MODULE-2

CHAPTER 1

WORK, POWER & ENERGY

Chapter Contents 01

IMPORTANT NOTES

CHAPTER 1

WORK, POWER & ENERGY

WORK

- Whenever a force acting on a body, displaces it in its direction, work is said to be done by the force.
- Work done by a force is equal to scalar product of force applied and displacement of the point of application, $W = \vec{F}.\vec{d}$
- Work is a scalar quantity.

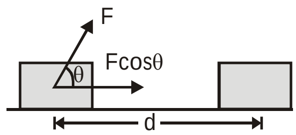
Work done by a constant force :

If the direction and magnitude of a force applied on a body is constant, the force is said to be constant. Work done by a constant force, W = Force × component of displacement along force

= displacement × component of force along displacement.

The work done will, be $W = (F \cos\theta)\, d$

$= F\,(d \cos\theta)$



In vector form, $W = \vec{F}\cdot\vec{d}$

Note : The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

Work done by a variable force

If the force applying on a body is changing its direction or magnitude or both, the force is said to be variable. Suppose a variable force causes displacement in a body from position P_1 to position P_2. To calculate the work done by the force the path from P_1 to P_2 can be divided into infinitesimal element, each element is so small that during displacement of body through it, the force is supposed to be constant. If $d\vec{r}$ be small displacement of point of application and $\vec{F}$ be the force applied on the body, the work done by force is $dW = \vec{F}\cdot d\vec{r}$

The total work done in displacing body from P_1 to P_2 is given by $\int dW = \int_{P_1}^{P_2} \vec{F}.d\vec{r} \Rightarrow W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{r}$

If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and P_2 respectively, the total work done

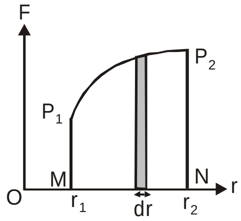
$$W = \int_{r_1}^{r_2} \vec{F}.d\vec{r}$$

Note : When we consider a block attached to a spring, the force on the block is k times the elongation of the spring, where k is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

Here : $W_s = \int_{x_i}^{x_f} -k\,x\,dx = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$

Calculation of work done from force displacement graph :

Suppose a body, whose initial position is r_1, is acted upon by a variable force $\vec{F}$ and consequently the body acquires its final position r_2. From position r to r + dr or for small displacement dr, the work done will be $\vec{F}.d\vec{r}$ whose value will be the area of the shaded strip of width dr. The work done on the body in displacing it from position r_1 to r_2 will be equal to the sum of areas of all such strips



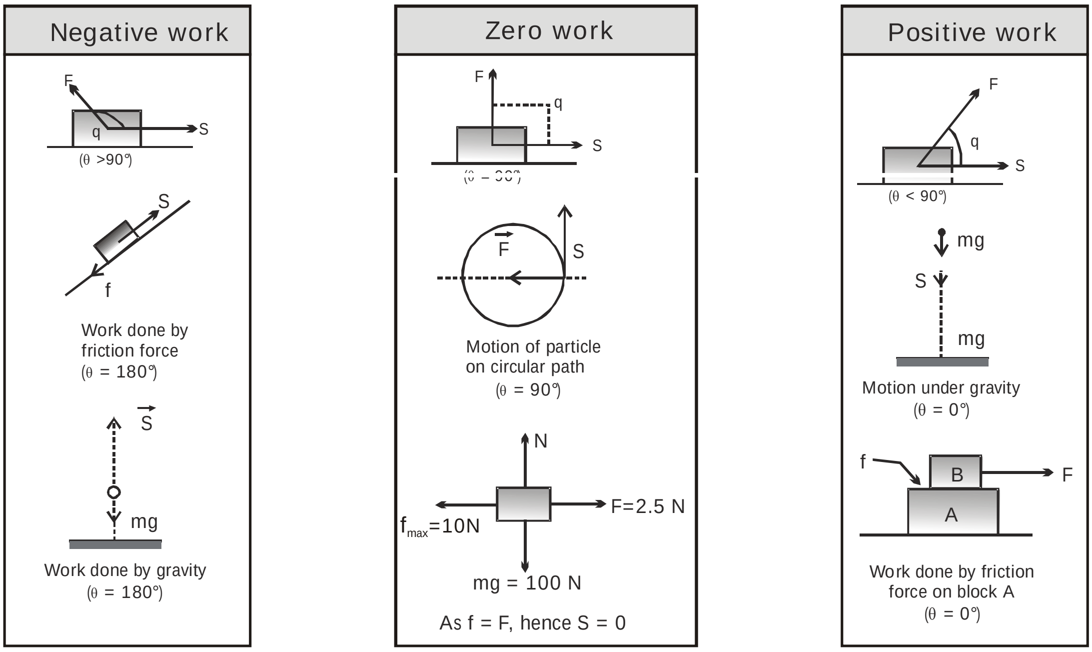
Thus, total work done, $W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} F.dr$ = Area of P_1P_2NM

The area of the graph between curve and displacement axis is equal to the work done.

Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cos θ and r.

Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero



UNITS :

SI Unit : joule (J).

joule : One joule of work is said to be done when a force of one newton displaces a body by one meter in the direction of force.

1 joule = 1 newton × 1 meter = 1 kgm²s⁻²

erg : One erg of work is said to be done when a force of one dyne displaces a body by one cm. in the direction of force.

1 erg = 1 dyne × 1 cm. = 1 gm. cm² s⁻²

Other Units :
(a) 1 joule = 10^7 erg
(b) 1 erg = 10^{-7} joule
(c) 1 eV = 1.6×10^{-19} joule
(d) 1 joule = 6.25×10^{18} eV
(e) 1 MeV = 1.6×10^{-13} J
(f) 1 J = 6.25×10^{12} MeV
(g) 1 kilo watt hour (kWh) = 3.6×10^6 joule

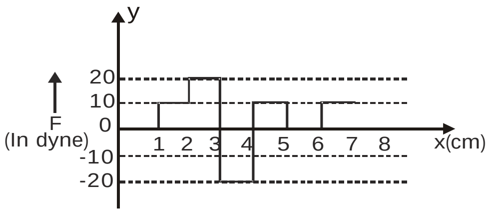
DIMENSIONS :

[Work] = [Force] [Displacement]= $[MLT^{-2}][L] = [ML^2T^{-2}]$

Ex. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joule.

Sol. $W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2)dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = 135J$

Ex. For the force displacement diagram shown in adjoining diagram. Calculate the work done by the force in displacing the body from x = 1 cm to x = 5 cm.



Sol. Work = Area under the curve and displacement axis = 10 + 20 – 20 + 10 = 20 erg

Ex. Calculate work done to move a body of mass 10 kg along a smooth inclined plane (θ = 30°) with constant velocity through a distance of 10 m.

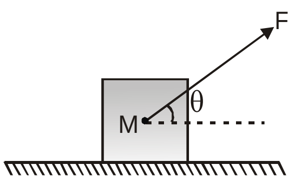
Sol. Here the motion is not accelerated, the resultant force parallel to the plane must be zero. So

$F - Mg \sin 30° = 0 \Rightarrow F = Mg \sin 30°$ & d=10m

$W = Fd\cos\theta = (Mg\sin 30°)d \cos 0° = 10 \times 10 \times \frac{1}{2} \times 10 \times 1 = 500$ J

Ex. Calculate work done in pulling an object with a constant force F as shown in figure.

(given that the ground is rough with coefficient of friction μ)



Sol. From the figure $F \sin\theta + N = Mg$

$\therefore$ $N = Mg - F\sin\theta$

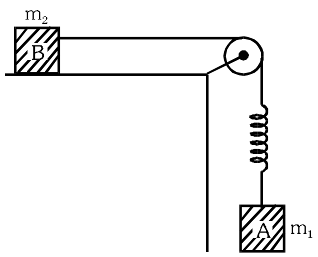
$F\cos\theta = f = \mu N = \mu[Mg - F\sin\theta]$

$F(\cos\theta + \mu\sin\theta) = \mu Mg$

$\therefore$ $F = \frac{\mu Mgd}{\cos\theta + \mu\sin\theta}$ = force required to pull an object

Work done in pulling an object $W = Fd = \frac{\mu Mgd}{\cos\theta + \mu\sin\theta}$

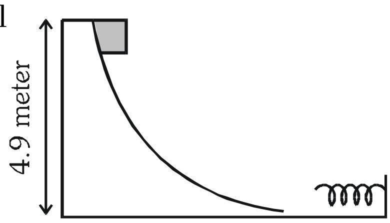
Ex. If the two blocks moves with a constant uniform speed then find coefficient of friction between the surface of the block B and the table. The spring is massless and the pulley is frictionless.



Sol. **For block B** : $T = f = \mu m_2 g$ and **For block A** : $T = m_1 g$

By solving above equations $\mu = \dfrac{m_1}{m_2}$

Ex. Figure shows a smooth curved track terminating in a smooth horizontal part, A spring of force constant 400 N/m is attached at one end to a wedge fixed rigidly with a horizontal part. A 40 g mass is released from rest at a height of 4.9 m on the curved track. Find the maximum compression of the spring.



Sol. From the law of conservation of mechanical energy $\Rightarrow mgh = \dfrac{1}{2}kx^2$

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times (0.04) \times 9.8 \times 4.9}{400}} = 9.8 \text{ cm.}$$

ENERGY

- The energy of a body is defined as the capacity of doing work.
- There are various form of energy

  (i) mechanical energy (ii) chemical energy (iii) electrical energy (iv) sound energy
  (v) light energy etc (vi) magnetic energy (vii) nuclear energy
- Energy of an isolated system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another

Examples

Electrical energy $\xrightarrow{\text{Motor}}$ Mechanical energy

Mechanical energy $\xrightarrow{\text{Generator}}$ Electrical energy

Light energy $\xrightarrow{\text{Photocell}}$ Electrical energy

Electrical energy $\xrightarrow{\text{Heater}}$ Heat energy

Electrical energy $\xrightarrow{\text{Radio / Speaker}}$ Sound energy

Nuclear energy $\xrightarrow{\text{Nuclear Reactor}}$ Electrical energy

Chemical energy $\xrightarrow{\text{Cell}}$ Electrical energy

Electrical energy $\xrightarrow[\text{Chargeable}]{\text{Secondary Cell}}$ Chemical energy

- Energy is a scalar quantity
- Unit : Its unit is same as that of work or torque. In MKS : joule, watt second ; In CGS : erg

  Note : 1 eV = 1.6×10^{-19} joule; 1 kWh = 3.6×10^{6} joule; 10^{7} erg = 1 joule.
- Dimension $[M^1L^2T^{-2}]$
- According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other. Energy equivalent of mass m is, $E = mc^2$ where, m : mass of the particle, c : velocity of light , E : equivalent energy corresponding to mass m.
- In mechanics we are concerned with mechanical energy only which is of two type :

  (i) kinetic energy (ii) potential energy

Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.
- If a body of mass m is moving with velocity v, its kinetic energy KE = 1/2 mv^2.
- Kinetic energy is always positive.
- If linear momentum of body is p, the kinetic energy for translatory motion is KE= $\frac{p^2}{2m} = \frac{1}{2}mv^2$.

Ex. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Sol. Initial kinetic energy, $K_i = \frac{1}{2} \times \frac{50}{1000} \times 200 \times 200 J = 1000J$

Final kinetic energy, $K_f = \frac{10}{100} \times 1000J = 100J$

If v_f is emergent speed of the bullet, then $\frac{1}{2} \times \frac{50}{1000} \times v_f^2 = 100$

$\Rightarrow v_f^2 = 4000 \Rightarrow v_f = 63.2\ ms^{-1}$.

Note that the speed is reduced by approximately 68% and not 90%.

Work Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in it's kinetic energy. So work done by all the forces = change in kinetic energy

$$W = \Delta KE = \frac{1}{2} mv_f^2 - \frac{1}{2}mv_i^2$$

Ex. A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Sol. Work done by all forces $= W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

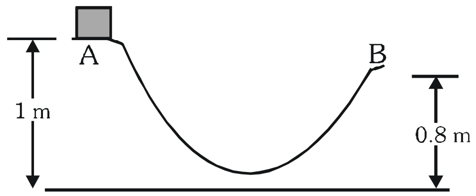
Here $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$, So $W = \frac{1}{2}ma^2d - 0 = \frac{1}{2}ma^2 d$

Ex. The displacement x of a body of mass 1 kg on horizontal smooth surface as a function of time t is given by $x = \frac{t^3}{3}$. Find the work done by the external agent for the first one second.

Sol. $\because x = \frac{t^3}{3} \quad \therefore v = \frac{dx}{dt} = t^2$, Velocity at t = 0, u = 0 and at t = 1s v = 1 m/s

Using work energy theorem : $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}1(1)^2 = 0.5\ J$

Ex. A block of mass 1kg is placed at the point A of a rough track shown in figure. If it slightly pushed towards right it stops at the point B of the track. Calculate the workdone by the frictional force on the block during its transit from A to B.



Sol. $W_c + W_{nc} + W_{ext} = \Delta K$

$mg(1 - 0.8) + W_{nc} + 0 = 0 \Rightarrow W_{nc} = -1 \times 9.8 \times 0.2 = -1.96\ J$

POWER

When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile.

In many cases it is useful to know not just the total amount of work being done, but how fast the work is done. We define power as the rate at which work is being done.

$$\text{Average Power} = \frac{\text{Work done}}{\text{Time taken to do work}} = \frac{\text{Total change in kinetic energy}}{\text{Total change in time}}$$

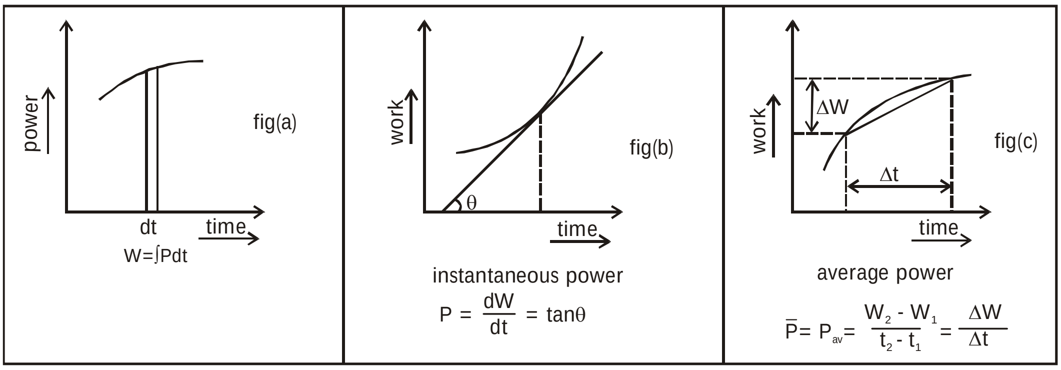
If ΔW is the amount of work done in the time interval Δt. Then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

When work is measured in joules and t is in seconds, the unit for power is the joule per second, which is called watt. For motors and engines, power is usually measured in horsepower, where horsepower is 1 hp = 746 W. The definition of power is applicable to all types of work like mechanical, electrical, thermal.

$$\text{Instanteneous power } P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$$

Where v is the instantaneous velocity of the particle and dot product is used as only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

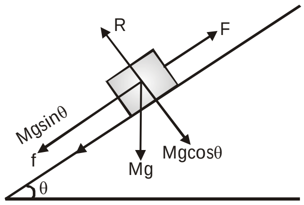
- Power is a scalar quantity with dimension $M^1L^2T^{-3}$
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt



- Area under power–time graph gives the work done. $W = \int Pdt$ (See Fig. a)
- The slope of tangent at a point on work time graph gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work time graph gives average power between two points (See Fig. c)
- For a system of varying mass $F = \frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$
- If v = constant then $F = v\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$

Ex. A truck pulls a mass of 1200 kg at constant speed of 10m/s on a level road. The tension in coupling is 1000 N. What is the power spent on the mass. Find tension when truck moves up a road with inclination 1 in 6.

Sol. Force applied by truck f = 1000 N



Power spent in pulling the mass $P = fv = 1000 \times 10 = 10^4$ W

Here sinθ=1/6, the required force for truck to move up is

$F = f + Mg\sin\theta$

$F = 1000\text{ N} + 1200 \times 9.8 \times \frac{1}{6} = 2960\text{ N}$

CONSERVATIVE FORCE

- A force is said to be conservative if the work done by or against the force is independent of path and depends only on initial and final positions
- It does not depend on the nature of path followed between the initial and final positions.

Examples of Conservative Force

All central forces are conservative like gravitational, electrostatic, elastic force, restoring force due to spring etc.

- In presence of conservative forces mechanical energy remains constant.
- Work done along a closed path or in a cyclic process is zero. i.e. $\oint \vec{F}.d\vec{r} = 0$

CENTRAL FORCE

The force whose line of action always passes through a fixed point (which is known as centre of force) and magnitude of force depends only on the distance from this point is known as **central force.**

$$\vec{F} = F(r)\hat{r}$$

All forces following inverse square law are called central forces.

$\vec{F} = \frac{k}{r^2}\hat{r}$ is central force like Gravitational force and Coulomb force.

- All central forces are conservative forces
- Central forces are function of position

NON CONSERVATIVE FORCE

A force is said to be non–conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non–conservative force field.

The frictional forces are non–conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Examples of non–conservative force

The velocity–dependent forces such as air resistance, viscous force etc. are non–conservative forces.

Conservative Forces	**Non–conservative Forces**
• Work done does not depend upon path.	• Work done depends upon path.
• Work done in a round trip is zero.	• Work done in a round trip is not zero.
• Central forces, spring forces etc. are conservative forces	• Force are velocity–dependent & retarding in nature e.g. friction, viscous force etc.
• When only a conservative force acts within a system, the kinetic energy and potential energy can change. However, their sum, the mechanical energy of the system, doesnot change.	• Work done against a non–conservative force may be dissipated as heat energy.
• Work done is completely recoverable.	• Work done is not completely recoverable.

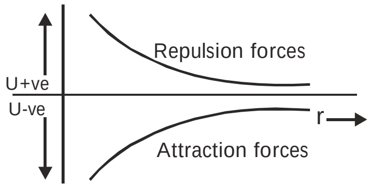
Potential energy

- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the workdone by an external agent against the action of conservative force in order to shift it from reference point. (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the body in moving from its present position to reference position.
- At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.
- Relationship between conservative force field and potential energy :

$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

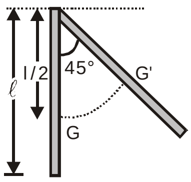
- If force varies only with one dimension (along x-axis) then $F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F dx$

- Potential energy may be positive or negative
  (i) Potential energy is positive, if force field is repulsive in nature
  (ii) Potential energy is negative, if force field is attractive in nature
- If r $\uparrow$ (separation between body and force centre), U $\uparrow$, force field is attractive or vice–versa.



- If r $\uparrow$, U $\downarrow$, force field is repulsive in nature.

Ex. A meter scale of mass m initially vertical is displaced at 45^0 keeping the upper end fixed. Find out the change in potential energy.



Sol. $\Delta U = mg\,\Delta h_{cm} = mg\frac{\ell}{2}(1-\cos\theta) = mg \times \frac{1}{2}\,(1\text{–}\cos 45^0) = \frac{mg}{2}\left(1-\frac{1}{\sqrt{2}}\right)$

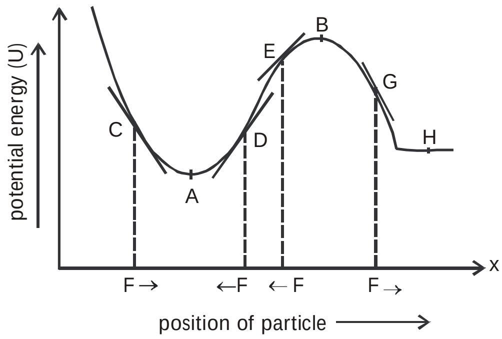
Ex. A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. Calculate the work done in keeping it vertical with one of its ends touching the ground.

Sol. As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length = $\ell/2$

Work done $W = mgh = mg\,\frac{\ell}{2} = (20)(9.8)\left(\frac{4}{2}\right) = 392$ J

POTENTIAL ENERGY CURVE AND EQUILIBRIUM

It is a curve which shows change in potential energy with postion of a particle.



Stable Equilibrium :

When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium

then it is said to be in stable equilibrium

At point **C :** slope $\frac{dU}{dx}$ is negative so F is positive

At point **D :** slope $\frac{dU}{dx}$ is positive so F is negative

At point **A :** it is the point of stable equilibrium.

At **A** $U = U_{min}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = positive

Unstable equilibrium :

When a particle is slightly displaced from equilbrium and it tends to move away from equilibrium position then it is said to be in unstable equilbirum

At point **E** : slope $\frac{dU}{dx}$ is positive so F is negative

At point **G** : slope $\frac{dU}{dx}$ is negative so F is postive

At point **B** : it is the point of unstable equilibrium.

At **B** $U = U_{max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = negative

Neutral equilibrium :

When a particle is slightly displaced from equilibrium position and no force acts on it then equilbirum is said to be neutral equilibrium

Point **H** is at neutral equilibrium $\Rightarrow U$ = constant ; $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

Ex. The potential energy for a conservative force system is given by $U = ax^2 - bx$. Where a and b are constants find out

(a) The expression of force

(b) Equilibrium position

(c) Potential energy at equilibrium.

Sol. (a) For conservative force $F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$

(b) At equilibrium $F = 0 \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$ (c) $U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$

Law of conservation of Mechanical energy

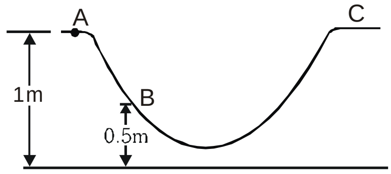
Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles and the work done by all other forces is zero.

From work energy theorem $W = \Delta KE$

For internal conservative forces $W_{int} = -\Delta U$

So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE \Rightarrow (KE + U) = 0 \Rightarrow KE + U = (\text{constant})$

Ex. A particle is placed at the point A of a frictionless track ABC. It is pushed slightly towards right. Find its speed when it reaches the point B. [Take g=10 m/s²]



Sol. $mg(1 - 0.5) = \frac{1}{2} \times m \times v^2 \Rightarrow v^2 = (2)(10)(0.5) \Rightarrow v = \sqrt{10}$ m/s

Ex. Determine the average force necessary to stop a bullet of mass 20 g and speed 250 ms^{-1} as it penetrates wood to a distance of 12 cm.

Sol. If F newton be the retarding force, then the work done by force $W = F \times s = F \times 0.12$ joule

Loss of kinetic energy $= \frac{1}{2} \times \frac{20}{1000} \times (250)^2 = 625$ joule

(This kinetic energy is consumed in stopping the bullet and is converted into heat energy)

Applying work–energy theorem, $F \times 0.12 = 625 \Rightarrow F = \frac{625}{0.12}$ N $= 5.2 \times 10^3$ N

It is interesting to note that the retarding force is nearly 30,000 times the weight of the bullet.

Ex. A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as $x = \frac{1}{2}t^2$ where x is in meter and t in sec. Find the work done by the force in first two seconds.

Sol. Work done = change in kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left(\frac{2t}{2}\right)^2 = \frac{1}{2}\times 8\times\left[\frac{2\times 2}{2}\right]^2 = 16J$

Ex. A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then determine height upto the body would reach.

Sol. $\frac{1}{4}mgh = mgh' \quad \therefore\ h' = \frac{h}{4} = \frac{1}{4}\times 20 = 5\text{ cm}$

Ex. Calculate the stopping distance for a vehicle of mass m moving with speed v along level road. (μ is the coefficient of friction between tyres and the road)

Sol. When the vehicle of mass m is moving with velocity v, the kinetic energy of the vehicle $K = \frac{1}{2}mv^2$ and if S is the stopping distance, work done by the friction

$W = FS\cos\theta = \mu mgS\cos 180^{O} = -\mu mgS$

So by Work–Energy theorem, $W = \Delta K = K_f - K_i \Rightarrow -\mu mgS = 0 - 1/2\, mv^2 \Rightarrow S = \frac{v^2}{2\mu g}$

Ex. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $(-k/r^2)$, where k is constant. Calculate the total energy of the particle.

Sol. As the particle is moving in a circle, so $\frac{mv^2}{r} = \frac{k}{r^2}$. Now $K.E = \frac{1}{2}mv^2 = \frac{k}{2r}$

As $U = -\int_{\infty}^{r} F.dr = \int_{\infty}^{r} +\left(\frac{k}{r^2}\right)dr = -\frac{k}{r}$. So total energy = U + K.E. $= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$

Negative energy means that particle is in bound state.

Ex. A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save?

Sol. In first case, $W_1 = \frac{1}{2}m(v_1)^2 + mgh = \frac{1}{2}m(12)^2 + m\times 10\times 12 = 72\text{ m} + 120\text{ m} = 192\text{m}$

and in second case, $W_2 = mgh = 120\text{ m}$

The percentage of energy saved $= \frac{192m - 120m}{192m}\times 100 = 38\%$

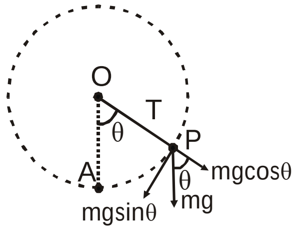
CIRCULAR MOTION IN VERTICAL PLANE

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point B of the circle as shown in figure. Here,

$h = R(1 - \cos\theta)$ ...(i)

From conservation of mechanical energy

$\frac{1}{2}m(u^2 - v^2) = mgh \Rightarrow v^2 = u^2 - 2gh$ ....(ii)



The necessary centripetal force is provided by the resultant of tension

T and mg cosθ $T - mg\cos\theta = \frac{mv^2}{R}$ ...(iii)

Since speed of the particle decreases with height, hence tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0^0$)

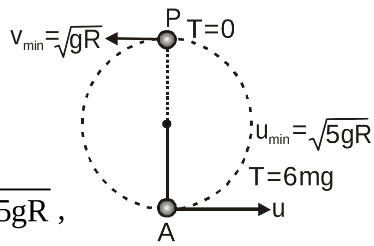
$\Rightarrow T_{max} = \frac{mv^2}{R} + mg$; $T_{min} = \frac{mv^2}{R} - mg$ at the top. Here, v' = speed of the particle at the top.

Condition of Looping the Loop $\left(u \geq \sqrt{5gR}\right)$

The particle will complete the circle if the string does not slack even at the highest point $(\theta = \pi)$. Thus, tension in the string should be greater than or equal to zero ($T \geq 0$) at $\theta = \pi$. In critical case substituting T = 0 and $\theta = \pi$ in Eq. (iii), we get $mg = \frac{mv_{min}^2}{R} \Rightarrow v_{min} = \sqrt{gR}$ (at highest point)

Substituting $\theta = \pi$ in Eq. (i), Therefore, from Eq. (ii)

$u_{min}^2 = v_{min}^2 + 2gh = gR + 2g(2R) = 5gR \Rightarrow u_{min} = \sqrt{5gR}$



Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle. At $u = \sqrt{5gR}$, velocity at highest point is $v = \sqrt{gR}$ and tension in the string is zero.

Substituting $\theta = 0^0$ and $v = \sqrt{5gR}$ in Eq. (iii), we get T = 6 mg or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure. If $u < \sqrt{5gR}$, following two cases are possible.

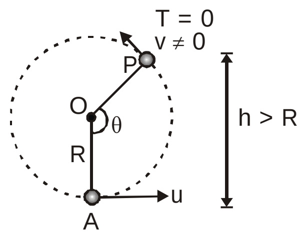
Condition of Leaving the Circle $\left(\sqrt{2gR} < u < \sqrt{5gR}\right)$

If $u < \sqrt{5gR}$, the tension in the string will become zero before reaching the highest point. From Eq. (iii), tension in the string becomes zero (T = 0) where, $\cos\theta = \frac{-v^2}{Rg} \Rightarrow \cos\theta = \frac{2gh - u^2}{Rg}$

Substituting, this value of $\cos\theta$ in Eq. (i), we get $\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R} \Rightarrow h = \frac{u^2 + Rg}{3g} = h_1$ (say)

or we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity of the particle becomes zero when $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = h_2$(say)...(v) i.e., at height h_2 velocity of particle becomes zero.

Now, the particle will leave the circle if tension in the string becomes zero but velocity is not zero. or T = 0 but $v \neq 0$. This is possible only when $h_1 < h_2$



$$\Rightarrow \frac{u^2 + Rg}{3g} < \frac{u^2}{2g} \Rightarrow 2u^2 + 2Rg < 3u^2 \Rightarrow u^2 > 2Rg \Rightarrow u > \sqrt{2Rg}$$

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

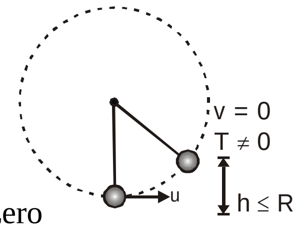
From Eq. (iv), we can see that h > R if $u^2 > 2gR$. Thus, the particle, will leave the circle when h > R or $90^0 < \theta < 180^0$. This situation is shown in the figure

$$\sqrt{2gR} < u < \sqrt{5gR} \text{ or } 90^0 < \theta < 180^0$$

Note : That after leaving the circle, the particle will follow a parabolic path.

Condition of Oscillation $\left(0 < u \leq \sqrt{2gR}\right)$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero or v = 0, but $T \neq 0$. This is possible when $h_2 < h_1$



$$\Rightarrow \frac{u^2}{2g} < \frac{u^2 + Rg}{3g} \Rightarrow 3u^2 < 2u^2 + 2Rg \Rightarrow u^2 < 2Rg \Rightarrow u < \sqrt{2Rg}$$

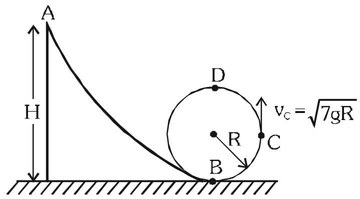
Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ and tension and velocity both becomes zero simultaneously. Further, from Eq. (iv), we can see that $h \leq R$ if $u \leq \sqrt{2Rg}$.

Thus, for $0 < u \leq \sqrt{2gR}$, particle oscillates in lower half of the circle $(0^0 < \theta \leq 90°)$

This situation is shown in the figure. $\quad 0 < u \leq \sqrt{2gR}$ or $0° < \theta \leq 90°$

Ex. Calculate following for shown situation :–

(a) Speed at D (b) Normal reaction at D (c) Height H



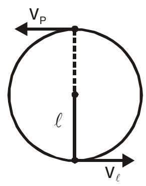
Sol. (a) $v_D^{\,2} = v_C^{\,2} - 2gR = 5gR \Rightarrow v_D = \sqrt{5gR}$

(b) $mg + N_D = \dfrac{mv_D^2}{R} \Rightarrow N_D = \dfrac{m(5gR)}{R} - mg = 4mg$

(c) by energy conservation between point A & C

$$mgH = \frac{1}{2}mv_C^2 + mgR = \frac{1}{2}m(5gR) + mg2R = \frac{9}{2}mgR \Rightarrow H = \frac{9}{2}R$$

Ex. A stone of mass 1 kg tied to a light string of length $\ell = \dfrac{10}{3}$ m is whirling in a circular path in vertical plane. If the ratio of the maximum to minimum tension in the string is 4, find the speed of the stone at the lowest and highest points

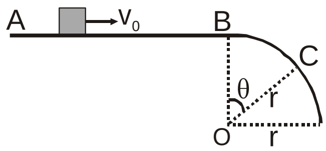


Sol. $\because \dfrac{T_{max}}{T_{min}} = 4 \therefore \dfrac{\dfrac{mv_\ell^2}{\ell} + mg}{\dfrac{mv_p^2}{\ell} - mg} = 4 \Rightarrow \dfrac{v_\ell^2 + g\ell}{v_p^2 - g\ell} = 4$

We know $v_\ell^2 = v_p^2 + 4g\ell \Rightarrow \dfrac{v_P^2 + 5g\ell}{v_P^2 - g\ell} = 4 \Rightarrow 3v_P^{\,2} = 9g\ell$

$\Rightarrow v_P^{\,2} = \sqrt{3g\ell} = \sqrt{3\times10\times\dfrac{10}{3}} = 10\ ms^{-1} \Rightarrow v_\ell = \sqrt{7g\ell} = \sqrt{7\times10\times\dfrac{10}{3}} = 15.2\ ms^{-1}$

Ex. A small block slides with velocity $0.5\sqrt{gr}$ on the horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. Calculate angle θ in the figure.

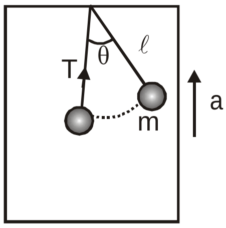


Sol. As block leaves the surface at C so at C, normal reaction = 0 $\Rightarrow mg\cos\theta = \dfrac{mv_C^2}{r}$

By energy conservation at point B & C $\dfrac{1}{2}mv_C^{\,2} - \dfrac{1}{2}mv_0^{\,2} = mgr\,(1-\cos\theta)$

$\Rightarrow \dfrac{1}{2}m\,(rg\cos\theta) - \dfrac{1}{2}m\left(0.5\sqrt{gr}\right)^2 = mgr\left(1-\cos\theta\right) \Rightarrow \cos\theta = \dfrac{3}{4} \Rightarrow \theta = \cos^{-1}\left(\dfrac{3}{4}\right)$

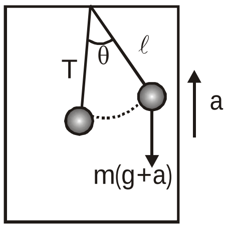
Ex. A particle of mass m is attached to the ceiling of a cabin with an inextensible light string of length ℓ. The cabin is moving upward with an acceleration 'a'. The particle is taken to a position such that the string makes an angle θ with vertical. When string becomes vertical, find the tension in the string.



Sol. In a frame associated with cabin work done on the particle when it comes in the vertical position = $mg\ell(1-\cos\theta)+ma(1-\cos\theta)$

By work energy theorem,

$$\frac{mv^2}{2}=(mg\ell+ma\ell)(1-\cos\theta)\Rightarrow\frac{v^2}{2}=(g+a)\,l\,(1-\cos\theta)$$



At vertical position, $T=(mg+ma)=\frac{mv^2}{\ell}$

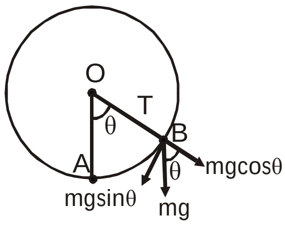
$\Rightarrow T=(mg+ma)+2m(g+a)(1-\cos\theta)=mg\,(g+a)(3-2\cos\theta)$

Ex. A heavy particle hanging from a fixed point by a light inextensible string of length ℓ, is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Sol. Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

$$F_R=T-mg\cos\theta \quad ....(i)$$

If v_B be the speed of the particle at B, then



$$F_R=\frac{mv_B^2}{\ell} \quad ....(ii)$$

From (i) and (ii), we get, $T-mg\cos\theta=\frac{mv_B^2}{\ell}$ ....(iii)

Since at B, T = mg $\Rightarrow mg(1-\cos\theta)=\frac{mv_B^2}{\ell}\Rightarrow v_B^2=g\ell(1-\cos\theta)$ ...(iv)

Applying conservation of mechanical energy of the particle at point A and B, we have

$\frac{1}{2}mv_A{}^2=mg\ell\,(1-\cos\theta)+\frac{1}{2}mv_B^2$; where $v_A=\sqrt{g\ell}$ and $v_B=\sqrt{g\ell(1-\cos\theta)}$

$$\Rightarrow g\ell=2g\ell(1-\cos\theta)+g\ell\,(1-\cos\theta)\Rightarrow\cos\theta=\frac{2}{3}\Rightarrow\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Putting the value of $\cos\theta$ in equation (iv), we get : $v=\sqrt{\frac{g\ell}{3}}$

EXERCISE (S-1)

Work

1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative: **(NCERT)**

(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(b) work done by gravitational force in the above case,

(c) work done by friction on a body sliding down an inclined plane,

(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

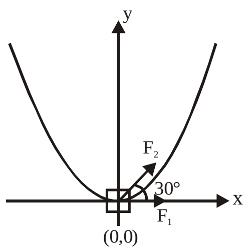
2. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by **(NCERT)**

$$F = -\hat{i} + 2\hat{j} + 3\hat{k}\text{N}$$

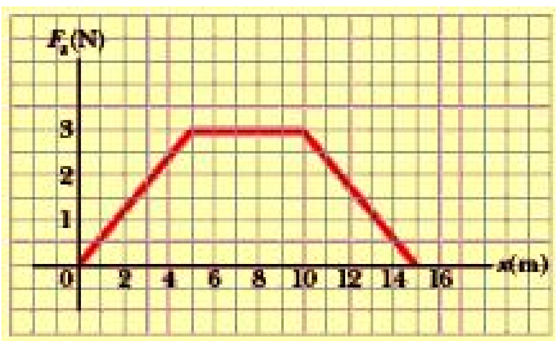
where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis ?

3. A point mass of 0.5 kg is moving along x-axis as $x = t^2 + 2t$, where, x is in meters and t is in seconds. Find the work done (in J) by all the forces acting on the body during the time interval [0, 2s].

4. A sleeve of mass 2 kg at origin can move on wire of parabolic shape $x^2 = 4y$. Two forces $F_1 = 6N$ and $F_2 = 8N$ are applied on the sleeve. F_1 is constant and is in x-direction. F_2 is constant in direction and magnitude. Body is displaced from origin to x = 4, then net work done by F_1 and F_2 is



5. A particle is subject to a force F_x that varies with position as in figure. Find the work done by the force on the body as it moves (a) from x = 0 to x = 5.00 m, (b) from x = 5.00 m to x = 10.0 m, and (c) from x = 10.0 m to x = 15.0 m. (d) What is the total work done by the force over the distance x = 0 to x = 15.0 m?



6. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2.

Find $\frac{W_2}{W_1}$.

Kinetic energy, Work energy theorem, Power

7. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the **(NCERT)**

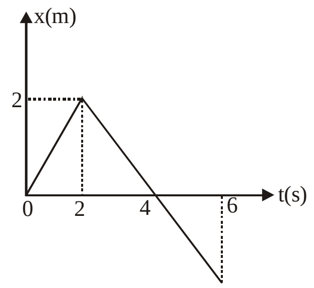
(a) work done by the applied force in 10 s,

(b) work done by friction in 10 s,

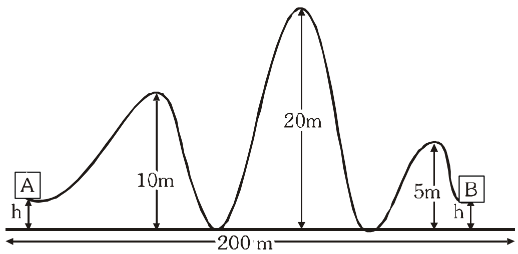
(c) work done by the net force on the body in 10 s,

(d) change in kinetic energy of the body in 10 s, and interpret your results.

8. Position-time graph of a particle of mass 2 kg is shown in figure. Total work done on the particle from t = 0 to t = 4s is



9. A point object of mass 2 kg is moved from point A to point B very slowly on a curved path by applying a tangential force on a curved path as shown in figure. Then find the work done by external force in moving the body. Given that $\mu_s = 0.3$, $\mu_k = 0.1$. [$g = 10\ m/s^2$]



10. A 4 kg particle moves along the X-axis. It's position x varies with time according to $x(t) = t + 2t^3$, where x is in m and t is in seconds. Compute:

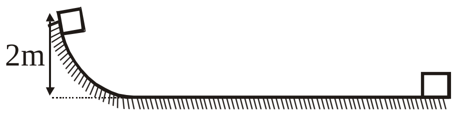
(i) The kinetic energy at time t.

(ii) The force acting on the particle at time t.

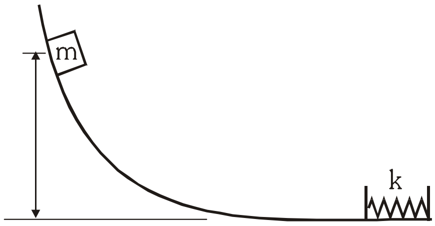
(iii) The power delivered to the particle at time t.

(iv) The work done on the particle from t = 0 to t = 2 seconds.

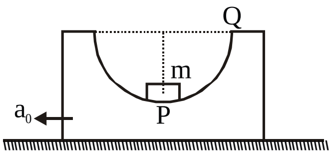
11. A block is released from rest from top of a rough curved track as shown in figure. It comes to rest at some point on the horizontal part. If its mass is 200 gm, calculate negative of work by friction in joules.



