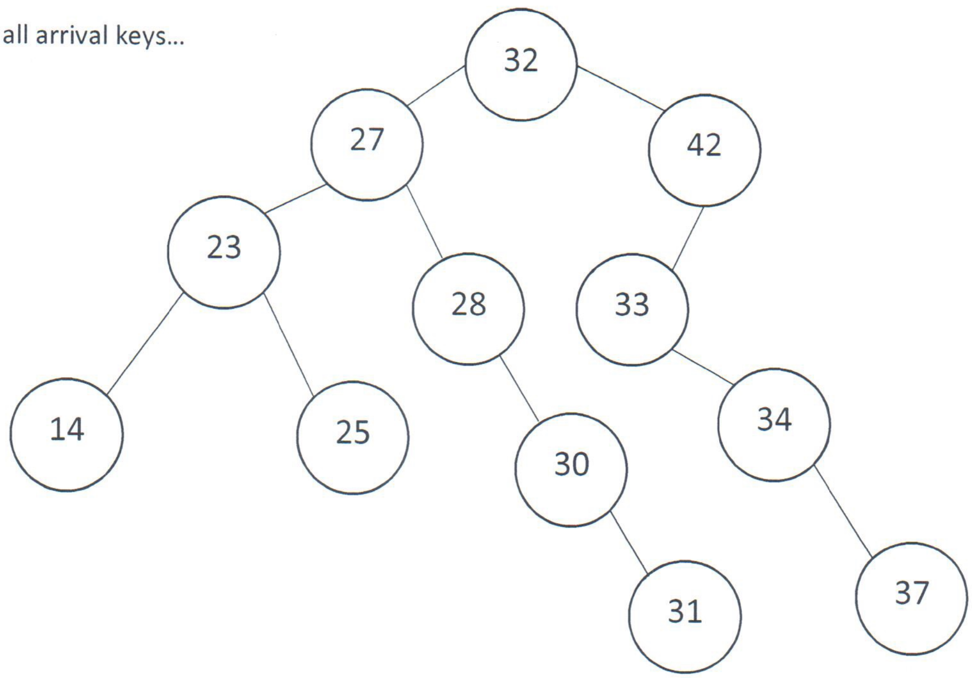
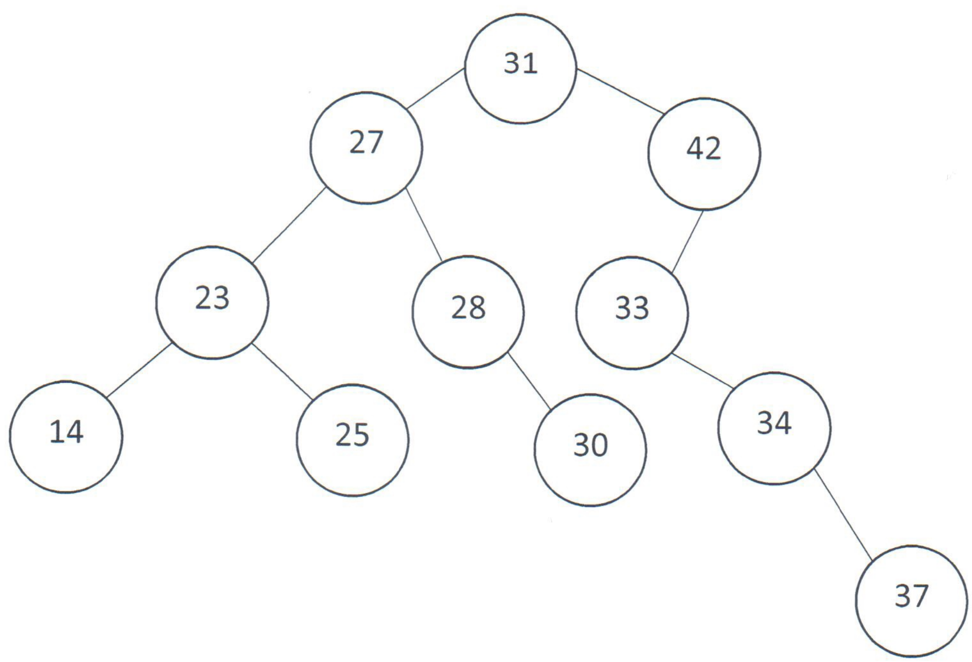
CSCE 500 Midterm Exam 2 11/4/2020

