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<u>CSCE 500</u> Midterm Exam #2

11/4/2020

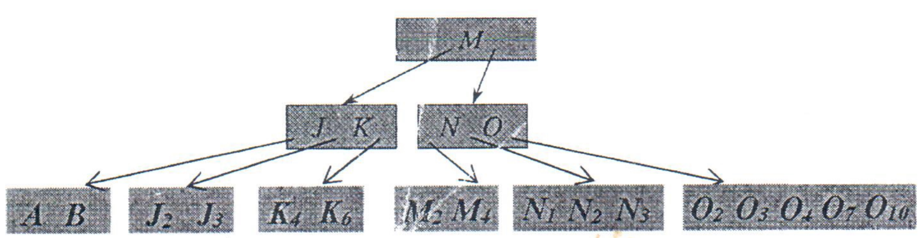
1. A binary search tree (T) is to be maintained following the in-order tree traversal order. Consider a sequence of arrival keys, {32, 27, 23, 42, 14, 25, 33, 34, 37, 28, 30, 31}, to T which is empty initially.
   (a) Show the resulting T after inserting all arrival keys. (8%)
   (b) Show the resulting T after its root node is then deleted. (4%)
   (c) Show the resulting T after deleting node with key of 27 from T obtained by (2) above. (4%)

2. For a node in a binary search tree (T), how do you find its predecessor? its successor? (8%)

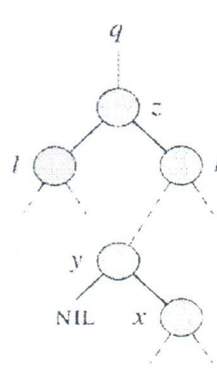
3. For any n-key B-tree of height h and with the minimum node degree of $t \geq 2$, show that h is no larger than $\log_t \frac{n+1}{2}$. (Hint: consider the number of keys stored in each tree level.) (16%)

4. For a given B-tree of height h and with the minimum node degree of $t \geq 2$, what is the maximum number of keys held in such a B-tree? (12%)

5. Given the initial B-tree with the minimum node degree of $t = 3$ below, show the results (a) after deleting the key of M_2, (b) followed by inserting the key of L, (c) then by deleting the key of J_2, (d) then by inserting the key of O_5, with $O_4 < O_5 < O_7$, and (e) then by deleting K. (Show the result after each deletion and after each insertion; 20%)

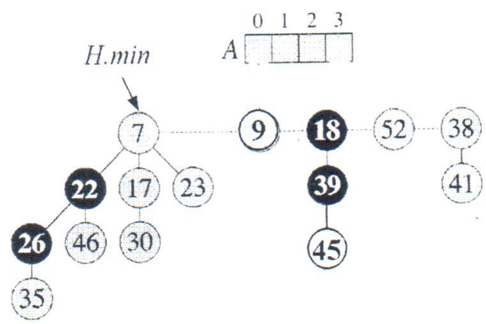


6. When the root of a binary subtree, z, is deleted, as depicted below, show the resulting subtree. (8%)



(7) A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge trees in the root list upon the operation of extracting the minimum node. Show steps involved and the resulting heap after *H.min* is extracted form the Fibonacci min-heap given below. (16%)

After consolidation is completed, show the resulting Fibonacci min-heap with key '46' decreased to 20. (4%)



Good Luck!

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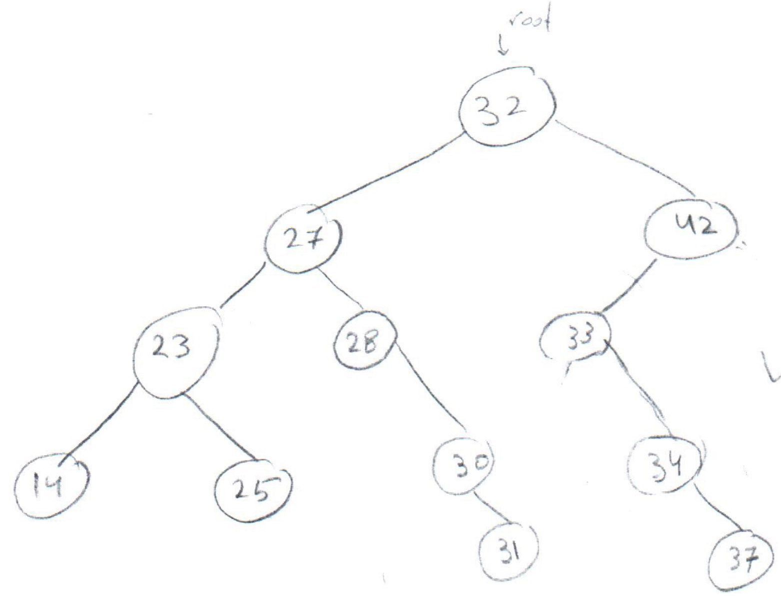
① 32, 27, 23, 42, 14, 25, 33, 34, 37, 28, 30, 31

