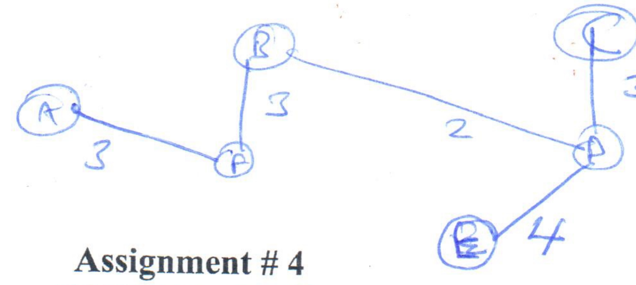
1) Prim's



Cost = 3+3+2+4+3 = 15

Assignment # 4
CSCE 500 CACS

reviewed

Fobe

1. Solve the following (showing the diagram as needed)
   a. Cost Matrix between pair of nodes is given below

	A	B	C	D	E	F
A	0	5	4	-	-	3
B	5	0	3	2	3	-
C	4	3	0	3	-	3
D	-	2	3	0	4	5
E	-	3	-	4	0	4
F	3	-	3	5	4	0

cost 15

Find the minimum spanning tree using Kruskal's and Prim's algorithm. (show the details)

   b. Rational Knapsack problem:

Objects	1	2	3	4	5
Weights	7	8	9	11	12
Profits:	13	15	16	23	24
Capacity of the knapsack is 26					
Profit/weight	1.85	1.875	1.77	2.09	2

Pick the item in the decreasing order of the values Profit/wt

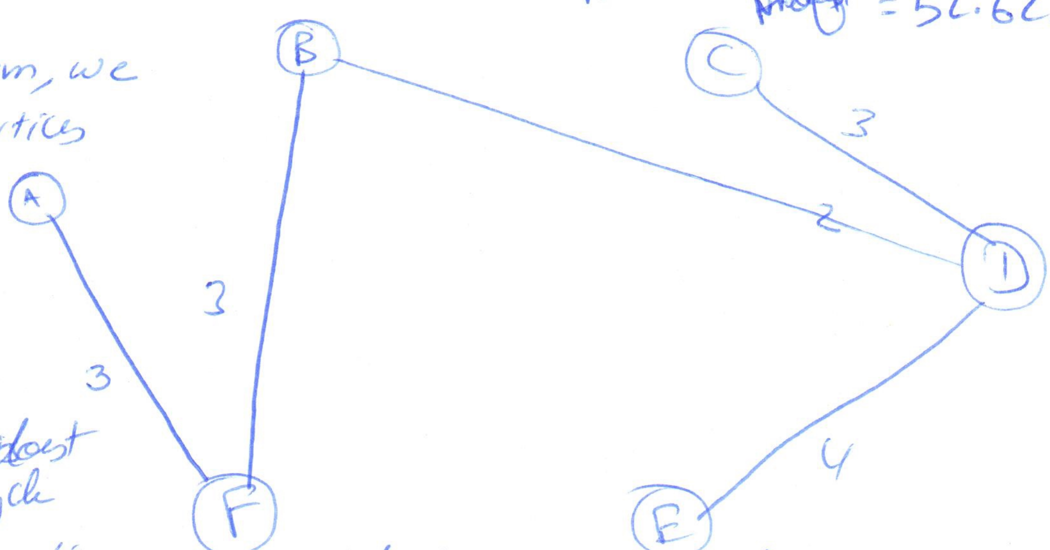
2. Using the distance matrix of a graph given in 1a, find the shortest distance from source node A to all other nodes using Dijkstra's algorithm.

3. Using the distance matrix of a graph given in 1a, find the shortest distance between all pair of nodes (follow the steps shown in the text book).

maximal Profit $= 11 \times 2.09 + 12 \times 2 + 3 \times \frac{15}{8} = 52.625$

Kruskal's:

In this algorithem, we put all the vertices from the original graph, then we start be selecting the minimum weighted edge such that it doest not form any cycle



& that the vertices are connected all together without any formation of cycles.

