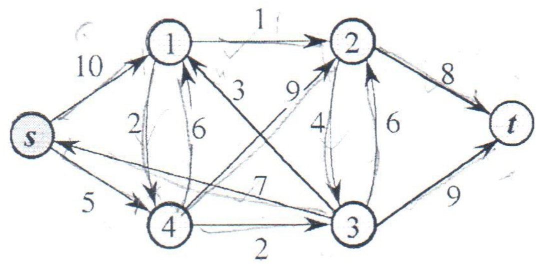
CSCE 500 Final Exam

12/3/2018

The next three questions are based on the following direct graph, which has no negative weights.

1. The Edmonds-Karp algorithm (*EK*) follows the basic Ford-Fulkerson method with breadth-first search to choose the shortest augmenting path (in terms of the number of edges involved) for computing the maximum flow iteratively from Vertex s to Vertex t below. Illustrate the maximum flow computation process (including the augmenting path chosen in each iteration and its resulting residual network) via *EK*. (10%)

   What is the time complexity of *EK* on the graph $G = (V, E)$? (2%)



2. The Floyd-Warshall algorithm (*FW*) obtains all pairs of shortest paths in a weighted directed graph. Consider the graph given in Problem 1 above, with Vertices s and t ignored. What is the recursive equation of $d_{i,j}^{(k)}$ for the shortest-path weight of any path between i and j with intermediate vertices $\in \{1, 2, 3, \ldots, k\}$? (2%)

   Derive all distance matrices $D^{(k)}$ following *FW* so that the $d_{i,j}^{(n)}$ element of final matrix $D^{(n)}$ denotes $\delta(i, j)$ for every vertex pair $\langle i, j\rangle$ for all $i, j \in \{1, 2, 3, 4\}$. (12%)

3. Follow depth-first search (*DFS*), starting from Node s, to traverse all six nodes of the graph shown in Problem 1 above. Mark (1) the type of every edge and (2) the discovery and the finish times of each node. (8%)

4. Many problems have been proved to be NP-complete. To prove NP-completeness, it is necessary in general to demonstrate two proof components. What are the two proof components to show a problem being NP-complete? (2%)

   Given that the Hamiltonian-cycle problem (HAM-CYCLE) belongs to NP-completeness, how do you prove that the traveling-salesman problem (TSP) is NP-complete? (1%)

   TSP has a 2-approximation solution in polynomial time based on establishing a minimum spanning tree (MST) rooted at the start/end vertex (in polynomial time following MST-PRIM), if the graph edge weights observe triangle inequality. Sketch a brief proof to demonstrate that such a solution satisfies 2-approximation. (7%)

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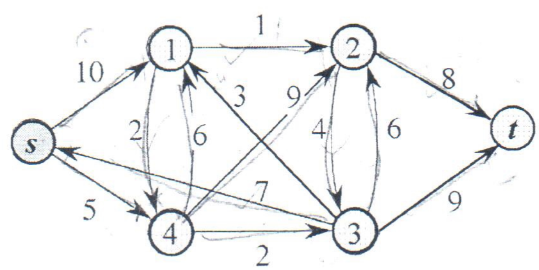

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10. Given the matrix-chain multiplication problem for four matrices sized 20×50, 50×10, 10×30, 30×15, follow the tabular, bottom-up method in the procedure of MATRIX-CHAIN-ORDER, which constructs a table to keep entry $m[i, j]$ for all $1\leq i, j\leq 4$ (with $m[i, j]$ denoting the minimum number of scalar multiplications needed to compute the result) and another table to hold corresponding entry $s[i, j]$ for $1\leq i\leq 3$ and $2\leq j\leq 4$.

(a) Construct both tables, with their entry values shown. (12%)

(b) Give the optimal parenthesized result, following s. (3%)

Good Luck!