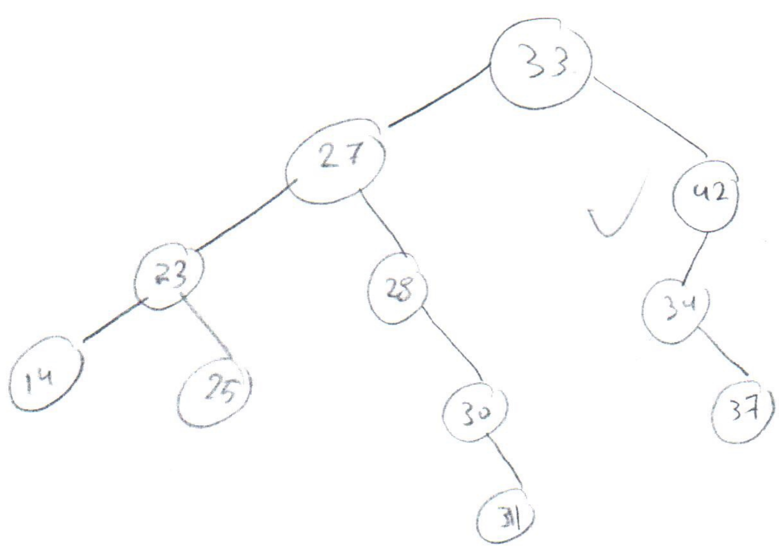
① 16
② 8
③ 15
④ 11
⑤ 12
⑥ 8
⑦ 20

90

a)