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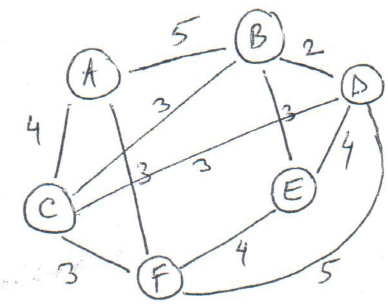
Assignment #4
CSCE 500

(1)

	A	B	C	D	E	F
A	0	5	4	–	–	3
B		0	3	2	3	–
C			0	3	–	3
D				0	4	5
E					0	4
F						0



a) MST using Kruskal & Prim

Kruskal: sort the edges by their cost ASC

Edges = $\{ \overset{2}{\langle B,D\rangle}, \overset{3}{\langle B,C\rangle}, \overset{3}{\langle B,E\rangle}, \overset{3}{\langle C,D\rangle}, \overset{3}{\langle C,F\rangle}, \overset{3}{\langle A,F\rangle}, \overset{4}{\langle A,C\rangle}, \overset{4}{\langle D,E\rangle}, \overset{4}{\langle E,F\rangle}, \overset{5}{\langle A,B\rangle}, \overset{5}{\langle D,F\rangle} \}$

each node will form an initial group: $\{A\}, \{B\}, \{C\}, \{D\}, \{E\}, \{F\}$

MTS = $\{ \}$ (final result)

For each pair of nodes taken in ascending order, if the 2 nodes belong to different groups, then – concatenate the groups
– add the edge to the MTS
– remove the edge from the list

Else, just remove the edge from list.

$\Rightarrow \overset{2}{\langle B,D\rangle}$: groups are $\{A\}, \{B,D\}, \{C\}, \{E\}, \{F\}$

MTS = $\{\langle B,D,2\rangle\}$

$\overset{3}{\langle B,C\rangle}$: groups will be $\{A\}, \{B,C,D\}, \{E\}, \{F\}$

MTS = $\{\langle B,D,2\rangle, \langle B,C,3\rangle\}$

$\overset{3}{\langle B,E\rangle}$: groups will be $\{A\}, \{B,C,D,E\}, \{F\}$

MTS = $\{\langle B,D,2\rangle, \langle B,C,3\rangle, \langle B,E,3\rangle\}$

$\overset{3}{\langle C,D\rangle}$: C, D are from the same group

$\overset{3}{\langle C,F\rangle}$: groups will be $\{A\}, \{B,C,D,E,F\}$

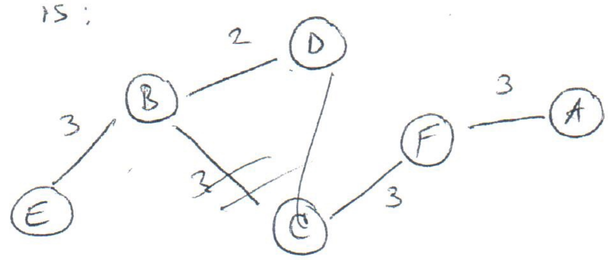
MTS = $\{\langle B,D,2\rangle, \langle B,C,3\rangle, \langle B,E,3\rangle, \langle C,F,3\rangle\}$

$\overset{3}{\langle A, F \rangle}$: groups will be $\{A, B, C, D, E, F\}$

$MTS = \{\langle B, D, 2\rangle, \langle B, C, 3\rangle, \langle B, E, 3\rangle, \langle C, F, 3\rangle, \langle A, F, 3\rangle\}$

Since we have a single final group, we can stop

$\Rightarrow$ MTS is:



Prim

$V = \{\ \}, \ E = \{\ \}$

Pick an arbitrary node and add in V $\Rightarrow$ $V = \{C\}$

while V does not contain all nodes

Choose the minimal cost edge $\langle a, b \rangle$ such that $a \in V$ and $b \notin V$

Add $V = V \cup \{b\}$ and $E = E \cup \{\langle a, b\rangle\}$

$\Downarrow$

Choose $\langle C, B \rangle$, $C \in V$ and $B \notin V$

$\Rightarrow V = \{C, B\}, \ E = \{\langle B, \overset{3}{C}\rangle\}$

Choose $\langle B, D \rangle$, $B \in V$ and $D \notin V$

$\Rightarrow V = \{B, C, D\}, \ E = \{\langle B, \overset{3}{C}\rangle, \langle B, \overset{2}{D}\rangle\}$

Choose $\langle B, E \rangle$, $B \in V$ and $E \notin V$

$\Rightarrow V = \{B, C, D, E\}, \ E = \{\langle B, C\rangle, \langle B, D\rangle, \langle B, E\rangle\}$

Choose $\langle C, F \rangle$, $C \in V$ and $F \notin V$

$\Rightarrow V = \{B, C, D, E, F\}, \ E = \{\langle B, C\rangle, \langle B, D\rangle, \langle B, E\rangle, \langle C, F\rangle\}$

Choose $\langle A, F \rangle$, $F \in V$ and $A \notin V$

$\Rightarrow V = \{A, B, C, D, E, F\}, \ E = \{\langle B, C\rangle, \langle B, D\rangle, \langle B, E\rangle, \langle C, F\rangle, \langle A, F$

Done. (V contains all the 6 nodes)

b) Rational Knapsack problem

Obj	1	2	3	4	5
Weights	7	8	9	11	12
Profits	13	15	16	23	24
Capacity =	26.				