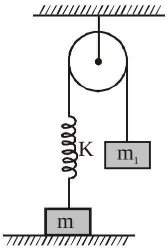
12. A mass m slides from rest at height h down a smooth curved surface which becomes horizontal at zero height (see figure). A spring is fixed horizontally on the level part of the surface. The spring constant is k N/m. When the mass encounters the spring it compresses it by an amount x = h/10. If m = 1 kg, h = 5m then find k /100.



13. A small block of mass m is lying at rest at point P of a wedge having a smooth semi circular track of radius R. The minimum value of horizontal acceleration a_0 (in m/s^2) of wedge so that mass can just reach the point Q, is



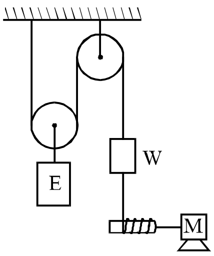
14. Initially spring is relaxed when m_1 is released from rest. Calculate for what minimum value of m_1 the block of mass m will just leave the contact with surface ? Give answer in terms of $\frac{m}{m_1}$.



15. The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor

(a) when the elevator is moving down at a constant speed of 3 m/s,

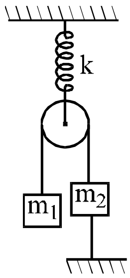
(b) when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s^2.



16. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between time t = 2 s and t = 4 s.

Conservative & non conservation forces, Potential energy, Conservation of energy

17. In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring $(m_1 > m_2)$.



18. The potential energy (in joules) function of a particle in a region of space is given as :

$$U = (2x^2 + 3y^3 + 2z)$$

Here x, y and z are in metres. Find the magnitude of x component of force (in newton) acting on the particle at point P (1m, 2m, 3m).

19. The P.E. of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J

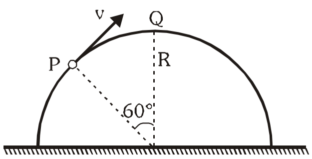
(i) Find the mean position

(ii) Find the max. K.E. of the particle

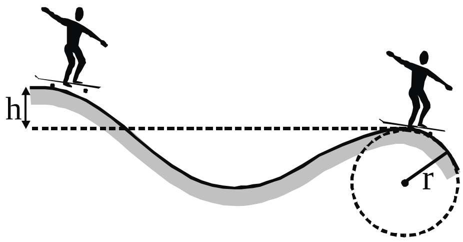
20. The potential energy of a 2 kg particle moving along the x axis is given by $U(x) = (4.0 J/m^2)x^2 + (1.0 J/m^4)x^4$. When the particle is at x = 1.0m, find its acceleration. [only conservative forces are acting]

Potential energy diagram, Equilibrium, Vertical circular motion

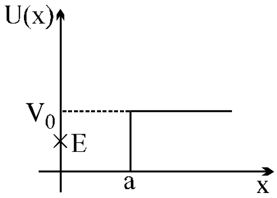
21. A particle is given a certain veloicty v at point P as shown on a hemispherical smooth surface. Find the value of v (in m/s), such that when particle reaches Q, the normal reaction of surface becomes equal to particle's weight. [R = 1.6 m, g = 10 m/s²]



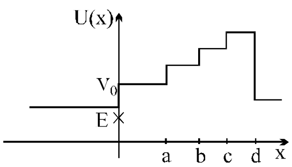
22. A skier starts from rest at the top of a hill. The skier coasts down the hill and up a second hill, as the drawing illustrates. The crest of the second hill is circular, with a radius of r = 36 m. Neglect friction and air resistance. What must be the height h (in m) of the first hill so that the skier just loses contact with the snow at the crest of the second hill?

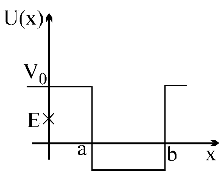


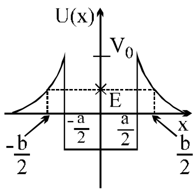
23. The given graph is a potential energy function in one dimension. The total energy of particle is indicated by cross on the ordinate axis. The graph of figure-1 is given as an example.From the figure-1, it can be interpreted that for the given total energy indicated by cross on the ordinate axis the particle cannot be found in the Region : x > a. Now, for the following potential functions in one dimensions, specify the regions, in which the particle cannot be found for the energy marked as E on graphs. Give your answer in the blocks shown.



(Figure -1)

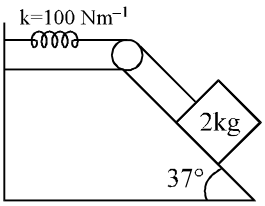
(Graph-1) 

(Graph-2) 

(Graph-3) 

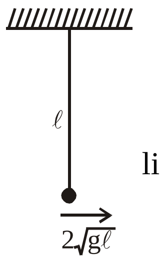
24. (a) A 2 kg block situated on a smooth fixed incline is connected to a spring of negligible mass, with spring constant k = 100 Nm^{-1}, via a frictionless pulley. The block is released from rest when the spring is unstretched. How far does the block move down the incline before coming (momentarily) to rest? What is its acceleration at its lowest point?

(b) The experiment is repeated on a rough incline. If the block is observed to move 0.20 m down along the incline before it comes to instantaneous rest, calculate the coefficient of kinetic friciton.



25. A ball is attached to a horizontal cord of length L whose other end is fixed, (a) If the ball is released, what will be its speed at the lowest point of its path ? (b) A peg is located a distance h directly below the point of attachment of the cord. If h = 0.75 L, what will be the speed of the ball when it reaches the top of its circular path about the peg ?

26. One end of a string of length $\ell = \frac{14}{9}$ m is fixed and a mass of 1 kg is tied to the other end. The ball is given a velocity $2\sqrt{g\ell}$ at the bottom most point as shown in figure. The string is cut when the ball becomes horizontal. Find the distance (in m) travelled till it stop for the 1st time (Take $\pi = \frac{22}{7}$).



EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Work

1. A force of magnitude of 30 N acting along $\hat{i}+\hat{j}+\hat{k}$, displaces a particle from point (2, 4, 1) to (3, 5, 2). The work done during this displacement is :-

(A) 90 J (B) 30 J (C) $30\sqrt{3}$ J (D) $30/\sqrt{3}$ J.

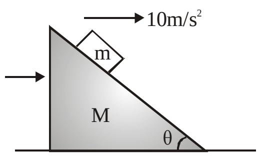
2. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5cm to 15 cm is- **[AIEEE - 2002]**

(A) 16 J (B) 8 J (C) 32 J (D) 24 J

3. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is :

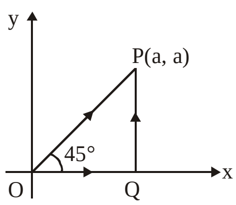
(A) Mgx (B) ½ Mgx^2 (C) – ½ Mgx (D) Mgx^2

4. In the figure shown all the surfaces are frictionless, and mass of the block, m = 1 kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds is :



(A) 30J (B) 60 J (C) 150 J (D) $100\sqrt{3}$ J

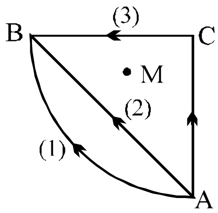
5. A particle is moved from (0, 0) to (a, a) under a force $\vec{F}=(3\hat{i}+4\hat{j})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then :



(A) $W_1 = W_2$ (B) $W_1 = 2W_2$ (C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

6. In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then

[IIT-JEE (Scr.)'2003]



(A) $W_1 = W_2 = W_3$ (B) $W_1 = W_2 > W_3$ (C) $W_1 > W_2 > W_3$ (D) $W_1 < W_2 < W_3$

Kinetic energy, Work energy theorem, Power

7. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}s^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

(A) 1.5 J (B) 50 J (C) 10 J (D) 100 J

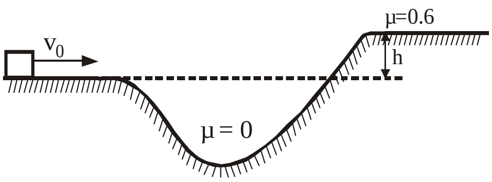
8. A man who is running has half the kinetic energy of the boy of half his mass. The man speeds up by1 m/s and then has the same kinetic energy as the boy. The original speed of the man was

(A) $\sqrt{2}$ m/s (B) $(\sqrt{2} - 1)$ m/s (C) 2 m/s (D) $(\sqrt{2} + 1)$ m/s

9. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be

(A) 250 π^2 joule (B) $100\pi^2$ joule (C) $5\pi^2$ joule (D) 0 joule

10. In the figure, a block slides along a track from one level to a higher level, by moving through an intermediate valley. The track is frictionless untill the block reaches the higher level. There a frictional force stops the block in a distance d. The block's initial speed v_0 is 6 m/s, the height difference h is 1.1m and the coefficient of kinetic friction μ is 0.6. The value of d is



(A) 1.17 m (B) 1.71 m (C) 7.11 m (D) 11.7 m

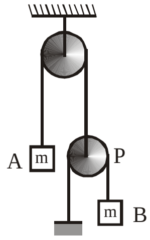
11. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m/s at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s^2, the kinetic energy of the shotput when it just reaches the ground will be

(A) 2.5 J (B) 5.0 J (C) 52.5 J (D) 155.0 J

12. A particle moves on a rough horizontal ground with some initial velocity say v_0. If 3/4 of its kinetic energy is lost in friction in time t_0 then coefficient of friction between the particle and the ground is:-

(A) $\frac{v_0}{2gt_0}$ (B) $\frac{v_0}{4gt_0}$ (C) $\frac{3v_0}{4gt_0}$ (D) $\frac{v_0}{gt_0}$

13. In the figure shown, the system is released from rest. Find the velocity of block A when block B has fallen a distance 'ℓ'. Assume all pulleys to be massless and frictionless.

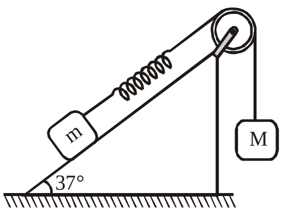


(A) $\sqrt{\frac{g\ell}{5}}$ (B) $\sqrt{g\ell}$ (C) $\sqrt{5g\ell}$ (D) None of these

14. A block of mass m is hung vertically from an elastic thread of force constant mg/a. Initially the thread was at its natural length and the block is allowed to fall freely. The kinetic energy of the block when it passes through the equilibrium position will be :

(A) mga (B) mga/2 (C) zero (D) 2mga

15. A block of mass m is attached with a massless spring of force constant k. The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The minimum value of M required to move the block up the plane is : (Neglect mass of string and pulley and friction in pulley)



(A) $\frac{3}{5}$ m (B) $\frac{4}{5}$ m (C) 2 m (D) $\frac{3}{2}$ m

16. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to- **[AIEEE - 2003]**

(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

17. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to- **[AIEEE - 2004]**

(A) x^2 (B) e^x (C) x (D) $\log_e x$

18. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is- **[AIEEE - 2004]**

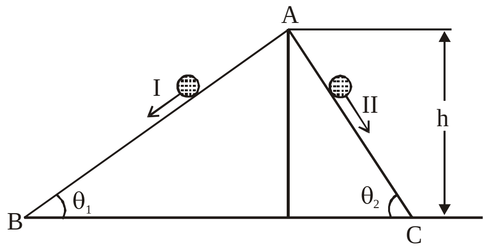
(A) $\frac{mv_1t}{t_1}$ (B) $\frac{mv_1^2t}{t_1^2}$ (C) $\frac{mv_1t^2}{t_1}$ (D) $\frac{mv_1^2t}{t_1}$

19. Assume the aerodynamic drag force on a car is proportional to its speed. If the power output from the engine is doubled, then the maximum speed of the car.

(A) is unchanged
(B) increases by a factor of $\sqrt{2}$
(C) is also doubled
(D) increases by a factor of four.

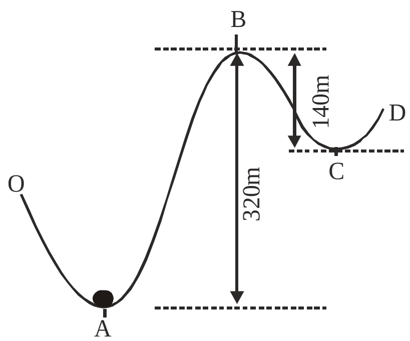
Conservative & non conservative forces, Potential energy, Conservation of energy

20. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure. Which of the following statement is correct?



(A) Both the stones reach the bottom at the same time but not with the same speed.
(B) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(C) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(D) Both the stones reach the bottom at different times and with different speeds.

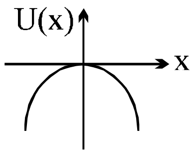
21. Track OABCD (as shown is figure) is smooth and fixed in vertical plane. What minimum speed has to be given to a particle lying at point A, so that it can reach point C?

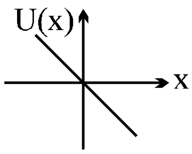


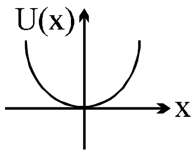
(A) 60 m/s (B) 100 m/s (C) 70 m/s (D) 80 m/s

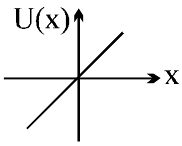
22. A particle is placed at the origin and a force F = kx is acting on it (where k is a positive constant). If U(0) = 0, the graph of U(x) versus x will be (where U is the potential energy function)

[IIT-JEE' 2004(Scr)]

(A) 

(B) 

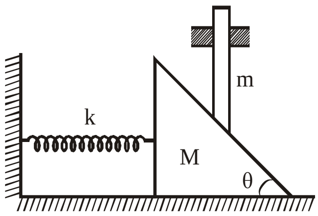
(C) 

(D) 

23. When a conservative force does positive work on a body

(A) the potential energy increases
(B) the potential energy decreases
(C) total energy increases
(D) total energy decreases

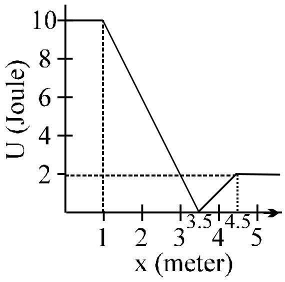
24. A wedge of mass M fitted with a spring of stiffness 'k' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium. Assuming that all surfaces are smooth, the potential energy stored in the spring is:



(A) $\dfrac{mg^2 \tan^2\theta}{2k}$
(B) $\dfrac{m^2 g \tan^2\theta}{2k}$
(C) $\dfrac{m^2 g^2 \tan^2\theta}{2k}$
(D) $\dfrac{m^2 g^2 \tan^2\theta}{k}$

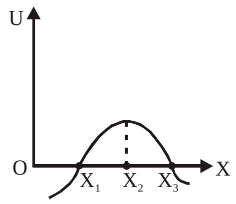
Potential energy diagram, Equilibrium, Vertical circular motion

25. A body with mass 2 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at x = 2m, then its speed when it crosses x = 5 m is



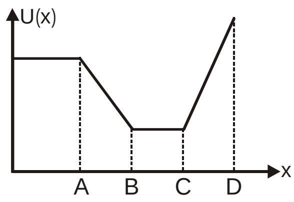
(A) zero
(B) 1 ms^{-1}
(C) 2 ms^{-1}
(D) 3 ms^{-1}

26. In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at :



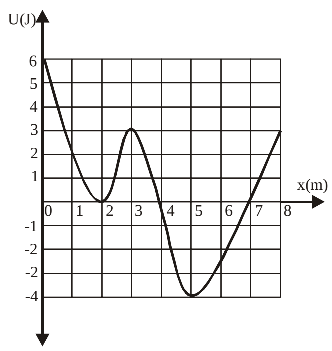
(A) x_1 is in stable equilibrium
(B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium
(D) None of these

27. As a particle moves along the x–axis it is acted upon by a conservative force. The potential energy is shown below as a function of the coordinate x of the particle. Rank the labelled regions according to the magnitude of the force, least to greatest.



(A) AB, BC, CD
(B) AB, CD, BC
(C) BC, CD, AB
(D) BC, AB, CD

28. Potential energy curve U of a particle as function of the position of a particle is shown. The particle has total mechanical energy E of 3.0 joules.



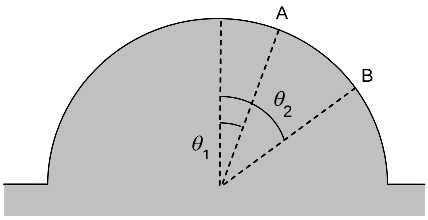
(A) It can never be present at x= 0 m.

(B) It can never be present at x=5 m

(C) At x=2 its kinetic energy is 0 J

(D) At x =1 its kinetic energy 3 J

29. A small block slides down from rest at point A on the surface of a smooth circular cylinder, as shown. At point B, the block falls off (leaves) the cylinder. The equation relating the angles θ_1 and θ_2 is given by



(A) $\sin\theta_2 = \frac{2}{3}\sin\theta_1$ (B) $\sin\theta_2 = \frac{3}{2}\sin\theta_1$ (C) $\cos\theta_2 = \frac{2}{3}\cos\theta_1$ (D) $\cos\theta_2 = \frac{3}{2}\cos\theta_1$

MULTIPLE CORRECT TYPE QUESTIONS

30. Which of the following statements is TRUE for a system comprising of two bodies in contact exerting frictional force on each other :

(A) total work done by static friction on whole system is always zero.

(B) work done by static friction on a body is always zero

(C) work done by kinetic friction on a body is always negative

(D) total work done by internal kinetic friction on whole system is always negative

31. A particle of mass *m* is at rest in a train moving with constant velocity with respect to ground. Now the particle is accelerated by a constant force F_0 acting along the direction of motion of train for time t_0. A girl in the train and a boy on the ground measure the work done by this force. Which of the following are **INCORRECT**?

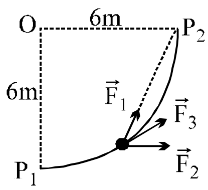
(A) Both will measure the same work

(B) Boy will measure higher value than the girl

(C) Girl will measure higher value than the boy

(D) Data are insufficient for the measurement of work done by the force F_0

32. A smooth track in the form of a quarter circle of radius 6 m lies in the vertical plane. A particle moves from P_1 to P_2 under the action of forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. Force $\vec{F}_1$ is always toward P_2 and is always 20N in magnitude. Force $\vec{F}_2$ always acts horizontally and is always 30 N in magnitude. Force $\vec{F}_3$ always acts tangentially to the track and is of magnitude 15 N. Select the correct alternative(s)



(A) work done by $\vec{F}_1$ is 120 J

(B) work done by $\vec{F}_2$ is 180 J

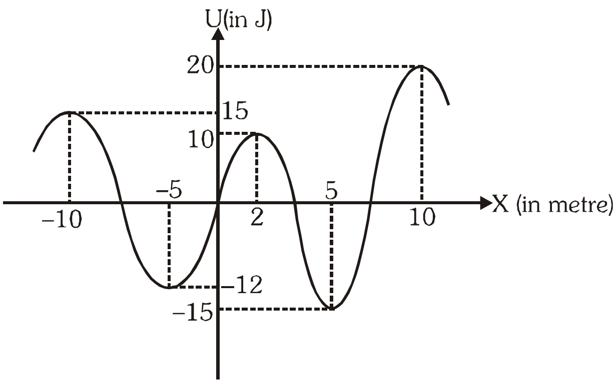
(C) work done by $\vec{F}_3$ is 45 π

(D) $\vec{F}_1$ is conservative in nature

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 33 to 35

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



33. If the particle is released at the origin then :

(A) It will move towards positive x-axis.

(B) It will move towards negative x-axis.

(C) It will remain stationary at the origin.

(D) Its subsequent motion cannot be decided due to lack of information.

34. If the particle is released at $x = 2 + \Delta$ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be-

(A) $\sqrt{22}$ m/s (B) $\sqrt{25}$ m/s (C) $\sqrt{24}$ m/s (D) $\sqrt{23}$ m/s

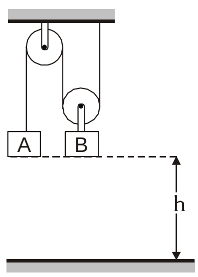
35. x = –5 m and x = 10 m positions of the particle are respectively of-

(A) Neutral and stable equilibrium (B) Neutral and unstable equilibrium

(C) Unstable and stable equilibrium (D) Stable and unstable equilibrium.

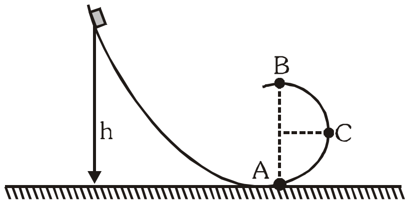
MATRIX MATCH TYPE QUESTIONS

36. In the figure shown are two blocks A and B of same mass connected with pulley and string to each other. Initially both of them are at a height of h = 0.5 m from ground. After they are released they move in either direction and one of them strike the ground. For, the interval from releasing to when one of them strike, some physical quantities are in column I and their modulus values in SI units are in column II.



	Column I		Column II
(A)	Velocity of A immediately before any one of them strike ground.	(P)	1
(B)	Velocity of B immediately before any one of them strike ground.	(Q)	2
(C)	Ratio of work done by gravity on A to workdone by gravity on B.	(R)	3
(D)	Acceleration of block A before any one of them strike ground.	(S)	4
		(T)	5

37. A block of mass m is released from top of a smooth track as shown in the figure. The end part of the track is a circle in vertical plane of radius R. N is normal reaction of the track at any point of the track. Match the entries of column I with entries of column–II.

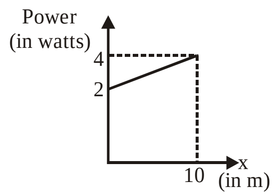


Column I		Column II	
(A)	$h = \frac{5}{2} R$	(P)	Net force on the block at C is mg
(B)	$h = \frac{9}{2} R$	(Q)	$N_A - N_B = 6\ mg$
(C)	$h = R$	(R)	Block leaves contact before B
(D)	$h = 2R$	(S)	Block will keep contact with the track is region between A & B.
		(T)	$N_C > mg$

EXERCISE (O-2)

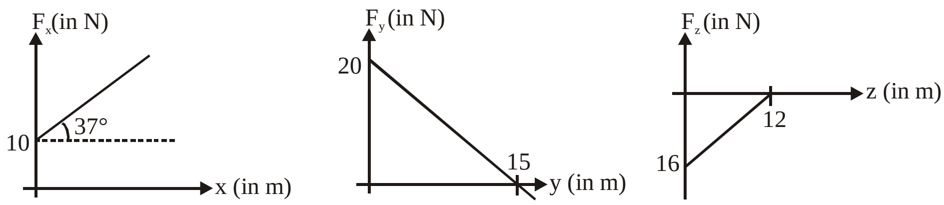
SINGLE CORRECT TYPE QUESTIONS

1. A particle 'A' of mass $\frac{10}{7}$ kg is moving in the positive x–direction. Its initial position is x = 0 & initial velocity is 1 m/s. The velocity at x = 10m is : (use the graph given)



(A) 4 m/s (B) 2 m/s (C) $3\sqrt{2}\ m/s$ (D) 100/3 m/s

2. The components of a force acting on a particle are varying according to the graphs shown. To reach at point B (8, 20, 0) from point A(0, 5, 12) the particle moves on paths parallel to x-axis then y-axis and then z-axis, then work done by this force is :-

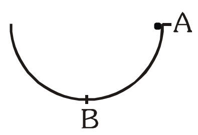


(A) 192 J (B) 58 J (C) 250 J (D) 125 J

3. A light spring of length 20 cm and force constant 2 N/cm is placed vertically on a table. A small block of mass 1 kg falls on it. The length h from the surface of the table at which the block will have the maximum velocity is :

(A) 20 cm (B) 15 cm (C) 10 cm (D) 5cm

4. A small ball can move in a vertical plane along a semi–circle of radius r without friction. At what speed is the ball to launch from point A so that its acceleration is 3g at point B ?



(A) $(3gr)^{1/2}$ (B) $(2gr)^{1/2}$ (C) $(gr)^{1/2}$ (D) $2(gr)^{1/2}$

MULTIPLE CORRECT TYPE QUESTIONS

5. A particle of mass 2 kg is projected with an initial speed u = 10 m/sec at an angle $\theta = 30°$ with the horizontal

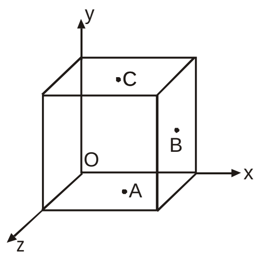
(A) Total work done on the particle during the first half of the total time of flight of the particle is (–25) J.

(B) Total work done on the particle during the total time of flight of the particle is 0 J.

(C) Average power delivered to the particle during the first half of the flight is (–50) watt.

(D) The radius of curvature of the trajectory of the particle at the highest point of the projectile is 7.5m.

6. A particle is shifted from A to B and then from B to C where A, B and C are the midpoints of the corresponding faces of a cube of side 2m. If a force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})$ N is continuously acting on the particle, then select correct alternative :-



(A) work done from A to B is 7 J

(B) work done from B to C is 1 J

(C) work done A to C is 8 J

(D) $\vec{F}$ is a conservative force

7. Which of the following is/are conservative force(s)?

(A) $\vec{F} = 2r^3\hat{r}$

(B) $\vec{F} = -\frac{5}{r}\hat{r}$

(C) $\vec{F} = \frac{3(x\,i + y\,\hat{j})}{(x^2+y^2)^{3/2}}$

(D) $\vec{F} = \frac{3(y\,i + x\,\hat{j})}{(x^2+y^2)^{3/2}}$

8. If one of the forces acting on a particle is conservative then :

(A) Work done by this force is zero when the particle moves exactly once around any closed path.

(B) Work done by this force equals the change in the kinetic energy of the particle.

(C) It obeys Newton's second law.

(D) Work done by this force depends on the end points of the motion, not on the path in between.

9. A particle of mass m = 1 kg lying on x–axis experiences a force given by law F=x(3x–2) Newton, where x is the x–coordinate of the particle in meters. The points on x–axis where the particle is in equilibrium are :

(A) x = 0 (B) x = 1/3 (C) x = 2/3 (D) x = 1

10. A particle is given a velocity at the bottom most position in a smooth spherical shell of radius 2m. It just complete a vertical circular motion. Then

(A) acceleration of particle when the velocity of the particle is in vertically upward direction is $g\sqrt{10}$

(B) acceleration of particle when the velocity of the particle is in vertically downward direction is $g\sqrt{10}$

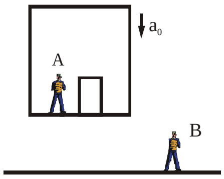
(C) acceleration of the particle at the top most point of path is g

(D) acceleration of the particle at the bottom most point is 5 g

COMPREHENSION TYPE QUESTIONS

Comprehension for Question no. 11 to 14

A block of mass m is kept in an elevator which starts moving downward with an acceleration a_0 as shown in figure. The block is observed by two observers A and B for a time interval t_0.



11. The observer B finds that the work done by gravity is

(A) $\frac{1}{2}mg^2t_0^2$ (B) $-\frac{1}{2}mg^2t_0^2$ (C) $\frac{1}{2}mga_0t_0^2$ (D) $-\frac{1}{2}mga_0t_0^2$

12. The observer B finds that work done by normal reaction N is :-

(A) zero (B) $-Na_0t_0^2$ (C) $\frac{Na_0t^2}{2}$ (D) None of these

13. According to observer B, the net work done on the block is

(A) $-\frac{1}{2}ma_0t_0^2$ (B) $\frac{1}{2}ma_0^2t_0^2$ (C) $\frac{1}{2}mga_0t_0^2$ (D) $-\frac{1}{2}mga_0t_0^2$

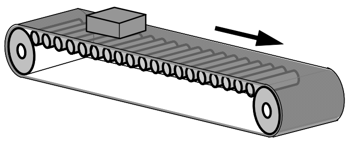
14 According to the observer A

(A) the work done by gravity is zero (B) the work done by normal reaction is zero

(C) the work done by pseudo force is zero (D) all the above

Paragraph for Question 15 & 16

An object of mass M is gently placed on a horizontal conveyor belt, which is moving with uniform velocity v_o as shown in the figure. The coefficient of static friction is μ_s, the coefficient of kinetic friction is μ_k, and the acceleration of gravity is g. Initially the object slips for a while but finally moves without slipping together with the belt.



15. How far the conveyor belt moves while the object is slipping?

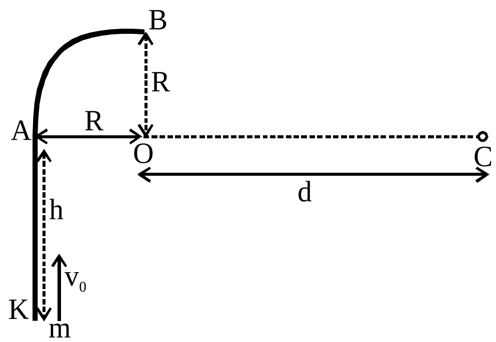
(A) $\frac{v_o^2}{\mu_k g}$ (B) $\frac{v_o^2}{2\mu_k g}$ (C) $\frac{v_o^2}{\mu_s g}$ (D) $\frac{v_o^2}{2\mu_s g}$

16. Work done on the object by friction force relative to the reference frame moving with the conveyer belt is

(A) $\frac{1}{2}Mv_o^2$ (B) $-\frac{1}{2}Mv_o^2$ (C) Mv_o^2 (D) zero

Paragraph for Question Nos.17 to 19

A point like object of mass m starts from point K as shown in the figure. It slides inside along the full length of the smooth track of radius R, and then moves freely and travels to point C. [The track is kept in vertical plane]



17. Determine the vertical initial velocity of the pointlike object.

(A) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{R}}$

(B) $v_0 = \sqrt{2g(h-R)+\frac{d^2g}{2R}}$

(C) $v_0 = \sqrt{2g(h+R)+\frac{d^2g}{2R}}$

(D) $v_0 = \sqrt{2g(h+R)+\frac{dg}{2R}}$

18. What is the minimum possible distance OC = d, necessary for the object to slide along the entire length of the track ?

(A) $d_{min} = R\sqrt{2}$ (B) $d_{min} = \sqrt{3}R$ (C) $d_{min} = \frac{R}{\sqrt{2}}$ (D) $d_{min} = \frac{R}{\sqrt{3}}$

19. Find the normal force exerted by the track at point A.

(A) $F_A = mg\left(\frac{d^2}{2R^2}-2\right)$

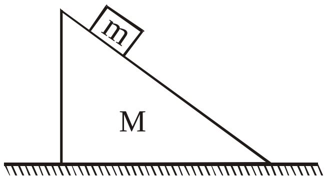
(B) $F_A = mg\left(\frac{d^2}{2R^2}+2\right)$

(C) $F_A = mg\left(\frac{3d^2}{R^2}+2\right)$

(D) $F_A = mg\left(\frac{d}{3R^2}+2\right)$

MATRIX MATCH TYPE QUESTIONS

20. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column–I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge, g is acceleration due to gravity).



Column I	**Column II**
(A) Work done by normal reaction acting on the block is	(p) Positive
(B) Work done by normal reaction (exerted by block) acting on wedge is	(q) Negative
(C) The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r) Zero
(D) Net work done by all forces on block is	(s) Less than mgh in magnitude

EXERCISE (JM)

1. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constant and x is the distance between the atoms. if the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is : **[AIEEE-2010]**

(1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$ (3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

2. At time t = 0s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :- **[AIEEE-2011]**

(1) $\sqrt{t}$ (2) constant (3) t (4) $\frac{1}{\sqrt{t}}$

3. **This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.** **[AIEEE-2012]**

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement-1: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement-2 : $k_1 < k_2$

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

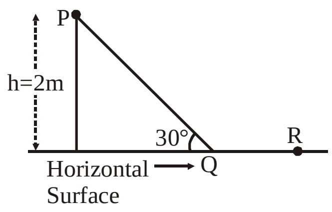
4. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:- **[JEE-Main-2014]**

(1) $\frac{aL^2}{2}+\frac{bL^3}{3}$ (2) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$ (3) $aL^2 + bL^3$ (4) $\frac{1}{2}(aL^2+bL^3)$

5. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$:- **[JEE-Main-2016]**

(1) 12.89×10^{-3} kg (2) 2.45×10^{-3} kg (3) 6.45×10^{-3} kg (4) 9.89×10^{-3} kg

6. A point particle of mass, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and PR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x(=QR) are, respecitvely close to :- **[JEE-Main-2016]**



(1) 0.29 and 6.5 m (2) 0.2 and 6.5 m (3) 0.2 and 3.5 m (4) 0.29 and 3.5 m

7. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be :- **[JEE-Main-2017]**

(1) 10^{-4} kg m^{-1} (2) 10^{-1} kg m^{-1} s^{-1} (3) 10^{-3} kg m^{-1} (4) 10^{-3} kg s^{-1}

8. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be : **[JEE-Main-2017]**

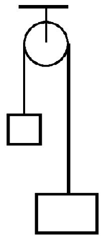
(1) 9 J (2) 18 J (3) 4.5 J (4) 22 J

9. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is :- **[JEE-Main-2018]**

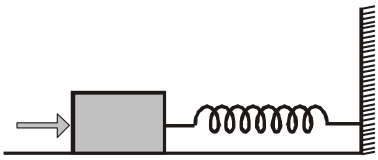
(1) $\frac{k}{2a^2}$ (2) Zero (3) $-\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

EXERCISE (JA)

1. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done (in **joules)** by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[IIT-JEE-2009]**



2. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V= N/10. Then N is **[IIT-JEE-2011]**



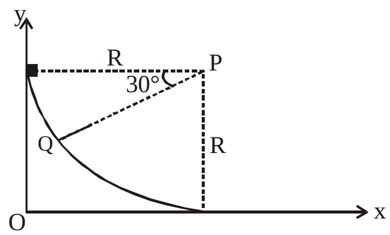
3. The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is :- **[JEE-Advance-2013]**

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

4. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is. **[JEE-Advance-2013]**

Paragraph for Questions 5 and 6

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, g = 10 m s^{-2}) **[JEE-Advance-2013]**



5. The magnitude of the normal reaction that acts on the block at the point Q is

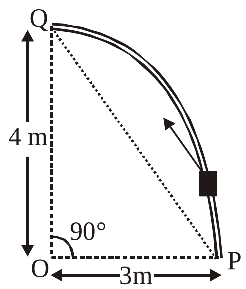
(A) 7.5 N (B) 8.6 N (C) 11.5 N (D) 22.5 N

6. The speed of the block when it reaches the point Q is

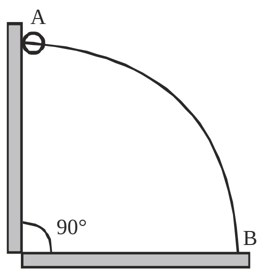
(A) 5 ms^{-1} (B) 10 ms^{-1} (C) $10\sqrt{3}ms^{-1}$ (D) 20 ms^{-1}

7. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})

[JEE-Advance-2014]



8. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[JEE Advanced-2014]**



(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

9. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

[JEE Advanced-2018]

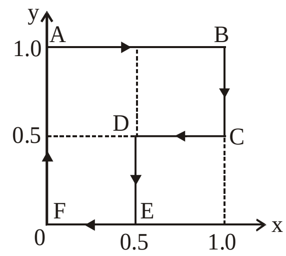
(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

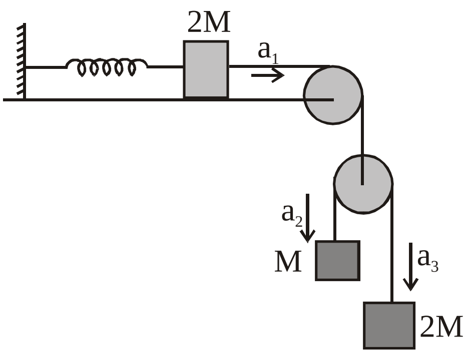
(C) The distance of the particle from the origin increses linerarly with time

(D) The force is conservative

10. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right) N$, where x and y are in meter and $\alpha = -1$ N/m^{-1}. The work done on the particle by this force $\vec{F}$ will be ____ Joule. **[JEE Advanced-2019]**



11. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction] **[JEE Advanced-2019]**



(1) $x_0 = \dfrac{4Mg}{k}$

(2) When spring achieves an extension of $\dfrac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\dfrac{M}{5k}}$

(3) $a_2 - a_1 = a_1 - a_3$

(4) At an extension of $\dfrac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\dfrac{3g}{10}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) +ve (b) –ve (c) –ve (d) +ve (e) –ve   2. Ans. 12 J   3. Ans. 8
4. Ans. 67.7 J   5. Ans. (a) 7.5 J; (b) 15 J; (c) 7.5 J; (d) 30 J   6. Ans. 3
7. Ans. (a) 875 J; (b) –250 J; (c) 625 J ; (d) 625 J;   8. Ans. 0
9. Ans. 400 J
10. Ans. (i) $2 + 24t^2 + 72t^4$ J, (ii) 48 t N, (iii) $48t + 288t^3$ W, (iv) 1248 J   11. Ans. 4
12. Ans. 4   13. Ans. 10   14. Ans. $m_1 = m/2$
15. Ans. (a) –30 kW, 19.5 kW   16. Ans. 46 J   18. Ans. 4
19. Ans. (i) x =2, (ii) 16 J   20. Ans. –6 m/s²   21. Ans. 4   22. Ans. 018
23. Ans. Graph-1 : For all x, Graph-2 : x < a & x > b, Graph-3 : $-\frac{b}{2} < x < -\frac{a}{2}$ & $\frac{a}{2} < x < \frac{b}{2}$
24. Ans. (a) s = 0.24 m, a = 6 m/s², (b) $\mu = 1/8$ 25. Ans. $\sqrt{2gL}$, $\sqrt{gL}$ 26. Ans. 4

EXERCISE (O-1)

1. Ans. (C)	2. Ans. (B)	3. Ans. (C)	4. Ans. (C)	5. Ans. (A)
6. Ans. (A)	7. Ans. (B)	8. Ans. (D)	9. Ans. (A)	10. Ans. (A)
11. Ans. (D)	12. Ans. (A)	13. Ans. (A)	14. Ans. (B)	15. Ans. (A)
16. Ans. (B)	17. Ans. (A)	18. Ans. (B)	19. Ans. (B)	20. Ans. (C)
21. Ans. (D)	22. Ans. (A)	23. Ans. (B)	24. Ans. (C)	25. Ans. (C)
26. Ans. (D)	27. Ans. (D)	28. Ans. (A)	29. Ans. (C)	30. Ans. (A,D)
31. Ans. (A,C)	32. Ans. (B,C,D)	33. Ans. (B)	34. Ans. (B)	35. Ans. (D)

36. Ans. (A) Q; (B) P; (C) Q; (D) S   37. Ans. (A) Q,S,T; (B) Q,S,T; (C) P; (D) R,T

EXERCISE (O-2)

1. Ans. (A)	2. Ans. (C)	3. Ans. (B)	4. Ans. (C)	5.Ans. (A,B,C,D)
6. Ans. (ABCD)	7. Ans. (A,B,C)	8. Ans. (A,C,D)	9. Ans. (A,C)	10.Ans. (A,B,C,D)
11. Ans. (C)	12. Ans. (D)	13. Ans. (B)	14. Ans. (D)	15. Ans. (A)
16. Ans. (B)	17. Ans. (C)	18. Ans. (A)	19. Ans. (B)	

20. Ans. (A) Q,S; (B) P,S; (C) R, S; (D) P,S

EXERCISE (JM)

1. Ans. (4)	2. Ans. (4)	3. Ans. (2)	4. Ans. (1)	5. Ans. (1)
6. Ans. (4)	7. Ans. (1)	8. Ans. (3)	9. Ans. (2)	

EXERCISE (JA)

1. Ans. 8	2. Ans. 4	3. Ans. (D)	4. Ans. 5	5. Ans. (A)
6. Ans. (B)	7. Ans. 5	8. Ans. (D)	9. Ans. (A,B,D)	10. Ans. 0.75
11. Ans. (3)				

CHAPTER 2

CIRCULAR MOTION

Chapter Contents 02

IMPORTANT NOTES

CHAPTER 2

CIRCULAR MOTION

KEY CONCEPT

Cross product (Vector Product) of two vectors :

Vector product of two vectors $\vec{A}$ and $\vec{B}$, also called the cross product, is denoted by $\vec{A}\times\vec{B}$. As the name suggests, the vector product is itself a vector. We will use this product to describe torque and angular momentum and extensively to describe magnetic fields forces.

