PHYSICS

INDEX

MODULE-1

S.No.	CHAPTER NAME	Pg.No.
1.	UNIT & DIMENSION, BASIC MATHS AND VECTOR	01-50
2.	KINEMATICS-1D & CALCULUS	51-104
3.	KINEMATICS-2D	105-152
4.	NEWTON'S LAWS OF MOTION & FRICTION	153-225

ALLEN CAREER INSTITUTE OVERSEAS

CHAPTER 1

UNIT & DIMENSION, BASIC MATHS AND VECTOR

Chapter Contents 01

01.	THEORY	3
02.	EXERCISE (S-1)	23
03.	EXERCISE (O-1)	27
04.	EXERCISE (O-2)	36
05.	EXERCISE (JM)	45
06.	EXERCISE (JA)	46
07.	ANSWER KEY	48

IMPORTANT NOTES

CHAPTER 1

UNIT & DIMENSION, BASIC MATHS AND VECTOR

KEY CONCEPT

PHYSICAL QUANTITIES AND UNITS

Physical quantities :

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationship.

Measurement :

Measurement is the comparison of a quantity with a standard of the same physical quantity.

Classification :

Physical quantities can be classified on the following bases :

I. Based on their directional properties

1. **Scalars:** The physical quantities which have only magnitude but no direction are called *scalar quantities*. **Ex.** mass, density, volume, time, etc.

2. **Vectors :** The physical quantities which have both magnitude and direction and obey laws of vector algebra are called *vector quantities*. **Ex.** displacement, force, velocity, etc.

II. Based on their dependency

1. **Fundamental or base quantities :** A set of physical quantities which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

2. **Derived quantities :** The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities.* **Ex.** Speed (= distance/time), volume, acceleration, force, pressure, etc.

Physical quantities can also be classified as dimensional and dimensionless quantities or constants and variables.

Ex. Classify the quantities displacement, mass, force, time, speed, velocity, acceleration, moment of inertia, pressure and work under the following categories :

(a) base and scalar (b) base and vector

(c) derived and scalar (d) derived and vector

Ans. (a) mass, time (b) displacement

(c) speed, pressure, work (d) force, velocity, acceleration

UNITS OF PHYSICAL QUANTITIES

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity. Four basic properties of units are :

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

System of Units :

1. FPS or British Engineering system :

In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

2. CGS or Gaussian system :

In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).

3. MKS system :

In this system also the fundamental quantities are length, mass and time but their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

4. International system (SI) of units :

This system is modification over the MKS system and so it is also known as *Rationalised MKS system. Besides* the three base units of MKS system four fundamental and two supplementary units are also included in this system.

Classification of Units : The units of physical quantities can be classified as follows :

1. Fundamental or base units :

The units of fundamental quantities are called *base units*. In SI there are seven base units.

SI BASE QUANTITIES AND THEIR UNITS

S.No.	Physical quantity	SI unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Temperature	Kelvin	K
5.	Electric current	ampere	A
6.	Luminous intensity	candela	cd
7.	Amount of substance	mole	mol

2. Derived units :

The units of derived quantities or the units that can be expressed in terms of the base units are called *derived* units

Ex. Unit of speed $= \dfrac{\text{unit of distance}}{\text{unit of time}} = \dfrac{\text{metre}}{\text{second}} = \text{ms}^{-1}$

Some derived units are named in honour of great scientists.

• Unit of force – newton (N) • Unit of frequency – hertz (Hz) etc.

UNITS OF SOME PHYSICAL QUANTITIES IN DIFFERENT SYSTEMS

Type of Physical Quantity	Physical Quantity	CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton(N)	poundal
	Work or Energy	erg	joule(J)	ft-poundal
	Power	erg/s	watt(W)	ft-poundal/s

DIMENSIONS :

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. To make it clear, consider the physical quantity force. As we shall learn later, force is equal to mass times acceleration. Acceleration is change in velocity divided by time interval. Velocity is length divided by time interval. Thus,

force = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}} = \text{mass} \times \frac{\text{length / time}}{\text{time}} = \text{mass} \times \text{length} \times (\text{time})^{-2}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero.

1. Dimensional formula :

The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to remind that the equation is among the dimensions and not among the magnitudes. Thus above equation may be written as [force]= MLT^{-2}.

Such an expression for a physical quantity in terms of the base quantities is called the dimensional formula. Thus, the dimensional formula of force is MLT^{-2}. The two versions given below are equivalent and are used interchangeably.

(a) The dimensional formula of force is MLT^{-2}.

(b) The dimensions of force are 1 in mass, 1 in length and –2 in time.

The *dimensional formula* of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

Ex. Dimensional formula of mass is $[M^1L^0\,T^0]$ and that of speed (= distance/time) is $[M^0L^1T^{-1}]$

2. Applications of dimensional analysis :

(i) To convert a physical quantity from one system of units to the other :

This is based on a fact that *magnitude of a physical quantity remains same whatever system is used for measurement*

i.e. magnitude = numeric value (n) ×

unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $[M^aL^bT^c]$

n_2	=	numerical value in II system
n_1	=	numerical value in I system
M_1	=	unit of mass in I system
M_2	=	unit of mass in II system
L_1	=	unit of length in I system
L_2	=	unit of length in II system
T_1	=	unit of time in I system
T_2	=	unit of time in II system

Then $\boxed{n_2 = n_1\left[\frac{u_1}{u_2}\right] = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$

Ex. 1m = 100 cm= 3.28 ft = 39.4 inch

(SI) (CGS) (FPS)

Ex. The acceleration due to gravity is 9.8 m s^{-2}. Give its value in ft s^{-2}

Sol. As 1m = 3.2 ft $\therefore$ 9.8 m/s^2 = 9.8 × 3.28 ft/s^2 = 32.14 ft/s^2 $\approx$ 32 ft/s^2

Ex. Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Sol. The dimensional equation of force is $[F] = [M^1\,L^1\,T^{-2}]$

Therefore if n_1, u_1, and n_2, u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1 \times 1000 \times 100 \times 1 = 10^5$$

$\therefore$ 1 newton = 10^5 dyne.

Q. The value of Gravitational constant G in MKS system is 6.67×10^{-11} N–m²/kg² .

What will be its value in CGS system ?

Ans. 6.67×10^{-8} cm³/g s²

(ii) ***To check the dimensional correctness of a given physical relation :***

If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the *principle of homogeneity of dimensions*.

Ex. Check the accuracy of the relation $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.

Sol. The dimensions of LHS = the dimension of T = $[M^0 L^0 T^1]$

The dimensions of RHS = $\left(\dfrac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2}$ ($\because$ 2π is a dimensionless constant)

$$= \left(\frac{L}{LT^{-2}}\right)^{1/2} = (T^2)^{1/2} = [T] = [\,M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

(iii) ***To derive relationship between different physical quantities :***

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Ex. It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Sol. We have $[T] = [G]^a [M]^b [R]^c$

$$[M]^0 [L]^0 [T]^1 = [M]^{-a} [L]^{3a} [T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a} [L]^{c+3a} [T]^{-2a}$$

Comparing the exponents

For [T] : $1 = -2a \Rightarrow a = -\frac{1}{2}$

For [M] : $0 = b - a \Rightarrow b = a = -\frac{1}{2}$

For [L] : $0 = c + 3a \Rightarrow c = -3a = \frac{3}{2}$

Putting the values we get $T = G^{-1/2} M^{-1/2} R^{3/2} = \sqrt{\frac{R^3}{GM}}$

So the actual expression is $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Limitations of this method :

(i) In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.

(ii) This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + at^2/2$ also can't be derived.

(iii) The relation derived from this method gives no information about the dimensionless constants.

(iv) If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.

(v) It gives no information whether a physical quantity is a scalar or a vector.

Units and Dimensions of Physical Quantities			
Quantity	**Common Symbol**	**SI unit**	**Dimension**
Displacement	s	METRE (m)	L
Mass	m, M	KILOGRAM (kg)	M
Time	t	SECOND (s)	T
Area	A	m^2	L^2
Volume	V	m^3	L^3
Density	ρ	kg/m^3	M/L^3
Velocity	v, u	m/s	L/T
Acceleration	a	m/s^2	L/T^2
Force	F	newton (N)	ML/T^2
Work	W	joule (J) (=N – m)	ML^2/T^2
Energy	E, U, K	joule (J)	ML^2/T^2
Power	P	watt (W) (=J/s)	ML^2/T^3
Momentum	p	kg-m/s	ML/T
Gravitational constant	G	$N\text{-}m^2/kg^2$	L^3/MT^2
Angle	θ, φ	radian	
Angular velocity	ω	radian/s	T^{-1}
Angular acceleration	α	radian/s^2	T^{-2}
Angular momentum	L	$kg\text{-}m^2/s$	ML^2/T
Moment of inertia	I	$kg\text{-}m^2$	ML^2
Torque	τ	N-m	ML^2/T^2
Angular frequency	ω	radian/s	T^{-1}
Frequency	ν	hertz (Hz)	T^{-1}
Period	T	s	T
Young's modulus	Y	N/m^2	M/LT^2
Bulk modulus	B	N/m^2	M/LT^2
Shear modulus	η	N/m^2	M/LT^2
Surface tension	S	N/m	M/T^2
Coefficient of viscosity	η	$N\text{-}s/m^2$	M/LT
Pressure	P, p	N/m^2, Pa	M/LT^2
Wavelength	λ	m	L
Intensity of wave	I	W/m^2	M/T^3
Temperature	T	KELVIN (K)	K
Specific heat capacity	c	J/kg–K	L^2/T^2K
Stefan's constant	σ	$W/m^2\text{–}K^4$	M/T^3K^4
Heat	Q	J	ML^2/T^2
Thermal conductivity	K	W/m–K	ML/T^3K

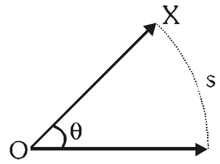
BASIC MATHEMATICS USED IN PHYSICS

Plane–angle

It is measure of change in direction.

If a line rotates in a plane about one of its ends, the other end sweeps an arc.

Angle (θ) between two orientation of the line is defined by ratio of the arc length(s) to length of the line(r) $\theta = \frac{s}{r}$ radian

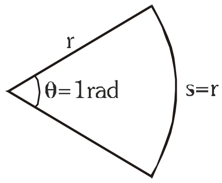


Angles measured in anticlockwise and clockwise directions are usually taken positive and negative respectively.

Angle is measured in radians (rad) or degrees. One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.

π rad $= 180°$ $\quad\quad$ $\pi = 3.1415 = \frac{22}{7}$

$1° = 60'$ (minute), $\quad$ $1'$ (minute) $= 60''$ (sec)

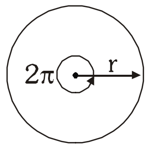


Example

Write expression for circumference of a circle of radius 'r'.

Solution

s = (Total angle about a point) r = $2\pi r$

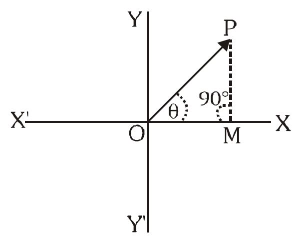


Trigonometrical ratios (or T–ratios)

Let two fixed lines XOX' and YOY' intersecting at right angles to each other at point O.

- Point O is called origin.
- Line XOX' is known as x–axis and YOY' as y–axis.
- Regions XOY, YOX', X'OY' and Y'OX are called I, II, III and IV quadrant respectively.

Consider a line OP making angle θ with OX as shown. Line PM is perpendicular drawn from P on OX. In the right angled triangle OPM, side OP is called hypotenuse, the side OM adjacent to angle θ is called base and the side PM opposite to angle θ is called the perpendicular.



Following ratios of the sides of a right angled triangle are known as trigonometrical ratios or T-ratio

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP}$ $\quad$ $\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$ $\quad$ $\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM}$

$\cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$ $\quad$ $\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM}$ $\quad$ $\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

Some trigonometric identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \text{cosec}^2\theta$

Example

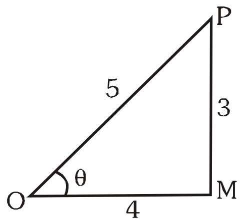
Given $\sin\theta = \frac{3}{5}$. Find all the other T–ratios, if θ lies in the first quadrant.

Solution

In Δ OMP , $\sin\theta = \frac{3}{5}$ So MP = 3 and OP = 5 ∵ $OM = \sqrt{(5)^2-(3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\tan\theta = \frac{MP}{OM} = \frac{3}{4}$ $\cot\theta = \frac{OM}{MP} = \frac{4}{3}$

$\sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\operatorname{cosec}\theta = \frac{OP}{MP} = \frac{5}{3}$



T–ratios of some commonly used angles

Angle (θ)	0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad
	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

sin (90°–θ)=cosθ cos (90°–θ)=sinθ tan (90°–θ)=cotθ	sin (180°–θ)=sinθ cos (180°–θ)=–cosθ tan (180°–θ)=–tanθ	sin (360°–θ)=–sinθ cos (360°–θ)= cosθ tan (360°–θ)=–tanθ
sin (90°+θ)=cosθ cos (90°+θ)=–sinθ tan (90°+θ)=–cotθ	sin (180°+θ)=–sinθ cos (180°+θ)=–cosθ tan (180°+θ)=tanθ	sin (–θ)=-sinθ cos (–θ)=cosθ tan (–θ)=–tanθ

- When θ is very small we can use following approximations : cos θ ≈ 1

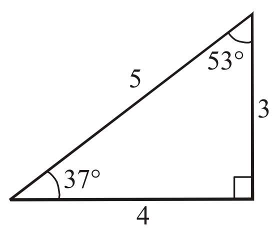
$$\left.\begin{matrix}\sin\theta \simeq \theta \\ \tan\theta \simeq \theta\end{matrix}\right\} \text{If } \theta \text{ is in radians}$$

$\tan\theta \approx \sin\theta$.

- In the given right angled triangle we have very commonly used T-ratios

$\sin 37° = \frac{3}{5}$ $\cos 37° = \frac{4}{5}$ $\tan 37° = \frac{3}{4}$

$\sin 53° = \frac{4}{5}$ $\cos 53° = \frac{3}{5}$ $\tan 53° = \frac{4}{3}$



Example

Find the value of

(i) cos (–60°) (ii) tan 210° (iii) sin 300° (iv) cos 120°

Solution

(i) $\cos(-60°) = \cos 60° = \frac{1}{2}$

(ii) $\tan 210° = \tan(180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

(iii) $\sin 300° = \sin(270° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(iv) $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

VECTORS

Precise description of laws of physics and physical phenomena requires expressing them in form of mathematical equations. In doing so we encounter several physical quantities, some of them have only magnitude and other have direction in addition to magnitude. Quantities of the former kind are referred as scalars and the latter as vectors and mathematical operations with vectors are collectively known as vector analysis.

Vectors

A vector has both magnitude and sense of direction, and follows triangle law of vector addition. For example, displacement, velocity, and force are vectors.

Vector quantities are usually denoted by putting an arrow over the corresponding letter, as $\vec{A}$ or $\vec{a}$. Sometimes in print work (books) vector quantities are usually denoted by boldface letters as **A** or **a**.

Magnitude of a vector $\vec{A}$ is a positive scalar and written as $|\vec{A}|$ or A.

Geometrical Representation of Vectors.

A vector is represented by a directed straight line, having the magnitude and direction of the quantity represented by it.

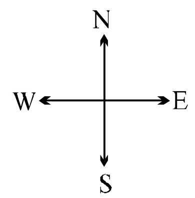
e.g. if we want to represent a force of 5 N acting 45° N of E

(i) We choose direction coordinates.

(ii) We choose a convenient scale like 1 cm ≡ 1N

(iii) We draw a line of length equal in magnitude and in the direction of vector to the chosen quantity.

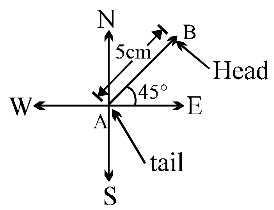
(iv) We put arrow in the direction of vector.



$\overrightarrow{AB}$

Magnitude of vector:

$|\overrightarrow{AB}| = 5N$

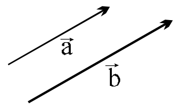


1 cm ≡ 1 N

By definition magnitude of a vector quantity is scalar and <u>is always positive</u>.

Terminology of vectors

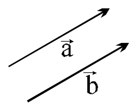
Parallel vector: If two vectors have same direction, they are parallel to each other. They may be located anywhere in the space.



Antiparallel vectors: When two vectors are in opposite direction they are said to be antiparallel vectors.

Equality of vectors: When two vectors have equal magnitude and are in same direction and represent the same physical quantity, they are equal.

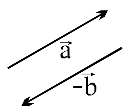
i.e. $\vec{a} = \vec{b}$



Thus when two parallel vectors have same magnitude they are equal. (Their initial point & terminal point may not be same)

Negative of a vector: When a vector have equal magnitude and is in opposite direction, it is said to be negative vector of the former.

i.e. $\vec{a} = -\vec{b}$ or $\vec{b} = -\vec{a}$

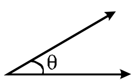


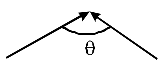
Thus when two antiparallel vectors have same magnitude they are negative of each other.

Remark : Vector shifting is allowed without change in their direction.

Angle Between two Vectors

It is the smaller angle formed when the initial points or the terminal points of the two vectors are brought together. It should be noted that $0° \le \theta \le 180°$.





Addition Of Vectors:

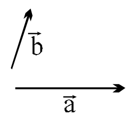
Parallelogram law of addition:

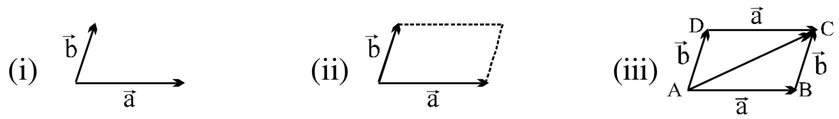
Steps:

(i) Keep two vectors such that there tails coincide.

(ii) Draw parallel vectors to both of them considering both of them as sides of a parallelogram.

(iii) Then the diagonal drawn from the point where tails coincide represents the sum of two vectors, with its tail at point of coincidence of the two vectors.

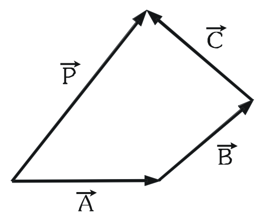




$\overrightarrow{AC} = \vec{a} + \vec{b}$

Addition of more than two Vectors

The triangle law can be extended to define addition of more than two vectors. Accordingly, if vectors to be added are drawn in head to tail fashion, resultant is defined by a vector drawn from the tail of the first vector to the head of the last vector. This is also known as the ***polygon rule for vector addition.***



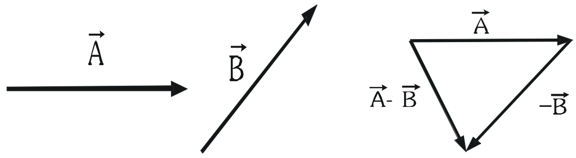
Operation of addition of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ and their resultant $\vec{P}$ are shown in figure.

$$\vec{A} + \vec{B} + \vec{C} = \vec{P}$$

Here it is not necessary that three or more vectors and their resultant are coplanar. In fact, the vectors to be added and their resultant may be in different planes. However if all the vectors to be added are coplanar, their resultant must also be in the same plane containing the vectors.

Subtraction of Vectors

A vector opposite in direction but equal in magnitude to another vector $\vec{A}$ is known as negative vector of $\vec{A}$. It is written as $-\vec{A}$. Addition of a vector and its negative vector results a vector of zero magnitude, which is known as a null vector. A null vector is denoted by arrowed zero $(\vec{0})$. The idea of negative vector explains operation of subtraction as addition of negative vector. Accordingly to subtract a vector from another consider vectors $\vec{A}$ and $\vec{B}$ shown in the figure. To subtract $\vec{B}$ from $\vec{A}$, the negative vector $-\vec{B}$ is added to $\vec{A}$ according to the triangle law as shown in figure-II.



If two vectors $\vec{a}$ & $\vec{b}$ are represented by $\overrightarrow{OA}$ & $\overrightarrow{OB}$ then their sum $\vec{a}+\vec{b}$ is a vector represented by $\overrightarrow{OC}$, where OC is the diagonal of the parallelogram OACB.

- $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ (commutative)
- $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ (associativity)
- $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$
- $\vec{a}+(-\vec{a})=\vec{0}=(-\vec{a})+\vec{a}$
- $|\vec{a}+\vec{b}|\le|\vec{a}|+|\vec{b}|$
- $|\vec{a}-\vec{b}|\ge||\vec{a}|-|\vec{b}||$
- $\left|\vec{a}\pm\vec{b}\right| = \sqrt{|\vec{a}|^2+|\vec{b}|^2\pm2|\vec{a}||\vec{b}|\cos\theta}$ where θ is the angle between the vectors

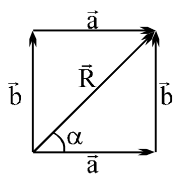
Some Important Results :

(1) If $\theta=0^\circ \Rightarrow \vec{a}\parallel\vec{b}$

then, $|\vec{R}|=|\vec{a}|+|\vec{b}|$ & $|\vec{R}|$ is maximum

(2) If $\theta=\pi \Rightarrow \vec{a}$ anti $\parallel$ $\vec{b}$

then, $|\vec{R}|=\left||\vec{a}|-|\vec{b}|\right|$ & $|\vec{R}|$ is minimum



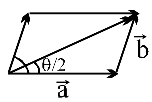
(3) If $\theta=\pi/2 \Rightarrow \vec{a}\perp\vec{b}$

$R=\sqrt{a^2+b^2}$

& $\tan\alpha = b/a$ (α is angle made by $\vec{R}$ with $\vec{a}$)

(4) $|\vec{a}|=|\vec{b}|=a$

$|\vec{R}| = 2a\cos\theta/2$ & $\alpha=\theta/2$



(5) If $|\vec{a}|=|\vec{b}|=a$ & $\theta=120^\circ$

then $|\vec{R}|=a$

Multiplication Of A Vector By A Scalar:

If $\vec{a}$ is a vector & m is a scalar, then m $\vec{a}$ is a vector parallel to $\vec{a}$ whose modulus is $|m|$ times that of $\vec{a}$. This multiplication is called **Scalar Multiplication**. If $\vec{a}$ and $\vec{b}$ are vectors & m, n are scalars, then:

$$m(\vec{a}) = (\vec{a})m = m\vec{a}$$

$$m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$$

$$(m+n)\vec{a} = m\vec{a} + n\vec{a}$$

$$m(a+\vec{b}) = m\vec{a} + m\vec{b}$$

Resolution of a Vector into Components

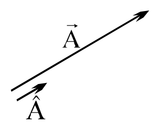
Following laws of vector addition, a vector can be represented as a sum of two (in two-dimensional space) or three (in three-dimensional space) vectors each along predetermined directions. These directions are called axes and parts of the original vector along these axes are called components of the vector.

UNIT VECTOR :

A unit vector is a vector of magnitude of 1, with no units. Its only purpose is to point, i.e. to describe a direction in space.

A unit vector in direction of vector $\vec{A}$ is represented as $\hat{A}$

& $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$



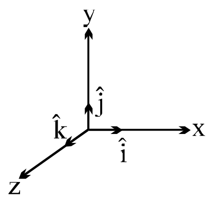
or $\vec{A}$ can be expressed in terms of a unit vector in its direction i.e. $\vec{A} = |\vec{A}|\hat{A}$

<ins>Unit Vectors along three coordinates axes</ins>:–

unit vector along x-axis is $\hat{i}$

unit vector along y-axis is $\hat{j}$

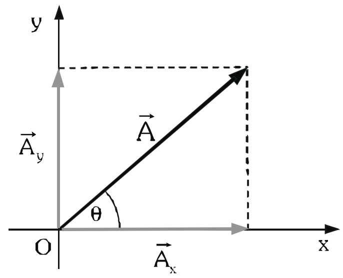
unit vector along z-axis is $\hat{k}$



Cartesian components in two dimensions

If a vector is resolved into its components along mutually perpendicular directions, the components are called Cartesian or rectangular components.

In figure is shown, a vector $\vec{A}$ resolved into its Cartesian components $\vec{A}_x$ and $\vec{A}_y$ along the x and y-axis. Magnitudes A_x and A_y of these components are given by the following equation.



$$A_x = A\cos\theta \quad \text{and} \quad A_y = A\sin\theta$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

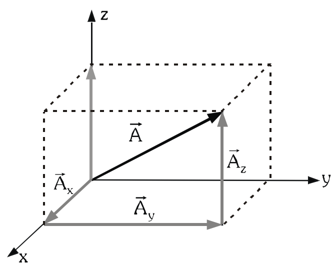
$$A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

Here $\hat{i}$ and $\hat{j}$ are the unit vectors for x and y coordinates respectively.

Mathematical operations e.g. addition, subtraction, differentiation and integration can be performed independently on these components. This is why in most of the problems use of Cartesian components becomes desirable.

Cartesian components in three dimensions

A vector $\vec{A}$ resolved into its three Cartesian components one along each of the directions x, y, and z-axis is shown in the figure.



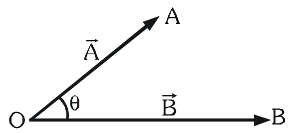
$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Product of Vectors

In all physical situation, whose description involve product of two vectors, only two categories are observed. One category where product is also a vector involves multiplication of magnitudes of two vectors and sine of the angle between them, while the other category where product is a scalar involves multiplication of magnitudes of two vectors and cosine of the angle between them. Accordingly, we define two kinds of product operation. The former category is known as vector or cross product and the latter category as scalar or dot product.

Scalar or dot product of two vectors

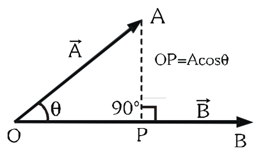


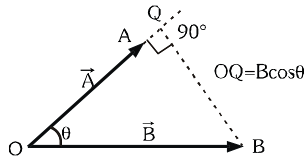
The scalar product of two vectors $\vec{A}$ and $\vec{B}$ equals to the product of their magnitudes and the cosine of the angle θ between them.

$\vec{A}\cdot\vec{B} = AB\cos\theta = OA \cdot OB \cdot \cos\theta$

The above equation can also be written in the following ways.

$\vec{A}\cdot\vec{B} = (A\cos\theta)B = OP \cdot OB$ $\qquad$ $\vec{A}\cdot\vec{B} = A(B\cos\theta) = OA \cdot OQ$

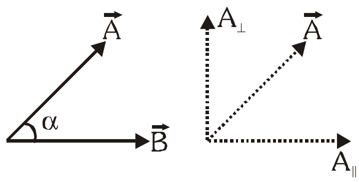




Above two equations and figures, suggest a scalar product as product of magnitude of the one vector and magnitude of the component of another vector in the direction of the former vector.

KEY POINTS

- Dot product of two vectors is commutative: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- If two vectors are perpendicular, their dot product is zero. $\vec{A}\cdot\vec{B} = 0$, if $\vec{A} \perp \vec{B}$
- Dot product of a vector by itself is known as self-product. $\vec{A}\cdot\vec{A} = A^2 \Rightarrow A = \sqrt{\vec{A}\cdot\vec{A}}$
- The angle between the vectors $\theta = \cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)$
- (a) Component of $\vec{A}$ in direction of $\vec{B}$



$$\vec{A}_{\parallel} = (|\vec{A}|\cos\theta)\hat{B} = |\vec{A}|\left(\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right)\hat{B} = \left(\frac{\vec{A}.\vec{B}}{|\vec{B}|}\right)\hat{B} = (\vec{A}.\hat{B})\hat{B}$$

(b) Component of $\vec{A}$ perpendicular to $\vec{B}$: $\vec{A}_{\perp} = \vec{A} - \vec{A}_{\parallel}$

- Dot product of Cartesian unit vectors: $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

  $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

- If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, their dot product is given by

  $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Solved Examples

1. Two displacement vectors of same magnitude are arranged in the following manner

Magnitude of resultant is maximum for

(A) case I (B) case II (C) case III (D) case IV

Ans. **(B)**

Sol. Magnitude of Resultant of $\vec{A}$ and $\vec{B}$ = $\sqrt{A^2+B^2+2AB\cos\theta}$ which is maximum for $\theta = 30^0$

2. Two vectors $\vec{P}$ and $\vec{Q}$ are added, the magnitude of resultant is 15 units. If $\vec{Q}$ is reversed and added to $\vec{P}$ resultant has a magnitude $\sqrt{113}$ units. The resultant of $\vec{P}$ and a vector perpendicular to $\vec{P}$ and equal in magnitude to $\vec{Q}$ has a magnitude

(A) 13 units (B) 17 units (C) 19 units (D) 20 units

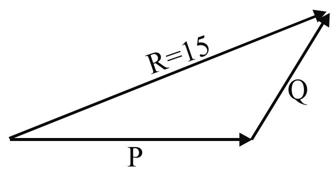
Ans. **(A)**

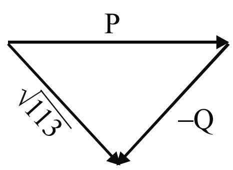
Sol. $P^2 + Q^2 + 2PQ\cos\theta = 225$ ...(i)

$P^2 + Q^2 - 2PQ\cos\theta = 113$ ...(ii)

By adding (i) & (ii) $2(P^2 + Q^2) = 338$

$P^2 + Q^2 = 169 \Rightarrow \sqrt{P^2+Q^2} = 13$





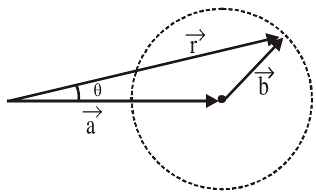
3. Three forces are acting on a body to make it in equilibrium, which set **can not** do it?

(A) 3 N, 3 N, 7 N (B) 2 N, 3 N, 6 N (C) 2 N, 1 N, 1 N (D) 8 N, 6 N, 1 N

Ans. **(A, B, D)**

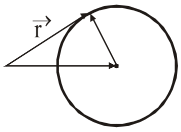
Sol. They must form a triangle. $(a + b \geq c)$

4. Keeping one vector constant, if direction of other to be added in the first vector is changed continuously, tip of the resultant vector describes a circle. In the following figure vector $\vec{a}$ is kept constant. When vector $\vec{b}$ added to $\vec{a}$ changes its direction, the tip of the resultant vector $\vec{r}=\vec{a}+\vec{b}$ describes circle of radius b with its center at the tip of vector $\vec{a}$. Maximum angle between vector $\vec{a}$ and the resultant $\vec{r}=\vec{a}+\vec{b}$ is



(A) $\tan^{-1}\left(\frac{b}{r}\right)$ (B) $\tan^{-1}\left(\frac{b}{\sqrt{a^2-b^2}}\right)$ (C) $\cos^{-1}$ (r/a) (D) $\cos^{-1}$ (a/r)

Ans. (A,B,C)

Sol. 

Angle between $\vec{r}$ and $\vec{b}$ is maximum when $\vec{r}$ is tangent to circle.

5. If $\vec{A}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=10\hat{i}+5\hat{j}+5\hat{k}$, if the magnitude of component of $\left(\vec{B}-\vec{A}\right)$ along $\vec{A}$ is $4\sqrt{x}$. Then x will be.

Ans. 6

Sol. $r=\vec{B}-\vec{A}=4\left(2\hat{i}+\hat{j}+\hat{k}\right)$

$$r\cos\theta=\frac{\vec{r}.\vec{A}}{|A|}=\frac{4(4+1+1)}{\sqrt{6}}=4\sqrt{6}$$

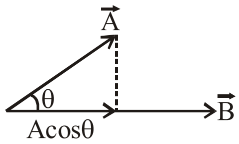
$x=6$

6. The component of $\vec{A}=\hat{i}+\hat{j}+5\hat{k}$ perpendicular to $\vec{B}=3\hat{i}+4\hat{j}$ is

(A) $-\frac{4}{25}\hat{i}+\frac{3}{25}\hat{j}+5\hat{k}$ (B) $-\frac{8}{25}\hat{i}-\frac{6}{25}\hat{j}+5\hat{k}$ (C) $\frac{4}{25}\hat{i}-\frac{3}{25}\hat{j}+5\hat{k}$ (D) $+\frac{8}{25}\hat{i}-\frac{6}{25}\hat{j}+5\hat{k}$

Ans. (C)

Sol.



$$\vec{A}_{\parallel} = A\cos\theta = A\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

$$= \frac{\vec{A}\cdot\vec{B}}{B} = \frac{3+4}{5} = \frac{7}{5}$$

$$\vec{A}_{\parallel} = \frac{7}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = \frac{7}{25}\left(3\hat{i}+4\hat{j}\right)$$

$$\vec{A}_{\parallel} = \frac{21}{25}\hat{i} + \frac{28}{25}\hat{j}$$

$$\vec{A}_{\perp} = \left(\hat{i}+\hat{j}+5\hat{k}\right) - \left(\frac{21}{25}\hat{i}+\frac{28}{25}\hat{j}\right)$$

$$= \frac{4}{25}\hat{i} - \frac{3}{25}\hat{j} + 5\hat{k}$$

ALGEBRA : SOME USEFUL FORMULAE

Quadratic equation and its solution

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation. General quadratic equation is $ax^2 + bx + c = 0$. The general solution of the above quadratic equation or value of variable x is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Example

Solve $2x^2 + 5x - 12 = 0$

Solution

By comparison with the standard quadratic equation $a = 2$, $b = 5$ and $c = -12$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4\times 2\times(-12)}}{2\times 2} = \frac{-5\pm\sqrt{121}}{4} = \frac{-5\pm 11}{4} = \frac{+6}{4}, \frac{-16}{4} \Rightarrow x = \frac{3}{2}, -4$$

Binomial approximation

In case, x is very small, then terms containing higher powers of x can be neglected. In such a case,

$(1 + x)^n = 1 + nx$

Also $(1 + x)^{-n} = 1 - nx$ and $(1 - x)^n = 1 - nx$ and $(1 - x)^{-n} = 1 + nx$

Exponential Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{........ and } e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \text{........}$$

Componendo and dividendo theorem :

If $\frac{p}{q} = \frac{a}{b}$ then by componendo and dividendo theorem $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

Determinant

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc, \text{ For example } \begin{vmatrix} -3 & 3 \\ -5 & 1 \end{vmatrix} = 12, \begin{vmatrix} 2 & -4 \\ -3 & 3 \end{vmatrix} = -6$$

$$D = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

Example

$$\begin{vmatrix} \overset{+}{5} & \overset{-}{4} & \overset{+}{3} \\ 2 & 1 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 5 \begin{vmatrix} 1 & 6 \\ 8 & 9 \end{vmatrix} - 4 \begin{vmatrix} 2 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 7 & 8 \end{vmatrix} = 5\,(9 - 48) - 4(18 - 42) + 3(16 - 7) = -72$$

Logarithm

$\log_e x = \ell n\, x$ (base e) $\log x = \log_{10} x$ (base 10)

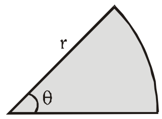
(a) Product formula $\log mn = \log m + \log n$ (b) Quotient formula $\log \frac{m}{n} = \log m - \log n$

(c) Power formula $\log m^n = n \log m$

GEOMETRY : SOME USEFUL FORMULAE

Formulae for determination of area :

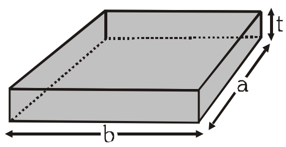
- Area of a square = $(\text{side})^2$
- Area of rectangle = length $\times$ breadth
- Area of a triangle = $(1/2) \times$ base $\times$ height
- Area of a trapezoid =$(1/2) \times$ (distance between parallel sides) $\times$ (sum of parallel sides)
- Area enclosed by a circle = πr^2 (where r = radius)
- Area of a sector a circle = $\frac{1}{2}\theta r^2$ (where r = is radius and θ is angle subtended at a centre)



- Area of ellipse = π ab (where a and b are semi major and semi minor axis respectively)
- Surface area of a sphere = $4\pi r^2$ (where r = radius)
- Area of a parallelogram = base $\times$ height
- Area of curved surface of cylinder = $2\pi r\ell$ (where r = radius and ℓ = length)
- Area of whole surface of cylinder = $2\pi r(r + \ell)$ (where ℓ = length)
- Surface area of a cube = $6(\text{side})^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$ [where $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ = lateral area (h=height)]

Formulae for determination of volume :

- Volume of a rectangular slab = length $\times$ breadth $\times$ height = abt
- Volume of a cube = $(\text{side})^3$



- Volume of a sphere = $\frac{4}{3}\pi r^3$ (where r = radius)
- Volume of a cylinder = $\pi r^2\ell$ (where r = radius and ℓ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ (where r = radius and h = height)

EXERCISE (S-1)

Units & Dimensions

1. A particle is in a unidirectional potential field where the potential energy (U) of a particle depends on the x-coordinate given by $U_x = k(1 - \cos ax)$ & k and 'a' are constants. Find the physical dimensions of 'a' & k.

2. The equation for the speed of sound in a gas states that $v = \sqrt{\gamma k_B T / m}$. Speed v is measured in m/s, γ is a dimensionless constant, T is temperature in kelvin (K), and m is mass in kg. Find the SI unit for the Boltzmann constant, k_B?

3. The time period (T) of a spring mass system depends upon mass (m) & spring constant (k) & length of the spring (ℓ) $\left[k = \frac{Force}{length}\right]$. Find the relation among T, m, ℓ & k using dimensional method.

4. The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by $x = a \sin\omega t$, where a & ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G). Using dimensional analysis find an expression for ω in terms of r, m and G.

5. A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon

(A) Radius of orbit (R)

(B) Mass of planet (M)

(C) Universal gravitation constant (G)

Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

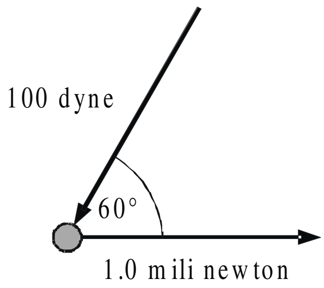
6. Assume that the largest stone of mass 'm' that can be moved by a flowing river depends upon the velocity of flow v, the density d & the acceleration due to gravity g. If 'm' varies as the Kth power of the velocity of flow, then find the value of K.

7. Given $\vec{F} = \frac{\vec{a}}{t}$ where symbols have their usual meaning. The dimensions of a is.

Addition of vectors

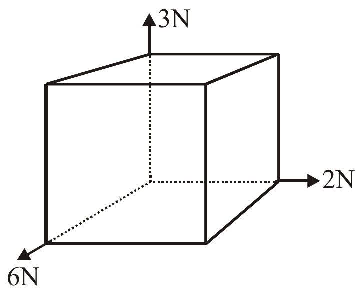
8. A block is applied two forces of magnitude 5N each. One force is acting towards East and the other acting along 60° North of East. The resultant of the two forces (in N) is of magnitude :-

9. Two forces act on a particle simultaneously as shown in the figure. Find net force in milli newton on the particle. [Dyne is the CGS unit of force]



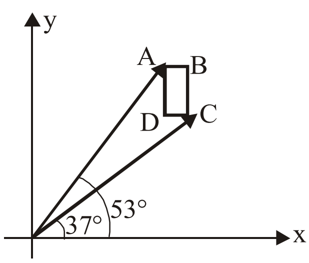
10. The maximum and minimum magnitudes of the resultant of two forces are 35 N and 5 N respectively. Find the magnitude of resultant force when act orthogonally to each other.

11. Three forces of magnitudes 2 N, 3 N and 6 N act at corners of a cube along three sides as shown in figure. Find the resultant of these forces in N.



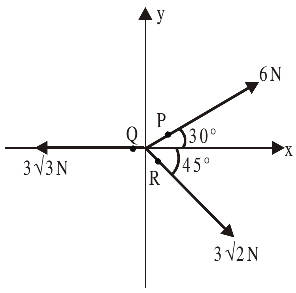
Resolution of vectors and unit vector

12. The farm house shown in figure has rectangular shape and has sides parallel to the chosen x and y axes. The position vector of corner A is 125 m at 53° and corner C is 100 m at 37° from x axis. Find the length of the fencing required in meter.



13. Vector B has x, y and z components of 4.00, 6.00 and 3.00 units, respectively. Calculate the magnitude of B and the angles that B makes with the coordinates axes.

14. Three ants P, Q and R are pulling a grain with forces of magnitude 6N, $3\sqrt{3}$N and $3\sqrt{2}$N as shown in the figure. Find the magnitude of resultant force (in N) acting on the grain.



15. Three boys are pushing horizontally a box placed on horizontal table. One is pushing towards north with a $5\sqrt{3}$N force. The second is pushing towards east and third pushes with a force 10 N such that the box is in equilibrium. Find the magnitude of the force, second boy is applying in newton.

Scalar product of vectors

16. Consider the two vectors : $\vec{L} = 1\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{l} = 4\hat{i} + 5\hat{j} + 6\hat{k}$. Find the value of the scalar α such that the vector $\vec{L} - \alpha\vec{l}$ is perpendicular to $\vec{L}$.

17. Find components of vector $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$ in directions parallel to and perpendicular to vector $\vec{b} = \hat{i} + \hat{j}$.

18. (a) Calculate $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ where $\vec{a} = 5\hat{i} + 4\hat{j} - 6\hat{k}$, $\vec{b} = -2\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 4\hat{i} + 3\hat{j} + 2\hat{k}$.

(b) Calculate the angle between $\vec{r}$ and the z-axis.

(c) Find the angle between $\vec{a}$ and $\vec{b}$

19. If the velocity of a particle is $(2\hat{i} + 3\hat{j} - 4\hat{k})$ and its acceleration is $(-\hat{i} + 2\hat{j} + \hat{k})$ and angle between them is $\frac{n\pi}{4}$. The value of n is.

Method of approximation

20. Quito, a city in Ecuador and Kampala, a city situated in Uganda both lie on the Equator. The longitude of Quito is 82°30' W and that of Kampala is 37°30' E. What is the distance from Quito to Kampala.

(a) along the shortest surface path

(b) along a direct (through-the-Earth) path? (The radius of the Earth is 6.4×10^6 m)

21. Use the approximation $(1+x)^n \approx 1+nx, |x| << 1$, to find approximate value for

(a) $\sqrt{99}$ (b) $\frac{1}{1.01}$

22. Use the small angle approximations to find approximate values for (A) sin 8° and (B) tan 5°

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Units & Dimensions

1. The dimensional formula for which of the following pair is not the same

(A) impulse and momentum (B) torque and work

(C) stress and pressure (D) momentum and angular momentum

2. Which of the following can be a set of fundamental quantities

(A) length, velocity, time (B) momentum, mass, velocity

(C) force, mass, velocity (D) momentum, time, frequency

3. Which of the following functions of A and B may be performed if A and B possess different dimensions?

(A) $\frac{A}{B}$ (B) A + B (C) A – B (D) None

4. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The dimensions of a, b and c are respectively :–

(A) LT^{-2}, L and T (B) L^2, T and LT^2 (C) LT^2, LT and L (D) L, LT and T^2

5. If area (A), velocity (v), and density (ρ) are base units, then the dimensional formula of force can be represented as :-

(A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

6. Density of wood is 0.5 g/ cc in the CGS system of units. The corresponding value in MKS units is :-

(A) 500 (B) 5 (C) 0.5 (D) 5000

7. In a book, the answer for a particular question is expressed as $b = \frac{ma}{k}\left[\sqrt{1+\frac{2k\ell}{ma}}\right]$ here m represents mass, a represents acceleration, ℓ represents length. The unit of b should be :-

(A) m/s (B) m/s^2 (C) meter (D) /sec

8. The frequency f of vibrations of a mass m suspended from a spring of spring constant k is given by $f=Cm^xk^y$, where C is a dimensionless constant. The values of x and y are, respectively

(A) $\frac{1}{2}, \frac{1}{2}$ (B) $-\frac{1}{2}, -\frac{1}{2}$ (C) $\frac{1}{2}, -\frac{1}{2}$ (D) $-\frac{1}{2}, \frac{1}{2}$

9. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:

(A) FT^2 (B) $F^{-1}A^2T^{-1}$ (C) FA^2T (D) AT^2

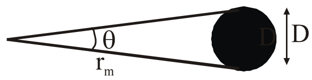
10. In a particular system the units of length, mass and time are chosen to be 10 cm, 10 g and 0.1 s respectively. The unit of force in this system will be equal to :-

(A) 0.1 N (B) 1 N (C) 10 N (D) 100 N

11. The units of three physical quantities x, y and z are $gcm^2\ s^{-5}$, gs^{-1} and cms^{-2} respectively. The relation among the units of x, y and z is :-

(A) $z = x^2y$ (B) $y^2 = xz$ (C) $x = yz^2$ (D) $x = y^2z$

12. The angle subtended by the moon's diameter at a point on the earth is about 0.50°. Use this and the fact that the moon is about 384000 km away to find the approximate diameter of the moon.



(A) 192000 km (B) 3350 km (C) 1600 km (D) 1920 km

13. **Statement 1 :** Method of dimensions cannot tell whether an equation is correct.

and

Statement 2 : A dimensionally incorrect equation may be correct.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

14. The equation of the stationary wave is $y = 2A\sin\left(\frac{2\pi ct}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$ which of the following statement is wrong?

(A) The unit of ct is same as that of λ.

(B) The unit of x is same as that of λ.

(C) The unit of $\frac{2\pi c}{\lambda}$ is same as that of $\frac{2\pi x}{\lambda t}$

(D) The unit of $\frac{c}{\lambda}$ is same as that of $\frac{x}{\lambda}$

15. Due to some unknown interaction, force F experienced by a particle is given by the following equation.

$$\vec{F} = -\frac{A}{r^3}\vec{r}$$

Where A is positive constant and r distance of the particle from origin of a coordinate system. Dimensions of constant A are :-

(A) ML^2T^{-2} (B) ML^3T^{-2} (C) ML^4T^{-2} (D) ML^0T^0

Addition of vectors

16. Three concurrent forces of the same magnitude are in equilibrium. What is the angle between the force? Also name the triangle formed by the force as sides :-

(A) 60° equilateral triangle
(B) 120° equilateral triangle
(C) 120°, 30°, 30° an isosceles triangle
(D) 120° an obtuse angled triangle

17. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between two forces is :-

(A) 150° (B) 90° (C) 60° (D) 120°

18. The resultant of two forces acting at an angle of 120° is 10 kg wt and is perpendicular to one of the forces. That force is :

(A) $10\sqrt{3}$ kg wt (B) $20\sqrt{3}$ kg wt (C) 10 kg wt (D) $\frac{10}{\sqrt{3}}$ kg wt

19. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60° is $\sqrt{7}$ Q, then P/Q is :-

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4

20. There are two force vectors, one of 5N and other of 12N at what angle the two vectors be added to get resultant vector of 17N, 7N and 13N respectively.

(A) 0°, 180° and 90°
(B) 0°, 90° and 180°
(C) 0°, 90° and 90°
(D) 180°, 0° and 90°

21. A body placed in free space, is simultaneously acted upon by three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. The body is in equilibrium and the forces $\vec{F}_1$ and $\vec{F}_2$ are known to be 36 N due north and 27 N due east respectively. Which of the following best describes the force $\vec{F}_3$?

(A) 36 N due south.
(B) 53 N due 60° south of east
(C) 45 N due 53° south of west
(D) 45 N due 37° north of west

22. The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{A}$ and $\vec{B}$ is 3 : 2. The relation between A and B is

(A) A = 5B (B) 5A = B (C) A = 3B (D) A = 4B

23. Find the resultant of the following two vectors $\vec{A}$ and $\vec{B}$.

$\vec{A}$: 40 units due east and ; $\vec{B}$: 25 units 37° north of west

(A) 25 units 37° north of west
(B) 25 units 37° north of east
(C) 40 units 53° north of west
(D) 40 units 53° north of east

24. Two vectors $\vec{a}$ and $\vec{b}$ add to give a resultant $\vec{c} = \vec{a} + \vec{b}$. In which of these cases angle between $\vec{a}$ and $\vec{b}$ is maximum: (a, b, c represent the magnitudes of respective vectors)

(A) $c = a + b$ (B) $c^2 = a^2 + b^2$

(C) $c = a - b$ (D) can not be determined

25. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60°, then $|\hat{a} - \hat{b}|$ is :-

(A) 0 (B) 1 (C) 2 (D) 4

26. A man moves towards 3 m north then 4 m towards east and finally 5 m towards 37° south of west. His displacement from origin is :-

(A) $5\sqrt{2}$m (B) 0m (C) 1 m (D) 12 m

Resolution of vectors and unit vector

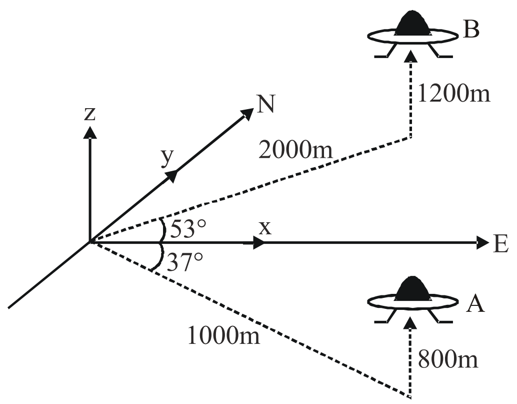
27. The projection of a vector, $\vec{r} = 3\hat{i} + \hat{j} + 2\hat{k}$, on the x–y plane has magnitude :–

(A) 3 (B) 4 (C) $\sqrt{14}$ (D) $\sqrt{10}$

28. A bird moves from point (1, –2) to (4, 2). If the speed of the bird is 10 m/s, then the velocity vector of the bird is

(A) $5(\hat{i} - 2\hat{j})$ (B) $5(4\hat{i} + 2\hat{j})$ (C) $0.6i + 0.8\hat{j}$ (D) $6\hat{i} + 8\hat{j}$

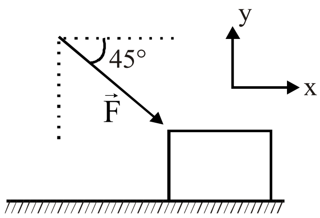
29. Personnel at an air post control tower track a UFO. At 11:02 am it was located at position A and at 11:12 am is was located at position B. Displacement vector of UFO is :



(A) $400\hat{i} + 2200\hat{j} + 400\hat{k}$ (B) $1200\hat{i} + 1000\hat{j} + 800\hat{k}$

(C) $2000\hat{i} + 2200\hat{j} + 2000\hat{k}$ (D) $400\hat{i} + 1000\hat{j} + 400\hat{k}$

30. A person pushes a box kept on a horizontal surface with force of 100 N. In unit vector notation force $\vec{F}$ can be expressed as :



(A) 100 $(\hat{i}+\hat{j})$ (B) 100 $(\hat{i}-\hat{j})$ (C) $50\sqrt{2}\ (\hat{i}-\hat{j})$ (D) $50\sqrt{2}\ (\hat{i}+\hat{j})$

31. For the given vector $\vec{A} = 3\hat{i} - 4\hat{j} + 10\hat{k}$, the ratio of magnitude of its component on the x-y plane and the component on z–axis is :-

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) None

32. After firing, a bullet is found to move at an angle of 37° to horizontal. Its acceleration is 10 m/s^2 downwards. Find the component of acceleration in the direction of the velocity.

(A) – 6 m/s^2 (B) – 4 m/s^2 (C) – 8 m/s^2 (D) – 5 m/s^2

Scalar product of vectors

33. In a methane (CH_4) molecule each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the centre. In coordinates where one of theC–H bonds is in the direction of $\hat{i}+\hat{j}+\hat{k}$, an adjacent C–H bond in the $\hat{i}-\hat{j}-\hat{k}$ direction. Then angle between these two bonds :-

(A) $\cos^{-1}\left(-\frac{2}{3}\right)$ (B) $\cos^{-1}\left(\frac{2}{3}\right)$ (C) $\cos^{-1}\left(-\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

34. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is :-

(A) 45° (B) 60° (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

35. The velocity of a particle is $\vec{v} = 6\hat{i}+2\hat{j}-2\hat{k}$. The component of the velocity of a particle parallel to vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ is :-

(A) $6\hat{i}+2\hat{j}+2\hat{k}$ (B) $2\hat{i}+2\hat{j}+2\hat{k}$ (C) $\hat{i}+\hat{j}+\hat{k}$ (D) $6\hat{i}+2\hat{j}-2\hat{k}$

36. A particle moves from a position $3\hat{i}+2\hat{j}-6\hat{k}$ to a position $14\hat{i}+13\hat{j}+9\hat{k}$ in m and a uniform force of $4\hat{i}+\hat{j}+3\hat{k}$ N acts on it. The work done by the force is :-

(A) 200 J (B) 100 J (C) 300 J (D) 500 J

37. Which of the following is perpendicular to $\hat{i}-\hat{j}-\hat{k}$?

(A) $\hat{i}+\hat{j}+\hat{k}$ (B) $-\hat{i}+\hat{j}+\hat{k}$ (C) $\hat{i}+\hat{j}-\hat{k}$ (D) none of these

MULTIPLE CORRECT TYPE QUESTIONS

38. Which of the following statements about the sum of the two vectors $\vec{A}$ and $\vec{B}$, is/are correct?

(A) $|\vec{A}+\vec{B}| \le A+B$ (B) $|\vec{A}+\vec{B}| \ge A+B$ (C) $|\vec{A}+\vec{B}| \ge |\vec{A}-\vec{B}|$ (D) $|\vec{A}+\vec{B}| \ge |A-B|$

39. Priya says that the sum of two vectors by the parallelogram method is $\vec{R}=5\hat{i}$. Subhangi says it is $\vec{R}=\hat{i}+4\hat{j}$. Both used the parallelogram method, but one used the wrong diagonal. Which one of the vector pairs below contains the original two vectors?

