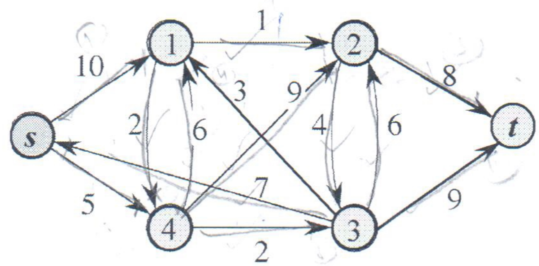
CSCE 500 Final Exam

12/3/2018

The next three questions are based on the following direct graph, which has no negative weights.

1. The Edmonds-Karp algorithm (*EK*) follows the basic Ford-Fulkerson method with breadth-first search to choose the shortest augmenting path (in terms of the number of edges involved) for computing the maximum flow iteratively from Vertex **s** to Vertex **t** below. Illustrate the maximum flow computation process (including the augmenting path chosen in each iteration and its resulting residual network) via *EK*. (10%)

   What is the time complexity of *EK* on the graph $G = (V, E)$? (2%)



2. The Floyd-Warshall algorithm (*FW*) obtains all pairs of shortest paths in a weighted directed graph. Consider the graph given in Problem 1 above, with Vertices **s** and **t** ignored. What is the recursive equation of $d_{i,j}^{(k)}$ for the shortest-path weight of any path between i and j with intermediate vertices $\in \{1, 2, 3, \dots, k\}$? (2%)

   Derive all distance matrices $D^{(k)}$ following *FW* so that the $d_{i,j}^{(n)}$ element of final matrix $D^{(n)}$ denotes $\delta(i, j)$ for every vertex pair $<i, j>$ for all $i, j \in \{1, 2, 3, 4\}$. (12%)

   $d_{ij}^{(k)} = w_{ij}$ if $k=0$

   $\min(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$ $k \geq 1$

3. Follow depth-first search (*DFS*), starting from Node **s**, to traverse all six nodes of the graph shown in Problem 1 above. Mark (1) the type of every edge and (2) the discovery and the finish times of each node. (8%)

4. Many problems have been proved to be NP-complete. To prove NP-completeness, it is necessary in general to demonstrate two proof components. What are the two proof components to show a problem being NP-complete? (2%)

   Given that the Hamiltonian-cycle problem (HAM-CYCLE) belongs to NP-completeness, how do you prove that the traveling-salesman problem (TSP) is NP-complete? (1%)

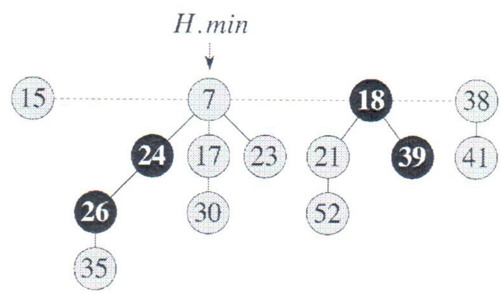
   TSP has a 2-approximation solution in polynomial time based on establishing a minimum spanning tree (MST) rooted at the start/end vertex (in polynomial time following MST-PRIM), if the graph edge weights observe triangle inequality. Sketch a brief proof to demonstrate that such a solution satisfies 2-approximation. (7%)

5. Solve the recurrence of $T(n) = 4 \cdot T(\sqrt{n}) + (\lg n)^2$ by either substitution or the recursion-tree method. (6%)

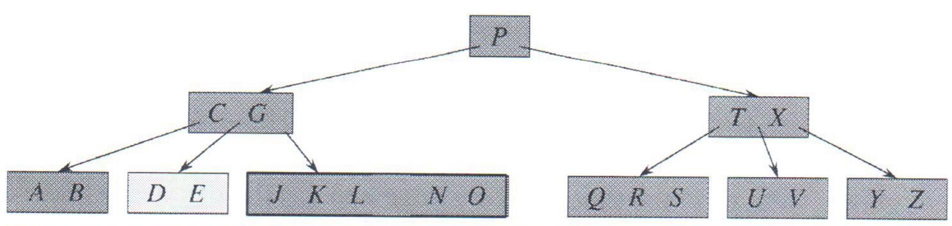
6. The utilization efficiency of a hash table depends heavily on its hash function(s) employed. Describe with a diagram to illustrate how a multiplication method of hashing works on a machine with the word size of w bits for a hash table with 2^p entries, $p < w$. (7%)

Briefly state how a hash function can be employed for de-duplication in data archival. (3%)

7. A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge trees in the root list upon the operation of extracting the minimum node. Given the Fibonacci heap below, show the resulting heap after *H.min* is extracted. (7%; illustrate key steps involved.)

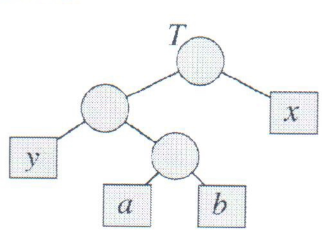


8. Given a B-tree with the minimum degree of $t = 3$ below, show the results after (i) deleting V, (ii) then followed by inserting M, and (iii) then followed by deleting B. (9%)



9. Sketch a proof of the Lemma below, using the tree provided. (8%)

Let C be an alphabet in which each character $c \in C$ has frequency $c.freq$. Let x and y be two characters in C having the lowest frequencies. Then there exists an optimal prefix code for C in which the codewords for x and y have the same length and differ only in the last bit.



