CSCE 500 Midterm Exam #2

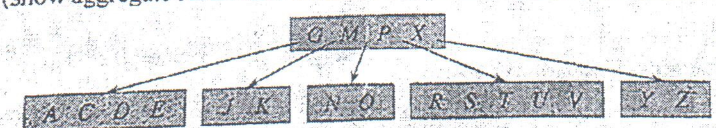
11/11/2014

1. The binary search tree (T) facilitates key search and it involves several operations to maintain the tree property when a node (z) is deleted, as shown in the following pseudo code, TREE-DELETE(T, z), where TRANSPLANT(T, u, v) replaces the subtree rooted at u with one rooted at v. Fill in the last three missing statements in the pseudo code. (10%)

```
TREE-DELETE(T, z)
if z.left == NIL
    TRANSPLANT(T, z, z.right)        // z has no left child
elseif z.right == NIL
    TRANSPLANT(T, z, z.left)         // z has just a left child
else // z has two children.
    y = TREE-MINIMUM(z.right)        // y is z's successor
    if y.p ≠ z
        // y lies within z's right subtree but is not the root of this subtree.
        TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    // Replace z by y.
```

2. For any n-key B-tree of height h and with the minimum node degree of $t \geq 2$, prove that h is no larger than $\log_t \frac{n+1}{2}$. (Hint: consider the number of keys stored in each tree level.) (10%)

3. Given the initial B-tree with the minimum node degree of $t = 3$ below, show the results (a) after inserting two keys in order: Q then W and (b) followed by deleting two keys in order: [illegible] then T. (show aggregate result after insertion and another result after deletion, 15%)



4. Given two sets A and B, each with n positive integers, we sort elements in each set in a monotonically decreasing order. Let the i^{th} element in sorted A (or B) be denoted by a_i (or b_i). Prove that the payoff of $\prod_{i=1}^{n} a_i^{b_i}$ is maximized. (10%)

5. Given the matrix-chain multiplication problem for four matrices sized 30×35, 35×10, 10×24, 24×50, follow the tabular, bottom-up method in the procedure of MATRIX-CHAIN-ORDER below to construct a table that keeps entry $m[i, j]$ for all $1 \le i, j \le 4$.

(a) Fill in the missing 4 statements in the procedure. (10%)

(b) Construct the table, with its entry values shown. (10%)

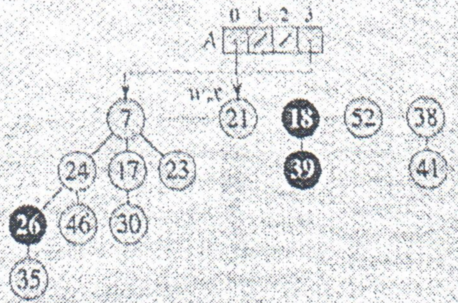
```
MATRIX-CHAIN-ORDER(p)
n = p.length - 1
let m[1..n, 1..n] and s[1..n-1, 2..n] be new tables
for i = 1 to n
    m[i, i] = 0
for l = 2 to n              // l is the chain length
    for i = 1 to n - l + 1
        j = i + l - 1
        m[i, j] = ∞
        for k = i to j - 1
            q =




return m and s
```

6. Show your construction of an optimal Huffman code for the set of 7 frequencies: **a**:2 **b**:3 **c**:5 **d**:8 **e**:13 **f**:21 **g**:34. (10%)

7. A Fibonacci min-heap relies on the procedure of CONSOLIDATE to merge trees in the root list upon the operation of extracting the minimum node. Given the following partially consolidated diagram, show every subsequent consolidation step till its completion. (10%)



8. For breadth-first search (BFS) over an undirected graph, prove that each cross edge (u, v) satisfying $v.d = u.d$ or $v.d = u.d + 1$. (10%)

Devise an algorithm to determine whether or not a given undirected graph $G = (V, E)$ contains a simple cycle. What is the time complexity of your algorithm? (5%)

c:5

a:2 b:3

e:13 f:21 g:34

Good Luck!

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