Vector product of two vectors $\vec{A}$ and $\vec{B}$ which are at an angle ϕ is defined as

$$\vec{C} = \vec{A} \times \vec{B},$$

then $\qquad C = AB\sin\phi\ \hat{n}$

where $\hat{n}$ is an unit vector perpendicular to plane containing vector $\vec{A}$ and $\vec{B}$. Direction of $\hat{n}$ is given by right hand rule as described below.

Note: the vector product of two parallel or antiparallel vectors is always zero. In particular, the vector product of any vector with itself is zero.

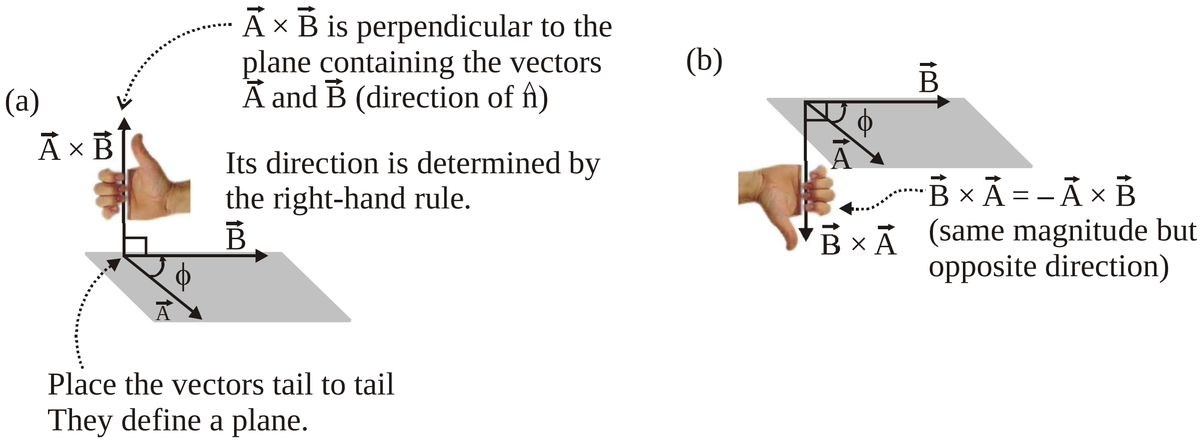
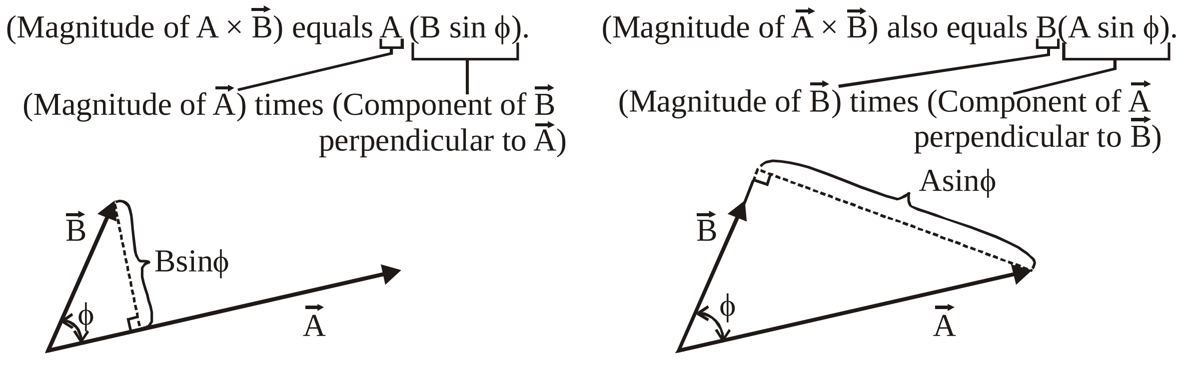


Figure : (a) The vector product $\vec{A}\times\vec{B}$. determined by the right-hand rule. (b) $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$

Note: The vector product is not commutative! In fact, for any two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$



(a) (b)

Figure: Calculating the magnitude AB sin ϕ of the vector product of two vector, $\vec{A}\times\vec{B}$.

Calculating the Vector Product Using Components

$$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$$

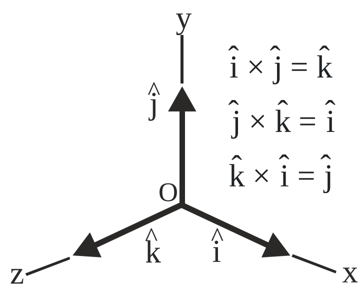
Using the right-hand rule, we find

$$\hat{i}\times\hat{j}=-\hat{j}\times\hat{i}=\hat{k};$$

$$\hat{j}\times\hat{k}=-\hat{k}\times\hat{j}=\hat{i};$$

$$\hat{k}\times\hat{i}=-\hat{i}\times\hat{k}=\hat{j}$$

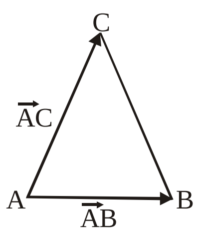
A right-handed coordinate system



Ex. Find the area of the triangle formed by the points having position vectors

$\vec{A}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{B}=4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$

Sol.



Let ABC is the triangle formed by the points A, B and C. Then

$$\overrightarrow{AB}=\vec{B}-\vec{A}=\left(4\hat{i}-3\hat{j}+\hat{k}\right)-\left(\hat{i}-\hat{j}-3\hat{k}\right)=3\hat{i}-2\hat{j}+4\hat{k}$$

$$\overrightarrow{AC}=\vec{C}-\vec{A}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)-\left(\hat{i}-\hat{j}-3\hat{k}\right)=2\hat{i}+5\hat{k}$$

Now, $\overrightarrow{AB}\times\overrightarrow{AC}=\left(3\hat{i}-2\hat{j}+4\hat{k}\right)\times\left(2\hat{i}+5\hat{k}\right)$

$$=\hat{i}\left(-10-0\right)+\hat{j}\left(8-15\right)+\hat{k}\left(0+4\right)=-10\hat{i}-7\hat{j}+4\hat{k}$$

Ex. Find unit vector perpendicular to the plane determined by the points P(1, –1, 2), Q(2, 0, –1) and R(0, 2, 1).

Sol. $\vec{P}Q$ = (Position vector of Q) – (Position vector of P)

$$= \left(2\hat{i}-\hat{k}\right)-\left(\hat{i}-\hat{j}+2\hat{k}\right)=\hat{i}+\hat{j}-3\hat{k}$$

Similarly, $\vec{P}R = \left(2\hat{j}+\hat{k}\right) - \left(\hat{i}-\hat{j}+2\hat{k}\right) = -\hat{i}+3\hat{j}-\hat{k}$

$\therefore \left|\vec{P}Q \times \vec{P}R\right| = \sqrt{8^2+4^2+4^2} = 4\sqrt{6}$

$\therefore$ Required unit vectors

$$= \pm \frac{1}{4\sqrt{6}}\left(8\hat{i}+4\hat{j}+4\hat{k}\right) = \pm \frac{1}{\sqrt{6}}\left(2\hat{i}+\hat{j}+9\hat{k}\right)$$

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point.

That fixed point is called centre and the distance is called radius of circular path.

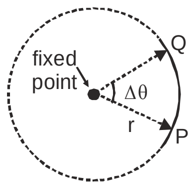
Radius Vector

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction.

KINEMATICS OF CIRCULAR MOTION

Angular Displacement

- Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.



$\Delta\theta$ = angular displacement

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \quad \text{or} \quad \Delta\theta = \frac{\text{Arc PQ}}{r}$$

- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \text{ But } \vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$$

- Its direction is perpendicular to plane of rotation and given by right hand screw rule.
- It is dimensionless and has S.I. unit "Radian" while other units are degree or revolution.

$$2\pi \text{ radian} = 360° = 1 \text{ revolution}$$

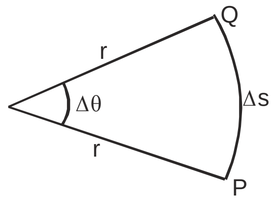
Frequency (n) : Number of revolutions describes by particle per second is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

Time Period (T) : It is time taken by particle to complete one revolution. $T = \frac{1}{n}$

Angular Velocity (ω) : It is defined as the rate of change of angular displacement of moving particle

$$\omega = \frac{\text{Angle traced}}{\text{Time taken}} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between linear and Angular velocity



$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}} \text{ or } \frac{\Delta s}{r} \qquad \Delta\theta = \frac{\Delta s}{r} \text{ or } \Delta s = r\Delta\theta$$

$$\therefore \quad \frac{\Delta s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \boxed{v = \omega r}$$

$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$ (direction of $\vec{v}$ is according to right hand thumb rule)

Average Angular Velocity (ω_{av})

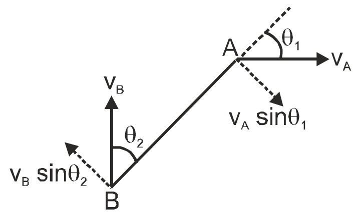
$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity

The angular velocity at some particular instant $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ or $\vec{\omega} = \frac{d\vec{\theta}}{dt}$

Relative Angular Velocity



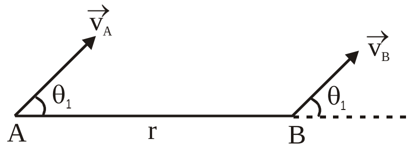
- Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B. That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$ $\qquad \therefore \ \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

- It is an axial vector quantity.
- Its direction is same as that of angular displacement i.e. perpendicular to the plane of rotation and along the axis according to right hand screw rule.
- Its unit is radian/second.

If particles A and B are moving with a velocity $\vec{v}_A$ and $\vec{v}_B$ and separated by a distance r at a given instant then

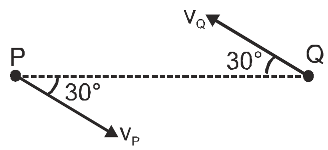


(i) $\frac{dr}{dt} = v_B\cos\theta_2 - v_A\cos\theta_1$ (ii) $\frac{d\theta_{AB}}{dt} = w_{AB} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$

Ex. A particle revolving in a circular path completes first one third of circumference in 2 s, while next one third in 1 s. Calculate the average angular velocity of particle.

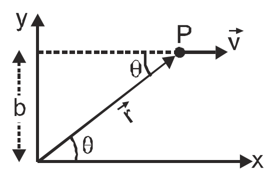
Sol. $\theta_1 = \frac{2\pi}{3}$ and $\theta_2 = \frac{2\pi}{3}$ total time T = 2 + 1 = 3 s $\therefore <\omega_{av}> = \frac{\theta_1+\theta_2}{T} = \frac{\frac{2\pi}{3}+\frac{2\pi}{3}}{3} = \frac{\frac{4\pi}{3}}{3} = \frac{4\pi}{9}$ rad/s

Ex. Two moving particles P and Q are 10 m apart at any instant. Velocity of P is 8 m/s at 30°, from line joining the P and Q and velocity of Q is 6m/s at 30°.Calculate the angular velocity of P w.r.t. Q

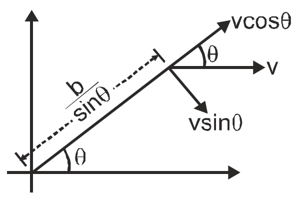


Sol. $\omega_{PQ} = \frac{8\sin 30^\circ - (-6\sin 30^\circ)}{10} = 0.7$ rad/s.

Ex. A particle moving parallel to x–axis as shown in fig. such that at all instant the y–axis component of its position vector is constant and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes angle θ from the x-axis.



Sol. $\therefore\ \omega_{PO} = \frac{v\sin\theta}{\frac{b}{\sin\theta}} = \frac{v}{b}\sin^2\theta$



Ex. The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, Find the time when its angular acceleration becomes zero.

Sol. $\alpha = \frac{d\omega}{dt} = 1.5 - 6\,t = 0 \Rightarrow t = 0.25$ s.

Ex. A disc starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.

Sol. $\frac{d\omega}{dt} = 3t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^{t}(3t - t^2)dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow$ at t = 2 s, $\omega = \frac{10}{3}$ rad/s

Angular Acceleration (α)

- Rate of change of angular velocity is called angular acceleration.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad \text{or} \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

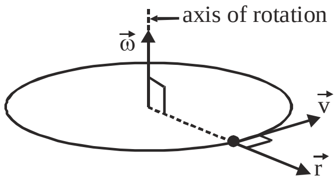
- Its an axial vector quantity. It direction is along the axis according to right hand screw rule.
- Unit $\rightarrow$ rad/s^2

Relation between Angular and Linear Acceleration

$\vec{v} = \vec{\omega} \times \vec{r}$ ($\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector.)

These three vectors are mutually perpendicular. but $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

or $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$ $\left(\frac{d\vec{\omega}}{dt} = \vec{\alpha} \text{ and } \frac{d\vec{r}}{dt} = \vec{v}\right) \Rightarrow \vec{a} = \vec{a}_T + \vec{a}_C$



($\vec{a}_T = \vec{\alpha} \times \vec{r}$ is tangential acceleration &

$\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

$\vec{a} = \vec{a}_T + \vec{a}_C$ ($\vec{a}_T$ and $\vec{a}_C$ are two component of net linear acceleration)

Tangential Acceleration

$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a}_T = \vec{\alpha} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that $\vec{v}$ and $\vec{a}_T$ are also parallel and along the tangential direction.

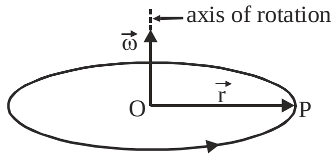
Magnitude of tangential acceleration in case of circulation motion.

$$a_T = \alpha \, r \sin 90° = \alpha r \quad (\vec{\alpha} \text{ is axial, } \vec{r} \text{ is radial so that } \vec{\alpha} \perp \vec{r})$$

As $\vec{a}_T$ is along the direction of motion (in the direction of $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of the particle. Its magnitude is rate of change of speed of the particle. If particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration

$\vec{a}_C = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_C = \vec{\omega} \times (\vec{\omega} \times \vec{r})$ ($\because$ $\vec{v} = \vec{\omega} \times \vec{r}$)



Let $\vec{r}$ is in $\hat{i}$ direction and $\vec{\omega}$ is in $\hat{j}$ direction then direction

of $\vec{a}_C$ is along $\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) = -\hat{i}$

opposite direction of $\vec{r}$ i.e., from P to O and it is

centripetal direction.Magnitude of centripetal acceleration, $a_C = \omega v = \frac{v^2}{r} = \omega^2 r$, $\vec{a}_C = \frac{v^2}{r}(-\hat{r})$

- Centripetal acceleration is always perpendicular to the velocity or displacement at each point.

Net Linear Acceleration :

$$\vec{a} = \vec{a}_T + \vec{a}_C \text{ and } \vec{a}_T \perp \vec{a}_C \text{ so that } |\vec{a}| = \sqrt{a_T^2 + a_C^2}$$

About uniform circular motion :–

- Position vector $(\vec{r})$ is always perpendicular to the velocity vector $(\vec{v})$ i.e. $\vec{r}.\vec{v} = 0$
- velocity vector is always perpendicular to the acceleration. $\vec{v}.\vec{a} = 0$
- $\because |\vec{v}|$ = constant so tangential acc. a_t=0 $\therefore$ $f_t = 0$
- Important difference between the projectile motion and uniform circular motion :

  In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

DYNAMICS OF CIRCULAR MOTION

If a particle moves with constant speed in a circle, motion is called uniform circular. In uniform circular motion a resultant non-zero force acts on the particle. The acceleration is due to the change in direction of the velocity vector. In uniform circular motion tangential acceleration (a_t) is zero. The acceleration of the particle is towards the centre and its magnitude is $\frac{v^2}{r}$. Here, v is the speed of the particle and r the radius of the circle. The direction of the resultant force F is therefore towards centre and its magnitude is $F = \frac{mv^2}{r} = mr\omega^2$ (as $v = r\omega$)

Here, ω is the angular speed of the particle. This force F is called the centripetal force. Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed. This force is provided by some external source such as friction, magnetic force, coulomb force, gravitation, tension, etc.

Circular Turning of Roads

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

• By friction only. • By banking of roads only. • By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

- **By Friction only**

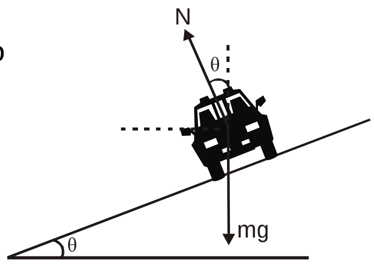
Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

Thus, $f = \dfrac{mv^2}{r}$ $\quad \because f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without sliding $\dfrac{mv^2}{r} \le f_{max} \Rightarrow \dfrac{mv^2}{r} \le \mu mg \Rightarrow v \le \sqrt{\mu rg}$

- **By Banking of Roads only**

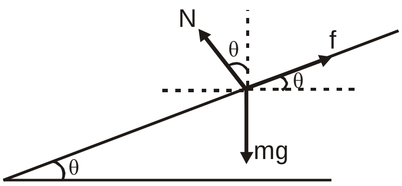
Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.



$N \sin\theta = \dfrac{mv^2}{r}$ and $N\cos\theta = mg \Rightarrow \tan\theta = \dfrac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$

- **Friction and Banking of Road both**

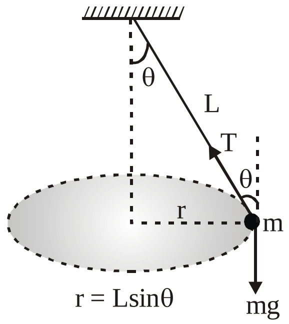
If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed



(perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, the magnitude of normal reaction N and direction plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\dfrac{mv^2}{r}$ towards the centre.

Conical Pendulum

If a small particle of mass m tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,



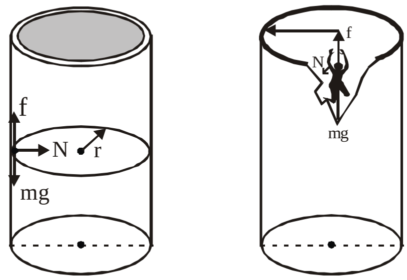
$$T\sin\theta = \frac{mv^2}{r} \quad \text{and} \quad T\cos\theta = mg \Rightarrow v = \sqrt{rg\tan\theta}$$

$$\therefore \text{ Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g\tan\theta}{r}}$$

So, the time period of pendulum is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

'Death Well' or Rotor

In case of 'death well' a person drives a motorcycle on a vertical surface of a large wooden well while in case of a rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotates.



Death well Rotor

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion,

i.e., $f = mg$ and $N = \frac{mv^2}{r} = mr\omega^2$

Ex. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s^2]

Sol. Here centripetal force is provided by friction so

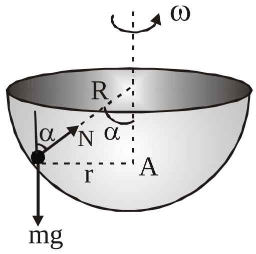
$$\frac{mv^2}{r} \le \mu mg \Rightarrow v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

Ex. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? (g = 10 m/s^2)

Sol. In case of banking $\tan\theta = \frac{v^2}{rg}$ Here v= 60 km/hr = $60 \times \frac{5}{18}$ ms^{-1} = $\frac{50}{3}$ ms^{-1} r = 0.1 km = 100 m

$$\text{So } \tan\theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{18}\right)$$

Ex. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.



Sol. $N\cos\alpha = mg$ and $N\sin\alpha = mr\omega^2$ but $r = R\sin\alpha$

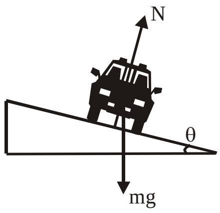
$\Rightarrow N\sin\alpha = mR\sin\alpha\omega^2 \Rightarrow N = mR\omega^2$

$$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$$

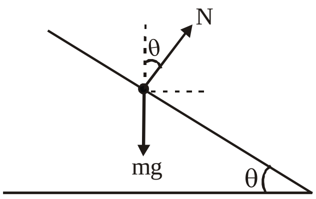
Ex. A car is moving along a banked road laid out as a circle of radius r.

(a) What should be the banking angle θ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn?

(b) The coefficients of friction between tyres and road are $\mu_s = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?

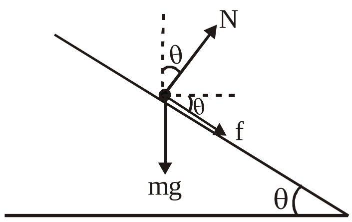


Sol. (a) $N\sin\theta = \frac{mv^2}{r}$ and $N\cos\theta = mg \Rightarrow \tan\theta = \frac{v^2}{rg}$



Note : In above case friction does not play any role in negotiating the turn.

(b) If the driver moves faster than the speed mentioned above, a friction force must act parallel to the road, inward towards centre of the turn.



$$\Rightarrow F\cos\theta + N\sin\theta = \frac{mv^2}{r} \text{ and } N\cos\theta = mg + f\sin\theta$$

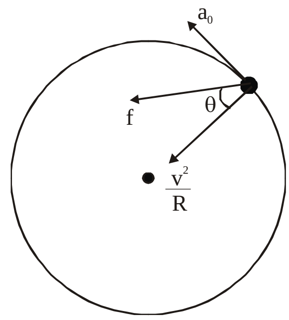
For maximum speed of $f = \mu N$

$$\Rightarrow N(\mu\cos\theta + \sin\theta) = \frac{mv^2}{r} \text{ and } N(\cos\theta - \mu\sin\theta) = mg$$

$$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \Rightarrow v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)rg}$$

Ex. A car starts from rest with a consant tangential acceleration a_0 in a circular path of radius r. At time t, the car skids, find the value of coefficient of friction.

Sol. The tangential and centripetal acceleration is provided only by the frictional force.



Thus, $f\sin\theta = ma_0$, $f\cos\theta = \frac{mv^2}{r} = \frac{m(a_0t)^2}{r}$

$$\Rightarrow f = m\sqrt{a_0^2 + \frac{(a_0t)^4}{r^2}} = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} = f_{max}$$

$$\mu mg = ma_0\sqrt{1+\frac{a_0^2t^4}{r^2}} \Rightarrow \mu = \frac{a_0}{g}\sqrt{1+\frac{a_0^2t^4}{r^2}}$$

Centrifugal force :-

Centrifugal force is a pseudo force which an observer needs to consider while making observations in a rotating frame. This force is non physical and arise from kinematics and not due to physical interactions. Centrifugal force is directed away from axis of rotation of rotating frame and its value is $m\omega^2r$, where ω is angular speed of rotating frame where observer has kept himself fixed and r is distance of object of mass m from axis of rotation.

EXERCISE (S-1)

Kinematics of circular motion

1. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?

2. A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At t = 0, it was subjected to a constant angular acceleration α and its angular speed increased to $(480/\pi)$ rotation per minute (rpm) in 2 sec. Particle then continues to move with attained speed. Calculate

(i) angular acceleration of the particle,

(ii) tangential velocity of the particle as a function of time.

(iii) acceleration of the particle at t = 0.5 second and at t = 3 second

(iv) angular displacement at t = 3 second.

3. A particle moves in a circle of radius 1.0 cm at a speed given by v = 2.0 t where v is in cm/s and t in seconds.

(a) Find the radial acceleration of the particle at t = 1s.

(b) Find the tangential acceleration at t = 1s.

(c) Find the magnitude of the acceleration at t = 1 s.

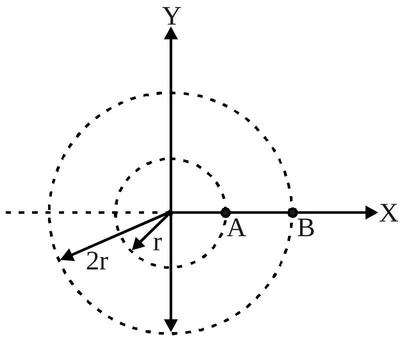
4. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37^{o} from the direction to the centre of the circle as seen from the particle

(i) At what rate is the speed of the particle increasing?

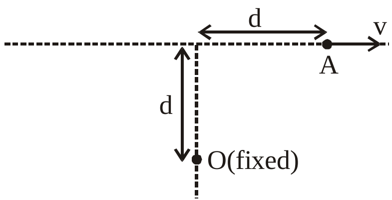
(ii) What is the magnitude of the acceleration?

5. A disk rotates about its central axis starting from rest and accelerates with constant angular acceleration. At one instant it is rotating at 12 rad/s and after 80 radian of more angular displacement, its angular speed becomes 28 rad/s. How much time (seconds) does the disk takes to complete the mentioned angular displacement of 80 radians.

6. Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω respectively. Find the relative velocity (in m/s) of B w.r.t. A after time $t = \pi/\omega$. They both start at the position as shown in figure. (Take ω = 3rad/sec, r = 2m)



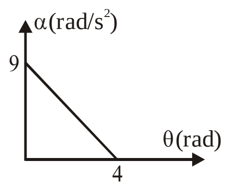
7. Find angular velocity of A with respect to O at the instant shown in the figure.



8. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.

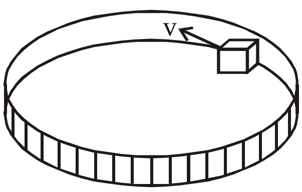
9. A particle moves in the x-y plane with the velocity $\vec{v} = a\hat{i} + bt\hat{j}$. At the instant $t = a\sqrt{3}/b$ the magnitude of tangential, normal and total acceleration are ______, ________, & __________.

10. A particle starts moving in a non-uniform circular motion, has angular acceleration as shown in figure. The angular velocity at the end of 4 radian is given by ω rad/s then find the value of ω.



Dynamics of circular motion

11. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



Find :

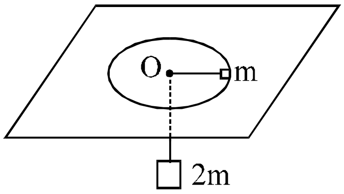
(i) normal reaction of the floor on the block.

(ii) normal reaction of the vertical wall on the block.

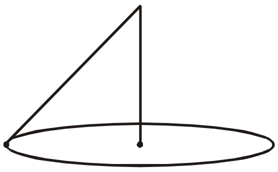
12. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co~efficient of static- friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

13. A stone of mass 0.25 kg tied to the end of a string is whirled round in a circle of radius 1.5 m with a speed of 40 rev/min in a horizontal plane. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N ?

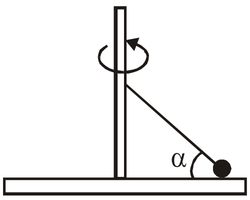
14. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass 2m, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.



15. Consider a conical pendulum having bob of mass m is suspended from a ceiling through a string of length L. The bob moves in a horizontal circle of radius r. Find (a) the angular speed of the bob and (b) the tension in the string.



16. A circular platform rotates around a vertical axis with angular velocity $\omega = 10$ rad/s. On the platform is a ball of mass 1 kg, attached to the long axis of the platform by a thin rod of length 10 cm ($\alpha = 30°$). Find normal force exerted by the ball on the platform (in newton). Friction is absent.



17. An aircraft executes a horizontal loop at a speed of 720 km/h with its wings banked at 15°. What is the radius of the loop ?

18. A block of mass m = 20 kg is kept at a distance R = 1m from central axis of rotation of a round turn table (A table whose surface can rotate about central axis). Table starts from rest and rotates with constant angular acceleration, $\alpha = 3$ rad/sec^2. The friction coefficient between block and table is $\mu = 0.5$. At time $t = \frac{x}{3}$ sec from starting of motion (i.e. t = 0 sec) the block is just about to slip. Find the value of x.

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Cross product of vectors

1. The area of parallelogram represented by the vectors $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + 4\hat{j}$ is :-

(A) 14 unit (B) 7.5 unit (C) 10 unit (D) 5 unit

2. What is the angle between $(\vec{P} + \vec{Q})$ and $(\vec{P} \times \vec{Q})$?

(A) 0 (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) π

3. The value of n so that vectors $2\hat{i} + 3\hat{j} - 2\hat{k}$, $5\hat{i} + n\hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$ may be coplanar, will be:-

(A) 18 (B) 28 (C) 9 (D) 36

4. If $\vec{a}$ and $\vec{b}$ are two vectors then the value of $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$ is :-

(A) $2(\vec{b} \times \vec{a})$ (B) $-2(\vec{b} \times \vec{a})$ (C) $\vec{b} \times \vec{a}$ (D) $\vec{a} \times \vec{b}$

5. If $|\vec{a} \cdot \vec{b}| = \sqrt{3}|\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :-

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Kinematics of circular motion

6. A body is executing circular motion in the vertical plane containing directions. If the direction of velocity $(\vec{v})$ at the top most point is towards west, what is the direction of angular velocity $(\vec{\omega})$?

(A) east (B) west (C) north (D) south

7. If the magnitude of velocity in the previous question is decreasing with time, what is the direction of angular acceleration $(\vec{\alpha})$?

(A) east (B) west (C) north (D) south

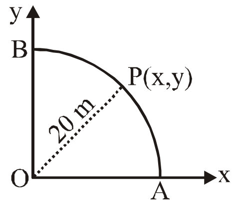
8. A mass is revolving in a circle which lies in a plane of paper. The direction of angular acceleration can be :-

(A) perpendicular to radius and velocity (B) towards the radius
(C) tangential (D) at right angle to angular velocity

9. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be

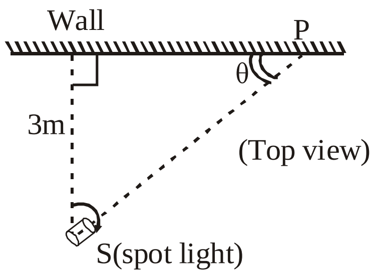
(A) 2π & 0 mm/s (B) $2\sqrt{2}\,\pi$ & 4.44 mm/s
(C) $2\sqrt{2}\,\pi$ & 2π mm/s (D) 2π & $2\sqrt{2}\,\pi$ mm/s

10. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^2 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 5\sqrt{\frac{3}{10}}$ seconds is nearly :



(A) 2 m/s^2 (B) 1.5 m/s^2 (C) 2.5 m/s^2 (D) 3 m/s^2

11. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3m. What is the velocity of the spot P when $\theta = 45°$?



(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

12. Which of the following statements is false for a particle moving in a circle with a constant angular speed ? **[AIEEE - 2004]**

(A) The velocity vector is tangent to the circle

(B) The acceleration vector is tangent to the circle

(C) the acceleration vector points to the centre of the circle

(D) The velocity and acceleration vectors are perpendicular to each other

Dynamics of circular motion

13. A particle is moving in a circle :

(A) the resultant force on the particle must be towards the centre

(B) the cross product of the tangential acceleration and the angular velocity will be zero

(C) the direction of the angular acceleration and the angular velocity must be the same

(D) the resultant force may be towards the centre

14. A particle of mass m is tied to a light string and rotated with a speed v along a circular path of radius r. If T = tension in the string and mg = gravitational force on the particle then the actual forces acting on the particle are :

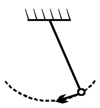
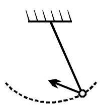
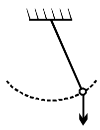
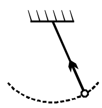
(A) mg and T only

(B) mg, T and an additional force of $\frac{mv^2}{r}$ directed inwards.

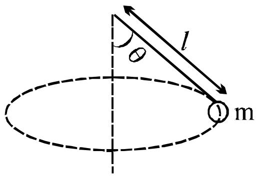
(C) mg, T and an additional force of $\frac{mv^2}{r}$ directed outwards.

(D) only a force $\frac{mv^2}{r}$ directed outwards.

15. Which vector in the figures best represents the acceleration of a pendulum mass at the intermediate point in its swing?

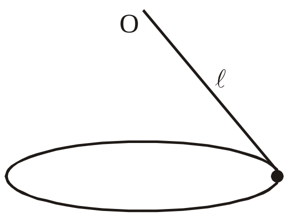
(A) 
(B) 
(C) 
(D) 

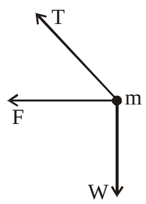
16. A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct ?

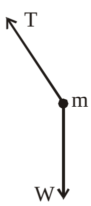


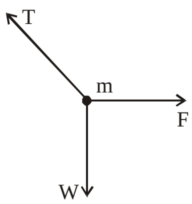
(A) $T = m\omega^2 l$ (B) $T \sin\theta = m\omega^2 l$ (C) $T = mg \cos\theta$ (D) $T = m\omega^2 l \sin\theta$

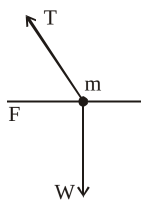
17. A point mass m is suspended from a light thread of length ℓ, fixed at O, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are :



(A) 

(B) 

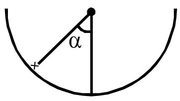
(C) 

(D) 

18. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α. If the coefficient of friction between the rod and bead is μ, and gravity is neglected, then the time after which the bead starts slipping is :– **[IIT-JEE 2000]**

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

19. A insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given **[IIT-JEE 2001]**



(A) $\cot \alpha = 3$ (B) $\tan \alpha = 3$ (C) $\sec \alpha = 3$ (D) $\text{cosec}\ \alpha = 3$

20. The maximum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is - **[AIEEE - 2002]**

(A) 60 (B) 30 (C) 15 (D) 25

21. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that- **[AIEEE - 2004]**

(A) Its velocity is constant

(B) Its acceleration is constant

(C) Its kinetic energy is constant

(D) It moves in a straight line

MULTIPLE CORRECT TYPE QUESTIONS

22. A car runs around a curve of radius 10 m at a constant speed of 10 ms^{-1}. Consider the time interval for which car covers a curve of 120° arc :-

(A) Resultant change in velocity of car is $10\sqrt{3}$ ms^{-1}

(B) Instantaneous acceleration of car is 10 ms^{-2}

(C) Average acceleration of car is $\frac{5}{24}$ ms^{-2}

(D) Instantaneous and average acceleration are same for the given period of motion.

23. A car is moving with constant speed on a rough banked road.

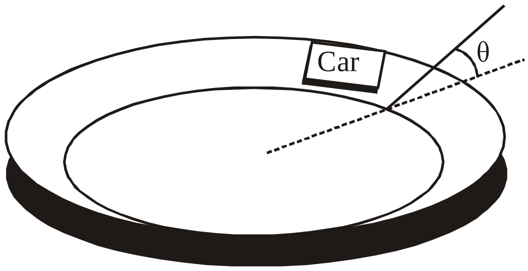
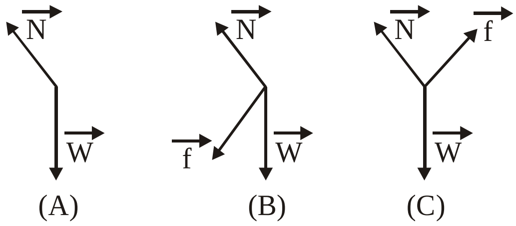


Figure shows the free body diagram of car in three situation A, B & C respectively:-



(A) Car in A has more speed than car in C
(B) Car in A has less speed than car in B
(C) FBD for car in A is not possible
(D) If $\mu > \tan\theta$ the FBD for car C is not possible

24. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60^o with the vertical. Then

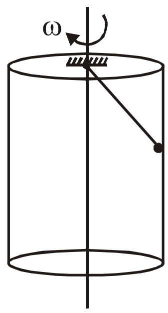
(A) its period of revolution is $\frac{4\pi}{7}$ sec.

(B) the tension in the string is double the weight of the particle

(C) the velocity of the particle = $2.8\sqrt{3}$ m/s

(D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

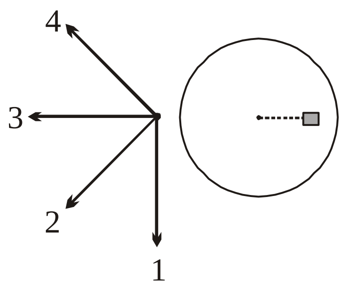
25. In the shown figure inside a fixed hollow cylinder with vertical axis a pendulum is rotating in a conical path with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving :-



(A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s
(B) Tension in the string is 5N for all values of angular velocity
(C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N
(D) For angular velocity of 10 rad/s, tension in the string is 20N

MATRIX MATCH TYPE QUESTION

26. A block is placed on a horizontal table which can rotate about its axis.The block is placed at a certain distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure.



Column-I		**Column-II**	
(A)	When rotation is clockwise with constant ω	(P)	1
(B)	When rotation is clock wise with decreasing ω	(Q)	2
(C)	When rotation is clockwise with increasing ω	(R)	3
(D)	Just after clockwise rotation begins from rest	(S)	4

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is :

(A) zero (B) $\frac{1}{2}m\omega^2 r^2$ (C) $m\omega^2 r^2$ (D) $mr\omega^2$

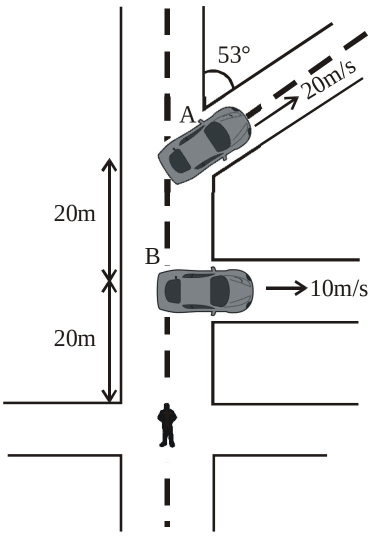
2. A uniform rod of mass m and length ℓ rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is :

(A) $\frac{1}{2}m\omega^2 x$ (B) $\frac{1}{2}m\omega^2 \frac{x^2}{\ell}$ (C) $\frac{1}{2}m\omega^2 \ell\left(1-\frac{x}{\ell}\right)$ (D) $\frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2 - x^2]$

3. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as :-

(A) 2 a sinωt (B) $2a \sin\frac{\omega t}{2}$ (C) 2a cos ωt (D) $2a \cos\frac{\omega t}{2}$

4. A traffic policeman standing at the intersection sees 2 cars A & B turning at angles 53° & 90° respectively (as shown in the figure). Their velocities are V_A = 20 m/s, V_B = 10 m/s. Then, which car appears to be moving faster to the traffic policeman :-

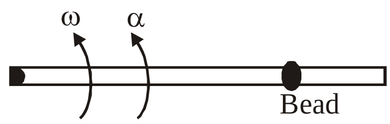


(A) A (B) B (C) Both equally fast (D) Insufficient info.

5. A bead is constrained to move on rod in gravity free space as shown in figure. The rod is rotating with angular velocity ω and angular acceleration α about its end. If μ is coefficient of friction. Mark the correct option. (Rod rotates in the plane of paper.)

(A) If $\mu = \frac{\omega^2}{\alpha}$ friction on bead is static in nature

(B) If $\mu > \frac{\omega^2}{\alpha}$ friction on bead is kinetic in nature



(C) If $\mu < \frac{\omega^2}{\alpha}$ friction is static

(D) If bead does not slide relative to rod. Friction will not exist between bead and rod.

6. A particle is moving in a circular path. The acceleration and momentum of the particle at a certain moment are $\vec{a} = (4\hat{i} + 3\hat{j})$ m/s² and $\vec{p} = (8\hat{i} - 6\hat{j})$ kg–m/s. The motion of the particle is

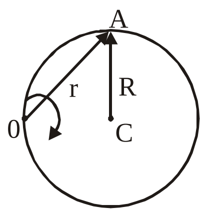
(A) uniform circular motion

(B) accelerated circular motion

(C) de–accelerated circular motion

(D) we can not say anything with $\vec{a}$ and $\vec{p}$ only

7. A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (figure) rotates with the constant angular velocity ω=0.40 rad/s. Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



(A) v= 0.4 m/s, a = 0.4 m/s²
(B) v = 0.32 m/s, a = 0.32 m/s²
(C) v = 0.32 m/s, a = 0.4 m/s²
(D) v = 0.4 m/s, a = 0.32 m/s²

MULTIPLE CORRECT TYPE QUESTIONS

8. For a curved track of radius R, banked at angle θ (Take $v_0 = \sqrt{Rg\tan\theta}$)

(A) a vehicle moving with a speed v_0 is able to negotiate the curve without calling friction into play at all

(B) a vehicle moving with any speed $v > v_0$ is always able to negotiate the curve, with friction called into play

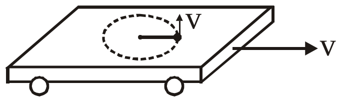
(C) a vehicle moving with any speed $v < v_0$ must have the force of friction into play

(D) the minimum value of the angle of banking for a vehicle parked on the banked road can stay there without slipping, is given by $\theta = \tan^{-1}\mu_0$ (μ_0 = coefficient of static friction)

9. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).

