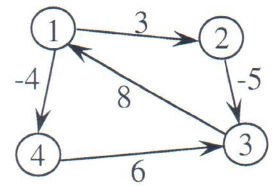
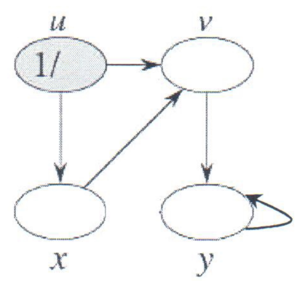
CSCE 500 Final Exam

1. All-pairs shortest paths (APSP) can be derived by extending the number of links per path repeatedly. A fast version for APSP doubles the number of links in each iteration.
   (a) Derive the resulting distance matrix of the directed graph below, following the fast APSP (Show the intermediate matrix of each iteration.)
   (b) How many iterations does it take to obtain the resulting matrix for a graph with n vertexes.

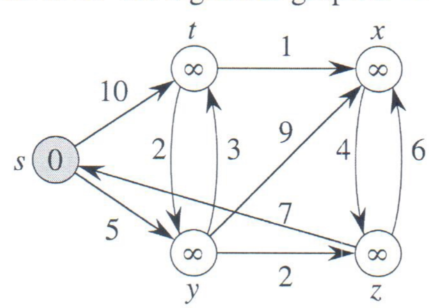


2. Depth First Search (DFS) colors every vertex of a given graph in white, gray, and black during the process to build a search tree, with a discovery time and a finish time kept at each vertex. All edges in a directed graph $G = (V, E)$ under DFS are classified into four types: tree, back, forward, and cross edges. For G given below, conduct DFS that starts from vertex u at time clock = 1 to
   (i) mark the discovery time and the finish time of every vertex, and
   (ii) classify the types of all edges. (Note: one intermediate result should be shown upon each edge classification for clarity.)



3. The Bellman-Ford algorithm (BF) solves the single-source shortest-path problem in a weighted directed graph $G = (V, E)$. Given the graph G below, follow BF to find shortest paths from vertex s to all other vertexes, with all predecessor edges shaded and estimated distance values from s to all vertexes provided at the end of each iteration.

   How many iterations are involved in BF for a general graph $G = (V, E)$?



4. Show your construction of an optimal Huffman code for the set of 6 frequencies: **a**:2 **b**:15 **c**:7 **d**:21 **e**:17 **f**:38.

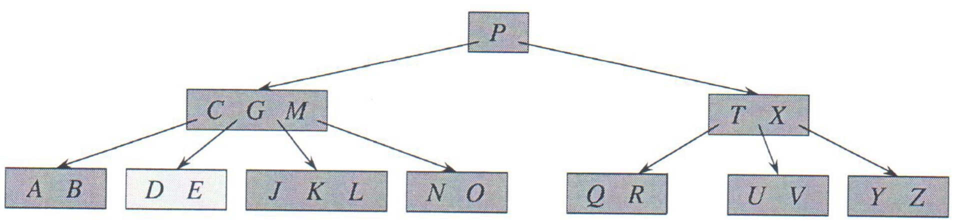
In general, for an alphabet set C, where each of its element c has frequency $c.f$, briefly prove that there exists an optimal prefix code for C such that the codewords of its two elements x and y have the same length and differ only in the last bit. (Hint: transform an arbitrary optimal prefix code tree where x and y are not under the same parent node to another tree in which they are.)

5. Solve the recurrence of $T(n) = 3 \cdot T(n/4) + n^{1/2}$.

6. The Given two hash functions of h_1 and h_2 for Cuckoo hashing under two tables, T_1 and T_2, describe the steps involved in inserting a record with the key of K_{new}.

   Cuckoo hashing can be analyzed by the Cuckoo graph, whose nodes denote table entries and links connect pairs of nodes where given keys can be held. State when a new key can be inserted successfully based on the Cuckoo graph.

7. Given a B-tree with the minimum degree of $t = 3$ below, show the results after
   (a) deleting T and then deleting P in order,
   (b) followed by inserting S.



8. Consider the matrix-chain multiplication problem for four matrices A_1, A_2, A_3, A_4, with their sizes being 30×10, 10×20, 20×50, and 50×40, respectively. Follow the tabular, bottom-up method in the procedure of MATRIX-CHAIN-ORDER below to construct tables that keep respectively entry $m[i, j]$ for all $1 \le i, j \le 4$ and entry $s[i, j]$ for $1 \le i \le 3$ and $2 \le j \le 4$ to get the optimal parenthesized multiplication result.
   (a) Construct the two tables, with their entry values shown.
   (b) Show the parenthesized multiplication of the matrix-chain.

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MATRIX-CHAIN-ORDER(p)
n = p.length - 1
let m[1..n, 1..n] and s[1..n-1, 2..n] be new tables
for i = 1 to n
    m[i,i] = 0
for l = 2 to n                // l is the chain length
    for i = 1 to n - l + 1
        j = i + l - 1
        m[i,j] = ∞
        for k = i to j - 1
            q = m[i,k] + m[k+1,j] + p_{i-1} p_k p_j
            if q < m[i,j]
                m[i,j] = q
                s[i,j] = k
return m and s
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