Ph.D. Comprehensive Examination, Spring, 2007

Design and Analysis of Algorithms

1 Short Questions

Answer 3 of 4 questions

[S1] Prove $h(n) \triangleq \sum_{i=1}^{n} \frac{1}{i} = \Theta(log_2 n)$.

[S2] Given the recursive equation

$$\begin{cases} T(n) = 5T(n-1) - 6T(n-2) + 7^n \\ T(0) = 1, \quad T(1) = 6 \end{cases}$$

Obtain a closed-form formula for $T(n)$ and determine its growth rate (Θ).

[S3] Formally define
(S3a) deterministic finite automaton (DFA);
(S3b) pushdown automaton (PDA);
(S3c) context free grammar (CFG);
Give the relations between the classes of languages of these devices (containment or equality).

[S4] Construct
(S4a) a finite automaton or a regular expression for the language

$$\{x \in \{0,1\}^* : \textit{any substring } 000 \textit{ in } x \textit{ is followed immediately by } 1\};$$

(S4b) a context free grammar or pushdown automaton for the language

$$\{a^n b^m c^{2n} : n, m > 0\}.$$

2 Long Questions

Answer 3 of 4 questions

[L1] Assume that the 3-dimensional matching problem (3-DM) has been proved NP-complete. Prove that the sub-set sum problem is NP-complete.

[L2] Consider the use of branch and bound method to solve the traveling salesman problem.

(L2a) Given a cost matrix $\mathbf{M}$, how to calculate the value $V = V(\mathbf{M})$ of the matrix $\mathbf{M}$?
(L2b) Consider a graph on 4 vertices corresponding to the following cost matrix $\mathbf{M}$

$$\mathbf{M} = \begin{bmatrix} \infty & 8 & 6 & 7 \\ 8 & \infty & 7 & 4 \\ 6 & 7 & \infty & 6 \\ 7 & 4 & 6 & \infty \end{bmatrix}.$$

Using a branch-and-bound method, find the minimum-cost Hamiltonian circuit.

[L3] Let A and B be in NP, and A be polynomial-time reducible to B. Briefly prove:
(L3a) If B is in P, then A is in P.
(L3b) If A is NP-complete, then B is NP-complete.
(L3c) If A is NP-complete and B is in P, then $P = NP$.

[L4]Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.
(L4a) $\{a^n b^m c^m : n, m \geq 0\}$;
(L4b) $\{a^{n^2} : n \geq 0\}$;
(L4c) $\{a^{2n+1} : n \geq 0\}$.