(A) Maximum speed of bob is 2 v in ground frame.

(B) Tension in string connecting bob is $\frac{4mv^2}{\ell}$ at an instant.



(C) Tension in string is $\frac{mv^2}{\ell}$ at all the moments.

(D) Minimum speed of bob is zero in ground frame.

10. Let $\vec{v}(t)$ be the velocity of a particle at time t. Then :

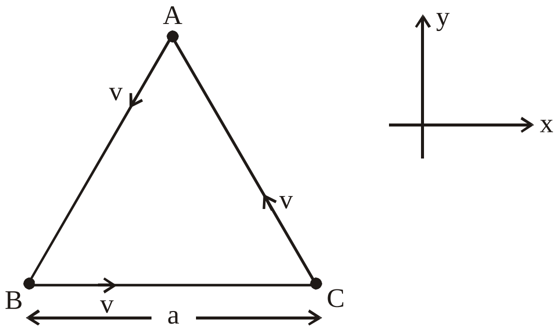
(A) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ are always equal

(B) $|d\vec{v}(t)/dt|$ and $d|\vec{v}(t)|/dt$ may be equal

(C) $d|\vec{v}(t)|/dt$ can be zero while $|d\vec{v}(t)/dt|$ is not zero

(D) $d|\vec{v}(t)|/dt \neq 0$ implies $|d\vec{v}(t)/dt| \neq 0$

11. Three particles A, B, C are located at the corners of an equilateral triangle as shown in figure. Each of the particle is moving with velocity v. Then at the instant shown, the relative angular velocity of



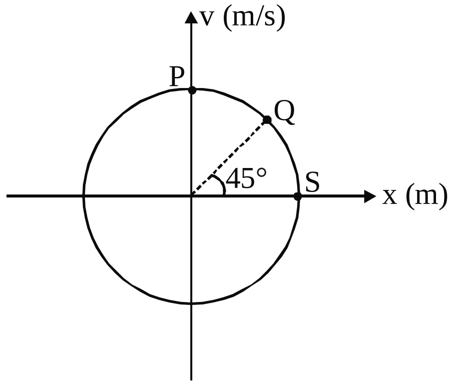
(A) A wrt B is $\frac{v\cos 30^\circ}{a}$ in z-direction

(B) B wrt C is $\frac{v\cos 30^\circ}{a}$ in z-direction

(C) A wrt C is $\frac{v\cos 30^\circ}{a}$ in z-direction

(D) B wrt A is $\frac{v\cos 30^\circ}{a}$ in –z direction

12. A particle is in motion on the x-axis. The variation of its velocity with position is as shown. The graph is circle and its equation is $x^2 + v^2 = 1$, where x is in m and v in m/s. The **CORRECT** statement(s) is/are :-



(A) When x is positive, acceleration is negative.

(B) When x is negative, acceleration is positive.

(C) At Q, acceleration has magnitude $\frac{1}{\sqrt{2}}$ m/s^2

(D) At S, acceleration is infinite.

13. An ant travels along a long rod with a constant velocity $\vec{u}$ relative to the rod starting from the origin. The rod is kept initially along the positive x-axis. At t = 0, the rod also starts rotating with an angular velocity ω (anticlockwise) in x-y plane about origin. Then

(A) the position of the ant at any time t is $\vec{r} = ut[\cos\omega t\hat{i} + \sin\omega t\hat{j}]$

(B) the speed of the ant at any time t is $u\sqrt{1+\omega^2t^2}$

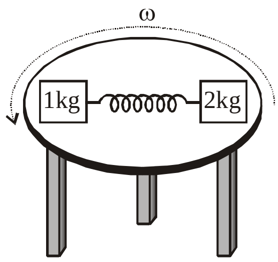
(C) the magnitude of the tangential acceleration of the ant at any time t is $\frac{\omega^2 tu}{\sqrt{1+\omega^2t^2}}$

(D) the speed of the ant at any time t is $\sqrt{1+2\omega^2t^2u}$

14. On a circular turn table rotating about its center horizontally with uniform angular velocity ω rad/s placed two blocks of mass 1 kg and 2 kg, on a diameter symmetrically about center. Their separation is 1m and friction is sufficient to avoid slipping. The spring between them as shown is stretched and applied force of 5N. If f_1 and f_2 are values of friction on 1 kg & 2kg block respectively:-

(A) For ω = 2 rad/s, f_1 = 3N & f_2 = 1N

(B) For ω = 3 rad/s, f_1 = 0.5 N & f_2 = 4N



(C) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 5N

(D) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 0N

15. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then

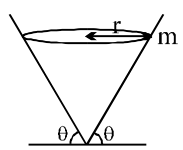
(A) they cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.

(B) they cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed.

(C) they cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.

(D) they cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.

16. A ball of mass 'm' is rotating in a circle of radius 'r' with speed v inside a smooth cone as shown in figure. Let N be the normal reaction on the ball by the cone, then choose the correct option.



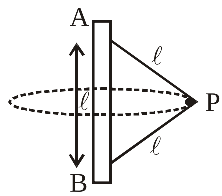
(A) $N = mg\cos\theta$

(B) $g\sin\theta = \frac{v^2}{r}\cos\theta$

(C) $N\sin\theta - \frac{mv^2}{r} = 0$

(D) none of these

17. A particle P of mass m is attached to a vertical axis by two strings AP and BP of length l each. The separation AB=l. P rotates around the axis with an angular velocity ω. The tensions in the two strings are T_1 and T_2



(A) $T_1=T_2$

(B) $T_1+T_2=m\omega^2 l$

(C) $T_1-T_2=2mg$

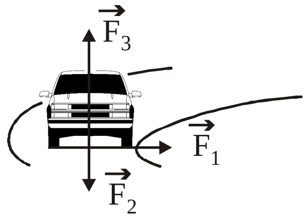
(D) BP will remain taut only if $\omega \geq \sqrt{\frac{2g}{\ell}}$

MATRIX MATCH TYPE QUESTION

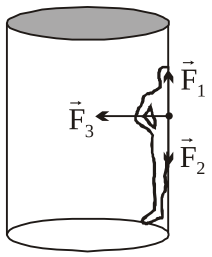
18. Column-I shows certain situations and column-2 shows information about forces.

Column - I

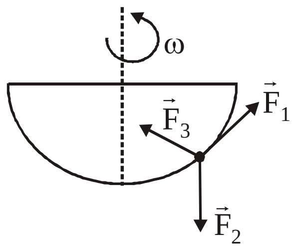
Situation

(A)


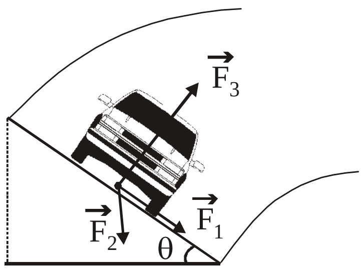
Front view of a car rounding a curve with constant speed.

(B)


Passengers in a rotor not sliding relative to rotor wall cylindrical rotor is rotating with constant angular velocity about its symmetry axis.

(C)


Particle kept on rough surface of a bowl, no relative motion of particle in bowl, bowl has constant angular velocity

(D)


Car moving on a banked road with constant speed, no sideways skidding

Column - II

(P) $\vec{F}_1+\vec{F}_2+\vec{F}_3$ is centripetal force.

(Q) $\vec{F}_1$ is static friction.

(R) $\vec{F}_1$ can be in direction opposite to that shown in figure.

(S) $\vec{F}_1+\vec{F}_2=\vec{0}$

(T) $\vec{F}_1+\vec{F}_2+\vec{F}_3=\vec{0}$

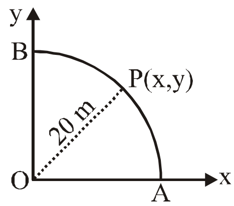
EXERCISE (JM)

1. For a pacticle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis). **[AIEEE - 2010]**

(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$ (2) $-\frac{v^2}{R}\cos\theta\ \hat{i} + \frac{v^2}{R}\sin\theta\ \hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\ \hat{i} + \frac{v^2}{R}\cos\theta\ \hat{j}$ (4) $-\frac{v^2}{R}\cos\theta\ \hat{i} - \frac{v^2}{R}\sin\theta\ \hat{j}$

2. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly : **[AIEEE - 2010]**



(1) 14 m/s² (2) 13 m/s² (3) 12 m/s² (4) 7.2 m/s²

3. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :- **[AIEEE - 2012]**

(1) 1 : 1 (2) $m_1 r_1 : m_2 r_2$ (3) $m_1 : m_2$ (4) $r_1 : r_2$

4. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to : (Take the radius of the drum to be 1.25 m and its axle to be horizontal): **[JEE Main (Online) - 2016]**

(1) 8.0 (2) 0.4 (3) 1.3 (4) 27.0

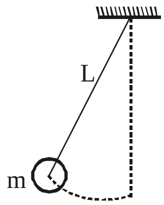
5. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then, **[JEE Main-2018]**

(1) $T \propto R^{\frac{n}{2}+1}$ (2) $T \propto R^{(n+1)/2}$ (3) $T \propto R^{n/2}$ (4) $T \propto R^{3/2}$ for any n

EXERCISE (JA)

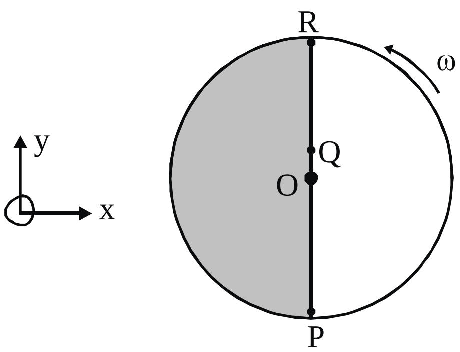
1. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is

[IIT-JEE-2011]



(A) 9 (B) 18 (C) 27 (D) 36

2. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then **[IIT-JEE-2012]**



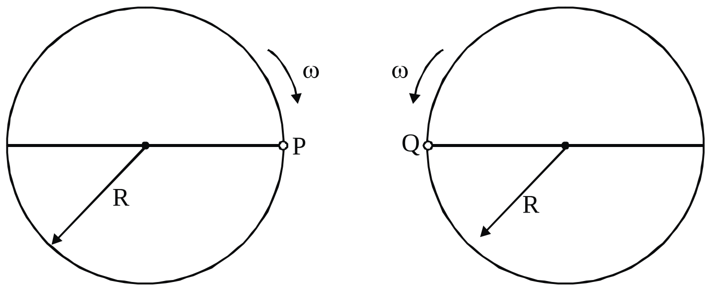
(A) P lands in the shaded region and Q in the unshaded region.

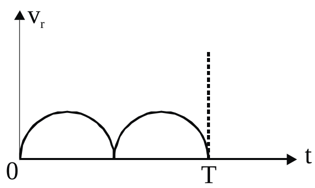
(B) P lands in the unshaded region and Q in the shaded region.

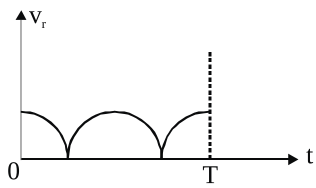
(C) Both P and Q land in the unshaded region.

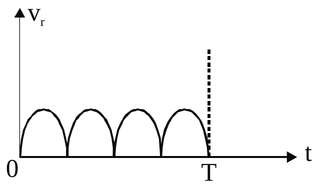
(D) Both P and Q land in the shaded region.

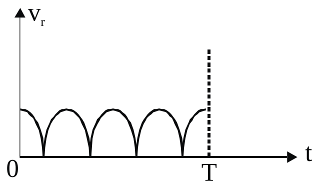
3. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t =0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[IIT-JEE 2012]**



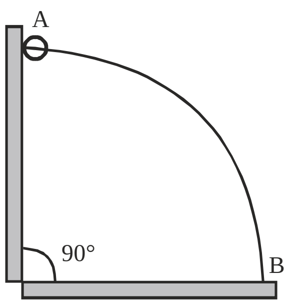
(A)


(B)


(C)


(D)


4. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is :- **[IIT-JEE Advanced 2014]**



(A) Always radially outwards

(B) Always radially inwards

(C) Radially outwards initially and radially inwards later.

(D) Radially inwards initially and radially outwards later.

Paragraph for Question No. 5 and 6

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference.

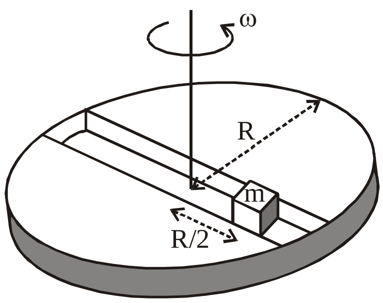
The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m\left(\vec{\upsilon}_{rot} \times \vec{\omega}\right) + m\left(\vec{\omega} \times \vec{r}\right) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $\left(\vec{\omega} = \omega\hat{k}\right)$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.

[IIT-JEE Advanced-2016]



5. The distance r of the block at time t is :

(A) $\frac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (B) $\frac{R}{2}\cos 2\omega t$ (C) $\frac{R}{2}\cos \omega t$ (D) $\frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

6. The net reaction of the disc on the block is :

(A) $-m\omega^2 R\cos \omega t\hat{j} - mg\hat{k}$

(B) $m\omega^2 R\sin \omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$

(D) $\frac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. 9.9 ms^{-2}, in radial direction towards the centre at all points.

2. Ans. (i) 2 rad/s^2 (ii) 12+2t for $t \le 2s$, 16 for $t \ge 2s$ (iii) $a = 169.01\ m/s^2$ (iv) 44 rad

3. Ans. (a) 4 cm/s^2 (b) 2 cm/s^2 (c) $\sqrt{20}\ cm/s^2$

4. Ans. (i) $75m/s^2$, (ii) $125m/s^2$

5. Ans. 4

6. Ans. 024

7. Ans. $\frac{v}{2d}$

8. Ans. $a_t = \frac{20}{\sqrt{13}}$, $a_n = \frac{30}{\sqrt{13}}$

9. Ans. $\sqrt{3}b/2$, $b/2$, b

10. Ans. 6

11. Ans. (i) mg, (ii) $\frac{mv^2}{r}$

12. Ans. Yes, $a_c = 8$, $\mu g = 1$

13. Ans. T = 6.6 N, $v_{max} = 35\ ms^{-1}$

14. Ans. $\sqrt{2g}$ rad/s

15. Ans. (a) $\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$ (b) $\frac{mgL}{\sqrt{L^2 - r^2}}$

16. Ans. 5

17. Ans. 15 km

18. Ans. 2

EXERCISE (O-1)

1. Ans. (D) **2. Ans. (B)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (A)** **6. Ans. (D)**

7. Ans. (C) **8. Ans. (A)** **9. Ans. (D)** **10. Ans. (C)** **11. Ans. (A)** **12. Ans. (B)**

13. Ans. (D) **14. Ans. (A)** **15. Ans. (B)** **16. Ans. (A)** **17. Ans. (C)** **18. Ans. (A)**

19. Ans. (A) **20. Ans. (B)** **21. Ans. (C)** **22. Ans. (A, B)**

23. Ans. (A, B) **24. Ans. (A,B,C,D)** **25. Ans. (A, B, C)**

26. Ans. (A)-R; (B)-S; (C)-Q; (D)-P

EXERCISE (O-2)

1. Ans. (C) **2. Ans. (D)** **3. Ans. (B)** **4. Ans. (B)**

5. Ans. (A) **6. Ans. (B)** **7. Ans. (D)**

8. Ans. (A, C) **9. Ans. (A,C,D)** **10. Ans. (B,C,D)** **11. Ans. (A,B,C)**

12. Ans. (A, B, C) **13. Ans. (A, B, C)** **14. Ans. (A, B, C)** **15. Ans. (B, C, D)**

16. Ans. (B, C) **17. Ans. (B, C, D)**

18. Ans. (A) P,Q (B) P,Q,S (C) P,Q,R (D) P,Q,R

EXERCISE (JM)

1. Ans. (4) **2. Ans. (1)** **3. Ans. (4)** **4. Ans. (4)**

5. Ans. (2)

EXERCISE (JA)

1. Ans. (D) **2. Ans. (C OR D)** **3. Ans. (A)** **4. Ans. (D)** **5. Ans. (D)**

6. Ans. (C)

CHAPTER 3

CENTER OF MASS, MOMENTUM & COLLISION

Chapter Contents 03

IMPORTANT NOTES

CHAPTER 3

CENTER OF MASS, MOMENTUM & COLLISION

KEY CONCEPTS

CENTRE OF MASS :

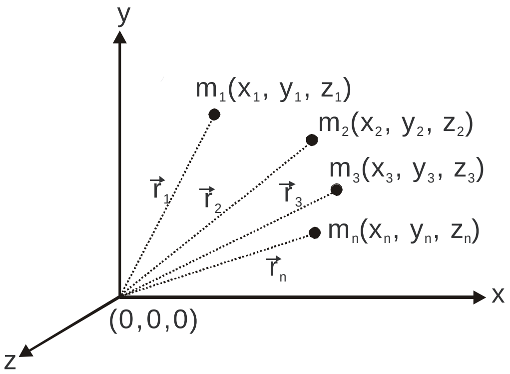
For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated.

The centre of mass of an object is a point that represents the entire body and moves in the same way as a point mass having mass equal to that of the object, when subjected to the same external forces that act on the object.

Centre of mass of system of discrete particles

Total mass of the body : $M = m_1 + m_2 + ... + m_n$

Then $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} = \dfrac{1}{M}\Sigma m_i\vec{r}_i$



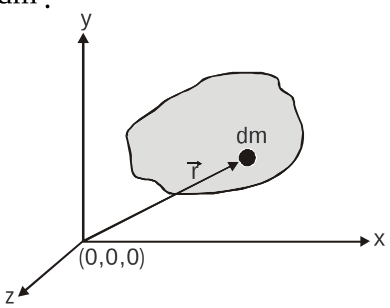
co-ordinates of centre of mass :

$x_{cm} = \dfrac{1}{M}\Sigma m_i x_i$, $y_{cm} = \dfrac{1}{M}\Sigma m_i y_i$ and $z_{cm} = \dfrac{1}{M}\Sigma m_i z_i$

Centre of mass of continuous distribution of particles

If the system has continuous distribution of mass, treating the mass element dm at position $\vec{r}$ as a point mass and replacing summation by integration. $\vec{R}_{CM} = \dfrac{1}{M}\int \vec{r}dm$.

So that $x_{cm} = \dfrac{1}{M}\int x\,dm$, $y_{cm} = \dfrac{1}{M}\int y\,dm$ and $z_{cm} = \dfrac{1}{M}\int z\,dm$



If co-ordinates of particles of mass m_1, m_2, ... are

(x_1, y_1, z_1), (x_2, y_2, z_2)....

then position vector of their centre of mass is

$$\vec{R}_{CM} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3\left(x_3\hat{i} + y_3\hat{j} + z_3\hat{k}\right) + ...}{m_1 + m_2 + m_3 + ...}$$

$$= \frac{(m_1x_1 + m_2x_2 + ...)\hat{i} + (m_1y_1 + m_2y_2...)\hat{j} + (m_1z_1 + m_2z_2 + ..)\hat{k}}{m_1 + m_2 + m_3 + ..}$$

So, $x_{cm} = \left(\dfrac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + m_3 + ...}\right)$, $y_{cm} = \left(\dfrac{m_1y_1 + m_2y_2 + ...}{m_1 + m_2 + ...}\right)$, $z_{cm} = \left(\dfrac{m_1z_1 + m_2z_2 + ...}{m_1 + m_2 + ...}\right)$

The centre of mass after removal of a part of a body

If a portion of a body is taken out, the remaining portion may be considered as,

Original mass (M) – mass of the removed part (m) = {original mass (M)} + { – mass of the removed part (m)}

The formula changes to : $x_{CM} = \dfrac{Mx - mx'}{M - m}$; $y_{CM} = \dfrac{My - my'}{M - m}$; $z_{CM} = \dfrac{Mz - mz'}{M - m}$

Where x', y' and z' represent the coordinates of the centre of mass of the removed part.

MOTION OF CENTRE OF MASS

As for a system of particles, position of centre of mass is $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$

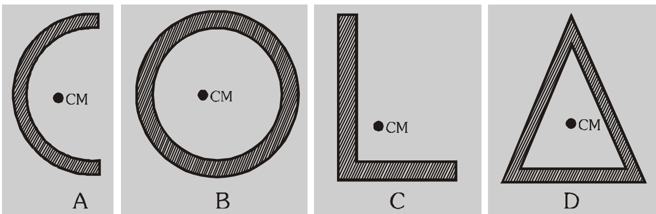
So $\dfrac{d}{dt}\left(\vec{R}_{CM}\right) = \dfrac{m_1\dfrac{d\vec{r}_1}{dt} + m_2\dfrac{d\vec{r}_2}{dt} + m_3\dfrac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...} \Rightarrow$

Similarly acceleration $\vec{a}_{CM} = \dfrac{d}{dt}\left(\vec{v}_{CM}\right) = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$

We can write $\quad M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...\ [\because \vec{p} = m\vec{v}\ ]$

$$M\vec{v}_{CM} = \vec{p}_{CM}\ [\because \Sigma\vec{p}_i = \vec{p}_{CM}]$$

IMPORTANT POINTS



- There may or may not be any mass present physically at centre of mass (See Figure A, B, C)
- Centre of mass may be inside or outside of the body (See figure A, B, C)
- Position of centre of mass depends on the shape of the body. (See figure A, B, C)
- For a given shape it depends on the distribution of mass of within the body and is closer to massive part. (See figure A,C)
- For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre. (See figure B,D).
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.

- It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.

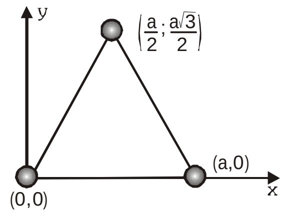
- If the origin of co-ordinate system is at centre of mass, i.e., $\vec{R}_{CM} = 0$, then by definition.

$$\frac{1}{M}\Sigma m_i \vec{r}_i = 0 \Rightarrow \Sigma m_i \vec{r}_i = 0$$

The sum of the moments of the masses of a system about its centre of mass is always zero.

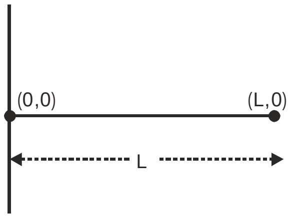
Ex. Three bodies of equal masses are placed at (0, 0), (a, 0) and at $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$.

Find out the co-ordinates of centre of mass.



Sol. $x_{CM} = \dfrac{0 \times m + a \times m + \frac{a}{2} \times m}{m + m + m} = \dfrac{a}{2}$, $y_{CM} = \dfrac{0 \times m + 0 \times m + \frac{a\sqrt{3}}{2} \times m}{m + m + m} = \dfrac{a\sqrt{3}}{6}$

Ex. Calculate the position of the centre of mass of a system consisting of two particles of masses m_1 and m_2 separated by a distance L apart, from m_1.



Sol. Treating the line joining the two particles as x axis

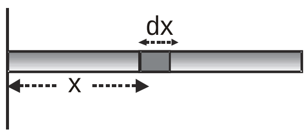
$$x_{CM} = \frac{m_1 \times 0 + m_2 \times L}{m_1 + m_2} = \frac{m_2 L}{m_1 + m_2}, y_{CM} = 0 \qquad z_{CM} = 0$$

Ex. If the linear density of a rod of length L varies as λ = A + Bx, compute position of its centre of mass.

Sol. Let the x–axis be along the length of the rod and origin at one of its end as shown in figure. As rod is along x–axis, for all points on it y and z will be zero so, $y_{CM} = 0$ and $z_{CM} = 0$ i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin, mass of this element $dm = \lambda dx = (A + Bx)dx$

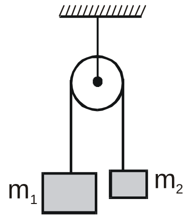
$$\text{so, } x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx} = \frac{\frac{AL^2}{2} + \frac{BL^3}{3}}{AL + \frac{BL^2}{2}} = \frac{L(3A+2BL)}{3(2A+BL)}$$



Note : (i) If the rod is of uniform density then λ = A = constant & B = 0 then X_{CM} = L/2

(ii) If the density of rod varies linearly with x, then λ = Bx and A = 0 then X_{CM} = 2L/3

Ex. Two bodies of masses m_1 and m_2 ($<m_1$) are connected to the ends of a massless cord and allowed to move as shown in. The pulley is both massless and frictionless. Calculate the acceleration of the centre of mass.



Sol. If $\vec{a}$ is the acceleration of m_1,then $-\vec{a}$ is the acceleration of m_2 then

acceleration of each body $a = \frac{\text{Net force}}{\text{Net mass}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$

$$\vec{a}_{cm} = \frac{m_1\vec{a} + m_2(-\vec{a})}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{a}$$

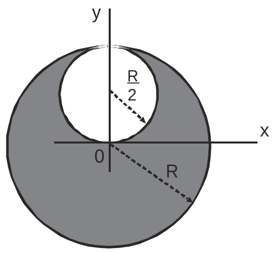
But $\vec{a} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{g}$ so $\vec{a}_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \vec{g}$

Ex. A circle of radius R is cut from a uniform thin sheet of metal. A circular hole of radius $\frac{R}{2}$ is now cut out of the circle, with the hole tangent to the rim. Find the distance of centre of mass from the centre of the original uncut circle to the centre of mass.

Sol. We treat the hole as a 'negative mass' object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass). By symmetry, the CM lies along the +y–axis in figure, so $x_{CM} = 0$. With the origin at the centre of the original circle whose mass is assumed to be m.

Mass of original uncut circle $m_1 = m$ & (0, 0)

Mass of hole of negative mass : $m_2 = \frac{m}{4}$; Location of CM $\left(0, \frac{R}{2}\right)$



Thus $y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$

So the centre of mass is at the point $\left(0, -\frac{R}{6}\right)$

Ex. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speeds of 2m/s and 6m/s, respectively, on a smooth horizontal surface. Find the speed of centre of mass of the system.

Sol. Velocity of centre of mass of the system $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$ Since the particles m_1 and m_2 are moving in same direction, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel. $\Rightarrow$ $|m_1\vec{v}_1 + m_2\vec{v}_2| = m_1v_1 + m_2v_2$

Therefore, $v_{cm} = \dfrac{|m_1\vec{v}_1 + m_2\vec{v}_2|}{m_1 + m_2} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2} = \dfrac{(1)(2) + \left(\frac{1}{2}\right)(6)}{\left(1 + \frac{1}{2}\right)} = 3.33\ \text{ms}^{-1}$

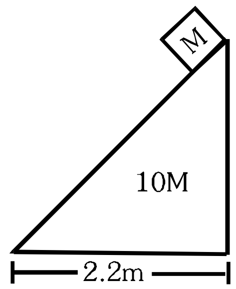
Ex. Two particles of masses 2 kg and 4 kg are approaching towards each other with acceleration 1 m/s^2 and 2 m/s^2, respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

Sol. The acceleration of centre of mass of the system $\vec{a}_{cm} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$ $\Rightarrow$ $a_{cm} = \dfrac{|m_1\vec{a}_1 + m_2\vec{a}_2|}{m_1 + m_2}$

Since $\vec{a}_1$ and $\vec{a}_2$ are anti–parallel, so $a_{cm} = \dfrac{|m_1a_1 - m_2a_2|}{m_1 + m_2} = \dfrac{|(2)(1) - (4)(2)|}{2 + 4} = 1\ \text{ms}^{-2}$

Since $m_2a_2 > m_1a_1$ so the direction of acceleration of centre of mass will be directed in the direction of a_2.

Ex. A block of mass M is placed on the top of a bigger block of mass 10 M as shown in figure. All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant the smaller block reaches the ground.



Sol. If the bigger block moves towards right by a distance (X), the smaller block will move towards left by a distance (2.2 –X) (taking the two blocks together as the system). The horizontal position of CM remains same $\Rightarrow$ M (2.2 – X) = 10 MX $\Rightarrow$ X = 0.2 m.

MOMENTUM

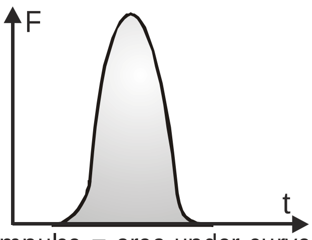
The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

IMPULSE

When a large force act for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. The total effect of force is called impulse (measure of the action of force). This type of force is generally variable in magnitude and is sometimes called impulsive force.

If a large force act on a body or particle for a small time then

Impulse = product of force with time.



impulse = area under curve

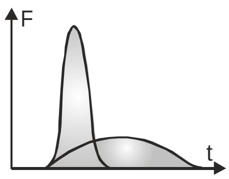
Suppose a force $\vec{F}$ acts for a short time dt then impulse = $\vec{F}dt$

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

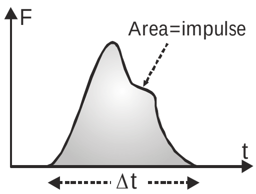
If constant force $\vec{F}$ acts for an interval Δt then Impulse = $\vec{F}\Delta t$

Impulse – Momentum theorem :

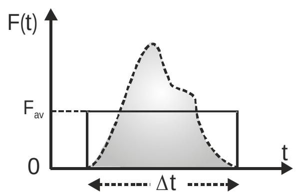
Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta\vec{p}}$



A large force acting for a short time and a small force acting for a long time may result in the same impulse, as indicated by the equal areas under the force-versus time curves.



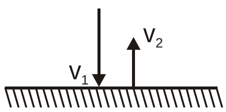
The force F(t) involved in the collision between a ball and bat. The interaction occurs during the time Δt. The impulse is equal in magnitude to the shaded area under the curve.



The constant force F_{av} acting for the same time interval Δt produces the same impulse as in figure since the areas under the two graphs are the same.

Ex. A ball of mass 50 g is dropped from a height h = 10 m. It rebounds losing 75 percent of its total mechanical energy. If it remains in contact with the ground for 0.01s, find the impulse of the impact force.

Sol. Impulse = change in momentum = $m(v_1+v_2)$



Here $v_1 = \sqrt{2gh}$ and for v_2, $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2\left(1-\frac{75}{100}\right) \Rightarrow v_2 = \frac{v_1}{2}$

So impulse = $m\left(v_1+\frac{v_1}{2}\right) = \frac{3mv_1}{2} = \frac{3}{2}m \times \sqrt{2gh} = \frac{3}{2} \times 50 \times 10^{-3} \times \sqrt{2\times9.8\times10} = 1.05$ N–s

LAW OF CONSERVATION OF LINEAR MOMENTUM

According to Newton's Second law of motion the rate of change of momentum is equal to the applied force.

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \text{if } \vec{F} = \vec{0} \text{ then } \frac{d\vec{p}}{dt} = \vec{0} \text{ i.e. } \vec{p} = \text{constant}$$

This leads to the law of conservation of momentum which is" In the absence of external forces, the total momentum of the system is conserved."

IMPORTANT POINTS

- For an isolated system, the initial momentum of the system is equal to the final momentum of the system. If the system consists of n bodies having momentum

  $\vec{p}_1, \vec{p}_2, \vec{p}_3, ...\vec{p}_n$, then $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...+ \vec{p}_n$ = constant

- As linear momentum depends on frame of reference. Observers in different frames would find different values of linear momentum of a given system but each would agree that his own value of linear momentum does not change with time provided. But the system should be isolated and closed, i.e., law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.

- Conservation of linear momentum is equivalent to Newton's III law of motion for a system of two particles in absence of external force by law of conservation of linear momentum.

  $\Rightarrow \quad \vec{p}_1 + \vec{p}_2$ = constant $\qquad$ i.e. $m_1\vec{v}_1 + m_2\vec{v}_2$ = constant

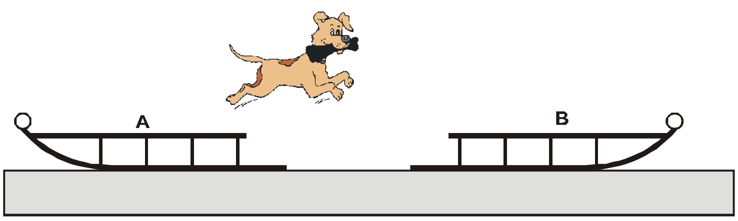
  Differentiating above with respect to time $m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = \vec{0}$ [as m is constant]

  $$\Rightarrow m_1\vec{a}_1 + m_2\vec{a}_2 = \vec{0} \ \left[\because \frac{d\vec{v}_1}{dt} = \vec{a}\right] \Rightarrow \vec{F}_1 + \vec{F}_2 = \vec{0} \qquad [\because \vec{F} = m\vec{a}] \qquad \Rightarrow \vec{F}_1 = -\vec{F}_2$$

  i.e., for every action there is equal and opposite reaction which is Newton's III law of motion.

- This law is universal, i.e., it applies to body macroscopic as well as microscopic systems.

Ex. Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure. A 3.63 kg dog, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 ms^{-1} relative to the ice. Find the final speeds of the two sleds.



Sol. Total momentum imparted to B $p_B = 2 \times 3.63 \times 3.05$ kg ms^{-1}.

Velocity of $B = \frac{p_B}{m_B} = \frac{2 \times 3.63 \times 3.05}{22.7} = 0.975$ ms^{-1}.

Velocity of A when the dog jumps away from $A = \frac{p_A}{m_A} = \frac{3.63 \times 3.05}{22.7} = 0.4877$ ms^{-1}.

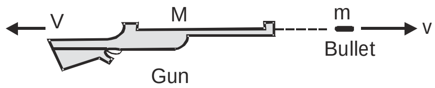
When the dog comes back to A, Velocity of $A = \frac{22.7 \times 0.4877 + 3.63 \times 3.05}{22.7 + 3.63} = 0.841$ ms^{-1}.

APPLICATIONS OF CONSERVATION OF LINEAR MOMENTUM

Firing a Bullet from a Gun :

- If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to mv. This is not the violation of law of conservation of linear momentum as linear momentum is conserved only in absence of external force .
- If the bullet and gun is the system, the force exerted by trigger will be internal so.

Total momentum of the system $\vec{p}_s = \vec{p}_B + \vec{p}_G$ = constant.



Now as initially both bullet and gun are at rest so $\vec{p}_B + \vec{p}_G = \vec{0}$ From this it is evident that:

- $\vec{p}_G = -\vec{p}_B$, i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.
- As $\vec{p} = m\vec{v}$, $m\vec{v} + M\vec{V} = \vec{0}$, i.e, $\vec{V} = -\frac{m}{M}\vec{v}$ i.e, if the bullet moves forward, gun 'recoils' or 'kicks' backward. Heavier the gun lesser will be the recoil velocity V.
- Kinetic energy $K = \frac{p^2}{2m}$ and $|\vec{p}_B| = |\vec{p}_G| = p$ Kinetic energy of gun $K_G = \frac{p^2}{2M}$,

  Kinetic energy of bullet $K_B = \frac{p^2}{2m}$ $\therefore \frac{K_G}{K_B} = \frac{m}{M} < 1$ ($\because$ M $\gg$ m) Thus kinetic energy of gun is smaller than bullet i.e., kinetic energy of bullet and gun will not be equal.
- Initial kinetic energy of the system is zero as both are at rest initially.

  Final kinetic energy of the system $[(1/2)(mv^2 + MV^2)] > 0$.

  So, here kinetic energy of the system is not constant but increases. If PE is assumed to be constant then Mechanical energy = (kinetic energy + potential energy) will also increase. However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

Ex. A bullet of mass 100g is fired by a gun of 10kg with a speed 2000 m/s. Find recoil velocity of gun.

Sol. According to conservation of momentum mv + MV = 0.

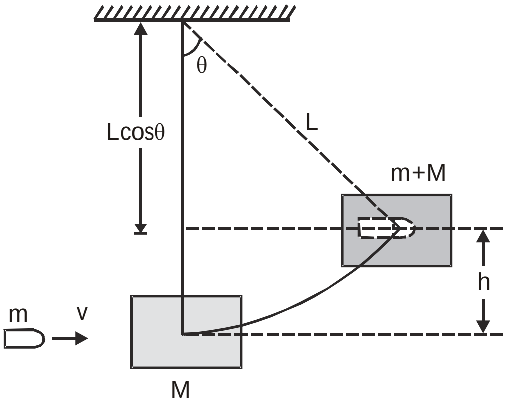
Velocity of gun $V = -\frac{mv}{M} = -\frac{0.1 \times 2000}{10} = -20 m/s$

Block Bullet System :

(a) When bullet remains in the block

Conserving momentum of bullet and block mv + 0 = (M+m) V

Velocity of block $V = \frac{mv}{M+m}$ ...(i)



By conservation of mechanical energy

$\frac{1}{2}(M+m)V^2 = (M+m)gh \Rightarrow V = \sqrt{2gh}$ ....(ii)

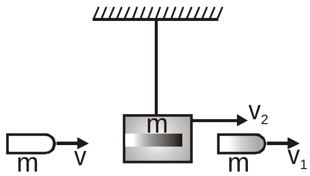
From eq^n. (i) and eq^n. (ii) $\frac{mv}{M+m} = \sqrt{2gh}$;

Speed of bullet $v = \frac{(M+m)\sqrt{2gh}}{m}$,

Maximum height gained by block $h = \frac{V^2}{2g} = \frac{m^2v^2}{2g(M+m)^2}$

$h = L - L\cos\theta \qquad \therefore \cos\theta = 1 - \frac{h}{L} \Rightarrow \theta = \cos^{-1}\left(1 - \frac{h}{L}\right)$

(b) If bullet moves out of the block



Conserving momentum $mv + 0 = mv_1 + Mv_2$

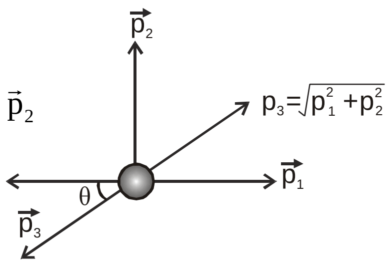
$m(v - v_1) = Mv_2$ ....(i)

Conserving energy $\frac{1}{2}Mv_2^2 = Mgh \Rightarrow v_2 = \sqrt{2gh}$ ....(ii)

From eq^n. (i) & eq^n. (ii) $m(v - v_1) = M\sqrt{2gh} \Rightarrow h = \frac{m^2(v-v_1)^2}{2gM^2}$

Explosion of a Bomb at rest

Conserving momentum



$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0} \Rightarrow \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2) \Rightarrow p_3 = \sqrt{p_1^2 + p_2^2}$ as $\vec{p}_1 \perp \vec{p}_2$

Angle made by $\vec{p}_3$ from $\vec{p}_1 = \pi + \theta$

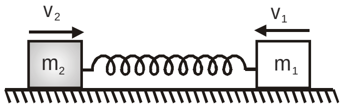
Angle made by $\vec{p}_3$ from $\vec{p}_2 = \frac{\pi}{2} + \theta$

Energy released in explosion $= K_f - K_i = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} - 0 = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3}$

Motion of Two Masses Connected to a Spring

Consider two blocks, resting on a frictionless surface and connected by a massless spring as shown in figure. If the spring is stretched (or compressed) and then released from rest,

Then $F_{ext}=0$ so $\vec{p}_s = \vec{p}_1 + \vec{p}_2$ = constant



However, initially both the blocks were at rest so, $\vec{p}_1 + \vec{p}_2 = \vec{0}$

It is clear that :

- $\vec{p}_2 = -\vec{p}_1$, i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (Though they have different values of momentum at different positions).

- As momentum $\vec{p} = m\vec{v}$, $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0} \Rightarrow \vec{v}_2 = -\left(\frac{m_1}{m_2}\right)\vec{v}_1$

  The two blocks always move in opposite directions with lighter block moving faster.

- Kinetic energy $KE = \frac{p^2}{2m}$ and $|\vec{p}_1| = |\vec{p}_2|$, $\frac{KE_1}{KE_2} = \frac{m_2}{m_1}$ or the kinetic energy of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater kinetic energy.