	W=0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	2
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
O_1	0	0	0	0	0	0	0	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	1
O_2	0	0	0	0	0	0	0	13	15	15	15	15	15	15	15	28	28	28	28	28	28	28	28	28	28	28	2
O_3	0	0	0	0	0	0	0	13	15	16	16	16	16	16	16	28	29	31	31	31	31	31	31	31	44	44	4
O_4	0	0	0	0	0	0	0	13	15	16	16	23	23	23	23	28	29	31	36	38	39	39	39	39	44	44	5
O_5	0	0	0	0	0	0	0	13	15	16	16	23	24	24	24	28	29	31	36	38	39	40	40	47	47	47	[illegible]

Discrete Knapsack

$Profit[i,j] = \max\{ Profit[i-1][j],\ p_i + Profit[i-1][W-w_i] \}$, $W=26$

Final profit $=51$, Objects taken: $\{O_4, O_2, O_1\}$, Capacity used $=26$

Rational Knapsack: sort the objects by $\frac{\text{Profit}}{\text{Weight}}$, descending order

$\Rightarrow$ O_1: $\frac{13}{7} \simeq 1.85$

O_2: $\frac{15}{8} \simeq 1.87$

O_3: $\frac{16}{9} \simeq 1.77$

O_4: $\frac{23}{11} \simeq 2.09$

$O_5 = \frac{24}{12} = 2$

$\Rightarrow$ Objects sorted are:

$O_4 > O_5 > O_2 > O_1 > O_3$

Take $O_4 \Rightarrow$ Capacity left $= 26 - 11 = 15$

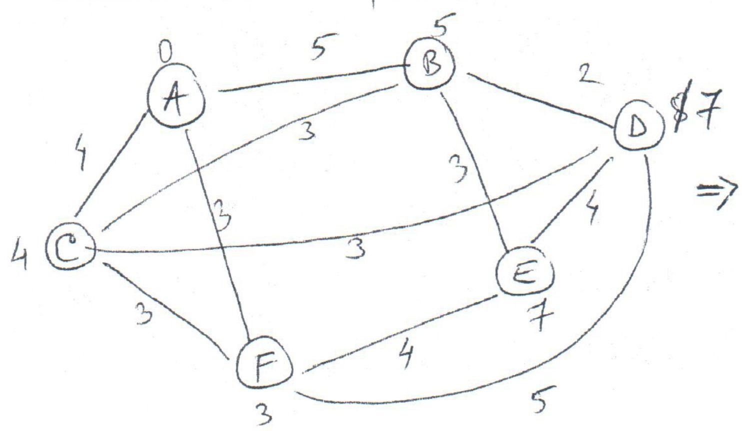
Take $O_5 \Rightarrow$ Capacity left $= 15 - 12 = 3$

We can take only $\frac{3}{w_2}$ out of $O_2 \Rightarrow \frac{3}{8} O_2$ and the capacity is 0.

$\Rightarrow$ Solution $= \{ O_4, O_5, \frac{3}{8} O_2 \}$

Final profit $= 23 + 24 + \frac{3}{8} \cdot 15 = 52.625 > 51$ (discrete case)

② Shortest distance from A to all nodes using Dijkstra.



$\Rightarrow$ L = table of adjacence

$L[A][C]$ = distance (A, C) or ∞

dist = retains the distances from A to all nodes

$dist[A] = 0$

Q = set of all nodes in graph = $\{B, C, D, E, F\}$

dist[Neighbors of A] = $L[A]$[Neighbor] $\Rightarrow$ $dist[C] = 4$, $dist[B] = 5$

dist [Non-neighbors] $= \infty$ $\Rightarrow$ $dist[E] = \infty$ $dist[F] = 3$

$dist[D] = \infty$

while Q not empty

Remove the closest node from Q = u (has the smallest value of dist)

