Algorithms and Theory of Computation 2009

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1 Short Questions

[S_1] Let $P(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$, where $a_k > 0$. Prove

$$P(n) = \Theta(n^k).$$

[S_2] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = & 4T(\frac{n}{2}) - 4T(\frac{n}{4}) \\ T(2) = 1, & T(4) = 4 \end{cases}.$$

[S_3] (a) Explain the difference between the claim that a problem is (1) computationally unsolvable and (2) NP-hard (or NP-complete).
(b) Give a specific example of a problem known to be in each class.
(c) What has been proved, and what is believed but yet proved, about the computational difficulty of an NP-complete problem.

[S_4] Construct
(a) A finite automaton or a regular expression for the language
$\{x \in \{0,1\}^* :$ "0,1" occurs in x an even number of times $\}$.
(b) A context free grammar or pushdown automaton for the language $\{ww^R : w \in \{0,1\}^*\}$.

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2 Long Questions

[L_1] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 \\ 3 & \infty & 6 & 4 & 5 \\ 5 & 6 & \infty & 2 & 4 \\ 8 & 4 & 2 & \infty & 7 \\ 1 & 5 & 4 & 7 & \infty \end{bmatrix}$$

Use backtracking with branch-and-bound to find the best solution and Draw the state space tree.

[L_2] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 3, 4; and weight 12, 9, 8, 4, 3, 2 (resp.).

[L_3] Briefly prove (if true) or disprove (if false):
(a) The intersection of two regular language is regular.
(b) The union of two context free languages is context free.
(c)The union of two languages in NP is in NP.

[L_4] Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.
(a) $\{a^{3n}b^{2m}c^{p} : n, m, p > 0\}$.
(b) $\{a^{n}b^{m}c^{m} : n > m > 0\}$.