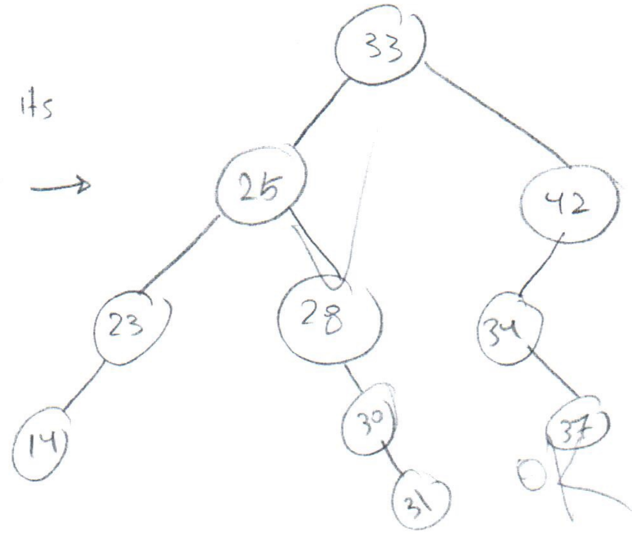


b) Delet the root 32, find its sucssor which is 33
replace the root with 33,

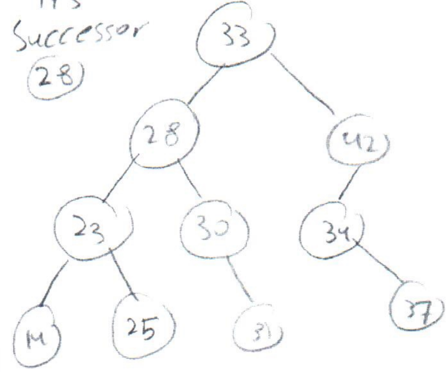


c) after delet 27

if replace 27 with its Predecessor 25 →

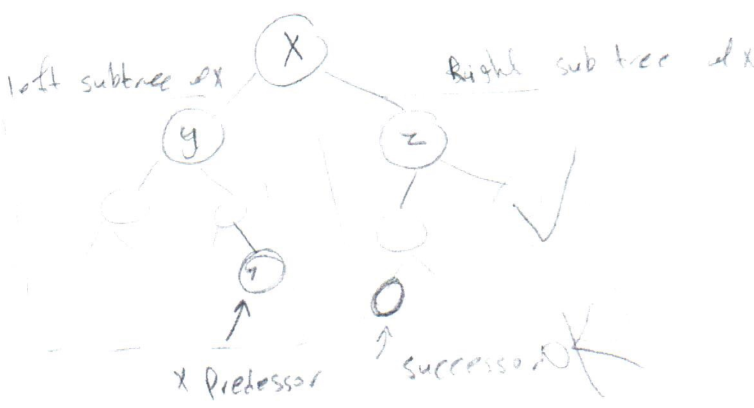


if replace nod 27 with its Successor 28

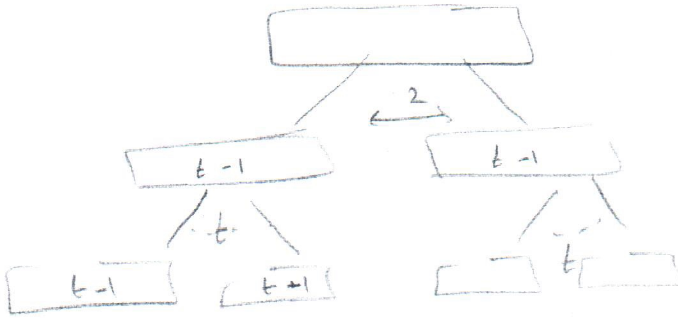


2) to find Predecessor for x, we gothrough the left subtree and find the most right node which is largest key in this left-sub tree.

to find sucessor of node x, we go throug the right sub tree of x, and find the ~~most~~ last node along left branc which is the smallest value (key) in the right sub tree of X



3)



depth	# of node
0	1
1	2
3	$2t^2$
$\vdots$	
i	$2t^{i-1}$

min degree of $t \geq 2$

- the root must have at least one key
- internal node at lest $t-1$
- at any depth h, the minimum # of node is $2t^{h-1}$ $\therefore$ total number will be $2t^0 + 2t^1 + 2t^2 + \ldots 2t^{i-1}$

$$n \geq 1 + (t-1) \cdot \sum_{i=1}^{h} 2t^{i-1}$$

$$\geq 1 + (t-1) \cdot 2 \cdot \frac{t^h - 1}{(t-1)}$$

$$n \geq 2t^h - 1$$

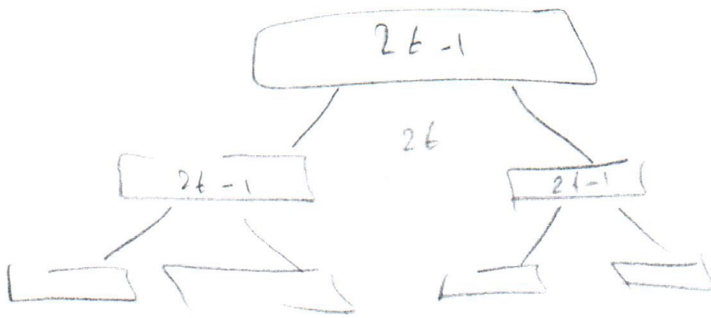
$$\frac{n+1}{2} \geq \frac{2}{2} t^h$$

$$\frac{n+1}{2} \geq t^h$$

$$h \leq \log_t \frac{n+1}{2}$$

-1

4)