10. Given the matrix-chain multiplication problem for four matrices sized 20×50, 50×10, 10×30, 30×15, follow the tabular, bottom-up method in the procedure of MATRIX-CHAIN-ORDER, which constructs a table to keep entry $m[i, j]$ for all $1 \le i, j \le 4$ (with $m[i, j]$ denoting the minimum number of scalar multiplications needed to compute the result) and another table to hold corresponding entry $s[i, j]$ for $1 \le i \le 3$ and $2 \le j \le 4$.

(a) Construct both tables, with their entry values shown. (12%)

(b) Give the optimal parenthesized result, following s. (3%)

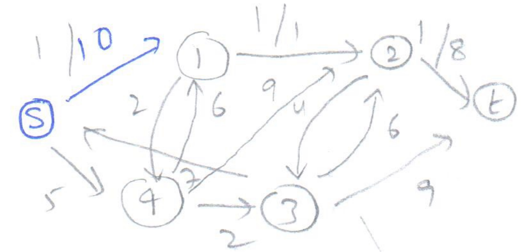
Good Luck!

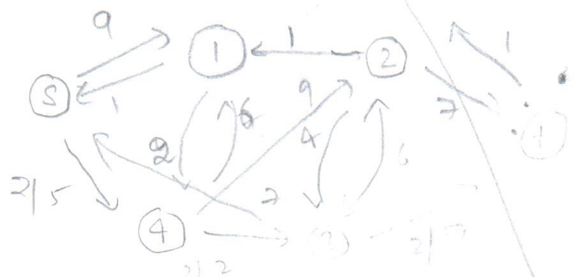
C00227093
Vijay srinivas Tide

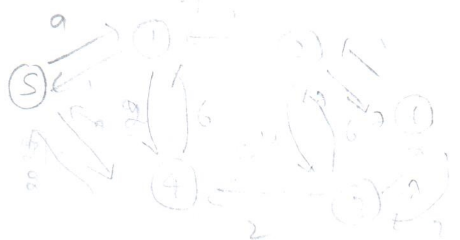
89

1	~~10~~ 12
2	14
3	8
4	10
5	6
6	8
7	3
8	6
9	8
10	14

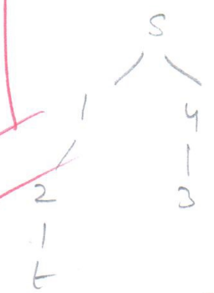
1



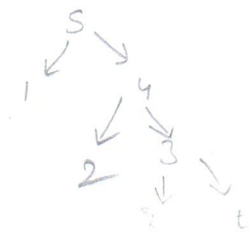




So the maximum flow from s → t using ... is 3.



