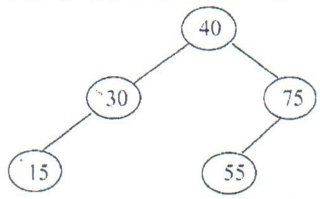
Quiz 1

[redacted]

1. Give big-O (upper bound) estimation for $f(n) = 2^5\log(n!) + n^2 + 2^{10}$. (10 scores)
2. Let $f(n) = 5n^3 + 2^5n^2$. Show that $f(n) = \theta(n^3)$. (10 scores)
3. Use the recursion tree technique to derive the upper bound of the recurrence: $T(n) = 3T\left(\frac{n}{3}\right) + n$. (10 scores)
4. Use the recursion tree technique to derive the upper bound of the recurrence: ~~$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{3}\right) + T\left(\frac{n}{4}\right)$~~ $+ n$. (10 scores)

   $T(n) = T(\frac{n}{4}) + T(\frac{3n}{4}) + n$
5. AVL Tree: a balanced binary search tree.



- Insert node with value 10 to the above AVL tree. Please show the position to insert node ~~20~~ 10. (5 scores) If the tree is not balanced after inserting the new node, please rotate it to a balanced one. Please describe the rotations such as right-rotation(node with value x) or left-rotation(node with value x). (10 scores)
- Insert another node with value 65 to the tree. Please show the position to insert node ~~35~~ 65. (5 scores) If the tree is not balanced after inserting the new node, please rotate it to a balanced one. Please describe the rotations such as right-rotation(node with value x) or left-rotation(node with value x). (10 scores)

6. Use hash table with size m=7 to store the set of keys: {3, 5, 10, 11, 13, 16, 23}
   a. Pick hash function $h(k) = k \bmod 7$ to hash. Please draw the table with keys hashed to each entry. If there exists collision in one entry, please use linear probing method with probe sequence $h(k,i) = (h(k) + i) \bmod 7$ to handle collision. (10 scores)
   b. Pick hash function $h(k) = k \bmod 7$ to hash. Please draw the table with keys hashed to each entry. If there exists collision in one entry, please use double hashing method with probe sequence $h(k,i) = (h_1(k) + i * h_2(k)) \bmod 7$ to handle collision, where $h_1(k) = h(k) = k \bmod 7$, and $h_2(k) = k \bmod 5 + 1$. (10 scores)
   c. What is the search cost of the two hashing methods, respectively in worst case? (10 scores)

Bonus Questions:

   d. For the double hashing method above, can we change $h_2(k) = k \bmod 5 + 1$ to $h_2(k) = k \bmod 5$? (~~10 scores~~) (5 scores)
   e. What is the average search cost of the two hashing methods, respectively? (10 scores) (5 score)

   Linear: $O(m)$ double hashing: $O(m^2)$

1. Given a sorted array and a number x, find a pair in array whose sum is closest to x with linear time complexity. (20 scores)
   Note that a simple solution is to consider every pair and keep track of the closest pair (absolute difference between pair sum and x is minimum). Finally print the closest pair. However, the time complexity of this solution is O(n^2).

Quiz #2

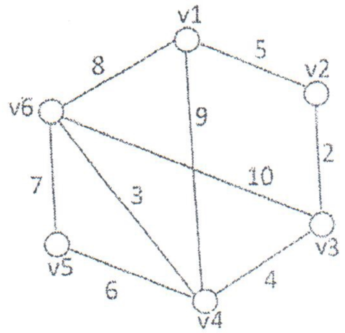
1. Minimum Number of Coins problem: If we want to make change for V cents, and we have infinite supply of each of {d[0], d[1], .. , d[m-1]} valued coins, what is the minimum number of coins to make the change?
   Please modify the following recursive implementation with exponential growth to one with linear time complexity using dynamic programing. (Either top-down or bottom-up) (30 scores)

```
int minCoins(int V)
{
        if (V == 0) return 0;              Table
        int res = INT_MAX;
        for (int i=0; i<m; i++)   // Try each coin
        {
                if (d[i] <= V)
                {
                        int sub_res = minCoins(V-d[i]);
                        if (sub_res + 1 < res)
                                res = sub_res + 1;
                }
        }
        return res;
}
```

2. Please follow the bottom up solution to ~~fine~~ find the longest palindrome that is a subsequence of string 'EEBEAEABE' by manually filling out a table. Please give the table and the longest palindrome. (40 scores)

```
int lps(char *str)
{
  int n = strlen(str);
  int i, j, sl;
  int L[n][n];  // Create a table to store results of subproblems
  for (i = 0; i < n; i++)
     L[i][i] = 1;
  for (sl=2; sl<=n; sl++)
  {
     for (i=0; i<n-sl+1; i++)
     {
        j = i+sl-1;
        if (str[i] == str[j] && sl == 2)
               L[i][j] = 2;
        else if (str[i] == str[j])
               L[i][j] = L[i+1][j-1] + 2;
           else
               L[i][j] = max(L[i][j-1], L[i+1][j]);
     }
  }
  return L[0][n-1];
}
```

3. Given graph G, use Prim's and Kruskal's Algorithms to compute its Minimum Spanning Tree, respectively. For each algorithm, please write down the edge picked in each step. For example, Step 1: (v1, v2). (30 scores)



4. Bonus question: Given n dice. Each dice has 6 faces, numbered from 1 to 6. Please find the number of ways to get sum X, where X is the summation of values on each top face when all the dice are thrown. (Note that $n < X < 6n$) (20 scores)