d	# of key
0	2
1	$2t^1$
2	$(2t)^2$
i	$(2t)^i$

max in each node is $2t-1$

at any depht h we have max node of $(2t)^i$

so $n \le (2t^0) + (2t^1) + (2t)^2 + \dots (2t)^i$

$$2t-1 \cdot \sum_{i=0}^{h} (2t)^i$$

$$\cancel{2t-1} \cdot \frac{(2t)^{h+1}-1}{\cancel{(2t-1)}}$$

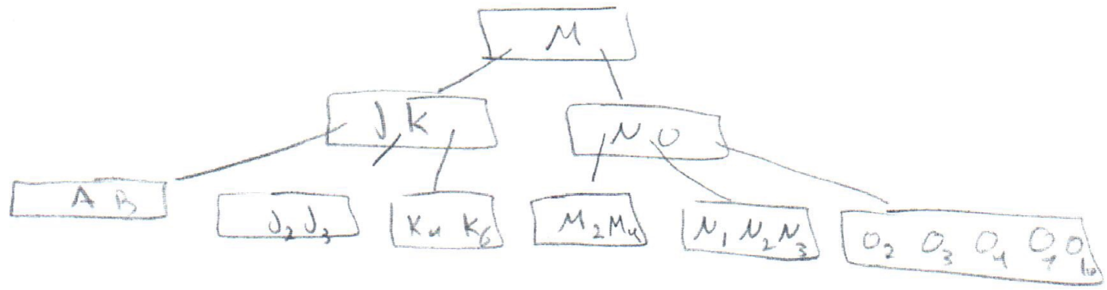
-1

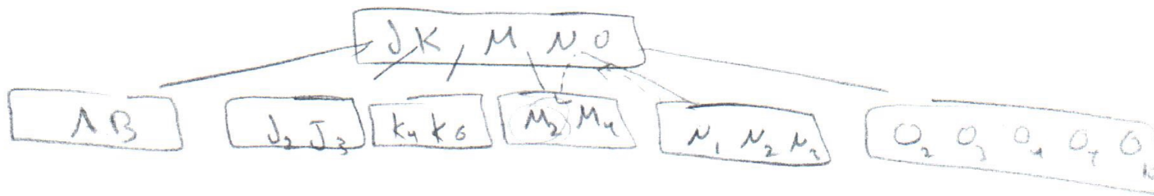
$n \le (2t)^{h+1} - 1$ is the max # of key

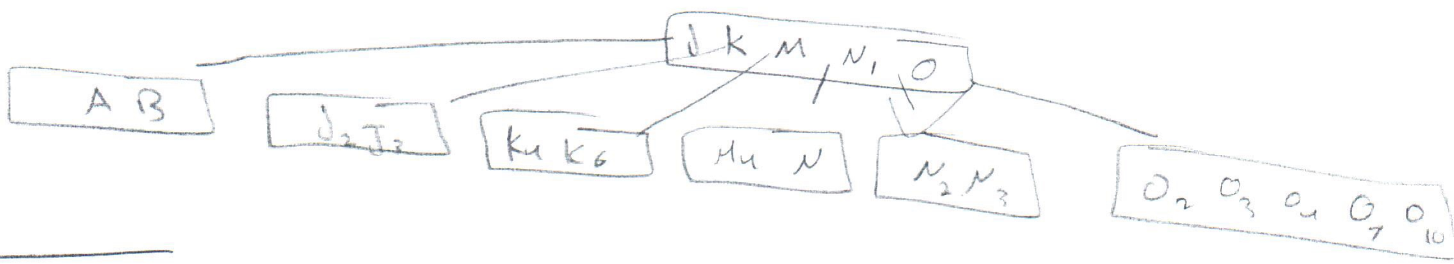
$t = 3 \rightarrow$ key $\underset{min}{2} \rightarrow \underset{max}{5}$

(b)