1. A binary search tree (T) is to be maintained following the in-order tree traversal order. Consider a sequence of arrival keys, {32, 27, 23, 42, 14, 25, 33, 34, 37, 28, 30, 31}, to T which is empty initially.
   a. Show the resulting T after inserting all arrival keys. (8%)
   b. Show the resulting T after its root node is then deleted (4%)
   c. Show the resulting T after deleting node with key of 27 from T obtained by (2) above. (4%)

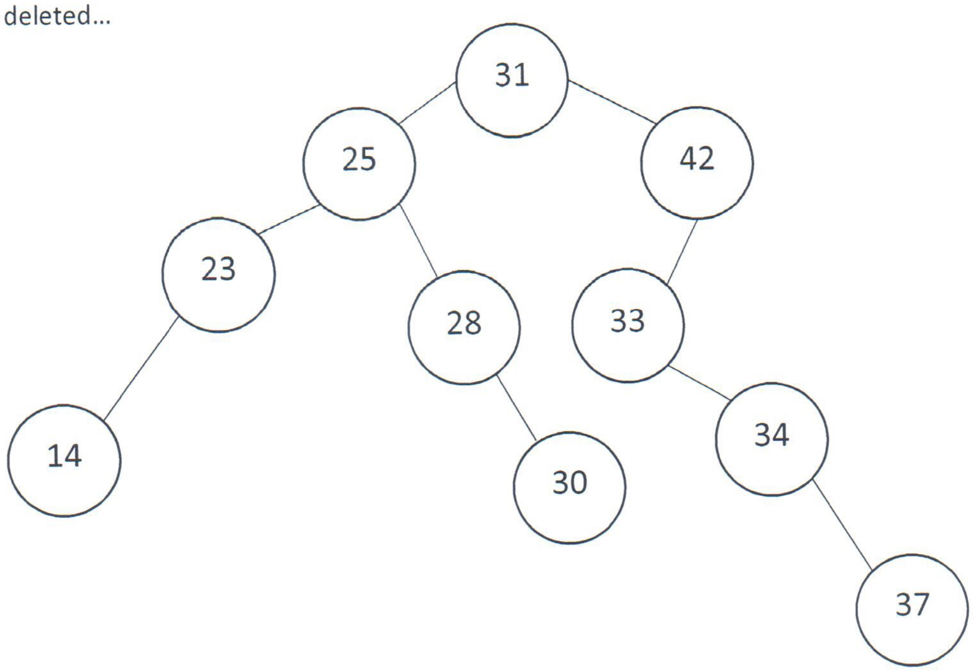
a. Inserting all arrival keys...



b. Root deleted...



c. Key 27 is deleted...

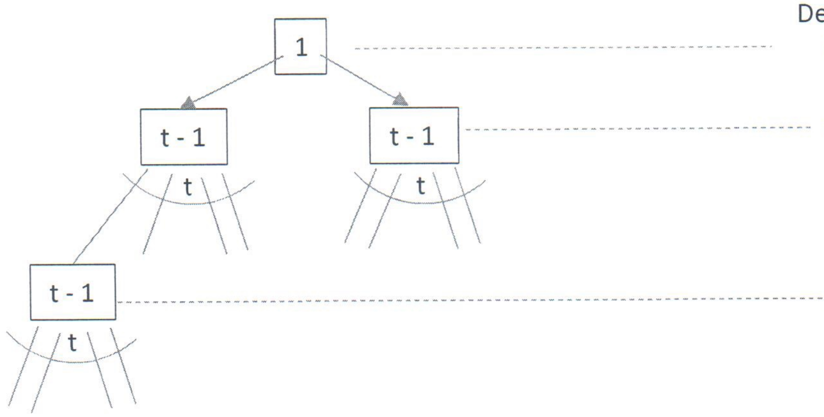


2. For a node in a binary search tree (T), how do you find its predecessor? Its successor? (8%)

   In a binary search tree (T), its predecessor is found by finding the maximum key of its left child. If it has no left child, its predecessor is the lowest ancestor whose left subtree it contains.