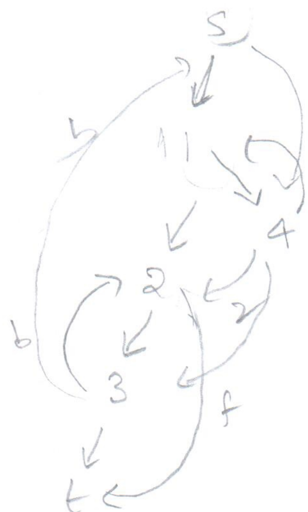
S → 1 → 2 → t

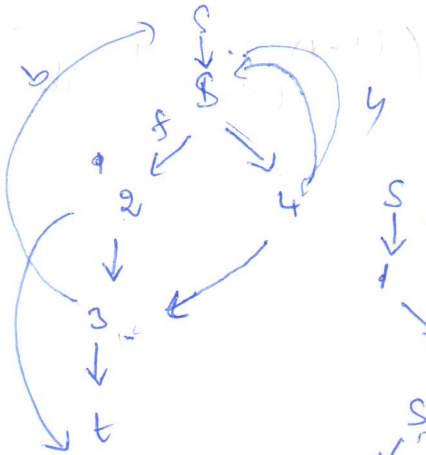


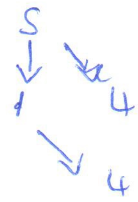
S → 4 → 3 → t

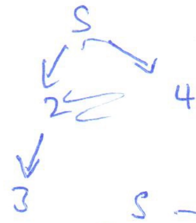
There are no more paths from s to t

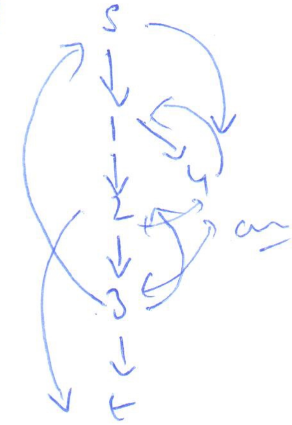
3

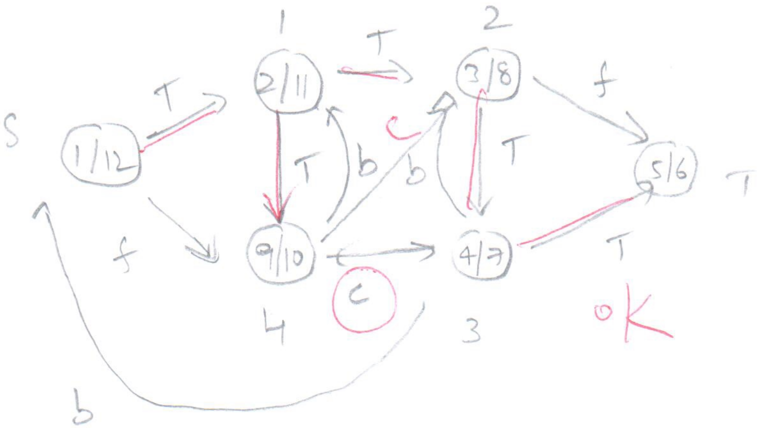












S → 1 , 1 → 2 , 1 → 4, 2 → 3, 3 → t are tree edges

3 → S , 4 → 1 , 3 → 2 backward edges

4 → 2, 4 → 3 cron edges

S → 4 ; 2 → T → foward ~~edges~~ OK

(6) K hash function $H(k) = \lfloor m(kA \bmod 1) \rfloor$ — kA mod 1 gives fractional part

The multiplication method has K i/p size of w bits. A is arbitary value p value such that $m = 2^p$

here m is choosen as power of 2.

K — w bits

x $s = A \times 2^w$

Suppose w = 3 bits

Realpart | fractional part — from w bits

2w bits — Extract p bits which gives h(K)

Eg: K = 100   K = 84

S   x 101   S = 5

```
        100
       000
      100
     ---------
     10 |100|
```

h(k) = 4

$A = 0.625$

$p = 3$

$H(k) = \lfloor 8 \times (4 \times 0.625) \rfloor$ — floor function

$= \lfloor 8 \times (0.5) \rfloor$ } Extracting

$= \lfloor 4.0 \rfloor$

$= 4$

if p = 2   m = 4

then

10 |10| 0 $H(k) = \lfloor 4 \times 0.5 \rfloor$

$= 2$

= 2 = h(k)   = 2

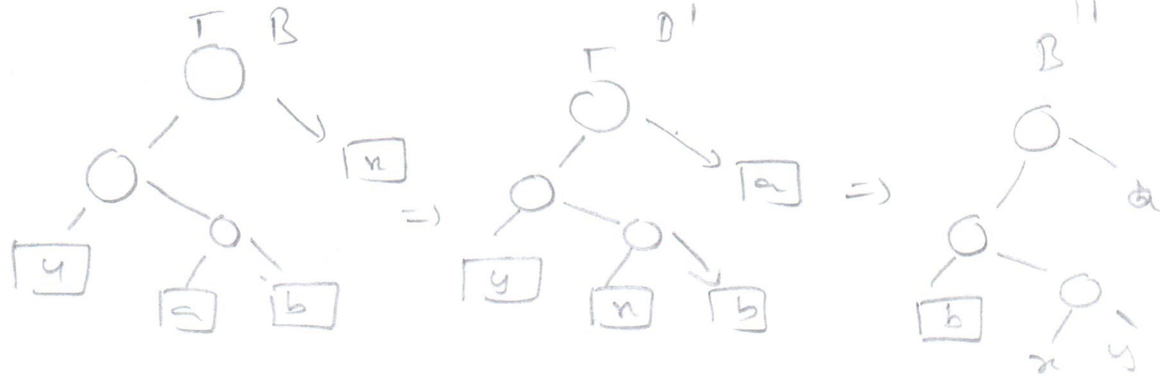
→ de duplication of data can be achieved using cuckoo hashing — any hashing function does

By means of having same data placed in one of slots. If ~~same~~ a data has same hash ~~function~~ result as other it ~~will be~~ is displaced.

→ hash function should be as efficient as can such that randomness should be more ~~for~~ eg. mapping every key to different slot results in ~~not~~ collision's

-2

(9)



Let

Total cost $B - B' = \sum_{c \in B} c.\text{freq}\, d_T(c) - \sum_{c \in B'} c.\text{freq}\, d_{T'}(c)$

$= x.\text{freq}\,(d_T(x)) + a.\text{freq}\, d_T(a) - x.\text{freq}\, d_{T'}(x) - a.\text{freq}\, d_{T'}(a)$

since $d_{T'}(x) = d_T(a)$
$d_{T'}(a) = d_T(x)$

$= x.\text{freq}\, d_T(x) + a.\text{freq}\, d_T(a) - x.\text{freq}\, d_T(a) - a.\text{freq}\, d_T(x)$

$= (a.\text{freq} - x.\text{freq})\,(d_T(a) - d_T(x))$

since $a.\text{freq} > x.\text{freq}$ $\quad d_T(a) > d_T(x)$ in B

OK

≥ 0

$B - B' \geq 0$ — ①

Let us ~~suppose~~ assume we've another tree optimal eg B''

$B' - B'' = \sum_{c \in} c.\text{freq}\, d_{T'}(c) - \sum_{c \in} c.\text{freq}\, d_{T''}(c)$

$= y.\text{freq}\, d_{T'}(y) + b.\text{freq}\, d_{T'}(b) - y.\text{freq}\, d_{T''}(y) - b.\text{freq}\,(d_{T''}(b))$

$= (y.\text{freq} - b.\text{freq})\,(d_{T'}(y) - d_{T'}(b))$ (||ly like before)