- Initially kinetic energy of the blocks is zero (as both are at rest) but after some time kinetic energy of the blocks is not zero (as both are in motion). So, kinetic energy is not constant but changes. Here during motion of blocks KE is converted into elastic potential energy of the spring and vice–versa but total mechanical energy of the system remain constant.

Kinetic energy + Potential energy = Mechanical Energy = Constant

Note – If $\vec{F}$ is the average of the time varying force during collision and Δt is the duration of collision then impulse $\vec{I} = \vec{F}\Delta t$.

Conservation of Linear Momentum During Impact :

If two bodies of masses m_1 and m_2 collide in air, the total external force acting on the system of bodies (m_1 + m_2) is equal to $\vec{F}_1 + m_1\vec{g} + \vec{F}_2 + m_2\vec{g} \Rightarrow F_{total} = m_1\vec{g} + m_2\vec{g} + \vec{F}_1 + \vec{F}_2$

During collision the impact forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction.

According to Newton's 3rd law of motion, $\vec{F}_1 + \vec{F}_2 = \vec{0} \Rightarrow$ $\vec{F}_{net}$ = $m_1\vec{g} + m_2\vec{g}$

So Impulse = $\vec{F}_{net}\Delta t = (m_1\vec{g} + m_2\vec{g})\Delta t$

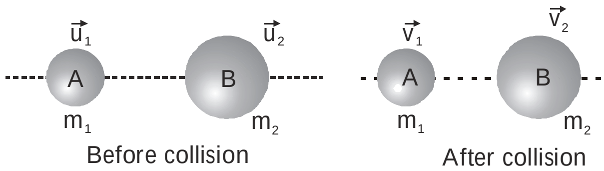
Since Δt is a very small time interval, the impulse $F(\Delta t)$ will be negligibly small. As impulse is equal to change in momentum of the system, a negligible impulse means negligible change of momentum. Let the change of momentum of 1 & 2 be $\Delta\vec{p}_1 \& \Delta\vec{p}_2$, respectively then the total change in momentum of the system $\Rightarrow \Delta\vec{p} = \Delta\vec{p}_1 + \Delta p_2 = \vec{F}_{net}.dt \approx \vec{0} \Rightarrow \Delta(\vec{p}_1 + \vec{p}_2) = 0 \Rightarrow \vec{p}_1 + \vec{p}_2 =$ constant.

Therefore, the net or total momentum of the colliding bodies remains practically unchanged along the line of action (impact) during the collision. In other words, the momentum of the system remains constant or conserved during the period of impact. Therefore, we can conveniently equate the net momentum of the colliding bodies at the beginning and at the end of the collision (or just before and just after the impact).

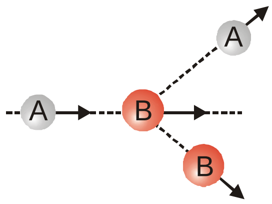
Note : Remember that the impact force F is not an external force for the system of colliding bodies. If no external force acts on the system, its momentum remains constant for all the times including the time of collision. Even if some external forces like gravitation and friction (known as non–impulsive forces in general) are present, we can conserved the momentum of the system during the impact, because the finite external forces cannot change the momentum of the system significantly in very short time. Therefore, the change in position of the system during infinitesimal time of impact can also be neglected.

- Types of collision according to the direction of collision :

(a) **Head on collision** : Direction of velocities of bodies is similar to the direction of collision.



(b) **Oblique collision** : Direction of velocities of bodies is not similar to the direction of collision.



- Types of collision according to the conservation law of kinetic energy :

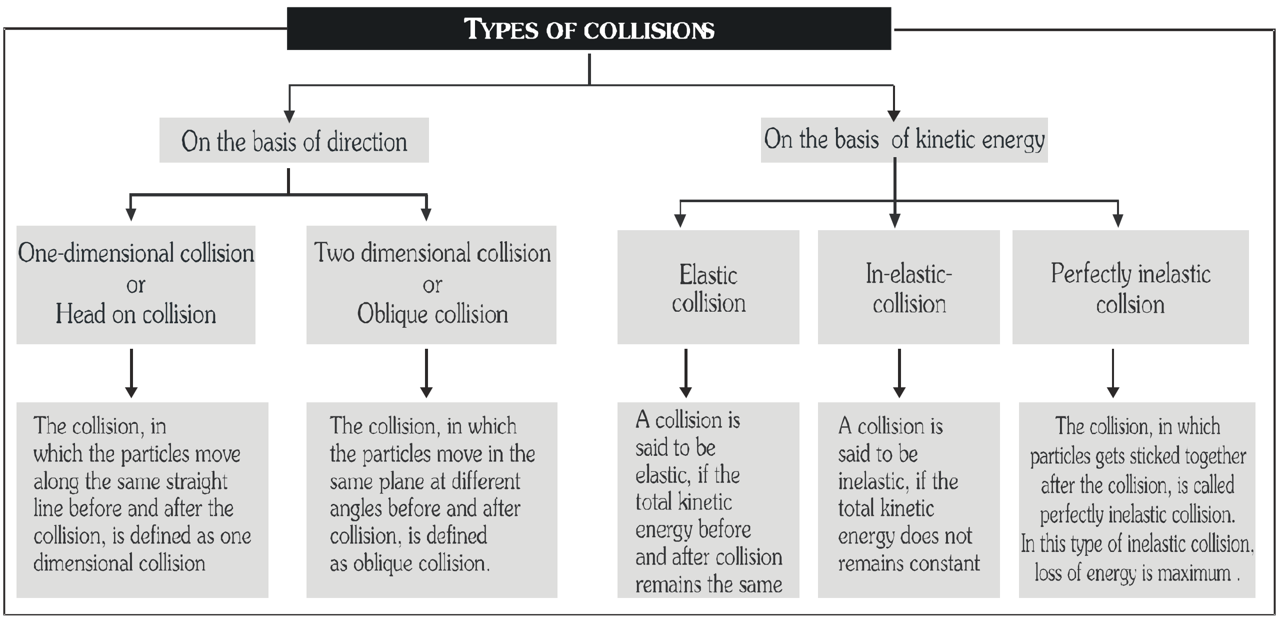
(a) **Elastic collision** : $KE_{\text{before collision}} = KE_{\text{after collision}}$

(b) **Inelastic collision** : kinetic energy is not conserved.

Some energy is lost in collision $KE_{\text{before collision}} > KE_{\text{after collision}}$

(c) **Perfect inelastic collision :** Two bodies stick together after the collision.

momentum remains conserved in all types of collisions.



Coefficient of restitution (e)

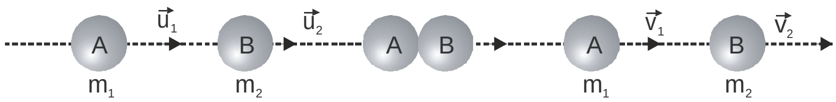
The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

$$e = -\frac{\text{impulse of recovery}}{\text{impulse of deformation}} \qquad e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

HEAD ON ELASTIC COLLISION

The elastic collision in which the colliding bodies move along the same straight line path before and after the collision.



Before collision Collision After collision

Assuming initial direction of motion to be positive and $u_1 > u_2$ (so that collision may take place) and applying law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad \text{...(i)}$$

For elastic collision, kinetic energy before collision must be equal to kinetic energy after collision,

i.e., $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \qquad \text{... (ii)}$$

Dividing equation (ii) by (i) $\quad u_1 + v_1 = v_2 + u_2 \Rightarrow (u_1 - u_2) = (v_2 - v_1) \qquad$ ...(iii)

In 1–D elastic collision 'velocity of approach' before collision is equal to the 'velocity of recession' after collision, no matter what the masses of the colliding particles be.

This law is called **Newton's law for elastic collision**

Now if we multiply equation (iii) by m_2 and subtracting it from (i)

$$(m_1 - m_2)\,u_1 + 2m_2u_2 = (m_1 + m_2)v_1 \;\Rightarrow\; v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \quad \text{... (iv)}$$

Similarly multiplying equation (iii) by m_1 and adding it to equation (i)

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_2 + m_1)v_2 \;\Rightarrow\; v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \quad \text{...(v)}$$

IMPORTANT POINTS

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.

- **If two bodies are of equal masses and second body is at rest.**

  $m_1 = m_2$ and initial velocity of second body $u_2 = 0$, $v_1 = 0$, $v_2 = u_1$

  When body A collides against body B of equal mass at rest, the body A comes to rest and the body B moves on with the velocity of the body A. In this case transfer of energy is hundred percent

  e.g.. Billiard's Ball, Nuclear moderation.

- **If the mass of a body is negligible as compared to other.**

  If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$

  When a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A.

  If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remains at rest.

Ex. Two ball of mass 5kg each is moving in opposite directions with equal speed 5m/s. collides head on with each other. Find out the final velocities of the balls if collision is elastic.

Sol. Here $m_1 = m_2 = 5\text{kg}$, $u_1 = 5$ m/s, $u_2 = -5$ m/s

In such type of condition velocity get interchange so $v_2 = u_1 = 5$ m/s & $v_1 = u_2 = -5$ m/s

Ex. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one third of its original speed. What is the mass of other ball ?

Sol. Here $m_1 = 0.1$ kg, $m_2 = ?$, $u_2 = 0$, $u_1 = u$, $v_1 = -u/3$

As $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u + \frac{2m_2u_2}{m_1+m_2} \Rightarrow -\frac{u}{3} = \left(\frac{0.1-m_2}{0.1+m_2}\right)u \Rightarrow m_2 = 0.2 \text{ kg}$$

HEAD ON INELASTIC COLLISION OF TWO PARTICLES

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law $\dfrac{v_2 - v_1}{u_2 - u_1} = -e$ ...(ii)

By solving eq. (i) and (ii)

$$v_1 = \left(\frac{m_1 - em_2}{m_1+m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1+m_2}\right)u_2 \;,\; v_2 = \left(\frac{m_2 - em_1}{m_1+m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1+m_2}\right)u_1$$

PERFECT INELASTIC COLLISION

In case of inelastic collision, after collision two bodies move with same velocity (or stick together). If two particles of masses m_1 and m_2, moving with velocity u_1 and u_2 ($u_2 < u_1$) respectively along the same line collide 'head on ' and after collision they have same common velocity v, then by conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v + m_2v \Rightarrow v = \frac{m_1u_1 + m_2u_2}{(m_1+m_2)} \quad ...(i)$$

Kinetic energy of the system before collision is $KE_I = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

And after collision is $KE_f = \frac{1}{2}(m_1+m_2)v^2$

Loss in KE during collision

$$\Delta KE = KE_i - KE_f = \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \frac{1}{2}(m_1+m_2)v^2 \quad ...(ii)$$

Substituting the value of v from eq. (i),

$$\Delta KE = \frac{1}{2}[(m_1u_1^2 + m_2u_2^2) - \frac{(m_1u_1+m_2u_2)^2}{(m_1+m_2)}]$$

$$\Rightarrow \Delta KE = \frac{1}{2}\left[\frac{m_1m_2(u_1^2+u_2^2-2u_1u_2)}{(m_1+m_2)}\right] \Rightarrow \boxed{\Delta KE = \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(u_1-u_2)^2}$$

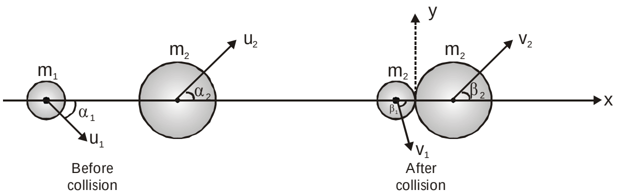
If the target is initially at rest $u_2 = 0$ and $u_1 = u$

$$\Delta KE = \frac{m_1m_2}{2(m_1+m_2)}u^2, \quad \frac{\Delta KE}{KE_i} = \frac{m_2}{(m_1+m_2)} \quad [\because KE_i = \frac{1}{2}m_1u_1^2]$$

Now if target is massive, i.e., $m_2 >> m_1$ then $\frac{\Delta KE}{KE_i} \approx 1$ so percentage loss in KE = 100%

i.e., if a light moving body strikes a heavy target at rest and sticks to it, practically all its KE is lost.

Oblique Collision



In oblique impact the relative velocity of approach of the bodies doesn't coincide with the line of impact. Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case)

$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$ and $m_2u_2\sin\alpha_2 - m_1u_1\sin\alpha_1 = m_2v_2\sin\beta_2 - m_1v_1\sin\beta_1$

Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved. $m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

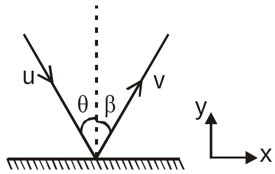
By using Newton's experimental law along the line of impact $e = \frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$

Oblique Impact on a Fixed Plane

Let a small ball collides with a smooth horizontal floor with a speed u at an angle θ to the vertical as shown in the figure. Just after the collision, let the ball leaves the floor with a speed v at angle β to vertical.

It is quite clear that the line of action is perpendicular to the floor. Therefore, the impact takes place along the (normal) vertical. Now we can use Newton's experimental law as

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$



$\Rightarrow$ e [velocity of approach] = velocity of separation

$\Rightarrow$ $e\,[u\cos\theta\,(-\hat{j})] = -[v\cos\beta\,(+\hat{j})] \Rightarrow v\cos\beta = e\,u\cos\theta$ ...(i)

Since impulsive force N acts on the body along the normal, we cannot conserve its momentum. Since along horizontal the component of N is zero, therefore we can conserve the horizontal momentum of the body.

Momentum $(p_x)_{body}$ = constant $\Rightarrow (p_x)_{initial} = (p_x)_{final}$

$\Rightarrow$ $m\,u\sin\theta = mv\sin\beta \Rightarrow v\sin\beta = u\sin\theta$ ..(ii)

Squaring equations(i) and (ii) and adding, $v^2\cos^2\beta + v^2\sin^2\beta = e^2u^2\cos^2\theta + u^2\sin^2\theta$

$\Rightarrow$ $v^2 = u^2[e^2\cos^2\theta + \sin^2\theta] \quad \Rightarrow v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$

Dividing equation (i) by (ii)

$$\Rightarrow \frac{v\cos\beta}{v\sin\beta} = \frac{eu\cos\theta}{u\sin\theta} \Rightarrow \cot\beta = e\cot\theta \Rightarrow \beta = \cot^{-1}(e\cot\theta)$$

Impulse of the blow = change of momentum of the body

$= \{(mv\sin\beta)\,\hat{i} + (mv\cos\beta)\,\hat{j}\} - \{(mu\sin\theta)\,\hat{i} - (mu\cos\theta)\,\hat{j}\}$

$= (mv\sin\beta - mu\sin\theta)\,\hat{i} + (mv\cos\beta + mu\cos\theta)\,\hat{j}$

Since $v\sin\beta = u\sin\theta \Rightarrow$ Impulse $= m(v\cos\beta + u\cos\theta)\,\hat{j}$

Putting $v\cos\beta = eu\cos\theta$ from eq. (i),

Impulse $= m(1+e)\,u\cos\theta\hat{j}$ $\quad\therefore$ Magnitude of the impulse $= m(1+e)\,u\cos\theta$

Change in Kinetic energy: $\quad \Delta K.E. = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Putting the value of v we obtain

$$\Delta KE = \frac{1}{2}m\left[\left[\sqrt{u\left(\sin^2\theta + e^2\cos^2\theta\right)}\right]^2 - u^2\right] = \frac{1}{2}mu^2[\sin^2\theta + e^2\cos^2\theta - 1]$$

$$= -\frac{1}{2}mu^2[\cos^2\theta - e^2\cos^2\theta] = -\frac{1}{2}(1-e^2)\,mu^2\cos^2\theta$$

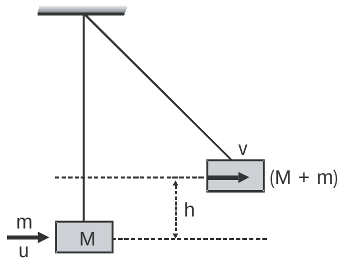
Negative sign indicates the loss of kinetic energy

IMPORTANT POINTS

- Momentum remains conserved in all types of collisions.
- Total energy remains conserved in all types of collisions.
- Only conservative forces works in elastic collisions.
- In inelastic collisions all the forces are not conservative.

Ex. A simple pendulum of length 1m has a wooden bob of mass 1kg. It is struck by a bullet of mass 10^{-2}kg moving with a speed of 2×10^2 m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

Sol. Applying principle of conservation of linear momentum



$$mu = (M + m)v \Rightarrow 10^{-2} \times (2 \times 10^2) = (1 + .01)v \Rightarrow v = \frac{2}{1.01}$$

KE_i of the block with bullet in it, is converted into P.E. as it rises through a height h

$$\frac{1}{2}(M+m)v^2 = (M+m)gh \Rightarrow v^2 = 2gh \Rightarrow h = \frac{v^2}{2g} = \left(\frac{2}{1.01}\right)^2 \times \frac{1}{2\times 9.8} = 0.2\text{ m}$$

Ex. A body falling on the ground from a height of 10m, rebounds to a height 2.5m calculate

(i) The percentage loss in K.E.

(ii) Ratio of the velocities of the body just before and just after the collision.

Sol. Let v_1 and v_2 be the velocity of the body just before and just after the collision

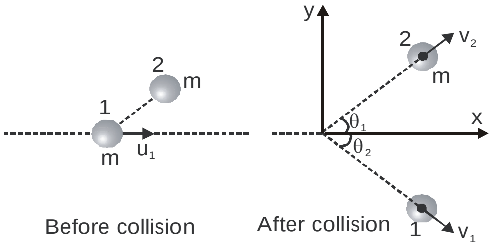
$$KE_1 = \frac{1}{2}mv_1^2 = mgh_1 \text{...(i) and } KE_2 = \frac{1}{2}mv_2^2 = mgh_2 \text{ ...(ii)}$$

$$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4 \Rightarrow \frac{v_1}{v_2} = 2$$

$$\text{Percentage loss in KE} = \frac{mg(h_1 - h_2)}{mgh_1} \times 100 = \frac{10 - 2.5}{10} \times 100 = 75\%$$

Ex. A body strikes obliquely with another identical stationary rest body elastically. Prove that they will move perpendicular to each other after collision.

Sol.



Conservation of linear momentum in x–direction gives

$mu_1 = mv_1\cos\theta_1 + mv_2\cos\theta_2 \quad \Rightarrow \quad u_1 = v_1\cos\theta_1 + v_2\cos\theta_2 \quad$ ... (i)

Conservation of linear momentum in y–direction gives

$0 = mv_1\sin\theta_1 - mv_2\sin\theta_2 \quad \Rightarrow \quad 0 = v_1\sin\theta_1 - v_2\sin\theta_2 \quad$ ...(ii)

Conservation of kinetic energy

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \quad \Rightarrow \quad u_1^2 = v_1^2 + v_2^2 \quad \text{...(iii)}$$

$(i)^2 + (ii)^2$

$\Rightarrow \quad u_1^2 + 0 = v_1^2\cos^2\theta_1 + v_2^2\cos^2\theta_2 + 2v_1v_2\cos\theta_1\cos\theta_2 + v_1^2\sin^2\theta_1 + v_2^2\sin^2\theta_2 - 2v_1v_2\sin\theta_1\sin\theta_2$

$\Rightarrow \quad u_1^2 = v_1^2(\cos^2\theta_1 + \sin^2\theta_1) + v_2^2(\cos^2\theta_2 + \sin^2\theta_2) + 2v_1v_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$\Rightarrow \quad u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2) \quad \{ \because u_1^2 = v_1^2 + v_2^2 \}$

$\Rightarrow \quad \cos(\theta_1 + \theta_2) = 0 \Rightarrow \theta_1 + \theta_2 = 90°$

Ex. A steel ball is dropped on a smooth horizontal plane from certain height h. Assuming coefficient of restitution of impact as e, find the average speed of the ball till it stops.

Sol. Since the ball falls through a height h, just before the first impact its speed v will be given as $v = \sqrt{2gh}$. Let its speed the v_1 just after the first impact.Then, Newton's experimental formula yields,

$$\frac{0 - v_1}{v} = e \Rightarrow v_1 = ev$$

Similarly, its speed just before 2nd impact, $v_1 = ev = e\sqrt{2gh}$

Speed just after n^{th} impact, $v_n = e^n v = e^n\sqrt{2gh}$

The maximum height attained after 1^{st} impact = $h_1 = \frac{v^2}{2g}$ = $(e\sqrt{2gh})^2$ = e^2h. Similarly, the maximum height attained after 2nd impact, $h_2 = e^4h$. Hence, the maximum height attained after n^{th} impact = $e^{2n}h$

The ball experiences infinite impacts till it becomes stationary. $\Rightarrow$ The total distance covered,

$d = h + 2h_1 + 2h_2 + ... d = h + 2e^2h + 2e^4h + ... = h[1+2(e^2 + e^4 + e^6 + ...)]$

$$= \left[1+2\left(\frac{e^2}{1-e^2}\right)\right]h = \left(\frac{1+e^2}{1-e^2}\right)h.$$

The total time taken by the ball till it stops bouncing $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ...$

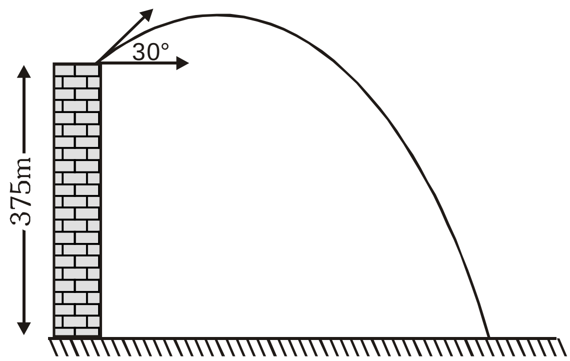
Putting $h_1 = e^2h$, $h_2 = e^4h$, $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2e^2h}{g}} + 2\sqrt{\frac{2e^4h}{g}} + ...$

$$\Rightarrow T = \sqrt{\frac{2h}{g}}\,[1 + 2(e + e^2 + ...)] = \sqrt{\frac{2h}{g}}\left[1+\frac{2e}{1-e}\right] = \frac{1+e}{1-e}\sqrt{\frac{2h}{g}}$$

Therefore, average speed of the ball for its total time of motion, $\vec{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{T}$

Putting the values of d and T, we obtain $\vec{v} = \frac{1+e^2}{(1+e)^2}\sqrt{\frac{gh}{2}}$

Ex. A particle of mass 1 kg is projected from a tower of height 375m with initial velocity 100 ms^{-1} at an angle 30° with the horizontal. Find out its kinetic energy in joule just after collision with ground if collision is inelastic with $e = \frac{1}{2}$ ($g = 10\ ms^{-2}$)



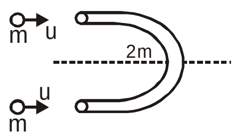
Sol. $v_y^2 = u_y^2 + 2gh \Rightarrow v_y = \sqrt{(50)^2 + 2\times10\times375} = 100\,ms^{-1}$

Horizontal velocity just after collision = $50\sqrt{3}$ ms^{-1}

Vertical velocity just after collision = $100 \times \frac{1}{2}$ = 50 ms^{-1}

Kinetic energy just after collision = $\frac{1}{2}\times1\times\left[\left(50\sqrt{3}\right)^2 + (50)^2\right] = 5000J$

Ex A U shaped tube of mass 2m is placed on a horizontal surface. Two spheres each of diameter d (just less than the inner diameter of tube) and mass m enter into the tube with a velocity u as shown in figure. Taking all collisions to be elastic and all surfaces smooth. Match the following-



Column-I	**Column-II**
(A) The speed of the tube with respect to ground, when spheres are just about to collide inside the tube.	(p) u
(B) The speed of spheres when spheres are just about to collide.	(q) u/2
(C) The speed of the spheres when they comes out the tube.	(r) $\frac{\sqrt{3}}{2}u$
(D) The speed of the tube when spheres comes out the	(s) zero

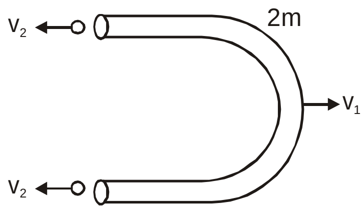
Sol. For (A) From conservation of linear momentum 2mu = (m + m)v $\Rightarrow$ v = $\frac{u}{2}$

For (B) Let v_1 be the velocity of spheres w.r.t. tube when they are just about to collide then by using conservation of kinetic energy $\frac{1}{2}$ (2m)u^2 = $\frac{1}{2}$ (4m) $\left(\frac{u}{2}\right)^2$ + 2 $\frac{1}{2}$ mv_1^2

$\Rightarrow$ $v_1 = \frac{u}{\sqrt{2}}$

$\Rightarrow$ Required speed of spheres = $\sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2} = \sqrt{\frac{u^2}{4} + \frac{u^2}{2}} = \frac{\sqrt{3}u}{2}$

For (C) $2mu = 2mv_1 - 2mv_2$



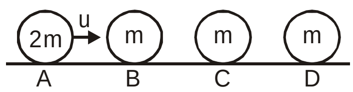
$$2\times\frac{1}{2}mu^2 = 2\times\frac{1}{2}mv_2^2 + \frac{1}{2}(2m)v_1^2$$

$\Rightarrow u^2 = v_1^{\,2} + v_2^{\,2} - 2v_1v_2$ & $u^2 = v_1^{\,2} + v_2^{\,2}$

$\Rightarrow\ v_1v_2 = 0$ but $v_1 \neq 0$ so $v_2 = 0$

For (D) Speed of tube $v_1 = u$

Ex. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B-

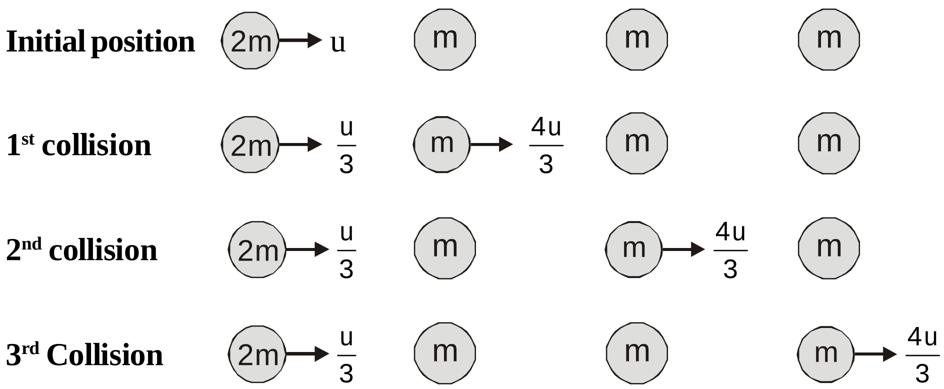


Column-I	**Column-II**
(A) Total impulse of all collisions on A	(p) $\frac{4mu}{9}$
(B) Total impulse of all collisions on B	(q) $\frac{4mu}{27}$
(C) Total impulse of all collision on C	(r) $\frac{4mu}{3}$
(D) Total impulse of all collisions on D	(s) $\frac{52}{27}mu$

Sol. In 1st collision between A & B

$$2mu = 2mv_A + 2mv_B \ \&\ e = 1 = \frac{v_B - v_A}{u} \ \Rightarrow\ v_A = \frac{u}{3},\ v_B = \frac{4u}{3}$$

Situation of all collisions is shown in figure.



4th Collision	$2m \rightarrow \frac{u}{9}$	$m \rightarrow \frac{4u}{9}$	m	$m \rightarrow \frac{4u}{3}$
5th collision	$2m \rightarrow \frac{u}{9}$	m	$m \rightarrow \frac{4u}{9}$	$m \rightarrow \frac{4u}{3}$
6th collision	$2m \rightarrow \frac{u}{27}$	$m \rightarrow \frac{4u}{27}$	$m \rightarrow \frac{4u}{9}$	$m \rightarrow \frac{4u}{3}$

For (A) Total impulse on A $= 2m\left(u - \frac{u}{27}\right) = \frac{52}{27}mu$

For (B) Total impulse on B $= m\left(\frac{4u}{27} - 0\right) = \frac{4}{27}mu$

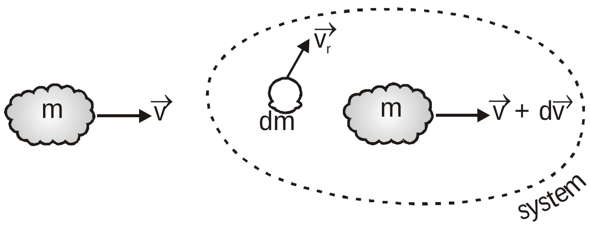
For (C) Total impulse on C $= m\left(\frac{4u}{9} - 0\right) = \frac{4}{9}mu$

For (D) Total impulse on D $= m\left(\frac{4u}{3} - 0\right) = \frac{4}{3}mu$

Variable mass system:

In previous discussion of the conservation of linear momentum, we assume that system's mass remains constant. Now we are consider those system whose mass is variable i.e. those in which mass enters or leaves the system. Suppose at some moment t = t mass of a body is m and its velocity is $\vec{v}$. After some time at t=t + dt its mass becomes (m–dm) and velocity becomes $\vec{v} + d\vec{v}$.

The mass dm is ejected with relative velocity $\vec{v}_r$.



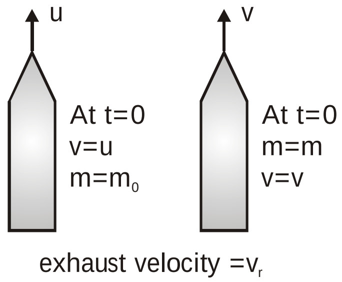
If no forces are acting on the system then the linear momentum of the system will remain conserved.

$$\Rightarrow \qquad \vec{F}_{ex}\,dt = (m - dm)(\vec{v} + d\vec{v}) + dm\,(\vec{v}_r + \vec{v} + d\vec{v}) - m\vec{v}$$

$$\because \qquad F_{ex} = 0 \Rightarrow m d\vec{v} = -\vec{v}_r dm \Rightarrow m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$$

Rocket propulsion :

Thrust force on the rocket = $v_r\left(-\frac{dm}{dt}\right)$



So for motion of rocket $m\frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg$

$\Rightarrow dv = v_r\left(-\frac{dm}{m}\right) - gdt \Rightarrow \int_{u_0}^{v} dv = -v_r\int_{m_0}^{m}\frac{dm}{m} - g\int_0^t dt$

$\Rightarrow v-u = v_r \ell n\left(\frac{m_0}{m}\right) - gt \Rightarrow v = u - gt + v_r \ell n\left(\frac{m_0}{m}\right)$

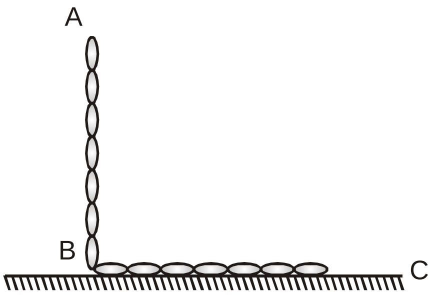
Ex An open topped rail road car of mass M has an initial velocity v_0 along a straight horizontal frictionless track. It suddenly starts raining at time t=0. The rain drops fall vertically with velocity u and add a mass m kg/sec of water. Find the velocity of car after t second (assuming that it is not completely filled with water).

Sol. According to law of conservation of momentum, $Mv_0 = (M + m \times t)\,v$. Where m is the mass of water added per second and v is the velocity of the car after t second. $\therefore\ v = \frac{Mv_0}{M+mt}$

Ex. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the chain on the table when half of its length has fallen on the table. The fallen part does not form heap.

Sol. At given condition force exerted by the chain on the table consists of two parts

(i) Weight of portion BC = $\frac{mg}{2}$



(ii) Thrust force = $v_r\left(-\frac{dm}{dt}\right) = v\left(\frac{m}{\ell}v\right) = \frac{m}{\ell}v^2$

but $v = \sqrt{2g\left(\frac{\ell}{2}\right)} = \sqrt{g\ell}$

$\Rightarrow$ Thrust force = $\frac{m}{\ell}(g\ell) = mg$

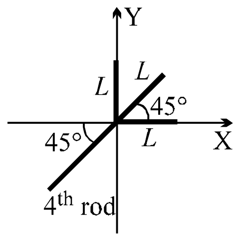
$\therefore$ Net force exerted by falling chain = $\frac{mg}{2} + mg = \frac{3mg}{2}$

EXERCISE (S-1)

1. Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (–1, 5), (2, –3), (–1, –4). Find the coordinates of their centre of mass.

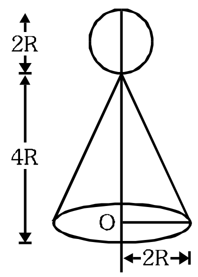
2. A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3kg mass is located at $\vec{r}_1 = (2\hat{i} + 5\hat{j})$ m and the 2 kg mass at $\vec{r}_2 = (4\hat{i} + 2\hat{j})$ m. Find the length of rod and the coordinates of the centre of mass.

3. Three identical uniform rods of the same mass *M* and length *L* are arranged in xy plane as shown in the figure. A fourth uniform rod of mass *3M* has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the center of mass of all the four rods lie at the origin?



4. From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is

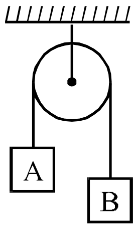
5. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times of the cone compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of h/4 from its base.]



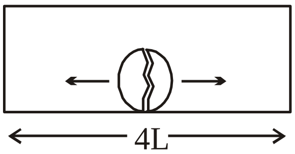
6. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at t = 1 s.

7. Mass centers of a system of three particles of masses 1, 2, 3 kg is at the point (1 m, 2 m, 3 m) and mass center of another group of two particles of masses 2 kg and 3 kg is at point (–1 m, 3 m, –2 m). Where a 5 kg particle should be placed, so that mass center of the system of all these six particles shifts to mass center of the first system?

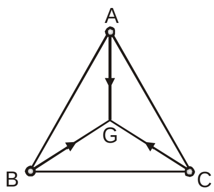
8. In the arrangement shown in the figure, $m_A = 2$ kg and $m_B = 1$ kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



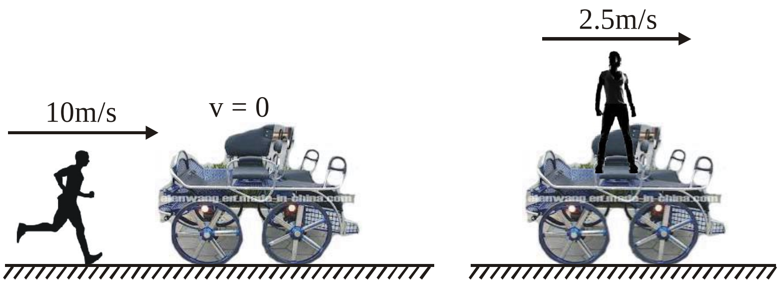
9. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at it's centre. It explodes in two parts of mass m & 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



10. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in fig. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v. What is the velocity of C ?



11. A 50 kg boy runs at a speed of 10 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart ? (Assuming friction is absent between cart and ground)



12. Two cars initially at rest are free to move in the x direction. Car A has mass 4 kg and car B has mass 2 kg. They are tied together, compressing a spring in between them. When the spring holding them together is burned, car A moves off with a speed of 2 m/s.

(i) With what speed does car B leave.

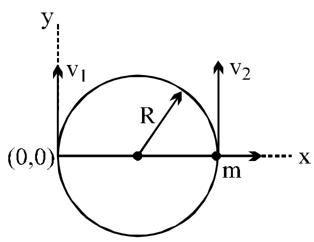
(ii) How much energy was stored in the spring before it was burned.

13. A 24 kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.

(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)

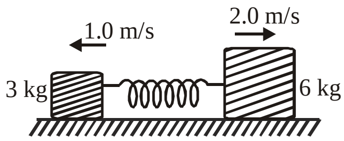
(ii) How much energy was released during the explosion?

14. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time. **[IIT-JEE' 2003]**

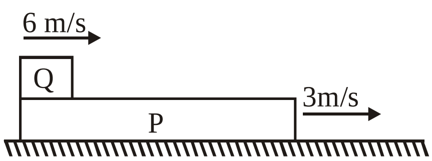


15. A spaceship is moving with constant speed v_0 in gravity free space along +Y-axis suddenly shoots out one third of its part with speed $2v_0$ along + X-axis. Find the speed of the remaining part.

16. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be:-

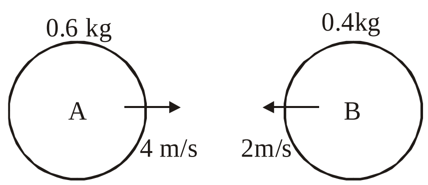


17. A plank P and block Q are arranged as shown on a smooth table top. They are given velocities 3 m/s and 6 m/s respectively. The length of plank is 1m and block is of negligible size. After some time when the block has reached the other end of plank it stops slipping on plank. Find the coefficient of friction between plank P and block Q if mass of plank is double of block).



18. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v, find the magnitude of the impulse acting on the bullet.

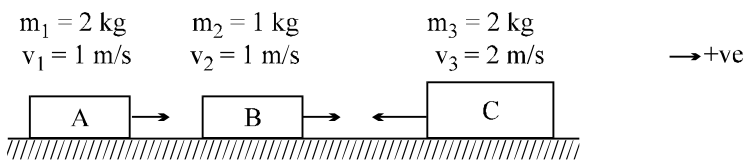
19. The velocities of two steel balls before impact are shown. If after head on impact the velocity of ball B is observed to be 3 m/s to the right, the coefficient of restitution is



20. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.

(i) Find loss of mechanical energy when B & C stick together.

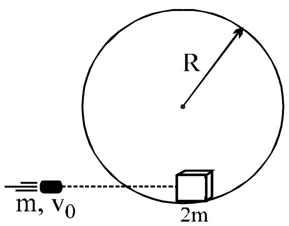
(ii) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).



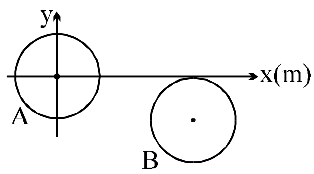
21. A small block of mass 2m initially rests at the bottom of a fixed circular, vertical track, which has a radius of R. The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of m, v_0, R and g.

(i) The speed of the masses immediately after the impact.

(ii) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop



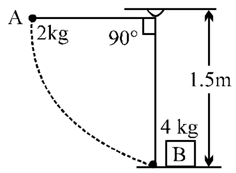
22. Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,–R,R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.



23. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If e = 0.75 between A and B and μ_k = 0.5 between B and the support, determine

(i) the velocity of A just after the impact

(ii) the maximum displacement of B after the impact.

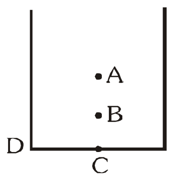


24. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is __________.

EXERCISE (O-1)

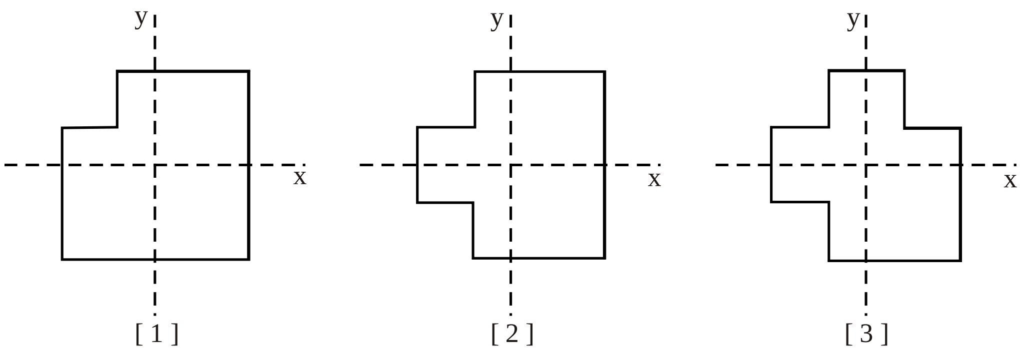
SINGLE CORRECT TYPE QUESTIONS

1. A thick uniform wire is bent into the shape of the letter "U" as shown. Which point indicates the location of the center of mass of this wire? A is the midpoint of the line joining mid points of two parallel sides of 'U' shaped wire.



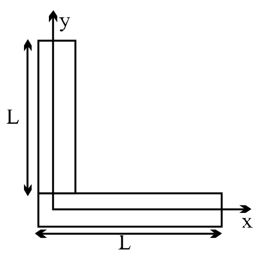
(A) D (B) A (C) B (D) C

2. A machinist starts with three identical square plates but cuts one corner from one of them, two corners from the second, and three corners from the third. Rank the three plates according to the x-coordinate of their centers of mass, from smallest to largest.