(a) Delet M_2

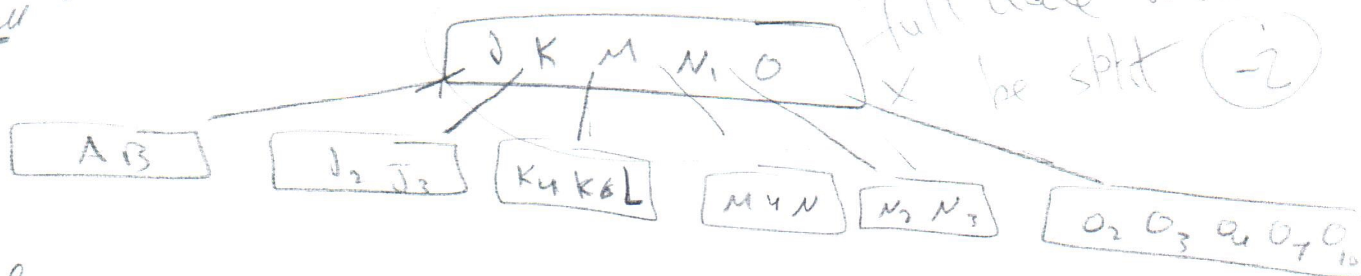






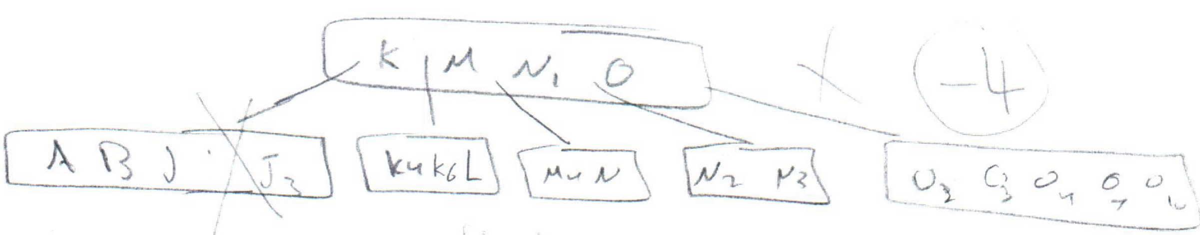
insert L

isertion → any node cant be full



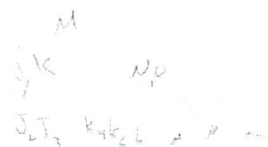
full node needs to be split ✗ (-2)

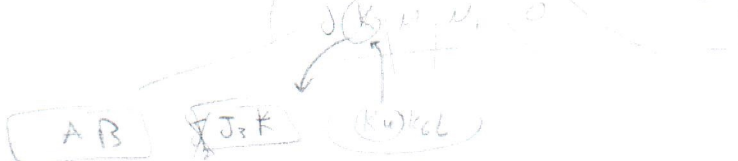
Delet J_2

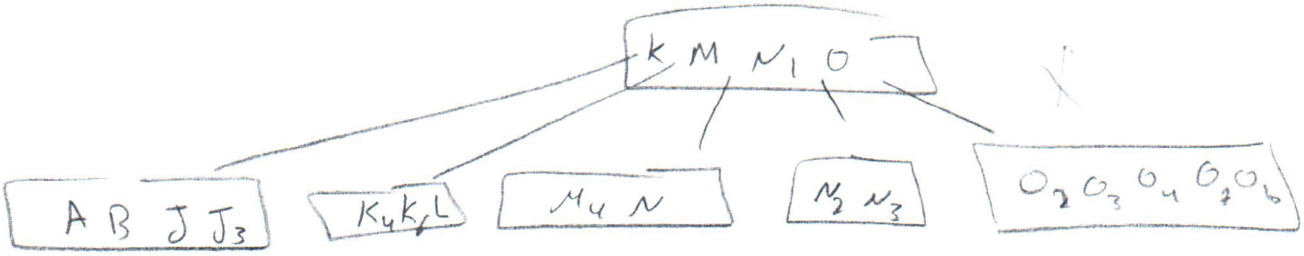


✗ (-4)

