EE5471: Assignment 2 on Minimization
Finding Minima of functions in n-D using Powell and Conjugate Gradient Methods

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1 Reading Assignment

Minimization chapter in Numerical Recipes.

2 Minimization of "nice" functions

The function to be minimized is

$$f(x,y,z) = 1.5 - J_0\left((x-0.5)^2 + (0.5x+y-0.5)^2 + (0.25x+0.5y+z-0.5)^2\right)$$

Clearly a global minimum is at $(0.5, 0.25, 0.25)$ since $J_0(x)$ is maximum when its (real) argument is zero. The initial point is the origin. Use

- Downhill Simplex
- Minimizing along Coordinate directions
- Powell,
- Minimization along the gradient
- Conjugate Gradient

to determine how the minimum is reached, and how quickly.

Obtain the Hessian for this problem and determine its eigenvalues and eigen directions. Compare them to the conjugate directions obtained by Powell and the Conjugate Gradient methods.

3 Minimization of difficult functions

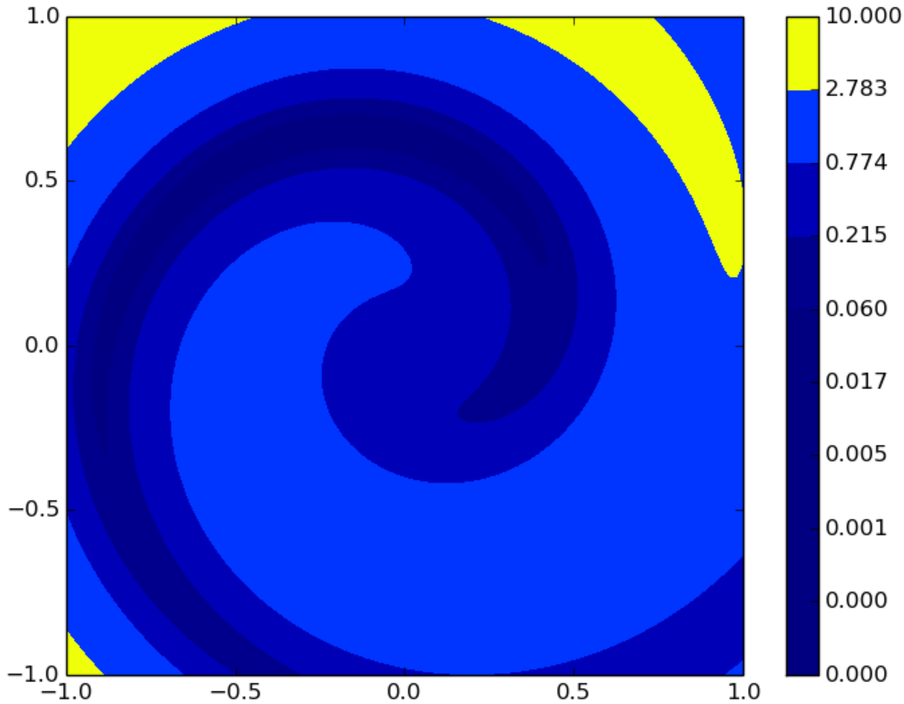
The function to be minimized is the following:

$$f(x,y) = u^2 + v^2 \tag{1}$$

where

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{2}$$

where $\alpha = 5\left(1.5\sqrt{x^2+y^2} - 1.5\right)$. You have already used this function with Downhill Simplex.



- Apply Downhill Simplex, Powell and Conjugate Gradient methods and find the approach to the minimum.
- In the approach region, see how much effect there is due to the $(g_{i+1} - g_i) \cdot g_{i+1}$ modification in the Conjugate Gradient method.
- In the final region where the function is nearly quadratic, study the way the g_i and h_i are generated and whether they satisfy the orthogonality conditions.
- Animate the solution process to show how the algorithms progress.

Note: The desired graphs should be overlaid on top of the contour plot obtained in part 1.