   In a binary search tree (T), its successor is found by finding the minimum key of its right child. If it has no right child, its successor is its parent.

3. For any n-key <u>B-tree of height h</u> and with the minimum node degree of t ≥ 2, show that h is no larger than $\log_t \frac{n+1}{2}$. (Hint: consider the number of keys stored in each tree level) (16%)



Depth	Number of Nodes
0	1
1	2
2	2t
h	$2t^{h-1}$

Let n = the total number of keys.

n ≥ number of root keys + (keys per nodes) * (total number of nodes)

$$n \geq 1 + (t-1) * \sum_{i=1}^{h} 2t^{i-1}$$

$$n \geq 1 + 2(t-1)\left[\frac{1-t^h}{1-t}\right]$$

$$n \geq 1 + 2(t-1)\left[\frac{1-t^h}{1-t}\right]$$

$$n \geq 1 + 2(t^h - 1)$$

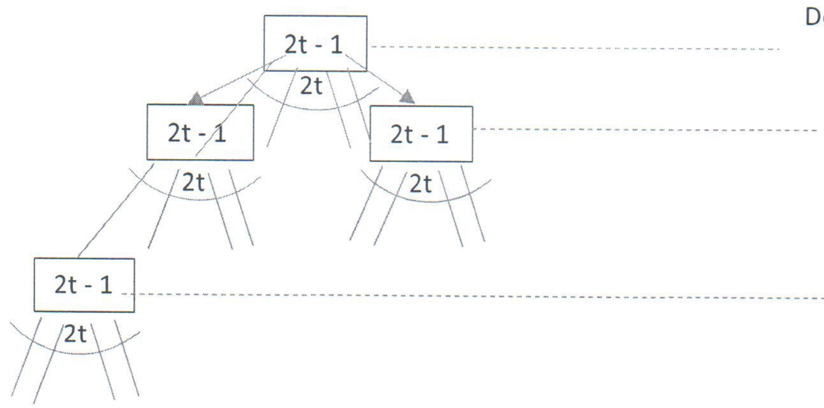
$$n \geq 1 + 2t^h - 2$$

$$n \geq 2t^h - 1$$

* Adding 1, dividing by 2 and taking the log base t yields the following result.

$$h \leq \log_t \frac{n+1}{2}.$$

4. For a given B-tree of height h and with the minimum node degree of t ≥ 2, what is the maximum number of keys held in such a B-tree? (12%)



Depth	Number of Nodes
0	1
1	$2t$
2	$(2t)^2$
j	$(2t)^j$

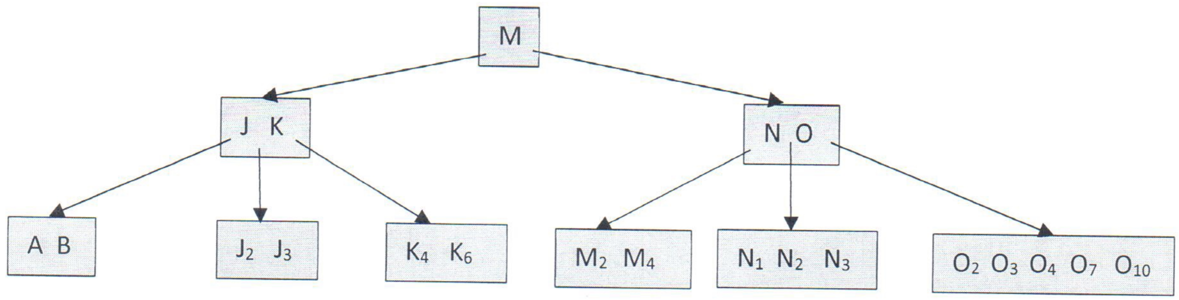
$$The\ maximum\ number\ of\ keys\colon n \leq (keys\ per\ node)(number\ of\ nodes)$$