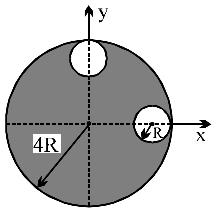
(A) 3, 1, 2 (B) 1, 3, 2 (C) 3, 2, 1 (D) 1 and 3 tie, then 2

3. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates



(A) (L/2, L/2) (B) (2L/3, L/2) (C) (L/3, L/3) (D) (L/3, L/6)

4. From the circular disc of radius 4R two small disc of radius R are cut off. The centre of mass of the new structure will be :



(A) $i\frac{R}{5}+j\frac{R}{5}$ (B) $-i\frac{R}{5}+j\frac{R}{5}$ (C) $\frac{-3R}{14}(\hat{i}+\hat{j})$ (D) None of these

5. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds

(A) continues south at the original speed, but is now located some distance behind the flying birds

(B) continues south, but at 6/7 the original velocity

(C) continues south, but at 1/7 the original velocity

(D) stops with the dead bird

6. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time ?

(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

7. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment along the track.

(A) both C_1 and C_2 will move with respect to the ground.

(B) neither C_1 nor C_2 will move with respect to the ground.

(C) C_1 will move but C_2 will be stationary with respect to the ground.

(D) C_2 will move but C_1 will be stationary with respect to the ground.

8. A non–zero external force acts on a system of particles. The velocity and acceleration of the centre of mass are found to be v_0 and a_C respectively at any instant t. It is possible that

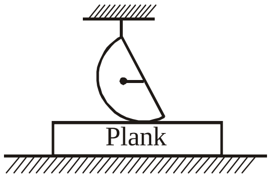
(i) $v_0 = 0, a_C = 0$ (ii) $v_0 \neq 0, a_C = 0$ (iii) $v_0 = 0, a_C \neq 0$ (iv) $v_0 \neq 0, a_C \neq 0$

Then

(A) (iii) and (iv) are true. (B) (i) and (ii) are true.

(C) (i) and (iii) are true. (D) (ii), (iii) and (iv) are true.

9. Lower surface of a plank is rough and lying at rest on a rough horizontal surface. Upper surface of the plank is smooth and has a smooth hemisphere placed over it through a light string as shown in the figure. After the string is burnt, trajectory of centre of mass of the sphere is :-



(A) a circle
(B) an ellipse
(C) a straight line
(D) a parabola

10. Three interacting particles of masses 100 g, 200 g and 400 g each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $\left(10\hat{j}+5\hat{k}\right)$. What is the velocity of the first particle?

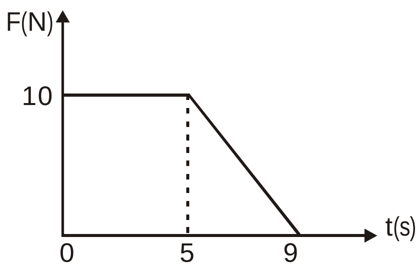
(A) $20\hat{i}+20\hat{j}+70\hat{k}$
(B) $10\hat{i}+20\hat{j}+8\hat{k}$
(C) $30\hat{i}+10\hat{j}+7\hat{k}$
(D) $15\hat{i}+5\hat{j}+60\hat{k}$

11. A system of N particles is free from any external forces.

(i) Which of the following is true for the magnitude of the total momentum of the system?
(A) It must be zero
(B) It could be non–zero, but it must be constant
(C) It could be non–zero, and it might not be constant
(D) The answer depends on the nature of the internal forces in the system

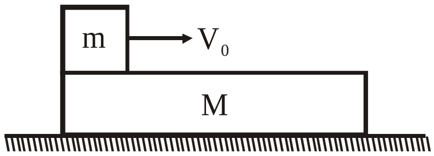
(ii) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system?
(A) It must be zero
(B) It could be non–zero, but it must be constant
(C) It could be non–zero, and it might not be constant
(D) It could be zero, even if the magnitude of the total momentum is not zero

12. A body of mass 4 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is



(A) 280 N–s
(B) 140 N–s
(C) 70 N–s
(D) 210 N–s

13. The coefficient of friction between the block and plank is μ and ground is smooth. The value of μ is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following statement is **incorrect.**



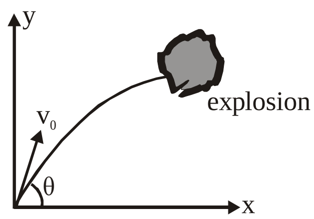
(A) The work done by friction on the block is negative.

(B) The work done by friction on the plank is positive .

(C) The net work done by friction is negative.

(D) Net work done by the friction is zero.

14. A projectile is projected in x-y plane with velocity v_0. At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion had not taken place. Which of the following can not be position vectors of two pieces, when they land on ground.



(A) $\frac{R}{2}\hat{i}, \frac{3R}{2}\hat{i}$ (B) $0\hat{i}, 2R\hat{i}$ (C) $R\hat{i} - R\hat{k}, R\hat{i} + R\hat{k}$ (D) $2R\hat{i} + \frac{R}{2}\hat{k}, R\hat{i} - \frac{R}{2}\hat{k}$

15. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

(A) half the original impulse (B) the original impulse

(C) twice the original impulse (D) four times the original impulse

16. Two balls of same mass are dropped from the same height h, on to the floor. The first ball bounces to a height h/4 ,after the collision & the second ball to a height h/16. The impulse applied by the first & second ball on the floor are I_1 and I_2 respectively. Then

(A) $5I_1 = 6I_2$ (B) $6I_1 = 5I_2$ (C) $I_1 = 2I_2$ (D) $2I_1 = I_2$

17. Ball A of mass 5.0 kilograms moving at 20 m/s collides with ball B of unknown mass moving at 10m/s in the same direction. After the collision, ball A moves at 10 m/s and ball B at 15 m/s, both still in the same direction. What is the mass of ball B?

(A) 6.0 kg (B) 10. kg (C) 2.0 kg (D) 12 kg

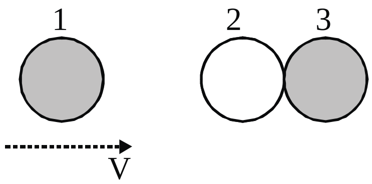
18. A smooth small spherical ball of mass m, moving with velocity u collides head on with another small spherical ball of mass 3m, which was initially at rest. Two-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

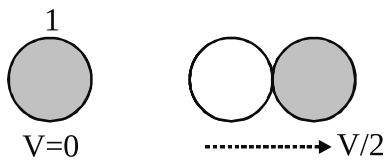
(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) zero

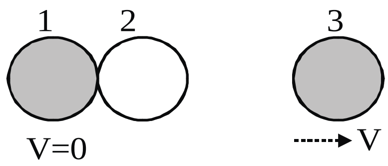
19. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \le 1$).

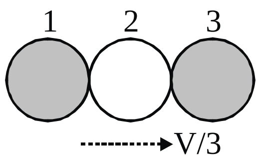
(A) 45° (B) 30° (C) 53° (D) 60°

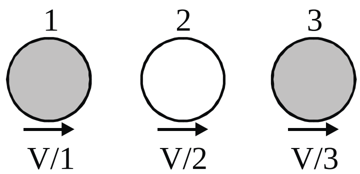
20. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed *V* as shown in figure (i). If the collision is elastic, which of the following is a possible result after collision?

fig. (i)


(A)


(B)


(C)


(D)


21. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collison with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v\sin\theta}{g}$ (B) $\frac{2v\cos\theta}{g}$ (C) $\frac{v\sin 2\theta}{g}$ (D) $\frac{v\cos\theta}{g}$

22. A ball is thrown downwards with initial speed = 6 m/s, from a point at height = 3.2 m above a horizontalfloor. If the ball rebounds back to the same height then coefficient of restitution equals to

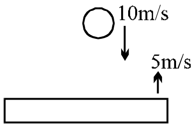
(A) 1/2 (B) 0.75 (C) 0.8 (D) None

23. A particle is projected from a smooth horizontal surface with velocity v at an angle θ from horizontal. Coefficient of restitution between the surface and ball is e. The distance of the point where ball strikes the surface second time from the point of projection is

(A) $\frac{v^2 \sin 2\theta(1+e^2)}{g}$ (B) $\frac{v^2 \sin 2\theta(1+e^4)}{g}$

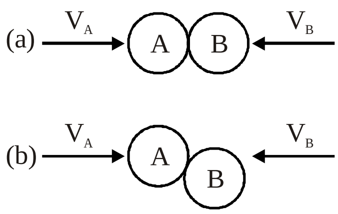
(C) $\frac{v^2 \sin 2\theta(1+e^3)}{g}$ (D) $\frac{v^2 \sin 2\theta(1+e)}{g}$

24. A ball of mass 1kg strikes a heavy platform, elastically, moving upwards with a velocity of 5m/s. The speed of the ball just before the collision is 10m/s downwards. Then the impulse imparted by the platform on the ball is :-



(A) 15 N – s (B) 10 N – s (C) 20 N – s (D) 30 N – s

25. Two bodies, A and B, collide as shown in figures a and b below. Circle the true statement :



(A) They exert equal and opposite forces on each other in (a) but not in (b)
(B) They exert equal and opposite force on each other in (b) but not in (a)
(C) They exert equal and opposite force on each other in both (a) and (b)
(D) The forces are equal and opposite to each other in (a), but only the components of the forces parallel to the velocities are equal in (b).

26. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass at rest. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision :-

(A) $\frac{2v}{\sqrt{3}}$

(B) $\frac{v}{\sqrt{3}}$

(C) $v\sqrt{\frac{2}{3}}$

(D) the situation of the problem is not possible without external impulse

MULTIPLE CORRECT TYPE QUESTIONS

27. Two charges moving under their only own mutual attraction separated by large distance initially. Then choose the correct statement(s)

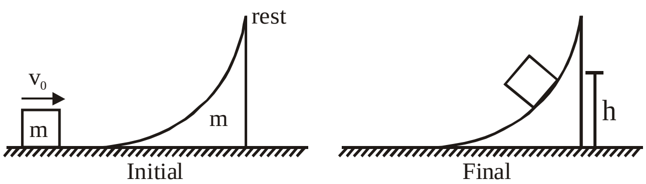
(A) If both are free, mechanical energy is conserved.

(B) If one is fixed and other is free, mechanical energy is conserved.

(C) If one is fixed and other is free, momentum is conserved.

(D) If both are free momentum is conserved.

28. In the arrangement shown, horizontal surface is smooth, but friction is present between the block and the surface of the wedge. Block is given velocity v_0 at t = 0. After achieving height 'h' on the wedge, block comes to rest with respect to wedge at $t = t_0$. Then from t = 0 to $t = t_0$:-



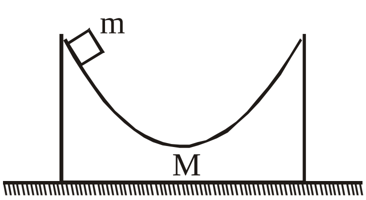
(A) Work done by friction on the block is negative

(B) Work done by friction on the wedge is negative

(C) Work done by block on the wedge is positive

(D) Work done by wedge on the block is positive

29. Figure shows a wedge on which a small block is released from rest. All the surfaces are smooth system comprises of wedge and blocks. Mark the correct statement(s) regarding motion of block on wedge till block attains maximum height on wedge.



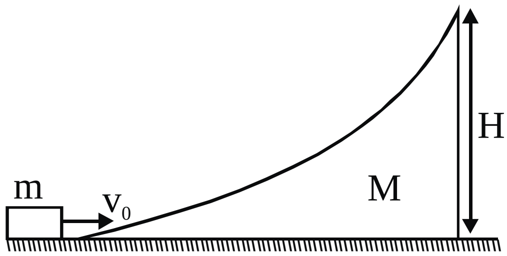
(A) Acceleration of centre of mass of system is initially vertically down then vertically up.

(B) Initially centre of mass moves down and then up.

(C) At the maximum height block and wedge move with common velocity.

(D) Centre of mass of wedge moves towards left then right

30. Figure shows a block of mass m projected with velocity v_0 towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (over come) wedge. Mark the correct option(s)



(A) when block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system

(B) wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.

(C) momentum of wedge and block is conserved at all times

(D) centre of mass of wedge and block remains stationary

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 31 and 32

A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of 53° with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

31. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:

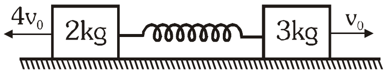
(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 4 : 9

32. The distance between the pieces of the projectile when they reach the ground are:

(A) 240 (B) 360 (C) 120 (D) none

Paragraph for Question 33 to 35

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



33. The maximum energy stored in the spring in the subsequent motion will be

(A) $5v_0^2$ (B) $15v_0^2$ (C) zero (D) $10v_0^2$

34. Maximum speed of 3 kg block in the subsequent motion will be

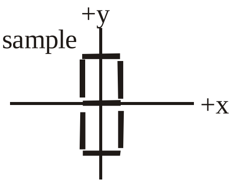
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

35. Maximum speed of 2 kg block in the subsequent motion will be

(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

MATRIX MATCH TYPE QUESTIONS

36. On the left are statements about the location of the center of mass of the objects depicted on the right. The objects on the right are symbols constructed out of sticks of equal length and mass. The location of the center of mass is described using the coordinate system depicted in the sample.



The centre of mass lies at x = 0, y = 0

Column I

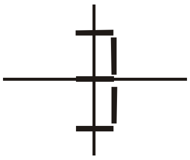
(A) The center of mass is at $x > 0$ and $y = 0$

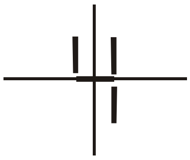
(B) The center of mass is at $x = 0$ and $y > 0$

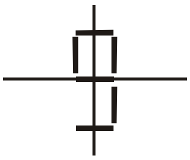
(C) The center of mass is at $x > 0$ and $y > 0$

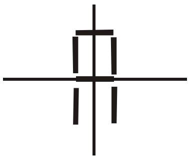
(D) The center of mass is at $x = 0$ and $y = 0$

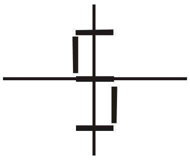
Column II

(P) 

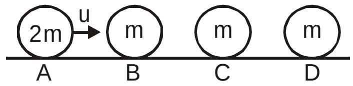
(Q) 

(R) 

(S) 

(T) 

37. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B- (Assume each collision to be elastic)



Column-I		**Column-II**	
(A)	Total impulse of all collisions on A	(p)	$\frac{4mu}{9}$
(B)	Total impulse of all collisions on B	(q)	$\frac{4mu}{27}$
(C)	Total impulse of all collision on C	(r)	$\frac{4mu}{3}$
(D)	Total impulse of all collisions on D	(s)	$\frac{52}{27}mu$

38. In Column-I, 4 situations are depicted and in column-II, 4 possible kinds of collision are listed. Match the situation with type of collision.

Column-I **Before** | **Column-II** **After**

(A) 

(P) Elastic

(B) 

(Q) Perfectly Inelastic

(C) 

(R) Partially elastic

(S) Collision is not possible

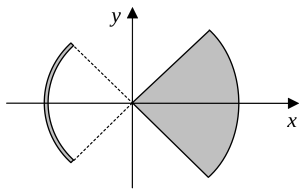
39. A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass 2 m at rest. After collision :

Column I		**Column II**	
(A)	Momentum of first particle	(P)	3/4 p
(B)	Momentum of second particle	(Q)	– K/9
(C)	Kinetic energy of first particle	(R)	– p/3
(D)	Kinetic energy of second particle	(S)	$\frac{8K}{9}$
		(T)	None

EXERCISE (O-2)

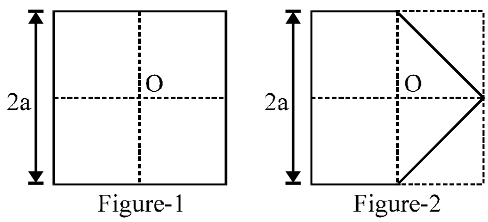
SINGLE CORRECT TYPE QUESTIONS

1. A sector cut from a uniform disk of radius 12 cm and a uniform rod of the same mass bent into shape of an arc are arranged facing each other as shown in the figure. If center of mass of the combination is at the origin, what is the radius of the arc?



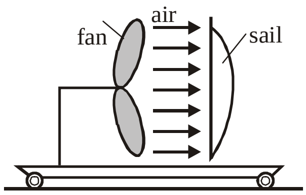
(A) 8 cm (B) 9 cm (C) 12 cm (D) 18 cm

2. A piece of paper (shown in figure-1) is in form of a square. Two corners of this square are folded to make it appear like figure-2. Both corners are put together at centre of square 'O'. If O is taken to be (0, 0), the centre of mass of new system will be at



(A) $\left(\frac{-a}{8}, 0\right)$ (B) $\left(\frac{-a}{6}, 0\right)$ (C) $\left(\frac{a}{12}, 0\right)$ (D) $\left(\frac{-a}{12}, 0\right)$

3. A fan and a sail are mounted vertically on a cart that is initially at rest on a horizontal table as shown in the diagram. When the fan is turned on, an air stream is blown towards the right and is incident on the sail. The cart is free to move with negligible resistance forces. After the fan has been turned on the cart will



(A) move to the right and then to the left (B) remain at rest

(C) move towards the right (D) move towards the left

4. Two identical carts constrained to move on a straight line, on which sit two twins of same mass, are moving with same velocity. At some time snow begins to drop uniformly vertically downward. Ram, sitting on one of the trolleys, throws off the falling snow sideways with respect to himself and in the second cart shyam is asleep. (Assume that friction is absent)

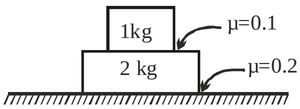
(A) Cart carrying Ram will speed up while cart carrying shyam will slow down

(B) Cart carrying Ram will remain at the same speed while cart carrying shyam will slow down

(C) Cart carrying Ram will speed up while cart carrying shyam will remain at the same speed

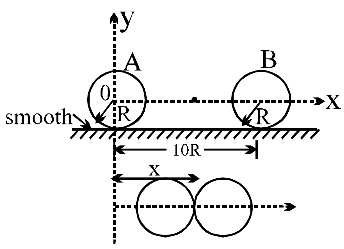
(D) Cart carrying Ram as well as shyam will slow down

5. If both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If a_{cm} be the subsequent acceleration of the centre of mass of the system of blocks then a_{cm} equals



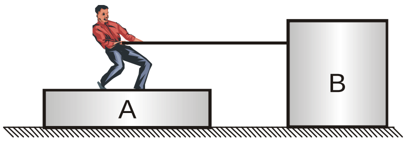
(A) 0 m/s^2 (B) $\frac{5}{3}$ m/s^2 (C) $\frac{7}{3}$ m/s^2 (D) 2 m/s^2

6. Two uniform non conducting balls A & B have identical size having radius R but made of different density material (density of A = 2 density of B). The ball A is +vely charged & ball B is –vely charged. The balls are released on the horizontal smooth surface at the separation 10R as shown in figure. Because of mutual attraction the balls start moving towards each other. They will collide at a point.



(A) $x = \frac{10R}{3}$ (B) $x = \frac{11R}{3}$ (C) x = 5R (D) $x = \frac{7R}{5}$

7. In adjacent figure a boy, on a horizontal platform A, kept on a smooth horizontal surface, holds a rope attached to a box B. Boy pulls the rope with a constant force of 50N. The coefficient of friction between boy and platform is 0.5. (Mass of boy = 80 kg, mass of platform = 120kg and mass of box = 100 kg)



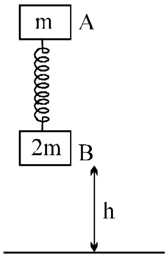
(A) Velocity of platform relative to box after 4 sec. is 2m/s

(B) Velocity of boy relative to platform after 4sec is 2m/s

(C) Friction force between boy and platform is 30N

(D) Friction force between boy and platform is 50N

8. From what minimum height h must the system be released when spring is unstretched so that after perfectly inelastic collision (e = 0) with ground, B may be lifted off the ground (Spring constant = k).

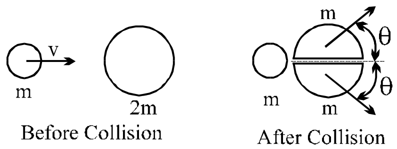


(A) mg/(4k) (B) 4mg/k (C) mg/(2k) (D) none

9. An isolated particle of mass m is moving in horizontal plane (x–y), along the x–axis, at a certain height above the ground. It suddenly explodes into two fragment of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at y = +15 cm. The larger fragment at this instant is at :–

(A) y = –5 cm (B) y = +20 cm (C) y = +5 cm (D) y = –20 cm

10. A particle of mass m is moving along the x-axis with speed v when it collides with a particle of mass 2m initially at rest. After the collisions, the first particle has come to rest, and the second particle has split into two equal-mass pieces that move at equal angles θ > 0 with the x-axis, as shown in the figure. Which of the following statements correctly describes the speeds of the two pieces?



(A) Each piece moves with speed v

(B) One of the pieces moves with speed v, the other moves with speed less than v

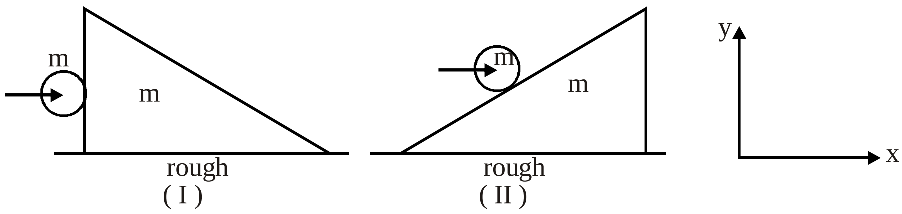
(C) Each piece moves with speed $v/2$

(D) Each piece moves with speed greater than $v/2$

11. A ball of mass m collides horizontally with a stationary wedge on a rough horizontal surface, in the two orientations as shown. Neglect friction between ball and wedge. Two student comment on system of ball and wedge in these situations

Saurav : Momentum of system in x-direction will change by significant amount in both cases.

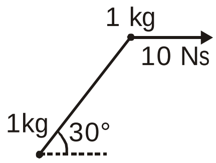
Rahul : There are no impulsive external forces in y-direction in both cases hence the total momentum of system in y-direction can be treated as conserved in both cases.



(A) **Saurav** is wrong and **Rahul** is correct (B) **Saurav** is correct and **Rahul** is wrong

(C) Both are correct (D) Both are wrong

12. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 Ns is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is :-

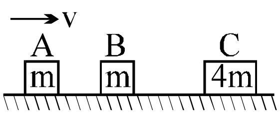


(A) 6Ns (B) $\frac{5}{2}\sqrt{3}$Ns (C) 5 Ns (D) $\frac{5}{\sqrt{3}}$Ns

13. A force exerts an impulse I on a particle changing its speed from u to 2u. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(A) $\frac{3}{2}Iu$ (B) $\frac{1}{2}Iu$ (C) I u (D) 2 I u

14. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass 4m are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



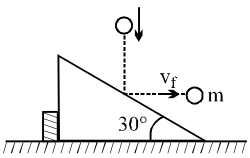
(A) 0.6v to the left (B) 1.4v to the left (C) v to the left (D) 0.4v to the right

15. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured?

(A) inelastic (B) elastic

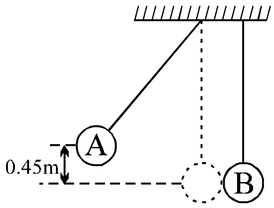
(C) completely inelastic (D) cannot be determined from the information given

16. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If $\angle$ of inclined is 30°, the velocity v_f will be



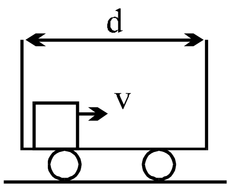
(A) 3 m/s (B) $\sqrt{3}$ m/s (C) $1/\sqrt{3}$ m/s (D) this is not possible

17. Two massless string of length 5 m hang from the ceiling very near to each other as shown in the figure. Two balls A and B of masses 0.25 kg and 0.5 kg are attached to the string. The ball A is released from rest at a height 0.45 m as shown in the figure. The collision between two balls is completely elastic. Immediately after the collision, the kinetic energy of ball B is 1 J. The velocity of ball A just after the collision is



(A) 5 ms^{-1} to the right (B) 5 ms^{-1} to the left (C) 1 ms^{-1} to the right (D) 1 ms^{-1} to the left

18. In a smooth stationary cart of length d, a small block is projected along it's length with velocity v towards front. Coefficient of restitution for each collision is e. The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is

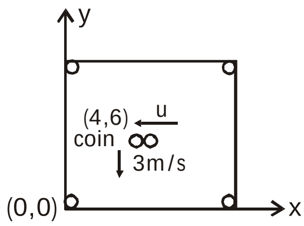


(A) $\dfrac{ed}{(1-e)v}$ (B) $\dfrac{ed}{(1+e)v}$ (C) $\dfrac{d}{e}$ (D) infinite

19. A smooth sphere is moving on a horizontal surface with a velocity vector $(2\hat{i}+2\hat{j})$ m/s immediately before it hit a vertical wall. The wall is parallel to vector $\hat{j}$ and coefficient of restitution between the sphere and the wall is e = $1/2$. The velocity of the sphere after it hits the wall is

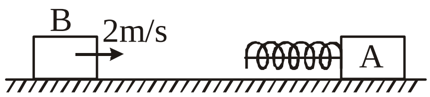
(A) $\hat{i}-\hat{j}$ (B) $-\hat{i}+2\hat{j}$ (C) $-\hat{i}-\hat{j}$ (D) $2\hat{i}-\hat{j}$

20. On a smooth carom board, a coin moving in negative y–direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x–axis. The line joining centres of the coin and the striker just before the collision is parallel to x–axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be



(A) (1.2, 0) (B) (2, 0) (C) (3, 0) (D) None of these

21. Figure shows a block A of mass 5 kg kept at rest on a horizontal smooth surface. A spring (K = 200 N/m) which is compressed by 10 cm and tied with the help of a string to maintain the compression is attached to block A as shown in figure. Block B also of mass 5 kg moving with 2 m/s collides with A, as shown. During the collision the string breaks and after the collision the spring is in its natural state. Assume the bodies to be elastic and let the velocities of A and B be v_1 and v_2 respectively assuming positive direction towards right, after collision. Then



(A) $v_1 + v_2 > 2$

(B) Initial kinetic energy of system = final kinetic energy of system

(C) $v_1^2 + v_2^2 = 4.4\ (m/s)^2$

(D) $v_1 - v_2 = 2$

22. An open water tight railway wagon of mass 5×10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

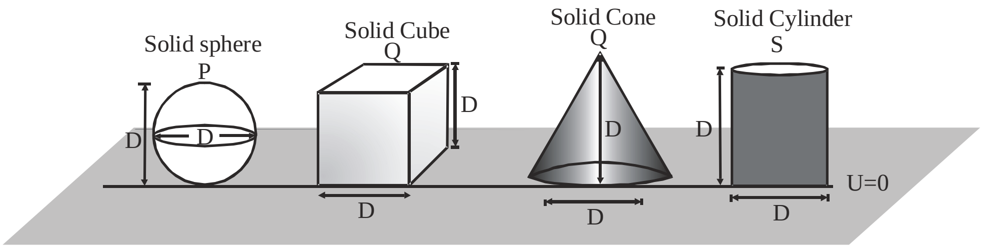
(A) 0.5 m/s (B) 2 m/s (C) 1 m/s (D) 1.5 m/s

23. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude 19.6 m/s^2.
[Exhaust speed of fuel = 980 m/s.]

(A) 240 kg s^{-1} (B) 60 kg s^{-1} (C) 120 kg s^{-1} (D) None

MULTIPLE CORRECT TYPE QUESTIONS

24. Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.

U_P = Potential energy of solid sphere

U_Q = Potential energy of solid cube

U_R = Potential energy of solid cone

U_S = Potential energy of solid cylinder

(A) $U_S > U_P$ (B) $U_Q > U_S$ (C) $U_P > U_Q$ (D) $U_S > U_R$

25. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

(A) All the three pieces share the energy of blast equally

(B) The speed of bigger mass is $\sqrt{2}$ times the speed of either of the smaller mass

(C) The direction of motion of bigger mass makes an angle of 135° with the direction of smaller pieces

(D) The bigger piece carries double the energy of either piece.

26. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.

(A) The minimum kinetic energy of the system is 1 joule.

(B) The maximum elastic potential energy of the system is 2 joule.

(C) Momentum and total kinetic energy of the system are conserved at every instant.

(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

27. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.

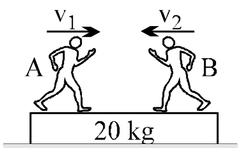
(A) The total momentum of the 'A plus B' system is p before and after the impact, and (p-J) during the impact.

(B) During the impact A gives impulse of magnitude J to B

(C) The coefficient of restitution is $\frac{2J}{p}-1$

(D) The coefficient of restitution is $\frac{J}{p}+1$

28. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds v_1 and v_2 respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec. (Here v_1 and v_2 are given with respect to the plank). Then the possible condition(s) can be



(A) $v_1 = 0$ m/s, $v_2 = 10$ m/s

(B) $v_1 = 5$ m/s, $v_2 = 15$ m/s

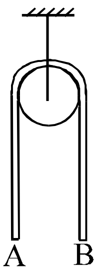
(C) $v_1 = 10$ m/s, $v_2 = 20$ m/s

(D) $v_1 = 2$ m/s, $v_2 = 12$ m/s

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 29 and 30

A uniform chain of length 2L is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible



29. The acceleration of end B when it has been displaced by distance x, is

(A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$ (C) $\frac{x}{2}g$ (D) g

30. The velocity v of the string when it slips out of the pulley (height of pulley from floor > 2L)

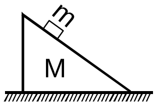
(A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$ (C) $\sqrt{gL}$ (D) none of these

MATRIX MATCH TYPE QUESTION

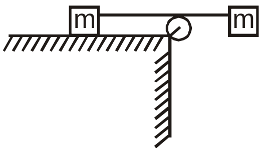
31. In each situation of column–I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column–I with the corresponding results in column–II

Column I

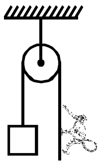
(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system



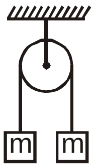
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system



(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey+block system



(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards



Column II

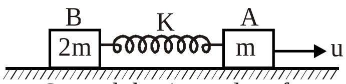
(P) Shifts towards right

(Q) Shifts downwards

(R) Shifts upwards

(S) Does not shift

32. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant.



Smooth horizontal surface

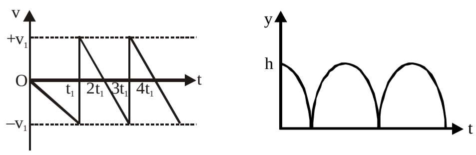
Column-I	**Column-II**
(A) The velocity of block A	(P) can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two block	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is maximum at maximum extension of spring

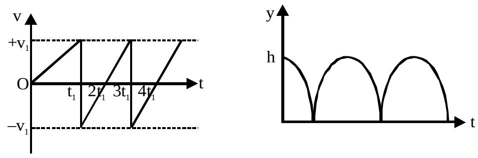
EXERCISE (JM)

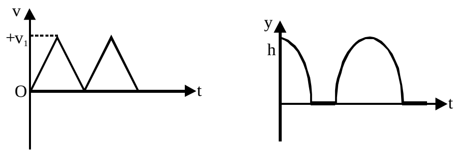
1. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.

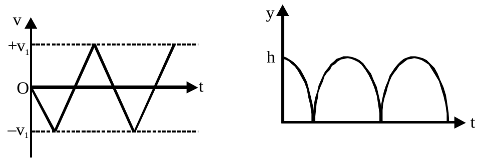
Then the velocity as a function of time and the height as a function of time will be :-

[AIEEE - 2009]

(1) 

(2) 

(3) 

(4) 

Directions : Question number 4 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best discribes the two statements.

2. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE - 2010]

(1) Statement–1 is true, Statement–2 is false

(2) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1

(3) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1

(4) Statement–1 is false, Statement–2 is true

3. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. **[JEE Main-2013]**

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement–I is true, Statement–II is true, Statement–II is a correct explanation of Statement–I.

(2) Statement–I is true, Statement–II is true, Statement–II is a not correct explanation of Statement–I.

(3) Statement–I is true, Statement–II is false.

(4) Statement–I is false, Statement–II is true

4. A particle of mass m moving in the x direction with speed 2υ is hit by another particle of mass 2m moving in the y direction with speed υ. If the collisions perfectly inelastic , the percentage loss in the energy during the collision is close to : **[JEE Main-2015]**

(1) 56 % (2) 62% (3) 44% (4) 50%

5. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :- **[JEE Main-2015]**

(1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

6. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively : **[JEE Main-2018]**

(1) (·28, ·89) (2) (0, 0) (3) (0, 1) (4) (·89, ·28)

7. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is : **[JEE Main-2018]**

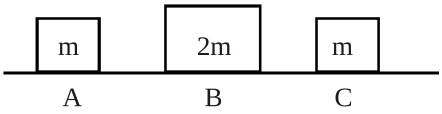
(1) $\sqrt{2}\,v_0$ (2) $\frac{v_0}{2}$ (3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

8. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly : **[JEE Main-2018]**

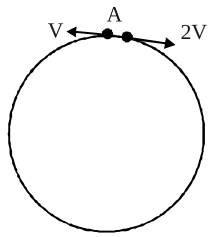
(1) 4.70×10^3 N/m^2 (2) 2.35×10^2 N/m^2 (3) 4.70×10^2 N/m^2 (4) 2.35×10^3 N/m^2

EXERCISE (JA)

1. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C. **[IIT-JEE-2009]**

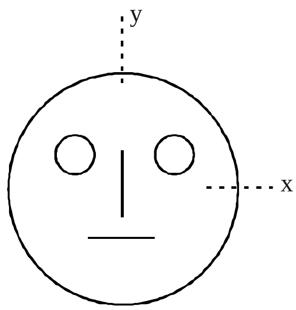


2. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(A) 4 (B) 3 (C) 2 (D) 1 **[IIT-JEE-2009]**

3. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$ **[IIT-JEE-2009]**

4. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. Aftr their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m/s. Which of the following statemet(s) is (are) correct for the system of these two masses ? **[IIT-JEE 2010]**

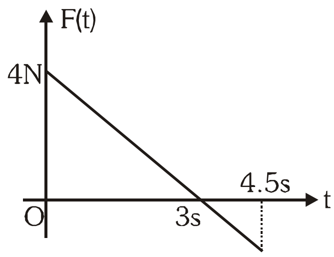
(A) Total momentum of the system is 3 kg m/s.

(B) Momentum of 5 kg mass after collision is 4 kg m/s.

(C) Kinetic energy of the centre of mass is 0.75 J.

(D) Total kinetic energy of the system is 4 J.

5. A block of mass 2 kg is free to move along the x–axis. It is at rest and from t=0 onwards it is subjected to a time–dependent force F(t) in the x–direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is

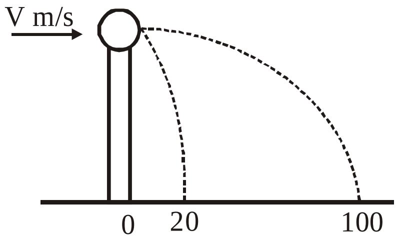


(A) 4.50 J (B) 7.50 J

(C) 5.06 J (D) 14.06 J **[IIT-JEE-2010]**

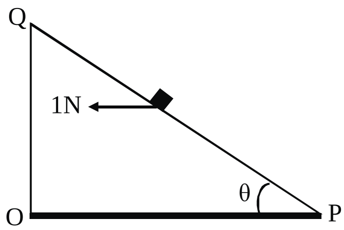
6. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is +

[IIT-JEE 2011]



(A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

7. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2) **[IIT-JEE 2012]**



(A) θ = 45°

(B) θ > 45° and a frictional force acts on the block towards P

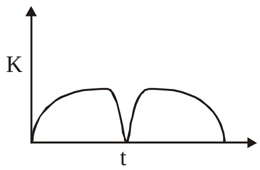
(C) θ > 45° and a frictional force acts on the block towards Q

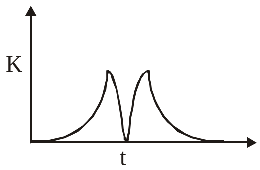
(D) θ < 45° and a frictional force acts on the block towards Q

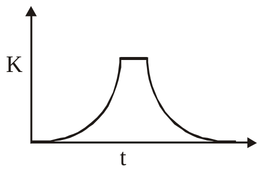
8. A bob of mass m, suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum sped required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is. **[JEE Advanced-2013]**

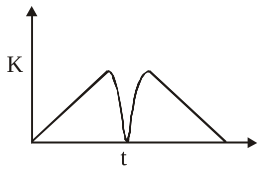
9. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately ? The figures are only illustrative and not to the scale.

[JEE Advanced-2014]

(A) 

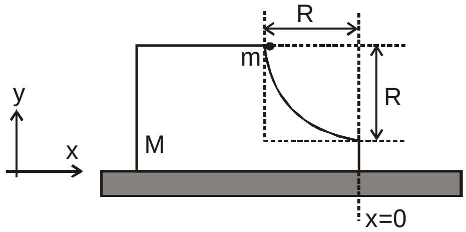
(B) 

(C) 

(D) 

10. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a *co-ordinate system fixed to the table.* A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are **correct**? **[JEE Advanced-2017]**

(A) The x component of displacement of the centre of mass of the block M is : $-\dfrac{mR}{M+m}$



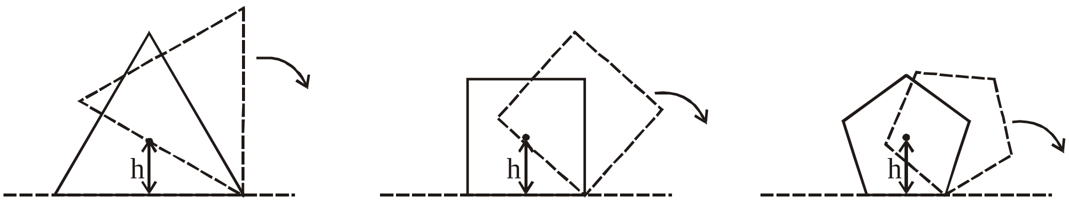
(B) The position of the point mass is : $x=-\sqrt{2}\dfrac{mR}{M+m}$

(C) The velocity of the point mass m is : $v=\sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(D) The velocity of the block M is : $V=-\dfrac{m}{M}\sqrt{2gR}$

11. Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as :

[JEE Advanced-2017]



(A) $\Delta = h\sin^2\left(\dfrac{\pi}{n}\right)$

(B) $\Delta = h\sin\left(\dfrac{2\pi}{n}\right)$

(C) $\Delta = h\left(\dfrac{1}{\cos\left(\dfrac{\pi}{n}\right)}-1\right)$

(D) $\Delta = h\tan^2\left(\dfrac{\pi}{2n}\right)$

12. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4$kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to $t = 0$ so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is.................. Take $e^{-1} = 0.37$?