(A) $\vec{A}=+3\hat{i}-2\hat{j}$; $\vec{B}=-2\hat{i}+2\hat{j}$ (B) $\vec{A}=-3\hat{i}-2\hat{j}$; $\vec{B}=+2\hat{i}+2\hat{j}$

(C) $\vec{A}=+3\hat{i}+2\hat{j}$; $\vec{B}=+2\hat{i}-2\hat{j}$ (D) $\vec{A}=+3\hat{i}+2\hat{j}$; $\vec{B}=-2\hat{i}+2\hat{j}$

40. For the equation $x = AC\sin(Bt) + D\,e^{(BCt)}$, where x and t represent position and time respectively, which of the following is/are **CORRECT** :-

(A) Dimension of AC is LT^{-1} (B) Dimension of B is T^{-1}

(C) Dimension of AC and D are same (D) Dimension of C is T^{-1}

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 41 to 43

In a certain system of absolute units the acceleration produced by gravity in a body falling freely is denoted by 5, the kinetic energy of a 500 kg shot moving with velocity 400 metres per second is denoted by 2000 & its momentum by 100.

41. The unit of length is :-

(A) 15 m (B) 50 m (C) 25 m (D) 100 m

42. The unit of time is :-

(A) 10 s (B) 20 s (C) 5 s (D) 15 s

43. The unit of mass is :-

(A) 200 kg (B) 400 kg (C) 800 kg (D) 1200 kg

Paragraph for Question Nos. 44 and 45

For any particle moving with some velocity $(\vec{v})$ & acceleration $(\vec{a})$, tangential acceleration & normal acceleration are defined as follows

Tangential acceleration – The component of acceleration in the direction of velocity.

Normal acceleration – The component of acceleration in the direction perpendicular to velocity.

If at a given instant, velocity & acceleration of a particle are given by.

$\vec{v} = 4\hat{i} + 3\hat{j}$

$\vec{a} = 10\hat{i} + 15\hat{j} + 20\hat{k}$

44. Find the tangential acceleration of the particle at the given instant :-

(A) $17(4\hat{i}+3\hat{j})$ (B) $\frac{17}{5}(4\hat{i}+3\hat{j})$ (C) $17(4\hat{i}-3\hat{j})$ (D) $\frac{17}{5}(4\hat{i}-3\hat{j})$

45. Find the normal acceleration of the particle at the given instant :-

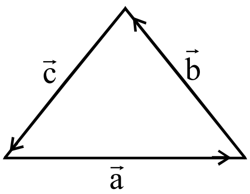
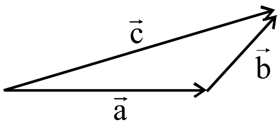
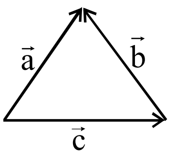
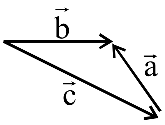
(A) $\frac{-9\hat{i}+12\hat{j}+50\hat{k}}{5}$ (B) $\frac{9\hat{i}-12\hat{j}-50\hat{k}}{5}$ (C) $\frac{-18\hat{i}+24\hat{j}+100\hat{k}}{5}$ (D) $\frac{18\hat{i}-24\hat{j}-100\hat{k}}{5}$

MATRIX MATCH TYPE QUESTION

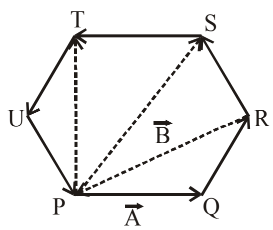
46. Two particles A and B start from origin of a coordinate system towards point P (10, 20) and Q (20, 10) respectively with speed $5\sqrt{5}$ each. Both continue their motion for 10 s and then stop. There after particle B moves towards particle A with speed $2\sqrt{2}$ and after particle B meets particle A, they both return to origin following a straight line path with speed $5\sqrt{5}$. Match the items of column-I with suitable items of Column-II.

Column-I		Column-II	
(A)	Initial velocity vector of A	(P)	$(-5\hat{i}-10\hat{j})$
(B)	Initial velocity of B	(Q)	$(5\hat{i}+10\hat{j})$
(C)	Velocity vector of B while it moves towards A	(R)	$(10\hat{i}+5\hat{j})$
(D)	Velocity vector of A and B while they return to origin	(S)	$(2\hat{i}-2\hat{j})$
		(T)	$(-2\hat{i}+2\hat{j})$

47. Column-I show vector diagram relating three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. Match the vector equation in column-II, with vector diagram in column-I :

Column-I	**Column-II**
(A) 	(P) $\vec{a}-(\vec{b}+\vec{c})=0$
(B) 	(Q) $\vec{b}-\vec{c}=\vec{a}$
(C) 	(R) $\vec{a}+\vec{b}=-\vec{c}$
(D) 	(S) $\vec{a}+\vec{b}=\vec{c}$

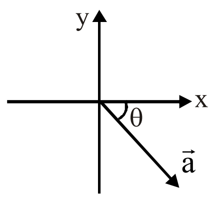
48. In a regular hexagon two vectors $\overrightarrow{PQ}=\vec{A}$, $\overrightarrow{RP}=\vec{B}$. Express other vector's in term of them :-



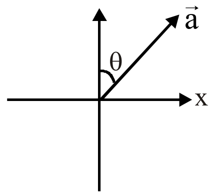
Column–I	**Column–II**
(A) $\overrightarrow{PS}$	(P) $-2\vec{B}-3\vec{A}$
(B) $\overrightarrow{PT}$	(Q) $-\vec{B}-\vec{A}$
(C) $\overrightarrow{RS}$	(R) $-\vec{B}-2\vec{A}$
(D) $\overrightarrow{TS}$	(S) $-2(\vec{B}+\vec{A})$
	(T) $\vec{A}$

49. Show a vector $\vec{a}$ at angle θ as shown in the figure column-II. Show its unit vector representation.

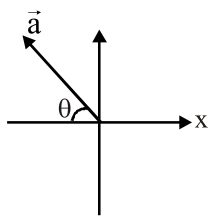
Column–I **Column–II**

(A) 

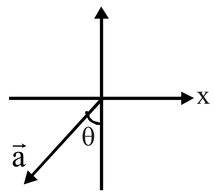
(P) $\vec{a} = a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j}$

(B) 

(Q) $\vec{a} = -a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$

(C) 

(R) $\vec{a} = -a\sin\theta\,\hat{i} - a\cos\theta\,\hat{j}$

(D) 

(S) $\vec{a} = a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j}$

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and 1 unit of length is 10 m. In this system, one unit of power will correspond to :-

(A) 16 watts (B) $\frac{1}{16}$ watts (C) 25 watts (D) none of these

2. If the unit of length be doubled then the numerical value of the universal gravitation constant G will become (with respect to present value)

(A) Double (B) Half (C) 8 times (D) 1/8 times

3. If in a system, the force of attraction between two point masses of 1 kg each situated 1 km apart is taken as a unit of force and is called notwen (newton written in reverse order) & if $G = 6.67\times 10^{-11}$ N–m^2 kg^{-2} in SI units then which of the following is true?

(A) 1 notwen = 6.67×10^{-11} newton (B) 1 newton = 6.67×10^{-17} notwen

(C) 1 notwen = 6.67×10^{-17} newton (D) 1 newton = 6.67×10^{-12} notwen

4. In two different systems of units an acceleration is represented by the same number, while a velocity is represented by numbers in the ratio 1 : 3. The ratios of unit of length and time are respectively

(A) 3, 9 (B) 9, 3 (C) 1, 1 (D) None of these

5. **Statement–1 :** Whenever the unit of measurement of a quantity is changed, its numerical value changes.

and

Statement–2 : Smaller the unit of measurement smaller is its numerical value.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

6. Forces proportional to AB, BC and 2CA act along the sides of triangle ABC in order. Their resultant represented in magnitude and direction as

(A) CA (B) AC (C) BC (D) CB

7. A man rows a boat with a speed of 18 km/hr in north–west direction. The shoreline makes an angle of 15° south of west. Obtain the component of the velocity of the boat along the shoreline.

(A) 9 km/hr
(B) $18\frac{\sqrt{3}}{2}$ km/hr
(C) 18 cos (15°) km/hr
(D) 18 cos (75°) km/hr

8. **Statement 1** : Unit vector has a unit though its magnitude is one

and

Statement 2 : Unit vector is obtained by dividing a vector by its own magnitude.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

9. A vector of magnitude 10 m in the direction 37° south of west has its initial point at (5 m, 2 m). If positive x–axis represents the east and positive y–axis the north, the coordinates of its terminal point are

(A) (–3 m, –4 m)
(B) (3 m, 4 m)
(C) (–4 m, 6 m)
(D) (–4 m,–6 m)

10. A plumber steps down 1 m out of his truck and walks 50 m east and then 25 m south, and then takes an elevator to the basement of the building 9 m below street level. If the east, the north and the upward direction are represented by the positive x, y and z–axes, which one of the following represents displacement (meters) of the plumber?

(A) $50\hat{i}-25\hat{j}-9\hat{k}$
(B) $50\hat{i}+25\hat{j}-9\hat{k}$
(C) $50\hat{i}-25\hat{j}-10\hat{k}$
(D) $50\hat{i}+25\hat{j}-10\hat{k}$

11. A body moves in anticlockwise direction on a circular path in the x-y plane. The radius of the circular path is 5 m and its centre is at the origin. In a certain interval of time, displacement of the body is observed to be 6 m in the positive y-direction. Which of the following is true?

(A) Its initial position vector is $5\hat{i}$ m.
(B) Its initial position vector is $(-3\hat{i}+4\hat{j})$ m.
(C) Its final position vector is $(4\hat{i}+3\hat{j})$ m.
(D) Its final position vector is $6\hat{j}$ m.

12. A boy A is standing 20√3 m away in a direction 30° north of east from his friend B. Another boy C standing somewhere east of B can reach A, if he walks in a direction 60° north of east. In a Cartesian coordinate system with its x–axis towards the east, the position of C with respect to A is

(A) $\left(-20\hat{i}+-10\hat{j}\right)$ m
(B) $\left(-10\hat{i}-10\sqrt{3}\hat{j}\right)$ m
(C) $\left(10\hat{i}+10\sqrt{3}\hat{j}\right)$ m
(D) It depends on where we chose the origin.

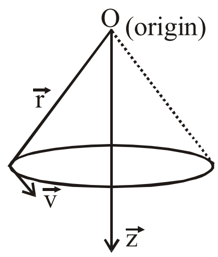
13. Find the component of $\vec{r}$ in the direction of $\vec{a}$:–

(A) $\dfrac{(\vec{r}.\vec{a})\vec{a}}{a^2}$
(B) $\dfrac{(\vec{r}.\vec{a})\vec{a}}{a}$
(C) $\dfrac{(\vec{r}\cdot\vec{a})\hat{r}}{r}$
(D) $\dfrac{(\vec{r}\cdot\vec{a})\hat{r}}{r^2}$

14. Consider three vectors $\vec{A}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{B}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{C}=2\hat{i}-3\hat{j}+4\hat{k}$. A vector $\vec{X}$ of the form $\alpha\vec{A}+\beta\vec{B}$ (α and β are numbers) is perpendicular to $\vec{C}$. The ratio of α and β is

(A) 1 : 1
(B) 2 : 1
(C) –1 : 1
(D) 3 : 1

15. A string connected with bob is suspended by the point (O) such that it sweeps out conical surface in horizontal plane. Here $\vec{r}$ is the position vector of bob, $\vec{v}$ is its velocity and $\vec{z}$ is the axis of swept cone as shown. Select **INCORRECT** statement :-



(A) $\vec{r}.\vec{z}$ is always zero
(B) $\vec{r}.\vec{v}$ is always zero
(C) $\vec{z}\cdot\vec{v}$ is always constant
(D) $\vec{r}\cdot\vec{z}$ is always non zero constant

16. x-component of a vector $\vec{A}$ is twice of its y-component and $\sqrt{2}$ times of its z-component. Find out the angle made by the vector from y-axis.

(A) $\cos^{-1}\left(\dfrac{2}{\sqrt{7}}\right)$
(B) $\cos^{-1}\left(\dfrac{1}{\sqrt{7}}\right)$
(C) $\cos^{-1}\left(\dfrac{1}{\sqrt{6}}\right)$
(D) $\cos^{-1}\left(\dfrac{2}{\sqrt{6}}\right)$

17. Given the vectors $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$; $\vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}$ & $\vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$. Find the angle between $(\vec{A} - \vec{B})$ & $\vec{C}$

(A) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (C) $\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) none of these

MULTIPLE CORRECT TYPE QUESTIONS

18. Four forces acting on a particle keep it in equilibrium, then :-

(A) the force must be coplanar.

(B) the forces cannot be coplanar.

(C) the forces may or may not be coplanar.

(D) if three of these forces are coplanar, so must be the fourth.

19. A man is standing at point (x = 5 m, y = 0). Then he walks along straight line to (x = 0, y = 5m). A second man walks from the same initial position along the x-axis to the origin and then along the y-axis to (x = 0, y = 5m). Mark the **CORRECT** statement(s) :

(A) Displacement vector of first man and second man are equal

(B) Distance travelled by second man is greater

(C) Magnitude of displacement of second man is same that of first man but direction is different

(D) Magnitude of displacement is $\sqrt{50}$ m for 2^{nd} man.

20. The vector i + xj + 3k is rotated through an angle θ and doubled in magnitude, then it becomes 4i + (4x – 2) j + 2k. The values of x are

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

21. The value of $|\vec{A} + \vec{B} - \vec{C} + \vec{D}|$ can be zero if :–

(A) $|\vec{A}| = 5, |\vec{B}| = 3, |\vec{C}| = 4; |\vec{D}| = 13$ (B) $|\vec{A}| = 2\sqrt{2}, |\vec{B}| = 2, |\vec{C}| = 2; |\vec{D}| = 5$

(C) $|\vec{A}| = 2\sqrt{2}, |\vec{B}| = 2, |\vec{C}| = 2; |\vec{D}| = 10$ (D) $|\vec{A}| = 5, |\vec{B}| = 4, |\vec{C}| = 3; |\vec{D}| = 8$

22. The four pairs of force vectors are given, which pairs of force vectors cannot be added to give a resultant vector of magnitude 10 N?

(A) 2N, 13 N (B) 5N, 16 N

(C) 7N, 8N (D) 100N, 105 N

23. Select **CORRECT** statement(s) for three vectors $\vec{a} = -3\hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 4\hat{k}$

(A) The above vectors can form triangle.

(B) Component of $\vec{a}$ along $\vec{c}$ is 3.

(C) $\vec{a}$ makes angle $\cos^{-1}\sqrt{\frac{2}{7}}$ with y-axis.

(D) A vector having magnitude twice the vector $\vec{a}$ and anti parallel to vector $\vec{b}$ is $\sqrt{\frac{2}{5}}\left(-2\hat{i} + 6\hat{j} - 10\hat{k}\right)$

24. If a vector $\vec{P}$ makes an angle α, β, γ with x, y, z axis respectively then it can be represented as $\vec{P} = P\left[\cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}\right]$. Choose the **CORRECT** option(s) :-

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\vec{P}.\vec{P} = P^2$

(C) $\vec{P}.(\hat{i} - \hat{k}) = P(\cos\alpha - \cos\gamma)$

(D) $\vec{P}.\hat{i} = \cos\alpha$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 25 to 27

A boy lost in a jungle finds a note. In the note was written the following things.

Displacements

1. 300 m 53° South of East.
2. 400 m 37° North of East
3. 500 m North
4. 500 √2 m North-West
5. 500 m South

He starts walking at constant speed 2 m/s following these displacements in the given order.

25. How far and in which direction is he from the starting point after 5 min. and 50 s?

(A) 500 m due East

(B) 500 m due West

(C) 700 m due South-West

(D) 700 m due North-East

26. How far and in which direction is he from the starting point after 10 minutes?

(A) 500 √2 m due North

(B) 1200 m due North-East

(C) 500√2 m due North-East

(D) 900 m due 37° North of East

27. How far and in which direction has he finally displaced after all the displacements in the note?

(A) $500\sqrt{2}$ m due North-East
(B) 500 m due North
(C) 866 m due North-West
(D) $500\sqrt{3}$ m due 60° North of West

Paragraph for Question Nos. 28 to 30

A physical quantity is a physical property of a phenomenon, body, or substance, that can be quantified by measurement.

The magnitude of the components of a vector are to be considered dimensionally distinct. For example, rather than an undifferentiated length unit L, we may represent length in the x direction as L_x, and so forth. This requirement stems ultimately from the requirement that each component of a physically meaningful equation (scalar or vector) must be dimensionally consistent. As an example, suppose we wish to calculate the drift S of a swimmer crossing a river flowing with velocity V_x and of width D and he is swimming in direction perpendicular to the river flow with velocity V_y relative to river, assuming no use of directed lengths, the quantities of interest are then V_x, V_y both dimensioned as $\frac{L}{T}$, S the drift and D width of river both having dimension L. With these four quantities, we may conclude that the equation for the drift S may be written: $S \propto V_x^a V_y^b D^c$

Or dimensionally $L = \left(\frac{L}{T}\right)^{a+b} \times (L)^c$ from which we may deduce that $a+b+c=1$ and $a+b=0$, which leaves one of these exponents undetermined. If, however, we use directed length dimensions, then V_x will be dimensioned as $\frac{L_x}{T}$, V_y as $\frac{L_y}{T}$, S as L_x and D as L_y. The dimensional equation becomes : $L_x = \left(\frac{L_x}{T}\right)^a \left(\frac{L_y}{T}\right)^b (L_y)^c$ and we may solve completely as $a = 1$, $b = -1$ and $c = 1$.

The increase in deductive power gained by the use of directed length dimensions is apparent.

28. Which of the following is not a physical quantity

(A) Height of a boy
(B) Weight of a boy
(C) Fever of a boy
(D) Speed of a running boy

29. From the concept of directed dimension what is the formula for a range (R) of a cannon ball when it is fired with vertical velocity component V_y and a horizontal velocity component V_x assuming it is fired on a flat surface. [Range also depends upon acceleration due to gravity, g and k is numerical constant]

(A) $R = \frac{k(V_xV_y)}{g}$
(B) $R = \frac{k(V_x)^2}{g}$
(C) $R = \frac{k(V_x)^3}{V_yg}$
(D) $R = \frac{k(V_y)^3}{V_xg}$

30. A conveyer belt of width D is moving along x-axis with velocity V. A man moving with velocity U on the belt in the direction perpendicular to the belt's velocity with respect to belt wants to cross the belt. The correct expression for the drift (S) suffered by man is given by (k is numerical constant)

(A) $S = k\frac{UD}{V}$ (B) $S = k\frac{VD}{U}$ (C) $S = k\frac{U^2D}{V^2}$ (D) $S = k\frac{V^2D}{U^2}$

MATCHING LIST TYPE (4 × 4 × 4) MULTIPLE OPTION CORRECT

(THREE COLUMNS AND FOUR ROWS)

Answer Q.31, Q.32 and Q.33 by appropriately matching the information given in the three columns of the following table.

L, M and T are units of length, Mass and Time respectively in a system of units.

	Coloumn–1		Column-2		Column-3
(I)	L = 10 m	(i)	M = 100 gm	(P)	T = 0.1 sec
(II)	L = 10 cm	(ii)	M = 10 kg	(Q)	T = 10 ms
(III)	L = 0.1 mm	(iii)	M = 10 gm	(R)	T = 10 sec
(IV)	L = 1 km	(iv)	M = 1 tonne	(S)	T = 0.01 sec

31. In which of the following combinations unit of force is 10^6 dyne.

(A) (IV) (i) (P) (B) (II) (iii) (S) (C) (III) (iv) (P) (D) (I) (ii) (Q)

32. In which of the following system, unit of energy is 10^9 erg

(A) (III) (i) (S) (B) (IV) (iii) (R) (C) (II) (iv) (Q) (D) (I) (iii) (P)

33. In which of the following system, unit for coefficient of viscosity is 100 poiseuille

(A) (III) (ii) (S) (B) (II) (i) (Q) (C) (III) (iii) (R) (D) (IV) (iv) (P)

MATRIX MATCH TYPE QUESTION

34. In a new system of units known as RMP, length is measured in 'retem', mass is measured in 'marg' and time is measured in 'pal'.

100 retem = 1.0 meter

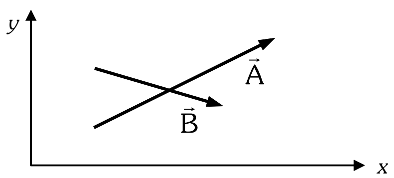
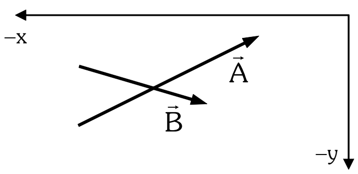
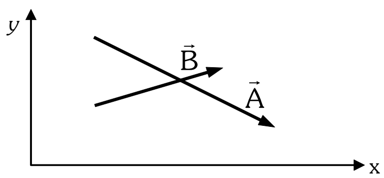
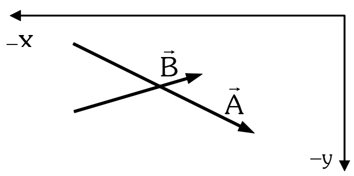
1.0 marg = 10^{-3} kilogram

10 pal = 1.0 second

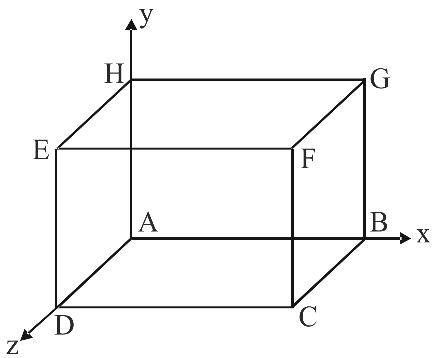
In the given table some unit conversion factors are given. Suggest suitable match.

Column-I	**Column-II**
(A) One SI unit of force	(P) 10^2 units of RMP
(B) One SI unit of potential energy	(Q) 10^3 units of RMP
(C) One SI unit of power	(R) 10^4 units of RMP
(D) One SI unit of momentum	(S) 10^5 units of RMP
	(T) 10^6 units of RMP

35. Refer the following table, where in the first column four pairs of two vectors are shown and in the second column some possible outcomes of basic mathematical operation on these vectors are given. Suggest suitable match(s).

Column-I	**Column-II**
(A) 	(P) X-component of $\vec{A}+\vec{B}$ is positive
(B) 	(Q) Y-component of $\vec{A}+\vec{B}$ is negative
(C) 	(R) X-component of $\vec{A}-\vec{B}$ is positive
(D) 	(S) Y-component of $\vec{A}-\vec{B}$ is negative
	(T) $\vec{B}.\vec{A}$ is positive

36. Figure shows a cube of edge length a.



Column-I	**Column-II**
(A) The angle between AF and x-axis	(P) 60°
(B) Angle between AF and DG	(Q) $\cos^{-1}\frac{1}{3}$
(C) Angle between AE and AG	(R) $\cos^{-1}\frac{1}{\sqrt{3}}$
	(S) $\cos^{-1}\sqrt{\frac{2}{3}}$

EXERCISE (J-M)

1. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency. **[JEE Main-2013]**

(1) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (2) $\frac{1}{2\pi}\frac{V_0 MP_0}{A^2\gamma}$ (3) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

EXERCISE (J-A)

1. Match List I with List II and select the correct answer using the codes given below the lists :

	List I		**List II**
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Planck constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conductivity	4.	$[ML^2T^{-2}K^{-1}]$

[JEE Advanced-2013]

Codes :

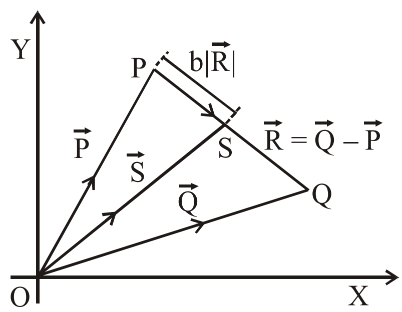
	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

2. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is. **[JEE Advanced-2014]**

3. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is(are) **[JEE Advanced-2015]**

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 cV$ (D) $\mu_0 cI = \varepsilon_0 V$

4. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is : **[JEE Advanced-2017]**



(A) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$ (B) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$ (D) $\vec{S} = (1-b^2)\vec{P} + b\vec{Q}$

5. Two vectors $\vec{A}$ and $\vec{B}$ are defined as $\vec{A} = a\hat{i}$ and $\vec{B} = a\left(\cos\omega t\hat{i} + \sin\omega t\hat{j}\right)$, whre a is a constant and $\omega = \pi/6$ rad s^{-1}. If $|\vec{A}+\vec{B}| = \sqrt{3}|\vec{A}-\vec{B}|$ at time $t = \tau$ for the first time, the value of τ, in seconds, is ______. **[JEE Advanced-2018]**

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\in_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

6. The relation between [E] and [B] is :- **[JEE Advanced-2018]**

(A) $[E] = [B][L][T]$ (B) $[E] = [B][L]^{-1}[T]$ (C) $[E] = [B][L][T]^{-1}$ (D) $[E] = [B][L]^{-1}[T]^{-1}$

PARAGRAPH "X"

In electromagnetic theory, the electric and magnetic phenomena are related to each other. Therefore, the dimensions of electric and magnetic quantities must also be related to each other. In the questions below, [E] and [B] stand for dimensions of electric and magnetic fields respectively, while $[\in_0]$ and $[\mu_0]$ stand for dimensions of the permittivity and permeability of free space respectively. [L] and [T] are dimensions of length and time respectively. All the quantities are given in SI units.

(There are two questions based on Paragraph "X", the question given below is one of them)

7. The relation between $[\in_0]$ and $[\mu_0]$ is :- **[JEE Advanced-2018]**

(A) $[\mu_0] = [\in_0][L]^2[T]^{-2}$ (B) $[\mu_0] = [\in_0][L]^{-2}[T]^2$

(C) $[\mu_0] = [\in_0]^{-1}[L]^2[T]^{-2}$ (D) $[\mu_0] = [\in_0]^{-1}[L]^{-2}[T]^2$

8. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following statement (s) is/are correct ?

[JEE Advanced-2019]

(1) The dimension of force is L^{-3}

(2) The dimension of energy is L^{-2}

(3) The dimension of power is L^{-5}

(4) The dimension of linear momentum is L^{-1}

ANSWER KEY

EXERCISE (S-1)

1. Ans. L^{-1}, ML^2T^{-2}

2. Ans. $kg.m^2.s^{-2}.K^{-1}$

3. Ans. $T = a\sqrt{\frac{m}{k}}$

4. Ans. $\omega = K\sqrt{\frac{GM}{r^3}}$

5. Ans. $v_0 = k\sqrt{\frac{GM}{R}}$

6. Ans. K = 6

7. Ans. $[MLT^{-1}]$

8. Ans. $\sqrt{75}$

9. Ans. 1

10. Ans. 25

11. Ans. 7

12. Ans. 90 m

13. Ans. $\cos\alpha = \left(\frac{4}{\sqrt{61}}\right)$, $\cos\beta = \left(\frac{6}{\sqrt{61}}\right)$ $\cos\gamma = \left(\frac{3}{\sqrt{61}}\right)$, $magnitude = \sqrt{61}$

14. Ans. 3

15. Ans. 5

16. Ans. $\frac{7}{16}$

17. Ans. $\hat{i} + \hat{j}$, $3\hat{k}$

18. Ans. (a) $11\hat{i} + 5\hat{j} - 7\hat{k}$, (b) $\cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$, (c) $\cos^{-1}\left(\frac{-20}{\sqrt{1309}}\right)$

19. Ans. 2

20. Ans. (a) $\frac{2\pi}{3} \times 6.4 \times 10^6 m$, (b) $\sqrt{3} \times 6.4 \times 10^6$ m

21. Ans. (a) 9.95, (b) 0.99

22. Ans. 0.14, 0.09

EXERCISE (O-1)

1. Ans. (D)	**2. Ans. (C)**	**3. Ans. (A)**	**4. Ans. (A)**	**5. Ans. (B)**	**6. Ans. (A)**
7. Ans. (C)	**8. Ans. (D)**	**9. Ans. (D)**	**10. Ans. (A)**	**11. Ans. (C)**	**12. Ans. (B)**
13. Ans. (C)	**14. Ans. (D)**	**15. Ans. (B)**	**16. Ans. (B)**	**17. Ans. (D)**	**18. Ans. (D)**
19. Ans. (C)	**20. Ans. (A)**	**21. Ans. (C)**	**22. Ans. (A)**	**23. Ans. (B)**	**24. Ans. (C)**
25. Ans. (B)	**26. Ans. (B)**	**27. Ans. (D)**	**28. Ans. (D)**	**29. Ans. (A)**	**30. Ans. (C)**
31. Ans. (B)	**32. Ans. (A)**	**33. Ans. (C)**	**34. Ans. (B)**	**35. Ans. (B)**	**36. Ans. (B)**
37. Ans. (D)	**38. Ans. (A,D)**	**39. Ans. (C,D)**	**40. Ans. (B, C)**	**41. Ans. (B)**	**42. Ans. (C)**
43. Ans. (A)	**44. Ans. (B)**	**45. Ans. (C)**			

46. Ans. (A) $\rightarrow$ (Q); (B) $\rightarrow$ (R); (C) $\rightarrow$ (T); (D) $\rightarrow$ (P)

47. Ans. (A)-R; (B)-S; (C)-P; (D)-Q

48. Ans. (A) $\rightarrow$ (S); (B) $\rightarrow$ (P); (C) $\rightarrow$ (R); (D) $\rightarrow$ (T)

49. Ans. (A)-S; (B)-P; (C)-Q; (D)-R

EXERCISE (O-2)

1. Ans. (A) 2. Ans. (D) 3. Ans. (C) 4. Ans. (B) 5. Ans. (C) 6. Ans. (A)
7. Ans. (A) 8. Ans. (D) 9. Ans. (A) 10. Ans. (C) 11. Ans. (C) 12. Ans. (B)
13. Ans. (A) 14. Ans. (A) 15. Ans. (A) 16. Ans. (B) 17. Ans. (C) 18. Ans. (C, D)
19. Ans. (A,B,D) 20. Ans. (A, D) 21. Ans. (B,D) 22. Ans. (A,B)
23.Ans. (A,C,D) 24. Ans. (A, B, C) 25. Ans. (A) 26. Ans. (C)
27. Ans. (B) 28. Ans. (C) 29. Ans. (A) 30. Ans. (B) 31. Ans. (B,C) 32. Ans. (B,D)
33. Ans. (B)
34. Ans. (A) $\rightarrow$ (Q); (B) $\rightarrow$ (S); (C) $\rightarrow$ (R); (D) $\rightarrow$ (R)
35. Ans. (A) $\rightarrow$ (P,R,T); (B) $\rightarrow$ (P,R,T); (C) $\rightarrow$ (P,Q,R,S,T); (D) $\rightarrow$ (P,Q,R,S,T)
36. Ans. (A) R (B) Q (C) P

EXERCISE (J-M)

1. Ans. (3)

EXERCISE (J-A)

1. Ans. (C) 2. Ans. 3 3. Ans. (A,C) 4. Ans. (C) 5. Ans. 2 [1.99, 2.01]
6. Ans. (C) 7. Ans. (D) 8. Ans. (1,2,4)

IMPORTANT NOTES

CHAPTER 2

KINEMATICS-1D & CALCULUS

Chapter Contents 02

01.	THEORY	53
02.	EXERCISE (S-1)	80
03.	EXERCISE (O-1)	86
04.	EXERCISE (O-2)	94
05.	EXERCISE (JM)	101
06.	EXERCISE (JA)	102
07.	ANSWER KEY	103

IMPORTANT NOTES

CHAPTER 2

KINEMATICS-1D & CALCULUS

KEY CONCEPT

KINEMATICS

Study of motion of objects without taking into account the factor which cause the motion (i.e. nature of force).

Motion

If a body changes its position with time, it is said to be moving else it is at rest. Motion is always relative to the observer.

Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer. In other words absolute motion or rest is meaningless.

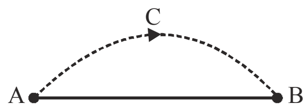
- To locate the position of a particle we need a reference frame. A commonly used reference frame is Cartesian coordinate system or simply coordinate system.

  The coordinates (x, y, z) of a particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.
- The reference frame is chosen according to the problems.
- If the frame is not mentioned, then ground is taken as the reference frame.

Distance and Displacement

Total length of path covered by the particle, in definite time interval is called distance.

Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.



But overall, body is displaced from A to B. A vector from A to B, i.e. $\overrightarrow{AB}$ is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

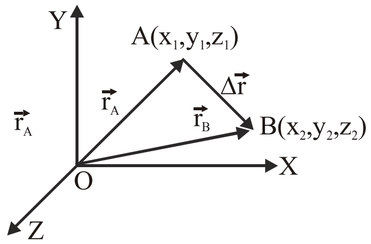
- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial position.
- For a moving body, distance can't have zero or negative values but displacement may be +ive, –ive or zero.
- For a moving/stationary object distance can't be decreasing.
- If motion is in straight line without change in direction then

  distance = |displacement| i.e. magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance.

  distance $\geq$ |displacement|

Displacement in terms of position vector

Let a body is displaced from A (x_1, y_1, z_1) to B (x_2, y_2, z_2) then its displacement is given by vector $\vec{AB}$.



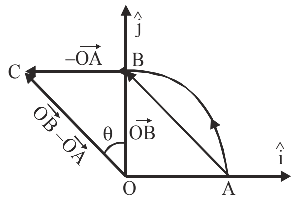
From ΔOAB $\vec{r}_A + \Delta\vec{r} = \vec{r}_B \Rightarrow \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \text{ and } \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \Rightarrow \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Ex. A particle goes along a quadrant from A to B of a circle radius 10m as shown in fig. Find the direction and magnitude of displacement and distance along path AB.

Sol. Displacement $\vec{AB} = \vec{OB} - \vec{OA} = 10\hat{j} - 10\hat{i}$



$|\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m

From ΔOBC $\tan\theta = \dfrac{OA}{OB} = \dfrac{10}{10} = 1 \Rightarrow \theta = 45°$

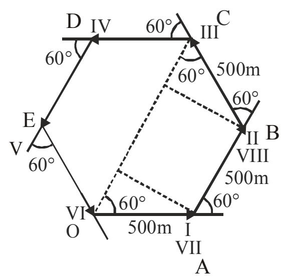
Angle between displacement vector $\vec{OC}$ and x–axis = 90° + 45° = 135°

Distance of path AB = $\dfrac{1}{4}$ (circumference) = $\dfrac{1}{4}(2\pi R)$ m = (5π) m

Ex. On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Sol. **At III turn**

Displacement $= \overline{OA} + \overline{AB} + \overline{BC} = \overline{OC}$



= 500 cos 60° + 500 + 500 cos 60°

$= 500 \times \dfrac{1}{2} + 500 + 500 \times \dfrac{1}{2} = 1000$ m from O to C

Distance = 500 + 500 + 500 = 1500 m. So $\dfrac{\text{Displacement}}{\text{Distance}} = \dfrac{1000}{1500} = \dfrac{2}{3}$

At VI turn

$\because$ initial and final positions are same so displacement = 0 and distance = 500 × 6 = 3000 m

$$\therefore \frac{\text{Displacement}}{\text{Distance}} = \frac{0}{3000} = 0$$

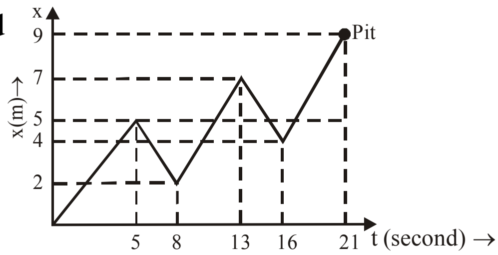
At VIII turn

$$\text{Displacement} = 2(500)\cos\left(\frac{60°}{2}\right) = 1000 \times \cos 30° = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}\ \text{m}$$

Distance = 500 × 8 = 4000 m

$$\therefore \frac{\text{Displacement}}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$$

Ex. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Plot the x–t graph of his motion. Determine graphically or otherwise how long the drunkard takes to fall in a pit 9m away from the start.



Sol. from x–t graph time taken = 21 s

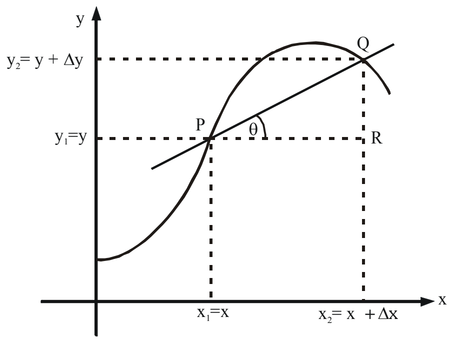
or (5m – 3m) + (5m – 3m) + 5m = 9m

$\Rightarrow$ total steps = 21 $\Rightarrow$ time = 21 s

DERIVATIVE OF A FUNCTION

Average Rate of Change

Let a function y=f(x) be plotted by an arbitrary graph as shown in the figure. Average rate of change in y with respect to x in an interval $[x_1, x_2]$ is defined as



$$\text{Average rate of change} = \frac{\text{Change in y}}{\text{Change in x}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

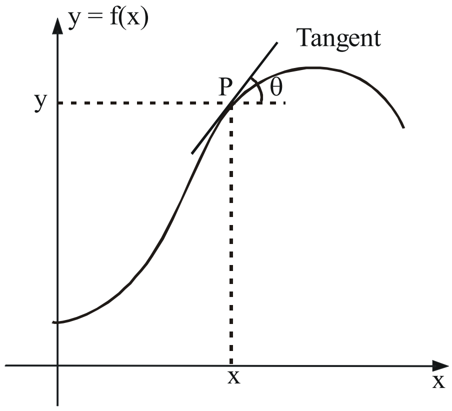
$$= \text{slope of chord}$$

If both the axes have equal scale

Average rate of change = slope of chord PQ = tanθ

Instantaneous Rate of Change : First derivative

It is defined as the rate of change in y with x at a particular value of x. It is measured graphically by the slope of the tangent drawn to the y-x graph at the point (x, y) and algebraically by the first derivative of the function y=f(x).



Instantaneous rate of change $= \frac{dy}{dx}$ = Slope of the tangent

If both the axes have equal scale then $\frac{dy}{dx} = \tan\theta$

$$\text{Instantaneous rate of change } = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Derivatives of Commonly Used Functions.

- $y = \text{constant} \Rightarrow \frac{dy}{dx} = 0$
- $y = \cos x \Rightarrow \frac{dy}{dx} = -\sin x$
- $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$
- $y = \tan x \Rightarrow \frac{dy}{dx} = \sec^2 x$
- $y = e^x \Rightarrow \frac{dy}{dx} = e^x$
- $y = \cot x \Rightarrow \frac{dy}{dx} = -\text{cosec}^2 x$
- $y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \text{cosec } x \Rightarrow \frac{dy}{dx} = -\text{cosec } x \cot x$
- $y = \sin x \Rightarrow \frac{dy}{dx} = \cos x$
- $y = \sec x \Rightarrow \frac{dy}{dx} = \sec x \tan x$

Method of Differentiation.

If y = f(x), let us denote $\frac{dy}{dx} = f'(x)$

- Sum or Subtraction of two functions $y = f(x) \pm g(x) \Rightarrow \frac{dy}{dx} = f'(x) \pm g'(x)$
- Product of two functions

$$y = f(x) \cdot g(x) \Rightarrow \frac{dy}{dx} = g(x)f'(x) + f(x)g'(x)$$

- Division of two functions.

$$y = \frac{f(x)}{g(x)} \Rightarrow \frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{\{g(x)\}^2}$$

- Chain Rule

$$y = f\{g(x)\} \Rightarrow \frac{dy}{dx} = g'(x)f'\{g(x)\}$$

Ex. Find $\frac{dy}{dx}$, when (i) $y = \sqrt{x}$ (ii) $y = x^5 + x^4 + 7$ (iii) $y = x^2 + 4x^{-1/2} - 3x^{-2}$

Sol. (i) $\frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

(ii) $\frac{dy}{dx} = \frac{d}{dx}(x^5 + x^4 + 7) = \frac{d}{dx}(x^5) + \frac{d}{dx}(x^4) + \frac{d}{dx}(7) = 5x^4 + 4x^3 + 0 = 5x^4 + 4x^3$

(iii) $\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2}) = \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$

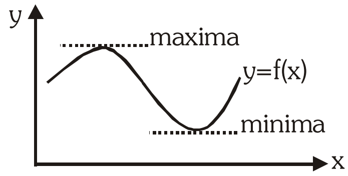
$$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2}) = 2x + 4\left(-\frac{1}{2}\right)x^{-3/2} - 3(-2)x^{-3} = 2x - 2x^{-3/2} + 6x^{-3}$$

Second Derivative and it's meaning

Second derivative of a function y = f(x) is defined as $\frac{d}{dx}\left[\frac{dy}{dx}\right]$. It is obtained by differentiating the function with respect to x two times successively. Geometrically it expresses rate of change in slope of graph of the function.

Maxima & Minima

Maxima & minima of a function y = f(x)



for maximum value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} =$ negative

for minimum value $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} =$ positive

Ex. Find minimum value of $y = 25x^2 - 10x + 5$.

Sol. For maximum/minimum value $\frac{dy}{dx} = 0 \Rightarrow 50x - 10 = 0 \Rightarrow x = \frac{1}{5}$

Now at $x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 50$ which is positive. So $y_{min} = 25\left(\frac{1}{5}\right)^2 - 10\left(\frac{1}{5}\right) + 5 = 1 - 2 + 5 = 4$

INTEGRATION

- This operation enables us to find sum of infinite number of infinitely small quantities.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{\infty} \Delta x_i = \int dx = x$$

- It is reverse operation of differentiation. If derivative, which is rate of change at a point, is given as a function $f(x) = \frac{dF(x)}{dx}$, operation of integration enables us to find original function F(x).

$$\int f(x)dx = F(x) + C$$

Here function f(x) is known as integrand, function F(x) as integral and C as constant of integration. Value of C is obtained by substituting initial, final or any other condition (known as boundary conditions) in the above equation.

This interpretation enables us to find integral of those functions whose derivative is known.

Integrand $f(x)=\dfrac{dF(x)}{dx}$	**Integral** $\int f(x)dx = F(x)+C$
k = Constant	kx+C
x^n	$\dfrac{x^{n+1}}{n+1}+C$ If $n \neq -1$
x^{-1}	$\ln x + C$
e^x	$e^x + C$
sin x	–cos x + C
cos x	sin x + C
f(ax+b)	$\dfrac{F(ax+b)}{a}+C$

Ex. Evaluate the following:

(i) $\int x^{-7}dx$ (ii) $\int x^{p/q}dx$

Sol. (i) $\int x^{-7}dx = \dfrac{x^{-7+1}}{-7+1}+c = -\dfrac{1}{6}x^{-6}+c$

(ii) $\int x^{\frac{p}{q}}dx = \dfrac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1}+c = \dfrac{q}{p+q}x^{(p+q)/q}+c$

Ex. Evaluate $\int\left(x^2-\cos x+\dfrac{1}{x}\right)dx$

Sol. $I = \int x^2dx - \int \cos x dx + \int \dfrac{1}{x}dx = \dfrac{x^{2+1}}{2+1} - \sin x + \log_e x + c = \dfrac{x^3}{3} - \sin x + \log_e x + c$

SPEED :

Speed is the rate of change of distance with respect to time.

Uniform speed :

An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, irrespective of duration of interval. The uniform speed is shown by straight line in distance–time graph.

For example, suppose a train travels 1000 m in 60 s. The train is said to be moving with uniform speed, if it travels 500 m. in 30s, 250 m in 15s, 125 m in 7.5s and so on.

Non Uniform speed :

An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, irrespective of duration of interval. For example, suppose a train travels first 1000 m in 60s next 1000 m in 120s and next 1000m in 50s, then the train is moving with variable speed.

Average speed :

Speed is distance travelled per unit time. Average speed of a trip $v_{av} = \dfrac{\text{Total travelled distance}}{\text{Total time taken}}$

If a particle travels a distance s in time t_1 to t_2, the average speed is $v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{s}{t_2 - t_1}$

Instantaneous speed

The speed at a particular instant is defined as instantaneous speed (or speed) while average speed is defined for a time interval.

If Δt approaches zero, average speed becomes instantaneous speed. $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

i.e. instantaneous speed is the time derivative of distance.

If a particle travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively, then total travelled distance

$$s = s_1 + s_2 + s_3 + \dots\dots\dots + s_n$$

Total time taken $t = \dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dfrac{s_3}{v_3} + \dots\dots + \dfrac{s_n}{v_n}$

No sign is needed for distance or speed.
They are always positive quantities.

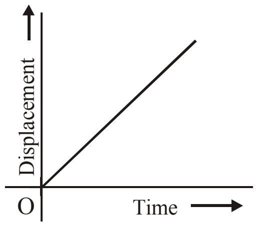
Average speed of the trip $= \dfrac{s_1 + s_2 + s_3 \dots\dots + s_n}{\left(\dfrac{s_1}{v_1} + \dfrac{s_2}{v_2} + \dots\dots + \dfrac{s_n}{v_n}\right)}$

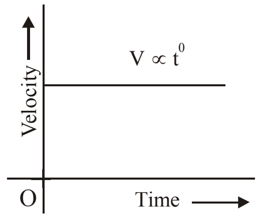
VELOCITY :

Velocity is the rate of change of displacement with respect to time.

Uniform Velocity :

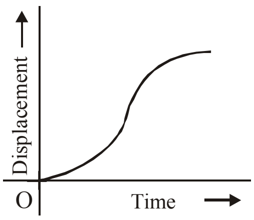
A body is said to move with uniform velocity, if it covers equal displacements in equal intervals of time, irrespective of duration of interval. When a body is moving with uniform velocity, then the magnitude and direction of the velocity of the body remains same at all points of its path.





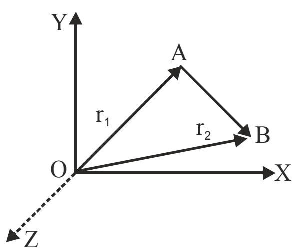
Non–uniform Velocity :

The particle is said to have non–uniform motion if it covers unequal displacements in equal intervals of time, irrespective of duration of interval. In this type of motion velocity does not remain constant.



Average Velocity

The average velocity of a particle in a time interval t_1 to t_2 is a defined as its displacement divided by the time interval. Let a particle is at a point A at time t_1 and B at time t_2. Position vectors of A and B are $\vec{r}_1$ and $\vec{r}_2$. The displacement in this time interval is the vector $\overrightarrow{AB} = (\vec{r}_2 - \vec{r}_1)$. The average velocity in this time interval is, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}}$

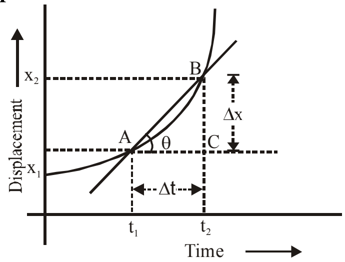


$$\vec{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

here $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$ = change in position vector.

Instantaneous velocity

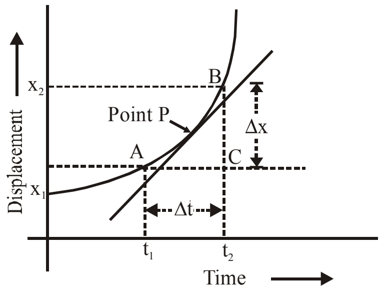
The velocity of the object at a given instant of time or at a given position during motion is called instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$



From fig., the average velocity between points A and B is

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \text{slope of secant AB} = \tan\theta$$

Average velocity is equal to slope of straight line joining two points on displacement time graph. If $\Delta t \to 0$, then average velocity becomes instantaneous velocity.



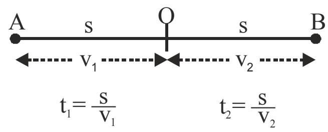
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \text{slope of tangent at P} = \tan\alpha$$

- Magnitude of instantaneous velocity is the instantaneous speed.

Note : When a particle moves with constant velocity, its magnitude of average velocity, its magnitude of instantaneous velocity and its speed all are equal.

Ex. If a particle travels the first half distance with speed v_1 and second half distance with speed v_2.Find its average speed during journey.

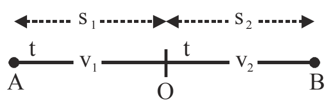
Sol. $v_{avg.} = \frac{s+s}{t_1+t_2} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$



Note :– Here $v_{avg.}$ is the harmonic mean of two speeds.

Ex. If a particle travels with speed v_1 during first half time interval and with v_2 speed during second half time interval. Find its average speed during its journey.

Sol. Total distance $= s_1 + s_2 = v_1t + v_2t = (v_1 + v_2)t$



Total time $= t + t = 2t$ $\qquad v_{avg.} = \frac{s_1+s_2}{t+t} = \frac{(v_1+v_2)t}{2t} = \frac{v_1+v_2}{2}$

Note :– here v_{avg} is arithmatic mean of two speeds.

Ex. A car travels a distance A to B at a speed of 40 km/h and returns to A at a speed of 30 km/h.

(i) What is the average speed for the whole journey?

(ii) What is the average velocity?

Sol. (i) Let AB = s, time taken to go from A to B, $t_1 = \frac{s}{40}$ h

and time taken to go from B to A, $t_2 = \frac{s}{30}$ h

$\therefore$ total time taken $= t_1 + t_2 = \frac{s}{40} + \frac{s}{30} = \frac{(3+4)\ s}{120} = \frac{7s}{120}$ h

Total distance travelled = s + s = 2s

$\therefore$ Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{2s}{\frac{7s}{120}} = \frac{120 \times 2}{7} = 34.3$ km/h.

(ii) Total displacement = zero, since the car returns to the original position.

Therefore, average velocity $= \frac{\text{total displacement}}{\text{time taken}} = \frac{0}{2t} = 0$

Ex. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. On reaching the market he instantly turns and walks back with a speed of 7.5 km/h. What is the (a) magnitude of average velocity and (b) average speed of the man, over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min (iii) 0 to 40 min.

Sol. Time taken by man to go from his home to market, $t_1 = \frac{\text{distance}}{\text{speed}} = \frac{2.5}{5} = \frac{1}{2}$ h

Time taken by man to go from market to his home, $t_2 = \frac{2.5}{7.5} = \frac{1}{3}$ h

$\therefore$ Total time taken $= t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ h $= 50$ min.

(i) 0 to 30 min

Average velocity $= \frac{\text{displacement}}{\text{time interval}} = \frac{2.5}{\frac{30}{60}} = 5$ km/h towards market

Average speed $= \frac{\text{distance}}{\text{time interval}} = \frac{2.5}{\frac{30}{60}} = 5$ km/h

(ii) **0 to 50 min**

Total displacement = zero so average velocity = 0

So, average speed = $\dfrac{5}{50/60}$ = 6 km/h

Total distance travelled = 2.5 + 2.5 = 5 km.

(iii) **0 to 40 min**

Distance moved in 30 min (from home to market) = 2.5 km.

Distance moved in 10 min (from market to home) with speed 7.5 km/h = $7.5 \times \dfrac{10}{60}$ = 1.25 km

So, displacement = 2.5 – 1.25 = 1.25 km (towards market)

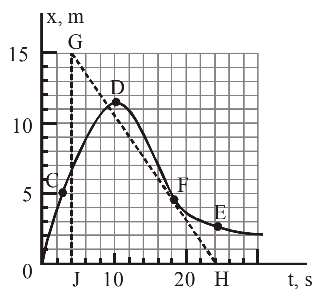
Distance travelled = 2.5 + 1.25 = 3.75 km

Average velocity $= \dfrac{1.25}{\frac{40}{60}}$ = 1.875 km/h. (towards market)

Average speed $= \dfrac{3.75}{\frac{40}{60}}$ = 5.625 km/h.

