

# Algorithm and Theory of Computation 2011 Jan.

## 1 Short Questions

[S<sub>1</sub>] Let  $P(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$ , where  $a_k > 0$ . Prove

$$P(n) = \Theta(n^k).$$

[S<sub>2</sub>] Ordering by asymptotic growth rate from slowest to fastest:

$$7^{n^{1.008}}, (100n + 2)^2, n^3, \lg^5 n, e^n, \log_5 n, 8^{n^{1/007}}.$$

[S<sub>3</sub>] Calculate  $\sum_{i=1}^n i \times (i + 1) \times (i + 2) \times (i + 3) \times (i + 4) \times (i + 5)$ .

[S<sub>4</sub>] Define and briefly explain: (a) an example of a problem proven to be computationally unsolvable; (b) an example of a problem solvable and believed, but not yet proved, to be computationally intractable; (c) the difference between unsolvable and intractable.

## 2 Long Questions

[L<sub>1</sub>] From the following recurrence determine the growth rate of  $T(n)$ :

$$\begin{cases} T(n) = T(n-1) + T(n-2) \\ T(0) = 1 \quad T(1) = 1 \end{cases}.$$

[L<sub>2</sub>] Using dynamic programming algorithm to calculate the matrix product  $A_1 \times A_2 \times A_3 \times A_4 \times A_5$ , where  $A_1 : 30 \times 35$ ,  $A_2 : 35 \times 15$ ,  $A_3 : 15 \times 5$ ,  $A_4 : 5 \times 10$  and  $A_5 : 10 \times 20$ .

[L<sub>3</sub>] Suppose we have an instance of *TSP* given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Give the partial solution  $X = (5, -, -, -, -)$ , calculate  $B(X)$  using the reducing technique on the matrix.

b) For  $X$  as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.

[L<sub>4</sub>] Prove or disprove: (a) The concatenation of two regular languages is regular. (b) The concatenation of two context free languages is context free.