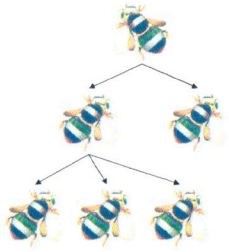
CSE 326: Data Structures
Lecture #9
Big, Bad
B-Trees



Steve Wolfman
Winter Quarter 2000

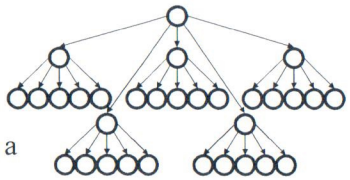
Today's Outline

- Hand in the homework (**GO! GO! GO!**)
- B-Trees

M-ary Search Tree

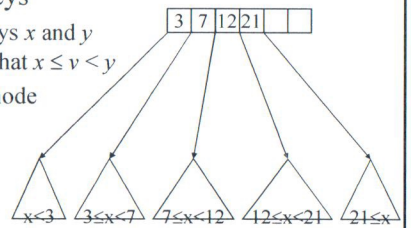
- Maximum branching factor of M
- Complete tree has depth = $\log_M N$
- Each internal node in a complete tree has $M - 1$ keys

runtime:



B-Trees

- B-Trees are specialized *M*-ary search trees
- Each node has many keys
  - subtree between two keys x and y contains values v such that $x \le v < y$
  - binary search within a node to find correct subtree
- Each node takes one full {*page, block, line*} of memory



B-Tree Properties‡

- Properties
  - maximum branching factor of M
  - the root has between 2 and M children *or* at most L keys
  - other internal nodes have between $\lceil M/2 \rceil$ and M children
  - internal nodes contain only search keys (no data)
  - smallest datum between search keys x and y equals x
  - each (non-root) leaf contains between $\lceil L/2 \rceil$ and L keys
  - all leaves are at the same depth
- Result
  - tree is $\Theta(\log n)$ deep
  - all operations run in $\Theta(\log n)$ time
  - operations pull in about M or L items at a time

‡These are technically B^+-Trees

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  - all operations run in $\Theta(\log_M n)$ time
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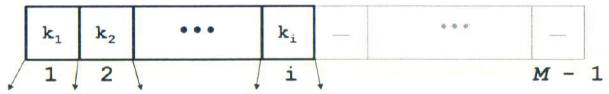
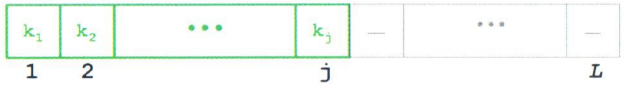
B-Tree Nodes

- Internal node
  - `i` search keys; `i+1` subtrees; ***M*** `- i - 1` inactive entries

