CSCE 500 Midterm Exam #2

10/25/2021

1. Given that for an open-address hash table with load factor $\alpha = n/m < 1$, the expected number of probes in unsuccessful search under uniform hashing is at most $1/(1 - \alpha)$, prove the expected number of probes in a successful probe under uniform hashing being at most $(1/\alpha) \cdot \ln(1-\alpha)^{-1}$ by giving a proof sketch which explains how many probes are needed to locate existing keys. (15%).

2. Use perfect hashing to store the set of K = {10, 40, 64, 91}, with its outer hash function of h(k) = ((a·k + b) mod p) mod m, where a = 3, b = 45, p = 137, and m (i.e., the outer hash table size) = 8. Illustrate the perfect hashing result under K after devising the appropriate inner hash function(s) as needed. (15%)

3. (a) Explain briefly how Cuckoo hashing works under two hash functions of h_1 and h_2. (10%)

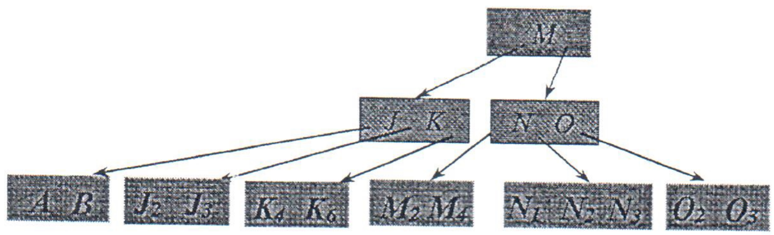
   (b) State the situation when a new key cannot be inserted in a Cuckoo hash table successfully; provide two solutions for key insertion failures and contrast them in terms of advantages/disadvantages. (8%)

4. Deletion in a binary search tree relies on TRANSPLANT procedure given below, where the subtree rooted at u is replaced by the subtree rooted at v. Complete the three missing statements of the procedure and provide an illustrative figure to show the resulting figure after the procedure is conducted. (12%)

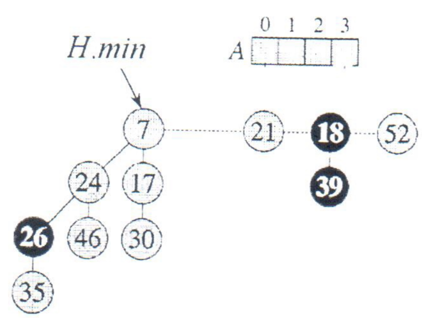
```
TRANSPLANT(T, u, v)
if u.p == NIL
    T.root = v
elseif u == u.p.left
    u.p.left = v
else
    u.p.right = v
if v ≠ NIL
    v.p = u.p
```

5. For a B-tree of height h with the minimum node degree of $t \geq 2$, derive the maximum number of keys that can be stored in such a B-tree. (10%)

6. Given the initial B-tree with the minimum node degree of $t = 3$ below, show the results (a) after deleting the key of M_2, (b) followed by inserting the key of L, (c) then by deleting the key of J_2, (d) then by inserting the key of O_1, with $O < O_1 < O_2$, (e) then by deleting K, and (f) then by deleting M. (Show the result after each deletion and after each insertion. 18%)



7. A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge min-heaps in the root list upon the operation of extracting the minimum node. Given the following Fibonacci min-heap, show every consolidation step and the final heap result after $H.min$ is extracted, with the aid of A. (12%)



Good Luck!