Note : Moving body with uniform speed may have variable velocity. e.g. in uniform circular motion speed is constant but velocity is non–uniform.

Ex. Refer to figure for the motion of an object along the x-axis.



What is the instantaneous velocity of the object at (a) F (b) D

Sol. **(a)** The tangent at F is the dashed line GH. Taking triangle GHJ,

$\Delta t = 24 - 4 = 20$ s $\quad \Delta x = 0 - 15 = -15$m

Hence slope at F is $\quad v_F = \dfrac{\Delta x}{\Delta t} = \dfrac{-15\text{m}}{20\ \text{s}} = -0.75$ m/s

The negative sign tells us that the object is moving in the –x direction.

(b) At point D slope of curve is zero so v = $\dfrac{dx}{dt}$ = 0.

ACCELERATION :

Acceleration

The acceleration is rate of change of velocity or change in velocity per unit time interval.

Velocity is a vector quantity hence a change in its magnitude or in direction or in both, will change the velocity .

Uniform acceleration :

An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time, irrespective of duration of intervals.

Variable acceleration :

An object is said to be moving with a variable acceleration if its velocity changes by unequal amounts in equal intervals of time, irrespective of duration of intervals..

Average Acceleration :

When an object is moving with a variable acceleration, then the average acceleration of the object for the given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken

$$\text{Average Acceleration} = \frac{\text{Total change in velocity}}{\text{total time taken}}$$

Suppose the velocity of a particle is $\vec{v}_1$ at time t_1 and $\vec{v}_2$ at time t_2. Then $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$

Instantaneous Acceleration :

The acceleration of the object at a given instant of time or at a given point of motion, is called its instantaneous acceleration. Suppose the velocity of a particle at time $t_1 = t$ is $\vec{v}_1 = \vec{v}$ and becomes

$\vec{v}_2 = \vec{v} + \Delta\vec{v}$ at time $t_2 = t + \Delta t$, Then, $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$

If Δt approaches to zero then the rate of change of velocity will be instantaneous acceleration.

$$\text{Instantaneous acceleration } \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

Ex. An athlete takes 2 second to reach the maximum speed of 18 km/h from rest. What is the magnitude of his average acceleration.

Sol. Here, Initial velocity u = 0, v = (v_{max}) = 18 km/h = $18 \times \frac{5}{18}$ = 5 m/s, $t_1 = 0$s, $t_2 = 2$s.

$$a_{av} = \frac{v - u}{t_2 - t_1} = \frac{5.0}{2} = 2.5 \text{ m/s}^2$$

Ex. A car moving with a velocity of 20 ms^{-1} is brought to rest in 5 seconds by applying brakes. Calculate the retardation of the car.

Sol. Here, $u = 20\ ms^{-1}$, $v = 0$, $t = 5s$. acceleration $a = \dfrac{v-u}{t} = \dfrac{(0-20)}{5} = -4 m/s^2$

–ve acceleration is known as retardation. Thus, retardation of the car = 4 ms^{-2}.

Use of Mathematical Tools in Solving Problems of One–Dimensional Motion

If displacement–time equation is given, we can get velocity–time equation with the help of differentiation. Again, we can get acceleration–time equation with the help of differentiation. If acceleration–time equation is given, we can get velocity–time equation by integration. From velocity equation, we can get displacement–time equation by integration.

$$\boxed{\textbf{Displacement}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\textbf{Velocity}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\textbf{Acceleration}}$$

Ex. The velocity of any particle is related with its displacement as; $x = \sqrt{v+1}$, Calculate acceleration at $x = 5$ m.

Sol. $\therefore x = \sqrt{v+1}$ $\therefore x^2 = v + 1 \Rightarrow v = (x^2-1)$

Therefore $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(x^2-1\right) = 2x\dfrac{dx}{dt} = 2x\ \ v = 2x\left(x^2-1\right)$

At x = 5 m, $a = 2 \times 5\ (25-1) = 240\ m/s^2$

Ex. The velocity of a particle moving in the positive direction of x–axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment t = 0, the particle was located at x = 0 find, (i) the time dependance of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Sol. (i) Given that $v = \alpha\sqrt{x}$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \ \therefore \frac{dx}{\sqrt{x}} = \alpha\,dt \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\,dt \quad 2\sqrt{x} = \alpha t \Rightarrow x = \left(\alpha^2 t^2/4\right)$$

Velocity $\dfrac{dx}{dt} = \dfrac{1}{2}\alpha^2 t$ and Acceleration $\dfrac{d^2x}{dt^2} = \dfrac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres $s = \dfrac{\alpha^2 t^2}{4} \Rightarrow t^2 = \dfrac{4s}{\alpha^2} \Rightarrow t = \dfrac{2\sqrt{s}}{\alpha}$;

average velocity $= \dfrac{\text{total displacement}}{\text{total time}} = \dfrac{s\alpha}{2\sqrt{s}} = \dfrac{1}{2}\sqrt{s}\alpha$

Ex. A particle moves in the plane xy with constant acceleration a directed along the negative y–axis. The equation of motion of the particle has the form $y = px - qx^2$ where p and q are positive constants. Find the velocity of the partcle at the origin of coordinates.

Sol. Given that $y = px - qx^2$

$$\therefore \frac{dy}{dt} = p\frac{dx}{dt} - q.2x\frac{dx}{dt} \text{ and } \frac{d^2y}{dt^2} = p\frac{d^2x}{dt^2} - 2qx\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2 = (p-2qx)\frac{d^2x}{dt^2} - 2q\left(\frac{dx}{dt}\right)^2$$

$$\because \frac{d^2x}{dt^2} = 0 \text{ (no acceleration along x–axis) and } \frac{d^2y}{dt^2} = -a$$

$$\therefore v_x^2 = \frac{a}{2q} \Rightarrow v_x = \sqrt{\frac{a}{2q}} \text{ Further, } \left(\frac{dy}{dt}\right)_{x=0} = p\frac{dx}{dt} \Rightarrow v_y = p\sqrt{\left(\frac{a}{2q}\right)}$$

$$\text{Now } v = \sqrt{\left(v_x^2 + v_y^2\right)} = \sqrt{\left(\frac{a}{2q} + \frac{ap^2}{2q}\right)} \Rightarrow v = \sqrt{\left[\frac{a\left(p^2+1\right)}{2q}\right]}$$

Ex. A particle moves along a straight line path such that its magnitude of velocity is given by $v = (3t^2 - 6t)\ ms^{-1}$, where t is the time in seconds. If it is initially located at the origin O then determine the magnitude of particle's average velocity and average speed in time interval from t = 0 to t = 4s.

Sol.

$$\text{Average velocity } = \frac{\int v dt}{\int dt} = \frac{\int_0^4 \left(3t^2 - 6t\right)dt}{\int_0^4 dt} = \frac{\left(t^3 - 3t^2\right)_0^4}{(t)_0^4} = 4\ ms^{-1}$$

$$\text{Average speed} = \frac{\int |v| dt}{\int dt} = \frac{\int_0^4 |3t^2 - 6t| dt}{\int_0^4 dt} = \frac{\int_0^2 \left(6t - 3t^2\right)dt + \int_2^4 \left(3t^2 - 6t\right)dt}{\int_0^4 dt}$$

$$= \frac{\left(3t^2 - t^3\right)_0^2 + \left(t^3 - 3t^2\right)_2^4}{(t)_0^4} = \frac{24}{4} = 6\ ms^{-1}$$

Ex. The coordinates a particle moving in a plane are given by x = 3 cos2t and y = 4sin2t.

(i) Find the equation of the path of the particle.

(ii) Find the angle between $\vec{r}$ and $\vec{v}$ at $t = \frac{\pi}{4}$.

(iii) Prove that acceleration of the particle is always directed towards a fixed point.

Sol. (i) Eliminating t from x = 3 cos2t & y = 4sin2t. We get $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$

(ii) $\vec{r} = x\hat{i} + y\hat{j} = 3\cos 2t\,\hat{i} + 4\sin 2t\,\hat{j} \Rightarrow \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = -6\sin 2t\,\hat{i} + 8\cos 2t\,\hat{j}$

At $t = \frac{\pi}{4}$, $\vec{r} = 4\hat{j}$, $\vec{v} = -6\hat{i}$

Angle between $\vec{r}$ and $\vec{v}$ $\cos\theta = \frac{\vec{r}.\vec{v}}{rv} = \frac{(4\hat{j}).(-6\hat{i})}{(4)(6)} = 0 \Rightarrow \theta = \frac{\pi}{2}$

(iii) $\vec{a} = \frac{d\vec{v}}{dt} = -12\cos 2t\,\hat{i} - 16\sin 2t\,\hat{j} = -4(3\cos 2t\,\hat{i} + 4\sin 2t\,\hat{j}) = -4\vec{r}$

So acceleation is always directed toward origin (a fixed point)

Ex. A particle moves in a straight line according to the relation $x = \frac{t^3}{3} - \frac{5t^2}{2} + 6t$.

Find the displacement and distance travelled by the particle upto t = 4 sec.

Sol. $v = \frac{dx}{dt} = t^2 - 5t + 6$

The particle turns when $v = 0 = t^2 - 5t + 6 \Rightarrow (t-2)(t-3) = 0$ i.e. t = 2 sec, 3 secs

Displacement = $x(4) - x(0) = \frac{64}{3} - \frac{80}{2} + 24 = \frac{16}{3}$ m

Distance = $|x(2) - x(0)| + |x(3) - x(2)| + |x(4) - x(3)|$

$= \left[\frac{8}{3} - 10 + 12\right] + \left|\left(9 - \frac{45}{2} + 18 - \frac{14}{3}\right)\right| + \left|\frac{16}{3} - \frac{9}{2}\right| = \frac{2}{3} + \frac{1}{6} + \frac{5}{6} = \frac{5}{3}$ m

Alter : distance = $\int_0^4 |v|\,dt$

Equations of motion (motion with constant acceleration)

If a particle moves with acceleration $\vec{a}$, then by definition $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}dt$. Let at starting

(t = 0) initial velocity of the particle $\vec{u}$ and at time t its final velocity = $\vec{v}$ then $\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}dt$

If acceleration is constant

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \vec{a}\int_0^t dt \Rightarrow [\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{a}[t]_0^t \Rightarrow \vec{v} - \vec{u} = \vec{a}\,t \Rightarrow \vec{v} = \vec{u} + \vec{a}t \qquad ....(1)$$

Now by definition of velocity, equation (1) reduces to

$$\vec{v} = \frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t \Rightarrow \int_0^{\vec{s}} d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)\,dt \Rightarrow \vec{s} = \left[\vec{u}t + \frac{1}{2}\vec{a}t^2\right]_0^t \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \qquad ....(2)$$

Now substituting the value of t from equation (1) to equation (2)

$$s = u\,\frac{(v-u)}{a} + \frac{1}{2}a\left[\frac{v-u}{a}\right]^2 \Rightarrow 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv \Rightarrow v^2 = u^2 + 2as \qquad ....(iii)$$

vector form of equation (iii) $\boxed{v^2 = u^2 + 2\vec{a}.\vec{s}}$ ....(3)

These three equation are called equations of motion and are applicable only and only when acceleration is constant.

Distance travalled by the body in n^{th} second

$$s_{n^{th}} = s_n - s_{n-1} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2 = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{a}{2}$$

vector form of equation (iv)

$$\boxed{s_{n^{th}} = u + \frac{a}{2}(2n-1)} \qquad ....(4)$$

Ex. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of $6.0 m/s^2$, find the distance travelled by the car after he sees the need to put the brakes on

Sol. Distance covered by the car during the application of brakes by driver –

$$s_1 = ut = \left(54 \times \frac{5}{18}\right)(0.2) = 15 \times 0.2 = 3.0 \text{ meter}$$

After applying the brakes; $v = 0$ $u = 15$ m/s, $a = 6$ m/s^2 $s_2 = ?$

Using $v^2 = u^2 - 2as \Rightarrow 0 = (15)^2 - 2 \times 6 \times s_2 \Rightarrow 12\ s_2 = 225 \Rightarrow s_2 = \frac{225}{12} = 18.75$ metre

Distance travelled by the car after driver sees the need for it $s = s_1 + s_2 = 3 + 18.75 = 21.75$ metre.

Ex. A passenger is standing d distance away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed u towards the bus. What must be the minimum speed of the passenger so that he may catch the bus?

Sol. Let the passenger catch the bus after time t.

The distance travelled by the bus, $s_1 = 0 + \frac{1}{2}at^2$ ....(1)

and the distance travelled by the passenger $s_2 = ut + 0$ ....(2)

Now the passenger will catch the bus if $d + s_1 = s_2$ ....(3)

$$\Rightarrow d + \frac{1}{2}at^2 = ut \Rightarrow \frac{1}{2}at^2 - ut + d = 0 \Rightarrow t = \frac{[u \pm \sqrt{u^2 - 2ad}]}{a}$$

So the passenger will catch the bus if t is real, i.e., $u^2 \geq 2ad \Rightarrow u \geq \sqrt{2ad}$

So the minimum speed of passenger for catching the bus is $\sqrt{2ad}$.

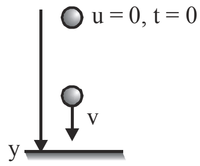
Vertical motion under gravity

If air resistance is neglected and a body is freely moving along vertical line near the earth surface then an acceleration downward which is $9.8 m/s^2$ or $980 cm/s^2$ or $32 ft/s^2$ is experienced by the body.

Freely falling bodies from a height h above the ground

Taking initial position as origin and direction of motion (i.e. downward direction) positive y axis, as body is just released/dropped $u = 0$

acceleration along +Y axis $a = g$

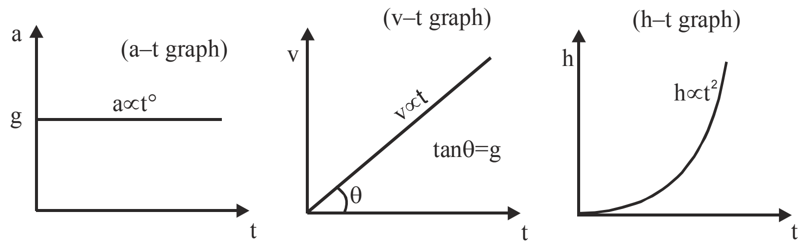


Use equations of motion to describe the motion, i.e.

$$v = u + at, \quad y = ut + \frac{1}{2}at^2, \; v^2 = u^2 + 2ay$$

Let the body acquires velocity v(downward) after falling a distance h in time t, then

$$v = gt \Rightarrow t = v/g \because h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}, \; v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

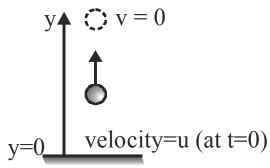


Body is projected vertically upward : With velocity u take initial position as origin and direction of motion

(i.e. vertically upward) as positive y–axis.

$v = 0$ at maximum height, at $t = T$,

$a = -g$ (because directed downward)



Put the values in equation of motion

$$v = u + at \Rightarrow 0 = u - gT \Rightarrow u = gT$$

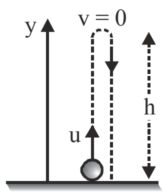
$$s = ut + \frac{1}{2}at^2 \qquad \Rightarrow h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow h_{max} = uT - \frac{1}{2}gT^2 \qquad \Rightarrow h_{max} = (gT)T - \frac{1}{2}gT^2 = \frac{1}{2}gT^2$$

$$v^2 = u^2 + 2as \qquad \Rightarrow v^2 = u^2 - 2gh \Rightarrow 0 = u^2 - 2g\,h_{max} \Rightarrow u^2 = 2g\,h_{max} \Rightarrow u = \sqrt{2g\,h_{max}}$$

After attaining maximum height body turns and come back at ground. During complete flight acceleration is constant,

Time taken during up flight and down flight are equal

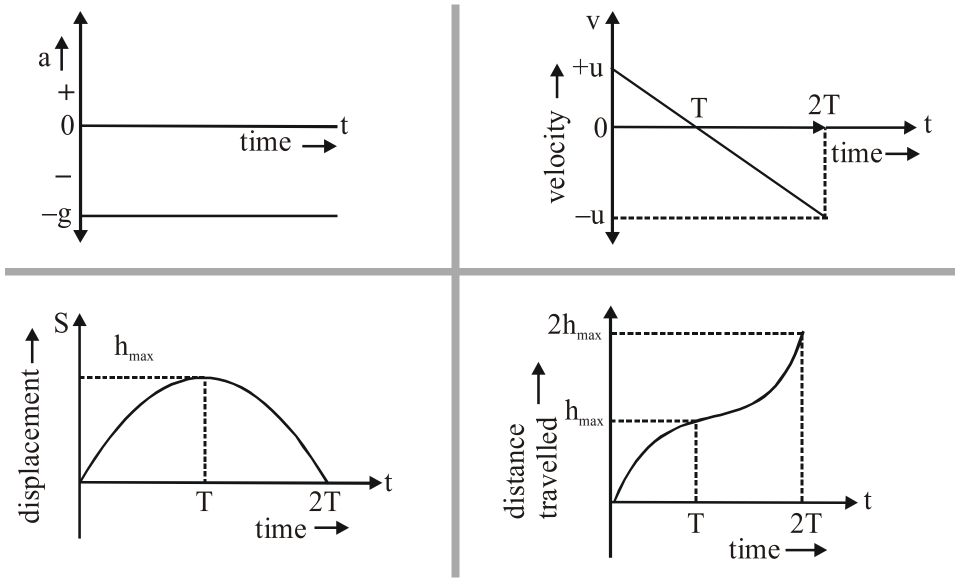


Time for one side $T = \frac{u}{g}$ and total flight time $= 2T = \frac{2u}{g}$

At each equal height from ground speed of body will be same either going up or coming down.

SOME RELATED GRAPHS FOR ABOVE MOTION'S

GRAPHS



Ex. A body is freely dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1^{st} second, in 2^{nd} second, in 3^{rd} second etc.

Sol. From second equation of motion, i.e. $h = \frac{1}{2}gt^2$ $(h = ut + \frac{1}{2}gt^2$ and $u = 0)$

$$h_1 : h_2 : h_3 \ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 = 1^2 : 2^2 : 3^2 \ldots\ldots\ldots = 1 : 4 : 9 : \ldots\ldots .$$

Now from the of distance travelled in n^{th} second

$$s_n = u + \frac{1}{2}a\,(2n - 1) \text{ here } u = 0,\ a = g \Rightarrow s_n = \frac{1}{2}g\,(2n - 1)$$

$$\Rightarrow s_1 : s_2 : s_3 \ldots\ldots\ldots = \frac{1}{2}g\,(2 \times 1 - 1) : \frac{1}{2}g\,(2 \times 2 - 1) : \frac{1}{2}g(2 \times 3 - 1) = 1 : 3 : 5 \ldots\ldots\ldots$$

Ex. A rocket is fired vertically up from the ground with a resultant vertical acceleration of $10 m/s^2$. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After finishing fuel, calculate the time for which it continues its upwards motion.

(Take $g = 10\ m/s^2$)

Sol. (a) The distance travelled by the rocket during burning interval (1minute= 60s) in which resultant acceleration 10 m/s^2 is vertically upwards will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m = 18 km

and velocity acquired by it will be $v = 0 + 10 \times 60 = 600$ m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to

gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero such that

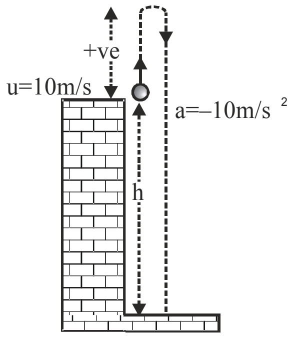
$0 = (600)^2 - 2gh_2 \Rightarrow h_2 = 18000$ m = 18 km [g = 10 ms^{-2}]

So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36$ km

(b) As after burning of fuel the initial velocity 600m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till it velocity v =0

$0 = 600 - gt \Rightarrow t = 60$ s

Ex. A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground (g = 10 m/s^2)



Sol. In the problem u = + 10 m/s, a = – 10 m/s^2 and s = –40m

(at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

$-40 = 10t - 5t^2 \Rightarrow 5t^2 - 10t - 40 = 0 \Rightarrow t^2 - 2t - 8 = 0$

Solving this we have t = 4 s and – 2s. Taking the positive value t = 4s.

Ex. The acceleration of a particle moving in a straight line varies with its displacement as, a = 2s velocity of the particle is zero at zero displacement. Find the corresponding velocity displacement equation.

Sol. $a = 2s \Rightarrow \frac{dv}{dt} = 2s \Rightarrow \frac{dv}{ds}.\frac{ds}{dt} = 2s \Rightarrow \frac{dv}{ds}.v = 2s$

$$\Rightarrow \int vdv = 2\int sds \Rightarrow \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{s^2}{2}\right)_0^s$$

$$\Rightarrow \frac{v^2}{2} = s^2 \Rightarrow v = s\sqrt{2}$$

Ex. If a body travels half its total path in the last second of its fall from rest, find :

(a) The time and

(b) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. ($g = 9.8\ m/s^2$)

Sol. If the body falls a height h in time t, then

$$h = \frac{1}{2}gt^2 \text{ [} u = 0 \text{ as the body starts from rest]} \quad \text{... (1)}$$

Now, as the distance covered in $(t - 1)$ second is $h' = \frac{1}{2}g(t-1)^2$ ... (2)

So from Equations (1) and (2) distance travelled in the last second.

$h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-1)^2$ i.e., $h - h' = \frac{1}{2}g(2t-1)$

But according to given problem as $(h - h') = \frac{h}{2}$

i.e., $\left(\frac{1}{2}\right)h = \left(\frac{1}{2}\right)g(2t-1)$ or $\left(\frac{1}{2}\right)gt^2 = g(2t-1)$ [as from equation (1) $h = \left(\frac{1}{2}\right)gt^2$]

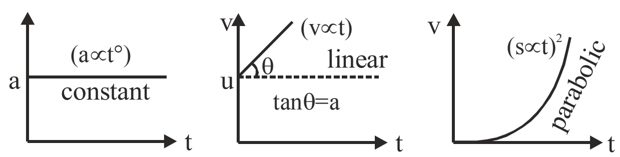
or $t^2 - 4t + 2 = 0$ or $t = [4 \pm \sqrt{(4^2 - 4\times 2)}]/2$ or $t = 2 \pm \sqrt{2}$ $\Rightarrow t = 0.59$ s or 3.41 s

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1s.

so $t = 3.41$s and $h = \frac{1}{2} \times (9.8) \times (3.41)^2 = 57$ m

Graphs based on 1–D

For constant acceleration, a/t, v/t and s/t curve from equations of motion are –



In case of constant acceleration motion in a straight line, scalar form of equations of motion can be applied and problem becomes fairly simple.

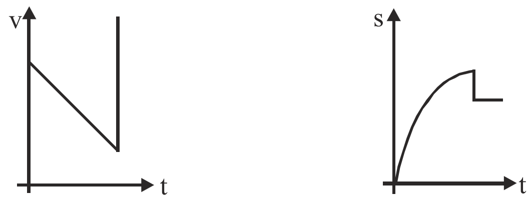
As $d\vec{v} = \vec{a}dt$ or $[\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{v} - \vec{u} = \int_{t_1}^{t_2} \vec{a}dt$ = Area between curve and time axis from t_1 to t_2.

Area under the curve of a – t graph always gives the change in velocity.

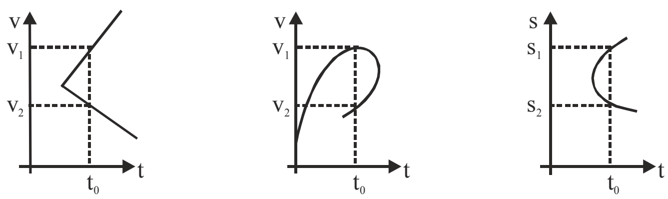
Similarly $d\vec{s} = \int \vec{v}dt$ or $\vec{s} = \int_{t_1}^{t_2} \vec{v}dt$ = Area between curve and time axis from t_1 to t_2.

Here $\vec{s}$ is the displacement of particle in time interval t_1 to t_2, i.e. area under the curve of v/t graph always gives the displacement. If only magnitude of area is taken into account then sum of all area is the total distance travelled by the particle.

- Slopes of v–t or s–t graphs can never be infinite at any point, because infinite slope of v–t graph means infinite acceleration. Similarly, infinite slope of s–t graph means infinite velocity. Hence, the following graphs are not possible.



- At one time, two values of velocity or displacement are not possible Hence, the following graphs are not acceptable.



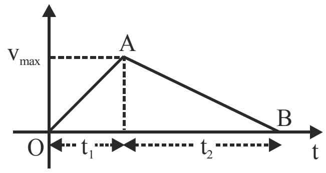
- The slope of velocity–time graph of uniform motion is zero.
- When a body is having uniform motion along a straight line in a given direction, the magnitude of the displacement of body is equal to the actual distance travelled by the body in the given time.
- The average and instantaneous velocity in a uniform motion are equal in magnitude.
- In a uniform motion along a straight line, the slope of position–time graph gives the velocity of the body.
- The position–time graph of a body moving along a straight line can never be a straight line parallel to position axis because it will indicate infinite velocity.
- The speed of a body can never be negative
- Medium effects the motion of a body falling freely under gravity due to thrust and viscous drag.

Ex. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t evaluate (a) the maximum velocity attained and (b) the total distance travelled.

Sol. (a) Let the car accelerates for time t_1 and decelerates for time t_2 then

$t = t_1 + t_2$ ......(i)

and corresponding velocity–time graph will be as shown in. fig.



From the graph α = slope of line AB $= \dfrac{v_{max}}{t_1} \Rightarrow t_1 = \dfrac{v_{max}}{\alpha}$

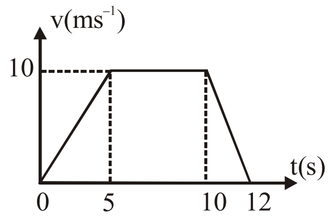
and β = – slope of line OB $= \dfrac{v_{max}}{t_2} \Rightarrow t_2 = \dfrac{v_{max}}{\beta}$

$$\Rightarrow \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t \Rightarrow v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t \Rightarrow v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(b) Total distance = area under v–t graph $= \dfrac{1}{2} \times t \times v_{max} = \dfrac{1}{2} \times t \times \dfrac{\alpha\beta t}{\alpha+\beta} = \dfrac{1}{2}\left(\dfrac{\alpha\beta t^2}{\alpha+\beta}\right)$

Note: This problem can also be solved by using equations of motion ($v = u + at$, etc.).

Ex. Draw displacement time and acceleration – time graph for the given velocity–time graph



Sol. For $0 \le t \le 5$ $v \propto t \Rightarrow s \propto t^2$ and a_1 = constant $\dfrac{10}{5} = 2\ ms^{-2}$

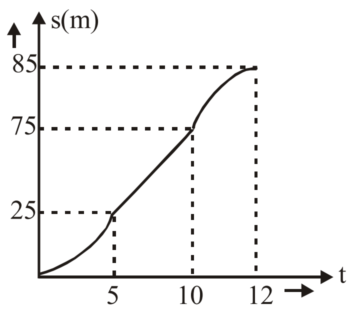
for whole interval s_1 = Area under the curve $= \dfrac{1}{2} \times 5 \times 10 = 25$ m

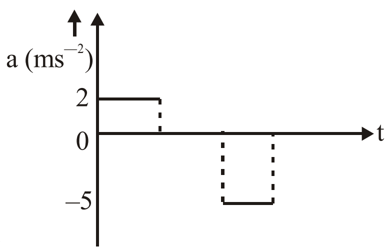
For $5 \le t \le 10$ $v = 10ms^{-1}$ $\Rightarrow a = 0$

for whole interval s_2 = Area under the curve $= \dfrac{1}{2} \times 5 \times 10 = 50$ m

For $10 \le t \le 12$ v linearly decreases with time $\Rightarrow a_3 = -\dfrac{10}{2} = -5\ ms^{-1}$

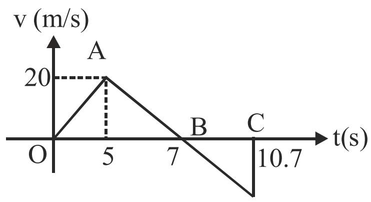
for whole interval s_3= Area under the curve $= \dfrac{1}{2} \times 2 \times 10 = 10$ m

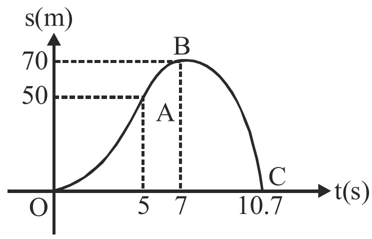




Ex. A rocket is fired upwards vertically with a net acceleration of 4 m/s^2 and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity–time and displacement–time graphs for the complete journey. Take g = 10 m/s^2.

Sol.





In the graphs, $v_A = at_{OA} = (4)(5) = 20$ m/s $v_B = 0 = v_A - gt_{AB}$

$\therefore t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2s$ $\therefore t_{OAB} = (5 + 2)s = 7s$

Now, s_{OAB} = area under v–t graph between 0 to 7 s = $\frac{1}{2}(7)(20) = 70$ m

Now, $s_{OAB} = s_{BC} = \frac{1}{2}gt^2_{BC}$ $\therefore 70 = \frac{1}{2}(10)\,t^2_{BC}$

$\therefore \quad t_{BC} = \sqrt{14} = 3.7s$ $\therefore t_{OAB} = 7 + 3.7 = 10.7s$

Also s_{OA} = area under v–t graph between OA = $\frac{1}{2}(5)(20) = 50$ m

Ex. At the height of 500m, a particle A is thrown up with $v = 75\ ms^{-1}$ and particle B is released from rest. Draw, accelearation –time, velocity–time, speed–time and displacement–time graph of each particle.

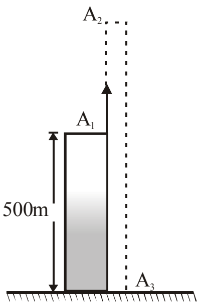
For particle A :

Time of flight

$$-500 = +75\,t - \frac{1}{2} \times 10t^2$$

$\Rightarrow t^2 - 15t - 100 = 0$

$\Rightarrow t = 20$ s



Time taken for A_1A_2

$= 75 - 10t \Rightarrow t = 7.5$ s

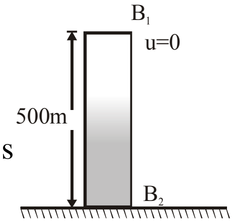
Velocity at A_3, $v = 75 - 10 \times 20 = -125\ ms^{-1}$

$$\text{Height } A_2A_1 = \frac{1}{2}(10)(7.5)^2 = 281.25 \text{ m}$$

For Particle B

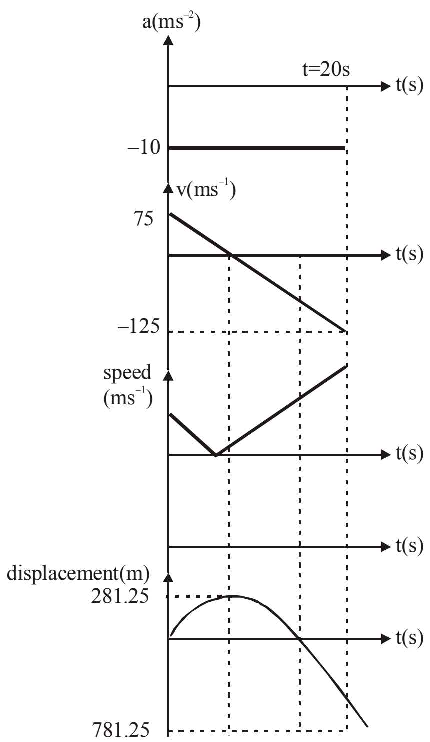
Time of flight

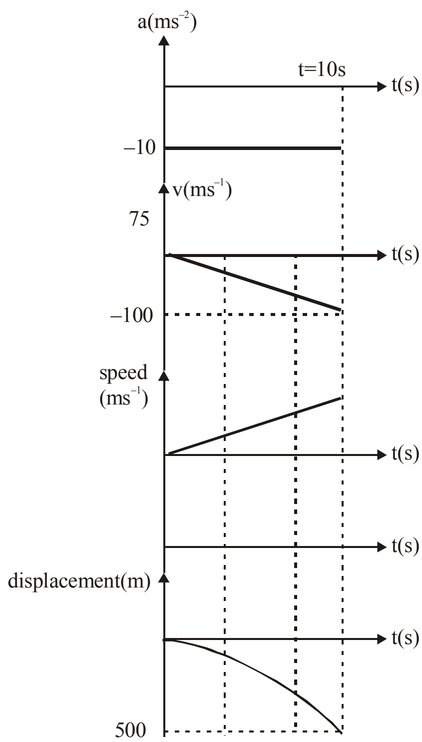
$$500 = \frac{1}{2}(10)t^2 \Rightarrow t = 10 \text{ s}$$

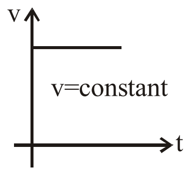
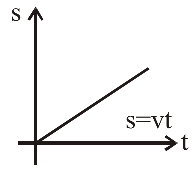
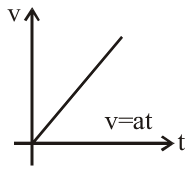
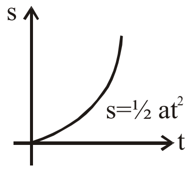
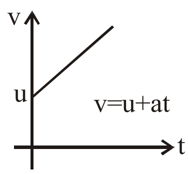
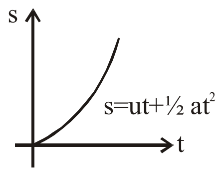
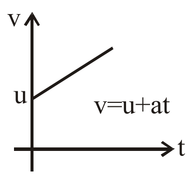
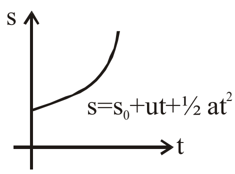
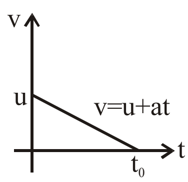
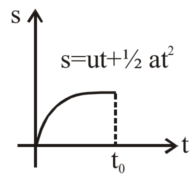
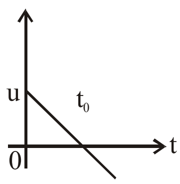
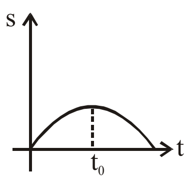


Velocity at B_2

$v = 0 - (10)(10) = -100\ ms^{-1}$



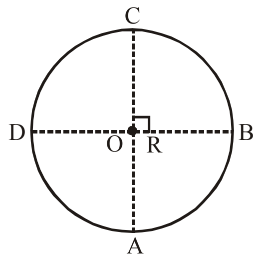


S.N.	Different Cases	v–t graph	s–t graph	Important Points
1.	Uniform motion			(i) Slope of s–t graph = v = constant (ii) In s–t graph s = 0 at t = 0
2.	Uniformly accelerated motion with u = 0 at t = 0			(i) u = 0, i.e. v = 0 at t = 0 (ii) u = 0, i.e., slope of s–t graph at t = u, should be zero (iii) a or slope of v – t graph is constant
3.	Uniformly accelerated motion with $u \neq 0$ at t = 0			(i) $u \neq 0$, i.e., v or slope of s – t graph at t = 0 is not zero (ii) v or slope of s – t graph gradually goes on increasing.
4.	Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at t = 0			(i) $s = s_0$ at t = 0
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s – t graph at t = 0 gives u (ii) Slope of s – t graph at $t = t_0$ becomes zero (iii) In this case u can't be zero.
6.	Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, v = 0 or slope of s – t graph is zero (ii) In s – t graph slope or velocity first decreases then increases with opposite sign.

EXERCISE (S-1)

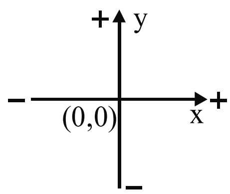
Definitions of kinematics variables

1. A particles starts from point A with constant speed v on a circle of radius R. Find magnitude of average velocity during its journey from :-



(a) A to B (b) A to C (c) A to D

2. A particle is moving along x-axis. Initially it is located 5 m left of origin and it is moving away from the origin and slowing down. In this coordinate system, what are the signs of the initial velocity and acceleration.



v_0	a

Motion with constant acceleration

3. A car accelerates with uniform rate from rest on a straight road. The distance travelled in the last second of a three second interval from the start is 15 m then find the distance travelled in first second in m.

4. A particle moving in one-dimension with constant acceleration of 10 m/s^2 is observed to cover a distance of 100 m during a 4s interval. How far will the particle move in the next 4s?

5. A particle starts from rest at t = 0 and x = 0 to move with a constant acceleration = +2 m/s^2, for 20 seconds. After that, it moves with –4 m/s^2 for the next 20 seconds. Finally, it moves with positive acceleration for 10 seconds until its velocity becomes zero.
(a) What is the value of the acceleration in the last phase of motion?
(b) What is the final x-coordinate of the particle?
(c) Find the total distance covered by the particle during the whole motion.

6. A body moving with uniform acceleration has a velocity of –11 cm/s when its x coordinate is 3.00 cm. If its x coordinate 2 s later is – 5 cm, what is the magnitude in cm/s^2 of its acceleration?

7. A driver travelling at speed 36 kmh^{-1} sees the light turn red at the intersection. If his reaction time is 0.6s, and then the car can deaccelerate at 4ms^{-2}. Find the stopping distance of the car.

8. The window of the fourth floor of SANKALP building is 5 m high. A man looking out of the window sees an object moving up and down the height of window for 2 sec. Find the height that the object reaches from the top end of the window.

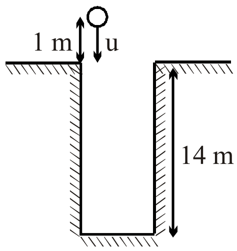
9. A body is dropped from a height of 300 m. Exactly at the same instant another body is projected from the ground level vertically up with a velocity of 150 ms^{-1}. Find when they will meet.

10. A stone is dropped from the top of a tall cliff, and 1s later a second stone is thrown vertically downward with a velocity of 20 ms^{-1}. How far below the top of the cliff will the second stone overtake the first?

11. Speed of train is increasing linearly with time. The train passes a hut with speed 2 m/s and acquires a speed of 12 m/s after 10 s. What is the speed of the train in m/s, 5 s after passing the hut?

12. Two particle A and B are moving in same direction on same straight line. A is ahead of B by 20m. A has constant speed 5 m/sec and B has initial speed 30 m/sec and retardation of 10 m/sec^2. Then if x (in m) is total distance travelled by B as it meets A for second time. Then value of x will be.

13. A boy throws a ball with speed u in a well of depth 14 m as shown. On bounce with bottom of the well the speed of the ball gets halved. What should be the minimum value of u (in m/s) such that the ball may be able to reach his hand again? It is given that his hands are at 1 m height from top of the well while throwing and catching.



14. From the top of a tower, a ball is thrown vertically upwards. When the ball reaches h below the tower, its speed is double of what it was at height h above the tower. Find the greatest height attained by the ball from the tower.

15. A rocket is fired vertically upwards with initial velocity 40 m/s at the ground level. Its engines then fired and it is accelerated at 2 m/s^2 until it reaches an altitude of 1000 m. At that point the engines shut off and the rocket goes into free-fall. If the velocity (in m/s) just before it collides with the ground is 40α. Then fill the value of α. Disregard air resistance ($g = 10 m/s^2$).

16. A balloon rises from rest on the ground with constant acceleration $\frac{g}{3}$. A stone is dropped when the balloon has rises to a height 60 metre. The time taken by the stone to reach the ground is.

Motion with variable acceleration & calculus

17. The position x of a particle w.r.t. time t along x–axis is given by $x = 9t^2 - t^3$ where x is in metre and t in second. Find

(a) Maximum speed along +x direction

(b) Position of turning point

(c) Displacement in first ten seconds

(d) Distance travelled in first ten seconds

18. The momentum of a particle moving in straight line is given by $p = \ln t + \frac{1}{t}$ (in kg m/s) find the time $t > 0$ at which the net force acting on particle is 0 and it's momentum at that time. [Hint : $F = \frac{dp}{dt}$]

19. The velocity of the particle is given as $v = 3t^3 + t - \frac{1}{t^2}$. Calculate the net force acting on the body at time t = 2 sec, if the mass of the body is 5 kg.

20. A wheel rotates so that the angle of rotation is proportional to the square of time. The first revolution was performed by the wheel for 8 sec. Find the angular velocity ω, 32 sec after the wheel started. [**Hint:** Consider $\theta = kt^2$, find k]

21. The charge flowing through a conductor beginning with time t = 0 is given by the formula $q = 2t^2 + 3t + 1$ (coulombs). Find the current $i = \frac{dq}{dt}$ at the end of the 5th second.

22. The angle θ through which a pulley turns with time t is specified by the function $\theta = t^2 + 3t - 5$. Find the angular velocity $\omega = \frac{d\theta}{dt}$ at t = 5 sec.

23. The motion of a particle in a straight line is defined by the relation $x = t^4 - 12t^2 - 40$ where *x* is in meters and t is in sec. Determine the position x, velocity v and acceleration a of the particle at t = 2sec.

24. A point moves in a straight line so that its distance from the start in time t is equal to

$$s = \frac{1}{4}t^4 - 4t^3 + 16t^2.$$

(a) At what times was the point at its starting position?

(b) At what times is its velocity equal to zero?

25. A body whose mass is 3 kg performs rectilinear motion according to the formula $s = 1 + t + t^2$, where *s* is measured in centimetres & t in seconds. Determine the kinetic energy $\frac{1}{2}mv^2$ of the body in 5 sec after its start.

26. A force of 40N is responsible for the motion of a body governed by the equation $s = 2t + 2t^2$ where s is in meters and t in sec. What is the momentum of the body at t = 2 sec?
[**Hint**: Find acc. then m = F/a & p = mv]

27. The angle rotated by a disc is given by $\theta = \frac{2}{3}t^3 - \frac{25}{2}t^2 + 77t + 5$, where θ is in rad and t in seconds.
(a) Find the times at which the angular velocity of the disc is zero.
(b) Its angular acceleration at these times.

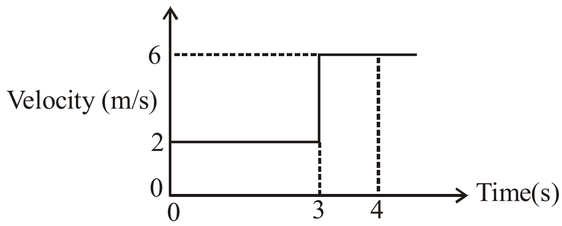
28. The acceleration of a particle starting from rest vary with respect to time is given by a = (2t – 6), where t is in seconds. Find the time (in seconds) at which velocity of particle in negative direction is maximum.

29. Acceleration of a particle is defined as $a = \left(75V^2 - 30V + 3\right)$ (m/s^2). If the constant speed achieved by the particle is given by V_C, then find the value of $10V_C$.

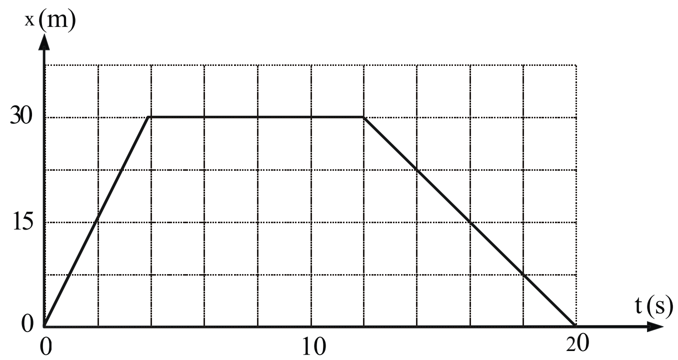
30. Position vector of a particle is given by $\vec{r} = 3t^3\hat{i} + 4t\hat{j} + t^2\hat{k}$. Find avg acceleration of particle from t = 1 to t = 2sec.

Question based on graph

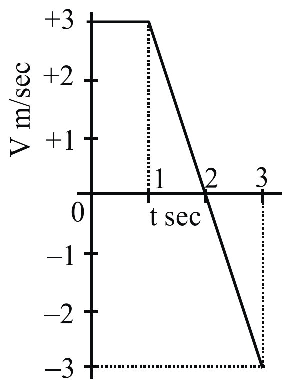
31. In the following graph variation with time (t), in velocity (v) of a particle moving rectilinearly is shown. What is average velocity in m/s of the particle in time interval from 0 s to 4 s?



32. The graph illustrates motion of a bucket being lowered into a well from the top at the instant t = 0, down to the water level, filled with water and drawn up again. Here 'x' is the depth. Find the average speed of the bucket in m/s during whole operation.



33. A particle moves along a straight line, x. At time t = 0, its position is at x = 0. The velocity, V, of the object changes as a function of time t, as indicated in the figure; t is in seconds, V in m/sec and x in meters.



(a) What is x at t = 3 sec?

(b) What is the instantaneous acceleration (in m/sec^2) at t = 2 sec?

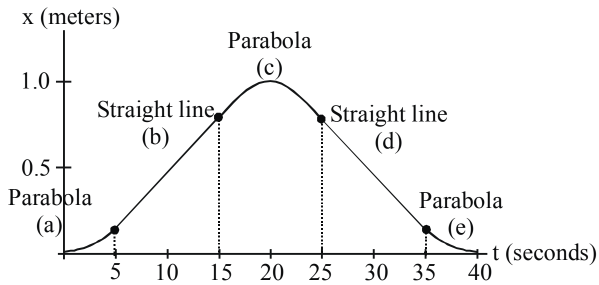
(c) What is the average velocity (in m/sec) between t = 0 and t = 3 sec?

(d) What is the average speed (in m/sec) between t = 1 and t = 3 sec?

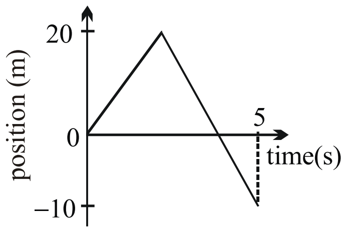
34. The figure below is a displacement vs time plot for the motion of an object, answer questions (i) & (ii) with the letter of appropriate section of the graph.

(i) Which section represents motion in the forward direction with positive acceleration?

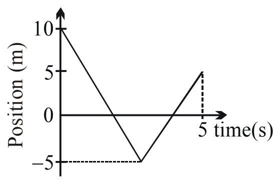
(ii) Which section represents uniform motion backwards (–x direction)?



35. **(a)** The diagram shows the position-time graph for a particle moving in a straight line. Find the average velocity for the interval from t = 0 to t = 5.

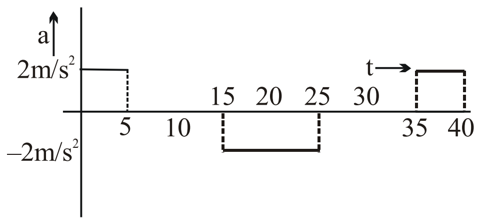


(b) The diagram shows the position-time graph for a particle moving in a straight line. Find the average speed for the interval from t = 0 to t = 5.



36. Figure shows a graph of acceleration of a particle moving on the x-axis. Plot the following graphs if the particle is at origin and at rest at t = 0.

(i) velocity-time graph (ii) displacement-time graph (iii) distance-time graph.

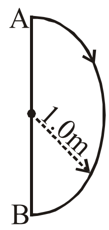


EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Definitions of kinematics variables

1. In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is : **[JEE '99]**



(A) 3.14 m/sec (B) 2.0 m/sec (C) 1.0 m/sec (D) zero

2. An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?

(A) On the way up $a_y > 0$, on the way down $a_y > 0$

(B) On the way up $a_y < 0$, on the way down $a_y > 0$

(C) On the way up $a_y > 0$, on the way down $a_y < 0$

(D) On the way up $a_y < 0$, on the way down $a_y < 0$

Motion with constant acceleration

3. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as :-

(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

4. If a body starts from rest and travels 120 cm in the 6th second, with constant acceleration then what is the acceleration :

(A) 0.20 m/s^2 (B) 0.027 m/s^2 (C) 0.218 m/s^2 (D) 0.03 m/s^2

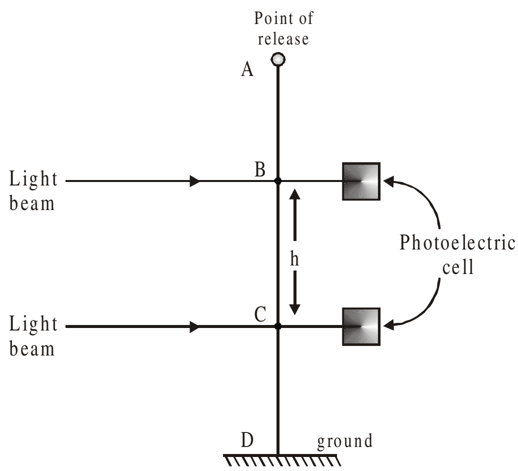
5. A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec.

(A) 8.3 m (B) 9.3 m (C) 10.3 m (D) None of above

6. The engine of a motorcycle can produce a maximum acceleration 5 m/s^2. Its brakes can produce a maximum retardation 10 m/s^2. If motorcyclist start from point A and reach at point B. What is the minimum time in which it can cover if distance between A and B is 1.5 km.
(Given : that motorcycle comes to rest at B)

(A) 30 sec (B) 15 sec (C) 10 sec (D) 5 sec

7. The acceleration of free fall at a planet is determined by timing the fall of a steel ball photo –electrically. The ball passes B and C at times t_1 and t_2 after release from A. The acceleration of free fall is given by



(A) $\frac{2h}{t_2 - t_1}$ (B) $\frac{h}{t_2^2 - t_1^2}$ (C) $\frac{2h}{t_2^2 - t_1^2}$ (D) $\frac{2h}{t_2^2 + t_1^2}$

8. A particle has an initial velocity of 9 m/s due east and a constant acceleration of 2 m/s^2 due west. The distance covered by the particle in the fifth second of its motion is :-

(A) 0 (B) 0.5 m (C) 2 m (D) none of these

9. A physics teacher finds a scrap of paper on which one of his students has written the following equation: $0^2 - 5^2 = 2 \times (-9.8) \times x$; of which of the following problem would this equation be part of the correct solution?

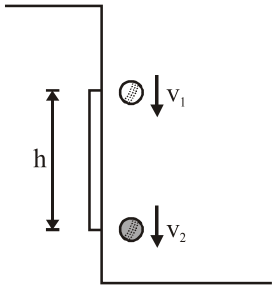
(A) Find the speed of an object 5 seconds after it was dropped from rest.

(B) Find the distance of an object has fallen 5 seconds after it was released from rest on Earth.

(C) Find the height from which a ball when released will strike the ground with a speed of 5 m/s.

(D) Find the maximum height to which a ball will rise if it is thrown upward with an initial speed of 5 m/s.

10. A ball dropped from the top of a building passes past a window of height h in time t. If its speeds at the top and the bottom edges of the window are denoted by v_1 and v_2 respectively, which of the following set of equations are correct?



(A) $v_2 - v_1 = gt$ and $(v_2 - v_1)t = h$ (B) $v_2 - v_1 = gt$ and $(v_2 + v_1)t = 2h$
(C) $v_2 + v_1 = gt$ and $(v_2 - v_1)t = h$ (D) None of the above.

11. A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of :

(A) 3 s (B) 5 s (C) 7 s (D) 9 s

12. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec taking origin at the point of projection, vertical up as positive y-axis and horizontal as x-axis:-

(A) (0, 25) (B) (0, 20) (C) (0, 45) (D) (0, 5)

13. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is [g = 10 m/s^2]

(A) 1 : 1 (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{2}}$

14. A body of mass 'm' is travelling with a velocity 'u'. When a constant retarding force 'F' is applied, it comes to rest after travelling a distance 's_1'. If the initial velocity is '2u', with the same force 'F', the distance travelled before it comes to rest is 's_2'. Then

(A) $s_2 = 2s_1$ (B) $s_2 = \frac{s_1}{2}$ (C) $s_2 = s_1$ (D) $s_2 = 4\,s_1$

15. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec, taking origin at 45 m above the point of projection, vertical up as positive y-axis and horizontal as x-axis :-

(A) (0, –25) (B) (0, –20) (C) (0, –45) (D) (0, –5)

Motion with variable acceleration & calculus

16. If $s = 2t^3 + 3t^2 + 2t + 8$ then the time at which acceleration is zero, is :-

(A) $t = \frac{1}{2}$ (B) $t = 2$ (C) $t = \frac{1}{2\sqrt{2}}$ (D) Never

17. Velocity of a particle varies with time as $v = 4t$. The displacement of particle between $t = 2$ to $t = 4$ sec, is :-

(A) 12 m (B) 36 m (C) 24 m (D) 6 m

18. A point mass moves with velocity $v = (5t - t^2)\ ms^{-1}$ in a straight line. Find the distance travelled (i.e. $\int v dt$) in fourth second.

