

Comprehensive Examination Algorithms and Theory

17 August 2007

1 Short Questions: Answer any 3 of 4.

[S₁] Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ -function, prove that

$$\max(f(n), g(n)) = \Theta(f(n) + g(n)).$$

[S₂] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = 2T(\frac{n}{2}) + n^3 \\ T(1) = 1 \end{cases}$$

[S₃] Give a regular expression or a finite automaton for each language:

1. $\{x \in \{a, b\}^* : abb \text{ occurs at least twice in } x\}$
2. $\{x \in \{a, b\}^* : abb \text{ occurs exactly once in } x\}$

[S₄] Give a context free grammar for each language:

1. $\{a^n b^{n+m} c^m : n, m \geq 0\}$
2. $\{xc^n d^n x^R : x \in \{a, b\}^*, n \geq 0\}$

2 Long Questions: Answer any 3 of 4.

[L₁] Suppose we have an instance of *TSP* given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Give the partial solution $X = (5, 2, -, -, -)$; calculate $B(X)$ using the reducing technique on the matrix.

b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.

[L₂] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 4, 3; and weight 7, 5, 4, 3, 2, 1 (resp.).

[L₃] Classify each of the following languages as regular, context free but not regular, or in P but not context free. Prove your answers.

1. $\{a^n b^{n^2} : n > 0\}$
2. $\{a^{2n} b^{2n+1} : n > 0\}$
3. $\{a^{2n} b^{2m+1} : n, m > 0\}$

[L₄] Briefly prove or disprove:

1. The intersection of two context free languages is context free.
2. The intersection of two Turing acceptable languages is Turing acceptable.
3. The intersection of two NP languages is NP .