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CSCE 500 Midterm Exam #2

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Just un.

1. Procedure EXTENDED-BOTTOM-UP-CUT-ROD(p, n) below exhibits low time complexity by utilizing two auxiliary arrays $r[0 .. n]$ and $s[0 .. n]$ to keep solutions for sub-problems obtained so far. (1) Fill in the three missing statements in the procedure, (2) give its time complexity, and (3) if EXTENDED-BOTTOM-UP-CUT-ROD(p, 8) returns r and s as follows, show your resulting cut of the rod with 8 units in length for the maximal profit. (15%) — $\Theta(n^2)$

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EXTENDED-BOTTOM-UP-CUT-ROD(p, n)
let r[0..n] and s[0..n] be new arrays
r[0] = 0
for j = 1 to n
    q = -∞
    for i = 1 to j
        if q  1. < p[i] + r[j-i]
              2. q = "
              3. s(j) = i
    r[j] = q
return r and s
```

while n > 0

s(n)

s'n = n - s(n)

i	0	1	2	3	4	5	6	7	8
$r[i]$	0	1	5	8	10	13	15	16	25
$s[i]$	0	1	2	3	2	2	4	1	2

16

8 − S[8]

6, 6 − S[6]

4

2

2

7

1 2 6

SnFS

2. Given the matrix-chain multiplication problem for four matrices sized 30×50, 50×10, 10×25, 25×15, follow the tabular, bottom-up method in the procedure of MATRIX-CHAIN-ORDER below to construct a table that keeps entry $m[i, j]$ for all $1 \le i, j \le 4$, where $m[i, j]$ denotes the minimum number of scalar multiplications needed to compute the result, and another table that keeps corresponding entry $s[i, j]$ for $1 \le i \le 3$ and $2 \le j \le 4$.

   (a) Construct both tables, with their entry values shown. (20%)

   (b) Give the optimal parenthesized result, following s. (5%)

```
MATRIX-CHAIN-ORDER(p)
n = p.length - 1
let m[1..n, 1..n] and s[1..n-1, 2..n] be new tables
for i = 1 to n
    m[i, i] = 0
for l = 2 to n              // l is the chain length
    for i = 1 to n - l + 1
        j = i + l - 1
        m[i, j] = ∞
        for k = i to j - 1
            q = m[i, k] + m[k+1, j] + p_{i-1} p_k p_j
            if q < m[i, j]
                m[i, j] = q
                s[i, j] = k
return m and s
```

3. Show your construction of an optimal Huffman code for the set of 7 frequencies: **a**: 2, **b**: 13, **c**: 5, **d**: 28, **e**: 16, **f**: 31, **g**: 18. (10%)

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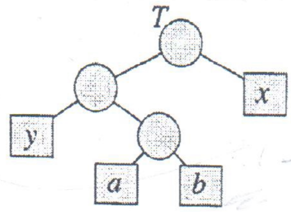
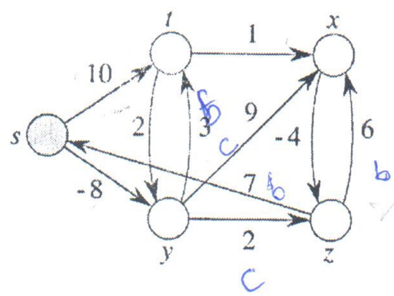
4. Sketch a proof of the Lemma below, using the tree provided. (15%)
Let C be an alphabet in which each character $c \in C$ has frequency $c.freq$. Let x and y be two characters in C having the lowest frequencies. Then there exists an optimal prefix code for C in which the codewords for x and y have the same length and differ only in the last bit.



5. Follow depth-first search (*DFS*), starting from Node *s*, to traverse the graph shown below, with its edge weights all ignored and the start time equal to 1. Mark (1) the type of every edge and (2) the discovery and the finish times of each node. (12%) $O(2V+E)$

Follow breadth-first search (*BFS*), starting from Node *s*, to traverse the graph shown below, with its edge weights all ignored. Show the predecessor tree rooted at Node *s* after *BFS*, with the number of links (i.e., distance) from Node *s* to every other node indicated. (8%) $O(V+E)$



6. Derive the single-source shortest paths from Node *s* to all other nodes in the graph depicted above, following the Bellman-Ford algorithm. Demonstrate the outcome after each relaxation iteration. (15%)

Good Luck!

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