(A) $\frac{31}{6}$ m (B) $\frac{29}{6}$ m (C) $\frac{37}{6}$ m (D) None of these

19. A particle is projected with velocity v_0 along x–axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a = -\alpha x^2$. The distance at which the particle stops is:–

(A) $\sqrt{\frac{3v_0}{2\alpha}}$ (B) $\left(\frac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$ (C) $\sqrt{\frac{3v_0^2}{2\alpha}}$ (D) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

20. The acceleration vector along x–axis of a particle having initial speed v_0 changes with distance as $a = \sqrt{x}$. The distance covered by the particle, when its speed becomes twice that of initial speed is:-

(A) $\left(\frac{9}{4}v_0\right)^{\frac{4}{3}}$ (B) $\left(\frac{3}{2}v_0\right)^{\frac{4}{3}}$ (C) $\left(\frac{2}{3}v_0\right)^{\frac{4}{3}}$ (D) $2v_0$

21. For a particle moving in a straight line the position of the particle at time (t) is given by $x = \frac{t^3}{6} - t^2 - 9t + 18$ m. What is the velocity of the particle when its acceleration is zero :-

(A) 18 m/s (B) –9 m/s (C) –11 m/s (D) 6 m/s

22. A particle moves along a straight line such that at time t its displacement from a fixed point O on the line is $3t^2 - 2$. The velocity of the particle when $t = 2$ is:

(A) 8 ms^{-1} (B) 4 ms^{-1} (C) 12 ms^{-1} (D) 0

23. Temperature of a body varies with time as $T = (T_0 + \alpha t^2 + \beta \sin t)K$, where T_0 is the temperature in Kelvin at $t = 0$ sec. & $\alpha = 2/\pi$. K/s^2 & $\beta = -4$ K, then rate of change of temperature at $t = \pi$sec. is

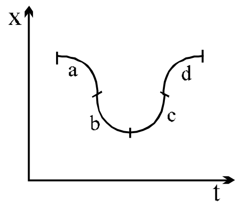
(A) 8 K (B) 8^0K (C) 8K/sec (D) 8^0K/sec

24. The velocity of a particle moving on the x-axis is given by $v = x^2 + x$ where v is in m/s and x is in m. Find its acceleration in m/s^2 when passing through the point x = 2m

(A) 0 (B) 5 (C) 11 (D) 30

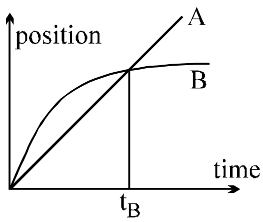
Question based on graph

25. The graph shown is a plot of position versus time. For which labeled region is the velocity positive and the acceleration negative?



(A) a (B) b (C) c (D) d

26. The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



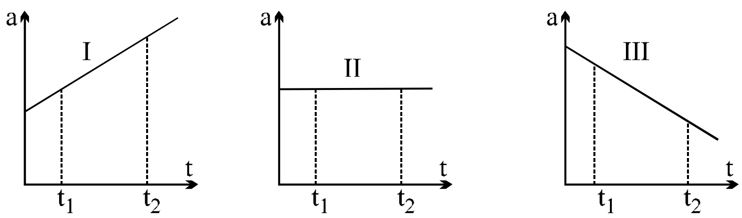
(A) At time t_B, both trains have the same velocity.

(B) Both trains have the same velocity at some time after t_B.

(C) Both trains have the same velocity at some time before t_B.

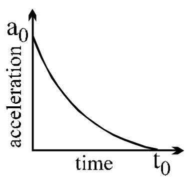
(D) Somewhere on the graph, both trains have the same acceleration.

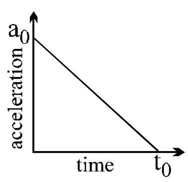
27. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?

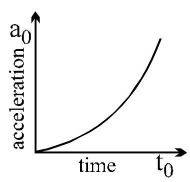


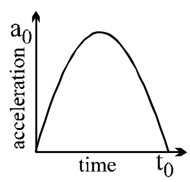
(A) graph I, only
(B) graphs I and II, only
(C) graphs I and III, only
(D) graphs I, II, and III

28. Acceleration versus time graphs for four objects are shown below. All axes have the same scale. Which object had the greatest change in velocity during the interval?

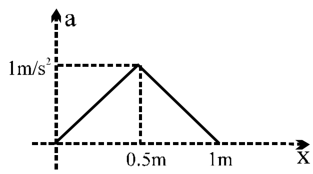
(A) 

(B) 

(C) 

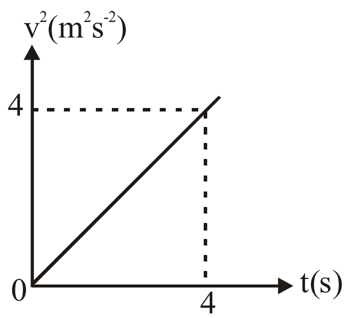
(D) 

29. A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is :-



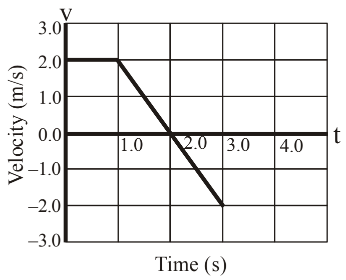
(A) 1 m/s
(B) 6 m/s
(C) 2 m/s
(D) none

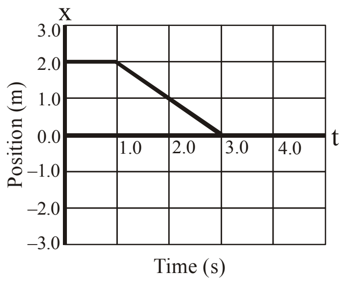
30. A particle is moving along a straight line such that square of its velocity varies with time as shown in the figure. What is the acceleration of the particle at t = 4 s?

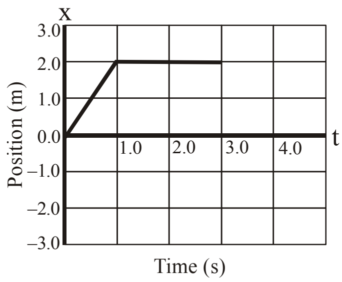


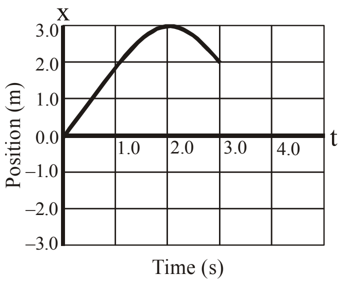
(A) 4 m/s^2
(B) 1/4 m/s^2
(C) 1/2 m/s^2
(D) 0

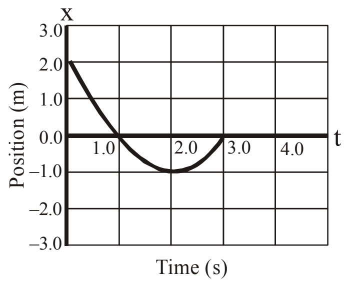
31. The graph below shows the velocity of a particle moving in a straight line. At t = 0, the particle is located at x = 0. Which of the following graphs shows the position of the particle with respect to time, x(t) ?



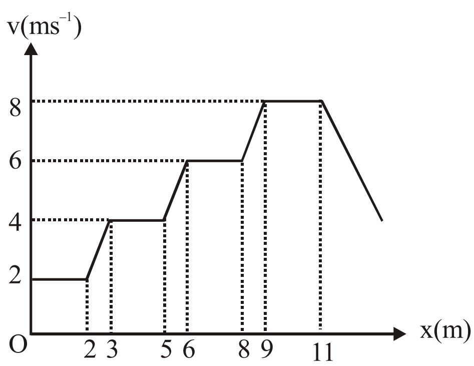
(A) 

(B) 

(C) 

(D) 

32. The velocity of a particle that moves in the positive x-direction varies with its position as shown in figure. The acceleration of the particle when x = 5.5 m is-



(A) 0 (B) 5 ms^{-2} (C) 10 ms^{-2} (D) 20 ms^{-2}

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 33 and 34

A particle is moving in a straight line along positive y-axis. Its displacement from origin at any time t is given by $y = 5t^2 - 10t + 5$ where y is in meters and t is in seconds.

33. The velocity at t = 2s will be :

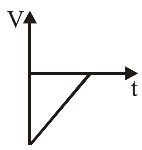
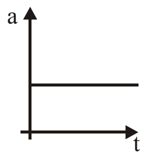
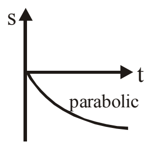
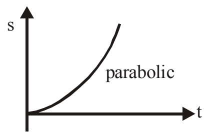
(A) 20 ms^{-1} (B) 10 ms^{-1} (C) 5 ms^{-1} (D) 15 ms^{-1}

34. Displacement of particle when its velocity is zero, is

(A) 2.5 m (B) 1.25 m (C) 5m (D) 0m

MATRIX MATCH TYPE QUESTION

35. v, a, s and t denote velocity, acceleration, displacement and time respectively. Match the columns :-

Column-I	Column-II
(A) 	(P) Velocity of the particle is in positive direction, acceleration in negative direction
(B) 	(Q) Both velocity and acceleration of the particle are in negative directions.
(C) 	(R) Velocity of the particle is in negative direction and acceleration in positive direction
(D) 	(S) Velocity and acceleration both in positive direction
	(T) Acceleration is constant

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

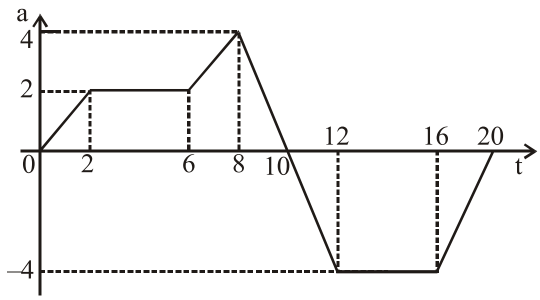
1. A parachutist jumps out of an airplane and accelerates with gravity for 6 seconds. He then pulls the parachute cord and after a 4 s deceleration period, descends at 10 m/s for 60 seconds, reaching the ground. From what height did the parachutist jump? Assume acceleration due to gravity to be 10 m/s^2 throughout the motion.

(A) 840 m (B) 920 m (C) 980 m (D) 1020 m

2. A train moving with a speed of 60 km/hr is slowed down uniformly to 30 km/hr for repair purposes during running. After this it was accelerated uniformly to reach to its original speed. If the distance covered during constant retardation be 2 km and that covered during constant acceleration be 1 km, find the time lost in the above journey

(A) 1 min (B) 2 min (C) 4 min (D) 5 min

3. If initial velocity of particle is 2 m/s, the maximum velocity of particle from t = 0 to t = 20 sec is :



(A) 20 m/s (B) 18 m/s (C) 22 m/s (D) 24 m/s

ASSERTION & REASON

These questions contains, Statement-1 (assertion) and Statement-2 (reason).

(A) Statement-1 is true, Statement-2 is true ; Statement-2 is correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true ; Statement-2 is NOT a correct explanation for statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

(E) Both Statement-1 and Statement-2 are false.

4. **Statement I :** When velocity of a particle is zero then acceleration of particle is zero.

and

Statement II : Acceleration is equal to rate of change of velocity.

5. **Statement-I :** A particle moves in a straight line with constant acceleration. The average velocity of this particle cannot be zero in any time interval.

and

Statement-II : For a particle moving in straight line with constant acceleration, the average velocity in a time interval is $\frac{u+v}{2}$, where u and v are initial and final velocity of the particle of the given time interval.

6. A particle moves in a straight line, according to the law $x = 4a\,[t + a\sin\left(\frac{t}{a}\right)]$, where x is its position in meters, t in sec. & a is some constants, then the velocity is zero at :-

(A) $x = 4a^2\pi$ meters (B) $t = \pi$ sec. (C) $t = 0$ sec (D) none

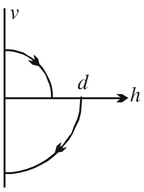
7. A particle moving on the x-axis with constant acceleration has displacements of 6 m from t = 4 s to t = 7 s and 3 m from t = 5 s to t = 8 s. The distance covered from t = 6 s to t = 9 s is

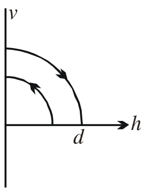
(A) 1.75 m (B) 2.25 m (C) 3.0 m (D) 0

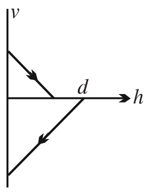
8. A point moves in a straight line so that its displacement is x m at time t sec, given by $x^2 = t^2 + 1$. Its acceleration in m/s^2 at time t sec is :

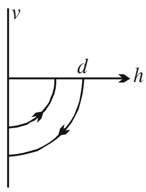
(A) $\frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{x^2}$ (C) $-\frac{t}{x^2}$ (D) $\frac{1}{x^3}$

9. A ball is dropped vertically from a height *d* above the ground, hits the ground and bounces up vertically to a height *d*/2. Neglecting subsequent motion and air resistances, its velocity *v* varies with the height *h* above the ground as :- **[IIT-JEE'2000 (Scr)]**

(A) 

(B) 

(C) 

(D) 

MULTIPLE CORRECT TYPE QUESTIONS

10. A particle moving along a straight line with uniform acceleration has velocities 7m/s at A and 17 m/s at C. B is the mid point of AC. Then

(A) The velocity at B is 12 m/s.

(B) The average velocity between A and B is 10 m/s.

(C) The ratio of the time to go from A to B to that from B to C is 3 : 2.

(D) The average velocity between B and C is 15 m/s.

11. A particle moves along the X–axis as $x = u(t - 2s) + a(t - 2s)^2$

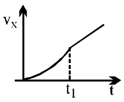
(A) The initial velocity of the particle is u
(B) The acceleration of the particle is a
(C) The acceleration of the particle is 2a
(D) At t = 2s particle is at the origin.

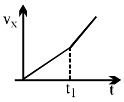
12. The position of a particle with time is given by

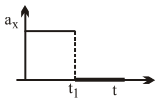
$$(x, y) = (8t^2, 3) \text{ for } t \le t_1$$
$$= (8t\,t_1, 3) \text{ for } t > t_1$$

Choose the **CORRECT** alternative.

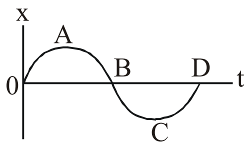
(A) Particle moves along a straight line parallel to x axis.

(B) 

(C) 

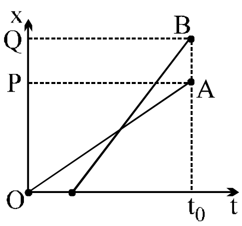
(D) 

13. A particle has a rectilinear motion and the figure gives its displacement as a function of time. Which of the following statements are true with respect to the motion



(A) in the motion between O and A the velocity is positive and acceleration is negative
(B) between A and B the velocity and acceleration are positive
(C) between B and C the velocity is negative and acceleration is positive
(D) between C and D the acceleration is positive

14. The position–time (x–t) graphs for two children A and B returning from their school O to their homes P and Q respectively along straight line path (taken as x axis) are shown in figure below. Choose the **CORRECT** statement (s):



(A) A lives closer to the school than B
(B) A starts from the school earlier than B
(C) A and B have equal average velocities from 0 to t_0.
(D) B overtakes A on the way

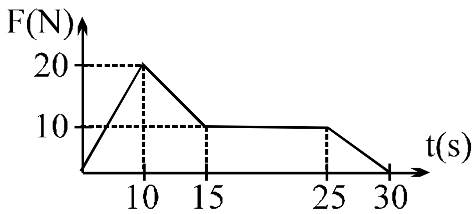
15. A ball is dropped from a building. Somewhere down it crosses a window of length 4 m in 0.5 sec. Speed of ball at top of window is v_1 and at bottom v_2, then choose the **CORRECT** option(s) ($g = 10\ m/s^2$) :-

(A) $v_2 - v_1 = 5$ m/s (B) $v_2 + v_1 = 16$ m/s (C) $\frac{v_2}{v_1} = 9$ (D) $\frac{v_2}{v_1} = \frac{21}{11}$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 16 to 19

In figure shown, the graph shows the variation of a unidirectional force F acting on a body of mass 10kg (in gravity free space), with time t. The velocity of the body at t = 0 is zero. (Area under F-t curve gives change in momentum)



16. The velocity of the body at t = 30 s is

(A) 30 m/s (B) 20 m/s (C) 40 m/s (D) none

17. The power of the force at t = 12 s is (Power = force × velocity)

(A) 225.0 W (B) 217.6 W (C) 226.7 W (D) none

18. The average acceleration of the body from t = 0 to t = 15 s is :-

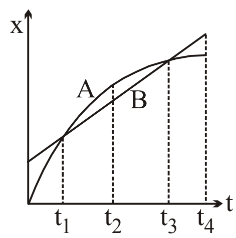
(A) 1.25 m/s^2 (B) 4/7 m/s^2 (C) 5/6 m/s^2 (D) 7/6 m/s^2

19. The change in momentum of the body between the time t = 10 s to 15 s is :-

(A) 100 kg.m/s (B) 75 kg.m/s (C) 125 kg.m/s (D) none

Paragraph for Question nos. 20 to 23

The graph given shows the **POSITION** of two cars, A and B, as a function of time. The cars move along the x-axis on parallel but separate tracks, so that they can pass each other's position without colliding.



20. At which instant in time is car-A overtaking the car-B ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

21. At time t_3, which car is moving faster?

(A) car A (B) car B (C) same speed (D) None of these

22. At which instant do the two cars have the same velocity ?

(A) t_1 (B) t_2 (C) t_3 (D) t_4

23. Which one of the following best describes the motion of car A as shown on the graphs?

(A) speeding up (B) constant velocity

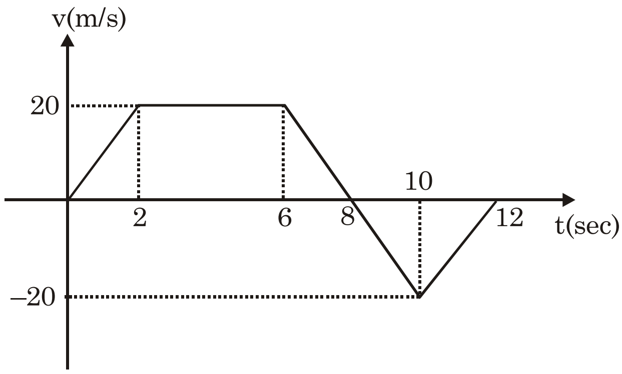
(C) slowing down (D) first speeding up, then slowing down

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT

(THREE COLUMNS AND FOUR ROWS)

Answer Q.24, Q.25 and Q.26 by appropriately matching the information given in the three columns of the following table.

The velocity-time graph of an object moving along a straight line is given below.



	Column-I Time interval		Column-II Average velocity		Column-III Average acceleration
(I)	0 to 2 sec	(i)	10 m/s	(P)	zero
(II)	2 to 6 sec	(ii)	$-\frac{10}{3}$ m/s	(Q)	–2 m/s^2
(III)	0 to 10 sec	(iii)	15 m/s	(R)	$-\frac{10}{3}$ m/s^2
(IV)	6 to 12 sec	(iv)	20 m/s	(S)	–4 m/s^2

24. Which of the following combination is correctly matched :-

(A) (III) (iv) (S) (B) (III) (i) (R) (C) (III) (i) (S) (D) (III) (i) (Q)

25. Which of the following combination is correctly matched :-

(A) (II) (iv) (P) (B) (I) (i) (Q) (C) (II) (iv) (Q) (D) (I) (i) (S)

26. Which of the following combination is correctly matched :-

(A) (IV) (ii) (S) (B) (IV) (iii) (R) (C) (IV) (i) (S) (D) (IV) (ii) (R)

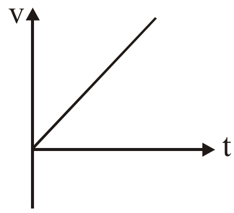
MATRIX MATCH TYPE QUESTION

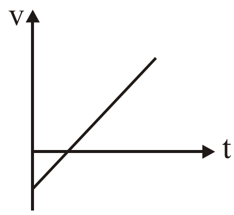
27. A balloon starts rising up from ground with constant net acceleration of $10 m/s^2$. After 2 s a particle drops from the balloon. After further 2 s match the following : ($g = 10 m/s^2$)

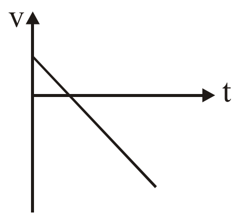
	Column-I		**Column-II**
(A)	Height of particle from ground	(P)	Zero
(B)	Speed of particle	(Q)	10 SI units
(C)	Displacement of Particle	(R)	40 SI units
(D)	Acceleration of particle	(S)	20 SI units

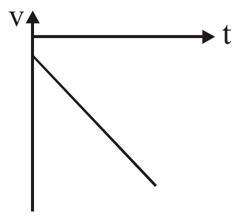
28. In the first column of the given table, some velocity-time (v-t) graphs and in the second column some position-time (x-t) graphs are shown. Sugest suitable match or matches.

Column-I

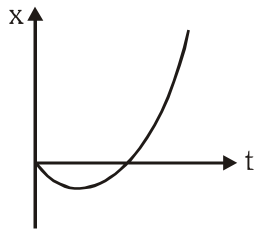
(A) 

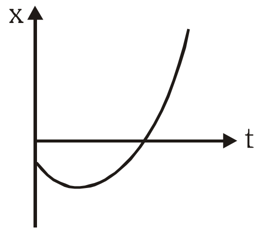
(B) 

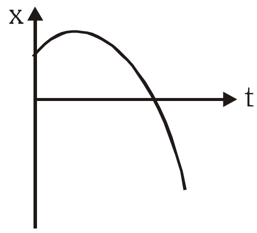
(C) 

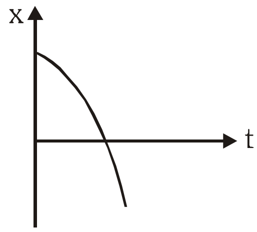
(D) 

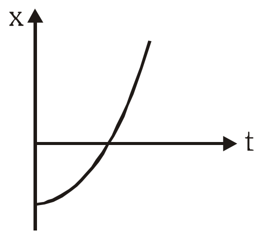
Column-II

(P) 

(Q) 

(R) 

(S) 

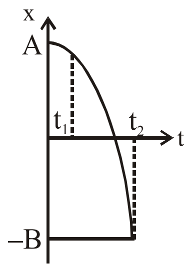
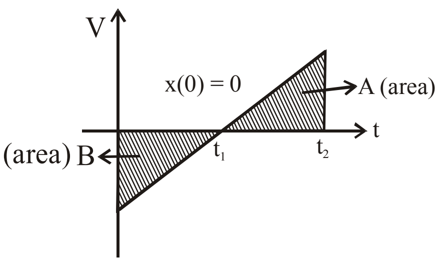
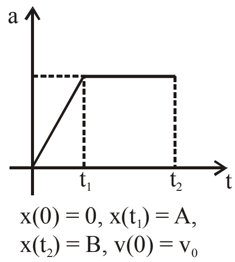
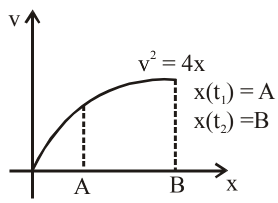
(T) 

29. Match the column :-

Column-I : Shows graph of One Dimension motion of a particle. Symbols have their usual meaning such as $x(0)$ = initial position, $x(t_1)$ = position at $t = t_1$, $v(0)$ = initial velocity.

Column-II : Shows physical quantities. Displacement and distance are asked for $0 < t < t_2$, and average values are asked for $0 < t < t_2$

V_0, A & B are positive constant

Column-I	Column-II
(A) 	(P) |Displacement| = Distance
(B) 	(Q) |Instantaneous velocity| = |Instantaneous speed|
(C) 	(R) |Average velocity| ≤ Average speed
(D) 	(S) Instantaneous acceleration = Average acceleration
	(T) Displacement = A – B and Distance = A + B

EXERCISE (JM)

1. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :-

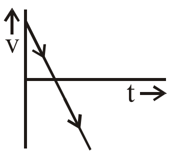
[AIEEE-2011]

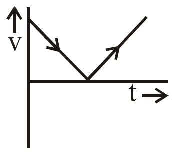
(1) 4 s (2) 8 s (3) 1 s (4) 2 s

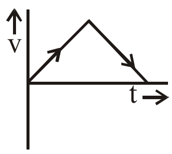
2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is : **[JEE-Main-2014]**

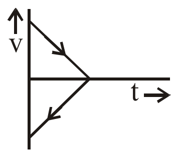
(1) $2g\,H = nu^2(n-2)$ (2) $g\,H = (n-2)u^2$ (3) $2g\,H = n^2u^2$ (4) $g\,H = (n-2)^2u^2$

3. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[JEE-Main-2017]**

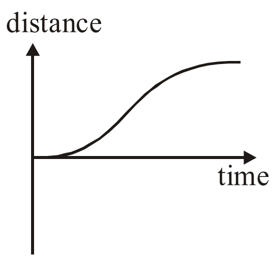
(1) 

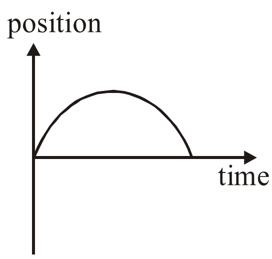
(2) 

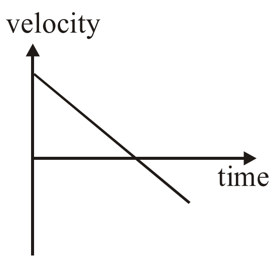
(3) 

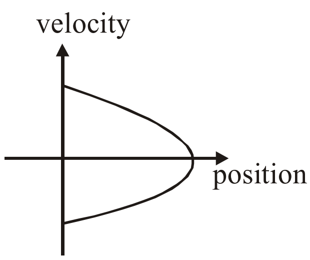
(4) 

4. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE-Main-2018]**

(1) 

(2) 

(3) 

(4) 

EXERCISE (J-A)

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to : **[JEE Advanced-2017]**

(A) R^3 (B) $\frac{1}{R}$ (C) R (D) $R^{2/3}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) $2\sqrt{2}\frac{v}{\pi}$, (b) $\frac{2v}{\pi}$, (c) $\frac{2\sqrt{2}v}{3\pi}$

2. Ans.

v_0	a
—	+

Because particle is slowing down so velocity & acceleration are in opposite direction.

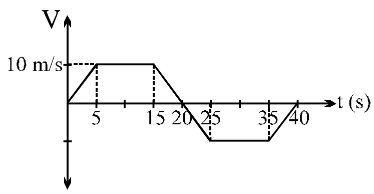
3. Ans. 3
4. Ans. 260 m
5. Ans. (a) 4 m/s^2, (b) 200, (c) 1000 m
6. Ans. 7
7. Ans. 18.5 m
8. Ans. Zero
9. Ans. 2 sec. after body is dropped
10. Ans. $\frac{45}{4}$m
11. Ans. 7
12. Ans. 50
13. Ans. 30
14. Ans. 5h/3
15. Ans. 4
16. Ans. 6
17. Ans. (a) 27 m/s, (b) 108 m, (c) –100 m, (d) 316 m
18. Ans. 1 kg m/sec.
19. Ans. 186.25 N
20. Ans. 2π rad/sec.
21. Ans. 23 amp
22. Ans. 13 rad/s
23. Ans. –72, –16, 24
24. Ans. (a) 0, 8 sec (b) 0, 4, 8 sec
25. Ans. 1.815×10^5 ergs.
26. Ans. 100 kgm/s
27. Ans. (a) 7, $\frac{11}{2}$ (b) 3, –3
28. Ans. 3
29. Ans. 2
30. Ans. $\left(27\hat{i}+2\hat{k}\right)m/s^2$
31. Ans. 3
32. Ans. 3
33. Ans. (a) 3 m; (b) –3 m/s^2; (c) 1m/s; (d) 3/2 m/s

34. Ans. (i) section (a) as slope = v = $\frac{dx}{dt}$ is positive and increasing.

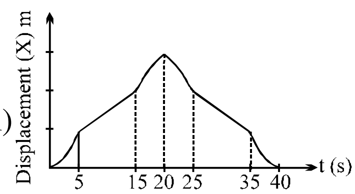
(ii) section (d) as slope = v = $\frac{dx}{dt}$ is negative and constant.

35. Ans. (a) – 2 ms^{-1} (b) 5 m/s

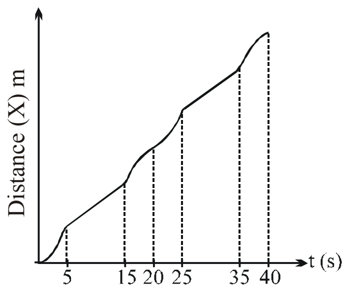
36. Ans. (i)



(ii)



(iii)



EXERCISE (O-1)

1. Ans. (B) 2. Ans. (D) 3. Ans. (C) 4. Ans. (C) 5. Ans. (A) 6. Ans. (A)

7. Ans. (C) 8. Ans. (B) 9. Ans. (D) 10. Ans. (B) 11. Ans. (B) 12. Ans. (A)

13. Ans. (B) 14. Ans. (D) 15. Ans. (B) 16. Ans. (D) 17. Ans. (C) 18. Ans. (A)

19. Ans. (D) 20. Ans. (B) 21. Ans. (C) 22. Ans. (C) 23. Ans. (C) 24. Ans. (D)

25. Ans. (D) 26. Ans. (C) 27. Ans. (D) 28. Ans. (D) 29. Ans. (A) 30. Ans. (B)

31. Ans. (C) 32. Ans. (C) 33. Ans. (B) 34. Ans. (D)

35. Ans. (A) $\rightarrow$ (R,T) ; (B) $\rightarrow$ (T) ; (C) $\rightarrow$ (R, T) ; (D) $\rightarrow$ (S, T)

EXERCISE (O-2)

1. Ans. (B) 2. Ans. (A) 3. Ans. (C) 4. Ans. (D) 5. Ans. (D) 6. Ans. (A)

7. Ans. (B) 8. Ans. (D) 9. Ans. (A) 10. Ans. (B,C,D) 11. Ans. (C,D)

12. Ans. (A,D) 13. Ans. (A,C,D) 14. Ans. (A,B,D)

15. Ans. (A,B,D) 16. Ans. (A) 17. Ans. (B) 18. Ans. (D) 19. Ans. (B)

20. Ans. (A) 21. Ans. (B) 22. Ans. (B) 23. Ans. (C) 24. Ans. (D) 25. Ans. (A)

26. Ans. (D)

27. Ans. (A) - (R); (B) - (P); (C) - (S); (D) - (Q)

28. Ans. (A) $\rightarrow$ (T); (B) $\rightarrow$ (P,Q); (C) $\rightarrow$ (R); (D) $\rightarrow$ (S)

29. Ans. (A) $\rightarrow$ (P,Q,R) ; (B) $\rightarrow$ (Q,R,S,T) ; (C) $\rightarrow$ (P,Q,R) ; (D) $\rightarrow$ (P,Q,R,S)

EXERCISE (JM)

1. Ans. (4) 2. Ans. (1) 3. Ans. (1) 4. Ans. (1)

EXERCISE (JA)

1. Ans. (C)

CHAPTER 3

KINEMATICS-2D

Chapter Contents 03

01.	THEORY	107
02.	EXERCISE (S-1)	121
03.	EXERCISE (O-1)	127
04.	EXERCISE (O-2)	135
05.	EXERCISE (JM)	148
06.	EXERCISE (JA)	149
07.	ANSWER KEY	150

IMPORTANT NOTES

CHAPTER 3

KINEMATICS-2D

KEY CONCEPT

MOTION IN TWO AND THREE DIMENSIONS : (Using 3rd dimension z is optional)

When a particle is moving in space then its motion can be broken up in three co-ordinate axes (x, y& z). The motion in these three directions is governed only by velocity & acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions.

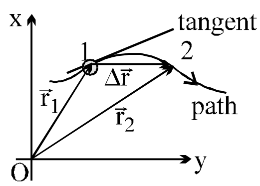
Lets say a particle is moving in space

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Gives position of particle in space.

VELOCITY

Using the language of calculus, we may write $\vec{v}$ as the derivative



$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k}$$

where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt},\ v_z = \frac{dz}{dt}$$

Differentiating $\vec{r}$ w.r.t. time gives us velocity vector of particle at that time.

ACCELRATION

Similarly, if we differentiate $\vec{V}$ w.r.t. time we get acceleration of particle $\vec{a} = \frac{d\vec{V}}{dt}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

where the scalar components of $\vec{a}$ are

$$a_x = \frac{dv_x}{dt},\ a_y = \frac{dv_y}{dt},\ a_z = \frac{dv_z}{dt}$$

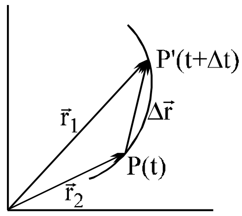
Now, collecting equations of motion relating to x & y axes separately

x-axis	y-axis
$V_x = \frac{dx}{dt}$	$V_y = \frac{dy}{dt}$
$a_x = \frac{dV_x}{dt}$	$a_y = \frac{dV_y}{dt}$

Thus we can see that motion in plane is composed of two straight line motions. **These motions are completely independent of each other**. Only thing connecting them is fact that they are occuring simultaneously.

Velocity is along tangent of path

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle position. $\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$



$\Delta\vec{r}$ will be along tangent

The result is the same in three dimensions:

Ex. A particle with velocity $\vec{v}_0 = -2\hat{i} + 4\hat{j}$ (in meters per second) at t = 0 undergoes a constant acceleration $\vec{a}$ of magnitude a = 3 m/s^2 at an angle θ = 127° from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at t = 5 sec, in unit vector notation ?

Sol. We know that $v = v_0 + at$

now $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$

$a_x = a\cos\theta = (3\ m/s^2)(\cos 127°) = -1.80\ m/s^2$

$a_y = a\sin\theta = (3\ m/s^2)(\sin 127°) = +2.40\ m/s^2$

at time t = 5 sec

$v_x = -2\ m/s + (-1.80\ m/s^2)(5\ sec) = -11\ m/s$

$v_y = 4\ m/s + (2.40\ m/s^2)(5\ sec) = 16\ m/s$

Thus, at t = 5 sec,

$\vec{v} = (-11 m/s)\hat{i} + (16 m/s)\hat{j}$ **Ans.**

Ex. A particle moves in the x–y plane according to the law x = at; y = at(1–αt) where a and α are positive constants and t is time. Find the velocity and acceleration vector. The moment t_0 at which the velocity vector forms angle of 90° with acceleration vector.

Sol. $V_x = a$; $V_y = a - 2a\alpha t \Rightarrow \vec{V} = a\hat{i} + (a - 2a\alpha t)\hat{j}$

$a_x = 0$; $a_y = -2a\alpha \Rightarrow \vec{a} = -2a\alpha\hat{j}$

for 90°, $\vec{V}\cdot\vec{a} = 0$

$-2a\alpha(a - 2a\alpha t) = 0$

$1 - 2\alpha t = 0 \Rightarrow t = 1/(2\alpha)$ sec.

PROJECTILE MOTION

We next consider a special case of two-dimensional motion: A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a projectile (meaning that it is projected or launched) and its motion is called **projectile motion**.

Assumptions:–

Particle remains close to earth's surface, so acceleration due to gravity remains constant.

Air resistance is neglected.

Distance that projectile travels is small so that earth can be treated as plane surface.

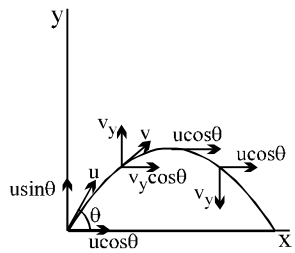
Two straight line motions:–

Our goal here is to analyse projectile motion using the tools for two dimensional motion. This feature allows us to break up a problem involving two dimensional motion into two separate and easier one-dimensional problems,

(a) The horizontal motion is motion with uniform velocity (no effect of gravity)

(b) The vertical motion is motion of uniform acceleration, or freely falling bodies.

Note: In projectile motion, the horizontal motion and the vertical motion are independent of each other, that is either motion does not affects the other.



Treating as two straight line motions:–

The horizontal Motion(x axis):

Because there is no accelration in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion,

The vertical motion(y axis):

The vertical motion is the motion we discussed for a particle in free fall.

As is illustrated in figure and equation (1.3), the vertical component behaves just as for a ball thrown vertically upward. It is directed upward initially and its magnitude steadily decreasing to zero, which marks the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

x-axis	**y-axis**
Initial velocity$(u_x) = u\cos\theta$	Initial velocity$(u_y) = u\sin\theta$
acceleration$(a_x) = 0$	acceleration$(a_y) = -g$
Thus, velocity after time t	Thus, velocity after time t
$v_x = u\cos\theta$	$v_y = u\sin\theta - gt$
Displacement after time t	Displacement after time t
$x = u\cos\theta t$	$y = u\sin\theta t - gt^2/2$

Resultant velocity

$$(\vec{V}_R) = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{V}_R| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

& $\tan\alpha = \dfrac{u\sin\theta - gt}{u\cos\theta}$

where α is angle that velocity vector makes with horizontal. Also known as direction or angle of motion

Time of flight(T)

$$T = \frac{2u\sin\theta}{g}$$

Considering vertical motion

$s_y = 0$; $u_y = v\sin\theta$; $a_y = -g$

$$0 = u\sin\theta T - gT^2/2 \Rightarrow T = \frac{2u\sin\theta}{g}$$

Maximum Height(H)

$$H = \frac{u^2\sin^2\theta}{2g}$$

Vertical velocity at maximum height $v_y = 0$

$$0 = u^2\sin^2\theta - 2gH \Rightarrow H = \frac{u^2\sin^2\theta}{2g}$$

Horizontal Range(R)

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

Total time $T = \dfrac{2u\sin\theta}{g}$

Velocity in horizontal direction $u_x = u\cos\theta$

Total displacement in horizontal direction $R = u\cos\theta T$

$$R = \frac{u^2\sin 2\theta}{g}$$

Note:– *For complementry angles i.e. $\theta + \alpha = 90°$, the range is same for same projection speed but maximum height and time of flight are different.*

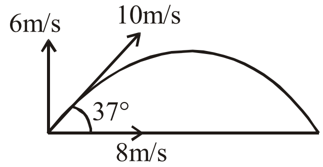
Ex. A body is thrown with initial velocity 10m/sec. at an angle 37° from horizontal. Find

(i) Time of flight (ii) Maximum height.

(iii) Range (iv) Position vector after t = 1 sec.

Ans. (i) 1.2 sec, (ii) 1.8 m, (iii) 9.6 m, (iv) $(16\hat{i}-8\hat{j})-(8\hat{i}+\hat{j})$

Sol. (i) Time of flight $T=\dfrac{2u\sin\theta}{g}=\dfrac{2\times 10\times\frac{3}{5}}{10}=\dfrac{6}{5}=1.2$ sec



(ii) Maximum height $H=\dfrac{u^2\sin^2\theta}{2g}=\dfrac{100\left(\frac{9}{25}\right)}{2\times 10}=\dfrac{9}{5}=1.8$ m

(iii) $R=\dfrac{u^2\sin 2\theta}{g}=\dfrac{100\times 2\times\frac{3}{5}\times\frac{4}{5}}{10}=\dfrac{240}{25}=9.6\text{m}$

(iv) $x = 8\times 1 = 8$ m

$y=6\times 1-\dfrac{1}{2}\times 10\times(1)^2=1\text{m}$

$\vec{r}=8\hat{i}+\hat{j}$

Caution: This equation does not given the horizontal distance travelled by a projectile when the final height is not the launch height.

Maximum Range

$$R=\frac{u^2\sin 2\theta}{g}$$

for $\theta = 45°$, R is maximum

$$R_{max}=\frac{u^2}{g}$$

Ex. A person can throw a ball vertically upto maximum height of 20 mt. How far can he throw the ball.

Sol. $H=\dfrac{u^2}{2g}$

$\therefore$ u = 20 m/s

$$R_{max}=\frac{u^2}{g}=40\text{ m}$$

Ex. A particle is projected with a speed u at an angle θ with horizontal. Find the average velocity of projectile for the period during which it crosses half of maximum height.

Sol. ucosθ along horizontal

avg. velocity is a vector

First we will find vertical component

$$\vec{V}_y = \frac{\text{Total Displacement}}{\text{Total time}} = 0$$

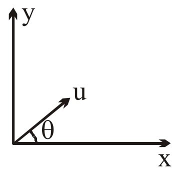
Horizontal

$$V_{1x} = V_{2x} = u\cos\theta$$

$$\overline{V}_x = u\cos\theta$$

EQUATION OF TRAJECTORY

Lets say point of projection is our origin and horizontal direction is x-axis and vertically upwards is positive y-axis.



We know x = u cosθt

$\therefore \quad t = \dfrac{x}{u\cos\theta}$ .....(1)

also $y = u\sin\theta t - \dfrac{1}{2}gt^2$ .....(2)

Putting value of 't' from eq. (1) in eq. (2), we get

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Ex. A particle is projected with a velocity 10 m/s at an angle 37° to the horizontal. Find the location at which the particle is at a height 1m from point of projection.

Ans. 1.6 m, 8 m.

Sol. $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

for y = 1; θ = 37°; u = 10 m/s

$$1 = \frac{3}{4}x - \frac{10x^2}{2\times 100\times\left(\frac{16}{25}\right)}$$

$$1 = \frac{3}{4}x - \frac{5}{64}x^2$$

$5x^2 - 48x + 64 = 0$

$5x^2 - 40x - 8x + 64 = 0$

x = 8m, 1.6m

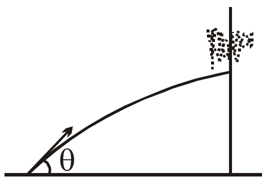
Ex. We have a hose pipe which disposes water at the speed of 10 ms^{-1}. The safe distance from a building on fire, on ground is 5 m. How high can this water go? (take : $g = 10\ ms^{-2}$)

Sol. Here we must understand that taking range of projectile as 10m and making projectile hit the building when it is at maximum height is wrong. By doing this we are not acheiving maximum y for given x = 5m. This just makes highest pt. of path to like on x = 5, But there may be other path for which y will be maximum for given x. This problem will be solved by using equation of trajectory by putting x = 5m and maximising y by varying θ.

$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Putting we get x = 5m

$$y = 5\tan\theta - \frac{10\times 25\sec^2\theta}{2\times 100}$$



$5\tan^2\theta - 20\tan\theta + (4y + 5) = 0$

for real roots discriminant must be positive.

$400 - 4 \times 5\,(4y + 5) > 0$

Solving $3.75 \geq y$

hence maximum y = 3.75 m

If we have taken range as 10 m then angle of projection will be $\theta = 45°$ corsponding maximum hight H = 2.5m which is smaller than our answer.

Ex. A projectile is fired horizontaly with a velocity of 98 m/s from the top of a hill 490 m high. Find

(i) Time to reach ground

(ii) The horizontal distance from foot of hill to ground.

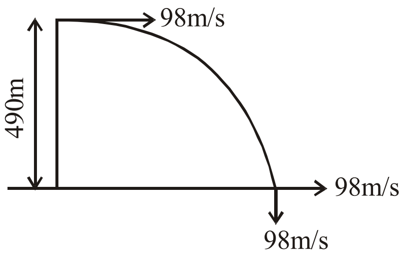
(iii) The speed with which it hits the ground.

Ans. (i) 10 sec, (ii) 980 m, (iii) $98\sqrt{2}$ m/se

Sol. $u_y = 0$; $u_x = 98$ m/s

on y-axis

$$s = ut + \frac{1}{2}at^2$$



$$-490 = 0 - \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{490\times 2}{9.8}} = 10 \text{ sec}$$

on x-axis for horizontal distance,

$R = u_x t = 98 \times 10 = 980$ m

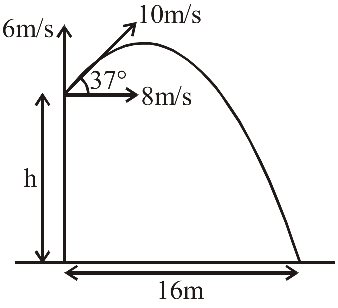
$v_x = 98$ m/s

$v_y = 0 - 9.8 \times 10 = -98$ m/s

So speed = $98\sqrt{2}$ m/s

Ex. A ball is thrown from the top of a tower with an initial velocity of 10m/s at an angle 37° above the horizontal, hits the ground at a distance 16m from the base of tower. Calculate height of tower. [$g = 10\ m/s^2$]

Ans. 8 m



Sol. So time to reach ground is $= \frac{16}{8} = 2$ sec

on y-axis (for height of tower)

$$-h = 6\times2-\frac{1}{2}\times10\times(2)^2$$

$-h = 12 - 22$

$h = 8$ m

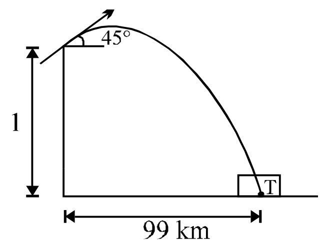
Ex. Prithvi missile is fired to destroy an enemy military base situated on same horizontal level, situated 99 km away. The missile rises vertically for 1 km & then for remainder of flight, it followes parbolic path like a free body under earth's gravity, at an angle of 45°. Calculate its velocity at beginning of parabolic path. ($g = 10\ ms^{-2}$)

Sol. for horizontal motion time t

$$t = \frac{99\times10^3}{u\cos45°}$$

for vertical

$$-1\times10^3 = u\sin45°\, t - \frac{1}{2}\times10\times t^2$$



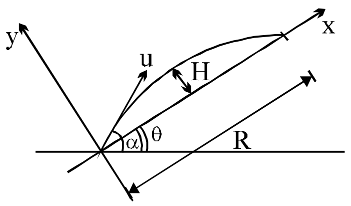
$$1\times10^3 + \frac{u\sin45°}{u\sin45°}\times99\times10^3 = \frac{10}{2}\times\frac{(99\times10^3)^2\times2}{u^2}$$

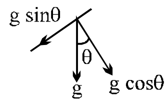
$$u^2 = \frac{(99\times10^3)^2\times10}{100\times10^3}$$

$$u = 99\times10^3\sqrt{\frac{1}{10^4}} = 990\ ms^{-1}$$

PROJECTION ON INCLINED PLANE

There is an inclined plane making an angle θ with horizontal. A particle is projected at an angle α from horizontal.





x-axis

$u_x = u\cos(\alpha - \theta)$

$a_x = -g\sin\theta$

vel. at any time t

$v_x = u\cos(\alpha - \theta) - g\sin\theta\, t$

y-axis

$u_y = u\sin(\alpha - \theta)$

$a_y = -g\cos\theta$

vel. at any time t

$v_y = u\sin(\alpha - \theta) - g\cos\theta\, t$

Time of flight

Displacement in y direction $s_y = 0$

$$0 = u\sin(a - \theta)\,T - \frac{1}{2}g\cos\theta\,T^2$$

$$T = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

Maximum distance of particle from inclined plane

Pt. where $v_y = 0$ is max. height

$(0)^2 = u^2\sin^2(\alpha - \theta) - 2g\cos\theta\,H$

$$H = \frac{u^2\sin^2(\alpha-\theta)}{2g\cos\theta}$$

Range along the inclined plane

$$s_x = u_xT + \frac{1}{2}a_x\,T^2$$

$$R = \frac{u\cos(\alpha-\theta)\times 2u\sin(\alpha-\theta)}{g\cos\theta} - \frac{2\sin\theta\times 2\times 2u^2\sin^2(\alpha-\theta)}{2g^2\cos^2\theta}$$

$$= \frac{2u^2\sin(\alpha-\theta)[\cos(\alpha-\theta)\cos\theta - \sin\theta\sin(\alpha-\theta)]}{g\cos^2\theta}$$

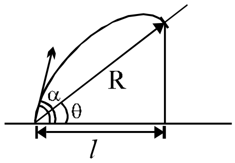
$$R = \frac{2u^2\sin(\alpha-\theta)\cos\alpha}{g\cos^2\theta}$$

$$R = \frac{u^2[\sin(2\alpha-\theta)-\sin\theta]}{g\cos^2\theta}$$

Alternate Method

$l = u \cos \alpha \, T$

$$R = \frac{l}{\cos\theta}$$



$$R = \frac{u\cos\alpha}{\cos\theta} \times \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$$R = \frac{2u^2\sin(\alpha-\theta)\cos\alpha}{g\cos^2\theta}$$

Note : Presence of incline plane does not affect the path of projectile in any way.

Maximum Range :

$$R = \frac{u^2[\sin(2\alpha-\theta)-\sin\theta]}{g\cos^2\theta}$$

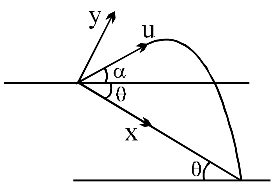
For max. range $2\alpha - \theta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4} + \frac{\theta}{2}$

so $R_{max} = \frac{u^2}{g(1+\sin\theta)}$

Projection from top of incline plane:

Incline plane is at an angle θ with horizontal and a particle is projected at an angle α from horizontal.

In all formulae replace θ with –θ



$$H = \frac{u^2\sin^2(\alpha+\theta)}{2g\cos\theta}$$

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$

$$R = \frac{2u^2\sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$$

$R_{max} = \frac{u^2}{g(1-\sin\theta)}$ and $\alpha = \frac{\pi}{4} - \frac{\theta}{2}$

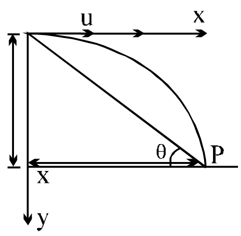
Note : If a particle strikes the incline plane ⊥ then its comp. of velocity along incline must be zero.

Ex. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far along the plane, from the point of projection will particle strike the plane?

Sol.

x-axis	**y-axis**
$u_x = u$	$u_x = 0$
$a_x = 0$	$a_x = g$
$x = ut$	$y = \frac{gt^2}{2}$

$\Rightarrow \quad y = \frac{gx^2}{2u^2}$



also $\frac{y}{x} = \tan\theta \Rightarrow x\tan\theta = \frac{gx^2}{2u^2}$

$x = 0, \frac{2u^2\tan\theta}{g}$

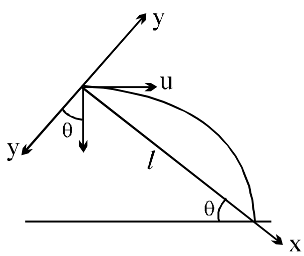
$x = \frac{2u^2\tan\theta}{g}$

$\Rightarrow y = \frac{2u^2\tan^2\theta}{g}$

dist. $l = \sqrt{x^2 + y^2}$

$l = \frac{2u^2\tan\theta\sec\theta}{g}$

Alternate Method.

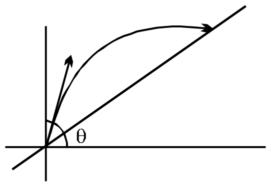


$R = \frac{2u^2\sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$

$R = 2u^2\tan\theta\sec\theta$

Ex. A particle is projected up an inclined plane. Plane is inclined at an angle θ with horizontal and particle is projected at an angle α with horizontal. If particle strikes the plane horizontally prove that $\tan\alpha = 2\tan\theta$

Sol. We know time of flight



$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$

considering vertical motion

$u = v\sin\alpha$

$a = -g$

$v = 0$

$$\therefore\ T = \frac{u\sin\alpha}{g} = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$\sin\alpha\cos\theta = 2\sin\alpha\ \cos\theta - 2\cos\alpha\sin\theta$

$2\cos\alpha\sin\theta = \sin\alpha\cos\theta$

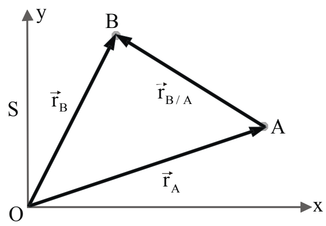
$2\tan\theta = \tan\alpha$

RELATIVE MOTION

Motion of a body can only be observed, when it changes its position with respect to some other body. In this sense, motion is a relative concept. To analyze motion of a body say A, therefore we have to fix our reference frame to some other body say B. The result obtained is motion of body A relative to body B.

