Algorithm and Theory of Computation 2013 Jan.

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1 Short Questions

[S_1] Let $P(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$, where $a_k > 0$. Prove

$$P(n) = \Theta(n^k).$$

[S_2] Ordering by asymptotic growth rate from slowest to fastest:

$$7^{n^{1.008}}, (100n+2)^2, n^3, lg^5 n, e^n, log_5 n, 8^{n^{1.007}}$$

[S_3] Calculate $S_2 = \sum_{i=1}^{n} i^2$ and $S_3 = \sum_{i=1}^{n} i^3$

2 Long Questions

[L_1] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = & 5T(n-1) - 6T(n-2) + 7^n \\ T(0) = 1 & T(1) = 6 \end{cases}.$$

[L_2] Using dynamic programming algorithm to calculate the matrix product $A_1 \times A_2 \times A_3 \times A_4 \times A_5$, where $A_1 : 30 \times 35$, $A_2 : 35 \times 15$, $A_3 : 15 \times 8$, $A_4 : 8 \times 10$ and $A_5 : 10 \times 20$.

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[L_3] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Give the partial solution $X = (5, -, -, -, -)$, calculate $B(X)$ using the reducing technique on the matrix.
b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.