CSCE 500 Midterm Exam #1

10/10/2018

1. Solve the two recurrences below:

   $T(n) = 2 \cdot T(n/4) + n^{1/2}$ (6%)

   $T(n) = 2 \cdot T(n^{1/2}) + \lg_2 n$ (10%)

2. The algorithm for finding the maximum subarray that crosses the midpoint of Array $A[1 \ldots n]$ includes the main routine of FIND-MAXIMUM-SUBARRAY(A, *low*, *high*), which calls FIND-MAX-CROSSING-SUBARRAY(A, *low*, *mid*, *high*), as follows. Complete the six (6) missing statements in FIND-MAX-CROSSING-SUBARRAY below. (12%)

```
FIND-MAX-CROSSING-SUBARRAY(A, low, mid, high)
// Find a maximum subarray of the form A[i .. mid].
left-sum = −∞
sum = 0
for i = mid downto low
    sum = sum + A[i]
    [                    ]
    [                    ]
    [                    ]
// Find a maximum subarray of the form A[mid + 1 .. j].
right-sum = −∞
sum = 0
for j = mid + 1 to high
    sum = sum + A[j]
    [                    ]
    [                    ]
    [                    ]
// Return the indices and the sum of the two subarrays.
return (max-left, max-right, left-sum + right-sum)
```

3. The hash table is a widely adopted data structure. Explain briefly

   (1) how perfect hashing works. (8 %)

   (2) how Cuckoo hashing works under two hash functions of h_1 and h_2. (10%)

   What is the situation when a new key cannot be inserted in a Cuckoo hash table successfully? (2%)

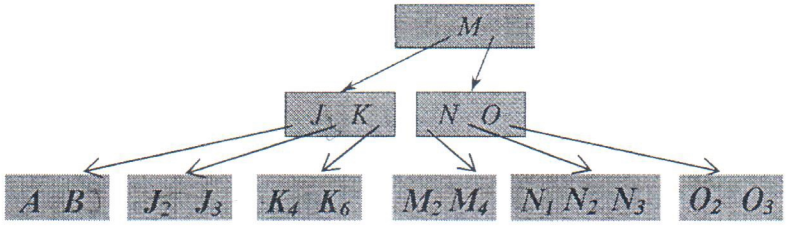
4. A binary search tree (T) is to be maintained following the in-order tree traversal order. Consider a sequence of arrival keys, {25, 23, 14, 7, 9, 21, 31, 34, 28, 24}, to T which has just the root node with its key = 20 initially.

   (1) Show the resulting T after inserting all arrival keys. (5%)

   (2) Show the resulting T after its root node is then deleted. (5%)

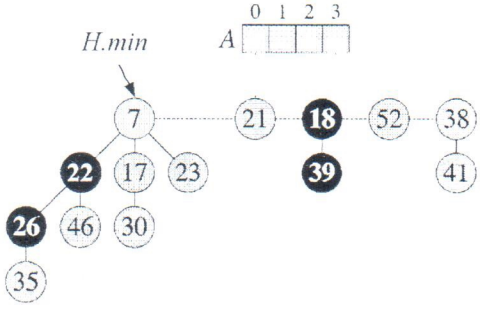
5. For any n-key B-tree of height h and with the minimum node degree of $t \geq 2$, prove that h is no larger than $\log_t \frac{n+1}{2}$. (Hint: consider the number of keys stored in each tree level.) (12%)

6. Given the initial B-tree with the minimum node degree of $t = 3$ below, show the results (a) after deleting the key of M_2 , (b) followed by inserting the key of L, (c) then by deleting the key of J_2, (d) then by inserting the key of O_1, with $O < O_1 < O_2$, and (e) then by deleting K. (Show the result after each deletion and after each insertion; 15%)



7. A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge trees in the root list upon the operation of extracting the minimum node. Given the following partially consolidated diagram, show every subsequent consolidation step till its completion. (10%)

After consolidation is completed, show the resulting Fibonacci min-heap with key '35' decreased to 4. (5%)



Good Luck!