Ph.D. Comprehensive Examination
Algorithms and Theory of Computation

Spring, 2010

Short Questions

Answer 3 of 4 questions.

[S_1] Prove
(a) $2^n < n!$ for $n \geq 4$.
(b) Prove that the function $2^{100}\sqrt{n}$ is $O(n)$.

[S_2] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = & 2T(\frac{n}{2}) + n^3 \\ T(1) = 1 & T(2) = 4 \end{cases}$$

[S3]

(a) Explain the difference between a nondeterministic and a deterministic automaton.

(b) When converting a nondeterministic automaton of n states to a deterministic automaton, what is the maximum number of states of the deterministic automaton, and briefly why?

(c) Explain the difference between the classes P and NP. What is known and not known about their relationship?

[S4] Construct

(a) a deterministic finite automaton and a regular expression for the language

$\{ x \in \{0,1\}^* :$ neither "00" nor "11" occurs in $x \}$.

(b) a context free grammar for the language

$\{ a^n b^m : n \neq m \}$

Long Questions

Answer 3 of 4 questions.

[L_1] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Give the partial solution $X = (5, 2, -, -, -)$, calculate $B(X)$ using the reducing technique on the matrix.
b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.

[L_2] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 4, 3; and weight 7, 5, 4, 3, 2, 1 (resp.).

[L3]

(a) Outline an algorithm to convert a regular expression to a deterministic finite automaton.

(b) Illustrate the algorithm with the regular expression $(aa* + bb*)ba(bb)*$.

[L4] Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.

(a) $\{ a^n b^m c^{n+m} : n, m > 0 \}$

(b) $\{ a^n b^m c^n : n > m > 0 \}$