CSCE 500

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Assignment # 2

1. a) $T(n) = T(n-1) + 1/n$

$T(n) = T(n-2) + 1/(n-1) + 1/n$
$T(n) = \Sigma(1/i)$

$\therefore T(n) = \Theta(\log n)$ ✓

b) $T(n) = T(n-1) + \log(n)$

$T(n) = \log n + \log(n-1) + \cancel{\log} \cdots + \log(1)$
$T(n) = \log(n!)$

$\therefore T(n) = \Theta(n \log n)$ ✓

c) $T(n) = n^{0.5} \, T(n^{0.5}) + n$
Dividing by n on both side
$\frac{T(n)}{n} = \frac{Tn^{0.5}}{n^{0.5}} + 1$

Rename $S(n) = T(n)/n$
$\Rightarrow S(n) = S(n^{0.5}) + 1$
let $k = \log n$, then $S(k) = T(2^k)$
$\Rightarrow S(k) = S(k/2) + 1$
$\sum_{i=1}^{h} 1 = S(k) = h$

$h = \Theta(\log k) \Rightarrow S(k) = \Theta(\log k) = S(2^k) = S(n)$

$\Rightarrow S(n) = \Theta(\log k) = \Theta(\log \log n)$

$\Rightarrow T(n) = nS(n) = \Theta(n \log \log n)$ ✓

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2. a) $T(n) = 2T(n/2) + n^3$

$T(n) = aT(\frac{n}{b}) + cf(n)$

$a = 2 \qquad f(n) = n^3$
$b = 2$

$n^{\log_b a} = n^{\log_2 2} = n$

$\Rightarrow f(n) = \Omega(n^{1+\epsilon})$ (case 3)
and $af(\frac{n}{b}) \leq cf(n)$

$\Longleftrightarrow \exists c < 1$
$2f(\frac{n}{2}) \leq c \cdot n^3$

$\cancel{2} \cdot \frac{n^3}{\cancel{8}4} \leq cn^3 \rightarrow c = \frac{1}{3}$ for $n > 0$

$\therefore T(n) = \Theta(n^3)$ ✓

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b) $T(n) = 16T(n/4) + n^2$

$a = 16$
$b = 4$ $\quad f(n) = n^2$

$\log_b a = \log_4 16 = 2 \Rightarrow f(n) = \Theta(n^2)$ (Case 2)

$\Rightarrow T(n) = \Theta(n^2 \log n)$ ✓

c) $T(n) = 7T(n/2) + n^2$

$a = 7$ $\quad f(n) = n^2$
$b = 2$

$2 < \log_b a = \log_2 7 < \log_2 8 = 3$

$\Rightarrow f(n) = O(n^{\log_2 7 - \epsilon})$

$\exists \epsilon, \epsilon = 0.5 > 0.\ \log 7 = 2.85$

$\Rightarrow n^2 = O(n^{2.85 - 0.5}) = O(n^{2.35})$ (Case 1)

$\Rightarrow T(n) = \Theta(n^{2.85})$ ✓

d) $T(n) = 2T(n/4) + n^{0.5}$

$a = 2$ $\quad f(n) = \sqrt{n} = n^{0.5}$

$b = 4$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$ -10

$\Rightarrow f(n) = \Theta(n^{\log_4 2})$ (Case (2))

$\Rightarrow T(n) = \Theta(n^{0.5} \log n)$

$T(n) = \Theta(\sqrt{n} \log n)$ X $T(n) = \Theta(n^{0.5})$

e) $T(n) = 3T(n/2) + n \log n$

$a = 3$

$b = 2$ $\quad f(n) = n \log n$

$\log_b a = \log_2 3 = 1.584$

$n \log n = O(n^{1.584 - \epsilon})$, $\epsilon > 0$

$\Rightarrow$ ~~$T(n) = \Theta(n \log$~~

$\Rightarrow T(n) = \Theta(n^{\log_2 3})$ ✓ (Case 1)