$$n \leq (2t-1)\sum_{i=1}^{h}(2t)^i$$

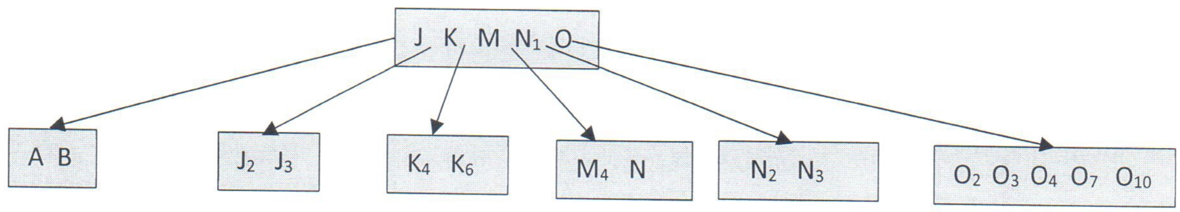
$$n \leq (2t-1)\frac{1-(2t)^{h+1}}{1-2t}$$

$$n \leq (2t)^{h+1} - 1$$

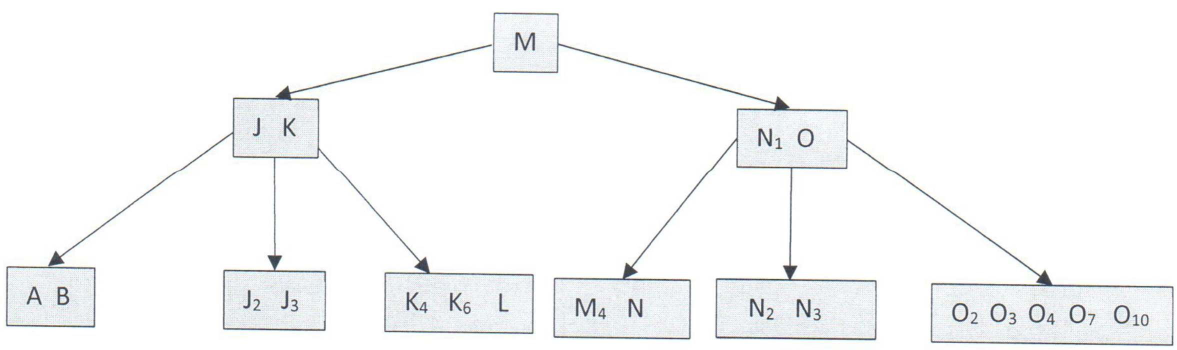
5. Given the initial B-tree with the minimum node degree of $t = 3$ below, show the results (a) after deleting the key of M_2, (b) followed by inserting the key of L (c) then by deleting the key of J_2, (d) then by inserting the key of O_5, with $O_4 < O_5 < O_7$, and (e) then by deleting K. (Show the result after each deletion and after each insertion. (20%)



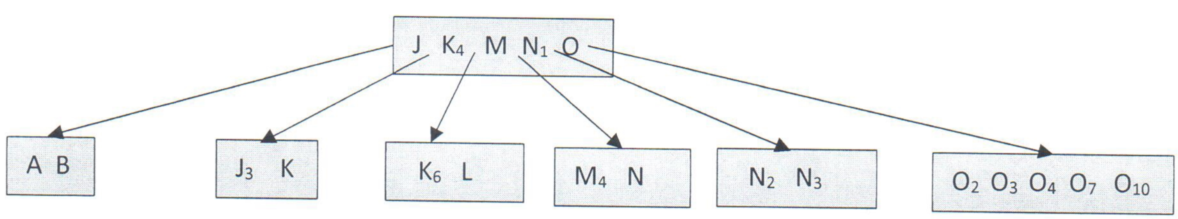
(a) Deleting M_2...



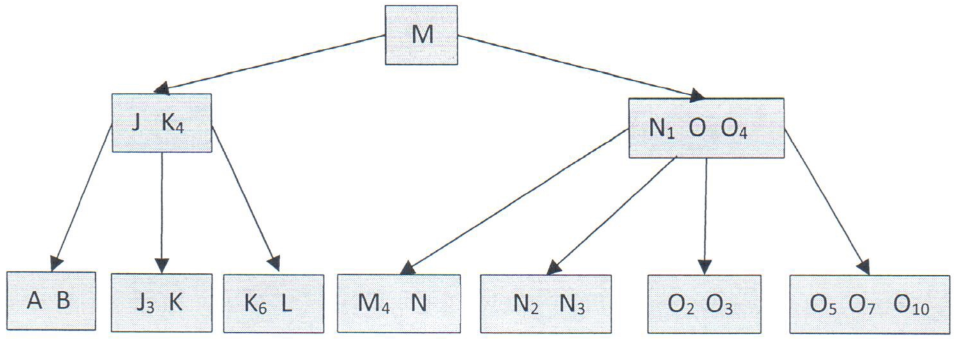
(b) Inserting L...