For each neighbor v of u (which is also in Q):

if $dist[v] > dist[u] + L[u][v]$ then $dist[v] = dist[u] + L[u][v]$

update previous$[v] = u$

$u = F$, $Q = \{B, C, D, E\}$

neighbors (F) $= \{C, E, D\} \Rightarrow v \in \{C, E, D\}$

Update: $dist[E] = dist[F] + L[E][F]$ & $prev[E] = F$

$= 3 + 4 = 7$

$dist[D] = dist[F] + L[D][F]$ & $prev[D] = F$

$= 3 + 5 = 8$

$u = C$, $Q = \{B, D, E\}$

neighbors (C) $= \{B, D\}$

Update: $dist[D] = dist[C] + L[D][C]$ & $prev[D] = C$

$= 4 + 3 = 7$

$u = B$, $Q = \{D, E\}$

neighbors(B) $= \{D, E\}$ $\Rightarrow$ no update

$u = D$, $Q = \{E\}$
neighbors(D) = {E} => No update

$u = E$, $Q = \emptyset$
neighbors(E) = $\emptyset$ => Done.

Distances:		Via:
$A \to B = 5$	,	$A \to B$
$A \to C = 4$	,	$A \to C$
$A \to D = 7$	,	$A \to C \to D$
$A \to E = 7$	,	$A \to F \to E$
$A \to F = 3$	,	$A \to F$

(3) Distances between all pairs of nodes?
→ Floyd Warshall

```
for all vertices:
    dist[v][v] = 0
for all edges <u,v>:
    dist[u][v] = L[u][v]        (from the adjacency matrix).
for k = 1 to |V|:
    for i = 1 to |V|:
        for j = 1 to |V|:
            if dist[i][j] > dist[i][k] + dist[k][j]:
                dist[i][j] = dist[i][k] + dist[k][j]
```

Use the steps in books!!

$\begin{cases} A \to 1 \\ B \to 2 \\ \vdots \\ F \to 6 \end{cases}$

=> dist:

	A	B	C	D	E	F
A	0	5	4	$\cancel{\infty}$ 7	$\cancel{\infty}$ $\cancel{8}$ 7	3
B		0	3	2	3	$\cancel{\infty}$ $\cancel{8}$ 6
C			0	3	$\cancel{\infty}$ 6	3
D				0	4	5
E					0	4
F						0

It make sense when $k \neq i$ and $k \neq j$ and $i \neq j$.

$k = A$

$i = B$

$j = \cancel{C}\ \cancel{D}\ \cancel{E}$ F => dist[B][F] = dist[B][A] + dist[A][F]
=> dist[B][F] = 8

$i = C$

$j = \cancel{D}\ \cancel{E}$ F => dist[C][E] $= 4 + \infty = \infty$

$i = D$

$j = E$ F

$i = E$

$j = F$

k=B

i=A

j=$\cancel{C}$ D => dist[A][D] = dist[A][B] + dist[B][D]
= 5 + 2 = 7

j=E => dist[A][E] = dist[A][B] + dist[B][E]
= 5 + 3 = 8

j=F

i=C

j=$\cancel{D}$ E => dist[C][E] = dist[C][B] + dist[B][E]
= 3 + 3 = 6.

j=$\cancel{F}$

i=D

j=$\cancel{E}$ $\cancel{F}$

i=E

j=F

k=C

i=A

j=$\cancel{B}$ $\cancel{C}$ $\cancel{D}$ $\cancel{E}$ $\cancel{F}$

i=B

j=$\cancel{D}$ $\cancel{E}$ F => dist[B][F] = dist[B][C] + dist[C][F]
= 3 + 3 = 6

i=D

j=$\cancel{E}$ $\cancel{F}$

i=E

j=$\cancel{F}$

k=D

i=A

j=$\cancel{B}$ $\cancel{C}$ $\cancel{D}$ $\cancel{E}$ $\cancel{F}$

i=B

j=$\cancel{C}$ $\cancel{D}$ $\cancel{E}$ $\cancel{F}$

i=C

j=$\cancel{D}$ $\cancel{E}$ $\cancel{F}$

i=E

j=$\cancel{F}$

k=E

i=A

j=$\cancel{B}$ $\cancel{C}$ $\cancel{D}$ $\cancel{F}$

i=B

j=$\cancel{C}$ $\cancel{D}$ $\cancel{F}$

i=C

j=$\cancel{D}$ $\cancel{E}$ $\cancel{F}$

i=D

j=$\cancel{F}$

i=E

k=F

i=A

j=$\cancel{B}$ $\cancel{C}$ $\cancel{D}$ =>
E , F

dist[A][E] = dist[A][F] + dist[F][E]
= 3 + 4 = 7

i=B

j=$\cancel{C}$ $\cancel{D}$ $\cancel{E}$

i=C

j=$\cancel{D}$ $\cancel{E}$

i=D => j=$\cancel{E}$

Done.