Relative position, Relative Velocity and Relative Acceleration

Let two bodies represented by particles A and B at positions defined by position vectors $\vec{r}_A$ and $\vec{r}_B$, moving with velocities $\vec{v}_A$ and $\vec{v}_B$ and accelerations $\vec{a}_A$ and $\vec{a}_B$ with respect to a reference frame S. For analyzing motion of terrestrial bodies the reference frame S is fixed with the ground.



The vectors $\vec{r}_{B/A}$ denotes position vector of B relative to A. Following triangle law of vector addition, we have

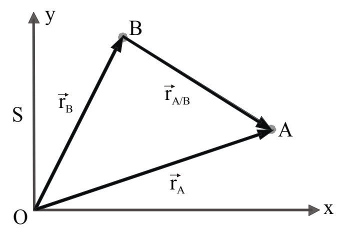
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \text{...(i)}$$

First derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to velocity of particle A and velocity of particle B relative to frame S and first derivative of $\vec{r}_{B/A}$ with respect to time defines velocity of B relative to A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \text{...(ii)}$$

Second derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to acceleration of particle A and acceleration of particle B relative to frame S and second derivative of $\vec{r}_{B/A}$ with respect to time defines acceleration of B relative to A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \text{...(iii)}$$



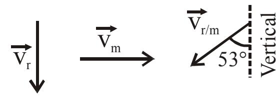
In similar fashion motion of particle A relative to particle B can be analyzed with the help of adjoining figure. You can observe in the figure that position vector of A relative to B is directed from B to A and therefore

$$\vec{r}_{B/A} = -\vec{r}_{A/B},\ \vec{v}_{B/A} = -\vec{v}_{A/B} \text{ and } \vec{a}_{B/A} = -\vec{a}_{A/B}.$$

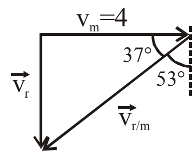
The above equations elucidate that how a body A appears moving to another body B is opposite to how body B appears moving to body A.

Ex. A man when standstill observes the rain falling vertically and when he walks at 4 km/h he has to hold his umbrella at an angle of 53° from the vertical. Find velocity of the raindrops.

Sol. Assigning usual symbols $\vec{v}_m$, $\vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following eq. $\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$



The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.



The addition of velocity vectors is represented according to the above equation is also represented. From the figure we have

$$v_r = v_m \tan 37° = 3 \text{ km/h} \ \textbf{Ans.}$$

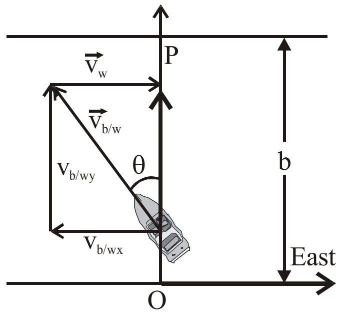
Ex. A boat can be rowed at 5 m/s on still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Sol. (a) Velocity of a boat on still water is its capacity to move on water surface and equals to its velocity relative to water.



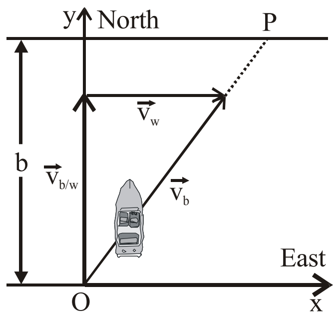
$\vec{v}_{b/w}$ = Velocity of boat relative to water = Velocity of boat on still water

On flowing water, the water carries the boat along with it. Thus velocity $\vec{v}_b$ of the boat relative to the ground equals to vector sum of $\vec{v}_{b/w}$ and $\vec{v}_w$. The boat crosses the river with the velocity $\vec{v}_b$.

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w$$

(b) To cross the river perpendicular to current the boat must be steered in a direction so that one of the components of its velocity ($\vec{v}_{b/w}$) relative to water becomes equal and opposite to water flow velocity $\vec{v}_w$ to neutralize its effect. It is possible only when velocity of boat relative to water is grater than water flow velocity. In the adjoining figure it is shown that the boat starts from the point O and moves along the line OP (y-axis) due north relative to ground with velocity $\vec{v}_b$. To achieve this it is steered at an angle θ with the y-axis.

$$v_{b/w} \sin\theta = v_w \rightarrow 5\sin\theta = 3 \Rightarrow \theta = 37° \textbf{ Ans.}$$



(c) The boat will cover river width b with velocity

$v_b = v_{b/wy} = v_{b/w} \sin 37° = 4$ m/s in time t, which is given by

$$t = b / v_b \rightarrow t = 50\text{s} \textbf{ Ans.}$$

(d) To cross the river in minimum time, the component perpendicular to current of its velocity relative to ground must be kept to maximum value. It is achieved by steering the boat always perpendicular to current as shown in the adjoining figure. The boat starts from O at the south bank and reaches point P on the north bank. Time t taken by the boat is given by

$$t = b / v_{b/w} \rightarrow t = 40\text{s} \textbf{ Ans.}$$

Drift is the displacement along the river current measured from the starting point. Thus, it is given by the following equation. We denote it by x_d.

$$x_d = v_{bx} t$$

Substituting $v_{bx} = v_w = 3$ m/s, from the figure, we have

x_d = 120 m **Ans.**

EXERCISE (S-1)

General 2-D motion

1. The vertical height y and horizontal distance x of a projectile on a certain planet are given by $x = (3t)m$, $y = (4t - 6t^2)$ m where t is in seconds. Find the speed of projection (in m/s).

2. The position of a particle is given by

$$r = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,k\ \text{m}$$

where t is in seconds and the coefficients have the proper units for r to be in metres.

(a) Find the v and a of the particle?

(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s ?

3. A particle moves in xy plane such that $v_x = 50 - 16\,t$ and $y = 100 - 4t^2$ where v_x is in m/s and y is in m. It is also known that x = 0 when t = 0. Determine (i) Acceleration of particle (ii) Velocity of particle when y = 0.

4. The position of a particle is given by $x = 7 + 3t^3$ m and $y = 13 + 5t - 9t^2$ m, where x and y are the position coordinates, and t is the time in s. Find the speed (magnitude of the velocity) when the x component of the acceleration is 36 m/s^2.

Projectile motion

5. A particle is projected with a speed of 10 m/s at an angle 37° with the vertical. Find (i) time of flight (ii) maximum height above ground (iii) horizontal range.

6. A particle is thrown with a speed 60 ms^{-1} at an angle 60° to the horizontal. When the particle makes an angle 30° with the horizontal in downward direction, it's speed at that instant is v. What is the value of v^2 in SI units ?

7. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?

8. A particle is projected upwards with a velocity of 100 m/s at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking g =10 m/s^2.

9. A particle is projected in x-y plane with y-axis along vertical, the point of projection is origin. The equation of a path is $y = \sqrt{3}\,x - \dfrac{gx^2}{2}$. Find angle of projection and speed of projection.

10. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{ox}}\right)$$

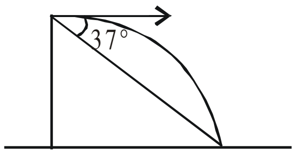
(b) Show that the projection angle θ_0 of a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

11. A particle is projected in the *x*-*y* plane with *y*-axis along vertical. Two second after projection the velocity of the particle makes an angle 45° with the *X*-axis. Four second after projection, it moves horizontally. Find the velocity of projection.

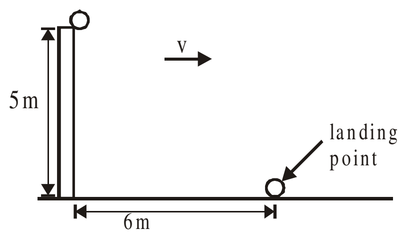
12. A ball is thrown horizontally from a cliff such that it strikes ground after 5 s. The line of sight from the point of projection to the point of landing makes an angle of 37° with the horizontal. What is the initial velocity of projection?



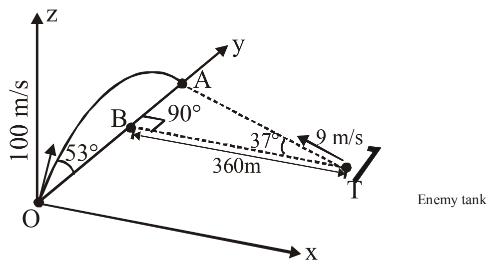
13. A Bomber flying upward at an angle of 53° with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 s after its release. Find: [Given sin 53° = 0.8; g = 10 m/s^2]
(i) The velocity of the bomber at the time of release of the bomb .
(ii) The maximum height attained by the bomb .
(iii) The horizontal distance travelled by the bomb before it strikes the ground
(iv) The velocity (magnitude & direction) of the bomb just when it strikes the ground.

14. A ball is projected at an angle of 30° above the horizontal from the top of a tower and strikes the ground in 5 s at an angle of 45° with the horizontal. Find the height of the tower and the speed with which it was projected.

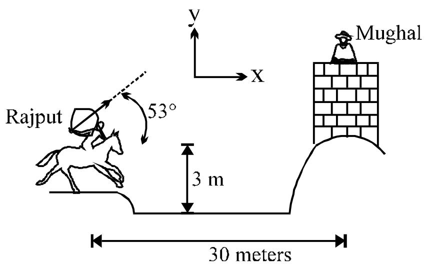
15. A ball is dropped from rest from a tower of height 5m. As a result of the wind it lands at a distance 6m from the bottom of the tower as shown. Assuming no air resistance but that the wind gives the ball a constant horizontal velocity v. Find value of v in m/s.



16. A tank is initially at a perpendicular distance BT = 360 m from the plane of firing as shown. The enemy tank is moving with a speed of 9 m/s in direction TA as shown in figure. A gun can fire shell in y-z plane only with a speed 100 m/s at an angle of 53° such that the shell lands at points A. If tank started at t = 0 then time interval (in sec) after which shell is to be fired to hit the tank is



17. A Rajput soldier, sits on a horse next to a river. Across the river there is a hill and atop the hill is a fortress. He sees a Mughal, sitting on the fortress's top wall. There is a full moon, so he angrily shoots an arrow at an angle 53° relative to the horizontal. The arrow hits Mughal after a 2 second flight. The horizontal distance from Rajput to Mughal is 30 meters. The arrow is 3 meters above the river when Rajput shoots it.

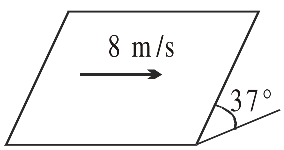


(a) What is the original velocity , $\vec{V}$, of the arrow when Rajput shoots it?

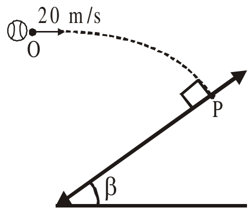
(b) What is Mughal elevation above the river?

(c) What is the flight direction of the arrow the instant before it strikes Mughal, i.e. what is the angle, θ, between its direction and the horizontal when it skins into Mughal's tender flesh?

18. A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of 8 m/s. Find it's speed after 1 s.



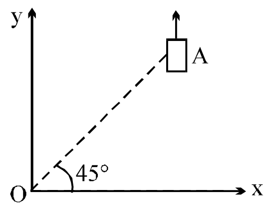
19. A ball is thrown horizontally from a point O with speed 20 m/s as shown. Ball strikes the incline plane along the normal to it after two seconds. Find value of x, if $\beta = \pi/x$ (where β is the angle of incline in degree).



Relative motion

20. A person decided to walk on an escalator which is moving at constant rate (speed). When he moves at the rate 1 step/sec, then he reaches top in 20 steps. Next day he goes 2 steps / sec. and reaches top in 32 steps. If speed of escalator is n steps / sec. Find the value of n.

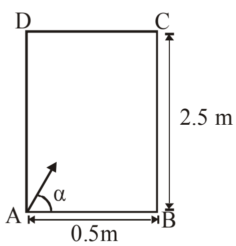
21. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley. A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



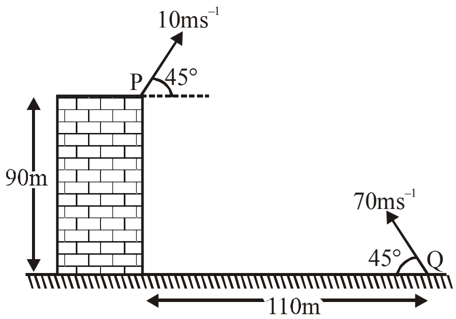
(a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \dfrac{4\theta}{3}$.

22. A cuboidal elevator cabin is shown in the figure. A ball is thrown from point A on the floor of cabin when the elevator is falling under gravity. The plane of motion is ABCD and the angle of projection of the ball with AB, relative to elevator, if the ball collides with point C, is α. Then find the value of $\tan\alpha$.



23. Two particles P and Q are launched simultaneously as shown in figure. Find the minimum distance between particles in meters.



24. A man crosses a river by a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$, find

(i) width of the river, (ii) velocity of the boat with respect to water ($v_{b/r}$) (iii) speed of the current (v_r)

25. Rain is falling vertically with a speed of 20 m/s relative to air. A person is running in the rain with a velocity of 5 m/s and a wind is also blowing with a speed of 15 m/s (both towards east). Find the angle with the vertical at which the person should hold his umbrella for best protection from rain.

26. A glass wind-screen of adjustable inclination is mounted on a car. The car moves horizontally with a speed of 6 m/s. At what angle α with the vertical should the wind screen be adjusted so that the rain drops falling vertically with 2 m/s strike the wind screen perpendicularly?

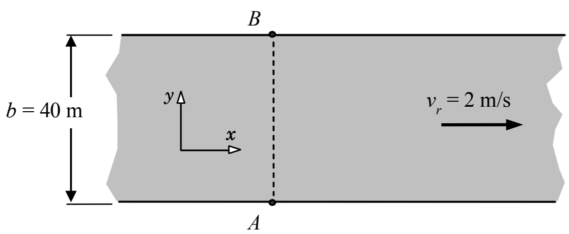
27. Boat moves with velocity 5m/s on still water. It is steered perpendicular to the river current.

(a) Will it reach point B or somewhere else on the other bank ?

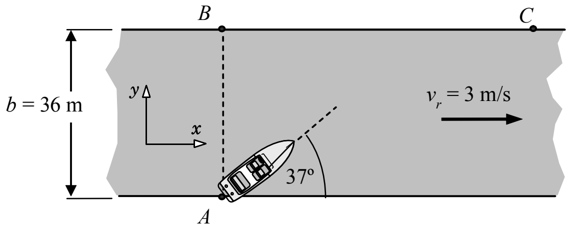
(b) How long will it take to cross the river ?

(c) How far down stream, will it reach the other bank ?

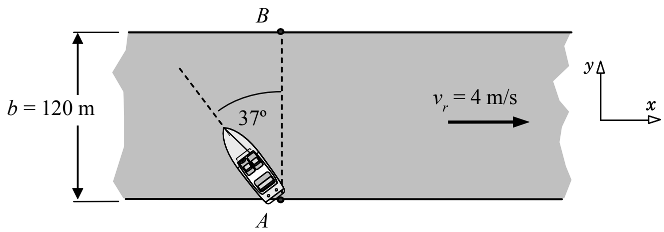
(d) Does it take minimum time in this way ?



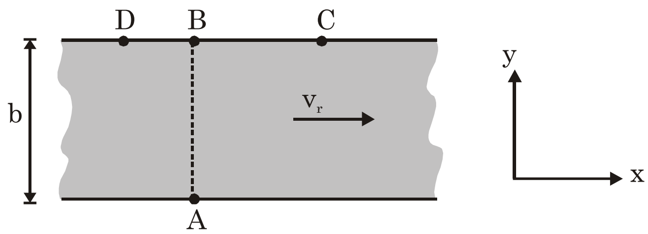
28. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown to reach point C. Find the time of the trip and distance between B and C.



29. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown. Where will it reach on the opposite bank?



30. Drift is distance along a river a boat covers in crossing the river. If the boat reaches point C, distance BC is called downstream drift and if the boat reaches point D, distance BD is called upstream drift. To cross a river without drift, what should be relation between v_{br} and v_r. If a boat crosses a river without drift, in which direction must it be steered.



EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

General 2-D motion

1. A particle is projected from a horizontal plane (x-z plane) such that its velocity vector at time t is given by $\vec{V} = a\hat{i} + (b - ct)\hat{j}$. Its range on the horizontal plane is given by :-

(A) $\frac{ba}{c}$ (B) $\frac{2ba}{c}$ (C) $\frac{3ba}{c}$ (D) None

2. A body of mass 5 kg starts from the origin with an initial velocity $\vec{u} = \left(30\hat{i} + 40\hat{j}\right)\text{ms}^{-1}$. If a constant force $\left(-6\hat{i} - 5\hat{j}\right)\text{N}$ acts on the body, the time in which the y component of the velocity becomes zero, is :-

(A) 5s (B) 20 s (C) 40 s (D) 80 s

Projectile motion

3. A boy throws a ball from shoulder height at an initial velocity of 30 m/s. Spending 4.8 s in air, the ball is caught by another boy as the same shoulder-height level. What is the angle of projection?

(A) 37° (B) 30° (C) 53° (D) 60°

4. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. They are given by

$$R = \frac{u^2 \sin 2\theta}{g} \qquad H = \frac{u^2 \sin^2\theta}{2g} \quad \text{and} \quad T = \frac{2u\sin\theta}{g}$$

Now keeping u fixed, θ is varied from 30° to 60°, then

(A) R will first increase then decrease, H will increase and T will decrease

(B) R will first increase then decrease while H and T both will increase

(C) R will decrease while H and T both will increase

(D) R will increase while H and T both will also increase

5. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?

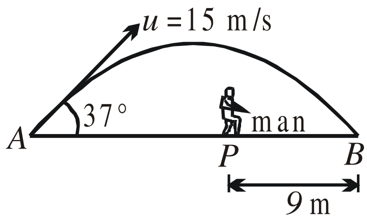
(A) The one with the farthest range.

(B) The one which reaches maximum height.

(C) The one with the greatest initial velocity.

(D) The one leaving the bat at 45° with respect to the ground.

6. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.

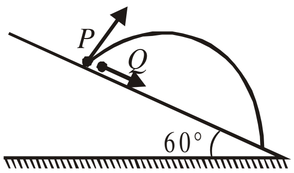


(A) 3 m/s (B) 5 m/s (C) 9 m/s (D) 12 m/s

7. A ball is projected horizontally. After 3 s from projection its velocity becomes 1.25 times of the velocity of projection. Its velocity of projection is :-

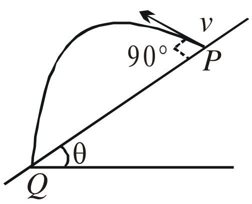
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

8. A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide after t = 4 s. The speed of projection of P is :-



(A) 5 m/s (B) 10 m/s (C) 15 m/s (D) 20 m/s

9. In the given figure, if time taken by the projectile to reach Q is T, than PQ =



(A) $Tv\sin\theta$ (B) $Tv\cos\theta$ (C) $Tv\sec\theta$ (D) $Tv\tan\theta$

10. A particle is projected up the incline such that its component of velocity along the incline is 10m/s. Time of flight is 2 second and maximum perpendicular distance during the motion from the incline is 5 m. Then velocity of projection will be :-

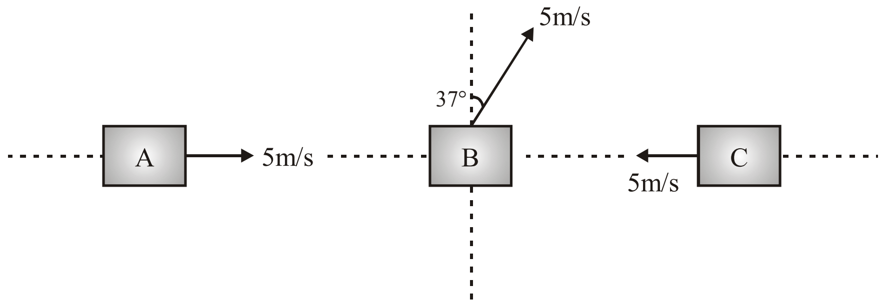
(A) 10 m/s (B) $10\sqrt{2}$ m/s (C) $5\sqrt{5}$ m/s (D) none of these

11. A particle A is projected with speed v_A from a point making an angle 60° with the horizontal. At the same instant, a second particle B is thrown vertically upward from a point directly below the maximum height point of parabolic path of A with velocity v_B. If the two particles collide then the ratio of v_A/v_B should be :-

(A) 1 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

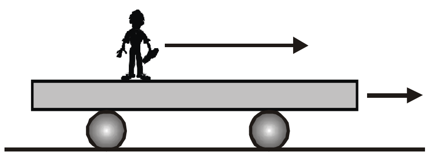
Relative motion

12. Consider the motion of three bodies as shown for an observer on B, what is the magnitude of relative velocity of A with respect to C ?



(A) 10 m/s (B) 0 (C) $\sqrt{20}$ m/s (D) 8 m/s

13. A trolley is moving horizontally with a constant velocity of v with respect to earth. A man starts running from one end of the trolley with a velocity 1.5 v with respect to trolley. After reaching the opposite end, the man returns back and continues running with a velocity of 1.5 v w.r.t. the trolley in the backward direction. If the length of the trolley is L then the displacement of the man with respect to earth during the process will be :-



(A) 2.5 L (B) 1.5 L (C) $\frac{5L}{3}$ (D) $\frac{4L}{3}$

14. An elevator car (lift) is moving upward with uniform acceleration of 2 m/s^2. At the instant, when its velocity is 2 m/s upwards a ball is thrown upward from its floor. The ball strikes back the floor 2 s after its projection. Find the velocity of projection of the ball relative to the lift.

(A) 10 m/s ↑ (B) 10 m/s ↓ (C) 12 m/s ↑ (D) 12 m/s ↓

15. A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction :-

(A) East (B) North-East (C) South-East (D) South-West

16. Three ships A, B & C are in motion. Ship A moves relative to B is with speed v towards North-East. Ship B moves relative to C with speed v towards the North-West. Then relative to A, C will be moving towards :-

(A) North (B) South (C) East (D) West

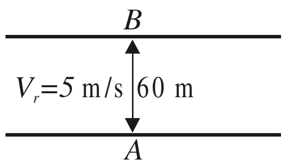
17. Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him :

(A) 2 m/s south (B) 2 m/s north (C) 4 m/s west (D) 4 m/s south

18. A boat having a speed of 5 km/hr in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in Km/hr.

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

19. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across the river at a distance of 60 m in 5 sec. His velocity in still water should be :-



(A) 12 m/s (B) 13 m/s (C) 5 m/s (D) 10 m/s

20. A motor boat is to reach at a point 30° upstream (w.r.t. normal) on other side of a river flowing with velocity 5m/s. The angle 30° is measured from a direction perpendicular to river flow. Velocity of motorboat with respect to water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle

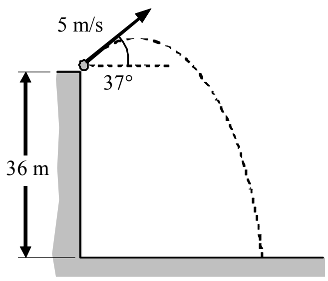
(A) 120° with respect to stream direction.

(B) 30° with respect to the perpendicular to the bank.

(C) 30° with respect to the line of destination from starting point.

(D) None of these.

21. A ball is thrown from the top of 36 m high tower with velocity 5 m/s at an angle 37° above the horizontal as shown. Its horizontal range on the ground is closest to [g = 10 m/s^2]



(A) 12 m (B) 18 m (C) 24 m (D) 30 m

MULTIPLE CORRECT TYPE QUESTIONS

22. A particle moves in the xy-plane and at time t is at the point $(t^2, t^3 - 2t)$. Then

(A) At $t = 2/3$ s, directions of velocity and acceleration are perpendicular

(B) At $t = 0$, directions of velocity and acceleration are perpendicular

(C) At $t = \sqrt{2/3}$ s, particle is moving parallel to x-axis

(D) Acceleration of the particle when it is at point (4, 4) is $\left(2\hat{i} + 12\hat{j}\right) m/s$

23. A particle moves in the x-y plane with a constant acceleration g in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following is/are correct?

(A) The x-component of its velocity is constant.

(B) At the origin, the y-component of its velocity is $a\sqrt{\frac{g}{2b}}$.

(C) At the origin, its velocity makes an angle $\tan^{-1}(a)$ with the x-axis.

(D) The particle moves exactly like a projectile.

24. Two particles A and B projected along different directions from the same point P on the ground with the same velocity of 70 m/s in the same vertical plane . They hit the ground at the same point Q such that $PQ = 480$ m. Then [$g = 9.8$ m/s^2]

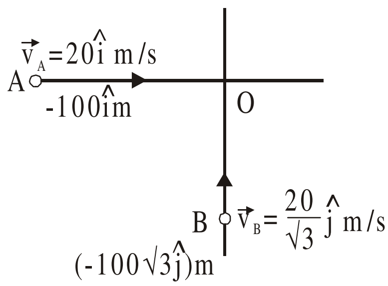
(A) Ratio of their times of flight is 4 : 5

(B) Ratio of their maximum heights is 9 : 16

(C) Ratio of their minimum speeds during flights is 4 : 3

(D) The bisector of the angle between their directions of projection makes 45° with horizontal

25. Positions of two vehicles A and B with reference to origin O and their velocities are as shown.



(A) they will collide

(B) distance of closest approach is 100 m.

(C) their relative speed is $\frac{40}{\sqrt{3}}$ m/s

(D) their relative velocity is $\frac{20}{\sqrt{3}}$ m/s

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 26 to 28

Two projectiles are thrown simultaneously in the same vertical plane from the same point. If their velocities of projection are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

26. The trajectory of particle 1 with respect to particle 2 will be

(A) a parabola (B) a straight line

(C) a vertical straight line (D) a horizontal straight line

27. If $v_1\cos\theta_1 = v_2\cos\theta_2$, then choose the ***incorrect*** statement

(A) One particle will remain exactly below or above the other particle

(B) The trajectory of one with respect to other during the flight will be a vertical straight line

(C) Both will have the same range

(D) Both will attain same maximum height

28. If $v_1\sin\theta_1 = v_2\sin\theta_2$, then choose the ***correct*** statement

(A) The time of flight of both the particles will be same

(B) The maximum height attained by the particles will be same

(C) The trajectory of one with respect to another during the flight will be a horizontal straight line

(D) None of these

Paragraph for Question Nos. 29 to 31

A particle leaves the origin with initial velocity $\vec{v}_0 = 11\hat{i} + 14\hat{j}\ \text{m/s}$. It undergoes a constant acceleration given by $\vec{a} = -\frac{22}{5}\hat{i} + \frac{2}{15}\hat{j}$ m/s^2.

29. When does the particle cross the y axis ?

(A) 2 sec (B) 4 sec (C) 5 sec (D) 7 sec

30. At the instant when particle crosses y-axis, direction in which particle is moving is :-

(A) At angle 37° from +x-axis towards +y-axis

(B) At angle 37° from –x-axis towards +y-axis

(C) At angle 53° from +x-axis towards +y-axis

(D) At angle 53° from –x-axis towards +y-axis

31. How far is it from the origin, at that time ?

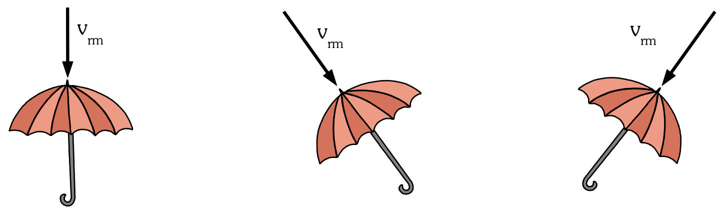
(A) 70 m (B) 71.67 m (C) 125 m (D) 15 m

Paragraph for Question no. 32 to 35: Rain and man

By the term velocity of rain, we mean velocity with which raindrops fall relative to the ground. In absence of wind, raindrops fall vertically and in presence of wind raindrops fall obliquely. Moreover raindrops acquire a constant terminal velocity due air resistance very quickly as they fall toward the earth. A moving man relative to himself observes an altered velocity of raindrops, which is known as velocity of rain relative to the man. It is given by the following equation.

$$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$$

A standstill man relative to himself observes rain falling with velocity, which is equal to velocity of the raindrops relative to the ground. To protect himself a man should hold his umbrella against velocity of raindrops relative to himself as shown in the following figure.



32. Rain is falling vertically with velocity 80 cm/s.

(a) How should you hold your umbrella?

(b) You start walking towards the east with velocity 60 cm/s. How should you hold your umbrella?

(c) You are walking towards the west with velocity 60 cm/s. How should you hold your umbrella?

(d) You are walking towards the north with velocity 60 cm/s. How should you hold your umbrella?

(e) You are walking towards the south with velocity 80 cm/s. How should you hold your umbrella?

33. When you are standstill in rain, you have to hold your umbrella vertically to protect yourself.

(a) When you walk with velocity 90 cm/s, you have to hold your umbrella at 53° above the horizontal. What is velocity of the raindrops relative to the ground and relative to you?

(b) If you walk with speed 160 cm/s, how should you hold your umbrella?

34. A man walks in rain at 72 cm/s due east and observes the rain falling vertically. When he stops, rain appears to strike his back at 37° from the vertical. Find velocity of raindrops relative to the ground.

35. When you walk in rain at 75 cm/s, you have to hold your umbrella vertically and when you double your speed in the same direction, you have to hold your umbrella at 53° above the horizontal. What is the rain velocity?

Paragraph for Question no. 36 to 38 : Flag in wind

When you are standstill holding a flag, the flag flutters in the direction of wind. When you start running the direction of fluttering of the flag changes in to the direction of the wind relative to you. In all case a flag flutters in the direction of the wind relative to the flag.

36. When you are standstill holding a flag the flag flutters in the north and when you run at 8 m/s due east, the flag flutters in direction 37° north of west. Find the wind velocity.

37. Wind is blowing uniformly due north everywhere with velocity 12 m/s. A car mounted with a flag starts running towards east. After 9 s from start the flag flutters in 53° north of west and after 16 s from the start the flag flutters in 37° north of west.

(a) Find velocity of the car 9 s after it starts.

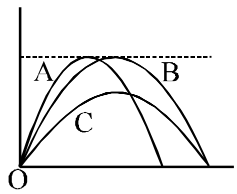
(b) Find velocity of the car 16 s after it starts.

(c) If the car maintains uniform acceleration, find acceleration of the car.

38. Holding a flag, when you run at 8 m/s due east, the flag flutters in the north and when you run at 2 m/s due south, the flag flutters in the northeast. If the wind velocity is uniform and remain constant, find the wind velocity.

MATRIX MATCH TYPE QUESTION

39. Trajectories are shown in figure for three kicked footballs. Initial vertical & horizontal velocity components are u_y and u_x respectively. Ignoring air resistance, choose the correct statement from **Column-II** for the value of variable in **Column-I**.



	Column-I		**Column-II**
(A)	time of flight	(P)	greatest for A only
(B)	u_y/u_x	(Q)	greatest for C only
(C)	u_x	(R)	equal for A and B
(D)	u_xu_y	(S)	equal for B and C

40. Trajectory of particle in a projectile motion is given as $y = x - \frac{x^2}{80}$. Here, x and y are in meters. For this projectile motion, match the following with $g = 10\ m/s^2$.

	Column–I		**Column–II**
(A)	Angle of projection (in degrees)	(P)	20
(B)	Angle of velocity with horizontal after 4s (in degrees)	(Q)	80
(C)	Maximum height (in metres)	(R)	45
(D)	Horizontal range (in metres)	(S)	30
		(T)	60

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A particle is moving in x-y plane. At certain instant, the components of its velocity and acceleration are as follows ; $V_x = 3$m/s, $V_y = 4$m/s, $a_x = 2$ m/s^2 and $a_y = 1$ m/s^2. The rate of change of speed at this moment is :-

(A) $\sqrt{10}\ \text{m/s}^2$ (B) 4 m/s^2 (C) 10 m/s^2 (D) 2 m/s^2

2. A particle leaves the origin with an initial velocity $\vec{v} = \left(3\hat{i} + 4\hat{j}\right) ms^{-1}$ and a constant acceleration $\vec{a} = \left(-\hat{i} - 0.5\hat{j}\right) ms^{-2}$. When the particle reaches its maximum x–coordinate, what is the y–coordinate?

(A) $\frac{27}{4}$m (B) $\frac{37}{4}$m (C) $\frac{29}{4}$m (D) $\frac{39}{4}$m

3. The position vector of a particle is determined by $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The distance travelled in first 10 sec is :-

(A) 100 m (B) 150 m (C) 500 m (D) 300 m

4. A point moves in x–y plane according to the law x = 4sin6t and y = 4(1 – cos6t). The distance traversed by the particle in 4 seconds is (x and y are in metres)

(A) 96 m (B) 48 m (C) 24 m (D) 108 m

5. A particle moves in the x-y plane. It x and y coordinates vary with time t according to equations $x = t^2 + 2t$ and $y = 2t$. Possible shape of path followed by the particle is

(A) Straight line (B) Circle

(C) Parabola (D) More information is required to decide.

6. Particle is dropped from the height of 20 m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 m/s^2. Find the horizontal displacement of the particle till it reaches ground.

(A) 6 m (B) 10 m (C) 12 m (D) 24 m

7. A projectile is fired with a velocity u making an angle θ with the horizontal. What is the magnitude of change in velocity when it is at the highest point ?

(A) u cos θ (B) u (C) u sin θ (D) u cos θ – u

8. A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4m from it. The height of the wall is

(A) 3/4 m (B) 2/3 m (C) 4/3m (D) 1/3 m

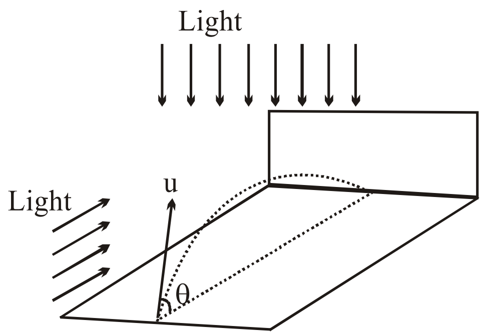
9. A ball was thrown by a boy A at angle 60° with horizontal at height 1m from ground. Boy B is running in the plane of motion of ball and catches the ball at height 1m from ground. He finds the ball falling vertically. If the boy is running at a speed 20 km/hr. Then the velocity of projection of ball is-

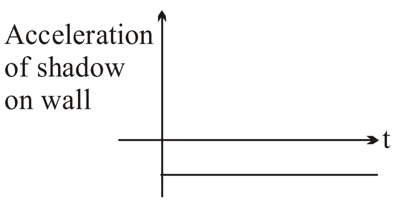
(A) 20 km/hr (B) 30 km/hr (C) 40 km/hr (D) 50 km/hr

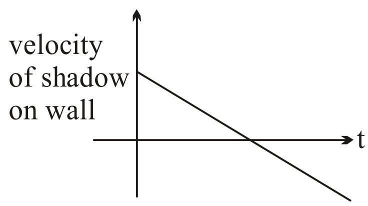
10. A light body is projected with a velocity $(10\hat{i}+20\hat{j}+20\hat{k})$ ms^{-1}. Wind blows along X-axis with an acceleration of 2.5 ms^{-2}. If Y-axis is vertical then the speed of particle after 2 second will be (g = 10 ms^{-2})

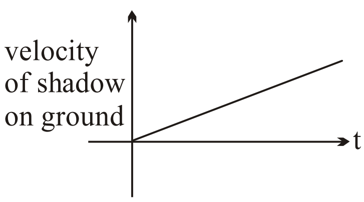
(A) 25 ms^{-1} (B) $10\sqrt{5}$ ms^{-1} (C) 30 ms^{-1} (D) None of these

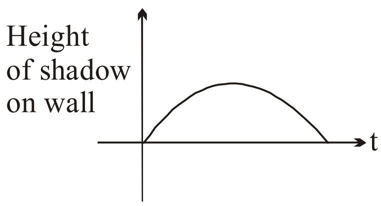
11. A projectile is projected as shown in figure. A proper light arrangement makes a shadow on the wall as well as on the floor ? Which of the following graphs is incorrect.



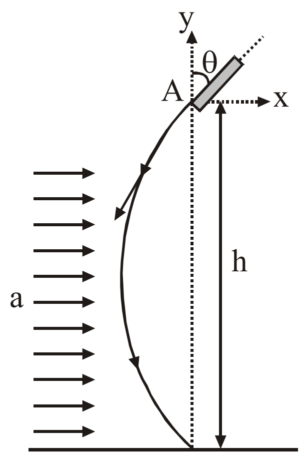
(A)


(B)


(C)


(D)


12. A particle is ejected from the tube at A with a velocity v at an angle θ with the vertical y–axis. A strong horizontal wind gives the particle a constant horizontal acceleration a in the x–directions. If the particle strikes the ground at a point directly under its released position and the downward y–acceleration is taken as g then



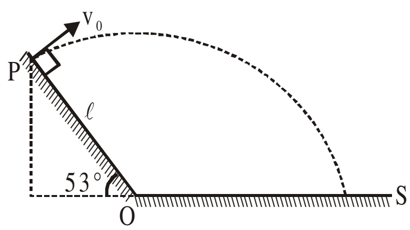
(A) $h = \dfrac{2v^2 \sin\theta \cos\theta}{a}$

(B) $h = \dfrac{2v^2 \sin\theta \cos\theta}{g}$

(C) $h = \dfrac{2v^2}{g} \sin\theta \left(\cos\theta + \dfrac{a}{g} \sin\theta \right)$

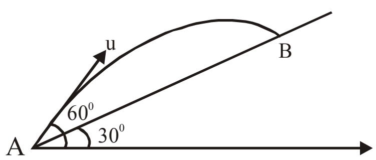
(D) $h = \dfrac{2v^2}{a} \sin\theta \left(\cos\theta + \dfrac{g}{a} \sin\theta \right)$

13. A stone is projected from point P on the inclined plane with velocity v_0 = 10 m/s directed perpendicular to the plane. The time taken by the stone to strike the horizontal ground S is (Given PO = ℓ = 10 meter)



(A) 1.5 sec (B) 1.4 sec (C) 2 sec (D) 2.3 sec

14. Time taken by the projectile to reach from A to B is t. Then the distance AB is equal to :-

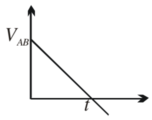


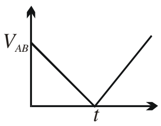
(A) $\dfrac{ut}{\sqrt{3}}$ (B) $\dfrac{\sqrt{3}ut}{2}$ (C) $\sqrt{3}$ ut (D) 2 ut

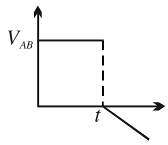
15. A particle is projected from a point P (2 m, 0 m, 0 m) with a velocity 10 m/s making an angle 45° with the horizontal. The plane of projectile motion passes through a horizontal line PQ which makes an angle of 37° with positive x–axis and xy plane is horizontal. The coordinates of the point where the particle will strike the line PQ is (g = 10 m/s^2)

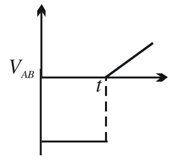
(A) (10 m, 6 m, 0 m) (B) (8 m, 6 m, 0 m) (C) (10 m, 8 m, 0 m) (D) (6 m, 10 m, 0 m)

16. A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h. Simultaneously another body B is dropped from height h. It strikes the ground and does not rebound. The velocity of A relative to B v/s time graph is best represented by (upward direction is positive)

(A) 

(B) 

(C) 

(D) 

17. An object moves to the East across a frictionless surface with constant speed. A person then applies a constant force to the North on the object. What is the resulting path that the object takes?

(A) A straight line path partly Eastward, partly Northward

(B) A straight line path totally to the North

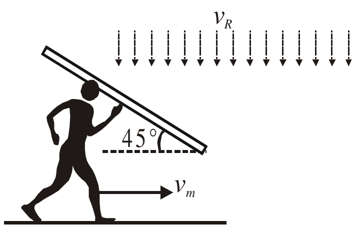
(C) A parabolic path opening toward the North

(D) A parabolic path opening toward the East

18. A particle is thrown from a stationary platform with velocity v at an angle of 60° with the horizontal. The range obtained is R. If the platform moves horizontally in the direction of target with velocity v, the range will increase to :

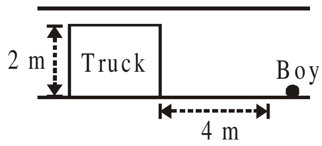
(A) $\frac{3R}{2}$ (B) $\frac{5R}{2}$ (C) 2 R (D) 3 R

19. On a particular day rain drops are falling vertically at a speed of 5 m/s. A man holding a plastic board is running to escape from rain as shown. The lower end of board is at a height half that of man and the board makes 45° with horizontal. The maximum speed of man so that his feet does not get wet, is



(A) 5 m/s (B) $5\sqrt{2}$ m/s (C) $5/\sqrt{2}$ m/s (D) zero

20. A 2 m wide truck is moving with a uniform speed of 8 m/s along a straight horizontal road. A pedestrian starts crossing the road at an instant when the truck is 4 m away from him. The minimum constant velocity with which he should run to avoid an accident is :-



(A) $1.6\sqrt{5}$ m/s (B) $1.2\sqrt{5}$ m/s (C) $1.2\sqrt{7}$ m/s (D) $1.6\sqrt{7}$ m/s

21. Two trucks are moving on parallel tracks. A person on one truck projects a ball vertically upward then path of the ball as seen by four observers: from the ground, from the second truck moving with same velocity as that first truck, from the second truck moving with speed greater than first one in same direction and from the second truck moving with speed less than the first truck in same direction are:

(A) Parabola, Parabola, Parabola and Parabola

(B) Straight line, Straight line, Parabola and Parabola

(C) Parabola, Straight line, Parabola and Parabola

(D) None of these

22. Man A sitting in a car moving at 54 km/hr observes a man B in front of the car crossing perpendicularly the road of width 15 m in three seconds. Then the velocity of man B will be

(A) $5\sqrt{10}$ towards the car (B) $5\sqrt{10}$ away from the car

(C) 5 m/s perpendicular to the road (D) None

23. A swimmer swims in still water at a speed = 5 km/hr. He enters a 200 m wide river, having river flow speed = 4 km/hr at point A and proceeds to swim at an angle of 127° with the river flow direction. Another point B is located directly across A on the other side. The swimmer lands on the other bank at a point C, from which he walks the distance CB with a speed = 3 km/hr. The total time in which he reaches from A to B is

(A) 5 minutes (B) 4 minutes (C) 3 minutes (D) None

24. A man wishes to swim across a river 400 m wide flowing with a speed of 3m/s, such that he reaches the point just infront on the other bank in time not greater than 100s. The angle made by the direction he swims and river flow direction is :-

(A) 90° (B) 127° (C) 150° (D) 143°

25. An observer on ground sees a boat cross a river of width 800 m perpendicular to its stream in 200seconds. He also finds a man on a raft floating at speed of 3 m/s with river. The distance travelled by boat as seen by man on the raft in crossing the river is-

(A) 800 m (B) 1000m (C) 1200m (D) 1600m

26. A boatman moves his boat with a velocity 'v' (relative to water) in river and finds to his surprise that velocity of river 'u' (with respect to ground) is more than 'v'. He has to reach a point directly opposite to the starting point on another bank by travelling minimum possible distance. Then

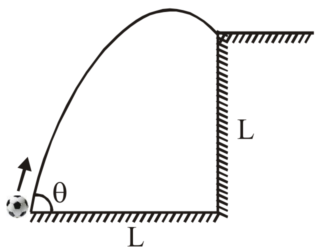
(A) he must steer the boat (with velocity v) at certain angle with river flow so that he can reach the opposite point on other bank directly.

(B) his velocity 'v' must be towards directly opposite point, So, that he can travel rest of distance by walking on other bank to reach the directly opposite point.

(C) boatman should maintain velocity v of boat at certain angle greater than 90° with direction of river flow to minimize drifting and then walk rest of distance on other bank.

(D) boat velocity 'v' should be at an angle less than 90° with direction of river flow to minimize the drift and then walk to the point.

27. A ball is thrown at an angle θ up to the top of a cliff of height L, from a point at a distance L from the base, as shown in figure. Assuming that one of the following quantities is the initial speed required to make the ball hit right at the edge of the cliff, which one is it :-



(A) $\sqrt{\dfrac{gL}{2(\tan\theta-1)}}$ (B) $\dfrac{1}{\cos\theta}\sqrt{\dfrac{gL}{2(\tan\theta-1)}}$ (C) $\dfrac{1}{\cos\theta}\sqrt{\dfrac{gL}{2(\tan\theta+1)}}$ (D) $\sqrt{\dfrac{gL\tan\theta}{2(\tan\theta+1)}}$

28. A particle is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options is/are incorrect (mass = 1kg) :

(A) The maximum height reached by the projectile can be 0.25 m.

(B) The minimum velocity during its motion can be $\sqrt{5}$ m/s

(C) The time taken for the flight can be $\sqrt{3/5}$ s.

(D) Minimum kinetic energy during its motion can be 6 J.

MULTIPLE CORRECT TYPE QUESTION

29. A particle is moving with a position vector, $\vec{r} = \left[a_0 \sin(2\pi t)\hat{i} + a_0 \cos(2\pi t)\hat{j}\right]$. Then-

(A) Magnitude of displacement of the particle between time t = 4 sec and t = 6 sec is zero

(B) Distance travelled by the particle in 1 sec is $2\pi a_0$

(C) The speed of particle in the whole motion is constant and equal to $2\pi a_0$.

(D) None of these

30. A point mass is moving in the x-y plane. Its acceleration is a constant vector perpendicular to the x-axis. Which of the following do/does not change with time?

(A) only y-component of its velocity vector

(B) only x-component of its velocity vector

(C) only y-component of its acceleration vector

(D) only x-component of its acceleration vector

31. A ball is thrown from ground such that it just crosses two poles of equal height kept 80 m apart. The maximum height attained by the ball is 80 m. When the ball passes the first pole, its velocity makes 45° with horizontal. The correct alternatives is/are :- ($g = 10\ m/s^2$)

(A) Time interval between the two poles is 4 s.

(B) Height of the pole is 60 m.

(C) Range of the ball is 160 m.

(D) Angle of projection is $\tan^{-1}(2)$ with horizontal.

32. Position vector of a particle is expressed as function of time by equation $\vec{r} = 2t^2 + (3t-1)\hat{j} + 5\hat{k}$, where r is in meters and t is in seconds.

(A) It always moves in a plane that is parallel to the x-y plane.

(B) At the instant $t = 0$ s, it is observed at point (0 m, –1 m, 5 m), moving with velocity 3 m/s in the positive y-direction.

(C) Its acceleration vector is uniform.

(D) It is an example of three dimensional motion.

33. A projectile is thrown with speed u into air from a point on the horizontal ground at an angle θ with horizontal. If the air exerts a constant horizontal resistive force on the projectile then select correct alternative(s).

(A) At the farthest point, the velocity is horizontal.

(B) The time for ascent equals the time for descent.

(C) The path of the projectile may be parabolic.

(D) The path of the projectile may be a straight line.

34. A block is thrown horizontally with a velocity of 2 m/s (relative to ground) on a belt, which is moving with velocity 4 m/s in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 s it was dropped then choose the correct statement(s) :-

(A) Displacement with respect to ground is zero after 2.66 s and magnitude of displacement with respect to ground is 12 m after 4 s.

(B) Magnitude of displacement with respect to ground in 4 s is 4 m.

(C) Magnitude of displacement with respect to belt in 4 s is 12 m.

(D) Displacement with respect to ground is zero in 8/3 s.

35. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself

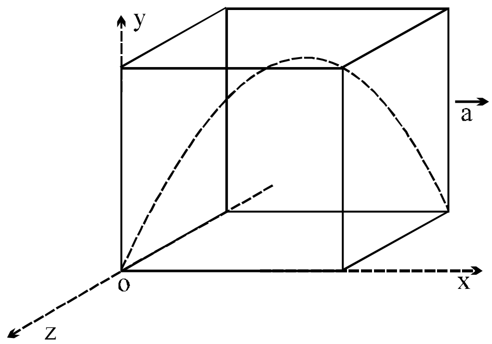
(A) The ball will always return to him.

(B) The ball will never return to him.

(C) The ball will return to him if the cart moves with constant velocity.

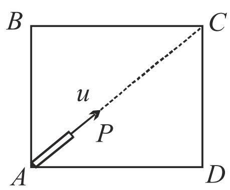
(D) The ball will fall behind him if the cart moves with some positive acceleration.

36. A cubical box dimension L = 5/4 metre starts moving with an acceleration $\vec{a} = 0.5\ \text{m/s}^2\ \hat{i}$ from the state of rest. At the same time, a stone is thrown from the origin with velocity $\vec{V} = v_1\hat{i} + v_2\hat{j} - v_3\hat{k}$ with respect to earth. Acceleration due to gravity $\vec{g} = 10\text{m/s}^2(-\hat{j})$. The stone just touches the roof of box and finally falls at the diagonally opposite point then :



(A) $v_1 = \dfrac{3}{2}$ (B) $v_2 = 5$ (C) $v_3 = \dfrac{5}{4}$ (D) $v_3 = \dfrac{5}{2}$

37. A large rectangular box moves vertically downward with an acceleration a. A toy gun fixed at *A* and aimed towards *C* fires a particle *P*.



(A) *P* will hit *C* if $a = g$

(B) *P* will hit the roof *BC*, if $a > g$

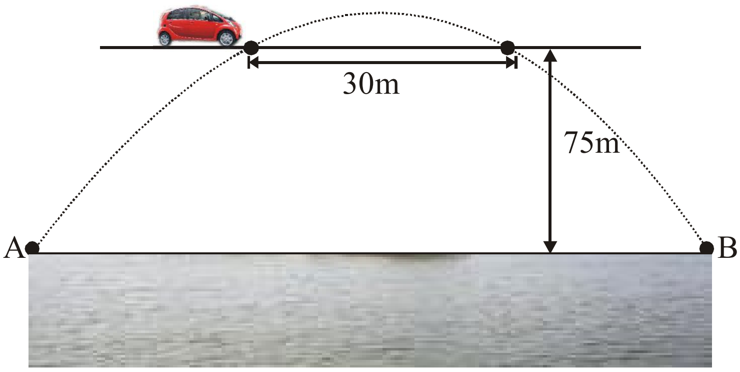
(C) *P* will hit the wall *CD* if $a < g$

(D) May be either (A), (B) or (C), depending on the speed of projection of *P*

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 38 to 40

In the figure shown there is a long horizontal bridge over a river 75 m high from water surface. A strong man throws a stone in the parallel plane of the bridge. A observer in a car travelling on the bridge finds the stone going pass by the car while ascending and also while descending between two points on the road 30 m away. The car is travelling at a speed of 15 m/s. The stone is thrown from the bank of river just at the same level of water.



38. What is the angle the velocity makes with the bridge when it goes past the car while ascending ?

(A) 30° (B) 45° (C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\tan^{-1}\left(\frac{3}{2}\right)$

39. Horizontal distance AB travelled by stone is :-

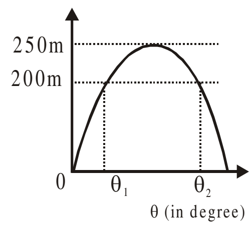
(A) 0 m (B) 75 m (C) 120 m (D) 240 m

40. What is the distance between car and stone at the instant when particle reaches at point B ?

(A) 0 m (B) 75 m (C) 120 m (D) 240 m

Paragraph for Question 41 & 42

From the ground level, a ball is to be shot with a certain speed. Graph shows the range (R) of the particle versus the angle of projection from horizontal (θ).