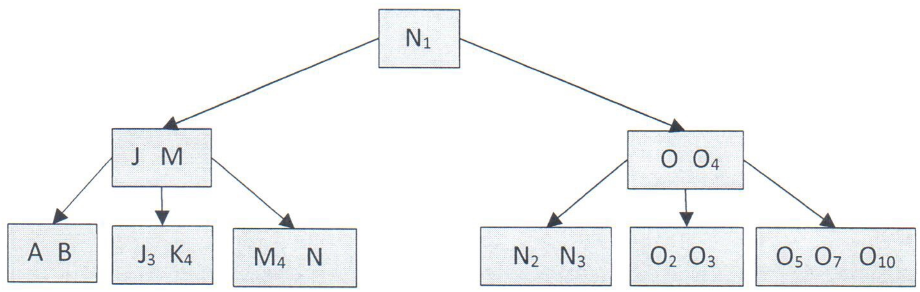
(c) Deleting J_2...



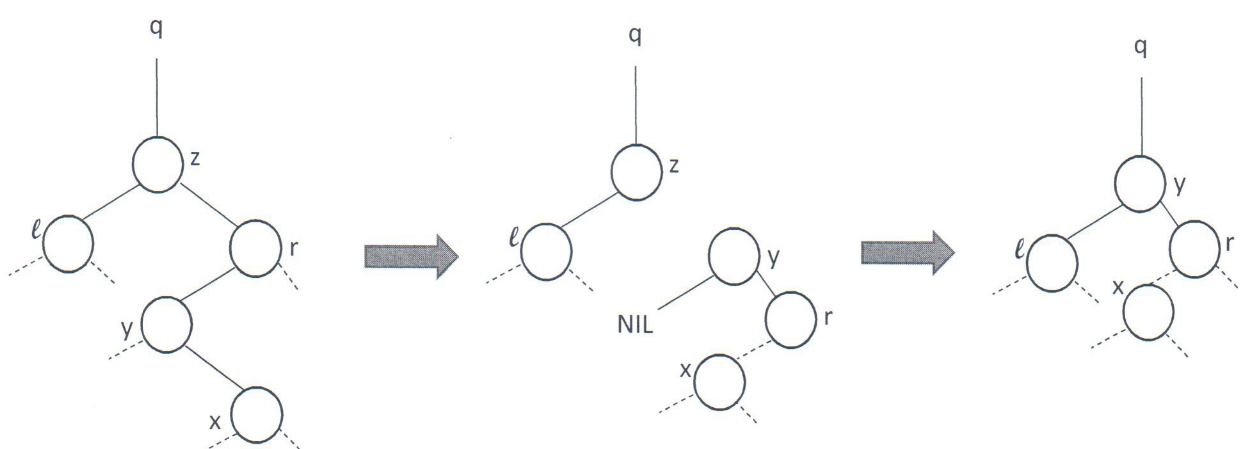
(d) Inserting key O_5...



(e) Deleting K...