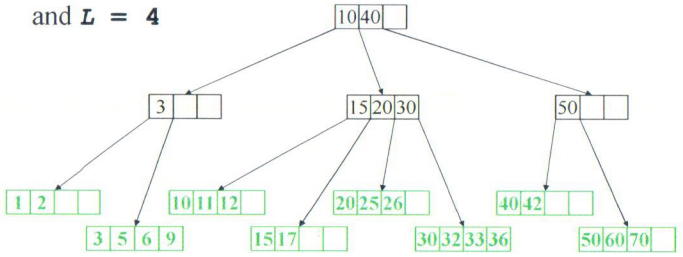


- Leaf
  - `j` data keys; ***L*** `- j` inactive entries

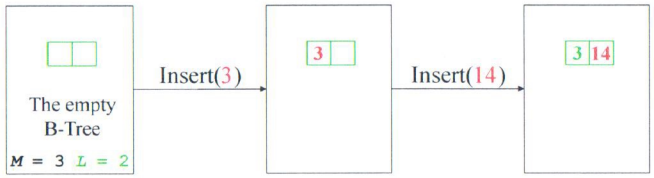


Example

B-Tree with ***M*** `= 4`
and ***L*** `= 4`

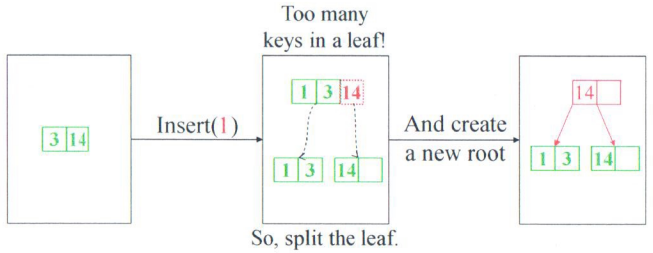


Making a B-Tree

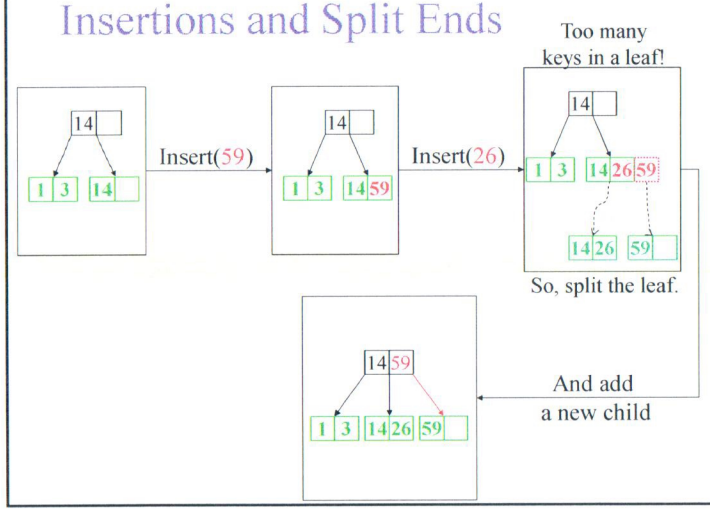


Now, Insert(1)?

