CSCE 500 Midterm Exam #1 10/5/2016

1. Is n^2 an asymptotically-tight bound of $n^2/(\lg n)$? of $(n^{2.5})/400$? (Briefly explain. 6%)

2. The algorithm for finding the maximum subarray that crosses the midpoint of Array $A[1 \ldots n]$ includes the main routine of FIND-MAXIMUM-SUBARRAY(A, *low*, *high*), which calls FIND-MAX-CROSSING-SUBARRAY(A, *low*, *mid*, *high*), as follows. Complete the six (6) missing statements in FIND-MAX-CROSSING-SUBARRAY below. (12%)

```
FIND-MAX-CROSSING-SUBARRAY(A, low, mid, high)
  // Find a maximum subarray of the form A[i .. mid].
  left-sum = −∞
  sum = 0
  for i = mid downto low
      sum = sum + A[i]
      □
          □
          □
  // Find a maximum subarray of the form A[mid + 1 .. j].
  right-sum = −∞
  sum = 0
  for j = mid + 1 to high
      sum = sum + A[j]
      □
          □
          □
  // Return the indices and the sum of the two subarrays.
  return (max-left, max-right, left-sum + right-sum)
```

3. Derive the tight lower and upper bounds of the following recurrences:
$T(n) = 2 \cdot T(n/4) + T(n/2) + c \cdot n$ (10%)
$T(n) = 2 \cdot T(n/2) + n \cdot \lg(n)$. (8%)

4. For any n-key B-tree of height h and with the minimum node degree of $t \geq 2$, prove that h is no larger than $\log_t \frac{n+1}{2}$. (Hint: consider the number of keys stored in each tree level.) (12%)

5. The utilization efficiency of a hash table depends heavily on its hashing function(s) employed. Describe with a diagram to illustrate how a multiplication method of hashing works on a machine with the word size of w bits for a hash table with 2^p entries, $p < w$. (10%)
Explain briefly (1) how perfect hashing works, and (2) how Cuckoo hashing works under two hash functions of h_1 and h_2. (12%)

Give an example that yields the worst-case time complexity under QUICKSORT and briefly describe a simple modification to QUICKSORT for curbing such worst-case scenarios. (10%)

6. RANDOMIZED-SELECT below is based on RANDOMIZED-PARTITION to pick the i^{th} ranked element among n array elements with linear time complexity on an average. Complete the missing 3 statements in the code. (8%)

```
RANDOMIZED-SELECT(A, p, r, i)
if p == r
    return A[p]
q = RANDOMIZED-PARTITION(A, p, r)
1.
if i == k            // pivot value is the answer
    return A[q]
elseif i < k
    2.
else
    3.
```

Briefly state how to select the i^{th} ranked element among n array elements with $O(n)$ time complexity in the worst case? (10%)

7. Given two hash functions of h_1 and h_2 for Cuckoo hashing under two tables, T_1 and T_2, describe the steps involved in inserting a record with the key of K_{new}. (10%)

Cuckoo hashing can be analyzed by the Cuckoo graph, whose nodes denote table entries and links connect pairs of nodes where given keys can be held. State when a new key can be inserted successfully based on the Cuckoo graph. (5%)

Good Luck!

Chance - Heap Sort example.