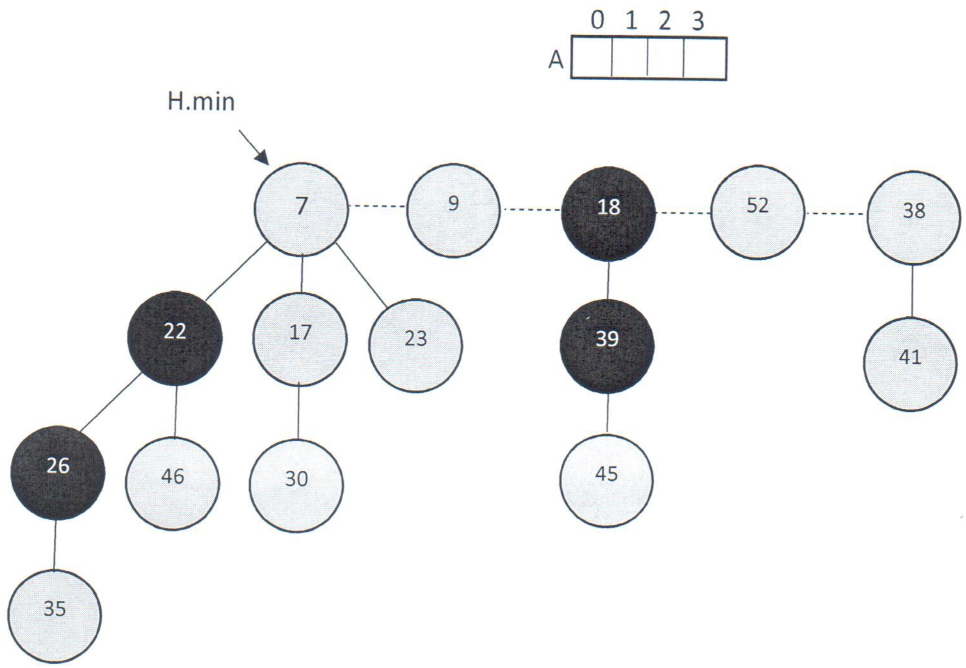


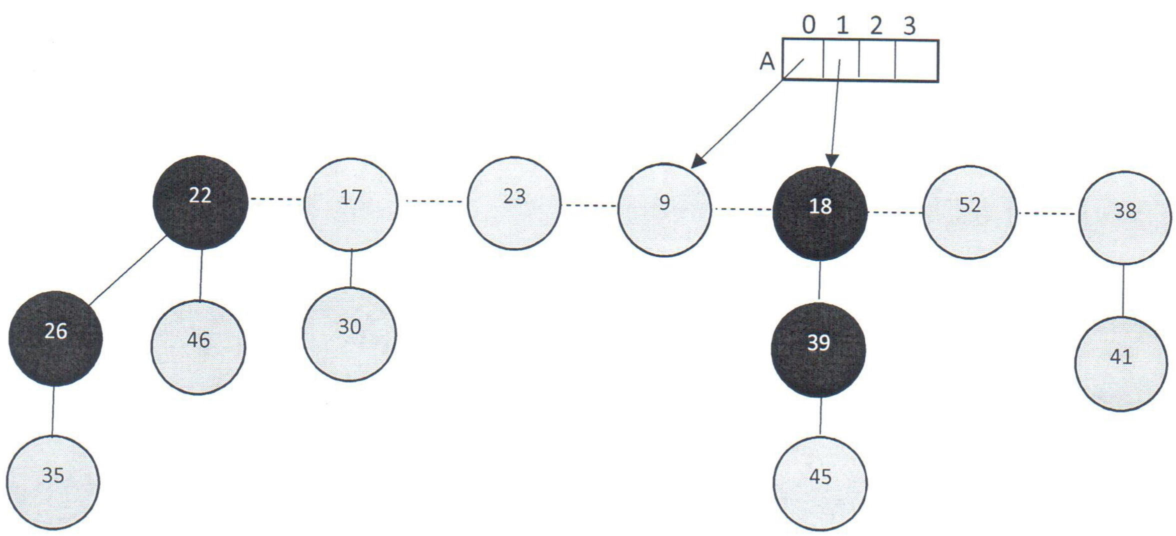
6. When the root of a binary subtree, z, is deleted, as depicted below, show the resulting subtree. (8%)