Cannot consolidate here

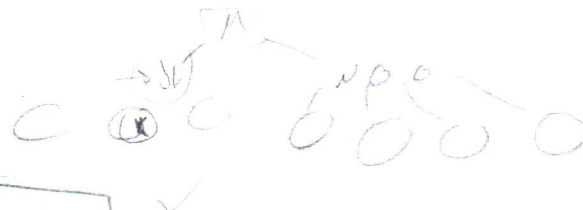


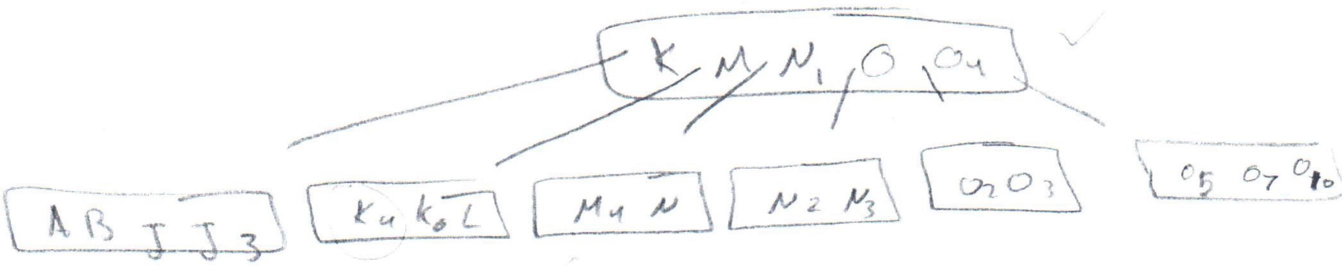




b) ~~at~~ insert O_5

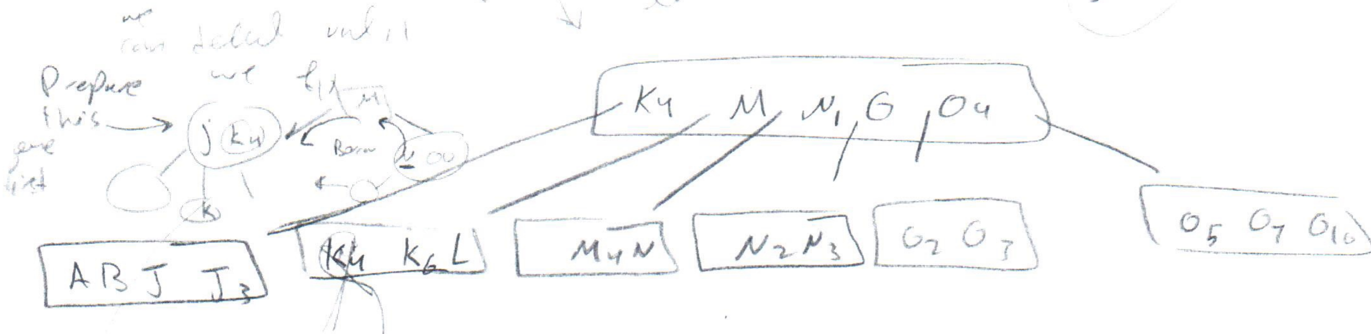
node is full → split





c) Delet K

You made this case easier



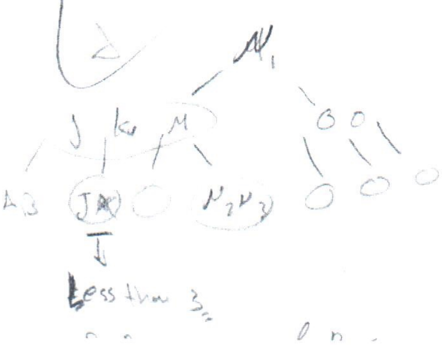
★ for insertion → node should not be full when we pass