$<0 \qquad\qquad <0$

both so >0

✓

$B' - B'' > 0$

$B' > B''$ — ②

OK

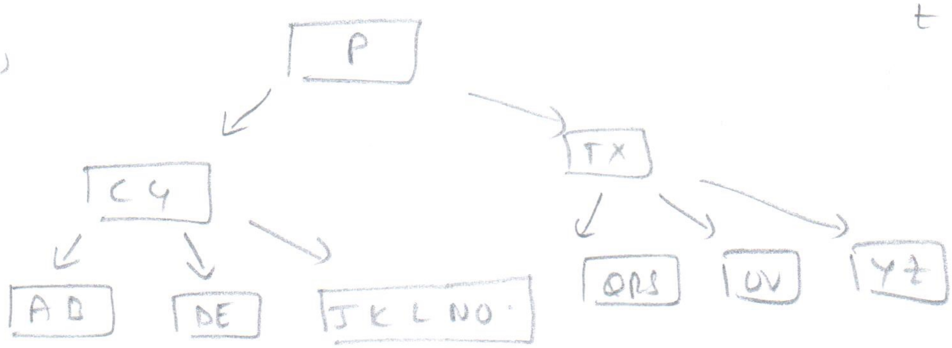
From ① & ② we can say that

B'' is the optimal tree.

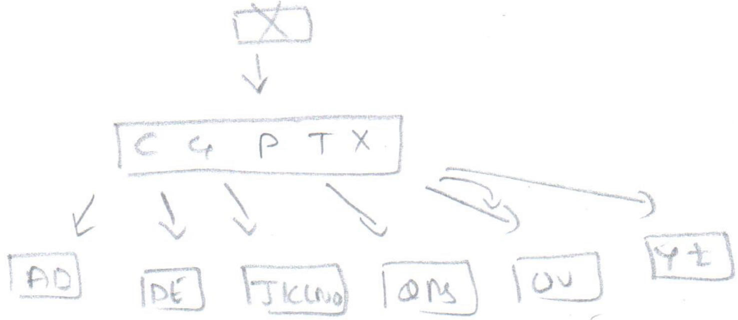
8 (i)