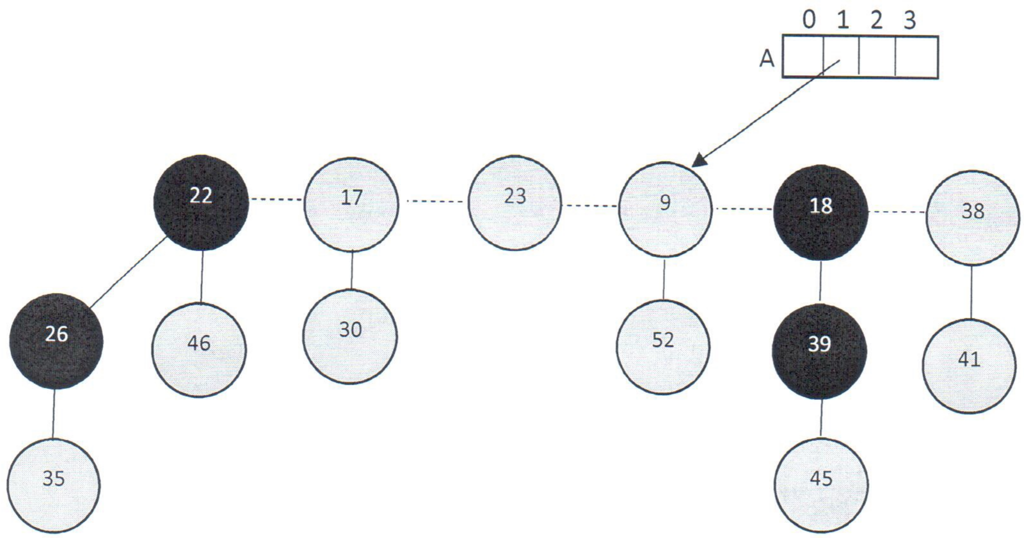


7. A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge trees in the root list upon the operation of extracting the minimum node. Show steps involved and the resulting heap after H.min is extracted form the Fibonacci min-heap given below (16%)

   After consolidation is completed, show the resulting Fibonacci min-heap with key '46' decreased to 20. (4%)







0 1 2 3
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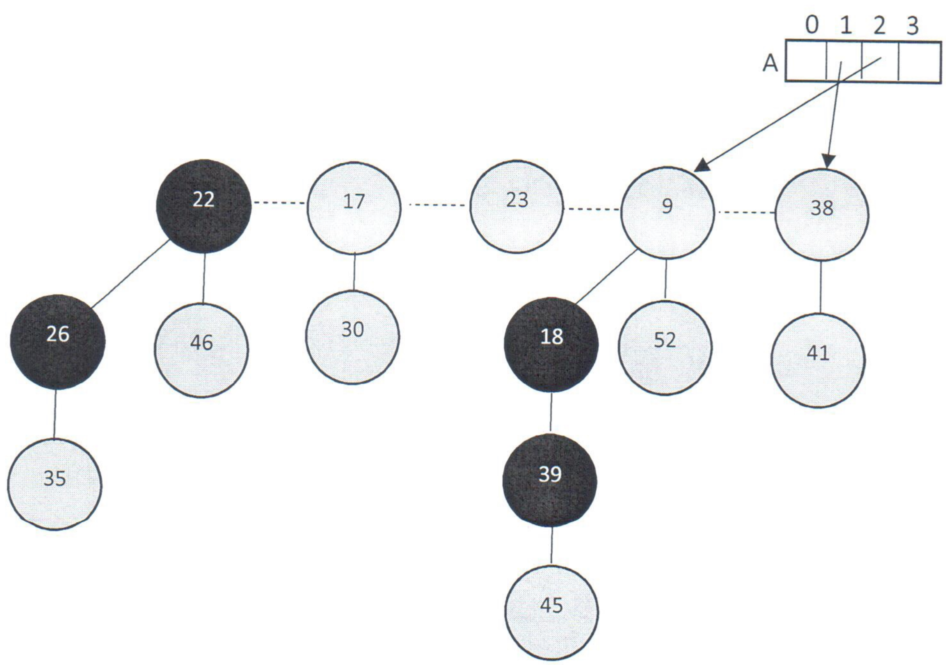