★ " Deletion → it is fine to be full

But its need to be have at least

t=3 ~~before deletion~~

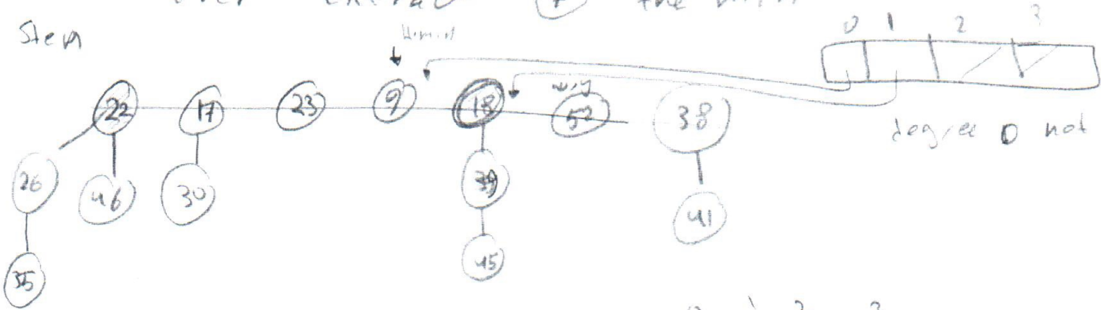
each node we go throug



(7) extract min

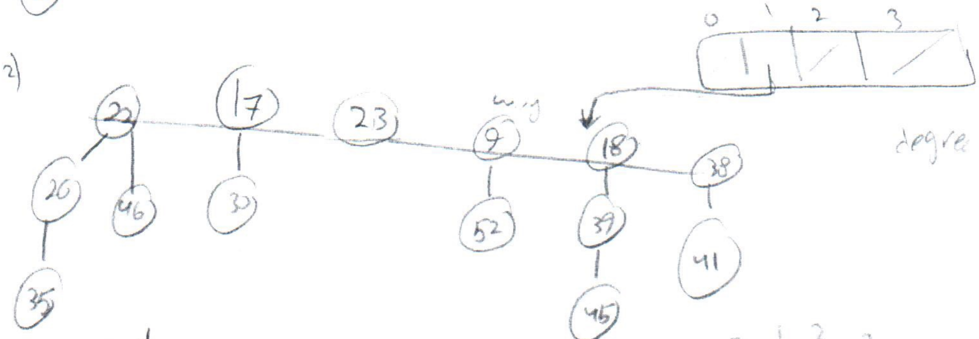
after extract (7) the min

Step



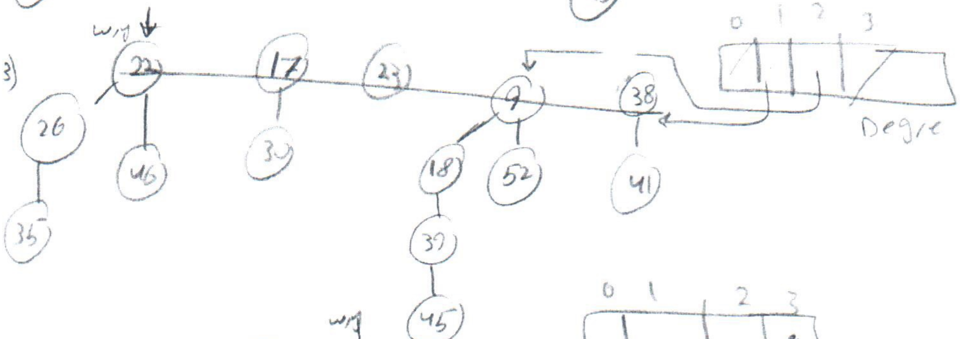
degree 0 not full => union

2)