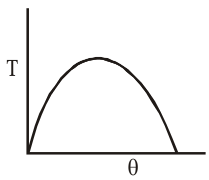


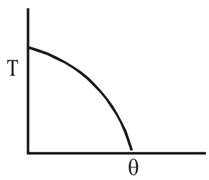
41. Values of θ_1 and θ_2 are

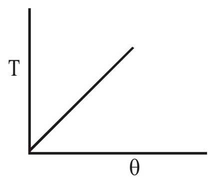
(A) 53° and 37° (B) 26.5° and 63.5°

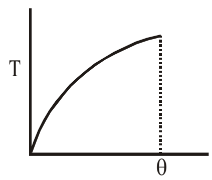
(C) 18.5° and 71.5 ° (D) 15° and 75°

42. The corresponding time of flight vs θ graph is :-

(A) 

(B) 

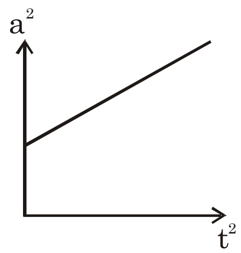
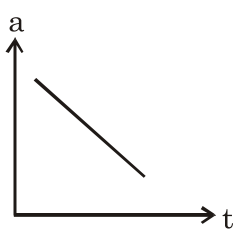
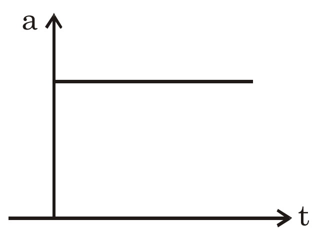
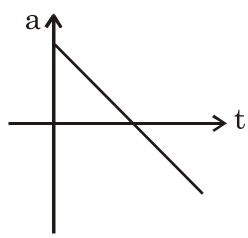
(C) 

(D) 

MATCHING LIST TYPE (4 × 4 × 4) SINGLE OPTION CORRECT
(THREE COLUMNS AND FOUR ROWS)

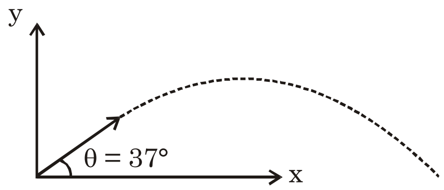
Answer Q.43, Q.44 and Q.45 by appropriately matching the information given in the three columns of the following table.

Match the following

Column–I **Time of flight (in sec)**	**Column-II** **Range (in m)** **Along the ground/ along the inclined plane**	**Column-III** **Graph**
(I) 2	(i) 3840	(P) 
(II) 1	(ii) $20\sqrt{2}$	(Q) 
(III) 10	(iii) 7	(R) 
(IV) 32	(iv) 3600	(S) 

43. A particle is projected with initial speed $u = \frac{25}{3}$ m/s as shown, here acceleration vector is given as

$a_x = 2t\hat{i}\ \text{m/s}^2;\ a_y = -10\hat{j}\ \text{m/s}^2$



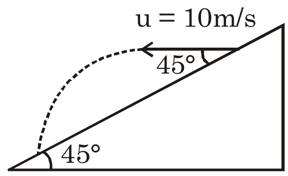
(A) (II) (iv) (P)

(B) (II) (iii) (P)

(C) (III) (iii) (S)

(D) (II) (iii) (S)

44. A particle is projected from a large–fixed incline plane as shown. Here $\vec{a} = g$ (Vertically downward) take g = 10 m/s^2.



(A) (II) (iv) (P)

(B) (IV) (ii) (S)

(C) (I) (ii) (R)

(D) (IV) (ii) (P)

45. In ground to ground projection a particle is projected at 53° from horizontal. At t = 25 sec after projection, its velocity vecotor becomes perpendicular to its initial veocity vector.

(Given $\vec{a} = g \downarrow = 10\text{m/s}^2$)

(A) (IV) (i) (R)

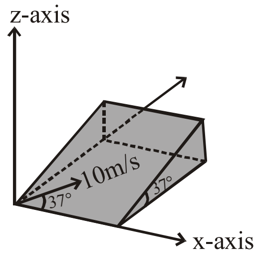
(B) (IV) (ii) (S)

(C) (II) (ii) (P)

(D) (I) (ii) (Q)

MATRIX MATCH TYPE QUESTION

46. A small ball is projected along the surface of a smooth inclined plane with speed 10m/s along the direction shown at t = 0. The point of projection is origin, z-axis is along vertical. The acceleration due to gravity is 10 m/s^2. Column-I lists values of certain parameters related to motion of ball and column-II lists different time instants. Match appropriately.

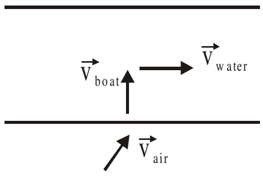


Column-I		Column-II	
(A)	Distance from x-axis is 2.25m	(P)	0.5 s
(B)	Speed is minimum	(Q)	1.0 s
(C)	Velocity makes angle 37° with x-axis	(R)	1.5 s
		(S)	2.0 s

47.

Column-I		Column-II	
(A)	Time for a boat to cross a river of width ℓ by the shortest distance ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water) $\lvert\vec{v}\rvert > \lvert\vec{u}\rvert$	(P)	$\dfrac{\ell}{\lvert\vec{v}+\vec{u}\rvert}$
(B)	Time for two particles moving with velocities $\vec{v}$ and $\vec{u}$ in opposite directions to meet each other. (initial separation of particles is ℓ)	(Q)	$\dfrac{\ell}{\sqrt{v^2-u^2}}$
(C)	Time for a boat to cross a river of width ℓ in the shortest time ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water)	(R)	$\dfrac{\ell}{\lvert\vec{v}\rvert+\lvert\vec{u}\rvert}$
(D)	Time for a boat to travel a distance ℓ downstream ($\vec{v}$ -velocity of boat with respect to water; $\vec{u}$ -velocity of water)	(S)	$\dfrac{\ell}{\lvert\vec{v}\rvert}$
		(T)	$\dfrac{\ell}{\sqrt{u^2+v^2}}$

48. A boat is being rowed in a river. Air is also blowing. Direction of velocity vectors of boat, water and air in ground frame are as shown in diagram.



Column-I	**Column-II (possible directions)**
(A) Direction in which boat is being steered	(P) $\nwarrow$
(B) Direction in which a flag on the boat may flutter	(Q) $\longrightarrow$
(C) Direction of velocity of water relative to boat	(R) $\uparrow$
(D) Direction of velocity of air relative to a piece of wood floating on river.	(S) $\searrow$

EXERCISE (JM)

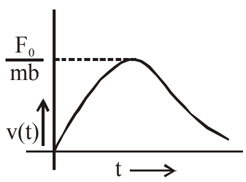
1. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is : **[AIEEE - 2010]**

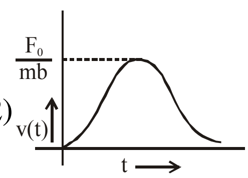
(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant (3) $y^2 = x$ + constant (4) xy = constant

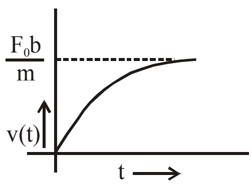
2. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :- **[AIEEE - 2011]**

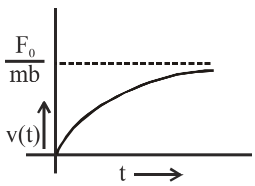
(1) $\frac{\pi}{2}\frac{v^4}{g^2}$ (2) $\pi\frac{v^2}{g^2}$ (3) $\pi\frac{v^2}{g}$ (4) $\pi\frac{v^4}{g^2}$

3. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves ? **[AIEEE - 2012]**

(1) 

(2) 

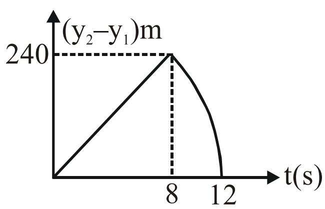
(3) 

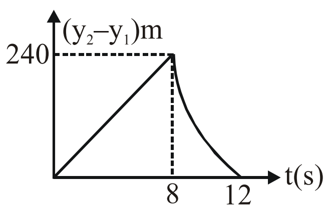
(4) 

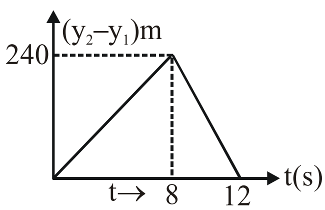
4. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s², the equation of its trajectory is : **[AIEEE - 2013]**

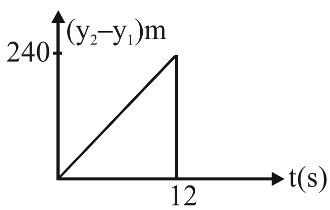
(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

5. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s²) *(The figure are schematic and not drawn to scale)* **[JEE Main-2015]**

(1) 

(2) 

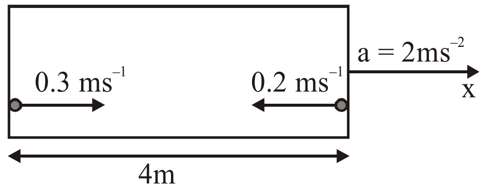
(3) 

(4) 

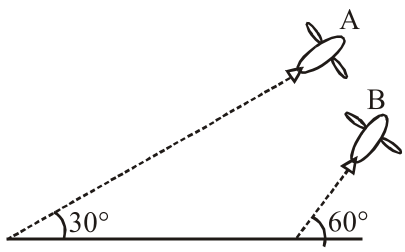
EXERCISE (JA)

1. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is **[IIT-JEE 2011]**

2. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[JEE Advanced 2014]**

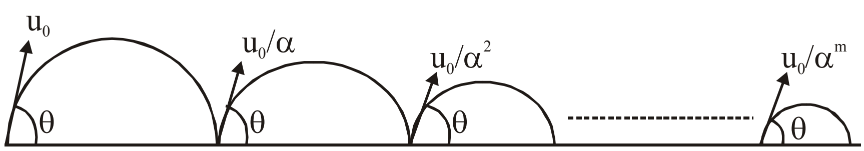


3. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is **[JEE Advanced-2014]**



4. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is................. **[JEE Advanced-2018]**

5. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1, the value of α is______ **[JEE Advanced-2019]**



ANSWER KEY

EXERCISE (S-1)

1. Ans. 5 **2. Ans.** (a) v (t) = $\left(3.0\hat{i}-4.0t\,\hat{j}\right)\hat{a}\left(t\right)=-4.0\,\hat{j}$ (b) 8.54 ms^{-1}, 70° with x-axis

3. Ans. (i) $\vec{a}=\left(-16\hat{i}-8\hat{j}\right)m/s^2$ (ii) $\vec{v}=\left(-30\hat{i}-40\hat{j}\right)m/s$ **4. Ans.** $\sqrt{2257}$ m/s

5. Ans. (i) 1.6 sec (ii) 3.2 m (iii) 9.6 m **6. Ans.** 1200 **7. Ans.** 50 m

8. Ans. 20 s **9. Ans.** 60°, 2 m/s. **11. Ans.** $20\sqrt{5}$ m/s **12. Ans.** $\frac{100}{3}$ m/s

13. Ans. (i) 100 m/s (ii) 980 m (iii) 1600 m (iv) $\left(80\hat{i}-140\hat{j}\right)$

14. Ans. $u=50(\sqrt{3}-1)$ m/s, $H=125\left(2-\sqrt{3}\right)$m **15. Ans.** 6 **16. Ans.** 34

17. Ans. (a) With $\hat{i}$ to right and $\hat{j}$ up $\vec{V}=(15\hat{i}+20\hat{j})$ m/s; (b) 23 meters; (c) It is horizontal. θ = 0]

18. Ans. 10 m/s **19. Ans.** 4 **20. Ans.** 3

21. Ans. (a) 45°, (b) 2 m/sec **22. Ans.** 5 **23. Ans.** 6

24. Ans. 200 m, 20 m/min, 12 m/min **25. Ans.** tan^{-1}(1/2) **26. Ans.** tan^{-1}(1/3)

27. Ans. (a) Somewhere down stream (b) 8 s (c) 16 m (d) Yes **28. Ans.** 6 s, 66 m

29. Ans. 30 m upstream **30. Ans.** $v_{br}>v_r,\ \theta=\sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream of line AB

EXERCISE (O-1)

1. Ans. (B) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (B)** **5. Ans. (B)** **6. Ans. (B)**

7. Ans. (D) **8. Ans. (B)** **9. Ans. (D)** **10. Ans. (B)** **11. Ans. (B)** **12. Ans. (A)**

13. Ans. (D) **14. Ans. (C)** **15. Ans. (C)** **16. Ans. (B)** **17. Ans. (B)** **18. Ans. (B)**

19. Ans. (B) **20. Ans. (C)** **21. Ans. (A)** **22. Ans. (A,B,C,D)**

23. Ans. (A,B,C,D) **24. Ans. (B,C,D)** **25. Ans. (B,C)** **26. Ans. (B)** **27. Ans. (C,D)**

28. Ans. (A,B,C) **29. Ans. (C)** **30. Ans. (D)** **31. Ans. (B)**

32. Ans. (a) Vertically (b) 53° above east (c) 53° above west (d) 53° above north (e) 45° above south

33. Ans. (a) 120 cm/s vertically150 cm/s 53° above horizontal (b) 37° above the horizontal.

34. Ans. 120 cm/s **35. Ans.** 125 cm/s at 37° from the vertical **36. Ans.** 6 m/s due north

37. Ans. (a) 9 m/s (b) 16 m/s (c) 1 m/s^2 **38. Ans.** 10 m/s, 37° north of east

39. Ans. (A) R; (B) P; (C) Q; (D) S **40. Ans. (A) → (R); (B) → (R); (C) → (P); (D) → (Q)**

EXERCISE (O-2)

1. Ans. (D)	2. Ans. (D)	3. Ans. (C)	4. Ans. (A)	5. Ans. (C)	6. Ans. (C)
7. Ans. (C)	8. Ans. (C)	9. Ans. (C)	10. Ans. (A)	11. Ans. (C)	12. Ans. (D)
13. Ans. (C)	14. Ans. (A)	15. Ans. (A)	16. Ans. (C)	17. Ans. (C)	18. Ans. (D)
19. Ans. (A)	20. Ans. (A)	21. Ans. (C)	22. Ans. (B)	23. Ans. (B)	24. Ans. (B)
25. Ans. (B)	26. Ans. (C)	27. Ans. (B)	28. Ans. (D)	29. Ans. (A,B,C)	

30. Ans. (B,C,D) **31. Ans. (A, B,C,D)** **32. Ans. (A,B,C)**

33. Ans. (B,C,D) **34. Ans. (B,C,D)** **35. Ans. (C,D)**

36. Ans. (A,B,C) **37. Ans. (A,B)** **38. Ans. (C)** **39. Ans. (C)** **40. Ans. (B)**

41. Ans. (B) **42. Ans. (D)** **43. Ans. (B)** **44. Ans. (C)** **45. Ans. (A)**

46. Ans. (A) - (P,R) ; (B) - (Q) ; (C) - (S)

47. Ans. (A) $\rightarrow$(P,Q); (B) $\rightarrow$(R); (C) $\rightarrow$(S); (D) $\rightarrow$(P,R)

48. Ans. (A)-P; (B)-Q, S; (C)-S; (D)-P,R

EXERCISE (JM)

1. Ans. (1) **2. Ans. (4)** **3. Ans. (4)** **4. Ans. (2)** **5. Ans. (1)**

EXERCISE (JA)

1. Ans. 5 **2. Ans. 8 or 2** **3. Ans. 5** **4. Ans. 30 [29.60, 30.40]** **5. Ans. 4.00**

IMPORTANT NOTES

CHAPTER 4

NEWTON'S LAWS OF MOTION & FRICTION

Chapter 04 Contents

01.	THEORY	155
02.	EXERCISE (S-1)	189
03.	EXERCISE (O-1)	195
04.	EXERCISE (O-2)	210
05.	EXERCISE (JM)	218
06.	EXERCISE (JA)	220
07.	ANSWER KEY	222

IMPORTANT NOTES

CHAPTER 4

NEWTON'S LAWS OF MOTION & FRICTION

KEY CONCEPTS

FORCE

A push or pull that one object exerts on another.

FORCES IN NATURE

There are four fundamental forces in nature :

1. Gravitational force
2. Electromagnetic force
3. Strong nuclear force
4. Weak force

TYPES OF FORCES ON MACROSCOPIC OBJECTS

(a) Field Forces or Range Forces :

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies. (ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each another.

Ex. (i) Normal forces. (ii) Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (OR GALILEO'S LAW OF INERTIA)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion *"Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".*

Inertia : Inertia is the property of the body due to which body oppose the change of its state.

Inertia of a body is measured by mass of the body. $\boxed{\text{inertia } \propto \text{mass}}$

TYPES OF INERTIA

Inertia of rest : It is the inability of a body to change, its state of rest by itself.

Examples :

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.
- When a stone hits a window pane, the glass is broken into a number of pieces whereas if the high speed bullet strikes the pane, it leaves a clean hole.

Inertia of motion : It is the inability of a body to change its state of uniform motion by itself.

Examples :

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.
- A ball thrown upwards in a running train continues to move along with the train.

Inertia of direction : It is the inability of a body to change its direction of motion by itself.

Examples :

- When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud, mudguard over the wheels stop this mud.
- A body released from a balloon rising up, continues to move in the direction of balloon.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

if $m = \text{constant}$ then $\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$

- Newton's Second Law Provides the Definition of the Concept of Force.
- **Definition of the 1 Newton (N) :–** If an object of mass one kilogram has an acceleration of $1\,ms^{-2}$ relative to an inertial reference frame, then the net force exerted on the object is one newton.

CONSEQUENCES OF NEWTON'S II LAW OF MOTION

- **Concept of inertial mass :** From Newton's II law of motion $a = \frac{F}{M}$

  i.e., the magnitude of acceleration produced by a given body is inversely proportional to mass i.e. greater the mass, smaller is the acceleration produced in the body. Thus, mass is the measure of inertia of the body. The mass given by above equation is therefore called the inertial mass.

- **An accelerated motion is the result of application of the force :**

  There may be two types of accelerated motion :

(i) When only the magnitude of velocity of the body changes : In this types of motion the force is applied along the direction of motion or opposite to the direction of motion.

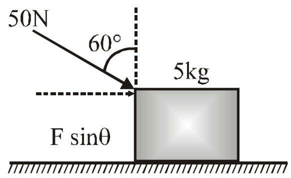
(ii) When only the direction of motion of the body changes : In this case the force is applied at right angles to the direction of motion of the body, e.g. uniform circular motion.

- **Acceleration produced in the body depends only on its mass and not on the final or initial velocity.**

Ex. A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})\,N$ produces acceleration of 1 ms^{-2} in a body. Calculate the mass of the body.

Sol. $\because$ Acceleration $a = \dfrac{|\vec{F}|}{m}$ $\qquad$ $\therefore$ mass $m = \dfrac{|\vec{F}|}{a} = \dfrac{\sqrt{6^2 + 8^2 + 10^2}}{1} = 10\sqrt{2}$ kg

Ex. A force of 50 N acts in the direction as shown in figure. The block of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.



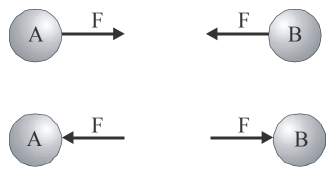
Sol. Horizontal component of the force = Fsinθ = 50sin60° = $\dfrac{50\sqrt{3}}{2}$N

acceleration of the block $a = \dfrac{F\sin\theta}{m} = \dfrac{50\sqrt{3}}{2} \times \dfrac{1}{5} = 5\sqrt{3}$ m/s^2

NEWTON'S THIRD LAW OF MOTION

The first and second laws are statements about a single object, whereas the third law is a statement about two objects.

- According to this law, every action has equal and opposite reaction. Action and reaction act on different bodies and they are simultaneous. There can be no reaction without action.
- If an object A exerts a force F on an object B, then B exerts an equal and opposite force (–F) on A.



The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive.

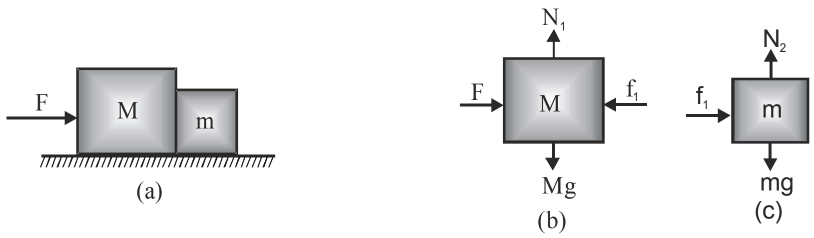
- Action and reaction never cancel each other, since they act on different bodies.

- **First law** : If no net force acts on a particle, then it is possible to select a set of reference frames, called inertial reference frames, observed from which the particle moves without any change in velocity.
- **Second law** : Observed from an inertial reference frame, the net force on a particle is proportional to the time rate of change of its linear momentum: $\dfrac{d(m\vec{v})}{dt}$
- **Third law** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.) In a specific problem, first we are required to choose a body and then we find the number of forces acting on it, and all the forces are drawn on the body, considering it as a point mass. The resulting diagram is known as free body diagram (FBD).

For example, if two bodies of masses m and M are in contact and a force F on M is applied from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figure (b) and (c).



Important Point :

Two forces in Newton's third law never occur in the same free–body diagram. This is because a free–body diagram shows forces acting on a single object, and the action–reaction pair in Newton's third law always act on different objects.

MOTION OF BODIES IN CONTACT

Case I :

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called Force of Contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass m_1 as shown in fig. (1) $F = (m_1 + m_2)\ a$.

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1 : $(F - f_1) = m_1\ a$

for body m_2 : $f_1 = m_2 a$

$\Rightarrow$ action of m_1 on m_2 : $f_1 = \dfrac{m_2 F}{m_1 + m_2}$

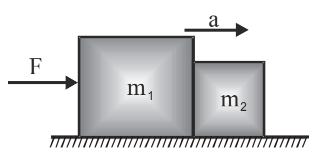


Fig.(1) : When the force F acts on mass m_1.

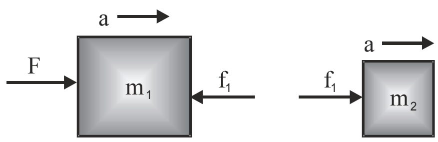


Fig. 1(a) : F.B.D. representation of action and reaction forces.

(ii) When the force F acts on the body with mass m_2 as shown in figure 2

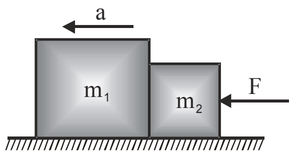


Fig. (2) : When the force F acts on mass m_2.

$F = (m_1 + m_2)\,a$

for body with mass m_2

$F - f_2 = m_2\,a$

for body m_1, $f_2 = m_1 a$

$\Rightarrow$ action on m_1 , $f_2 = \dfrac{m_1 F}{m_1 + m_2}$

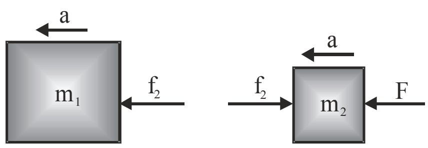
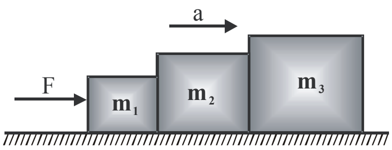


Fig. 2 (a) : F.B.D. representation of action and reaction forces.

Case II :

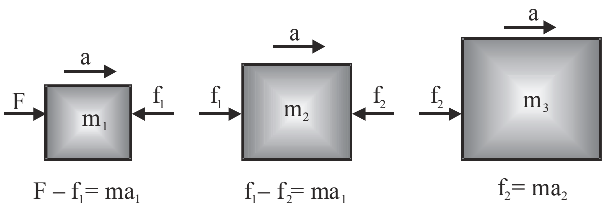
Three bodies of masses m_1, m_2 and m_3 placed one after another and in contact with each other. Suppose a force F is applied horizontally on mass m_1



then $F = (m_1 + m_2 + m_3)\,a \Rightarrow a = \dfrac{F}{(m_1 + m_2 + m_3)}$

$f_1 = \dfrac{(m_2 + m_3)F}{(m_1 + m_2 + m_3)}$

(action on both m_2 and m_3)

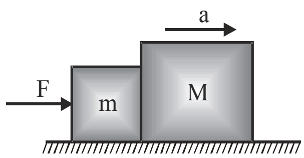


and $f_2 = \dfrac{m_3 F}{(m_1 + m_2 + m_3)}$ (action on m_3 alone)

when the force F is applied on m_3, then

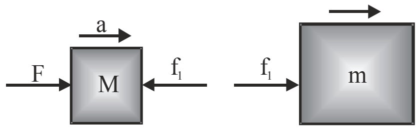
$f_1 = \dfrac{m_1 F}{(m_1 + m_2 + m_3)}$ (action on m_1 alone) and $f_2 = \dfrac{(m_1 + m_2)F}{(m_1 + m_2 + m_3)}$ (action on m_1 and m_2)

Ex. Two blocks of mass m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the block, will the force of contact remain same if F is applied to M ?



Sol. As the blocks are rigid under the action of a force F, both will move with same acceleration

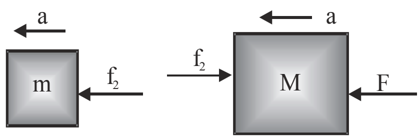
$$a = \frac{F}{m+M} = \frac{35}{2+5} = 5 \text{ m/s}^2$$



force of contact $f_1 = Ma = 5 \times 5 = 25$ N

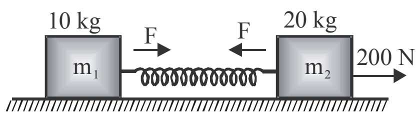
If the force is applied to M then its action on m will be

$f_2 = ma = 2 \times 5 = 10$ N.



From this problem it is clear that acceleration does not depends on the fact that whether the force is applied to m or M, but force of contact does.

Ex. Two masses 10 kg and 20 kg respectively are connected by a massless spring as shown in figure force of 200 N acts on the 20 kg mass. At the instant shown in figure the 10 kg mass has acceleration of 12 m/s², what is the acceleration of 20 kg mass?



Sol. Equation of motion for m_1 is $F = m_1a_1 = 10 \times 12 = 120$ N.

Force on 10 kg–mass is 120 N to the right. As action and reaction are equal and opposite, the reaction force F on 20 kg mass F = 120 N to the left.

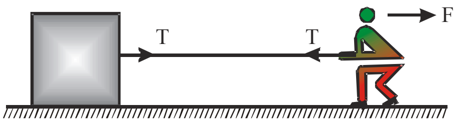
$\therefore$ Equation of motion for m_2 is $\quad 200 - F = 20\, a_2$

$\Rightarrow 200 - 120 = 20\, a_2 \qquad \Rightarrow 20\, a_2 = 80 \qquad \Rightarrow a_2 = \frac{80}{20} = 4 \text{ ms}^{-2}$

SYSTEM OF MASSES TIED BY STRINGS

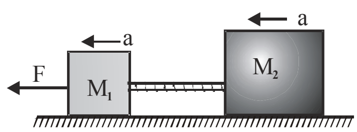
Tension in a String :

It is an intermolecular force between the atoms of a string, which acts or reacts when the string is streched.

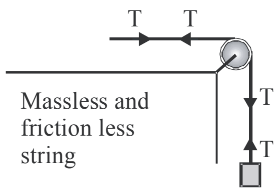


Important points about the tension in a string :

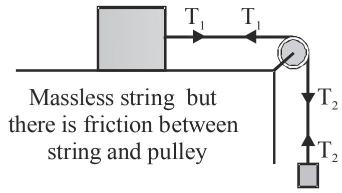
- Force of tension act on a body in the direction away from the point of contact or tied ends of the string.
- String is assumed to be inextensible so that the magnitude of accelerations of any number of masses connected through strings is always same.



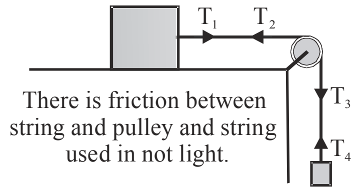
- If the string is extensible the acceleration of different masses connected through it will be different until the string can stretch.
- String is massless and frictionless so that tension throughout the string remains same.



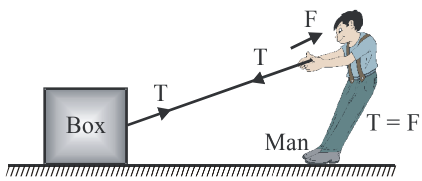
- If the string is massless but not frictionless, at every contact tension changes.



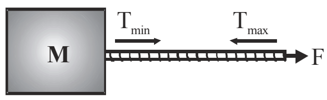
- If the string is not light, tension at each point will be different depending on the acceleration of the string.



- If a force is directly applied on a string as say man is pulling a tied string from the other end with some force the tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box will move or not, man will move or not.



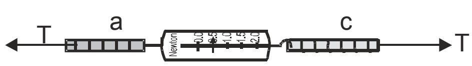
- String is assumed to be massless unless stated, hence tension in it everywhere remains the same and equal to applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.



- In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F. Here it must be noted that a string can never be compressed like a spring.

- If string is cut so that element b is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.

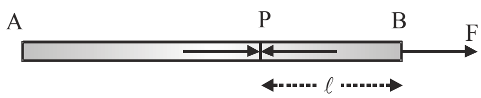
Three small elements
of a string, a, b, c



- Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.

Ex. A uniform rope of length L is pulled by a constant force F. What is the tension in the rope at a distance ℓ from the end where it is applied?

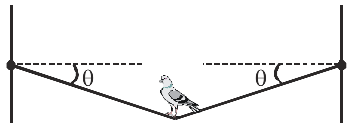
Sol. Let mass of rope is M and T be tension in the rope at point P, then. Acceleration of rope, $a = \dfrac{F}{M}$



Equation of motion of part PB is F – T = $(m\ell)$ a

$$\Rightarrow T = F - (m\ell)\,a \;=\; F - \left(\frac{M}{L}\right)(\ell)\left(\frac{F}{M}\right) = \left[1 - \frac{\ell}{L}\right] F$$

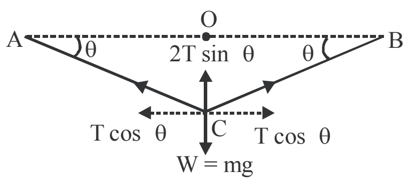
Ex. A bird with mass m perches at the middle of a stretched string



Show that the tension in the string is give by $T = \dfrac{mg}{2\sin\theta}$.

Assume that each half of the string is straight.

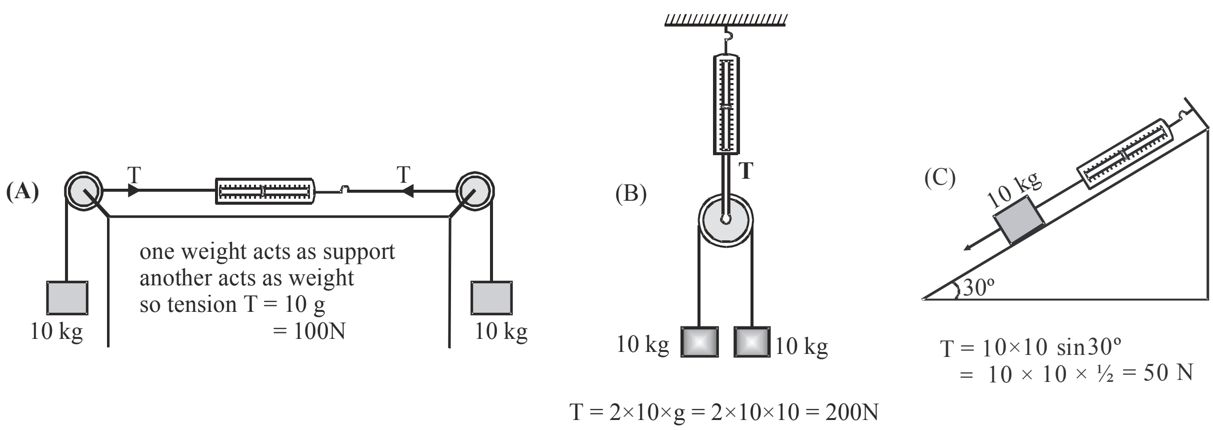
Sol. Initial position of wire = AOB. Final position of wire = ACB
Let θ be the angle made by wire with horizontal, which is very small. Resolving tension T of string in horizontal and vertical directions, we note that the horizontal components cancel while vertical components add and balance the weight.



For equilibrium 2T sinθ = W = mg $\Rightarrow T = \dfrac{W}{2\sin\theta}$ = mg/2sinθ

Ex. The system shown in figure are in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? (g = 10 m/s²)

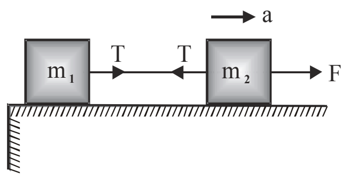
Sol.



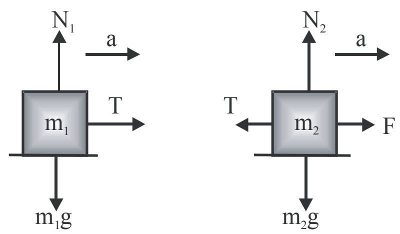
MOTION OF BODIES CONNECTED BY STRINGS

Case : I

Two bodies :



Let us consider the case of two bodies of masses m_1 and m_2 connected by a thread and placed on a smooth horizontal surface as shown in figure. A force F is applied on the body of mass m_2 in forward direction as shown. Our aim is to consider the acceleration of the system and the tension T in the thread. The forces acting separately on two bodies are also shown in the figure:



From figure $\quad T = m_1a$

and $\quad F - T = m_2a$

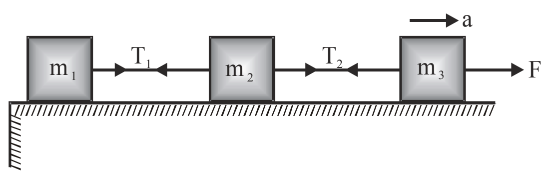
$\Rightarrow \quad F = (m_1 + m_2)a$

$\Rightarrow \quad a = \dfrac{F}{m_1 + m_2}$ & $T = \dfrac{m_1F}{m_1 + m_2}$

Case II:

Three bodies

In case of three bodies, the situation is shown in figure

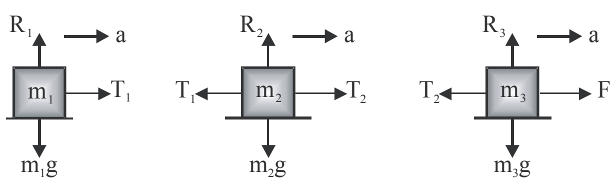


Acceleration $\quad a = \dfrac{F}{m_1 + m_2 + m_3}$,

$$T_1 = m_1a = \frac{m_1F}{m_1 + m_2 + m_3}$$

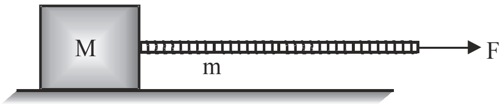
$\because$ for block of mass m_3 $\quad F - T_2 = m_3a$

$$\therefore \quad T_2 = F - \frac{m_3F}{m_1 + m_2 + m_3} = \frac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$$



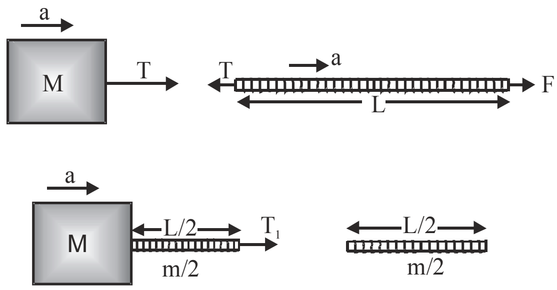
Ex. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid point.

Sol. (i) Accelaration $a = \dfrac{F}{(m+M)}$



(ii)Force exerted by rope $T = Ma = \dfrac{M.F}{(m+M)}$

(iii) $T_1 = \left(\dfrac{m}{2}+M\right)\ a = \left(\dfrac{m+2M}{2}\right)\left(\dfrac{F}{m+M}\right)$



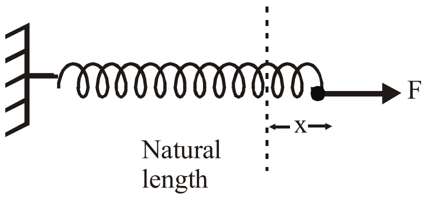
Tension in rope at midpiont $T_1 = \dfrac{(m+2M)F}{2(m+M)}$

Spring Force (According to Hooke's law) :

In equilibrium F = kx

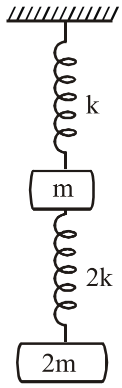
k is spring constant

Note : Spring force is non impulsive in nature.



Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

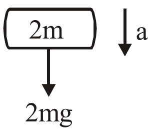
Sol. Intial stretches



$x_{upper} = \dfrac{3mg}{k}$ & $x_{lower} = \dfrac{mg}{k}$

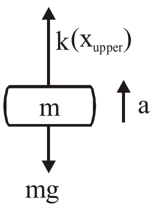
On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same. Thus,

Lower block :



$2mg = 2ma \Rightarrow a = g$

Upper block :



$k\left(\dfrac{3mg}{k}\right) - mg = ma \Rightarrow a = 2\ g$

FRAME OF REFERENCE

It is a conveniently chosen co–ordinate system which describes the position and motion of a body in space.

INERTIAL AND NON–INERTIAL FRAMES OF REFERENCE

Inertial frames of reference :

A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

- All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- All the fundamental laws of physics can be expressed as to have the same mathematical form in all the inertial frames of reference.
- The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results. It is called isotropic property of the inertial frame of reference.

Examples of inertial frames of reference :

- A frame of reference remaining fixed w.r.t. distant stars is an inertial frame of reference.
- A space–ship moving in outer space without spinning and with its engine cut–off is also inertial frame of reference.
- For practical purposes, a frame of reference fixed to the earth can be considered as an inertial frame. Strictly speaking, such a frame of reference is not an inertial frame of reference, because the motion of earth around the sun is accelerated motion due to its orbital and rotational motion. However, due to negligibly small effects of rotation and orbital motion, the motion of earth may be assumed to be uniform and hence a frame of reference fixed to it may be regarded as inertial frame of reference.

Non–inertial frame of reference :

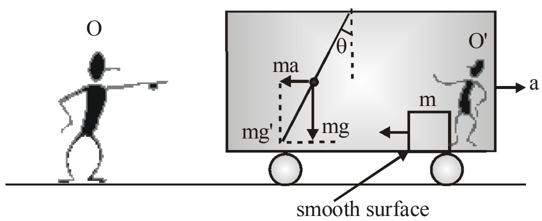
An accelerating frame of reference is called a non–inertial frame of reference.

Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Note : A rotating frame of references is a non–inertial frame of reference, because it is also an accelerating one due to its centripetal acceleration.

PSEUDO FORCE

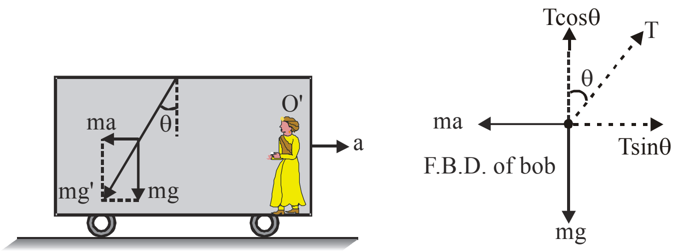
The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.



For observer O on ground train is moving with acceleration on "a" for observer O' in side the train there is pseudo force in opposite direction shown in figure.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass).
- But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

Ex. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.



Sol. Non–inertial frame of reference (Train)

F.B.D. of bob w.r.t. train. (real forces + pseudo force) : with respect to train, bob is in equilibrium

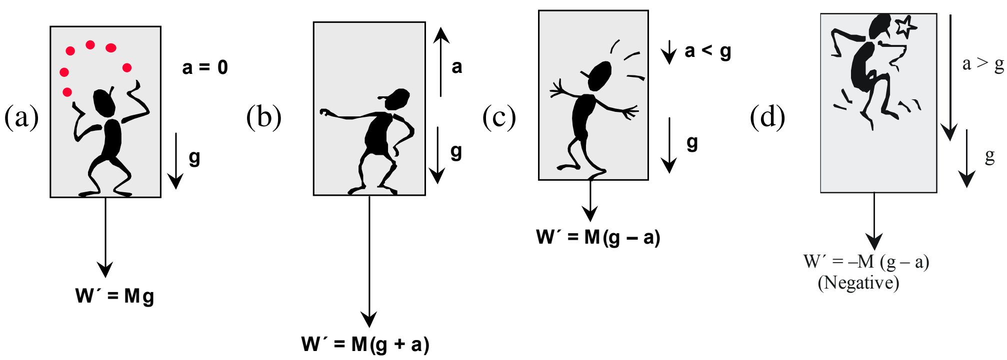
$\therefore \Sigma F_y = 0 \Rightarrow T \cos \theta = mg$ and

$\Sigma F_x = 0 \Rightarrow T \sin \theta = ma$

$$\Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$$

MOTION IN A LIFT

The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift. We consider the following cases :



Case (a) :

If the lift moving with constant velocity v upwards or downwards.

In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight $W' = Mg$ (Actual weight).

Case (b) :

If the lift is accelerated upward with constant acceleration a.

Then net forces acting on the man are (i) weight $W = Mg$ downward (ii) fictitious force $F_0 = Ma$ downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

Case (c) :

If the lift is accelerated downward with acceleration $a < g$

Then fictitious force $F_0 = Ma$ acts upward while weight of man $W = Mg$ always acts downward.

So apparent weight $W' = W + F_0 = Mg - Ma = M(g - a)$

Special Case : If $a = g$ then $W' = 0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

Case (d) :

If lift accelerates downward with acceleration $a > g$:

Then as in Case c . Apparent weight $W' = M(g - a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

Ex. A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?

Sol. (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.

(ii) In this case the acceleration is upward, the fictitious force ma acts downward,therefore apparent weight is more than actual weight i.e. $W' = m(g + a)$.

Hence scale shows a reading $= m(g + a) = \dfrac{mg\left(1+\dfrac{a}{g}\right)}{g} = \left(50+\dfrac{50a}{g}\right)$ kg wt.

Ex. Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?

Sol. Yes, since both masses experience equal fictitious forces in magnitude as well as direction.

Ex. A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?

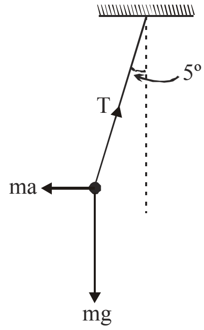
Sol. The ball is accelerated by the force T sin5°.

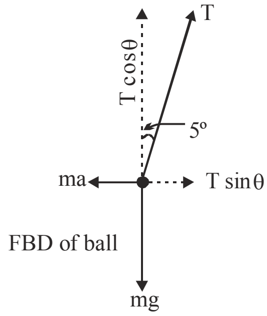
Therefore T sin5° = ma.

Vertical component $\Sigma F = 0$, so T cos5° = mg.

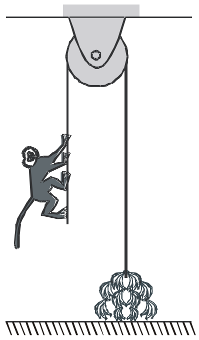
By solving a = g tan 5° = 0.0875 g

= 0.86 ms^{-2}.





Ex. A 12 kg monkey climbs a light rope as shown in figure. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas?



Sol. Effective weight of monkey $W_m = M_m(g + a)$

As per given condition $W_m = W_b$

$\Rightarrow$ $M_m (g + a) = M_b\, g$

$$\Rightarrow \quad a = \frac{(M_b - M_m)\ g}{M_m} = \left(\frac{16-12}{12}\right)\times 9.8$$

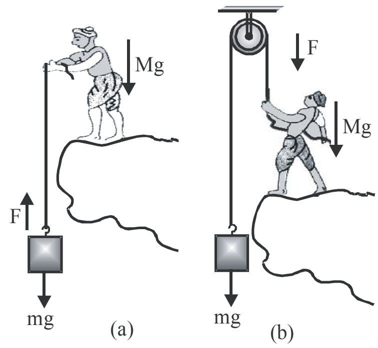
$$= \frac{9.8}{3} = 3.26 \ \ m/s^2$$

PULLEY SYSTEM

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

It is clear from example given below.

Ex. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases ? If the floor yields to a normal force of 700 N, which mode should be the man adopt to lift the block without the floor yielding ?



Sol. Mass of the block, m = 25 kg ; mass of the man, M = 50 kg

Force applied to lift the block F = mg = 25 × 9.8 = 245 N

Weight of the man, M g = 50 × 9.8 = 490 N

(a) When the block is raised by the man by applying force F in upward direction, reaction equal and opposite to F will act on the floor in addition to the weight of the man.

$\therefore$ action on the floor $\quad Mg + F = 490 + 245 = 735$ N

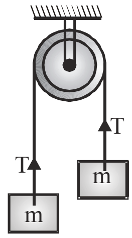
(b) When the block is raised by the mass applying force F over the rope (passed over the pulley) in downward direction, reaction equal and opposite to F will act on the floor,

$\therefore$ action on the floor $\quad Mg - F = 490 - 425 = 245$ N

floor yields to a normal force of 700 N, the mode (b) should be adopted by the man to lift block.

Some cases of pulley

I Case



$m_1 = m_2 = m$

Tension in the string $T = mg$

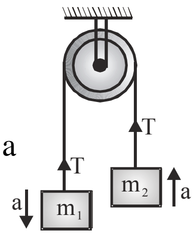
Acceleration 'a' = zero

Reaction at the suspension of the pulley

$R = 2T = 2$ mg.

$$\text{Acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

II Case



$m_1 > m_2$

now for mass m_1, $m_1 g - T = m_1 a$

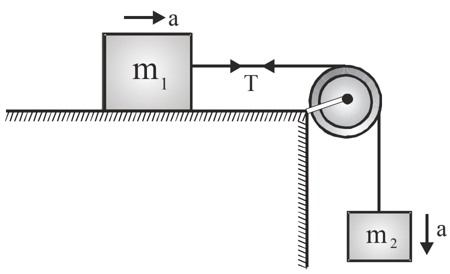
for mass m_2, $T - m_2 g = m_2 a$

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \quad \text{and} \quad T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$

$$\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

$$\text{Reaction at the suspension of pulley } R = 2T = \frac{4m_1 m_2 g}{(m_1 + m_2)}$$

III Case :

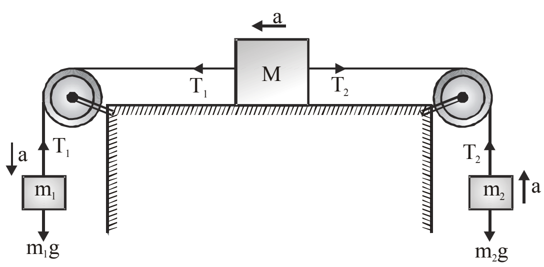


For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$

$$\text{acceleration } a = \frac{m_2 g}{(m_1 + m_2)} \quad \text{and} \quad T = \frac{m_1 m_2}{(m_1 + m_2)} g$$

IV Case :



$(m_1 > m_2)$

For m_1, $m_1 g - T_1 = m_1 a$

For m_2, $T_2 - m_2 g = m_2 a$

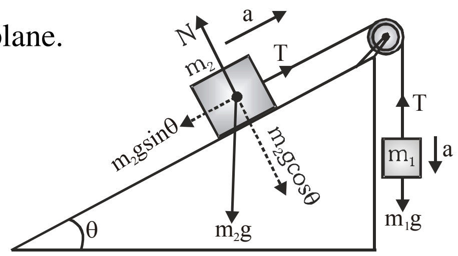
For M $T_1 - T_2 = Ma$

$$\Rightarrow a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} g,$$

$$T_1 = \frac{(2m_2 + M)m_1 g}{m_1 + m_2 + M}, \quad T_2 = \frac{(2m_1 + M)m_2 g}{m_1 + m_2 + M}$$

V Case :

Mass suspended over a pulley from another on an inclined plane.



For mass m_1 : $m_1g - T = m_1 a$

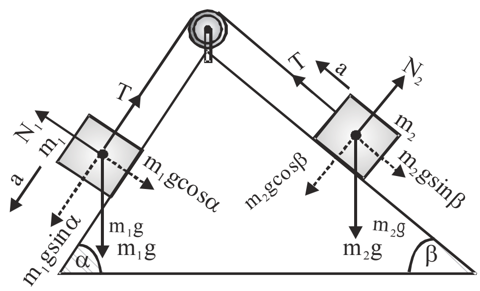
For mass m_2 : $T - m_2g\sin\theta = m_2 a$

$$\text{acceleration } a = \frac{(m_1 - m_2\sin\theta)}{(m_1 + m_2)} g$$

$$T = \frac{m_1 m_2 (1+\sin\theta)}{(m_1+m_2)} g$$

VI Case :

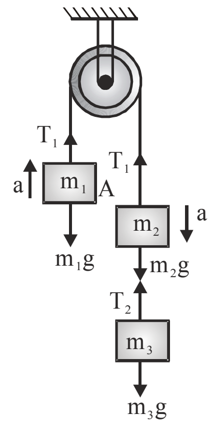
Masses m_1 and m_2 are connected by a string passing over a pulley $(m_1 > m_2)$



$$\text{Acceleration } a = \frac{(m_1\sin\alpha - m_2\sin\beta)}{(m_1+m_2)} g$$

$$\text{Tension } T = \frac{m_1 m_2(\sin\alpha + \sin\beta)}{(m_1+m_2)} g$$

VII Case :



For mass m_1 : $T_1 - m_1g = m_1a$

For mass m_2 : $m_2g + T_2 - T_1 = m_2a$

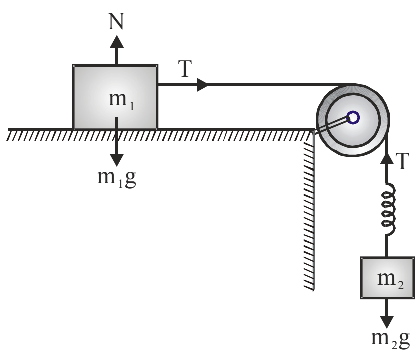
For mass m_3 : $m_3g - T_2 = m_3a$

$$\text{Acceleration } a = \frac{(m_2+m_3-m_1)}{(m_1+m_2+m_3)} g$$

we can calculate tensions T_1 and T_2 from above equations

VIII Case :

From case (iii)



$$\text{tension } T = \frac{m_1 m_2}{(m_1+m_2)} g$$

If x is the extension in the spring,

then $\quad T = kx$

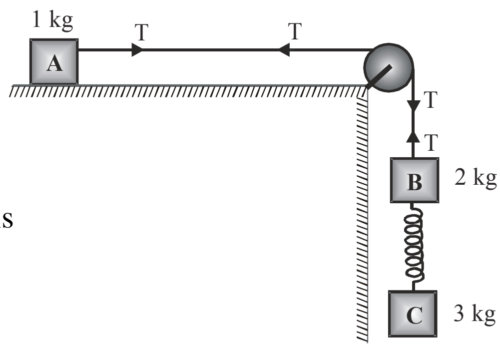
$$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1+m_2)}$$

Ex. In the system shown in figure all surface are smooth, string is massless and inextensible. Find:

(a) acceleration of the system

(b) tension in the string and

(c) extension in the spring if force constant of spring is $k = 50$ N/m (Take $g = 10$ m/s^2)

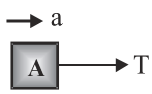


Sol. (a) In this case net pulling force $= m_c g + m_B g = 50$ N

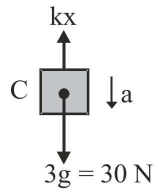
and total mass to be pulled is $(1 + 2 + 3)$ kg $= 6$ kg.

$\therefore$ Acceleration of the system is $a = \dfrac{50}{6}\ \text{ms}^{-2}$

(b) Free body diagram of 1 kg block gives $T = ma = (1)\left(\dfrac{50}{6}\right)\text{N} = \dfrac{50}{6}\text{N}$



(c) Free body diagram of 3 kg block gives



$30 - kx = ma$ but $ma = 3 \times \dfrac{50}{6} = 25\text{N}$

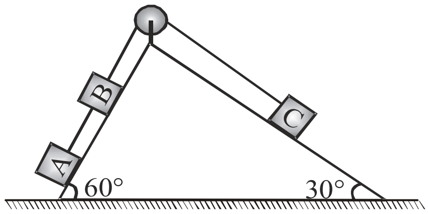
$x = \dfrac{30-25}{k} = \dfrac{5}{50} = 0.1$ m $= 10$ cm

Ex. In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find : (a) the acceleration of the system and

(b) tensions in the string

Neglect friction. ($g = 10$ ms^{-2})



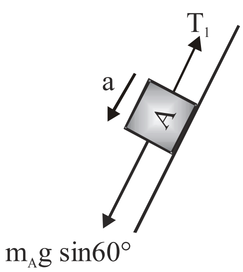
Sol. (a) In this case net pulling force

$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$= (m_A + m_B)\, g \sin 60° - m_C\, g \sin 30°$

$= (1 + 3) \times 10 \times \dfrac{\sqrt{3}}{2} - 2 \times 10 \times \dfrac{1}{2}$

$= 20\sqrt{3} - 10 = 20 \times 1.732 - 10 = 24.64$ N



Total mass being pulled $= 1 + 3 + 2 = 6$ kg

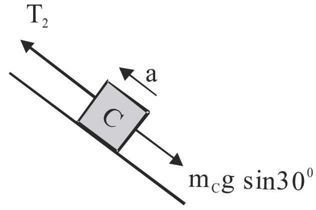
$\therefore$ Acceleration of the system $a = \dfrac{24.64}{6} = 4.1$ m/s^2

(b) For the tension T_1 in the string between A and B, $m_A g \sin 60° - T_1 = (m_A)(a)$

$\therefore T_1 = m_A g \sin 60° - m_A a = m_A (g \sin 60° - a)$

$$\Rightarrow T_1 = (1)\left(10 \times \frac{\sqrt{3}}{2} - 4.1\right) = 4.56 \text{ N}$$

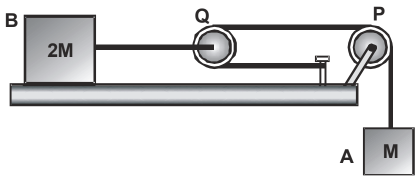
For the tension T_2 in the string between B and C.



$$T_2 - m_C g \sin 30° = m_C a$$

$$\Rightarrow T_2 = m_C (a + g \sin 30°) = 2\left[4.1 + 10\left(\frac{1}{2}\right)\right] = 18.2 \text{ N}$$

Ex.. Consider the situation shown in figure. Both the pulleys and the strings are light and all the surfaces are frictionless. Calculate (a) the acceleration of mass M,
(b) tension in the string PQ and
(c) force exerted by the clamp on the pulley P.



