5. Solve the recurrence of $T(n) = 4\,T(\sqrt{n}) + (\lg n)^2$ by either substitution or the recursion-tree method. (6%)

6. The utilization efficiency of a hash table depends heavily on its hash function(s) employed. Describe with a diagram to illustrate how a multiplication method of hashing works on a machine with the word size of w bits for a hash table with 2^p entries, $p < w$. (7%)

Briefly state how a hash function can be employed for de-duplication in data archival. (3%)

7. A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge trees in the root list upon the operation of extracting the minimum node. Given the Fibonacci heap below, show the resulting heap after $H.min$ is extracted. (7%, illustrate key steps involved.)
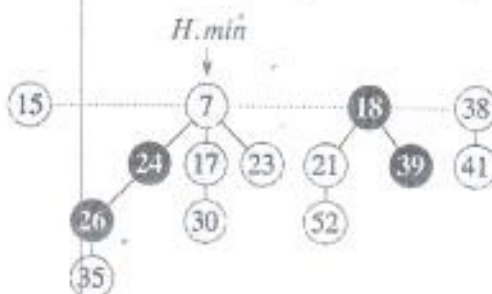
8. Given a B-tree with the minimum degree of $t = 3$ below, show the results after (i) deleting L, (ii) then followed by inserting M, and (iii) then followed by deleting B. (9%)
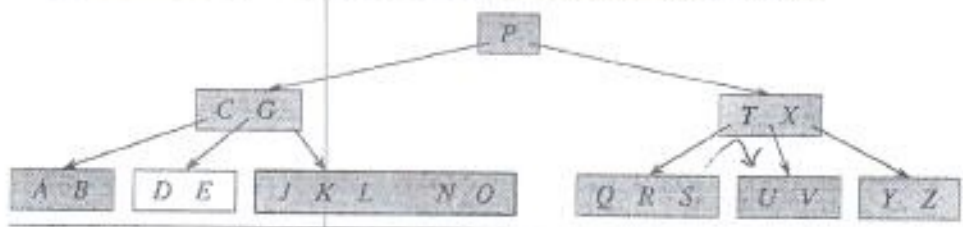
9. Sketch a proof of the Lemma below, using the tree provided. (8%)
   Let C be an alphabet in which each character $c \in C$ has frequency $c.freq$. Let x and y be two characters in C having the lowest frequencies. Then there exists an optimal prefix code for C in which the codewords for x and y have the same length and differ only in the last bit.