Answer 3 of 4 questions.

[L_1] Suppose we have an instance of TSP given by the cost matrix:

$$\begin{bmatrix} \infty & 3 & 5 & 8 & 1 & 2 \\ 3 & \infty & 6 & 4 & 5 & 9 \\ 5 & 6 & \infty & 2 & 4 & 1 \\ 8 & 4 & 2 & \infty & 7 & 5 \\ 1 & 5 & 4 & 7 & \infty & 6 \\ 2 & 9 & 1 & 5 & 6 & \infty \end{bmatrix}$$

a) Give the partial solution $X = (5, 2, -, -, -)$, calculate $B(X)$ using the reducing technique on the matrix.
b) For X as in a), use backtracking with branch-and-bound to find the best solution which is an extension of the given partial solution. Draw the portion of the state space tree you are investigating.

[L_2] Solve the instance of minimum tardy task weight with 6 objects, all of length 1, having deadlines 3, 2, 1, 2, 4, 3; and weight 7, 5, 4, 3, 2, 1 (resp.).

[L3] Regarding the classes of problems, P, NP, and NP-Complete:

[a] Define the classes and show their known relationships.

[b] Prove that if A is NP-complete, $B \in NP$, and $A \leq_p B$, then B is NP-complete.

[c] Prove that if A is NP-complete and A is in P, then $P = NP$.

[L4] Classify each of the following languages as regular, context free but not regular, or decidable but not context free. Prove your answers.

[a] $\{ a^{n+1} b^{n-1} c^m : n, m > 0 \}$

[b] $\{ a^{2n} b^{2m+1} : n, m \geq 0 \}$

[c] $\{ a^{n+1} b^{n-1} c^n : n > 0 \}$

Ph.D. Comprehensive Examination
Design and Analysis of Algorithms

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Short Questions

Answer 3 of 4 questions.

[S_1] Let $h(n) = \sum_{i=1}^{n} \frac{1}{i}$, prove

$$h(n) = \Theta(log_2\ n).$$

[S_2] From the following recurrence determine the growth rate of $T(n)$:

$$\begin{cases} T(n) = & 4T(n-1) - 4T(n-2) \\ T(1) = 1, & T(2) = 4 \end{cases}$$

[S3]

[a] Informally state the "Pumping Theorem" for regular languages.

[b] What is it used for? Give a simple example.

[c] Can it be used to show that a language is regular? Why or why not?

[S4] Construct

[a] a finite automaton or a regular expression for the language

{ $x \in \{0,1\}^*$: the first two characters of x are identical to the last two }.

[b] a context free grammar or pushdown automaton for the language

{ $a^{2n}b^{2n+1}$: $n > 0$ }