t = 3

for deletion t-1 2



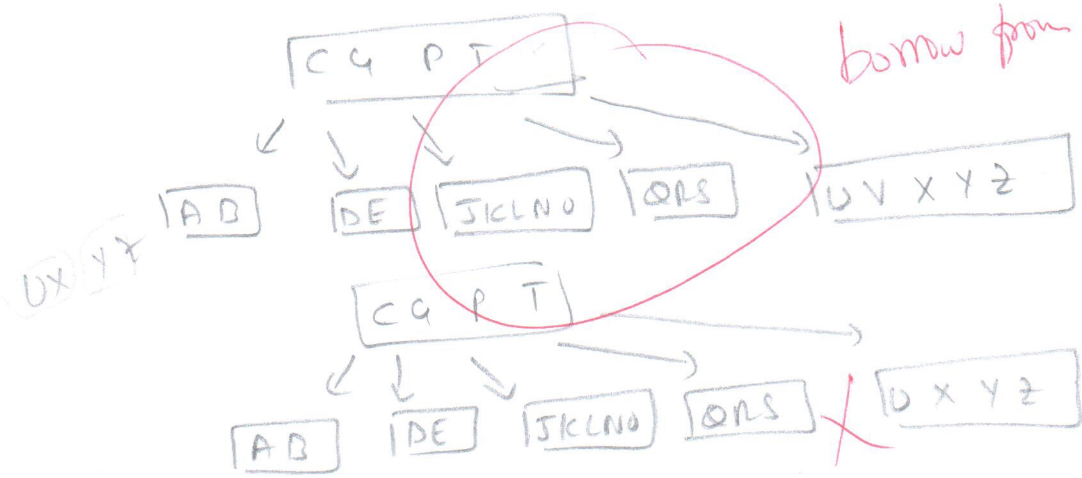
Merge



t-1

there are t-1

borrow from a sibling first



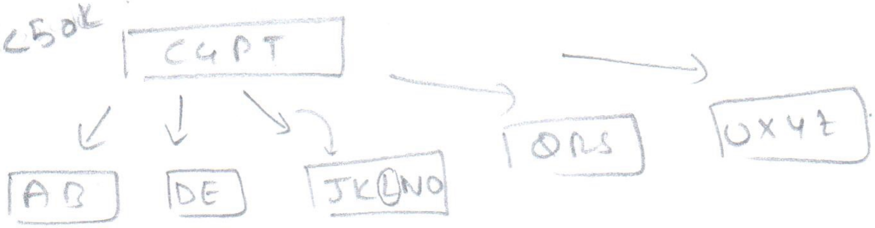
5 OK

delete V

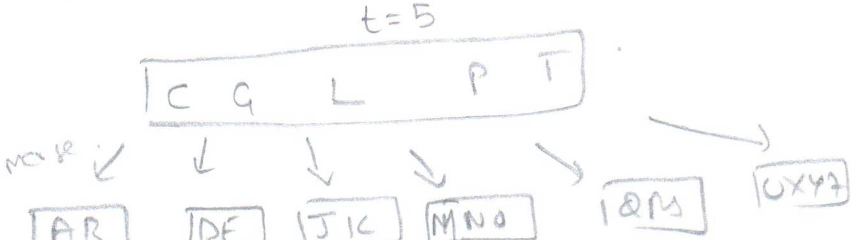
made yourself a simpler situation

2

ii, M insertion t<5 ok

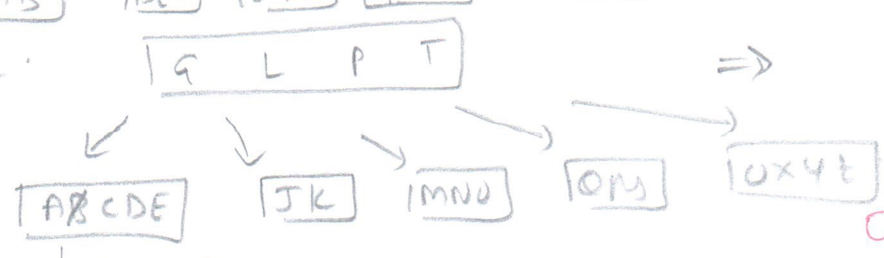


t=5



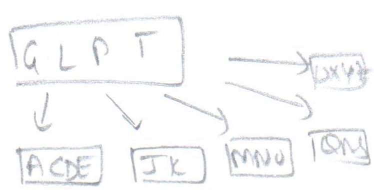
deleting B t=2 merge

iii



+ delete B

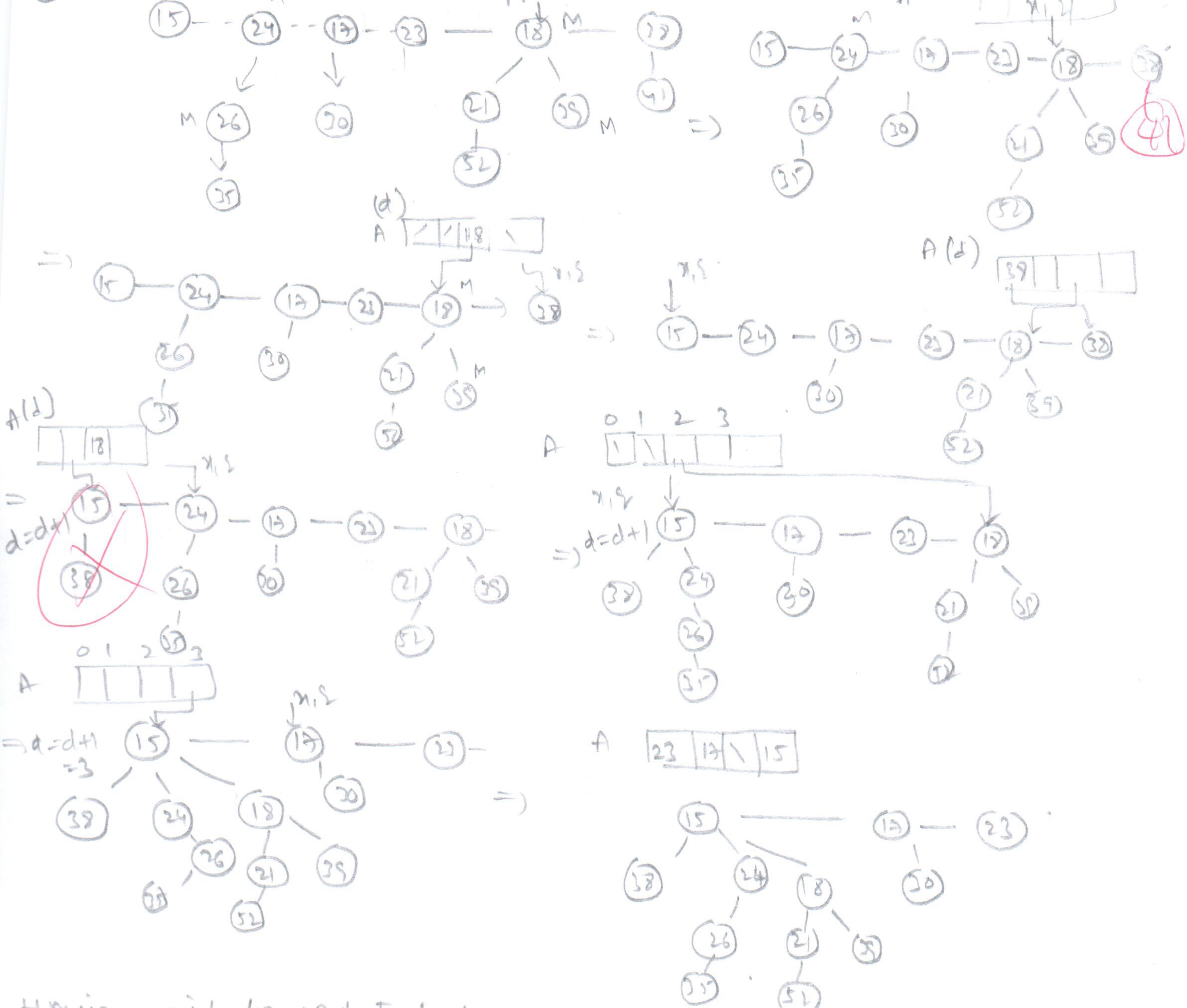
=>



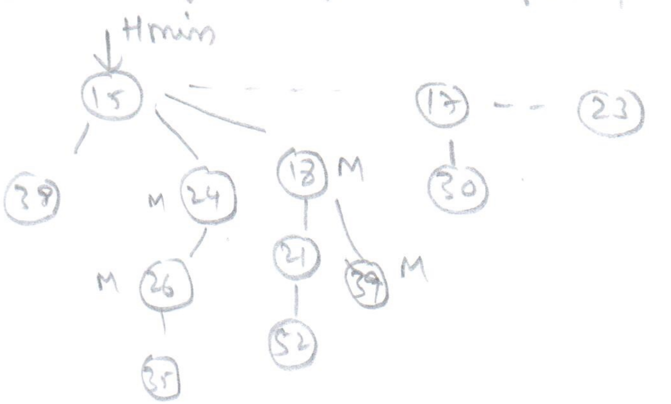
OK

3

7



Hmin will be updated by comparing 18 with all elements in A(d) array after comparing Hmin will be updated with 15