5. Bonus question: Given a rod of length n inches and an array of prices that contains prices of all pieces of size smaller than n. Determine the maximum value obtainable by cutting up the rod and selling the pieces. (20 scores)
For example, if length of the rod is 8 and the values of different pieces are given as following, then the maximum obtainable value is 22 (by cutting in two pieces of lengths 2 and 6)

Length	1	2	3	4	5	6	7	8
Price	1	5	8	9	10	17	17	20

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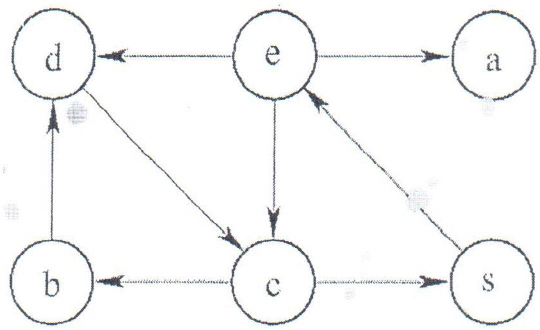
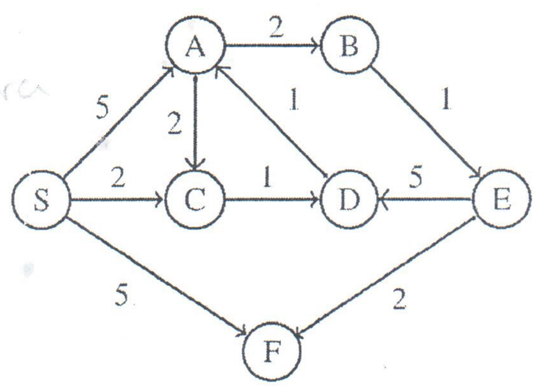
[redacted] Quiz#3

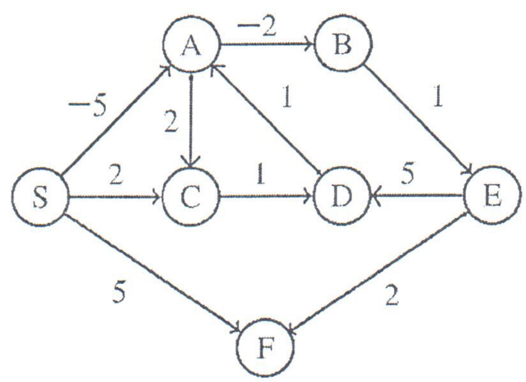
1. BFS/DFS: Give the visited node order for each type of graph search, starting with s. (10 scores)



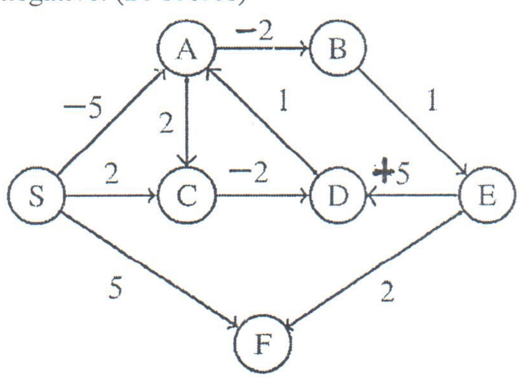
2. Shortest Path: Run Dijkstra's algorithm on the directed graph shown in below and left, starting at vertex S. Please show each step including the set S. (10 scores) Run Bellman-Ford algorithm on the directed graph shown in below and right, starting at vertex S. Please show each step. (10 scores)

Dijkstra

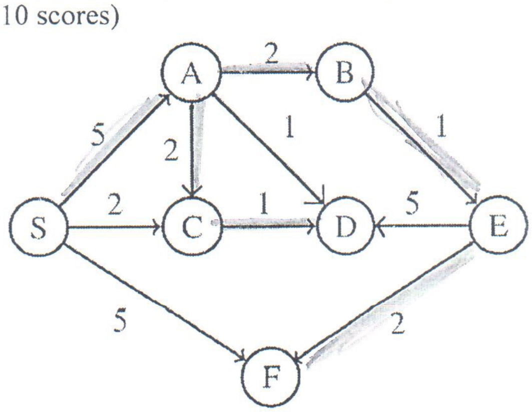




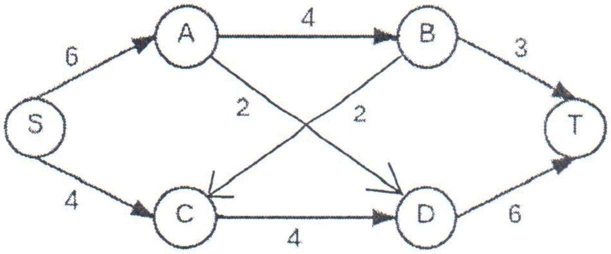
3. Please use Johnson's Algorithm to reweight the directed graph shown in below such that all edge weights are non-negative. (20 scores)



4. Longest Path: please find the longest distances from s to all other vertices in the following directed acyclic graph. (10 scores)



5. Run Ford-Fulkerson Algorithm to compute the max flow of the flow network shown in below. Please show each step to compute the mas flow. (15 scores) Please also show the min cut of the flow network. (5 scores)



6. Six students are assigned to three dorms. Student A prefers Rooms 1, 2, and 3. Student B prefers Rooms 1and 2. Student C prefers Rooms 1 and 2. Student D prefers Room 2. Student E prefers Room 2. Student F prefers Room 3. Please model the problem (5 scores), describe an algorithm to solve the problem (10 scores), and give an optimal assignment (5 scores).

7. Bonus Question: please find the number of simple paths from S to ~~E~~ D in the following directed acyclic graph (simple path: no repeated nodes in the path). (20 scores)