Answer 3 of 4 questions.

[S$_1$] Let $h(n) = \sum_{i=1}^{n} \frac{1}{i}$, prove $h(n) = \Theta(\log_2 n)$.

[S$_2$] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = 4\,T(n-1) - 4\,T(n-2) \\ T(1) = 1, \quad T(2) = 4 \end{cases}.$$

[S$_3$] (a) Order by asymptotic growth rate from slowest to fastest:

$$4^{n^{1.005}}, \quad (1200n+1)^2, \quad n^3, \quad \lg^2 n, \quad e^n, \quad \ln\ln n, \quad 5^{n^{1.004}}.$$

(b) Calculate $\sum_{i=1}^{n} i(i+1)(i+2)(i+3)(i+4)$.

[S$_4$] Construct

[a] a finite automaton or a regular expression for the language

$\{ x \in \{0,1\}^* : \text{the first two characters of } x \text{ are identical to the last two}\}$.

[b] a context free grammar or pushdown automaton for the language

$\{ a^{2n} b^{2n+1} : n > 0 \}$.

Long Questions

Answer 3 of 4 questions.

[L₁] Suppose we have an instance of TSP given by the cost matrix:

$$
\begin{bmatrix}
\infty & 3 & 5 & 8 & 1 & 2 \\
3 & \infty & 6 & 4 & 5 & 9 \\
5 & 6 & \infty & 2 & 4 & 1 \\
8 & 4 & 2 & \infty & 7 & 5 \\
1 & 5 & 4 & 7 & \infty & 6 \\
2 & 9 & 1 & 5 & 6 & \infty
\end{bmatrix}
$$

a) Given the partial solution $X = (5, 2, -, -, -)$, calculate $B(X)$ using the reducing technique on the matrix.

b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.

[L₂] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 4, 3; and weight 7, 5, 4, 3, 2, 1 (resp.).

[L₃] Keys a, b, c, d, e have frequencies 10, 3, 4, 7, 15, respectively. Find OBST using Dynamic programming algorithm.

[L₄] Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.

a) $\{a^{n+1}b^{n-1}c^m : \ n, m > 0\}$

b) $\{a^{2n}b^{2m+1} \ : \ n, m \geq 0\}$

c) $\{a^{n+1}b^{n-1}c^n : \ n > 0\}$