-5 +1

(4) 1) $L \in NP$ ✓ reducible to polynomial time

2) $L' \leq_p L$ where every $L' \in NP$

~~a some $L' \in NPC$~~

if (2) only satisfies then it is called NP hard ✓

if (1),2) satisfies it is NP complete

2) By means of reducing (it) hamiltan cycle problem ~~to~~ in polynomial time ~~to~~ to travelling sales man problem and prove that travelling salesman problem belong to NP class then we can say TSP is NP-complete problem.

OK

3) Let T and H^* be minimum span tree prim and optimal tree cost such that ① $C(T) < C(H^*)$. ✓

W be the some constant where each edge traverses two times

then ② $C(W) = 2C(T)$ ✓ $\Rightarrow C(W) < 2C(H^*)$ (free tree walk)

H be the cycle where repeated vertices are deleted then (tree walk).

③ $C(H) < C(W) \Rightarrow C(H) < 2C(H^*)$ which is by triangle inequality

OK

(10) 20 X 50   50 X 10   10 X 30   30 X 15

P_0 P_1 P_1 P_2 P_2 P_3 P_3 P_4

$P_0 = 20$ $P_1 = 50$ $P_2 = 10$ $P_3 = 30$ $P_4 = 15$

$$m(i,j) = \begin{cases} 0 & \text{if } i == j \\ \min\limits_{i \le k < j} m(i,k) + m(k+1,j) + P_{i-1} P_k P_j & \text{if } i < j \end{cases}$$

m

	1	2	3	4
1	0	10,000	16,000	?
2	X	0	15,000	12,000
3	X	X	0	4,500
4	X	X	X	0

17,500

-1

s

	2	3	4
1	1	2	2
2	X	2	2
3	X	X	3

$m(1,2) =$ $k=1$ $m(1,1) + m(2,2) + P_0 P_1 P_2 = 20 \times 50 \times 10 = 10,000$