[JEE Advanced-2018]

13. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[JEE Advanced-2019]**

(1) The rate at which the particle strikes the piston is v/L

(2) After each collision with the piston, the particle speed increases by 2V

(3) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(4) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. (1/7, 23/14)

2. Ans. $\sqrt{13}$ m, $\left(\frac{14}{5}, \frac{19}{5}\right)$

3. Ans. $L(\sqrt{2}+1)/3$

4. Ans. $\frac{a}{3(\pi-1)}$

5. Ans. 4R from O

6. Ans. x = 6m

7. Ans. (3 m, 1 m, 8 m)

$$\frac{6\left(\hat{i}+2\hat{j}+3\hat{k}\right)+5\left(-\hat{i}+3\hat{j}-2\hat{k}\right)+5\vec{r}}{16}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$\vec{r}=\left(3\hat{i}+\hat{j}+8\hat{k}\right)$$

8. Ans. g/9 downwards

9. Ans. $\frac{L}{3}$

10. Ans. $\vec{v}_C=-\vec{v}_B$

11. Ans. 150 kg

12. Ans. (i) 4 m/s, (ii) 24 J

13. Ans. (i) 360 m, (ii) 10800 J

14. Ans. $\vec{P}_{PM}=m\vec{v}_{PM}$

$$=-mv_2\sin\omega t\,\hat{i}+m(v_2\cos\omega t-v_1)\,\hat{j}$$

15. Ans. $\frac{\sqrt{13}}{2}v_0$

16. Ans. 30 cm

17. Ans. 0.3

18. Ans. $m\times\sqrt{u^2-uv+v^2}$

19. Ans. $\frac{7}{18}$

20. Ans. (i) 3 J, (ii) $\frac{12}{5}$ N-s

21. Ans. (i) $v_0/3$, (ii) $3\sqrt{5gR}$.

22. Ans. $(\hat{i}+\sqrt{3}\hat{j})$ m/s, $(3m\hat{i}-\sqrt{3}m\hat{j})$ kg-m/s

23. Ans. (i) $v_A=\sqrt{g/12}$ m/s, (ii) S_{max} = 49/48 m

24. Ans. 1 N

EXERCISE (O-1)

1. Ans. (C)	**2. Ans. (B)**	**3. Ans. (D)**	**4. Ans. (C)**	**5. Ans. (B)**	**6. Ans. (C)**
7. Ans. (C)	**8. Ans. (A)**	**9. Ans. (C)**	**10. Ans. (A)**	**11. Ans. (i) (B) (ii) (C)**	
12. Ans. (C)	**13. Ans. (D)**	**14. Ans. (D)**	**15. Ans. (C)**	**16. Ans. (A)**	**17. Ans. (B)**
18. Ans. (A)	**19. Ans. (B)**	**20. Ans. (B)**	**21. Ans. (B)**	**22. Ans. (C)**	**23. Ans. (D)**

24. Ans. (D) 25. Ans. (C) 26. Ans. (D) 27. Ans. (A, B,D) 28. Ans. (A, C)

29. Ans. (B,C) 30. Ans. (AB) 31. Ans. (A) 32. Ans. (A) 33. Ans. (B)

34. Ans. (C) 35. Ans. (D) 36. Ans. (A)-P; (B)-S; (C)-Q, R; (D)-T

37. Ans. A-(s), B-(q), C-(p), D-(r) 38. Ans. (A)-Q (B)-S (C)-P

39. Ans. (A) R, (B) T, (C) T, (D)S

EXERCISE (O-2)

1. Ans. (A) 2. Ans. (D) 3. Ans. (B) 4. Ans. (D) 5. Ans. (D) 6. Ans. (B)

7. Ans. (C) 8. Ans. (B) 9. Ans. (A) 10. Ans. (D) 11. Ans. (D) 12. Ans. (B)

13. Ans. (B) 14. Ans. (A) 15. Ans. (B) 16. Ans. (B) 17. Ans. (D) 18. Ans. (D)

19. Ans. (B) 20. Ans. (B) 21. Ans. (C) 22. Ans. (C) 23. Ans. (C)

24.Ans. (A,B,D) 25. Ans. (A,C) 26. Ans. (A,B,D)

27. Ans. (B,C) 28. Ans. (A,B,C,D)

29. Ans. (A) 30. Ans. (C) 31. Ans. (A)-Q; (B)-P, Q; (C)-R; (D) S

32. Ans. (A)-Q; (B)-Q; (C)-P,R;, (D)-Q,S

EXERCISE (JM)

1. Ans. (1) 2. Ans. (2) 3. Ans. (4) 4. Ans. (1) 5. Ans. (4) 6. Ans. (4)

7. Ans. (1) 8. Ans. (4)

EXERCISE (JA)

1. Ans. 4m/s 2. Ans. (C) 3. Ans. (A) 4. Ans. (A,C) 5. Ans. (C) 6. Ans. (D)

7. Ans. (A,C) 8. Ans. 5 9. Ans. (B) 10. Ans. (A,C) 11. Ans. (C)

12. Ans. 6.3 [6.29, 6.31] 13. Ans. (2, 3)

CHAPTER 4

KINEMATICS OF ROTATION MOTION

Chapter Contents 04

IMPORTANT NOTES

CHAPTER 4

KINEMATICS OF ROTATION MOTION

Rigid Body

A rigid body is an assemblage of a large number of material particles, which do not change their mutual distances under any circumstance or in other words, they are not deformed under any circumstance.

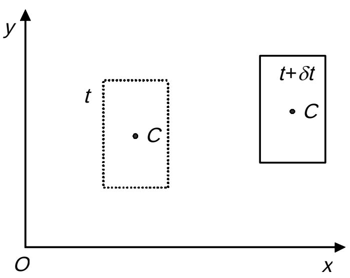
Actual material bodies are never perfectly rigid and are deformed under action of external forces. When these deformations are small enough to be considered during their course of motion, the body is assumed a rigid body. Hence, all solid objects such as stone, ball, vehicles etc are considered as rigid bodies while analyzing their translation as well as rotation motion.

To analyze rotation of a body relative motion between its particles cannot be neglected and size of the body becomes a considerable factor. This is why study of rotation motion is also known as *mechanics of rigid bodies*.

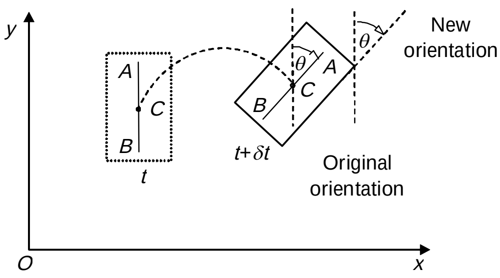
Rotation Motion of a Rigid Body

Any kind of motion of a body is identified by change in position or change in orientation or change in both. If a body changes its orientation during its motion it said to be in rotation motion.

In the following figures, a rectangular plate is shown moving in the *x*-*y* plane. The point *C* is its mass center. In the first case it does not changes orientation, therefore is in pure translation motion. In the second case it changes its orientation by during its motion. It is a combination of translation and rotation motion.



Pure Translation

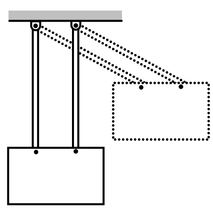


Combination of translation and rotation

Rotation i.e. change in orientation is identified by the angle through which a linear dimension or a straight line drawn on the body turns. In the figure this angle is shown by θ.

Ex. Identify Translation and rotation motion

A rectangular plate is suspended from the ceiling by two parallel rods each pivoted at one end on the plate and at the other end on the ceiling. The plate is given a side-push to oscillate in the vertical plane containing the plate. Identify motion of the plate and the rods.



Sol.

Neither of the linear dimensions of the plate turns during the motion. Therefore, the plate does not change its orientation. Here edges of the body easily fulfill our purpose to measure orientation; therefore, no line is drawn on it.

The plate is in *curvilinear translation motion* and the rods are in rotation motion.

Types of Motions involving Rotation

Motion of body involving rotation can be classified into following three categories.

I Rotation about a fixed axis.

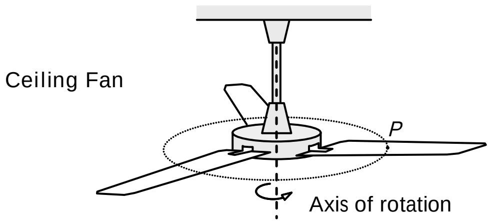
II Rotation about an axis in translation.

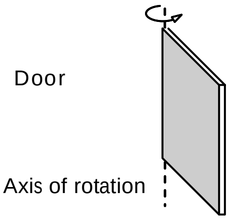
III Rotation about an axis in rotation

Rotation about a fixed axis

Rotation of ceiling fan, potter's wheel, opening and closing of doors and needles of a wall clock etc. come into this category.

When a ceiling fan it rotates, the vertical rod supporting it remains stationary and all the particles on the fan move on circular paths. Circular path of a particle P on one of its blades is shown by dotted circle. Centers of circular paths followed by every particle are on the central line through the rod. This central line is known as *axis of rotation* and is shown by a dashed line. All the particles on the axis of rotation are at rest, therefore the axis is stationary and the fan is in rotation about this fixed axis.

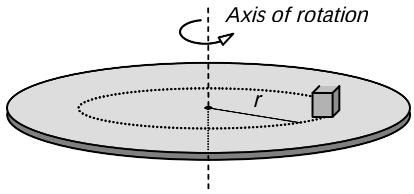




A door rotates about a vertical line that passes through its hinges. This vertical line is the axis of rotation. In the figure, the axis of rotation is shown by dashed line.

Axis of rotation

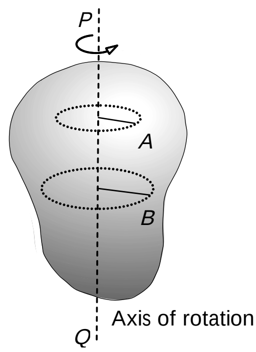
An imaginary line perpendicular to plane of circular paths of particles of a rigid body in rotation and containing the centers of all these circular paths is known as *axis of rotation*.



It is not necessary that the axis of rotation pass through the body. Consider system shown in the figure, where a block is fixed on a rotating disk. The axis of rotation passes through the center of the disk but not through the block.

Important observations

Let us consider a rigid body of arbitrary shape rotating about a fixed axis PQ passing through the body. Two of its particles A and B are shown moving on their circular paths.



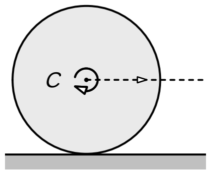
⇨ All of its particles, not on the axis of rotation, move on circular paths with centers on the axis or rotation. All these circular paths are in parallel planes that are perpendicular to the axis of rotation.

⇨ All the particles of the body cover same angular displacement in the same time interval, therefore all of them move with the same angular velocity and angular acceleration.

- ⇨ Particles moving on circular paths of different radii move with different speeds and different magnitudes of linear acceleration. Furthermore, no two particles in the same plane perpendicular to the axis of rotation have same velocity and acceleration vectors.
- ⇨ All the particles on a line parallel to the axis of rotation move circular paths of the same radius therefore have same velocity and acceleration vectors.
- ⇨ Consider two particles in a plane perpendicular to the rotational axis. Every such particle on a rigid body in rotation motion moves on circular path relative to another one. Radius of the circular path equals to the distance between the particles. In addition, angular velocity and angular acceleration equals to that of rotation motion of the body.

Rotation about an axis in translation

Rotation about an axis in translation includes a broad category of motions. Rolling is an example of this kind of motion. A rod lying on table when pushed from its one in its perpendicular direction also executes this kind of motion. To understand more let us discuss few examples.



Consider rolling of wheels of a vehicle, moving on straight level road. Relative to a reference frame, moving with the vehicle wheel appears rotating about its stationary axel. The rotation of the wheel from this frame is rotation about fixed axis. Relative to a reference frame fixed with the ground, the wheel appears rotating about the moving axel, therefore, rolling of a wheel is superposition two simultaneous but distinct motions – rotation about the axel fixed with the vehicle and translation of the axel together with the vehicle.

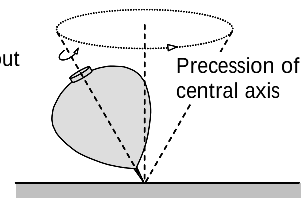
Important observations

- ⇨ Every particle of the body always remains in a plane perpendicular to the rotational axis. Therefore, this kind of motion is also known as *general plane motion*.
- ⇨ Relative to every particle another particle in a plane perpendicular to axis of rotation moves on circular path. Radius of the circular path equals to the distance between the particles and angular velocity and angular acceleration equals to that of rotation motion of the body.
- ⇨ Rotation about axis in translation is superposition of pure rotation about the axis and simultaneous translation motion of the axis.

Rotation about an axis in rotation.

In this kind of motion, the body rotates about an axis that also rotates about some other axis. Analysis of rotation about rotating axes is not in the scope of JEE, therefore we will discus it to have an elementary idea only.

As an example consider a rotating top. The top rotates about its central axis of symmetry and this axis sweeps a cone about a vertical axis. The central axis continuously changes its orientation, therefore is in rotation motion. This type of rotation in which the axis of rotation also rotates and sweeps out a cone is known as *precession*.

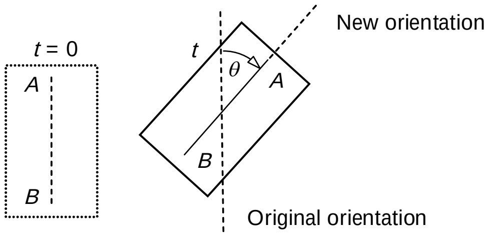


Another example of rotation about axis in rotation is a table-fan swinging while rotating. Table-fan rotates about its horizontal shaft along which axis of rotation passes. When the rotating table-fan swings, its shaft rotates about a vertical axis.

Angular displacement, angular velocity and angular acceleration

Rotation motion is the change in orientation of a rigid body with time.It is measured by turning of a linear dimension or a straight line drawn on the body.

In the figure is shown at two different instants $t = 0$ and t a rectangular plate moving in its own plane. Change in orientation during time t equals to the angle θ through which all the linear dimensions of the plate or a line AB turns.



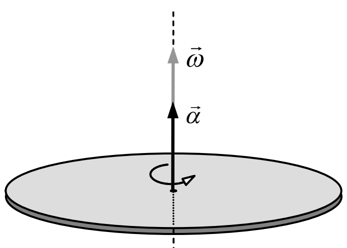
If the angle θ continuously changes with time t, instantaneous angular velocity ω and angular acceleration α for rotation of the body are defined by the following equations.

$$\omega = \frac{d\theta}{dt} \quad [1] \qquad\qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta} \quad [2]$$

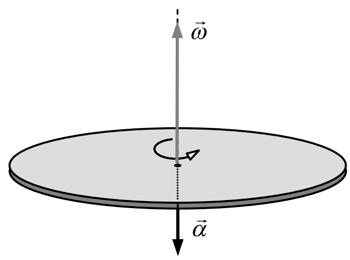
Direction of angular motion quantities

Angular displacement, angular velocity and angular acceleration are known as angular motion quantities. Infinitesimally small angular displacement, instantaneous angular velocity and angular acceleration are vector quantities. Direction of infinitesimally small angular displacement and instantaneous angular velocity is given by the right hand rule. For a disk rotating as shown in the figure, the angular velocity points upwards along the axis of rotation.

The direction of angular acceleration depends, whether angular velocity increases or decreases with time. For increasing angular velocity, the angular acceleration vector points in the direction of angular velocity vector and for decreasing angular velocity, the angular acceleration vector points opposite to the angular velocity vector.



Angular acceleration: Increasing angular speed



Angular acceleration: Decreasing angular speed

In rotation about fixed axis and rotation about axis in translation, the axis of rotation does not rotate and angular velocity and acceleration always point along the axis of rotation. Therefore, in dealing these kinds of motions, the angular motion quantities can used in scalar notations by assigning them positive sign for one direction and negative sign for the opposite direction.

These quantities have similar mathematical relations as position coordinate, velocity, acceleration and time have in rectilinear motion.

⇨ A body rotating with constant angular velocity ω and hence zero angular acceleration is said to be uniform rotation. Angular position θ is given by equation

$$\theta = \theta_o + \omega t \qquad [3]$$

⇨ Thus for a body rotating with uniform angular acceleration α, the angular position θ and angular velocity ω can be expressed by the following equation.

$$\omega = \omega_o + \alpha t \qquad [4]$$

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2 = \theta_o + \tfrac{1}{2}(\omega_o + \omega)t \qquad [5]$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o) \qquad [6]$$

Ex. A disk rotates about a fixed axis. Its angular velocity ω varies with time according to equation $\omega = at + b$. At the instant $t = 0$ its angular velocity is 1.0 rad/s at angular position is 2 rad and at the instant $t = 2$ s, angular velocity is 5.0 rad/s. Determine angular position θ and angular acceleration α when $t = 4$ s.

Sol. The given equation $\omega = at + b$ has form similar to eq.[4], therefore motion is rotation with uniform angular acceleration. Initial angular velocity = $\omega_o = b = 1.0$ rad/s, Angular acceleration $\alpha = a$,

$$\theta = \tfrac{1}{2}at^2 + t + c$$

Since at $t = 0$, $\omega = 1.0$ rad/s, we obtain the constant c.

Initial angular position = $\theta_o = c = 2.0$ rad

Since at $t = 2.0$ s angular velocity is 5.0 rad/s, from given expression of angular velocity, we have

$\omega = at + b \rightarrow$ Substituting $b = 1.0$ rad/s, $t = 2.0$ s and $\omega = 5.0$ rad/s, we have $a = 2.0$ rad/s

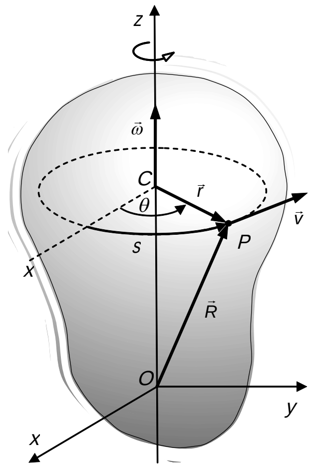
Now we can write expressions for angular position, angular velocity and angular acceleration.

$$\theta = t^2 + t + 2.0 \quad (1) \qquad \omega = 2.0t + 1.0 \quad (2)$$

From the above equations, we can calculate angular position, angular velocity and angular acceleration at $t = 4.0$ s

$\theta_4 = 22$ rad, $\omega_4 = 9.0$ rad/s, $\alpha = 2.0$ rad/s^2 **Ans.**

Kinematics of rotation about fixed axis



In figure is shown a rigid body of arbitrary shape rotating about the *z*-axis. In the selected frame (here the coordinate system) all the three axes are at rest, therefore the *z*-axis that is the axis of rotation is at rest and the body is in fixed axis rotation. All of its particles other than those on the *z*-axis move on circular paths with their centers on the *z*-axis. All these circular paths are parallel to the *x*-*y* plane. In the figure, one of its particles *P* is shown moving with velocity $\vec{v}$ on a circular path of radius *r* and center *C*. Its position vector is $\vec{R}$. It were at the line *Cx* at $t = 0$ and at the position shown at the instant *t*. During time interval *t*, it covers the circular arc of length *s* and its radius vector turns through angle θ.

In an infinitesimally small time interval *dt* let, the particle covers infinitesimally small distance *ds* along its circular path.

$$d\vec{s} = d\vec{\theta} \times \vec{r} = d\vec{\theta} \times \vec{R} \qquad [7]$$

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} \qquad [8]$$

From eq. [7] and [8] we have

$$\vec{v} = \vec{\omega} \times \vec{r} = \vec{\omega} \times \vec{R} \qquad [9]$$

The above equation tells us the relation between the liner and angular velocity. Now we explore relation between the linear and angular accelerations. For the purpose, differentiate the above equation with respect to time.

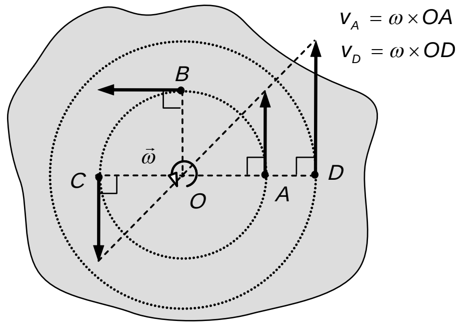
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \qquad [10]$$

The first term on the RHS points along the tangent in the direction of the velocity vector and it is known as tangential acceleration $\vec{a}_T$ same as we have in circular motion. In addition, the second term point towards the center *C*. It is known as centripetal acceleration of normal component $\vec{a}_n$ of acceleration same as in circular motion. Now we have

Tangential acceleration $\qquad \vec{a}_T = \vec{\alpha} \times \vec{r} \qquad$ [11]

Normal acceleration $\qquad \vec{a}_n = \vec{\omega} \times \vec{v} = -\omega^2 \vec{r} \qquad$ [12]

How to Locate Axis of Rotation

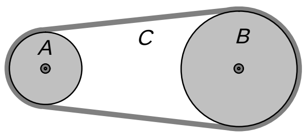


Every particle in a plane perpendicular to the axis of rotation move with different velocities and accelerations, moreover, they all have the same angular velocity and angular acceleration. Such a section of a body in rotation is shown here. The particles *A*, *B* and *C* at equal distance from the axis of rotation move with equal speeds v_A and the particle *D* moves with speed v_D on concentric circular paths. The location of rotational axis can be determined by any of the two graphical techniques.

⇨ Lines perpendicular to velocity vectors and passing through the particles, whose velocity vectors are neither parallel nor antiparallel intersect at the axis of rotation. See pairs of particles *A* and *B*, *B* and *C* and *B* and *D*.

⇨ Lines perpendicular to velocity vectors and passing through the particles, whose velocity vectors are either parallel or antiparallel, coincide and intersect the line joining tips of their velocity vectors at the axis of rotation. Refer pairs of particles *A* and *C*, *A* and *D* and *C* and *D*.

Ex. A belt moves over two pulleys *A* and *B* as shown in the figure. The pulleys are mounted on two fixed horizontal axels. Radii of the pulleys *A* and *B* are 50 cm and 80 cm respectively. Pulley *A* is driven at constant angular acceleration 0.8 rad/s^2 until the pulley *B* acquires an angular velocity of 10 rad/s. The belt does not slide on either of the pulleys.



(a) Find acceleration of a point *C* on the belt and angular acceleration of the pulley *B*.

(b) How long after the pulley *B* achieve angular velocity of 10 rad/s.

Sol. Since the belt does not slide on the pulleys, magnitude of velocity and acceleration of any point on the belt are same as velocity tangential acceleration of any point on periphery of either of the pulleys.

Using the above fact with eq.[11], we have

$\vec{a}_T = \vec{\alpha} \times \vec{r} \rightarrow$ $\qquad a_C = \alpha_A r_A = \alpha_B r_B$

Substituting $r_A = 0.5$ m, $r_B = 0.8$ m and $\alpha = 0.8$ rad/s^2, we have

$a_C = 5$ m/s^2 and $\alpha_B = \dfrac{a_C}{r_B} = \dfrac{\alpha_A r_A}{r_B} = 0.5$ rad/s^2 **Ans.**

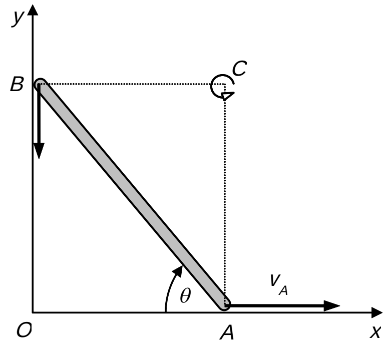
From eq. [4], we have

$\omega = \omega_o + \alpha t \rightarrow$ $\qquad t = \dfrac{\omega_B - \omega_{Bo}}{\alpha_B}$

Substituting $\omega_{Bo} = 0$, $\omega_B = 10$ rad/s and $\alpha_B = 0.5$ rad/s^2,

we have $t = 20$ s **Ans.**

Kinematics of rotation about axis in translation



In this kind of motion, the body rotates about an axis and the axis moves without rotation. Rolling is a very common example of this kind of motion.

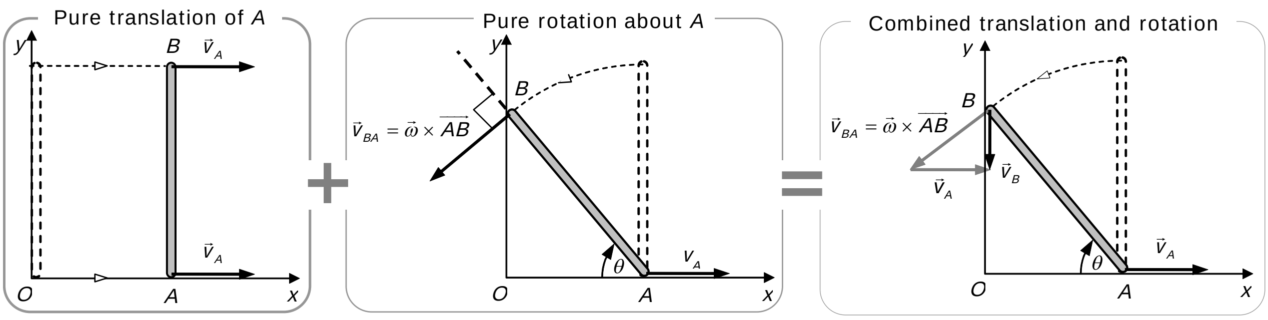
As an example consider a rod whose ends A and B are sliding on the x and y-axis as shown in the figure. Change in its orientation measured by change in angle θ indicates that the rod is in rotation. Perpendiculars drawn to velocity vector of its end points intersect at the axis of rotation, which is continuously changing it position.

Instantaneous Axis of Rotation (IAR)

It is a mathematical line about that a body in combined translation and rotation can be conceived in pure rotation at an instant. It continuously changes its location.

Now we explore how the combined translation and rotational motion of the rod is supper position of translation motion of any of its particle and pure rotation about an axis through that particle.

Consider motion of the rod from beginning when it was parallel to the y-axis. In the following figure translation motion of point A is superimposed with pure rotation about A.



The motion of the rod can be conceived as superposition of translation of point A and simultaneous rotation about an axis through A.

The same experiment can be repeated to demonstrate that motion of the rod can be conceived as superposition of translation of any of its particle and simultaneous rotation about an axis through that particle.

Considering translation of A and rotation about A this fact can be expressed by the following equation.

Combined Motion = Translation of point A + Pure rotation about point A

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad [13]$$

Since point B moves relative A moving on circular path its velocity relative to A is given by the equation

$$\vec{v}_{BA} = \vec{\omega} \times \overrightarrow{AB}.$$

Now we have $\quad \vec{v}_B = \vec{v}_A + \vec{\omega} \times \overrightarrow{AB} \quad$ [14]

The above fact is true for any rigid body in combined translation and rotation motion.

Rotation about an axis in translation of a rigid body can be conceived as well as analyzed as superposition of translation motion of any of its particle and simultaneous rotation about an axis passing through that particle provided that the axis is parallel to the actual one.

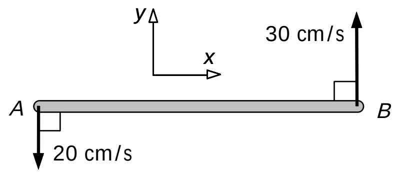
Similar to eq.[13], we can write equation for acceleration.

$$\left.\begin{aligned} \vec{a}_A &= \vec{a}_B + \vec{a}_{BA} \\ \vec{a}_A &= \vec{a}_B + \overbrace{\vec{a}_{BAT} + \vec{a}_{BAn}} \\ \vec{a}_A &= \vec{a}_B + \vec{\alpha} \times \overrightarrow{AB} + -\omega^2 \overrightarrow{AB} \end{aligned}\right\} \quad [15]$$

Ex. A 100 cm rod is moving on a horizontal surface. At an instant, when it is parallel to the *x*-axis its ends *A* and *B* have velocities 30 cm/s and 20 cm/s as shown in the figure.

(a) Find its angular velocity and velocity of its center.

(b) Locate its instantaneous axis of rotation.



Sol.

Let the rod is rotating anticlockwise, therefore its angular velocity is given by $\vec{\omega} = \omega\hat{k}$. Velocity vectors of all the points on the rod and its angular velocity must satisfy the relative motion eq.[14].

(a) Substituting velocities $\vec{v}_A = -20\hat{j}$ cm/s and $\vec{v}_A = 30\hat{j}$ cm/s and angular velocity $\vec{\omega}$ in eq.[14],

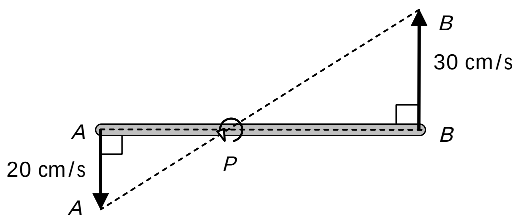
we have $\vec{v}_B = \vec{v}_A + \vec{\omega} \times \overrightarrow{AB} \rightarrow$

$\omega = 0.5$ rad/s **Ans.**

Velocity vector of the center C of the rod also satisfy the following equation.

$\vec{v}_C = \vec{v}_A + \vec{\omega} \times \overrightarrow{AC} \rightarrow \qquad \vec{v}_C = -20\hat{j} + 0.5\hat{k} \times 50\hat{i} = 5.0\hat{j}$ cm/s **Ans.**

(b) Here velocity vectors of the particles A and B are antiparallel, therefore the instantaneous axis of rotation passes through intersection of the common perpendicular to their velocity vectors and a line joining tips of the velocity vectors. The required geometrical construction is shown in the following figure.



Since triangles *AA'P* and *BB'P* are similar and $AB = 100$ cm, we have $AP = 40$ cm.

The instantaneous axis of rotation passes through the point *P*, which is 40 cm from *A*. **Ans.**

Analytical Approach.

The instantaneous center of rotation is at instantaneous rest. Using this fact in eq.[14], we have

$$\vec{v}_P = \vec{v}_A + \vec{\omega} \times \overrightarrow{AP} \rightarrow \qquad \vec{0} = -20\hat{j} + 0.5\hat{k} \times (AP)\hat{j} \Rightarrow AP = 40 \text{ cm} \qquad \textbf{Ans.}$$

Concept of Rotational Inertia (Moment of inertia)

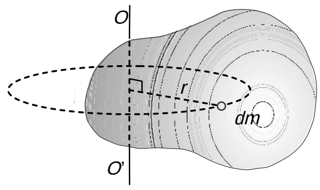
Total mass of a body in translation motion is the measure of its inertia to translation motion. Similarly if a point mass m is rotating about an axis at a distance r from the axis, then term mr^2 provides suitable measure of its inertia to rotation motion. *The inertia to rotation motion is known as rotational inertia or more commonly moment of inertia.*

Moment of inertia of a rigid body

A rigid body is continuous distribution of mass and can be assumed consisting of infinitely large number of point particles. If one of the point particle of infinitely small mass *dm* is at a distance *r* from the axis of rotation *OO'*, the moment of inertia of this point particle is given by

$$dI_o = r^2 dm$$

The moment of inertia of the whole body about the axis *OO'* can now be obtained by integrating term of the above equation over the limits to cover whole of the body.



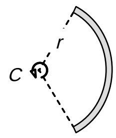
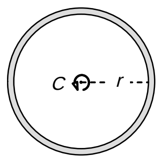
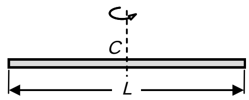
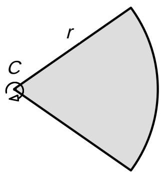
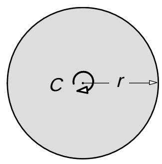
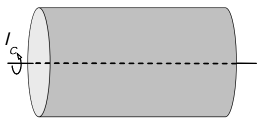
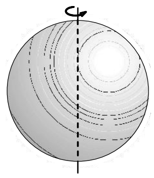
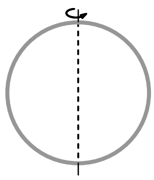
$$I_o = \int dI_o = \int r^2 dm$$

Expression for moment of inertia contains product of two terms. One of them is the mass of the body and the other is a characteristic dimension, which depends on the manner how mass of the body is distributed relative to the axis of rotation. Therefore moment of inertia of a rigid body depends on the mass of the body and distribution of the mass relative to the axis of rotation. Obviously for uniform bodies expression of moment of inertia depends on their shape and location and orientation of the axis of rotation. Based on these facts we can conclude

1. If mass distribution is similar for two bodies about an axis, expressions of their moment of inertia must be of the same form about that axis.

2. If the whole body or any of its portions is shifted parallel to the axis of rotation, moment of inertia remains unchanged.

Moment of Inertia for some commonly used bodies

Body	Axis	Moment of Inertia	
Uniform thin rod bent into shape of an arc of mass m		Passing through center and perpendicular to the plane containing the arc	$I_C = mr^2$
Uniform ring of mass m		Passing through center and perpendicular to the plane containing the arc or the centroidal axis.	$I_C = mr^2$
Straight uniform rod		Passing through center and perpendicular to the rod or the centroidal axis.	$I_C = \frac{mL^2}{12}$
Sector of a uniform disk of mass m		Passing through center and perpendicular to the plane containing the sector.	$I_C = \frac{mr^2}{2}$
Uniform disk of mass m		Passing through center and perpendicular to the plane containing the disk or the centroidal axis.	$I_C = \frac{mr^2}{2}$
Homogeneous cylinder of mass m		Axis of the cylinder or the centroidal axis.	$I_C = \frac{mr^2}{2}$
Homogeneous sphere of mass m		Diameter or the centroidal axis	$I_C = \frac{2}{5}mR^2$
Spherical shell of mass m		Diameter or the centroidal axis	$I = \frac{2}{3}mR^2$

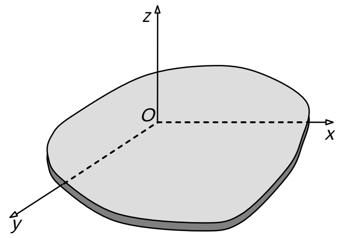
Theorems on Moment of Inertia

Moment of inertias of a rigid body about different axes may be different. There are two theorems known as *theorem of perpendicular axes* and *theorem of parallel axes*, which greatly simplify calculation of moment of inertia about an axis if moment of inertia of a body about another suitable axis is known.

Theorem of Perpendicular Axes

This theorem is applicable for a rigid body that lies entirely within a plane i.e. a laminar body or a rod bent into shape of a plane curve. The moment of inertia I_x, I_y and I_z of the body about the x, y and z-axis can be expressed by the following equations.

$$I_z = I_y + I_x$$

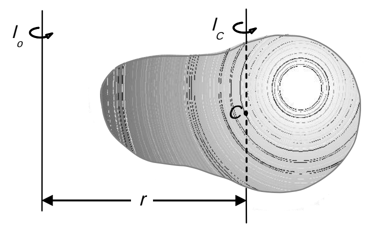


For a planar body, the moment of inertia about an axis perpendicular to the plane of the body is the sum of the moment of inertias about two perpendicular axes in the plane of the object provided that all the three axes are concurrent.

Theorem of Parallel Axes

This theorem also known as Steiner's theorem can be used to determine the moment of inertia of a rigid body about any axis, if the moment of inertia of the body about a parallel axis passing through mass center of the body and perpendicular distance between both the axes is known.

Consider a body of arbitrary shape and mass m shown in the figure. Its moment of inerta I_o and I_C are defined about two parallel axes. The axis about which moment of ienertia I_C is defined passes through the mass center C. Seperation between the axes is r. These two moment of inertias are related by the following equation.



$$I_O = I_C + Mx_C^2$$

The above equation is known as the theorem of parallel axes or Steiner's theorem.

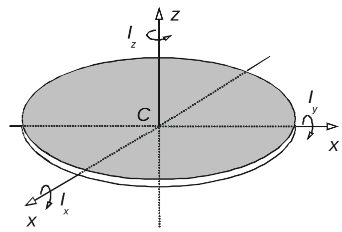
⇨ *The moment of inertia about any axis parallel to an axis through the mass center is given by sum of moment of inertia about the axis through the mass center and product term of mass of the body and square of the distance between the axes.*

⇨ *Among all the parallel axes the moment of inertia of a rigid body about the axis through the mass center is the minimum moment of inertia.*

The second term added to the moment of inertia I_C about the centroidal axis in the above equation can be recognized as the moment of inertia of a particle of mass equal to that of the body and located at its mass center. It again reveals that *the plane motion of a rigid body is superposition of pure rotation about the mass center or centroidal rotation and translation of its mass center*.

Ex. Find moment of inertia of a uniform disk of mass m and radius r about one of its diameter.

Sol. In the adjoining figure a disk is shown with two of its diameter perpendicular to each other. These diameters are along the x and the y-axis of a coordinate system. The x-axis is perpendicular to the plane of the disk and passes through its center is also shown.



Since the disk is symmetric about both the diameters, moment of inertias about both the diameters must be equal. Thus substituting this in the theorem of perpendicular axes, we have

$$I_z = I_y + I_x \rightarrow \qquad I_z = 2I_x = 2I_y$$

Moment of inertia of the disk about the z-axis is $I_z = \frac{1}{2}mr^2$. Substituting it in the above equation, we have

$$I_x = I_y = \tfrac{1}{2}I_z = \tfrac{1}{4}mr^2 \qquad \textbf{Ans.}$$

Radius of Gyration

It is the radial distance from a rotation axis at which the mass of an object could be concentrated without altering the moment of inertia of the body about that axis.

If the mass m of the body were actually concentrated at a distance k from the axis, the moment of inertia about that axis would be mk^2.

$$k = \sqrt{\frac{I}{m}}$$

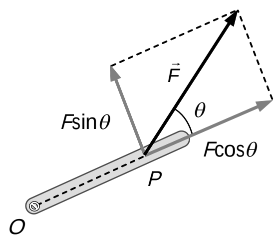
The radius of gyration has dimensions of length and is measured in appropriate units of length such as meters.

DYNAMICS OF RIGID BODY

Torque: Moment of a force

Torque is rotational analogue of force and expresses tendency of a force applied to an object to cause the object to rotate about a given point.

To investigate further let us discuss an experiment. Consider a rod pivoted at the point O. A force $\vec{F}$ is applied on it at the point P. The component $F\cos\theta$ of the force along the rod is counterbalanced by the reaction force of the pivot and cannot contribute in rotating the rod. It is the component $F\sin\theta$ of the force perpendicular to the rod, which is responsible for rotation

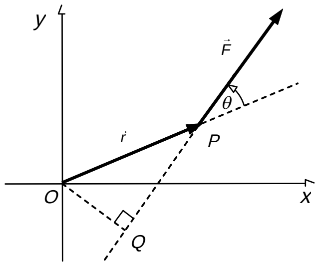


of the rod. Moreover, farther is the point P from O, where the force is applied easier is to rotate the rod. This is why handle on a door is attached as far away as possible from the hinges.

Magnitude of torque of a force is proportional to the product of distance of point of application of the force from the pivot and magnitude of the perpendicular component $F\sin\theta$ of the force. Denoting torque by symbol τ, the distance of point of application of force from the pivot by r, we can write

$$\tau_o \propto rF\sin\theta$$

Since rotation has sense of direction, torque should also be a vector. Its direction is given by right hand rule. Now we can express torque by the cross product of $\vec{r}$ and $\vec{F}$.



$$\vec{\tau}_o = \vec{r} \times \vec{F}$$

Here constant of proportionality has been assumed a dimensionless number unity because a unit of torque has been chosen as product of unit of force and unit of length.

The geometrical construction shown in figure suggests a simple way to calculate torque. The line OQ ($= r\sin\theta$) known as *moment arm*, is the length of perpendicular drawn from O on the line of action of the force. The magnitude of the torque equals to the product of OQ and magnitude of the force $\vec{F}$..

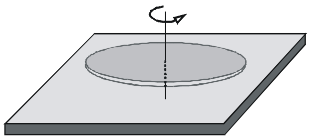
Torque about a Point and Torque about an Axes

We have defined *torque of a force about a point* as the moment of the force about that point. In dealing with rotation about a fixed axis we need to know torque about the axis rotation.

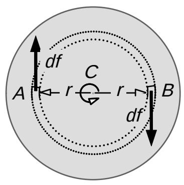
When a body is in plane motion the net torque of all the forces including the forces necessary to restrain rotation of the axis is along the axis of rotation. It is known as torque about the axis. Torque of a force about an axis of rotation equals to the moment of force about the point where plane of motion of the point of application of the force intersects the axis.

In analyzing plane motion we always consider torque about an axis under consideration and in rest of the book by the term torque of force we mean torque about an axis.

Ex. A uniform disk of mass M and radius R rotating about a vertical axis passing through its center and perpendicular to its plane is placed gently on a rough horizontal ground, where coefficient of friction is μ. Calculate torque of the frictional forces.



Sol. When the disk rotates on the ground, kinetic friction acts at every contact point. Since the gravity acts uniformly everywhere and the disk is also uniform, the normal reaction form the ground is uniformly distributed over the entire contact area. Consider two diametrically opposite identical portions A and B of the disk each of mass dm at distance r from the center as shown in the adjacent figure. The normal reaction form the ground on each of these portions equals to their weights and hence frictional forces are $df = \mu dmg$.



