CSCE 500 Midterm Exam #1

9/29/2021

1. Solve the following recurrences:

   $T(n) = 2 \cdot T(n/4) + T(n/2) + n^{1/3}$ (14%)

   $T(n) = T(n^{1/2}) + \lg_2 n$ (14%)

   $T(n) = 2 \cdot T(n/2) + n \cdot \lg_2 n$ (8%)

2. Validate the following non-asymptotically tight bounds:

   $n^2/\lg_2 n = o(n^2)$ and $n^{0.9999} \cdot \lg_2 n = \omega(n)$ (6%)

3. Merge sort follows the divide-and-conquer approach and involves a key procedure, as follows. Fill the missing statements to complete the procedure. (12%)

```
MERGE(A, p, q, r)
  n1 = q - p + 1
  n2 = r - q
  let L[1..n1 + 1] and R[1..n2 + 1] be new arrays
  for i = 1 to n1
      L[i] = A[p + i - 1]
  for j = 1 to n2
      R[j] = A[q + j]
  L[n1 + 1] = ∞
  R[n2 + 1] = ∞
  i = 1
  j = 1
  for k = p to r
      if L[i] ≤ R[j]
          [          ]
          [          ]
      else[          ]
          [          ]
```

4. The Master Method aims to solve $T(n) = a \cdot T(n/b) + f(n)$ with constants $a \geq 1$ and n being an exact power of b (> 1). Show by the recursion tree technique that $T(n)$ equals (14%)

$$\Theta(n^{\log_b a}) + \sum_{j=0}^{\log_b n - 1} a^j \cdot f(n/b^j)$$

5. A hash function realized by the division method takes the form of $h(k) = k \bmod m$, where m is the number of table entries.

   In general, what is a good choice for m, and in particular, what is a good m for the table to yield its load factor ≤ 0.5 when holding 120 distinct keys? (6%)

6. The utilization efficiency of a hash table depends heavily on its hashing function(s) employed. Describe with a diagram to illustrate how a multiplication method of hashing works on a machine with the word size of w bits for a hash table with 2^p entries, $p < w$. (16%)

7. Prove that for an open-address hash table with load factor α (< 1), the expected number of probes in unsuccessful search under uniform hashing is at most $1/(1-\alpha)$. (Hint: consider the probability of i probes to learn key absence.) (10%)

Good Luck!