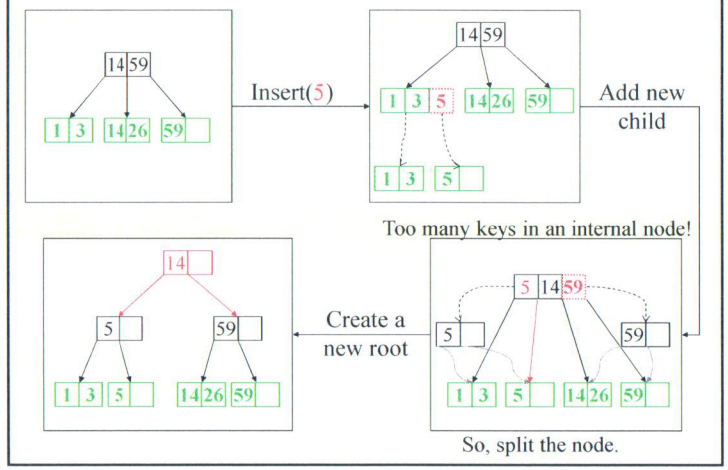
Splitting the Root



Insertions and Split Ends



Propagating Splits

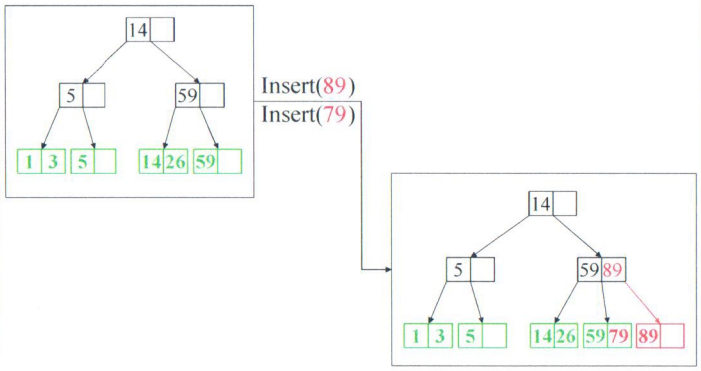


Insertion in Boring Text

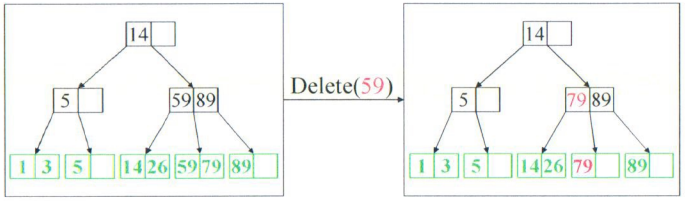
- Insert the key in its leaf
- If the leaf ends up with L+1 items, **overflow**!
  - Split the leaf into two nodes:
    - original with $\lceil (L+1)/2 \rceil$ items
    - new one with $\lfloor (L+1)/2 \rfloor$ items
  - Add the new child to the parent
  - If the parent ends up with $M+1$ items, **overflow**!
- If an internal node ends up with M+1 items, **overflow**!
  - Split the node into two nodes:
    - original with $\lceil (M+1)/2 \rceil$ items
    - new one with $\lfloor (M+1)/2 \rfloor$ items
  - Add the new child to the parent
  - If the parent ends up with $M+1$ items, **overflow**!
- Split an overflowed root in two and hang the new nodes under a new root

This makes the tree deeper!

