Ph.D. Comprehensive Exam
Algorithms

Fall 2016

Short Questions (Answer any 8 of the 10 questions below)

X 1. a) Define an upper and the tight time bound of an algorithm.

b) In which way the average time bound will add more value to the tight bound?

2. a) Briefly describe a quick sort algorithm for sorting objects in an ascending order of their keys.

b) What is the best and worst case time complexity of quick sort and the reason for such complexity.

3. Find the tight for the recurrence relations without using the master theorem.

a) $T(n) = T(n-2) + 2 \cdot \lg_2(n)$

b) $T(n) = 2 \cdot T(n^{1/2}) + \lg_2(n)$

4. a) What are the properties of min heap and max heaps.

b) What is the preferred data structure of implementing binary heap, also justify your answer.

c) What is the time complexity of merging two different min heaps with sizes of n and m.

5. Suppose there are n clauses and m variables (propositions) in a given 3-p sat problem.

a) How many possible interpretations are there?

b) Find the tight bound of checking for satisfiability of the n clauses.

6. a) Briefly define greedy strategy.

b) Will it always yield an optimal solution? If not, provide an algorithm that yields an optimal solution.

7. a) Define strongly connected components.

b) Does a strongly connected component graph cyclic or acyclic? Justify your answer.

8. Mark true/false (T/F) against the following statements:

(1) A binary search tree of size N will always find a key at most O(log N) time.

(2) A breadth first search can be considered as a special case of heuristic search algorithm.

(3) An optimal binary search tree is not necessarily being a balanced tree.

(4) A dynamic programming approach uses top-down problem solving strategy to solve optimization problem.

9. Given two hash functions of h_1 and h_2 for Cuckoo hashing under two tables, T_1 and T_2.
   a) Describe the steps involved in inserting a record with the key of K_{new}.
   b) Cuckoo hashing can be analyzed by the Cuckoo graph, whose nodes denote table entries and links connect pairs of nodes where given keys can be held. State when a new key can be inserted successfully based on the Cuckoo graph.

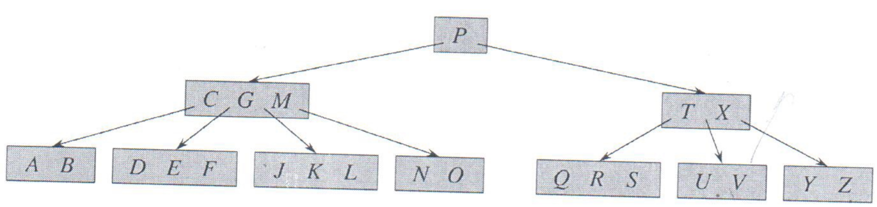
10. The recurrence of Procedure CUT-ROD(p, n) is given by $T(n) = 1 + \sum_{j=0}^{n-1} T(j)$, with $T(0) = 1$. Solve $T(n)$.

Long Questions (Answer any 3 of the 4 questions below)

1. a) Briefly describe NP-class, P-class, NP-complete and NP-hard.
   b) Show the conjectured relationship among the classes NP-class, P-class, NP-complete and NP-hard.
   c) Show that sorting n objects with integer key values belongs to NP-class.
   d) Provide the steps involved in showing whether a problem belongs to NP-complete or not.
   e) Illustrate the steps in step d by showing 3 proposition satisfiability (3-p sat) problem belongs to NP-complete.
   f) Provide a pseudo code that attempt to solve 3-p sat problem heuristically.

2. a) Explain what do you understand by "principle of optimality" in the context of dynamic programing.
   b) Characterize 0-1 knapsack problem in terms of objective function, constraints and the time and space complexity. (Assume that there are n objects. Suppose an object i has weight w_i and profit p_i. The overall capacity of the container is W).
   c) Show the 0-1 knapsack problem belong to NP-class.
   d) Does it belongs to P-class? (provide an explanation accordingly)
   e) Write down the basic rule that satisfy the principle of optimality and domain related constraints to the following problems:
      e.1) 0-1 knapsack problem.
      e.2) Pairwise shortest path problem.

3. Given an initial B-tree with the minimum node degree of $t = 2$ below, show the results
   a) after inserting the key of H, and
   b) then followed by deleting two keys in order: X then P. (show the result after insertion and the result after each deletion)



4. Given a set of 4 keys, with the following probabilities, determine the <u>cost</u> and the <u>structure</u> of an <u>optimal binary search tree</u>, following the <u>tabular, bottom-up method</u> realized in the procedure of OPTIMAL-BST below to construct and fill tables: $e[1..5,\ 0..4]$, $w[1..5,\ 0..4]$, and $root[1..4,\ 1..4]$.

i	0	1	2	3	4
p_i		0.12	0.08	0.14	0.20
q_i	0.05	0.08	0.07	0.16	0.10

Construct and fill <u>the three tables</u>, and show the <u>optimal BST obtained</u>.

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OPTIMAL-BST(p, q, n)
let e[1..n+1, 0..n], w[1..n+1, 0..n],
        and root[1..n, 1..n] be new tables
for i = 1 to n + 1
    e[i, i-1] = q_{i-1}
    w[i, i-1] = q_{i-1}
for l = 1 to n
    for i = 1 to n - l + 1
        j = i + l - 1
        e[i, j] = ∞
        w[i, j] = w[i, j-1] + p_j + q_j
        for r = i to j
            t = e[i, r-1] + e[r+1, j] + w[i, j]
            if t < e[i, j]
                e[i, j] = t
                root[i, j] = r
return e and root
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