$m(2,3) = P_1 P_2 P_3 = 50 \times 10 \times 30 = 15,000$

$m(3,4) = P_2 P_3 P_4 = 10 \times 30 \times 15 = 4,500$

$m(1,3) :$ $k=1$

$m(1,1) + m(2,3) + P_0 P_1 P_3$

$= 0 + 15,000 + 20 \times 50 \times 30 = 45,000$

$\boxed{k=2}$

$m(1,2) + m(3,3) + P_0 P_2 P_3$

$10,000 + 0 + 20 \times 10 \times 30 = 16,000$

$m(2,4) :$ $\boxed{k=2}$

$m(2,2) + m(3,4) + P_1 P_2 P_4$

$0 + 4,500 + 50 \times 10 \times 15 = 12,000$

$k=3$

$m(2,3) + m(4,4) + P_1 P_3 P_4$

$15,000 + 0 + 50 \times 30 \times 15 = 37,500$

$m(3,4) =$ $k=1$

$m(1,1) + m(2,4) + P_0 P_1 P_4$

$0 + 12{,}000 + 20 \times 50 \times 15 \quad = 27{,}000$

$\boxed{k=2}$

$m(1,2) + m(3,4) + P_0 P_2 P_4$

$10{,}000 + 4{,}500 + 20 \times 10 \times 15 \quad = 17{,}500$

$k=3$

$m(1,3) + m(4,4) + P_0 P_3 P_4$

$16{,}000 + 0 + 20 \times 30 \times 15 = 25{,}000$

(b) $(A1\,A2)\,(A3\,A4)$ ✓ ok

$20 \times 50 \times 10 \quad 10 \times 30 \times 15 = 14500$

$20 \times 10 \times 15 \quad = 17{,}500$

So the optimal parentisized $(A1\,A2)\,(A3\,A4)$

(5) $T(n) = 4T(\sqrt{n}) + (\lg n)^2$

$n = 2^m \qquad \lg n = m$

$T(2^m) = 4T(2^{m/2}) + m^2$

$T(m) = 4T(m/2) + m^2$

$T(m/2) = 4T(m/4) + \frac{m^2}{4}$

$T(m/4) = 4T(m/16) + (m/4)^2$

$2^2 T(m/4) = 4(4(T(m/4) + m^2/4) + m^2$

$2^4 T(m/2^3) = 16T(m/4) + m^2 + m^2 = 16T(m/4) + 2m^2$

$= 16\left(4T(m/16) + \frac{m^2}{16}\right) + 2m^2$

$T(m/2) =$ 16

$2^6 T(m/2^4) = 64T(m/16) + 3m^2$

$= T(m/$

$= 64\left(4T(m/32) + \frac{m^2}{8^2}\right) + 3m^2$

$= 256\,T(m/32) + 4m^2$

(5) $T(n) = 4T(\sqrt{n}) + (\lg n)^2$

$\Rightarrow 2^m = n$

$T(2^m) = 4T(2^{m/2}) + (\lg n)^2$

$T(m) = 4T(m/2) + m^2 \qquad 2^2 T(m/2) \qquad 2^2 T(m/2^1) + 1 \cdot m^2$

$= 4T(4(4T(m/4) + (\frac{m}{2})^2) + m^2$

$= 16T(m/4) + 2m^2$

$= 16T(m/4) + 2m^2$

$= 16(4T(m/8) + (m/4)^2)$

$64T(m/8) + 3m^2$

$= 2^2 T(m/2^1) + 1 \cdot m^2$

$= 2^4 T(m/2^2) + 2 \cdot m^2$

$= 2^6 T(m/2^3) + 3 \cdot m^2$

$= 2^{2i} T(m/2^i) + i m^2$

$\boxed{m/2^i = 1}$

$\Rightarrow m = 2^i$

$i = \log m$

$= (2^i)^2 T(1) + \log m \cdot (m^2)$

$= 2m^2 T(1) + \log m \cdot m^2$

$\therefore$ complexity of $T(m) = \Theta(T(m)) = m^2 \cdot \log m$

$\Rightarrow m = \log n$

$\Theta((\log_2 n)^2 \cdot \log_2(\log_2 n))$

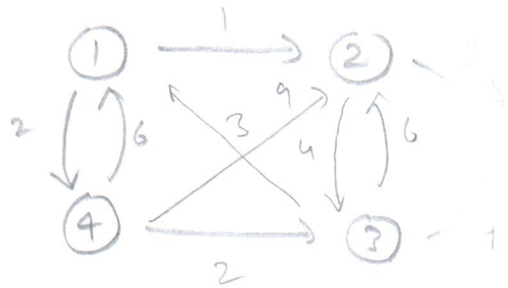
ok ✓ ok

②

$d_{ij}^{(v)} =$

for $k = 0$ to v

$d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$



$$D^{(0)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & \infty & 2 \\ 2 & \infty & 0 & 4 & \infty \\ 3 & 3 & 6 & 0 & \infty \\ 4 & 6 & 9 & 2 & 0 \end{array}$$

$$D^{(1)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & \infty & 2 \\ 2 & \infty & 0 & 4 & \infty \\ 3 & 3 & \boxed{4} & 0 & \boxed{5} \\ 4 & 6 & 7 & 2 & 0 \end{array}$$

✓

$d_{23}^{(1)} = \min\left(d_{23}^{(0)},\ d_{21}^{(0)} + d_{13}^{(0)}\right)$

$= 4,\ \infty$

$d_{24}^{(1)} = \min\left(d_{24}^{(0)},\ d_{21}^{(0)} + d_{14}^{(0)}\right)$

$\infty \qquad \infty + 0)$

$d_{32}^{(1)} = \min\ d_{32}^{(0)},\ d_{31} + d_{12}$

$6,\ 3 + 1$

$d_{34} = \min\left(d_{34}^{(0)},\ d_{31}^{(0)} + d_{14}\right)$

$\infty \qquad 3 + 2$

$d_{42} = \min\left(d_{42}^{(0)},\ 41^{(0)}\right.$

$9,\ 6 + 1$

$d_{43} = \min(2,\ 6 + \infty)$

$$D^{(2)} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 5 & 2 \\ 2 & \infty & 0 & 4 & \infty \\ 3 & 3 & 4 & 0 & \boxed{5} \\ 4 & 6 & 9 & 2 & 0 \end{array}$$

$d_{13}^{(2)} = \cancel{\infty},\ 1 + 4$

$d_{13}^{(2)} = 5$

$d_{14}^{(2)} = 2,\ 1 + 2$

$d_{31}^{(2)} = 3,\ 4 + 4$

$= 3,\ 8$

$d_{34}^{(2)} = 5,\ 4 + \infty$

$d_{41}^{(2)} = 6,\ 7 + \infty$

$d_{43}^{(2)} = 2,\ 7 + 4$

$$D^{(3)} = \begin{bmatrix} 0 & 1 & 5 & 2 \\ 7 & 0 & 4 & 9 \\ 3 & 4 & 0 & 5 \\ 5 & 6 & 2 & 0 \end{bmatrix}$$

$d_{12}^{(3)} = d_{12}^{(2)}, \; d_{13} + d_{32}$

$1, \; 5 + 4$

$d_{14} = 2, \; 2 + 0$

$d_{21}^{(3)} = \infty, \; d_{23} + d_{31}$

$4 + 3$

$d_{24}^{(3)} = \infty, \; d_{23} + d_{34}$

$4 + 5$

$d_{41} = 6, \; 2 + 3$

5

$d_{42} = 7, \; 2 + \cancel{2}4 \qquad d_{43} + d_{32}$

$6 \qquad 2 + 4$

$$D^{(4)} = \begin{bmatrix} 0 & 1 & 4 & 2 \\ 7 & 0 & 4 & 9 \\ 3 & 4 & 0 & 5 \\ 5 & 6 & 2 & 0 \end{bmatrix}$$