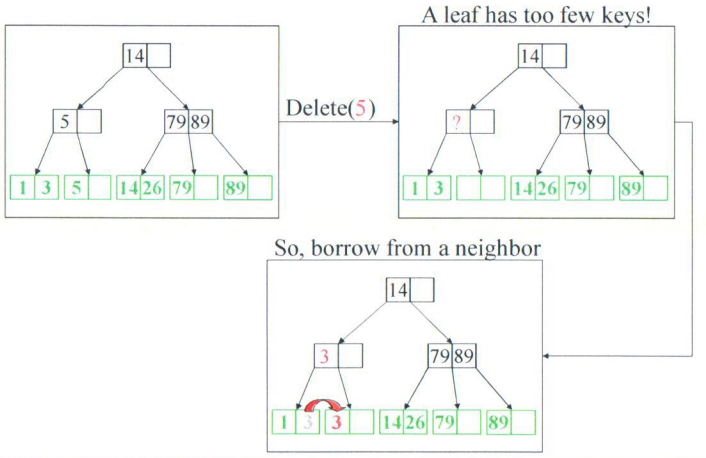
After More Routine Inserts



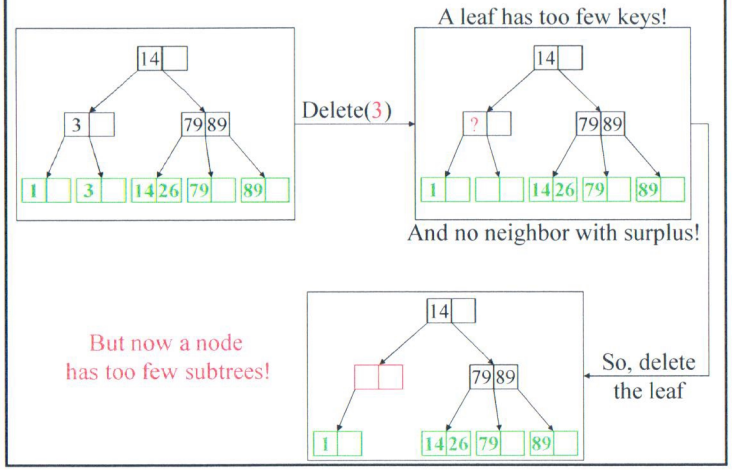
Deletion



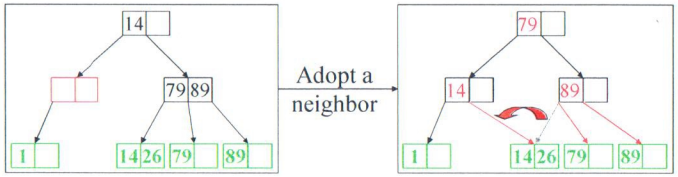
Deletion and Adoption



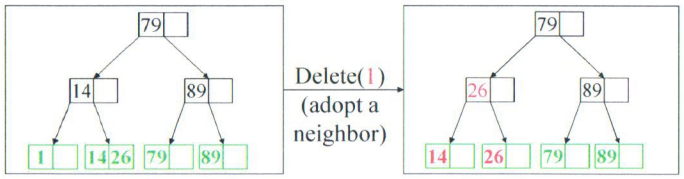
Deletion with Propagation



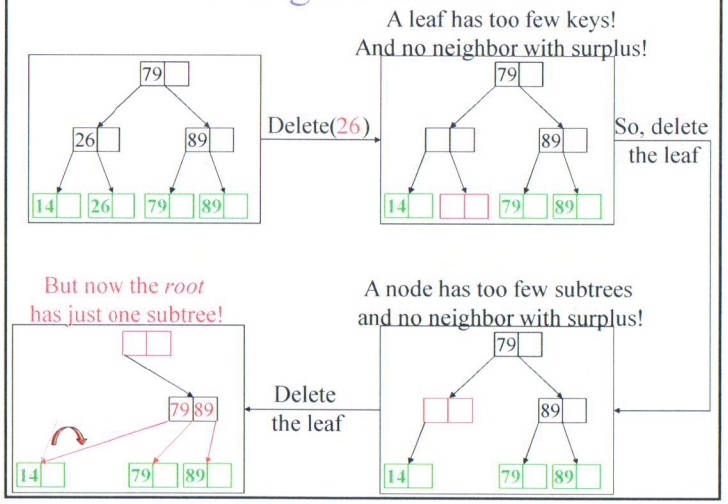
Finishing the Propagation (More Adoption)



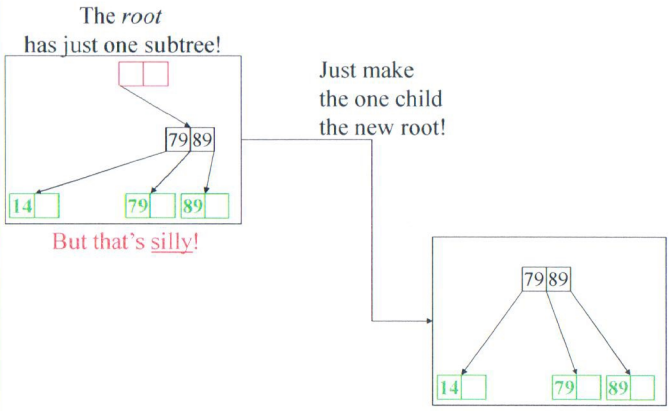
A Bit More Adoption



Pulling out the Root



Pulling out the Root (continued)



Deletion in *Two* Boring Slides of Text

- Remove the key from its leaf
- If the leaf ends up with fewer than $\lceil L/2 \rceil$ items, **underflow**!
  - Adopt data from a neighbor; update the parent
  - If borrowing won't work, delete node and divide keys between neighbors
  - If the parent ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!

Why will dumping keys always work if borrowing doesn't?

Deletion Slide Two

- If a node ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!
  - Adopt subtrees from a neighbor; update the parent
  - If borrowing won't work, delete node and divide subtrees between neighbors
  - If the parent ends up with fewer than $\lceil M/2 \rceil$ items, **underflow**!
- If the root ends up with only one child, make the child the new root of the tree

This reduces the height of the tree!

Thinking about B-Trees

- B-Tree insertion can cause (expensive) splitting and propagation
- B-Tree deletion can cause (cheap) borrowing or (expensive) deletion and propagation
- Propagation is rare if `M` and `L` are large *(Why?)*
- Repeated insertions and deletion can cause thrashing
- If `M = L = 128`, then a B-Tree of height 4 will store at least 30,000,000 items

A Tree with Any Other Name

FYI:

- B-Trees with `M = 3`, `L = x` are called 2-3 trees
- B-Trees with `M = 4`, `L = x` are called 2-3-4 trees

Why would we ever use these?

Another Way

- B-Trees are *hairy* in practice

hairy - Slang. Fraught with difficulties; hazardous: a hairy escape; hairy problems.

To Do

- Continue Project II
- Read chapter 4 in the book
- Look forward to no quiz/homework for two weeks!
- (And prepare for the midterm)

Coming Up

- Self-balancing Trees
  - AVL trees
  - Splay trees
- Second project due (January 31st)
- Midterm (February 4th)!