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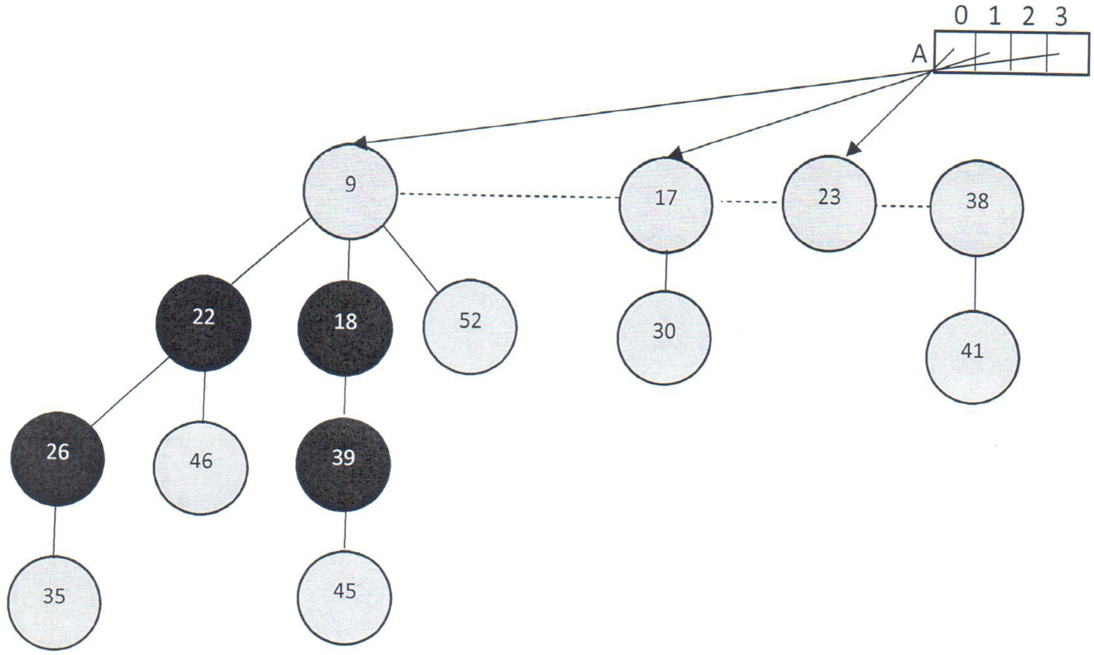
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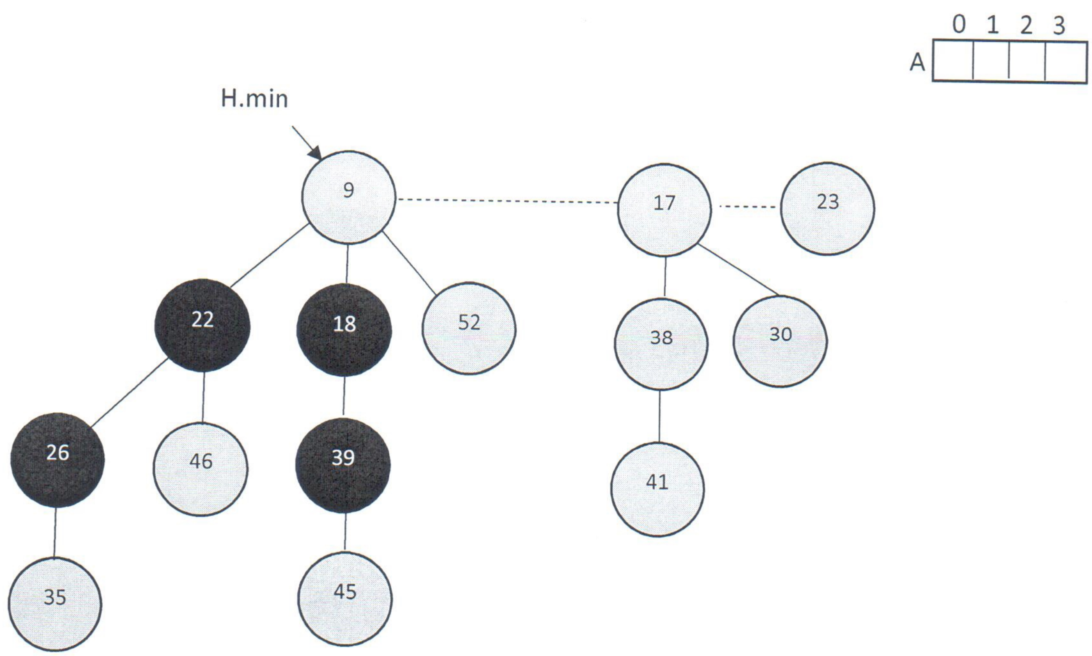
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