Sol. As pulley Q is not fixed so if it moves a distance d the length of string between P and Q will changes by 2d (d from above and d from below), i.e., M will move 2d. This in turn implies that if a is the acceleration of M the acceleration of Q and of 2M will be $\frac{a}{2}$.

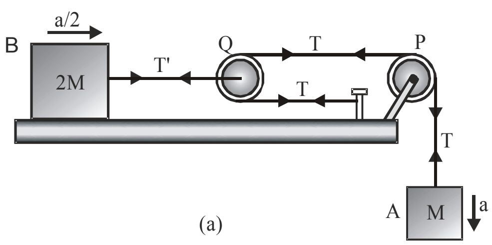
Now if we consider the motion of mass M, it is accelerated downward, so

$T = M(g - a)$ ....(i)

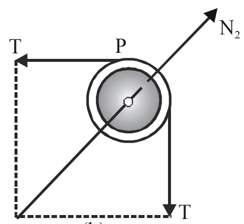
And for the motion of Q

$$2T - T' = 0 \times \frac{a}{2} = 0 \quad \Rightarrow T' = 2T \qquad \text{........(ii)}$$

And for the motion of mass 2M $\quad T' = 2M\left(\frac{a}{2}\right)$, $\Rightarrow T' = Ma$ ....(iii)



(a)



(b)

(a) From equation (ii) and (iii) as $T = \frac{1}{2}Ma$, so equation (i) reduces to

$$T = \frac{1}{2}\,Ma = M(g - a) \;\Rightarrow a = \frac{2}{3}\,g$$

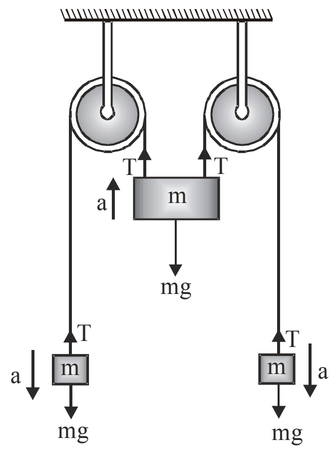
(b) So the acceleration of mass M is $\frac{2}{3}$ g while tension in the string PQ from equation (1) will be

$$T = M(g - \frac{2}{3}g) = \frac{1}{3}\,Mg$$

(c) Now from figure (b), it is clear that force on pulley by the clamp will be equal and opposite to the resultant of T and T at 90° to each other, i.e.,

$$(N_2) = \sqrt{T^2 + T^2} = \sqrt{2}\,T = \frac{\sqrt{2}}{3}\,Mg$$

Ex. Consider the double Atwood's machine as shown in the figure



(a) What is acceleration of the masses?

(b) What is the tension in each string?

Sol. (a) Here the system behaves as a rigid system, therefore every part of the system will move with same acceleration. Thus Applying newton's law

$mg - T = ma$ ...(1)

$2T - mg = ma$ ...(2)

Doubling the first equation and adding

$mg = 3\,ma \Rightarrow$ acceleration $a = \frac{1}{3}g$

(b) Tension in the string $T = m\,(g - a) = m\left(g - \frac{g}{3}\right) = \frac{2}{3}\,mg$

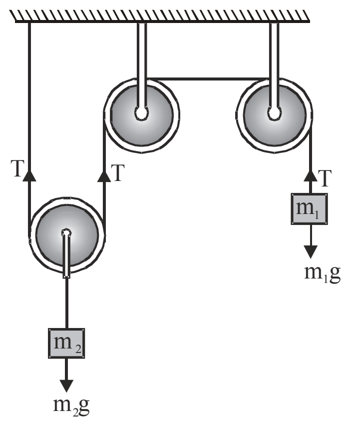
Ex. Consider the system of masses and pulleys shown in fig. with massless string and frictionless pulleys.

(a) Give the necessary relation between masses m_1 and m_2 such that system is in equilibrium and does not move.

(b) If $m_1 = 6$ kg and $m_2 = 8$ kg, calculate the magnitude and direction of the acceleration of m_1.

Sol. (a) Applying newton's law $m_2g - 2T = 0$ (because there is no acceleration) and $T - m_1g = 0$

$\Rightarrow (m_2 - 2m_1)\, g = 0 \Rightarrow m_2 = 2m_1$



(b) If the upwards acceleration of m_1 is a,

then acceleration of m_2 is $\frac{a}{2}$ downwards

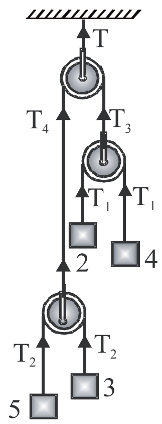
for mass m_2: $m_2g - 2T = m_2\left(\frac{a}{2}\right) \Rightarrow 2m_2g - 4T = m_2a$

for mass m_1 : $T - m_1g = m_1a$

$$\Rightarrow \quad a = \left(\frac{2m_2 - 4m_1}{m_2 + 4m_1}\right) g = \frac{2(8-12)}{8+24} g = -\frac{g}{4}$$

Negative sign shows that acceleration is opposite to considered direction i.e. it is downwards for m_1 and upwards for m_2.

Ex. In the given figure If $T_1 = 2T_2 = 50$ N then find the value of T.



Sol. As given in figure,

$$T_3 = 2T_1 = 2(2T_2) = 4T_2$$

and $\quad T_4 = 2T_2$

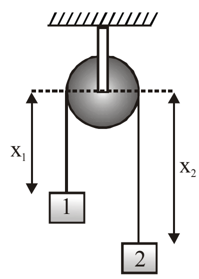
$\therefore \quad T = T_3 + T_4 = 4T_2 + 2T_2 = 6T_2$

$$= 6 \times \frac{50}{2} = 150\text{N}$$

CONSTRAINT RELATIONS

These equations establish the relation between accelerations (or velocities) of different masses attached by string(s). Normally number of constraint equations are equal to number of strings in the system under consideration.

Ex. Find the relation between acceleration of 1 and 2.



Sol. At any instant of time let x_1 and x_2 be the displacements of 1 and 2 from a fixed line. Then $x_1 + x_2$ = constant

Differentiating w.r.t. time, $v_1 + v_2 = 0$

Again differentiating w.r.t. time, $a_1 + a_2 = 0 \Rightarrow a_1 = -a_2$

So acceleration of 1 and 2 are equal but in opposite directions.

Ex. At certain moment of time, velocities of 1 and 2 both are 1 ms^{-1} upwards. Find the velocity of 3 at that moment.

Sol. $x_1 + x_4 = \ell_1$ (length of first string)

$x_2 - x_4 + x_3 - x_4 = \ell_2$ (length of second string)

$\Rightarrow\ v_1 + v_4 = 0\ \&\ v_2 + v_3 - 2v_4 = 0$

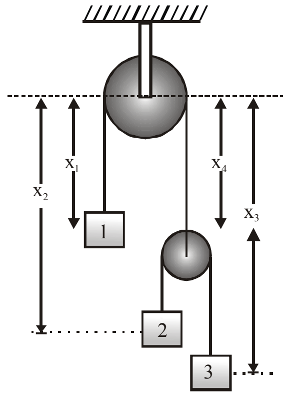
$\Rightarrow\ v_2 + v_3 + 2v_1 = 0$

Taking upward direction as positive

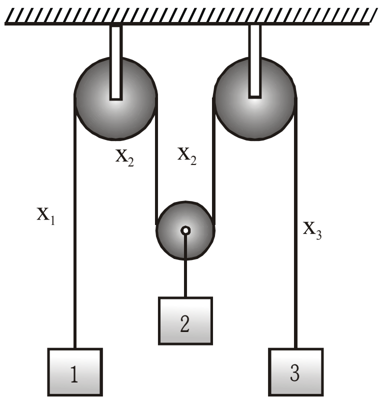
$v_1 = v_2 = 1$

so $1 + v_3 + 2 \times 1 = 0 \Rightarrow v_3 = -3\ ms^{-1}$

i.e. velocity of block 3 is 3 ms^{-1} downwards.



Ex. Find the relation between acceleration of blocks a_1, a_2 and a_3.

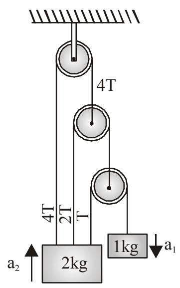


Sol. $x_1 + 2\,x_2 + x_3 = \ell$

$v_1 + 2v_2 + v_3 = 0$

$a_1 + 2a_2 + a_3 = 0$

Ex. Using contraint equation. Find the relation between a_1 and a_2.

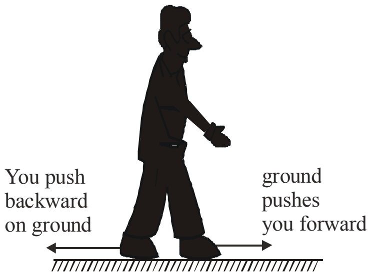


Sol. For this system $a_1T = a_2(4T+2T+T) \Rightarrow a_1=7a_2$

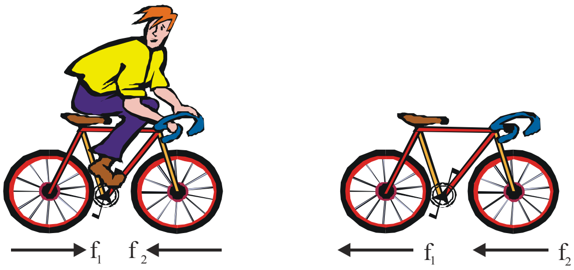
FRICTION

INTRODUCTION

Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.). Frictional forces may also exist between surfaces when there is no relative motion. Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.



- **Supporting :** Walking process can only take place because there is friction between the shoes and ground.
- **Opposing :** When a block slides over a surface the force of friction acts as an opposing force in the opposite direction of the motion
- **Both Supporting and Opposing :**
- **Pedaling :** When cyclist pedals the friction force on rear wheel acts as a supporting force and on front wheel as a opposing force.

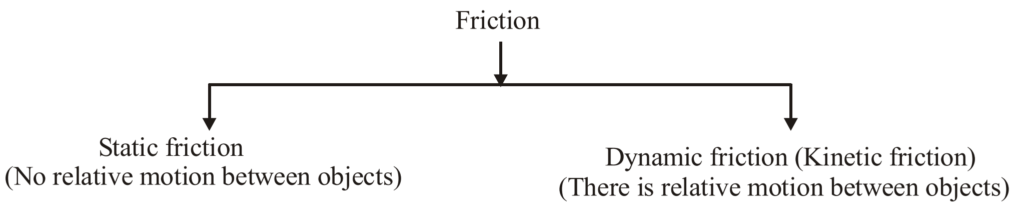


- **Non–Pedaling :** When cyclist not pedals the friction force on rear wheel & front wheel act as a opposing force.

CAUSE OF FRICTION

- **Old View :** When two bodies are in contact with each other, the irregularities in the surface of one body set interlocked in the irregularites of another surface. This locking opposes the tendency of motion.
- **Modern View :** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.

TYPES OF FRICTION



STATIC FRICTION

- It is the frictional force which is effective before motion starts between two planes in contact with each other.
- It's nature is self adjusting.
- Numerical value of static friction is equal to external force which creates the tendency of motion of body.
- Maximum value of static friction is called limiting friction. $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{applied}$

LAWS OF LIMITING FRICTION

- The magnitude of the force of limiting friction (F) between any two bodies in contact is directly proportional to the normal reaction (N) between them $F \propto N$
- The direction of the force of limiting friction is always opposite to the direction in which one body is on the verge of moving over the other.
- The force of limiting friction is independent of the apporent contact area, as long as normal reaction between the two bodies in contact remains the same.
- Limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Its value is more than to other types of friction force.

DYNAMIC FRICTION

The friction opposing the relative motion between two bodies is called dynamic or kinetic friction

$$\vec{f}_k = -(\mu_k N)$$

- This is always slightly less than the limitng friction

COEFFICIENT OF FRICTION

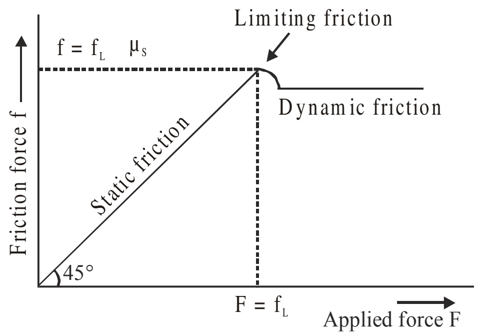
The frictional coefficient is a dimensionless scalar value which describes the ratio of the force of friction between two bodies and the force pressing them together.

- Static friction coefficient $\mu_s = \dfrac{F}{N}$
- Sliding friction coefficient $\mu_k = \dfrac{F_k}{N}$

The values of μ_s and μ_k depend on the nature of both the surfaces in contact.

GRAPH BETWEEN APPLIED FORCE AND FORCE OF FRICTION

If we slowly increase the force with which we are pulling the box, graph shows that the friction force increases with our force upto a certain critical value, f_L, the box suddenly begins to move, and as soon as it starts moving, a smaller force is required to maintain its motion as in motion friction is reduced. The friction value from 0 to f_L is known as static friction, which balances the external force on the body and prevent it from sliding. The value f_L is the maximum limit up to which the static friction acts is known as limiting friction,



after which body starts sliding and friction reduces to kinetic friction.

- When two highly polished surfaces are pressed hard, then a situation similar to welding occurs. It is called **cold welding.**
- When two copper plates are highly polished and placed in contact with each other, then instead of decreasing, the force of friction increases. This arises due to the fact that for two highly polished surfaces in contact, the number of molecules coming in contact increases and as a result the cohesive/adhesive forces increases. This in turn, increases the force of friction.

Net contact force is the resultant of normal reaction and frictional force.

APPROXIMATE COEFFICIENTS OF FRICTION

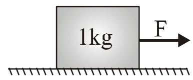
Materials	Coefficient of static friction, μ_s	Coefficien of kinetic friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Copper on cast iron	1.05	0.29
Brass on steel	0.51	0.44
Teflon on teflon	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

Ex. A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15, find the frictional forces if a horizontal force,

(a) F = 1N

(b) F = 1.96 N

(c) F = 2.5 N , is applied on a block



Sol. Maximum force of friction $f_{max} = 0.2 \times 1 \times 9.8\ N = 1.96\ N$

(a) for $F_{ext} = 1\ N$, $F_{ext} < f_{max}$

So, body is in rest means static friction is present and hence $f_s = F_{ext} = 1\ N$

(b) for $F_{ext} = 1.96\ N$, $F_{ext} = f_{max} = 1.96\ N$ so $f = 1.96\ N$

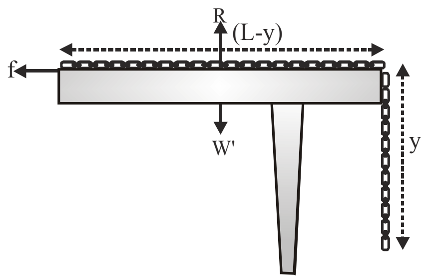
(c) for $F_{ext} = 2.5\ N$, so $F_{ext} > f_{max.}$

now body is in moving condition

$\therefore\ f_{max.} = f_k = \mu_k N = \mu_k\ mg = 0.15 \times 1 \times 9.8 = 1.47\ N$

Ex. Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can be hang from the table without sliding.

Sol. Let y be the maximum length of the chain can be hold out side the table without sliding.



Length of chain on the table = (L – y)

Weight of part of the chain on table $W' = \frac{M}{L}(L-y)g$

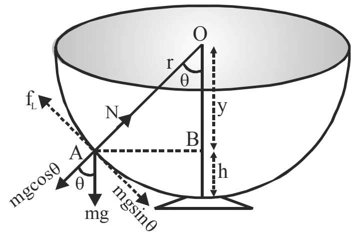
Weight of hanging part of the chain $W = \frac{M}{L}yg$

For equlibrium : limiting force of friction = weight of hanging part of the chain

$\mu R = W \Rightarrow \quad \mu W' = W \quad \Rightarrow \mu\frac{M}{L}(L-y)g = \frac{M}{L}yg \Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$

Ex. If the coefficient of friction between an insect and bowl is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bowl.

Sol. The insect will crawl up the bowl till the component of its weight along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the bowl and along the bowl,



$N = mg\cos\theta$, $f_L = mg\sin\theta \Rightarrow \tan\theta = \frac{f_L}{N} \Rightarrow \tan\theta = \mu\ [\because f_L = \mu N]$

$\Rightarrow \sqrt{\frac{(r^2-y^2)}{y}} = \mu \Rightarrow y = \frac{r}{\sqrt{1+\mu^2}}$ So $h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$

Ex. A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force F, but the body is not moving. What is the force by the surface on A.

Sol. Let f is the force of friction and N is the normal reaction,

then the net force by the surface on the body is $F=\sqrt{N^2+f^2}$

Let the applied force is F´ (varying), applied horizontally then $f \le \mu_s N$ (adjustable with f = F´).

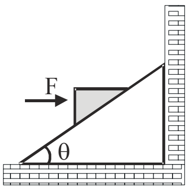
Now if F´ is zero, f = 0 and F_{min} = N = Mg

and when F´ is increased to maximum value permissible for no motion. $f = \mu_s N$,

giving $F_{max}=\sqrt{N^2+\mu_s^2 N^2} = Mg\sqrt{1+\mu_s^2}$

therefore we can write $Mg \le F \le Mg\sqrt{1+\mu_s^2}$

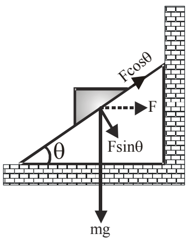
Ex. A block rest on a rough inclined plane as shown in fig. A horizontal force F is applied to it (a) Find out the force of reaction, (b) Can the force of friction be zero if yes when? and (c) Assuming that friction is not zero find its magnitude and direction of its limiting value.



Sol. (a) N = mg cos θ + F sin θ

(b) Yes, if mg sin θ = F cos θ

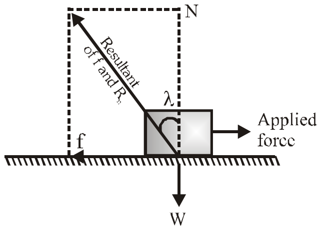
(c) f = μR = μ (mg cos θ + F sin θ); up the plane if the body has tendency to slide down and down the plane if the body has tendency to move up.



ANGLE OF FRICTION

The angle of friction is the angle which the resultant of limiting friction f_S and normal reaction N makes with the normal reaction.

It is represented by λ $\tan\lambda = \frac{f_S}{N} = \frac{\mu N}{N} = \mu$



- For smooth surface λ = 0

ANGLE OF REPOSE (θ)

If a body is placed on an inclined plane and if its angle of inclination is gradually increased, then at some angle of inclination θ the body will just on the point to slide down. The angle is called angle of repose (θ).

F_S = mg sinθ and N = mg cosθ

so $\frac{F_S}{N} = \tan\theta \Rightarrow \mu = \tan\theta$

Relation between angle of friction (λ) and angle of repose (θ)

tan λ = μ and μ = tan θ, hence tan λ = tan θ ⇒ θ = λ

Thus, angle of repose = angle of friction

Ex. A block of mass 2 kg slides down an inclined plane which makes an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.

(i) What force must be applied to the block so that the block moves down the plane without acceleration ?

(ii) What force should be applied to the block so that it can move up without any acceleration ?

Sol. Make a 'free–body' diagram of the block. Take the force of friction opposite to the direction of motion.

(i) Project forces along and perpendicular to the plane

perpendicular to plane $\quad N = mg\cos\theta$

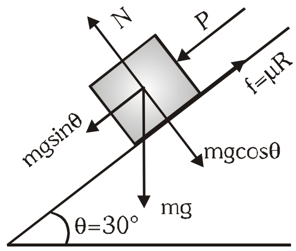
along the plane $\quad F + mg\sin\theta - f = 0$

($\because$ there is no acceleration along the plane)

$F + mg\sin\theta - \mu N = 0 \Rightarrow \quad F + mg\sin\theta = \mu\, mg\cos\theta$

$F = mg\,(\mu\cos\theta - \sin\theta) = 2 \times 9.8\,(\frac{\sqrt{3}}{2}\cos 30° - \sin 30°)$

$= 19.6\,(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2}) = 19.6\,(\frac{3}{4} - \frac{1}{2}) = 4.9\text{ N}$

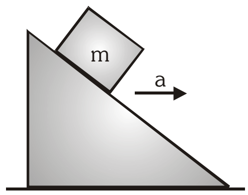


(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

$\therefore\; N = mg\cos\theta\,;\; F = mg\sin\theta + f$

$\Rightarrow F = mg\,(\mu\cos\theta + \sin\theta) = 19.6\,(\frac{3}{4} + \frac{1}{2}) = 24.5\text{ N}$

Ex. A block of mass 1 kg sits on an incline as shown in figure.



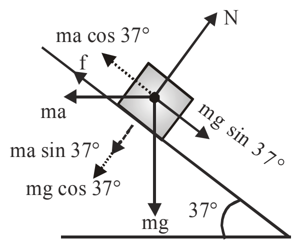
(a) What must be the frictional force between block and incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s^2 ?

(b) What is the least value μ_s can have for this to happen ?

Sol. $N = m\,(g\cos 37° + a\sin 37°) = 1(9.8 \times 0.8 + 3 \times 0.6) = 9.64\text{ N}$

$mg\sin 37° = ma\cos 37° + f$

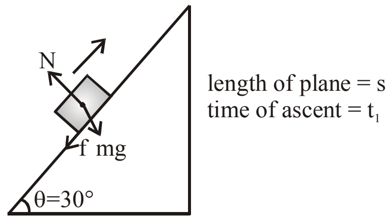
(a) $f = 1(9.8 \times 0.6 - 3 \times 0.8) = 3.48$



(b) $\because\; f = \mu N \qquad \therefore\; \mu = \frac{f}{N} = \frac{3.48}{9.64} = 0.36$

Ex. A body of mass 5×10^{-3} kg is launched up on a rough inclined plane making an angle of 30° with the horizontal. Obtain the coefficient of friction between the body and the plane if the time of ascent is half of the time of descent.

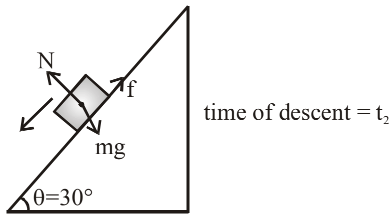
Sol. For upward motion: upward retardation $a_1 = \dfrac{\mu N + mg\sin\theta}{m}$



$a_1 = \mu g \cos 30° + g \sin 30° = (\sqrt{3}\,\mu + 1)\dfrac{g}{2}$

$\because\ s = \dfrac{1}{2} a_1 t_1^{\,2} \quad \therefore\ t_1 = \sqrt{\dfrac{2s}{a_1}} = \sqrt{\dfrac{4s}{(\sqrt{3}\mu + 1)g}}$

For downward motion: downward acceleration $a_2 = \dfrac{mg\sin\theta - \mu N}{m}$



$a_2 = g\sin 30° - g\cos 30° = (1 - \sqrt{3}\,\mu)\dfrac{g}{2}$

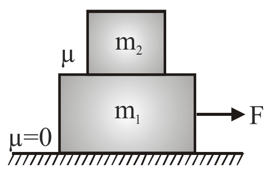
$\Rightarrow t_2 = \sqrt{\dfrac{2s}{a_2}} = \sqrt{\dfrac{4s}{(1 - \sqrt{3}\mu)g}}$

Now according to question $2t_1 = t_2$

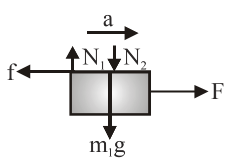
$\Rightarrow 2\sqrt{\dfrac{4s}{(\sqrt{3}\mu + 1)g}} = \sqrt{\dfrac{4s}{(1 - \sqrt{3}\mu)g}}$

$\Rightarrow \dfrac{1 - \sqrt{3}\,\mu}{1 + \sqrt{3}\mu} = \dfrac{1}{4} \quad \Rightarrow \mu = \dfrac{\sqrt{3}}{5}$

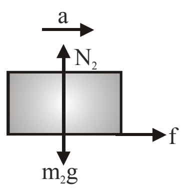
Ex. When force F applied on m_1 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ. What should be the minimum value of F so that there is no relative motion between m_1 and m_2



Sol. For m_1



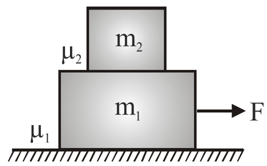
for m_2



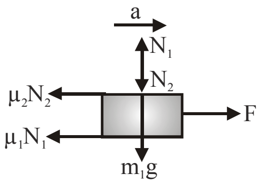
For system acceleration $a = \dfrac{F}{m_1 + m_2}$

For m_2 $\ f = m_2 a \Rightarrow \mu m_2 g = m_2\left(\dfrac{F}{m_1 + m_2}\right) \Rightarrow F_{min} = \mu(m_1 + m_2)\ g$

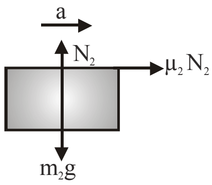
Ex. When force F applied on m_1 and the coefficient of friction between m_1 and surface. is μ_1 and the coefficient of friction between m_1 and m_2 is μ_2. What should be the minimum value of F so that there is no relative motion between m_1 and m_2.



Sol. For m_1



for m_2

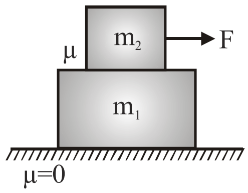


For system $a = \dfrac{F - \mu_1(m_1 + m_2)g}{m_1 + m_2}$

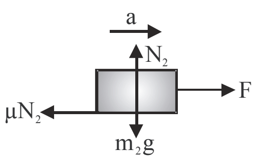
For m_2, $\mu_2\,(m_2 g) = m_2 a = m_2\left(\dfrac{F - \mu_1(m_1 + m_2)g}{m_1 + m_2}\right)$

$\Rightarrow \quad F_{min} = (m_1 + m_2)\,(\mu_1 + \mu_2)g$

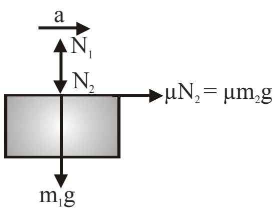
Ex. When force F applied on m_2 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ.. What should be the minimum value of F so that there is no relative motion between m_1 and m_2



Sol. For m_2



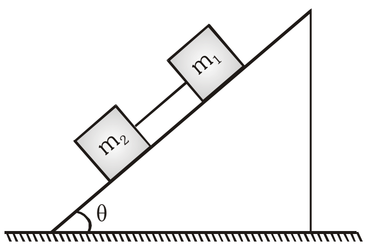
For m_1



for system : acceleration $= \dfrac{F}{m_1 + m_2}$

for m_1 : $\mu m_2 g = m_1 a = m_1\left(\dfrac{F}{m_1 m_2}\right)$, $F_{min} = (m_1 + m_2)\left(\dfrac{\mu m_2 g}{m_1}\right)$

Ex. Two blocks with masses m_1 = 1 kg and m_2 = 2 kg are connected by a string and side down a plane inclined at an angle $\theta = 45^0$ with the horizontal. The coefficient of sliding friction between m_1 and plane is μ_1 = 0.4, and that between m_2 and plane is μ_2 = 0.2. Calculate the common acceleration of the two blocks and the tension in the string.



Sol. As $\mu_2 < \mu_1$, block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration.

So acceleration of the blocks :

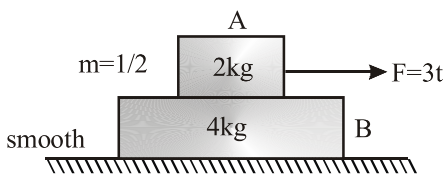
$$a = \frac{(m_1+m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1+m_2}$$

$$= \frac{(1+2)(10)\left(\frac{1}{\sqrt{2}}\right) - 0.4\times1\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}}}{1+2} = \frac{22}{3\sqrt{2}}\ ms^{-2}$$

For block m_2 : $m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - T = m_2 a \Rightarrow T = m_2 g\sin\theta - \mu_2 m_2 g\cos\theta - m_2 a$

$$= 2\times10\times\frac{1}{\sqrt{2}} - 0.2\times2\times10\times\frac{1}{\sqrt{2}} - 2\times\frac{22}{3\sqrt{2}} = \frac{2}{3\sqrt{2}}N$$

Ex. For shown situation draw a graph showing accelerations of A and B on y–axis and time on x–axis. $(g=10ms^{-2})$



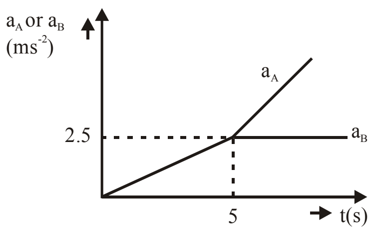
Sol. Limiting friction between A & B, $f_L = \mu m_A g = \left(\frac{1}{2}\right)(2)(10) = 10\ N$

Block B moves due to friction only. So maximum acceleration of B,

$$a_{max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5\ ms^{-2}.$$

So both the blocks move together till the common acceleration becomes 2.5 ms^{-2}, after that acceleations of B will become constant while that of A will go on increasing. Slipping will starts between A & B at 2.5 ms^{-2}

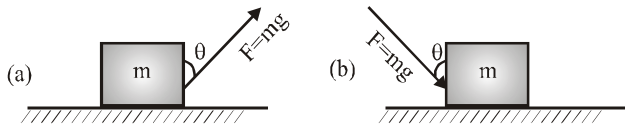
$$\Rightarrow 2.5 = \frac{F}{m_A+m_B} = \frac{3t}{6} \Rightarrow t = 5\ s$$



Hence for $t \le 5s$, $a_A = a_B = \frac{F}{m_A+m_B} = \frac{3t}{6} = \frac{t}{2}$

and for $t > 5$ s $a_B = 2.5\ ms^{-2}$, $a_A = \frac{F-f_L}{m_A} = \frac{3t-10}{2} = \frac{3}{2}t - 5$

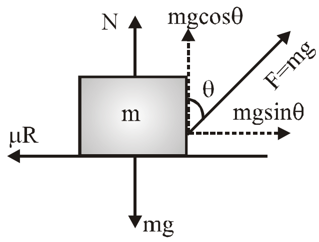
Ex. A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is μ. A force F = mg acting at an angle θ with the vertical side of the block. Find the condition for which block will move along the surface.



Sol. For (a) : normal reaction N = mg – mg cos θ, frictional force = $\mu N = \mu(mg - mg\cos\theta)$

Now block can be pulled when : Horizontal component of force $\geq$ frictional force

i.e. $mg\sin\theta \geq \mu(mg - mg\cos\theta)$



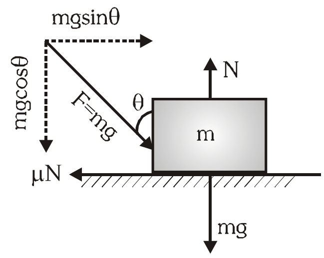
or $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq \mu(1-\cos\theta)$

or $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq 2\mu\sin^2\dfrac{\theta}{2}$ or $\cot\dfrac{\theta}{2} \geq \mu$

For (b) **:** Normal reaction N = mg + mg cos θ = mg (1 + cos θ)

Hence, block can be pushed along the horizontal surface when horizontal component of force $\geq$ frictional force

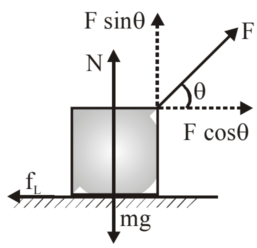
i.e. $mg\sin\theta \geq \mu\, mg(1+\cos\theta)$



or $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} \geq \mu\times 2\cos^2\dfrac{\theta}{2} \Rightarrow \tan\dfrac{\theta}{2} \geq \mu$

Ex. A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

Sol. Let the force F be applied at an angle θ with the horizontal as shown in figure.



For vertical equilibrium,

$R + F\sin\theta = mg \Rightarrow \quad N = mg - F\sin\theta$ (i)

for horizontal motion $F\cos\theta \geq f_L \Rightarrow F\cos\theta \geq \mu N$ [as $f_L = \mu N$] (ii)

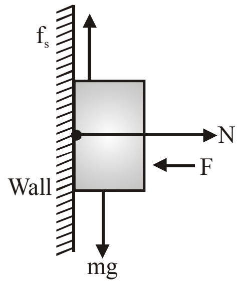
substituting value of R from equation (i) in (ii),

$$F\cos\theta \geq \mu(mg - F\sin\theta) \Rightarrow F \geq \frac{\mu mg}{(\cos\theta + \mu\sin\theta)} \quad \text{(iii)}$$

For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,

maximum value of $\cos\theta + \mu\sin\theta$ is $\sqrt{1+\mu^2}$ so that $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

Ex. A book of 1 kg is held against a wall by applying a perpendicular force F. If $\mu_S = 0.2$ then what is the minimum value of F ?



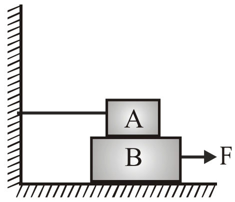
Sol. The situation is shown in fig. The forces acting on the book are–

For book to be at rest it is essential that $Mg = f_s$

But $f_{s\,max} = \mu_S N$ and $N = F$

$\therefore Mg = \mu_S F \Rightarrow F = \dfrac{Mg}{\mu_s} = \dfrac{1 \times 9.8}{0.2} = 49$ N

Ex. A is a 100 kg block and B is a 200 kg block. As shown in fig., the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. (take g =10 m/s^2).

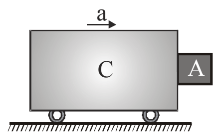


Sol. When B is tied to move, by applying a force F, then the frictional forces acting on the block B are f_1 and f_2 with limiting values, $f_1 = (\mu_S)_A m_A g$ and $f_2 = (\mu_S)_B (m_A + m_B)g$

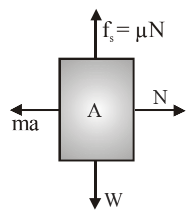
then minimum value of F should be (for just tending to move),

$F = f_1 + f_2 = 0.2 \times 100\ g + 0.3 \times 300\ g = 110\ g = 1100$ N

Ex. Consider the figure shown here of a moving cart C. If the coefficient of friction between the block A and the cart is μ, then calculate the minimum acceleration a of the cart C so that the block A does not fall.



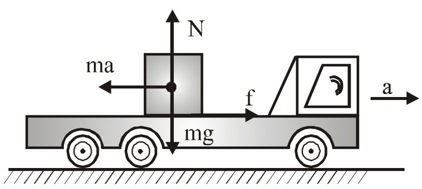
Sol. The forces acting on the block A (in block A's frame (i.e. non inertial frame) are :



For A to be at rest in block A's frame i.e. no fall,

we require $W = f_s$ $\Rightarrow mg = \mu(ma)$ Thus $a = \dfrac{g}{\mu}$

Ex. A block of mass 1kg lies on a horizontal surface in a truck, the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block. If the acceleration of the truck is 5 m/s^2.



Sol. Fictitious force on the block $F = ma = 1 \times 5 = 5N$

While the limiting friction force

$$F = \mu_s N = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88 \text{ N}$$

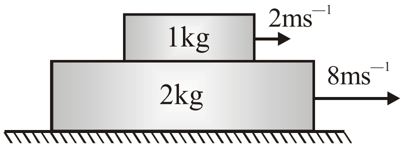
As required force F lesser than limiting friction force. The block will remain at rest in the truck and the force of friction will be equal to 5N and in the direction of acceleration of the truck.

Ex. Coefficient of friction between two blocks shown in figure is $\mu = 0.4$ The blocks are given velocities of $2ms^{-1}$ and $8\ ms^{-1}$ in the directions shown in figure. Find

(i) The time when relative motion between them will stop.

(ii) The common velocities of blocks upto that instant.

(iii) Displacement of blocks upto that instant ($g = 10\ ms^{-2}$)



Sol. (i) Frictional force between two blocks will oppose the relative motion. For 1 kg block friction support the motion & for 2 kg friction oppose the motion. Let common velocity be v then

for 1 kg $\quad v = 2 + a_1 t$ where $a_1 = \dfrac{\mu(1g)}{1} = \dfrac{0.4 \times 10}{1} = 4\ ms^{-2}$

for 2 kg $v = 8 - a_2\, t$ where $a_2 = \dfrac{\mu(1g)}{2} = \dfrac{0.4 \times 10}{2} = 2\ ms^{-2} \Rightarrow 2 + 4t = 8 - 2\,t \Rightarrow 6t = 6 \Rightarrow t = 1s$

(ii) $v = 2 + 4\,t = 2 + 4 \times 1 = 6\ ms^{-1}$

(iii) Displacement of 1 kg block from rest

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_1 = 2 \times 1 + \frac{1}{2} \times 4 \times 1^2 = 2 + 2 = 4m$$

Displacement of 2 kg block from rest

$$s = ut + \frac{1}{2}at^2 \Rightarrow s_2 = 8 \times 1 - \frac{1}{2} \times 2 \times 1^2 = 8 - 1 = 7\ m$$

Friction is a Necessary Evil :

Friction is a necessary evil. It means it has advantage as well as disadvantages. In other words, friction is not desirable but without friction, we cannot think of survival.

Disadvantages :

(i) A significant amount of energy of a moving object is wasted in the form of heat energy to overcome the force of friction.

(ii) The force of friction restricts the speed of moving vehicles like buses, trains, aeroplanes, rockets etc.

(iii) The efficiency of machines decreases due to the presence of force of friction.

(iv) The force of friction causes lot of wear and tear in the moving parts of a machine.

(v) Sometimes, a machine gets burnt due to the friction force between different moving parts.

Advantages :

(i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think of walking on the surface. That is why, we fall down while moving on a smooth surface.

(ii) The force of friction between the tip of a pen and the surface of paper helps us to write on the paper. It is not possible to write on the glazed paper as there is not force of friction.

(iii) The force of friction between the tyres of a vehicle and the road helps the vehicle to stop when brake is applied. In the absence of friction, the vehicle skid off the road when brake is applied.

(iv) moving belts remain on the rim of a wheel because of friction.

(v) The force of friction between a chalk and the black board helps us to write on the board.

Thus, we observe that inspect of various disadvantages of the friction, it is very difficult to part with it. So, friction is a necessary evil.

METHODS OF REDUCING FRICTION

As friction causes the wastage of energy so it becomes necessary to reduce the friction. Friction can be reduced by the following methods.

(i) Polishing the surface. We know, friction between rough surface is much more than between the polished surfaces. So we polish the surface to reduce the friction. The irregularities on the surface are filled with polish and hence the friction decreases.

(ii) Lubrication. To reduce friction, lubricants like oil or greese are used. When the oil or greese is put in between the two surfaces, the irregularities remain apart and do not interlock tightly. Thus, the surface can move over each other with less friction between them.

(iii) By providing the streamlined shape. When a body (e.g. bus, train, aeroplane etc.) moves with high speed, air resistance (friction) opposes its motion. The effect of air resistance on the motion of the objects (stated above) is decreased by providing them a streamlined shape.

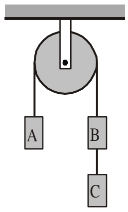
EXERCISE (S-1)

1. A force F applied to an object of mass m_1 produces an acceleration of 3.00 m/s^2. The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s^2.

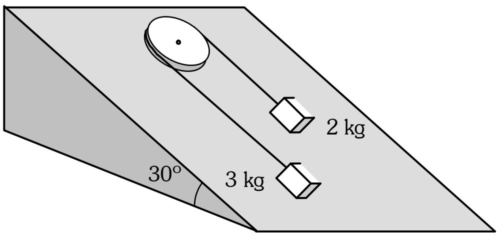
(i) What is the value of the ratio m_1/m_2 ?

(ii) If m_1 and m_2 are combined, find their acceleration under the action of the force F.

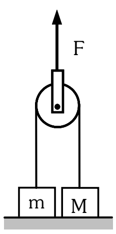
2. In the system shown, the blocks A, B and C are of weight 4W, W and W respectively. The system set free. The tension in the string connecting the blocks B and C is



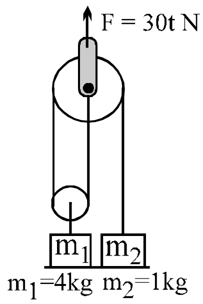
3. Two blocks of masses 2.0 kg and 3.0 kg are connected by light inextensible string. The string passes over an ideal pulley pivoted to a fixed axel on a smooth incline plane as shown in the figure. When the blocks are released, find magnitude of their accelerations.



4. In the system shown, pulley and strings are ideal. The vertically upward pull F is being increased gradually, find magnitude of F and acceleration of the 5 kg block at the moment the 10 kg block leaves the floor.

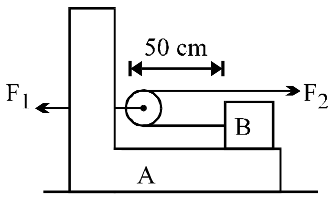


5. Force F is applied on upper pulley. If $F = 30t\,N$ where t is time in second. Find the time when m_1 loses contact with floor.

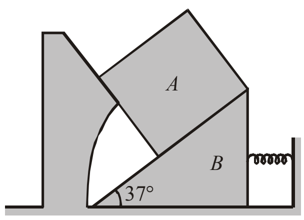


6. A 40 kg boy climbs a rope that passes over an ideal pulley. The other end of the rope is attached to a 60 kg weight placed on the ground. What is the maximum upward acceleration the boy can have without lifting the weight? If he climbs the rope with upward acceleration 2 g, with what acceleration the weight will rise up?

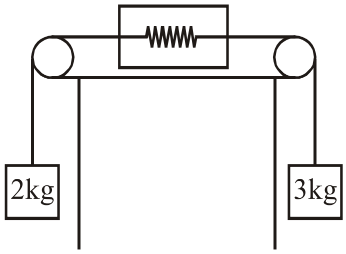
7. A 1kg block B rests as shown on a bracket A of same mass. Constant forces $F_1 = 20\text{N}$ and $F_2 = 8\text{N}$ start to act at time t = 0 when the distance of block B from pulley is 50cm. Time when block B reaches the pulley is _________.



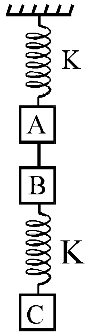
8. In the figure shown, all surfaces are smooth and block A and wedge B have mass 10 kg and 20 kg respectively. Find normal reaction between block A & B, spring force and normal reaction of ground on block B. (g= 10 m/s^2).



9. Find the reading of the massless spring balance in the given condition



10. The system shown adjacent is in equilibrium. Find the acceleration of the blocks *A*, *B* & *C* all of equal masses *m* at the instant when



(Assume springs to be ideal)

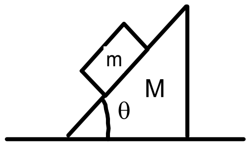
(i) The spring between ceiling & *A* is cut.

(ii) The string (inextensible) between *A* & *B* is cut.

(iii) The spring between *B* & *C* is cut.

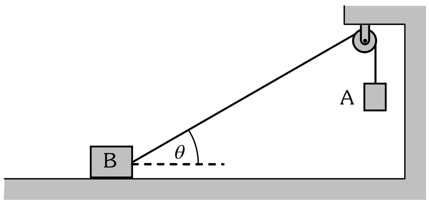
Also find the tension in the string when the system is at rest and in the above 3 cases.

11. A block of mass *m* lies on wedge of mass *M* as shown in figure.

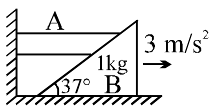


With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely.

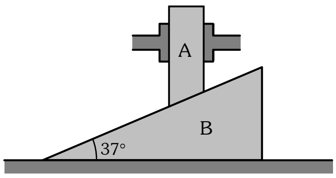
12. The block A is moving downward with constant velocity v_0. Find the velocity of the block B, when the string makes an angle θ with the horizontal



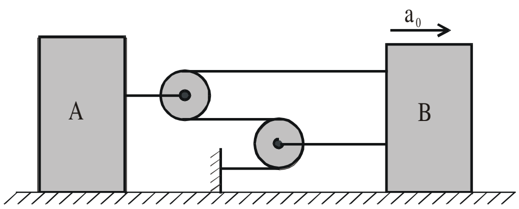
13. Find force in newton which mass *A* exerts on mass *B* if *B* is moving towards right with 3 m/s^2. Also find mass of *A*. (All surfaces are smooth)



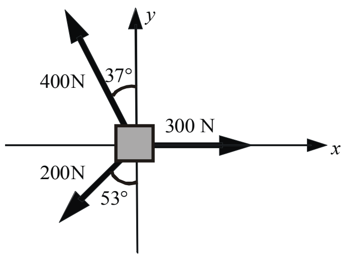
14. Rod A can slide in vertical direction pushing the triangular wedge B towards right. The wedge is moving toward right with uniform acceleration a_B. Find acceleration of the rod A.



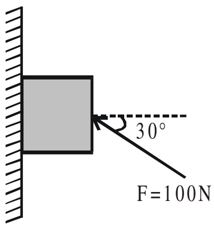
15. Calculate the relative acceleration of A w.r.t. B if B is moving with acceleration a_0 towards right.



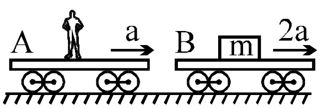
16. A block is placed on a rough horizontal plane. Three horizontal forces are applied on the block as shown in the figure. If the block is in equilibrium, find the friction force acting on the block.



17. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the wall and the surface of the block is 1/4. Calculate frictional force acting on the block.



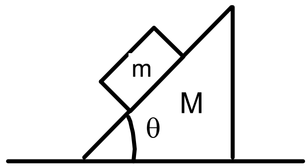
18. Two trolley A and B are moving with accelerations a and 2a respectively in the same direction. To an observer in trolley A, the magnitude of pseudo force acting on a block of mass m on the trolley B is



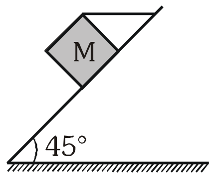
19. A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s^2. A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom.[g = 10 m/s^2]

20. A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has μ = 0.5. Plank rests on frictionless surface. Find the time when block comes to rest w.r.t. plank.

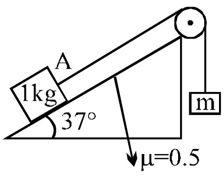
21. A block of mass *m* lies on wedge of mass *M* as shown in figure. Find the minimum friction coefficient required between wedge *M* and ground so that it does not move while block *m* slips down on it.



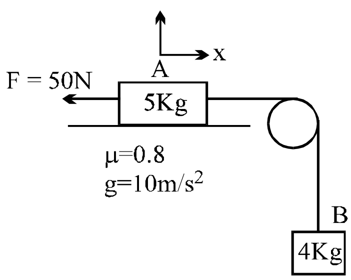
22. A block of mass 15 kg is resting on a rough inclined plane as shown in figure.The block is tied up by a horizontal string which has a tension of 50 N. Calculate the minimum coefficient of friction between the block and inclined plane.



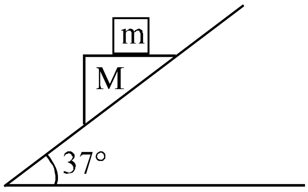
23. In the figure, what should be mass m so that block *A* slides up with a constant velocity?



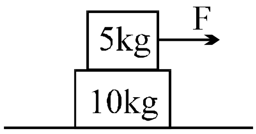
24. Find the acceleration of the blocks and magnitude & direction of frictional force between block *A* and table, if block *A* is pulled towards left with a force of 50 N.



25. Block M slides down on frictionless incline as shown. Find the minimum friction coefficient so that m does not slide with respect to M.



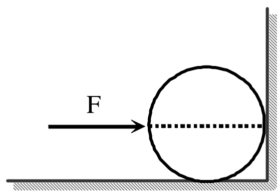
26. Coefficient of friction between 5 kg and 10 kg block is 0.5. If friction between them is 20 N. What is the value of force being applied on 5 kg. The floor is frictionless.

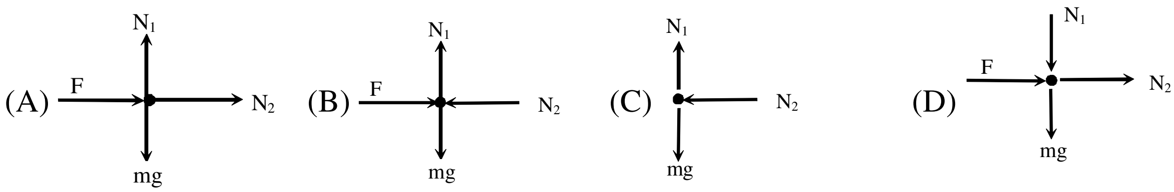


EXERCISE (O-1)

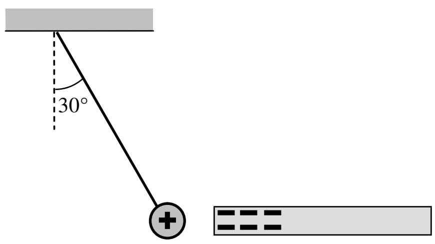
SINGLE CORRECT TYPE QUESTIONS

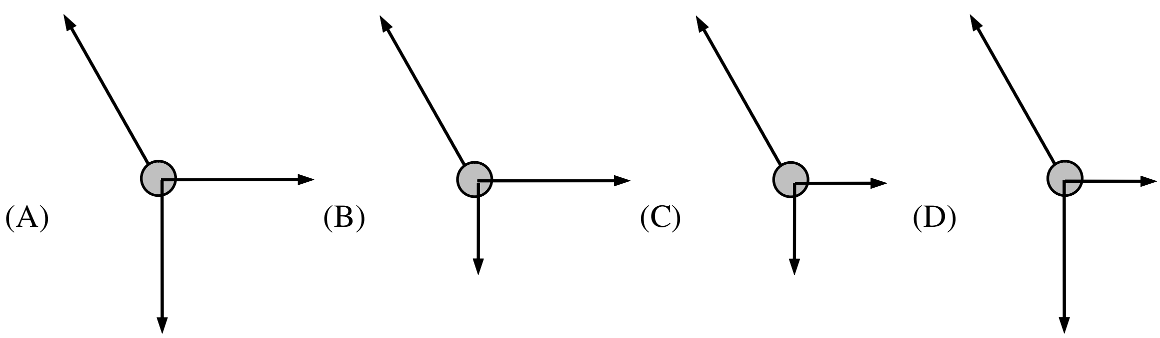
1. A ball of mass m kept at the corner as shown in the figure, is acted by a horizontal force F. The correct free body diagram of ball is





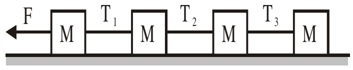
2. A small electrically charged sphere is suspended vertically from a thread. An oppositely charged rod is brought close to the sphere such that the sphere is in equilibrium displaced from the vertical by an angle of 30°. Which one of the following best represents the free body diagram for the sphere?