$d_{12}^{(4)} = 1, \; d_{14} + d_{42}$

$2 + 6$

$d_{13}^{(4)} = 5, \; d_{14} + d_{43}$

$2 + 2$

$d_{21} = 7, \; d_{24} + d_{41}$

$9 + 5$

$d_{23} = 4, \; 9 + 2$

$d_{31} = 3, \; 5 + 5$

$d_{32} = 4, \; 5 + 6$

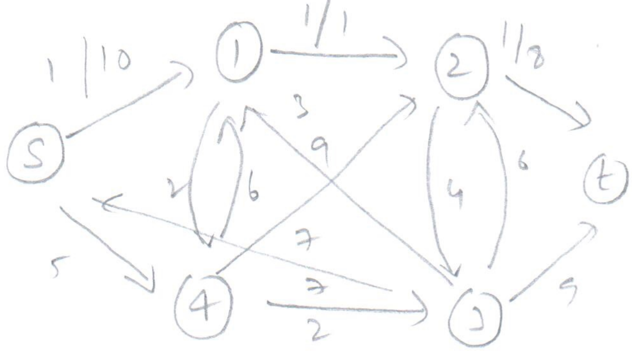
$$D^{(4)} = \begin{bmatrix} 0 & 1 & 4 & 2 \\ 7 & 0 & 4 & 9 \\ 3 & 4 & 0 & 5 \\ 5 & 6 & 2 & 0 \end{bmatrix}$$

ok

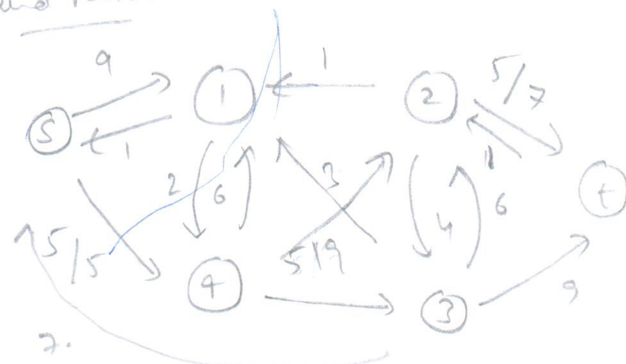
floyWarshall (D, i, j, k)

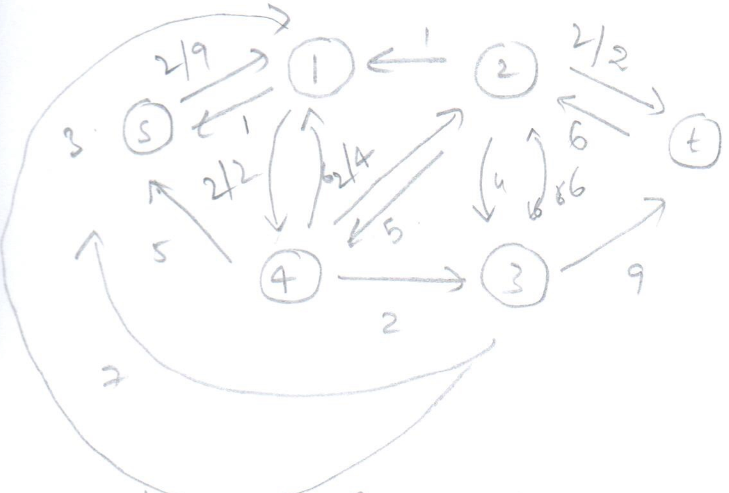
i) if (k > 1)
  return min ($d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$)
else
  return w(i)(j)

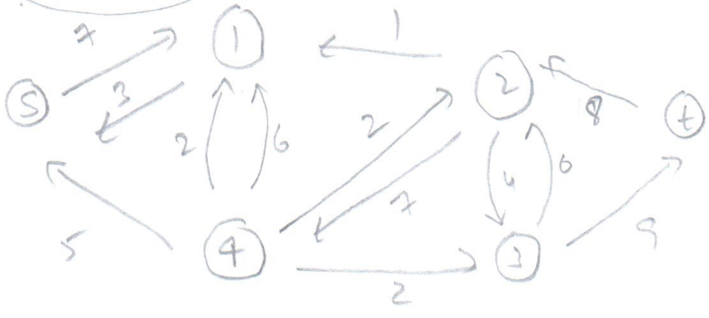
1

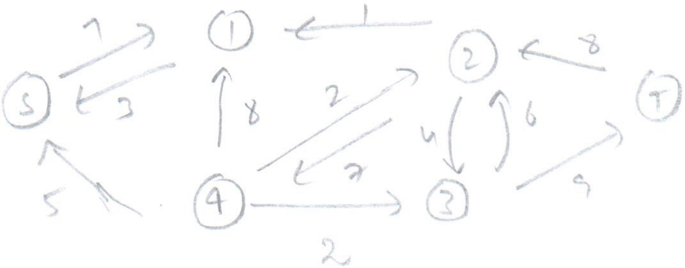


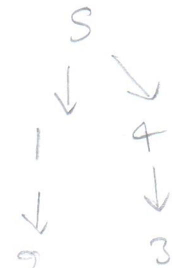
Residual network



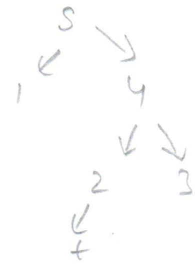




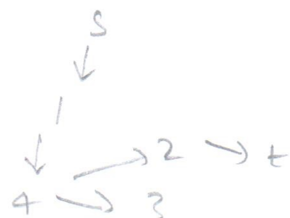




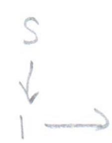
S → 1 → 2 → t



S → 4 → 2 → t



S → 1 → 4 → 2 → t



maximum flow will be 5+3 = 8

OK 10

b) time complexity of algorithm $O(VE^2)$

OK 2