CSCE 500 Midterm Exam #1

1. A problem with size n follows a typical divide-and-conquer approach to have its time complexity of $T(n) = T(n/4) + T(3n/4) + c \cdot n$. Solve $T(n)$. (Show your work, 10%)

2. Solve the two recurrence expressions below: (show your work)
   $T_1(n) = 2 \cdot T_1(n^{1/2}) + \lg(n)$ (12%)
   $T_2(n) = 2 \cdot T_2(n/2) + n \cdot \lg(n)$ (12%)

3. Is n^3 an asymptotically-tight bound of $(n^{2.99}) (\lg n)$? of $(n^{3.1})/(\lg n)$? (Briefly explain. 6%)

4. Given that for an open-address hash table with load factor $\alpha = n/m < 1$, the expected number of probes in unsuccessful search under uniform hashing is at most $1/(1 - \alpha)$, how do you prove the expected number of probes in a successful probe under uniform hashing being at most $(1/\alpha) \cdot \ln(1-\alpha)^{-1}$? (Just give a proof sketch, explaining how many probes are needed to locate existing keys. 10%).

5. The utilization efficiency of a hash table depends heavily on its hashing function(s) employed. Describe with a diagram to illustrate how a multiplication method of hashing functions works on a machine with the word size of w bits for a hash table with 2^p entries, $p < w$. (10%)

6. Deletion in a binary search tree relies on TRANSPLANT procedure given below, where the subtree rooted at u is replaced by the subtree rooted at v. Complete the three missing statements of the procedure. (10%)

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TRANSPLANT(T, u, v)
  if u.p == NIL
      T.root = v
  elseif u == u.p.left
      [                ]
  else [               ]
  if v ≠ NIL
      [                ]
```

7. Apply a sequence of deletion and addition operations to a given B-tree.