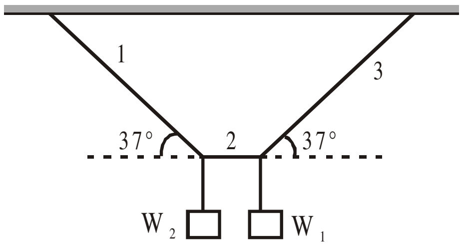
3. Under what condition(s) will an object be in equilibrium ?
(A) Only if it is at rest
(B) Only if it is moving with constant velocity
(C) Only if it is moving with constant acceleration
(D) If it is either at rest or moving with constant velocity

4. Four blocks of same mass connected by cords are pulled by force F on a smooth horizontal surface, as in figure. The tension T_1, T_2 and T_3 will be



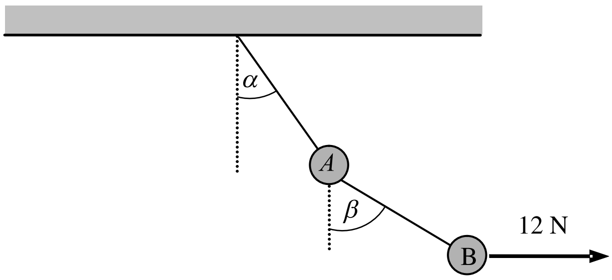
(A) $T_1 = F/4, T_2 = 3F/2, T_3 = F/4$ (B) $T_1 = F/4, T_2 = F/2, T_3 = F/2$
(C) $T_1 = 3F/4, T_2 = F/2, T_3 = F/4$ (D) $T_1 = 3F/4, T_2 = F/2, T_3 = F/2$

5. In a given figure system is in equilibrium. If $W_1 = 300$ N. Then W_2 is approximately equal to



(A) 500 N (B) 400 N (C) 670 N (D) 300 N

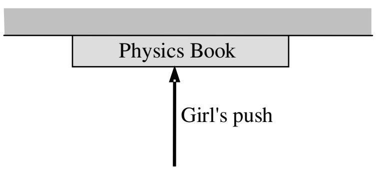
6. Two balls A and B weighing 7 N and 9 N are connected by a light cord. The system is suspended from a fixed support by connecting the ball A with another light cord. The ball B is pulled aside by a horizontal force 12 N and equilibrium is established. Angles α and β respectively are



(A) 30° and 60° (B) 60° and 30° (C) 37° and 53° (D) 53° and 37°

7. A girl pushes her physics book up against the horizontal ceiling of her room as shown in the figure.

The book weighs 20 N and she pushes upwards with a force of 25 N. The choices below list the magnitudes of the contact force F_{CB} between the ceiling and the book, and F_{BH} between the book and her hand. Select the correct pair.



(A) $F_{CB} = 20$ N and $F_{BH} = 25$ N (B) $F_{CB} = 25$ N and $F_{BH} = 45$ N
(C) $F_{CB} = 5$ N and $F_{BH} = 25$ N (D) $F_{CB} = 5$ N and $F_{BH} = 45$ N

8. Two astronauts A and B connected with a rope stay stationary in free space relative to their spaceship. Mass of A is more than that of B and the rope is straight. Astronaut A starts pulling the rope but astronaut B does not. If you were the third astronaut in the spaceship, what do you observe?

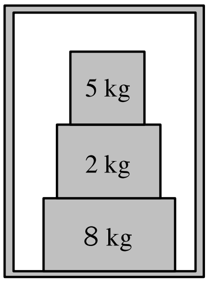
(A) Astronaut B accelerates towards A and A remains stationery.

(B) Both accelerate towards each other with equal accelerations of equal modulus.

(C) Both accelerate towards each other but acceleration of B is greater than that of A.

(D) Both accelerate towards each other but acceleration of B is smaller than that of A.

9. Three boxes are placed in a lift. When acceleration of the lift is 4 m/s^2, the net force on the 8 kg box is closest to

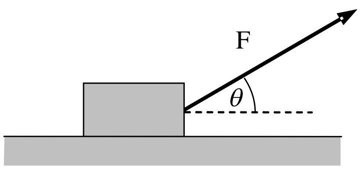


(A) 80 N (B) 48 N (C) 40 N (D) 32 N

10. A man is standing on a weighing machine with a block in his hand. The machine records w. When he takes the block upwards with some acceleration the machine records w_1. When he takes the block down with some acceleration, the machine records w_2.Then choose correct option

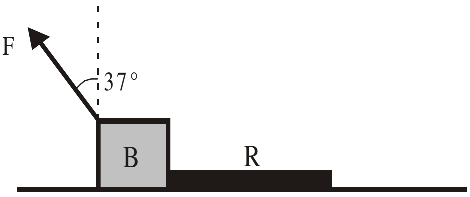
(A) $w_1 = w = w_2$ (B) $w_1 < w < w_2$ (C) $w_2 < w < w_1$ (D) $w_2 = w_1 > w$

11. A block is being pulled by a force F on a long frictionless level floor. Magnitude of the force is gradually increases from zero until the block lifts off the floor. Immediately before the block leaves the floor, its acceleration is



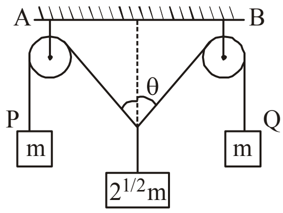
(A) gcosθ (B) gcotθ (C) gsinθ (D) g tanθ

12. A block B is tied to one end of a uniform rope R as shown. The mass of block is 2 kg and that of rope is 1 kg. A force F = 15 N is applied at angle 37° with vertical. The tension at the mid-point of rope is



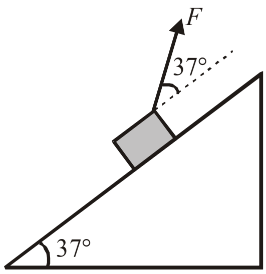
(A) 1.5 N (B) 2 N (C) 3N (D) 4.5 N

13. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be **[JEE (Scr) 2001]**



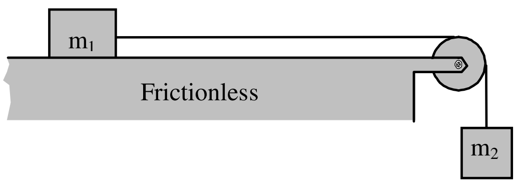
(A) 0° (B) 30° (C) 45° (D) 60°

14. A block resting on a smooth inclined plane is acted upon by a force F as shown. If mass of block is 2 kg and $F = 20$ N and sin 37°= 3/5, the acceleration of block is



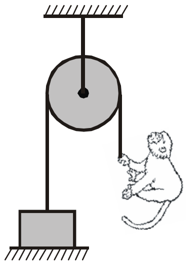
(A) 2 m/s^2 (B) 6 m/s^2 (C) 8 m/s^2 (D) zero

15. In the arrangement shown, the blocks of unequal masses are held at rest. When released, acceleration of the blocks is



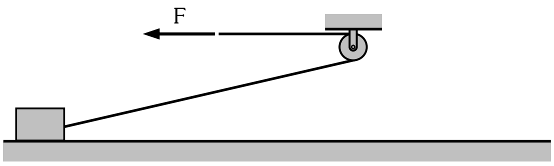
(A) g/2. (B) g.
(C) a value between zero and g. (D) a value that could be greater than g.

16. A monkey weighing 10 kg is climbing up a light rope and frictionless pulley attached to 15 kg mass at other end as in figure. In order to raise the 15 kg mass off the ground the monkey must climb-up



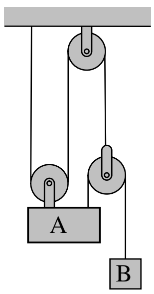
(A) with constant acceleration g/3.
(B) with an acceleration greater than g/2.
(C) with an acceleration greater than g/4.
(D) It is not possible because weight of monkey is lesser than the block.

17. A heavy cart is pulled by a constant force F along a horizontal track with the help of a rope that passes over a fixed pulley, as shown in the figure. Assume the tension in the rope and the frictional forces on the cart remain constant and consider motion of the cart until it reaches vertically below the pulley.
As the cart moves to the right, its acceleration



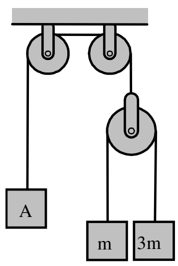
(A) decreases. (B) increases. (C) remains constant. (D) is zero

18. In arrangement shown the block A of mass 15 kg is supported in equilibrium by the block B. Mass of the block B is closest to



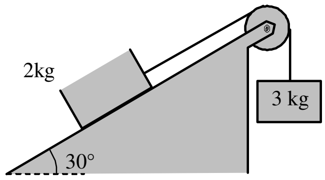
(A) 2 kg (B) 3 kg (C) 4 kg (D) 5 kg

19. In the given figure, find mass of the block A, if it remains at rest, when the system is released from rest. Pulleys and strings are massless. [$g = 10\ m/s^2$]



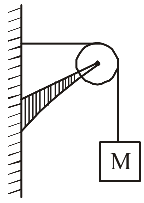
(A) m (B) 2m (C) 2.5m (D) 3m

20. In the arrangement shown, the 2 kg block is held to keep the system at rest. The string and pulley are ideal. When the 2 kg block is set free, by what amount the tension in the string changes? [g = 10 m/s^2]



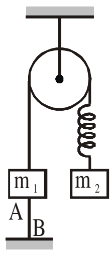
(A) Increase of 12 N (B) Decrease of 12 N (C) Increase of 18 N (D) Decrease of 18 N

21. A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given **[JEE (Scr) 2001]**



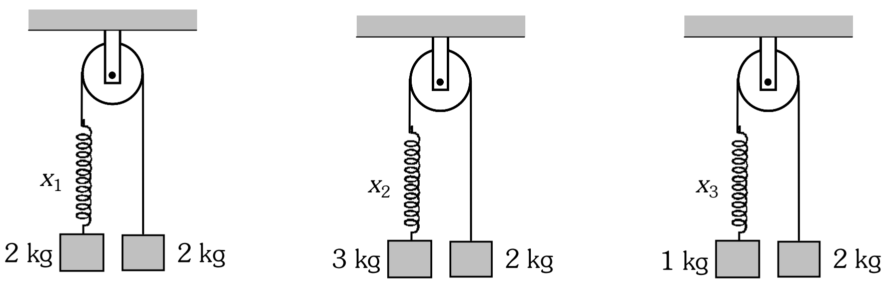
(A) $\sqrt{2}\,Mg$ (B) $\sqrt{2}\,mg$ (C) $\sqrt{(M+m)^2+m^2}$ g (D) $\sqrt{(M+m)^2+M^2}$ g

22. In a given figure two masses m_1 & m_2 ($m_2 > m_1$) are at rest in equilibrium position. Find the tension in string AB



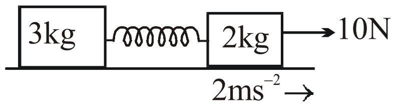
(A) m_1g (B) m_2g (C) $(m_1+m_2)g$ (D) $(m_2-m_1)g$

23. Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown. If x_1, x_2 and x_3 be the extensions in the spring in these three cases, when acceleration of both the blocks have same magnitude, then



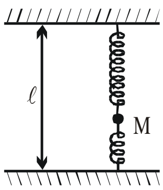
(A) $x_2 > x_3 > x_1$ (B) $x_2 > x_1 > x_3$ (C) $x_3 > x_1 > x_2$ (D) $x_1 > x_2 > x_3$

24. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(A) 3 ms^{-2} (B) 2 ms^{-2} (C) 0.5 ms^{-2} (D) zero

25. A small ball of mass M is held in equilibrium with two identical springs as shown in the figure. Force constant of each spring is k and relaxed length of each spring is $\ell/2$. What is distance between the ball and roof?

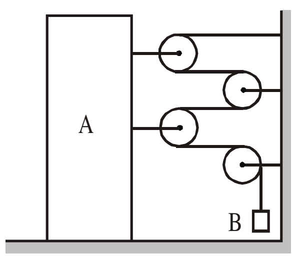


(A) $\frac{\ell}{2}+\frac{Mg}{k}$ (B) $\frac{\ell}{2}-\frac{Mg}{k}$ (C) $\frac{\ell}{2}+\frac{Mg}{2k}$ (D) $\frac{\ell}{2}-\frac{Mg}{2k}$

26. An elastic spring of relaxed length ℓ_o and force constant k is cut into two parts of lengths ℓ_1 and ℓ_2. The force constants of these parts are respectively

(A) $\frac{k\ell_o}{\ell_1}$ and $\frac{k\ell_o}{\ell_2}$ (B) $\frac{k\ell_1}{\ell_0}$ and $\frac{k\ell_2}{\ell_o}$ (C) $\frac{k\ell_0}{\ell_2}$ and $\frac{k\ell_0}{\ell_1}$ (D) $\frac{k\ell_2}{\ell_0}$ and $\frac{k\ell_1}{\ell_0}$

27. Block A is moving away from the wall at a speed v and acceleration a.



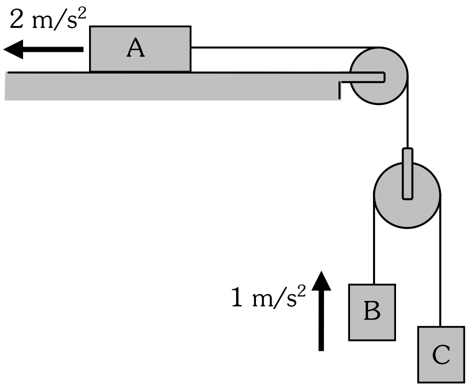
(A) Velocity of B is v with respect to A.

(B) Acceleration of B is a with respect to A.

(C) Acceleration of B is 4a with respect to A.

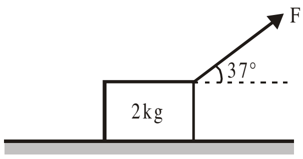
(D) Acceleration of B is $\sqrt{17}$a with respect to A.

28. In the setup shown, find acceleration of the block C.



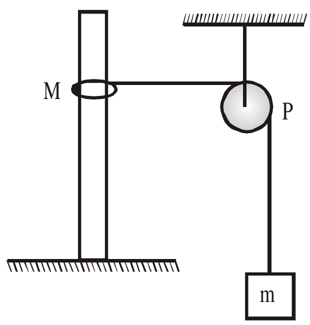
(A) 3 m/s² ↑ (B) 3 m/s² ↓ (C) 5 m/s² ↑ (D) 5 m/s² ↓

29. A block of mass 2 kg is kept on a rough horizontal floor and pulled with a force F. If the coefficient of friction is 0.5. then the minimum force required to move the block is :-



(A) 10 N (B) $\frac{100}{11}$ N (C) $\frac{100}{8}$ N (D) 20 N

30. In the figure shown a ring of mass M and a block of mass m are in equilibrium. The string is light and pulley P does not offer any friction and coefficient of friction between pole and M is μ. The frictional force offered by the pole on M is



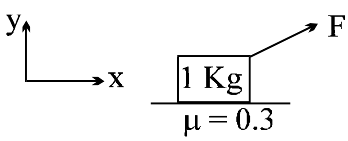
(A) Mg directed up

(B) μmg directed up

(C) $(M-m)g$ directed down

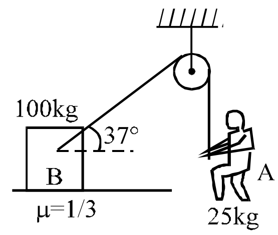
(D) μmg directed down

31. A force $\vec{F} = \hat{i} + 4\hat{j}$ acts on block shown. The force of friction acting on the block is :



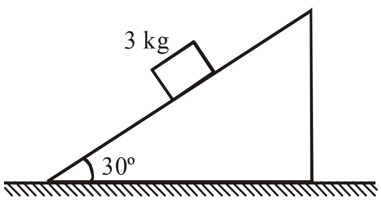
(A) $-\hat{i}$ (B) $-1.8\hat{i}$ (C) $-2.4\hat{i}$ (D) $-3\hat{i}$

32. Block *B* of mass 100 kg rests on a rough surface of friction coefficient μ = 1/3. A rope is tied to block *B* as shown in figure. The maximum acceleration with which boy *A* of 25 kg can climbs on rope without making block move is :



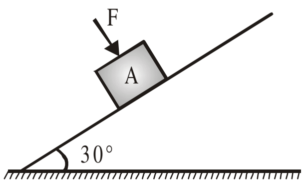
(A) $\frac{4g}{3}$ (B) $\frac{g}{3}$ (C) $\frac{g}{2}$ (D) $\frac{3g}{4}$

33. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be (g = 10 m/s^2)



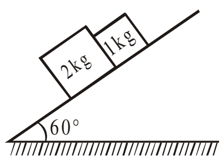
(A) 26 N (B) 19.5 N (C) 10 N (D) 30 N

34. A block of mass m = 2 kg is resting on a rough inclined plane of inclination 30° as shown in figure. The coefficient of friction between the block and the plane is μ = 0.5. What minimum force F should be applied perpendicular to the plane on the block, so that block does not slip on the plane (g=10m/s^2)



(A) zero (B) 6.24 N (C) 2.68 N (D) 4.34 N

35. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is μ_1=0.5 and μ_2 = 0.4 respectively, then



(A) both block will move together.

(B) both block will move separately.

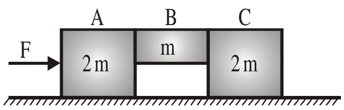
(C) there is a non zero contact force between two blocks.

(D) none of these

36. A block is pushed with some velocity up a rough inclined plane. It stops after ascending few meters and then reverses its direction and returns back to point from where it started. If angle of inclination is 37° and the time to climb up is half of the time to return back then coefficient of friction is

(A) $\frac{9}{20}$ (B) $\frac{7}{5}$ (C) $\frac{7}{12}$ (D) $\frac{5}{7}$

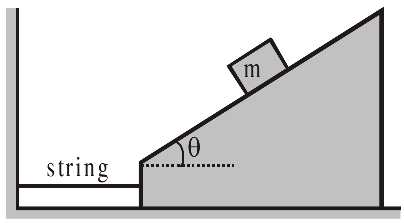
37. The system is pushed by a force *F* as shown in figure. All surfaces are smooth except between *B* and *C*. Friction coefficient between *B* and *C* is μ. Minimum value of *F* to prevent block *B* from downward slipping is :-



(A) $\left(\frac{3}{2\mu}\right)mg$ (B) $\left(\frac{5}{2\mu}\right)mg$ (C) $\left(\frac{5}{2}\right)\mu mg$ (D) $\left(\frac{3}{2}\right)\mu mg$

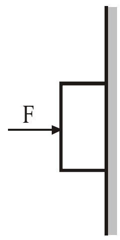
MULTIPLE CORRECT TYPE QUESTIONS

38. Refer the system shown in the figure. Block is sliding down the wedge. All surfaces are frictionless. Find correct statement(s)



(A) Acceleration of block is $g\sin\theta$
(B) Acceleration block is $g\cos\theta$
(C) Tension in the string is $mg\cos^2\theta$
(D) Tension in the string is $mg\sin\theta.\cos\theta$

39. A block of mass 1 kg is held at rest against a rough vertical surface by pushing by a force F horizontally. The coefficient of friction is 0.5. When



(A) $F = 40$ N, friction on the block is 20 N.
(B) $F = 30$ N, friction on the block is 10 N.
(C) $F = 20$ N, friction on the block is 10 N.
(D) Minimum value of force F to keep block at rest is 20 N.

40. A block is kept on a rough horizontal surface as shown. Its mass is 2 kg and coefficient of friction between block and surface (μ) = 0.5. A horizontal force F is acting on the block. When

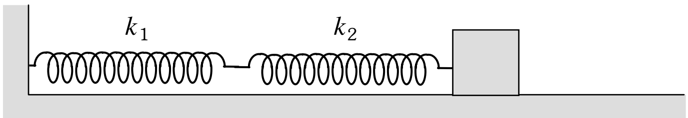
(A) $F = 4$ N, acceleration is zero.

(B) $F = 4$ N, friction is 10 N and acceleration is 3 m/s^2.

(C) $F = 14$ N, acceleration is 2 m/s^2.

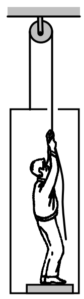
(D) $F = 14$ N, friction is 14 N.

41. The mass in the figure can slide on a frictionless surface. When the mass is pulled out, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2. The spring constants are k_1 and k_2 respectively. Magnitude of spring force pulling back on the mass is



(A) k_1x_1 (B) k_2x_2 (C) $(k_1x_1 + k_2x_2)$ (D) $0.5\,(k_1 + k_2)\,(x_1 + x_2)$

42. A carpenter of mass 50 kg is standing on a weighing machine placed in a lift of mass 20 kg. A light string is attached to the lift. The string passes over a smooth pulley and the other end is held by the carpenter as shown. When carpenter keeps the lift moving upward with constant velocity :- ($g = 10$ m/s^2)



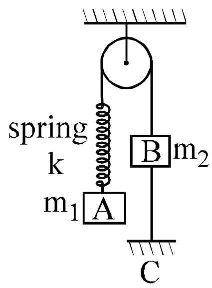
(A) the reading of weighing machine is 15 kg

(B) the man applies a force of 350 N on the string

(C) net force on the man is 150 N

(D) Net force on the weighing machine is 150 N

43. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt :



(A) initial acceleration of m_2 will be upwards

(B) magnitude of initial acceleration of both blocks will be equal to $\left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)$ g.

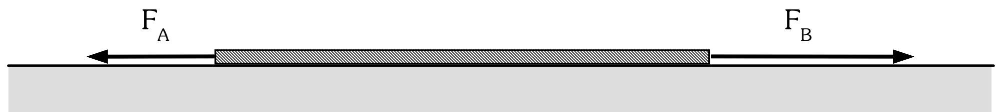
(C) initial acceleration of m_1 will be equal to zero

(D) magnitude of initial acceleration of two blocks will be non-zero and unequal.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 44 to 47

A uniform rope of mass (m) and length (L) placed on frictionless horizontal ground is being pulled by two forces F_A and F_B at its ends as shown in the figure. As a result, the rope accelerates toward the right.



44. Acceleration (A) of the rope is

(A) zero (B) $a = \frac{F_A + F_B}{m}$ (C) $a = \frac{F_A - F_B}{m}$ (D) $a = \frac{F_B - F_A}{m}$

45. Tension (T) at the mid point of the rope is

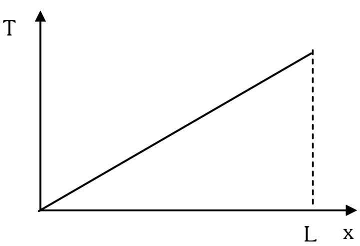
(A) $T = F_B - F_A$ (B) $T = F_A + F_B$ (C) $T = \frac{1}{2}(F_B - F_A)$ (D) $T = \frac{1}{2}(F_A + F_B)$

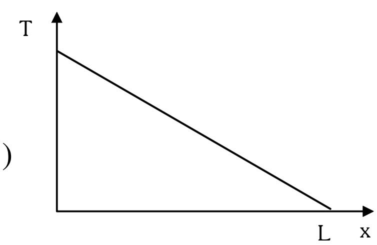
46. Expression (T_x) of tension at a point at distance x from the end A is

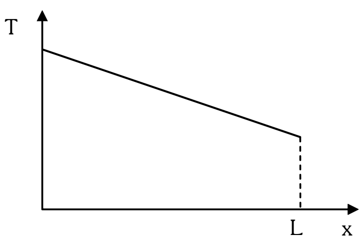
(A) $T_x = \left(\frac{F_B - F_A}{L}\right)x + F_A$ (B) $T_x = \left(\frac{F_B - F_A}{L}\right)x - F_A$

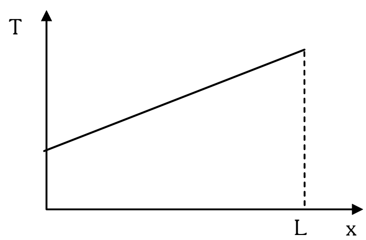
(C) $T_x = \left(\frac{F_B - F_A}{L}\right)x + F_B$ (D) $T_x = \left(\frac{F_B - F_A}{L}\right)x - F_B$

47. Which of the following graph best represents variation in tension at a point on the rope with distance x of the point from the end A?

(A)


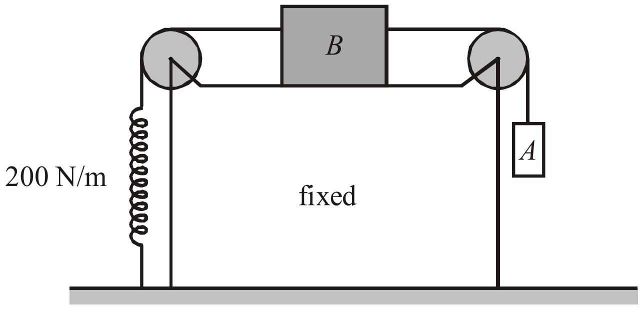
(B)


(B)


(D)


Paragraph for Question No. 48 to 50

The figure shown blocks A and B are of mass 2 kg and 8 kg and they are connected through strings to a spring connected to ground. The blocks are in equilibrium. ($g = 10\text{m/s}^2$)



48. The elongation of the spring is

(A) 1 cm (B) 10 cm (C) 0.1 cm (D) 1m

49. Now the block A is pulled downwards by a force gradually increasing to 20 N. The new elongation of spring is :-

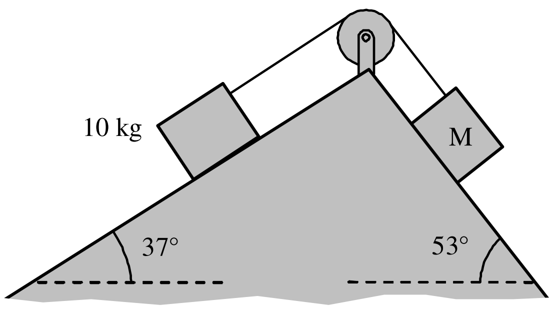
(A) 2 cm (B) 4 cm (C) 20 cm (D) 40 cm

50. Now the force on A is suddenly removed. The acceleration of block B becomes :-

(A) 1.0 m/s (B) 2.0 m/s^2 (C) 3.0 m/s^2 (D) 4.0 m/s^2

Paragraph for Question No. 51 to 53

The blocks are on frictionless inclined ramp and connected by a massless cord. The cord passes over an ideal pulley. [g = 10 m/s^2]



51. When set free, the 10 kg block slides down the ramp with acceleration of 2 m/s^2. Mass M is closest to

(A) 13 kg (B) 8 kg (C) 5 kg (D) 4 kg

52. When the 10 kg block slides down the ramp with acceleration of 2 m/s^2, tension in the cord is closest to :-

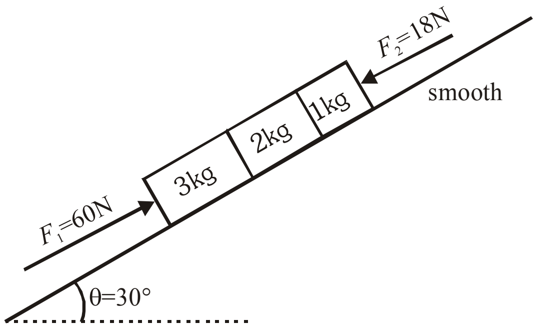
(A) 80 N (B) 60 N (C) 40 N (D) 30 N

53. What value of M would keep the system at rest.

(A) 10 kg (B) 8 kg (C) 7.5 kg (D) 6 kg

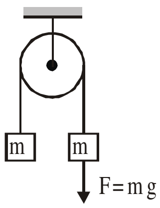
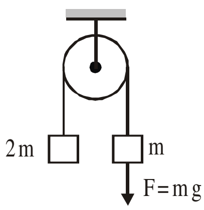
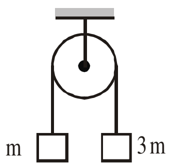
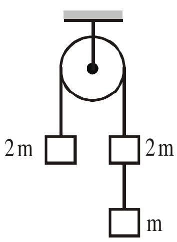
MATRIX MATCH TYPE QUESTION

54. In the diagram shown in figure ($g = 10$ m/s^2)

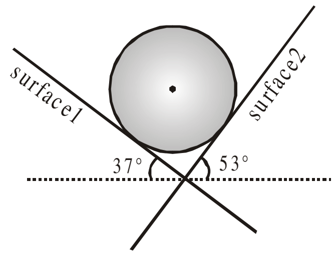


Column I		Column II	
(A)	Acceleration of 2 kg block in m/s^2	(P)	8
(B)	Net force on 3 kg block in newton	(Q)	25
(C)	Normal reaction between 2 kg and 1 kg in newton	(R)	2
(D)	Normal reaction between 3 kg and 2 kg in newton	(S)	45
		(T)	None

55. Match the situations in column I to the accelerations of blocks in the column II (acceleration due to gravity is g and F is an additional force applied to one of the blocks ?

Column I		Column II	
(A)		(P)	$\frac{g}{5}$
(B)		(Q)	$\frac{g}{3}$
(C)		(R)	$\frac{g}{2}$
(D)		(S)	$\frac{2g}{3}$
		(T)	zero

56. A sphere of mass 10 kg is placed in equilibrium in a V shaped groove plane made of two smooth surfaces 1 and 2 as shown in figure. ($g = 10\ ms^{-2}$)

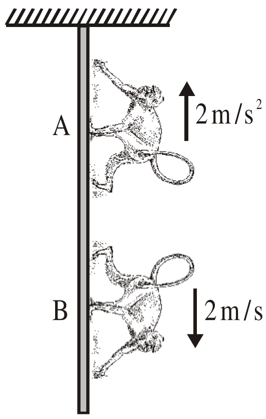


	Column I		**Column II**
(A)	Normal reaction by Surface 1	(P)	Zero
(B)	Normal reaction by surface 2	(Q)	60 N
(C)	Force on sphere by Earth	(R)	80 N
(D)	Net force on sphere	(S)	100 N
		(T)	120 N

EXERCISE (O-2)

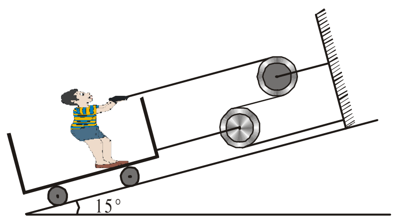
SINGLE CORRECT TYPE QUESTIONS

1. Two monkeys of masses 10 kg and 8 kg are moving along a vertical light rope, the former climbing up with an acceleration of 2 m/s^2, while the latter coming down with a uniform velocity of 2 m/s. Find tension in the rope at the fixed support.



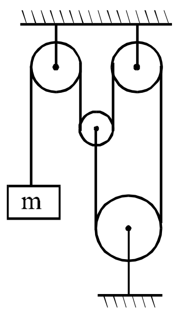
(A) 180 N (B) 200 N (C) 80 N (D) 216 N

2. A trolley is being pulled up an incline plane by a man sitting on it (as shown in figure). He applies a force of 250 N. If the combined mass of the man and trolley is 100 kg, the acceleration of the trolley will be [sin15° = 0.26]



(A) 2.4 m/s^2 (B) 9.4 m/s^2
(C) 6.9 m/s^2 (D) 4.9 m/s^2

3. If the string & all the pulleys are ideal, acceleration of mass *m* is :-

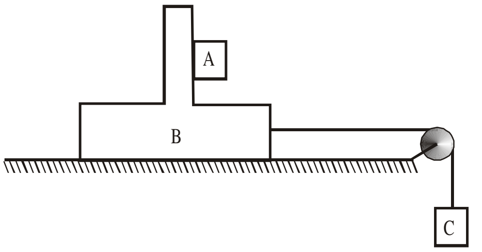


(A) $\frac{g}{2}$ (B) 0 (C) g (D) dependent on m

4. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end, $\mu = 0.15$ and $g = 10$ m/s^2. The truck starts from rest with an acceleration of 2m/s^2 on a straight road. The distance moved by the truck when box starts fall down is :-

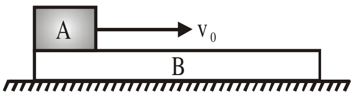
(A) 4 m (B) 8 m (C) 16 m (D) 32 m

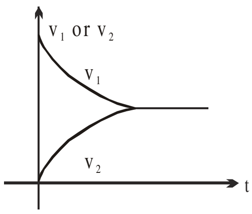
5. In the arrangement shown in the figure, mass of the block *B* and *A* is 2m and *m* respectively. Surface between *B* and floor is smooth. The block *B* is connected to the block *C* by means of a string-pulley system. If the whole system is released, then find the minimum value of mass of block *C* so that *A* remains stationary w.r.t. *B*. Coefficient of friction between *A* and *B* is μ.

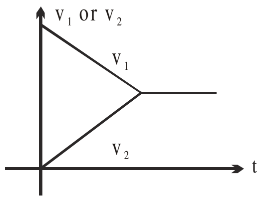


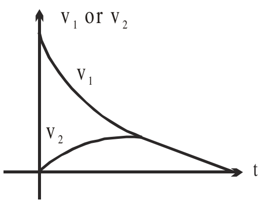
(A) $\frac{m}{\mu}$ (B) $\frac{2m+1}{\mu+1}$ (C) $\frac{3m}{\mu-1}$ (D) $\frac{6m}{\mu+1}$

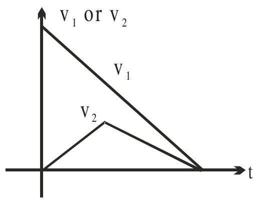
6. A block *A* is placed over a long rough plank *B* of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time t = 0, block *A* is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of *A* and *B* at time *t*. Then choose the correct graph between v_1 or v_2 and *t*.



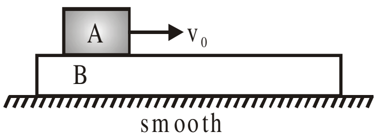
(A)


(B)


(C)


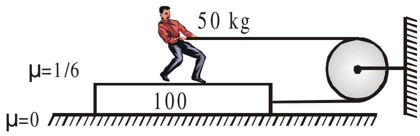
(D)


7. A block *A* of mass *m* is placed over a plank *B* of mass 2 m. Plank *B* is placed over a smooth horizontal surface. The coefficient of friction between *A* and *B* is 0.5. Block *A* is given a velocity v_0 towards right. Acceleration of *B* relative to *A* is :-



(A) $\frac{g}{2}$ (B) g (C) $\frac{3g}{4}$ (D) zero

8. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.



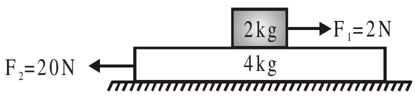
(A) $\frac{100}{3}$ N towards left

(B) $\frac{100}{3}$ N towards right

(C) $\frac{250}{3}$ N towards left

(D) $\frac{250}{3}$ N towards right

9. In the arrangement shown in figure, coefficient of friction between the two blocks is $\mu = 1/2$. The force of friction acting between the two blocks is :-

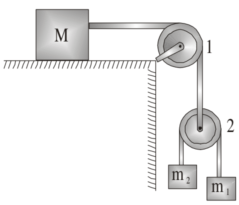


(A) 8 N (B) 10 N (C) 6 N (D) 4 N

10. A flexible chain of weight W hangs between two fixed points A & B which are at the same horizontal level. The inclination of the chain with the horizontal at both the points of support is θ. What is the tension of the chain at the mid point?

(A) $\frac{W}{2}$. *cosec* θ (B) $\frac{W}{2}$. *tan* θ (C) $\frac{W}{2}$ *cot* θ (D) none

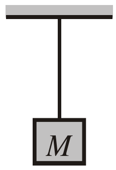
11. In the arrangement shown in figure $m_1 = 1\text{kg}$, $m_2 = 2\text{kg}$. Pulleys are massless and strings are light. For what value of M the mass m_1 moves with constant velocity (Neglect friction)



(A) 6 kg (B) 4 kg (C) 8 kg (D) 10 kg

MULTIPLE CORRECT TYPE QUESTIONS

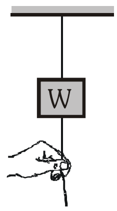
12. Consider a block suspended from a light string as shown in the figure.
Which of the following pairs of forces constitute Newton's third law pair?



(A) Force with which string pulls on the ceiling and the force with which string pulls on block
(B) Force with which string pulls on the block and weight of the block
(C) Force acting on block due to the earth and force the block exerts on the earth
(D) Force with which block pulls on string and force with which the string pulls on the block

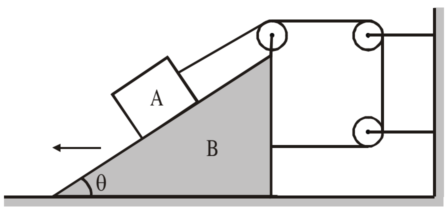
13. If a horizontal support exerts an upward force of 10 N on a block of weight 9.8 N placed on it, which of the following statements is/are correct. Assume acceleration due to gravity to be 9.8 m/s^2.
(A) The block exerts a force of 10 N on the support.
(B) The block exerts a force of 9.8 N on the support.
(C) The block has an upward acceleration.
(D) The block has a downward acceleration.

14. A block of mass m is suspended from a fixed support with the help of a cord. Another identical cord is attached to the bottom of the block. Which of the following statement is /are true?



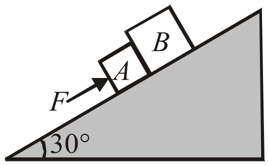
(A) If the lower cord is pulled suddenly, only the upper cord will break.
(B) If the lower cord is pulled suddenly, only the lower cord will break.
(C) If pull on the lower cord is increased gradually, only the lower cord will break.
(D) If pull on the lower cord is increased gradually, only the upper cord will break.

15. A block A and wedge B connected through a string as shown. The wedge B is moving away from the wall with acceleration 2 m/s^2 horizontally and acceleration of block A is vertical upwards. Then



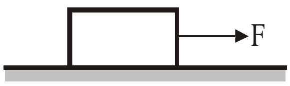
(A) Acceleration of A with respect to B is 4 m/s^2.
(B) Acceleration of A with respect to B is $2\sqrt{3}$ m/s^2.
(C) Angle θ is 60°.
(D) Acceleration of A is $2\sqrt{3}$ m/s^2.

16. Two blocks A and B of mass 2 kg and 4 kg respectively are placed on a smooth inclined plane and 2 kg block is pushed by a force F acting parallel to the plane as shown. If N be the magnitude of contact force applied on B by A, which of the following is/are correct?



(A) if $F = 0$ N, $N = 10$ N
(B) if F=15 N, N=10 N
(C) If $F = 30$ N, $N = 20$ N
(D) if F=45 N, N=30 N

17. A block is kept on a rough surface and applied with a horizontal force as shown which is gradually increasing from zero. The coefficient of static and kinetic friction are $1/\sqrt{3}$ then



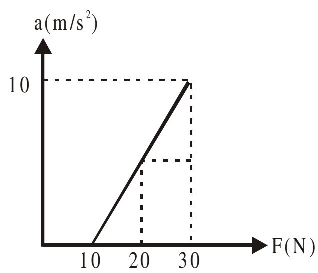
(A) When F is less than the limiting friction, angle made by net force on the block by the surface is less than 30° with vertical.

(B) When the block is just about to move, the angle made by net force by the surface on the block becomes equal to 30° with vertical.

(C) When the block starts to accelerate, the angle made by net force by the surface on the block becomes constant and equal to 30° vertical.

(D) The angle made by net force with vertical on the block by the surface, depends on the mass of the block.

18. A block placed on a rough horizontal surface is pushed with a force F acting horizontally on the block. The magnitude of F is increased and acceleration produced is plotted in the graph shown.



(A) Mass of the block is 2 kg.
(B) Coefficient of friction between block and surface is 0.5.
(C) Limiting friction between block and surface is 10 N.
(D) When $F = 8$ N, friction between block and surface is 10 N.

19. A block is placed over a plank. The coefficient of friction between the block and the plank is $\mu = 0.2$. Initially both are at rest, suddenly the plank starts moving with acceleration $a_0 = 4$ m/s^2. The displacement of the block in 1s is (g=10 m/s^2)

(A) 1 m relative to ground
(B) 1 m relative to plank
(C) zero relative to plank
(D) 2 m relative to ground

20. A block is released from rest from a point on a rough inclined place of inclination 37°. The coefficient of friction is 0.5.

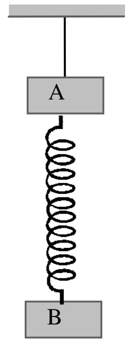
(A) The time taken to slide down 9 m on the plane is 3 s.

(B) The velocity of block after moving 4 m is 4 m/s.

(C) The block travels equal distances in equal intervals of time.

(D) The velocity of block increases linearly.

21. In the given figure both the blocks have equal mass. When the thread is cut, which of the following statements give correct description immediately after the thread is cut?



(A) Relative to the block A, acceleration of block B is 2g upwards.

(B) Relative to the block B, acceleration of block A is 2g downwards.

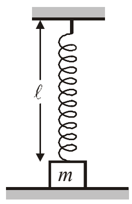
(C) Relative to the ground, accelerations of the blocks A and B are both g downwards.

(D) Relative to the ground, accelerations of the blocks A and B are 2g downwards and zero respectively.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 22 to 24

A block of mass m is placed on a smooth horizontal floor is attached to one end of spring. The other end of the spring is attached to fixed support. When spring is vertical it is relaxed. Now the block is pulled towards right by a force F, which is being increased gradually. When the spring makes angle 53° with the vertical, block leaves the floor.



22. When blocks leaves the table, the normal force on it from table is

(A) mg (B) zero (C) $\frac{4mg}{3}$ (D) $\frac{3mg}{4}$

23. Force constant of the spring is :-

(A) $\frac{5mg}{2\ell}$ (B) $\frac{15mg}{8\ell}$ (C) $\frac{5mg}{3\ell}$ (D) $\frac{5mg}{4\ell}$

24. When the block leaves the table, the force F is :-

(A) $\frac{3mg}{4}$ (B) $\frac{4mg}{3}$ (C) $\frac{3mg}{5}$ (D) $\frac{4mg}{5}$

Paragraph for Question No. 25 to 29

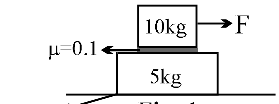


Fig. 1

25. When $F = 2$N, the frictional force between 5 kg block and ground is

(A) 2N (B) 0 (C) 8 N (D) 10 N

26. When $F = 2$N, the frictional force between 10 kg block and 5 kg block is

(A) 2N (B) 15 N (C) 10 N (D) None

27. The maximum F which will not cause motion of any of the blocks is

(A) 10 N (B) 15 N (C) data insufficient (D) None

28. The maximum acceleration of 5 kg block is :-

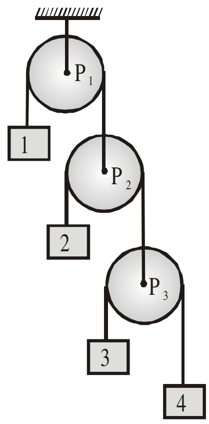
(A) 1 m/s^2 (B) 3 m/s^2 (C) 0 (D) None

29. The acceleration of 10 kg block when $F = 30$ N is

(A) 2 m/s^2 (B) 3 m/s^2 (C) 1 m/s^2 (D) None

MATRIX MATCH TYPE QUESTION

30. In the figure shown, acceleration of 1 is x (upwards). Acceleration of pulley P_3, w.r.t. pulley P_2 is y (downwards) and acceleration of 4 w.r.t. to pulley P_3 is z (upwards). Then



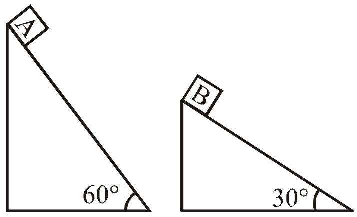
Column I	Column II
(A) Absolute acceleration of 2	(P) (y–x) downwards
(B) Absolute acceleration of 3	(Q) (z–x–y) upwards
(C) Absolute acceleration of 4	(R) (x+y+z) downwards
	(S) None

31. Velocity of three particles A, B and C varies with time t as, $\vec{v}_A = (2t\hat{i} + 6\hat{j})$ m/s; $\vec{v}_B = (3\hat{i} + 4\hat{j})$ m/s and $\vec{v}_C = (6\hat{i} - 4t\hat{j})$ m/s. Regarding the pseudo force match the following table

Column I	Column II
(A) On A as observed by B	(P) Along positive x–direction
(B) On B as observed by C	(Q) Along negative x–direction
(C) On A as observed by C	(R) Along positive y–direction
(D) On C as observed by A	(S) Along negative y–direction
	(T) Zero

EXERCISE (JM)

1. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ? **[AIEEE - 2010]**



(1) 4.9 ms^{-2} in vertical direction. (2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction (4) Zero

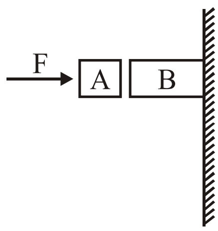
2. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that tanθ = 2μ then the ratio $\frac{F_1}{F_2}$ is :- **[AIEEE - 2011]**

(1) 4 (2) 1 (3) 2 (4) 3

3. A block of mass m is placed on a surface with a vertical cross section given by $y=\frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :- **[JEE-Main-2014]**

(1) $\frac{1}{3}$m (2) $\frac{1}{2}$m (3) $\frac{1}{6}$m (4) $\frac{2}{3}$m

4. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :- **[JEE-Main-2015]**

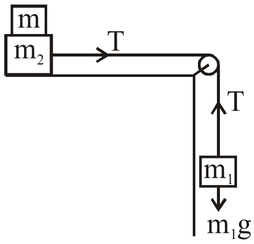


(1) 120 N (2) 150 N (3) 100 N (4) 80 N

5. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is: **[JEE-Main Online-2016]**

(1) 2 tan θ (2) 3 tan θ (3) tan θ (4) tan 2θ

6. Two masses m_1 = 5kg and m_2 = 10kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :- **[JEE-Main-2018]**



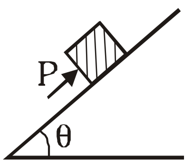
(1) 27.3 kg (2) 43.3 kg (3) 10.3 kg (4) 18.3 kg

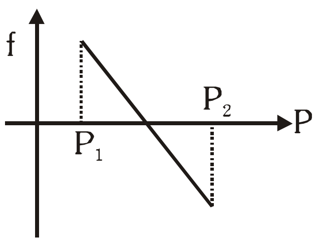
EXERCISE (JA)

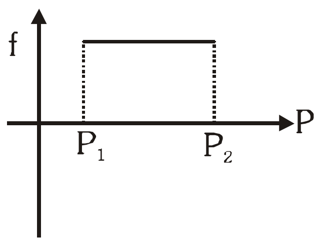
1. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is **[IIT-JEE-2009]**

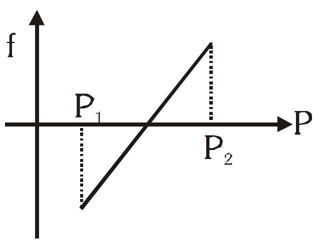
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

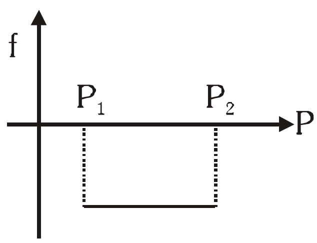
2. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and tanθ >μ. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like **[IIT-JEE-2010]**



(A)


(B)


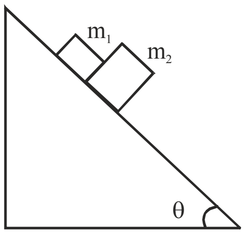
(C)


(D)


3. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N =10μ, then N is **[IIT-JEE-2011]**

4. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.

[**useful information :** $\tan(5.5°) \approx 0.1$; $\tan(11.5°) \approx 0.2$; $\tan(16.5°) \approx 0.3$] **[IIT-JEE-2014]**



	List–I		**List–II**
(P)	$\theta = 5°$	(1)	$m_2 g \sin\theta$
(Q)	$\theta = 10°$	(2)	$(m_1 + m_2)g \sin\theta$
(R)	$\theta = 15°$	(3)	$\mu m_2 g\cos\theta$
(S)	$\theta = 20°$	(4)	$\mu(m_1 + m_2)g \cos\theta$

Code :

(A) P-1, Q-1, R-1, S-3
(B) P-2. Q-2, R-2, S-3
(C) P-2,. Q-2, R-2, S-4
(D) P-2, Q-2, R-3, S-3

ANSWER KEY

EXERCISE (S-1)

1. Ans. (i) $\frac{m_1}{m_2}=\frac{1}{3}$ (ii) $a = 3/4$ m/s^2 **2. Ans.** $\frac{4}{3}W$ **3. Ans.** $\frac{g}{10}m/s^2$

4. Ans. 200 N, 10 m/s^2

5. Ans. 2 sec **6. Ans.** 0.5g, g **7. Ans.** 0.5 s **8. Ans.** 80 N, 48 N, 264 N **9. Ans. 24N**

10. Ans. (i) $a_A=\frac{3g\downarrow}{2}=a_B$; $a_C=0$; T $= mg/2$; (ii) $a_A= 2g\uparrow$, $a_B = 2g\downarrow$, $a_c = 0$, $T = 0$;

(iii) $a_A = a_B = g/2\uparrow$, $a_c = g\downarrow$, $T = \frac{3mg}{2}$;

11. Ans. $a = g \cot\theta$ **12. Ans.** $v_0/\cos\theta$ **13. Ans.** 5N, 16/31 kg **14. Ans.** $3a_B/4$

15. Ans. $\frac{a_0}{2}$ **16. Ans.** $(100\hat{i}-200\hat{j})$ N **17. Ans.** 20 N vertically downward

18. Ans. (ma) **19. Ans.** 1/2 s **20. Ans.** 4/3 s **21. Ans.** $\mu_{min}=\frac{m\sin\theta\cos\theta}{m\cos^2\theta+M}$

22. Ans. 0.5 **23. Ans.** 1 kg **24. Ans.** $10\hat{i}$ **25. Ans.** 3/4 **26. Ans.** 30 N

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (B)	**2. Ans. (D)**	**3. Ans. (D)**	**4. Ans. (C)**	**5. Ans. (D)**	**6. Ans. (C)**
7. Ans. (C)	**8. Ans. (C)**	**9. Ans. (D)**	**10. Ans. (C)**	**11. Ans. (B)**	**12. Ans. (A)**
13. Ans. (C)	**14. Ans. (A)**	**15. Ans. (C)**	**16. Ans. (B)**	**17. Ans. (A)**	**18. Ans. (B)**
19. Ans. (D)	**20. Ans. (B)**	**21. Ans. (D)**	**22. Ans. (D)**	**23. Ans. (B)**	**24. Ans. (B)**
25. Ans. (C)	**26. Ans. (A)**	**27. Ans. (D)**	**28. Ans. (A)**	**29. Ans. (B)**	**30. Ans. (A)**
31. Ans. (A)	**32. Ans. (B)**	**33. Ans. (D)**	**34. Ans. (C)**	**35. Ans. (B)**	**36. Ans. (A)**
37. Ans. (B)					

MULTIPLE CORRECT TYPE QUESTIONS

38. Ans. (A,D) **39. Ans. (B,C,D)** **40. Ans. (A,C)** **41. Ans. (A,B)** **42. Ans. (A,B)**

43. Ans. (A,C)

COMPREHENSION TYPE QUESTIONS

44. Ans. (D) **45. Ans. (D)** **46. Ans. (A)** **47. Ans. (D)** **48. Ans. (B)**

49. Ans. (C) **50. Ans. (B)** **51. Ans. (D)** **52. Ans. (C)** **53. Ans. (C)**

MATRIX MATCH TYPE QUESTION

54. Ans. (A) - (R); (B) - (T); (C) - (Q); (D) - (T) **55. Ans. (A) - (R); (B) - (T); (C) - (R); (D) - (P)**

56. Ans. (A)-R; (B)-Q; (C)-S; (D)-P

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (B) 2. Ans. (D) 3. Ans. (C) 4. Ans. (C) 5. Ans. (C) 6. Ans. (B)

7. Ans. (C) 8. Ans. (A) 9. Ans. (A) 10. Ans. (C) 11. Ans. (C)

MULTIPLE CORRECT TYPE QUESTIONS

12. Ans. (C,D) 13. Ans. (A,C) 14. Ans. (B,D)

15. Ans. (A,C,D) 16. Ans. (B,C,D) 17. Ans. (A,B,C,D)

18. Ans. (A,B,C) 19. Ans. (A,B) 20. Ans. (A,B,D) 21. Ans. (A,B,D)

COMPREHENSION TYPE QUESTIONS

22. Ans. (B) 23. Ans. (A) 24. Ans. (B) 25. Ans. (A) 26. Ans. (A) 27. Ans. (A)

28. Ans. (C) 29. Ans. (A)

MATRIX MATCH TYPE QUESTION

30. Ans. (A) - (S) ; (B) - (R) ; (C) - (Q) 31. Ans. (A) - (T); (B) - (R); (C) - (R); (D) - (Q)

EXERCISE (JM)

1. Ans. (1) 2. Ans. (4) 3. Ans. (3) 4. Ans. (1) 5. Ans. (3) 6. Ans. (1)

EXERCISE (JA)

1. Ans. (B) 2. Ans. (A) 3. Ans. 5 4. Ans. (D)

IMPORTANT NOTES

IMPORTANT NOTES