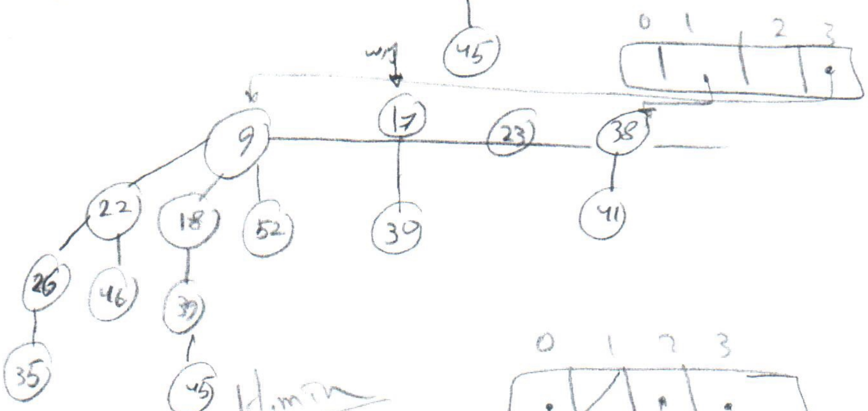


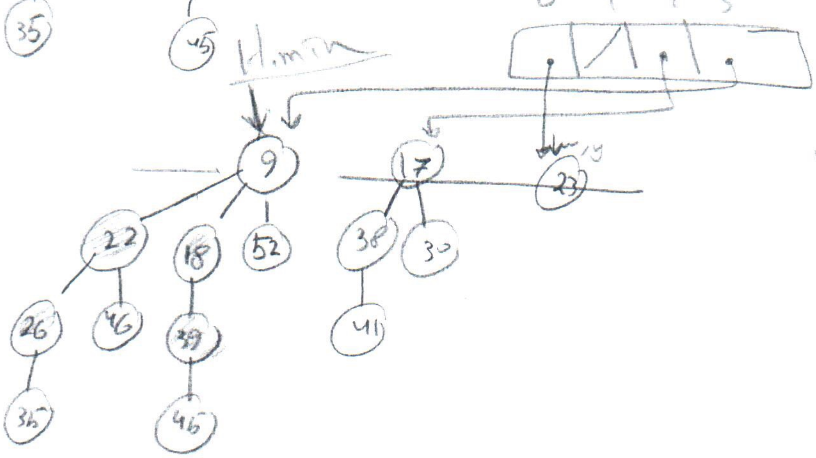
degree 1 not will => union

3)



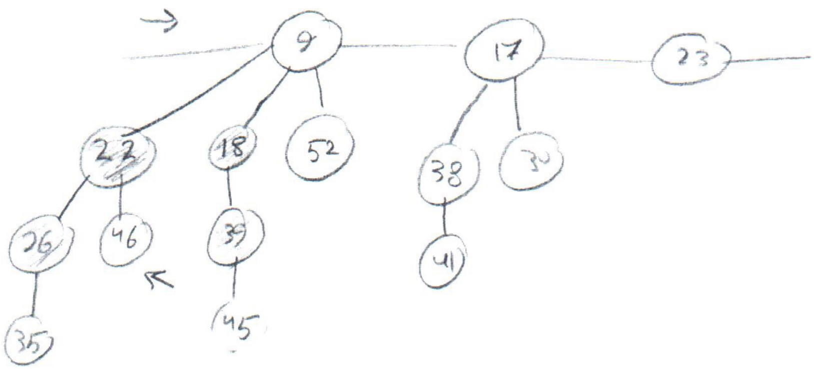
Degre 2, not nil => union



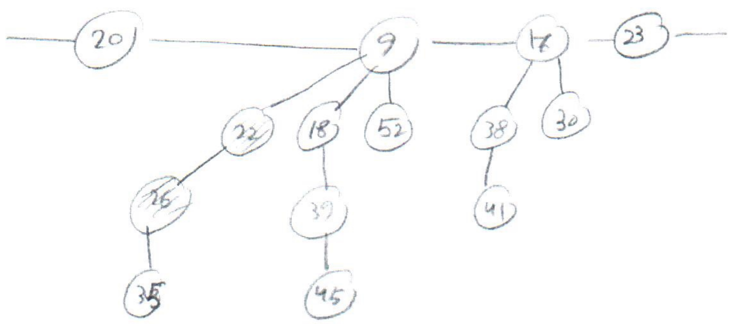


ok

decreased , $46 \xrightarrow{\text{to}} 20$

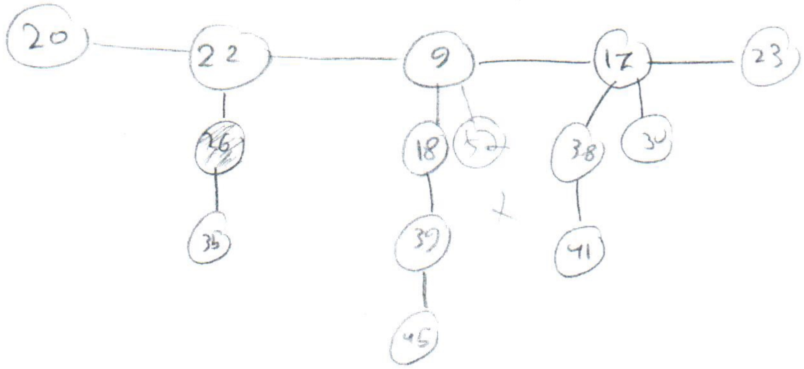


step $\underline{\underline{1}}$: remove $\underline{20}$ to the root , then check if Parent is marked



ok

Parent 22 is mark already ; so move it to the root



ok