Ph.D. Comprehensive Examination
Design and Analysis of Algorithms

Aug. 18, 06

Short Questions

(Do any 3 of the following 4 questions. Each question is worth 10 points)

[S1] (a) Write the recurrence equations for the worst and best case performance of : (1) quick sort; (2) merge sort; (3) bin-search. Briefly justify the role of each term in the recurrence equations.

(b) What is the essential difference between merge sort and quick sort ? How does it influence the worst-case running time functions ?

[S2] Given the recurrence relation

$$T(n) = 2T(\sqrt{n}) + \log_2 n ,$$

$$T(2) = 1 ,$$

obtain a closed-form formula for $T(n)$ and determine its growth rate (Θ). (Hint: let $n = 2^{2^i}$).

[S3] Construct a finite automaton or regular expression for each of the following languages:

(a) $\{ x \in \Sigma^*$: every bb in x is followed by a. $\}$

(b) $\{ x \in \Sigma^*$: the string ab occurs an odd number of times in x. $\}$

[S4]

(a) Define nondeterminism for a finite automaton and a Turing machine.

(b) What, in general, is the cost of converting a nondeterministic device to an equivalent deterministic device.

(c) Give an example of a type of device for which determinism and nondeterminism are equivalent, and one for which they are not.

Long Questions

(Do any 3 of the following 4 questions. Each question is worth 23 points)

[L1] (a) Define P, NP, and $NP - Complete$. Give one example for each case.

(b) Assuming $NP \neq P$, what is the relationship between P, NP, and $NP - Complete$?

(c) Prove that HC α TSP-decision, where HC is the Hamilton circuit problem and TSP-decision is the traveling salesman decision problem.

(d) Which of the two problems can be shown to be NP-complete, using the result of Part (c) ?

[L2] Given a set $P = \{ p_1, p_2, ..., p_n \}$, of n-files of length $\{ l_1, l_2, ..., l_n \}$ respectively, to be sorted on a tape whose length $L \geq \sum_{k=1}^{n} l_k$. These files are frequently accessed with an uniform probability.

(a) Provide a greedy algorithm to identify a permutation $I = \{ i_1, i_2, ..., i_n \}, i_k \in \{ 1, 2, ..., n \}$ for $1 \leq k \leq n$, and $i_k = i_h$ iff $k = h$, such that average seek-time is minimized, where average seek-time is given by $\frac{1}{n} \sum_{h=1}^{n} \sum_{k=1}^{h} l_{i_k}$.

(b) Prove that your greedy-policy results in an optimal (minimizes average seek-time) permutation I.

[L3] Classify each of the following languages as regular, context free but not regular, or Turing decidable but not context free. Prove your answers.

(a) $\{ a^n b^m a^k : n, m, k > 0 \}$.

(b) $\{ a^n b^m a^n : n > m > 0 \}$.

(c) $\{ a^n b^m a^n : n, m > 0 \}$.

[L4] Briefly prove or disprove that each of the following classes of languages is closed under concatention:

(a) regular languages,

(b) context free languages,

(c) P.