Consider a ring of radius r and width dr shown by dashed lines. Net torque $d\tau_C$ of friction force on this ring can easily be expressed by the following equation.

$$d\tau_C = r\mu(\text{mass of the ring})g = r\mu\left(\frac{\text{Mass of the disk}}{\text{Area of the disk}} \times \text{Area of the ring}\right)g = r\mu\left(\frac{2Mrdr}{R^2}\right)g$$

Integrating both sides of the above equation, we have

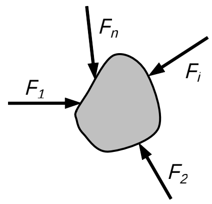
$$\tau_C = \frac{2\mu Mg}{R^2}\int_{r=0}^{R} r^2 dr = \tfrac{2}{3}\mu MgR$$ **Ans.**

Rotational equilibrium

A rigid body is said to be state of rotational equilibrium if its angular acceleration is zero. Therefore a body in rotational equilibrium must either be in rest or rotation with constant angular velocity.

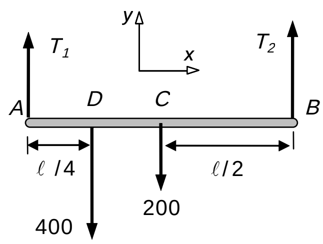
Since scope of JEE syllabus is confined only to rotation about a fixed axis or rotation about an axis in translation motion, the discussion regarding rotational equilibrium is limited here to situations involving only coplanar forces. Under these circumstances the necessary and sufficient condition for rotational equilibrium is

If a rigid body is in rotational equilibrium under the action of several coplanar forces, the resultant torque of all the forces about any axis perpendicular to the plane containing the forces must be zero.



In the figure a body is shown under the action of several external coplanar forces F_1, F_2, …… F_i, and F_n.

$$\sum \vec{\tau}_P = 0$$



Here P is a point in the plane of the forces about which we calculate torque of all the external forces acting on the body. The flexibility available in selection of the point P provides us with advantages that we can select such a point about which torques of several unknown forces will become zero or we can make as many number of equations as desired by selecting several different points. The first situation yields to a simpler equation to be solved and second situation though does not give independent equation, which can be used to determine additional unknowns yet may be used to check the solution.

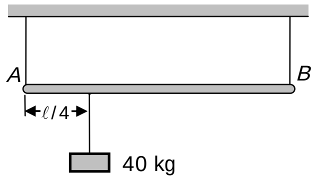
The above condition reveals that a body cannot be in rotational equilibrium under the action of a single force unless the line of action passes through the mass center of the body.

A case of particular interest arises where only three coplanar forces are involved and the body is in rotational equilibrium. It can be shown that *if a body is in rotational equilibrium under the action of three forces, the lines of action of the three forces must be either concurrent or parallel.* This condition provides us with a graphical technique to analyze rotational equilibrium.

Equilibrium of Rigid Bodies

A rigid body is said to be in equilibrium, if it is in translational as well as rotational equilibrium both. To analyze such problems conditions for both the equilibriums must be applied.

Ex. A uniform rod of 20 kg is hanging in horizontal position with the help of two threads. It also supports a 40 kg mass as shown in the figure. Find the tension developed in each thread.



Sol. Free body diagram of the rod is shown in the figure.

Translational equilibrium

$\Sigma F_y = 0 \rightarrow$ $\quad T_1 + T_2 = 400 + 200 = 600\ N \qquad (1)$

Rotational equilibrium: Applying the condition about A, we get T_2.

$\Sigma \vec{\tau}_A = \vec{0} \rightarrow$ $\quad 400(l/4) + 200(l/2) - T_2 l = 0$

$T_2 = 200$ N **Ans.**

Similarly writing torque equation about B, we have

$\Sigma \vec{\tau}_B = \vec{0} \rightarrow$ $\quad T_1 = 400$ N. **Ans.**

Force and Torque equations in General Plane Motion

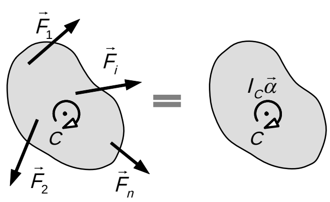
Motion of a rigid body either pure rotation or rotation about axis in translation can be thought and analyzed as superposition of translation of any of its particle and simultaneous rotation about an axis passing through that particle provided that the axis remain parallel to the original one. As far as kinematics in concerned this particle may or may not be the mass center. Whereas in dealing with kinetics, general plane motion is conceived as superposition of translation motion of the mass center and simultaneous centroidal rotation.

To make use the above idea and equations developed in the previous section we classify pure rotation i.e. rotation about fixed axis into two categories and deal with general plane motion as the third category.

Pure centroidal rotation: Rotation about fixed axis through mass centre

In this kind of rotation motion the axis of rotation passes through the mass center and remain fixed in space. Rotation of ceiling fan is a common example of this category. It is a subcategory of pure rotation. The axis of rotation passes through the mass center and remains fixed. In this kind of rotation the mass center of the body does not move.

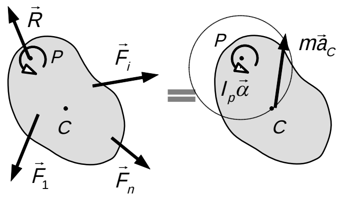
In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis passing through its mass center C is shown. The mass center of the body does not accelerate; therefore we only need to write the torque equation.



$$\Sigma \vec{\tau}_C = I_C \vec{\alpha}$$

Rotation about fixed axis not passing through mass center

In this kind of rotation the axis of rotation remains fixed and does not passes through the mass center. Rotation of door is a common example of this category. Doors are hinged about their edges; therefore their axis of rotation does not pass through the mass center. In this kind of rotation motion the mass center executes circular motion about the axis of rotation.



In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis through point P is shown. It is easy to conceive that as the body rotates its mass center moves on a circular path of radius $\vec{r}_{P/C}$. The mass center of the body is in translation motion with acceleration $\vec{a}_C$ on circular path of radius $r_{P/C}$. To deal with this kind of motion, we have to make use of both the force and the torque equations.

Translation of mass center $\quad \Sigma\vec{F}_i = M\vec{a}_C = M\vec{\alpha}\times\vec{r}_{C/P} - M\omega^2\vec{r}_{C/P}$

Centroidal Rotation $\quad \Sigma\vec{\tau}_C = I_C\vec{\alpha}$

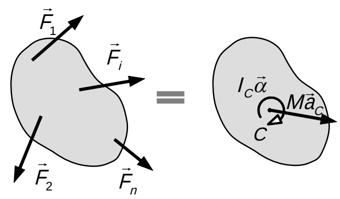
Making use of parallel axis theorem $\quad \left(I_P = Mr^2_{P/C} + I_C\right)$ and $\vec{a}_{C/P} = \vec{\alpha}\times\vec{r}_{C/P} - \omega^2\vec{r}_{C/P}$

we can write the following equation also.

Pure Rotation about P $\quad \Sigma\vec{\tau}_P = I_P\vec{\alpha}$

General Plane Motion: Rotation about axis in translation motion

Rotation of bodies about an axis in translation motion can be dealt with either as superposition of translation of mass center and centroidal rotation or assuming pure rotation about the instantaneous axis of rotation. In the figure is shown the free body diagram and kinetic diagram of a body in general plane motion.



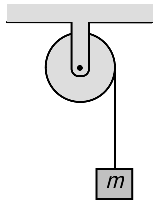
Translation of mass center $\quad \sum_{i=0}^{n}\vec{F}_i = M\vec{a}_C$

Centroidal Rotation $\quad \sum_{i-1}^{n}\vec{\tau}_C = I_C\vec{\alpha}$

This kind of situation can also be dealt with considering it rotation about IAR. It gives sometimes quick solutions, especially when IAR is known and forces if acting at the IAR are not required to be found.

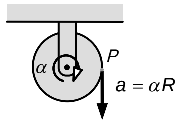
Example

A block of mass m is suspended with the help of a light cord wrapped over a cylindrical pulley of mass M and radius R as shown in the figure. The system is released from rest. Find the angular acceleration of the pulley and the acceleration of the block.



Solution.

After the system is released, the block is in translation motion and the pulley in rotation about an axis passing through its mass center i.e. in pure rotation.



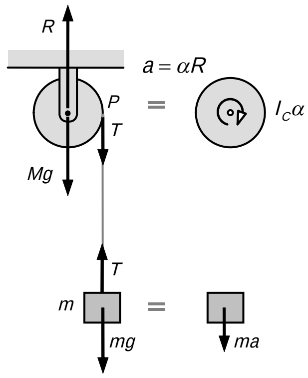
Let the block moves vertically down with acceleration a pulling the cord down and causing the pulley to rotate clockwise. Since the cord is inextensible every point on its vertical portion and point of contact P of the pulley move down with acceleration a as shown in the adjacent figure. It is the tangential acceleration of point P so the angular acceleration α of the pulley rotating in clockwise sense is given by

$$a = \alpha R \qquad (1)$$

The forces acting on the pulley and on the block are shown in their free-body diagrams along with the effective torque $I_c\alpha$ of the pulley and effective force ma of the block. Here T is the tension in the string, R is the reaction by the axel of the pulley, Mg is weight of the pulley and mg is weight of the block.

The pulley is in rotation about fixed axis through its mass center so we use eq. .

$$\sum \vec{\tau}_C = I_C\vec{\alpha} \rightarrow \qquad TR = I_c\alpha$$



After substituting $I_C = \frac{1}{2}MR^2$ and α from eq. (1), we have

$$T = \tfrac{1}{2}Ma \qquad (2)$$

The block is in translation motion, so we use Newton's second law

$$\sum \vec{F} = m\vec{a} \rightarrow \qquad mg - T = ma \qquad (3)$$

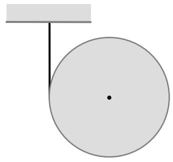
From equation (2) and (3), we have

Acceleration of the block $\qquad a = \dfrac{2mg}{M+2m}$ **Ans.**

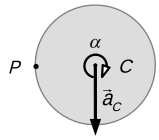
From eq. (1) and the above, we have

$$\alpha = \frac{2mg}{R(M+2m)}$$ **Ans.**

Ex. A thread is wrapped around a uniform disk of radius r and mass m. One end of the thread is attached to a fixed support on the ceiling and the disk is held stationary in vertical plane below the fixed support as shown in the figure. When the disk is set free, it rolls down due to gravity. Find the acceleration of the center of the disk and tension in the thread.



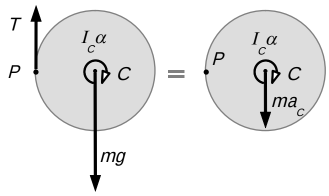
Sol. The point P, where the thread leaves the disk is always at instantaneous rest; therefore the disk can be assumed rolling without slipping with ICR at point P. Acceleration of the mass center $\vec{a}_C$ and angular acceleration of the disk are shown in the adjacent figure. Applying condition for rolling on stationary surface, we have



$$\vec{a}_C = \vec{\alpha} \times \vec{r}_{C/P} \rightarrow \qquad a_C = \alpha r \qquad (1)$$

The disk rolls down on the vertical stationary thread. Its motion can either be analyzed as superposition of translation of the mass center and simultaneous centroidal rotation or a pure rotation about ICR. Since tension, which acts at the ICR is asked; we prefer superposition of translation of the mass center and simultaneous centroidal rotation.

Forces acting on the disk are tension T applied by the thread at point P and weight of the disk. These forces and the effective force ma_C and effective torque $I_C\alpha$ are shown in the adjacent figure.



Applying Newton's second law for translation of mass center, we have

$$\sum \vec{F}_i = M\vec{a}_C \rightarrow \qquad mg - T = ma_C \qquad (2)$$

Applying torque equation for centroidal rotation, we have

$$\sum \vec{\tau}_C = I_C\vec{\alpha} \rightarrow \qquad Tr = I_C\alpha$$

Substituting $\frac{1}{2}mr^2$ for I_C and α from eq. (1), we have

$$T = \tfrac{1}{2}ma_C \qquad (3)$$

From eq. (2) and (3), we have

Acceleration of the mass center $\quad a_C = \frac{2}{3}g \quad$ **Ans.**

Tension in the string $\quad T = \frac{1}{3}mg \quad$ **Ans.**

Energy Methods

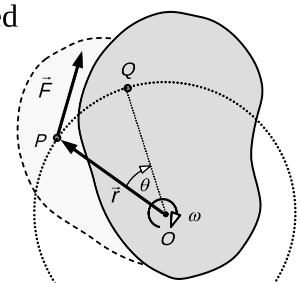
Newton's laws of motion tell us what is happening at an instant, while method of work and energy equips us to analyze what happens when a body moves from one place to other or a system changes its configuration. In this section, we introduce how to use methods of work and energy to analyze motion of rigid bodies.

Concept of Work in rotation motion

Work of a force is defined as the scalar product of the force vector and displacement vector of the point of application of the force. If during the action of a force $\vec{F}$ its point of application moves from position $\vec{r}_1$ to $\vec{r}_2$, the work $W_{1\to2}$ done by the force is expressed by the following equation.

$$W_{1\to2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

Either we can use of this idea to calculate work of a force or its modified version in terms of torque and angular displacement.

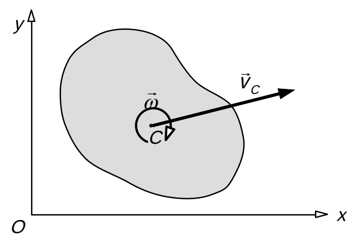


The work done by a torque during a finite rotation of the rigid body from initial value θ_i of the angle θ to final value θ_f, can be obtained by integrating both the sides of the equation given

$$W_{i\to f} = \int_{\theta_i}^{\theta_f} \vec{\tau}_o \cdot d\vec{\theta}$$

Kinetic Energy of a rigid body in rotation motion

A rigid body can be represented as a system of large number of particles, which keep their mutual distances unchanged in all circumstances. Kinetic energy of the whole body must be sum of kinetic energies of all of its particles. In this section we develop expressions for kinetic energy of a rigid body.



Kinetic Energy of a rigid body in plane motion

In the figure is shown a body in plane motion. Its mass center at an instant is moving with velocity $\vec{v}_C$ and rotating with angular velocity $\vec{\omega}$. Both these motions are shown superimposed in the given figure.

Kinetic energy too can be written as sum of kinetic energy $\left(\frac{1}{2}Mv_C^2\right)$ due to translation motion of the mass center and kinetic energy $\left(\frac{1}{2}I_C\omega^2\right)$ due to centroidal rotation.

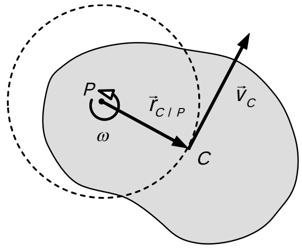
$$K = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_C\omega^2$$

If location of the instantaneous axis of rotation (IAR) is known, making use of the parallel axis theorem we can write kinetic energy by the following equation also.

$$K = \tfrac{1}{2}I_{IAR}\omega^2$$

Kinetic Energy of a rigid body in rotation about fixed axis not passing through the mass centre

In this kind of motion the mass center is in circular motion about the axis of rotation. In the figure is shown a body rotation with angular velocity ω about a fixed axis through pint P and perpendicular to plane of the paper. Mass center moves with speed $v_C = \omega r$. Kinetic energy of the body can now be expressed by the following equation.



$$K = \tfrac{1}{2} M v_C^2 + \tfrac{1}{2} I_C \omega^2$$

Making use of the parallel axis theorem $\left(I_P = M r_{P/C}^2 + I_C\right)$ we can write kinetic energy by the following equation also.

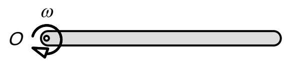
$$K = \tfrac{1}{2} I_P \omega^2$$

Kinetic Energy of a rigid body in pure centroidal rotation

In pure centroidal rotation the mass center remain at rest; therefore kinetic energy due to translation of mass center vanishes.

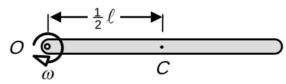
$$K = \tfrac{1}{2} I_C \omega^2$$

Ex. A rod of mass m and length ℓ is pivoted to a fixed support at one of its ends O. It is rotating with constant angular velocity ω. Write expression for its kinetic energy.



Sol. If the point C is the mass center of the rod, from theorem of parallel axes, the moment of inertia I_O of the rod about the fixed axis is

$$I_O = I_C + m(OC)^2 \rightarrow \quad I_O = I_C + \tfrac{1}{4} m\ell^2$$



Substituting $\frac{1}{12} m\ell^2$ for I_C, we have

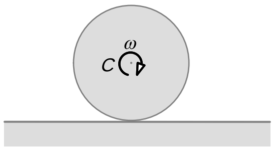
$$I_O = \tfrac{1}{3} m\ell^2$$

Kinetic energy of the rod equals to kinetic energy due to rotation about the fixed axis.

$K = \tfrac{1}{2} I_o \omega^2 \rightarrow$ Using above expression for I_O, we have

$$K = \tfrac{1}{6} m\ell^2 \omega^2$$ **Ans.**

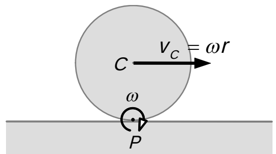
Ex. A uniform rigid body of mass m and round section of radius r is rolling on horizontal ground with angular velocity ω. Its radius of gyration about the centroidal axis is k.



(a) Write expression of its kinetic energy.

(b) Also express the kinetic energy as sum of kinetic energy due to translation of mass center and kinetic energy due to simultaneous centroidal rotation.

Sol. *(a)* The point of contact with ground of a body rolling on the ground is its ICR. Let the point P is the ICR as shown in the adjacent figure. The geometrical center C of a uniform body and the mass center coincide. Therefore moment of inertia I_P of the body about the ICR can be written by using the theorem of parallel axes.



$$I_P = I_C + m(PC)^2 \rightarrow \qquad I_P = I_C + mr^2$$

Substituting $I_C = mk^2$, we have

$$I_P = m\left(k^2 + r^2\right) \qquad (1)$$

Kinetic energy of a rigid body equals to kinetic energy due to rotation about the ICR.

$K = \frac{1}{2} I_P \omega^2 \rightarrow$ Substituting I_P from eq. (1), we have

$$K = \frac{1}{2} m\left(k^2 + r^2\right)\omega^2 \qquad \textbf{Ans.}$$

(b) Kinetic energy of the body also equals to sum of kinetic energy due to translation of its mass center and kinetic energy due to simultaneous centroidal rotation.

$K = \frac{1}{2} m v_C^2 + \frac{1}{2} I_C \omega^2 \rightarrow$Substituting condition for rolling $v_C = \omega r$ and $I_C = mk^2$, we have

$$K = \frac{1}{2} m(\omega r)^2 + \frac{1}{2} mk^2\omega^2 = \frac{1}{2} m\left(r^2 + k^2\right)\omega^2 \qquad \textbf{Ans.}$$

Rolling as rotation about an axis in translation

If the point of contact of the of the rolling body does not slide it is known as *rolling without slipping* or *pure rolling* or simply *rolling* and if the point of contact slides it is known as *rolling with slipping*.

All kind of rolling motion is examples of rotation abut an axis in translation.

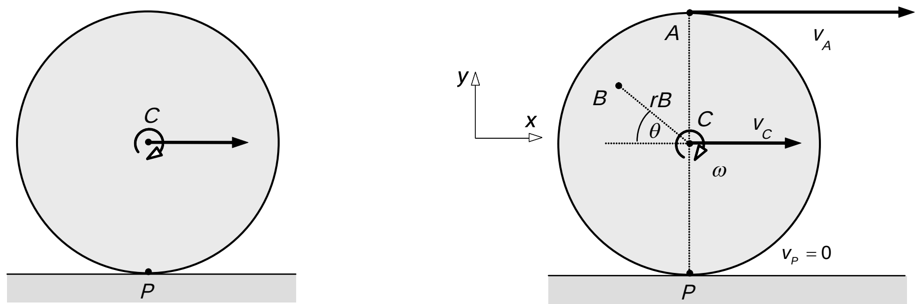
Rolling without slipping on stationary surface.

We first discuss velocity relations and thereafter accelerations relations of two points of a body of round section rolling on a stationary surface. For the purpose, we can use any of the following methods.

I	Analytical Method:	By using relative motion equations.
II	Superposition Method:	By superimposing translation of a point and pure rotation about that point.

Velocity relations by Analytical Method

Its point of contact P does not slide on the surface, therefore velocity of the point of contact relative to the surface is zero. In the next figure, velocity vectors of its center C and top point A are shown.



Velocity of the center C can be obtained with the help of relative motion equation.

$$\vec{v}_C = \vec{v}_P + \vec{\omega} \times \overrightarrow{PC} \rightarrow \quad \vec{v}_C = \vec{0} + \left(-\omega\hat{k}\right) \times R\hat{j}$$

$$\vec{v}_C = \omega R\hat{i} \qquad [16]$$

The above equation is used as condition of rolling without slipping on stationary surface.

Velocity of the top point A can be obtained by relative motion equation.

$$\vec{v}_A = \vec{v}_P + \vec{\omega} \times \overrightarrow{PA} \rightarrow \quad \vec{v}_C = \vec{0} + \left(-\omega\hat{k}\right) \times \left(2R\hat{j}\right)$$

$$\vec{v}_A = 2\omega R\hat{i} = 2\vec{v}_C \qquad [17]$$

Once velocity of the center is obtained, we can use relative motion between A and C as well.

$$\vec{v}_A = \vec{v}_C + \vec{\omega} \times \overrightarrow{CA} \rightarrow \quad \vec{v}_C = \omega R\hat{i} + \left(-\omega\hat{k}\right) \times \left(2R\hat{j}\right)$$

$$\vec{v}_A = 2\omega R\hat{i} = 2\vec{v}_C \qquad [18]$$

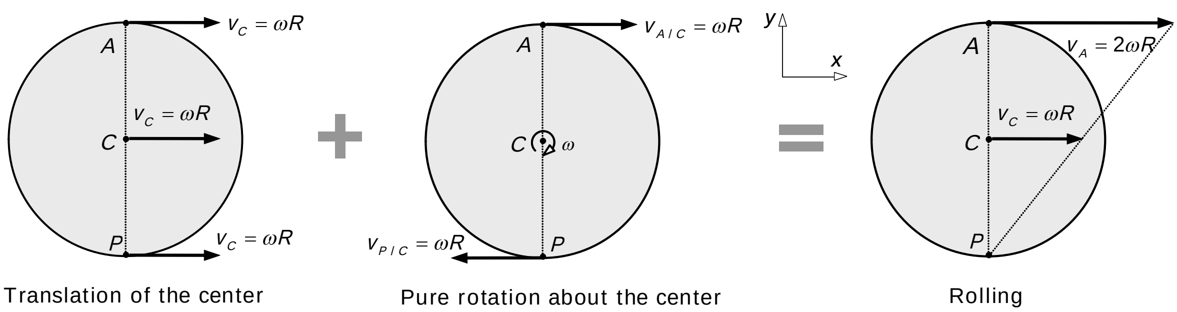
In similar fashion, velocity vector of an arbitrarily chosen point B.

$$\vec{v}_B = \vec{v}_C + \vec{\omega} \times \overrightarrow{CB} \rightarrow \quad \vec{v}_B = v_C\hat{i} + \left(-\omega\hat{k}\right) \times \left(-r\cos\theta\hat{i} + r\sin\theta\hat{j}\right)$$

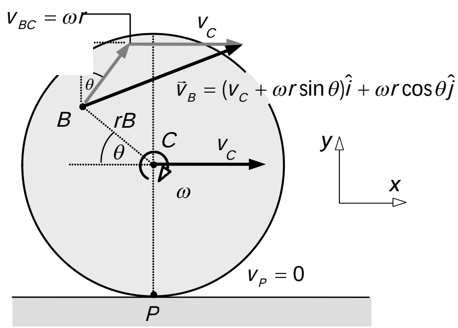
$$\vec{v}_B = \left(v_C + \omega r\sin\theta\right)\hat{i} + \omega r\cos\theta\hat{j} \qquad [19]$$

Velocity relations by Superposition Method

Now we will see that the above velocity relation can also be obtained by assuming rolling of the wheel as superposition of translation of its center and simultaneous rotation about the center.

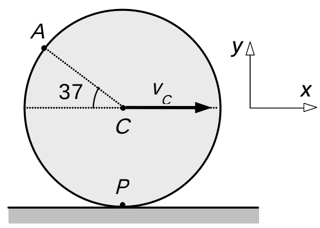


Velocity of an arbitrary point B as superposition of translation the center and rotation about the center.



Ex. A cylinder of radius 5 m rolls on a horizontal surface. Velocity of its center is 25 m/s. Find its angular velocity and velocity of the point A.

Sol. In rolling the angular velocity $\vec{\omega}$ and velocity of the center of a round section body satisfy condition described in the relative motion eq.[14]. So we have



$$\vec{v}_C = \vec{\omega} \times \vec{r}_{C/P} \to 25\hat{i} = \omega\hat{k} \times 5\hat{j} \Rightarrow \vec{\omega} = -5\hat{k} \text{ rad/s}$$

Angular velocity vector points in the negative z-axis so the cylinder rotates in clockwise sense.

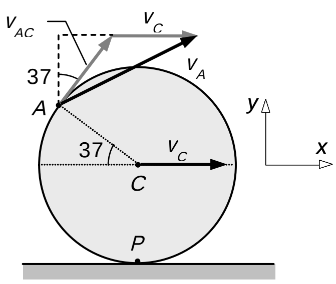
Velocity of the point A can be calculated by either analytical method, superposition method.

Analytical Method

$$\vec{v}_A = \vec{v}_C + \vec{\omega} \times \overrightarrow{AC} \to \quad \vec{v}_A = 25\hat{i} + \left(-5\hat{k}\right) \times \left(-5\cos 37°\hat{i} + 5\sin 37°\hat{j}\right)$$

$$\vec{v}_A = \left(40\hat{i} + 20\hat{j}\right) \text{ m/s}$$

Superposition Method

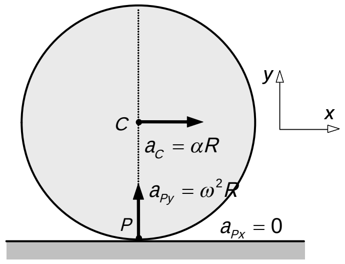


In rolling $v_C = v_{AC} = \omega R = 25 m/s$. The superposition i.e. vector addition of the terms of equation $\vec{v}_A = \vec{v}_C + \vec{v}_{AC}$ are shown in the following figure. Resolving $v_{AC} = \omega R = 25 m/s$ into its Cartesian components and adding to $\vec{v}_C$, we obtain

$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C} \to \quad \vec{v}_A = 25\hat{i} + 15\hat{i} + 20\hat{j} = \left(40\hat{i} + 20\hat{j}\right) \text{ m/s}$$

Acceleration relations by Analytical Method

The point of contact P does not slide on the surface, therefore component of its acceleration parallel to the surface must be zero. However, it has an acceleration component towards the center. The center always moves parallel to the horizontal surface and does not changes direction of its velocity; therefore, its acceleration can only be parallel to the surface.



Relation between acceleration of acceleration vector of the center C and point of contact P can be obtained with the help of relative motion [15] equation together with the above fact.

$$\vec{a}_C = \vec{a}_P + \vec{\alpha} \times \overrightarrow{PC} - \omega^2 \overrightarrow{PC} \rightarrow \qquad a_C\hat{i} = a_{Py}\hat{j} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j} = a_{Py}\hat{j} + \alpha R\hat{i} - \omega^2 R\hat{j}$$

Equating coefficients of x and y-components on both the sides of the above equation, we have

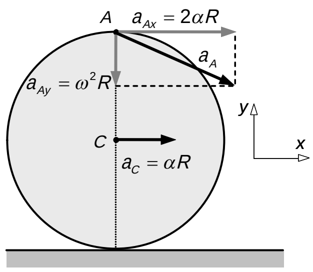
$$\vec{a}_C = \alpha R\hat{i} \qquad [20]$$

$$\vec{a}_P = \omega^2 R\hat{j} \qquad [21]$$

The eq. [20] *is used as condition for rolling without slipping together with eq.* [16]

In the given figure, acceleration vectors the point of contact; center and the top point are shown. Now we will see how these accelerations can be calculated by using relative motion equation.

Once velocity of the center is obtained, we can use relative motion between A and C as well. Now we calculate acceleration of the top point A.



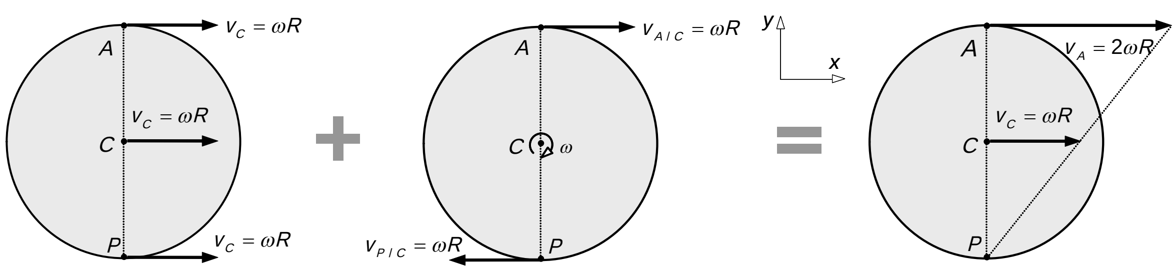
$$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \overrightarrow{CA} - \omega^2 \overrightarrow{CA} \rightarrow \qquad \vec{a}_A = \alpha R\hat{i} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j}$$

$$\vec{a}_A = 2\alpha R\hat{i} - \omega^2 R\hat{j} \qquad [22]$$

Acceleration vector of point A and its components are shown in the given figure.

Acceleration relations by Superposition Method

Now we see how acceleration relations are expressed for a rolling wheel by assuming its rolling as superposition of its translation with the velocity of center and simultaneous rotation about the centre.

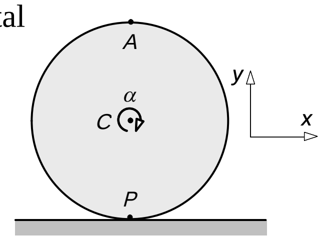


Pure Translation of the center | Pure rotation about the center | Rolling

Ex. A body of round section of radius 10 cm starts rolling on a horizontal stationary surface with uniform angular acceleration 2 rad/s^2.

(a) Find initial acceleration of the center C and top point A.

(b) Find expression for acceleration of the top point A as function of time.



Sol. Initially when the body starts, it has no angular velocity; therefore, the last term in relative motion equation [15] for acceleration vanishes and for a pair of two points A and B the equation reduces to

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \overrightarrow{BA}$$

The angular acceleration vector is $\vec{\alpha} = -2\hat{k}$ rad/s^2.

(a) Acceleration of the center C is obtained by using condition for rolling without slipping.

$\vec{a}_C = \vec{\alpha} \times \overrightarrow{PC} \rightarrow$ $\quad$ $\vec{a}_C = -2\hat{k} \times 10\hat{j} = 20\hat{i}$ cm/s^2 **Ans.**

Acceleration of the point A can be obtained either by analytical method, superposition method or by use of ICR. These methods for calculation of acceleration of the top point are already described; therefore, we use the result.

$\vec{a}_A = 2\alpha R\hat{i} \rightarrow$ $\quad$ $\vec{a}_A = 40\hat{i}$ cm/s^2 **Ans.**

(b) Initially at the instant $t = 0$, when the body starts, its angular velocity is zero. At latter time it acquires angular velocity $\vec{\omega}$, therefore acceleration of any point on the body, other than its center, has an additional component of acceleration. This additional component is accounted for by the last term in the relative motion equation [15].

Angular velocity acquired by the body at time t is obtained by eq.[4] used for a body rotating with constant angular acceleration.

$\vec{\omega} = \vec{\omega}_o + \vec{\alpha}t \rightarrow$ $\quad$ Substituting $\omega_o = 0$, we have

$$\vec{\omega} = -2t\hat{k}$$

Analytical Method

Using the relative motion equation for the pair of points C and A, we have

$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \overrightarrow{CA} - \omega^2 \overrightarrow{CA} \rightarrow$ $\quad$ $\vec{a}_A = \alpha R\hat{i} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j} = 2\alpha R\hat{i} - \omega^2 R\hat{j}$

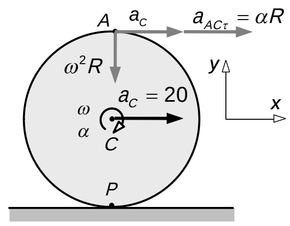
Substituting the known values

$\vec{\alpha} = -2\hat{k}$ rad/s^2, $\vec{\omega} = -2t\hat{k}$ rad/s and $R = 10$ cm

we have $\vec{a}_A = 40\hat{i} - 40t^2\hat{j}$ cm/s^2 **Ans.**

Superposition Method

We superimpose translation motion of the center and rotation motion about the center. In fact it is vector addition of terms of above equation used in analytical method.



From the above figure, we have

$$\vec{a}_A = (a_C + \alpha R)\hat{i} - \omega^2 R\hat{j}$$

Substituting known values $\vec{\alpha} = -2\hat{k}$ rad/s^2, $\vec{\omega} = -2t\hat{k}$ rad/s and R=10 cm,

we have $\vec{a}_A = 40\hat{i} - 40t^2\hat{j}$ cm/s^2 **Ans.**

Methods of Impulse and Momentum

Methods of impulse and momentum describe what happens over a time interval. When motion of a body involves rotation we have to consider angular impulse as well as angular momentum. In this section we discuss concept of angular impulse, angular momentum of rigid body, angular impulse momentum principle and conservation of angular momentum.

Angular Impulse

Like impulse of a force angular impulse of a constant torque equals to product of the torque and concerned time interval and if the torque is not constant it must be integrated with time over the concerned time interval.

If torque $\vec{\tau}_o$ about an axis passing through O is constant, its angular impulse during a time interval from t_1 to t_2 denoted by $\vec{J}_{o,1\to 2}$ is given by the following equation.

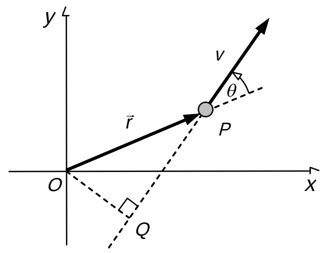
$$\vec{J}_{o,1\to 2} = \vec{\tau}_o(t_2 - t_1)$$

If torque $\vec{\tau}_o$ about an axis passing through O is time varying, its angular impulse during a time interval from t_1 to t_2 denoted by $\vec{J}_{o,1\to 2}$ is given by the following equation.

$$\vec{J}_{o,1\to 2} = \int_{t_1}^{t_2} \vec{\tau}_o dt$$

Angular momentum of a particle

Angular momentum $\vec{L}_o$ about the origin O of a particle of mass m moving with velocity $\vec{v}$ is defined as the moment of its linear momentum $\vec{p} = m\vec{v}$ about the point O.



$$\vec{L}_o = \vec{r} \times (m\vec{v})$$

Angular Momentum of a Rigid Body

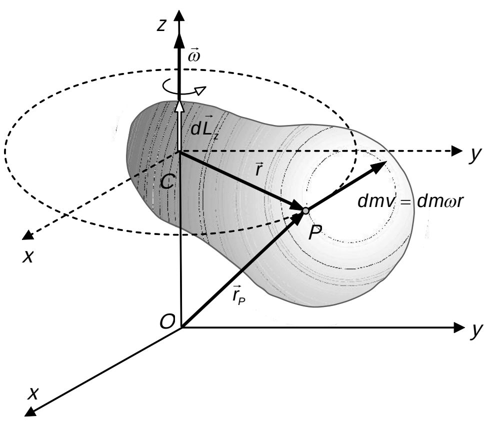
Angular momentum is quantity of rotation motion in a body. The angular momentum of a system of particles is the sum of angular momentum all the particles within the system. A rigid body is an assemblage of large number of particles maintaining their mutual distances intact under all circumstances, therefore angular momentum of a rigid body must be sum of angular momenta of all of its particles.

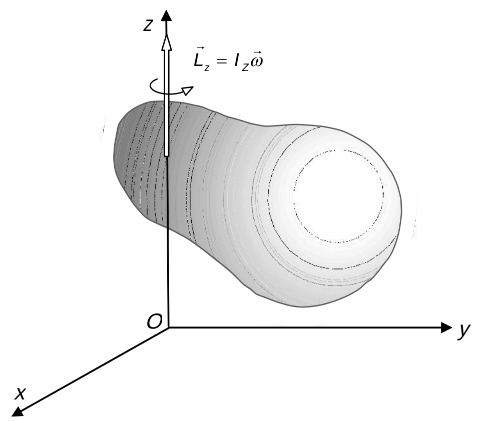
Angular Momentum about a point and about an axis

Angular momentum of a particle is not defined about an axis instead it is defined about a point. Therefore above idea of summing up angular momenta of all the particles about a point gives angular momentum of the rigid body about a point. But while dealing with fixed axis rotation or rotation about axis in translation we need angular momentum about an axis.

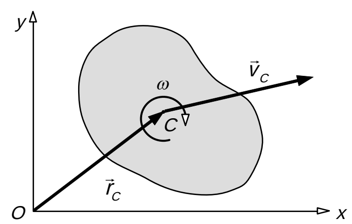
Angular momentum about an axis is calculated similar to torque abut an axis. To calculate angular momentum of a particle of rigid body about an axis we take moment of momentum of the particle about the point where plane of motion of the point of application of the force intersects the axis.

In the following figure is shown angular momentum $d\vec{L}_z = \vec{r} \times (dm\vec{v}) = r^2 dm\omega$ of a particle P of a rigid body rotating about the z-axis. It is along the z-axis i.e. axis of rotation. In the next figure total angular momentum $\vec{L}_z = \int d\vec{L}_z = I_z\vec{\omega}$ about the axis of rotation is shown. It is also along the axis of rotation.





Angular Momentum in general plane motion



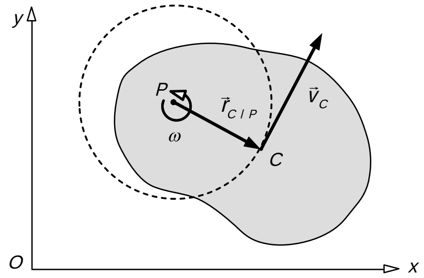
Angular momentum of a body in plane motion can also be written similar to torque equation or kinetic energy as sum of angular momentum about the axis due to translation of mass center and angular momentum of centroidal rotation about centroidal axis parallel to the original axis.

Consider a rigid body of mass M in plane motion. At the instant shown its mass center has velocity $\vec{v}$ and it is rotating with angular velocity $\vec{\omega}$ about an axis perpendicular to the plane of the figure. It angular momentum $\vec{L}_o$ about an axis passing though the origin and parallel to the original is expressed by the following equation.

$$\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$$

The first term of the above equation represent angular momentum due to translation of the mass center and the second term represents angular momentum in centroidal rotation.

Angular momentum in rotation about fixed axis

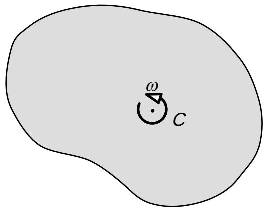


Consider a body of mass M rotating with angular velocity ω about a fixed axis perpendicular to plane of the figure passing through point P. Making use of the parallel axis theorem $I_P = Mr^2_{C/P} + I_C$ and equation $\vec{v}_C = \vec{\omega} \times \vec{r}_{C/P}$ we can express the angular momentum $\vec{L}_P$ of the body about the fixed rotational axis.

$$\vec{L}_P = I_P \vec{\omega}$$

The above equation reveals that the angular momentum of a rigid body in plane motion can also be expressed in a single term due to rotation about the instantaneous axis of rotation.

Angular momentum in pure centroidal rotation



In pure centroidal rotation, mass center remains at rest, therefore angular momentum due to translation of the mass center vanishes.

$$\vec{L}_C = I_C \vec{\omega}$$

Rotational Equivalent of the Newton's Laws of Motion

Differentiating terms on both the sides of equation $\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$ with respect to time, and making substitution of $\vec{v}_C = d\vec{r}_C/dt$, $\vec{a}_C = d\vec{v}_C/dt$ and $\vec{\alpha} = d\vec{\omega}/dt$ we have

$$\frac{d\vec{L}_o}{dt} = \vec{v}_C \times \left(M\vec{v}_C\right) + \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$$

The first on the right hand side vanishes, so we can write

$$\frac{d\vec{L}_o}{dt} = \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$$

Now comparing the above equation with torque equation $\Sigma\vec{\tau}_o = \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$, we have

$$\sum \vec{\tau}_o = \frac{d\vec{L}_o}{dt}$$

The above equation though developed for plane motion only yet is valid for rotation about an axis in rotation also. It states that the net torque about the origin of an inertial frame equals to the time rate of change in angular momentum about the origin and can be treated as a parallel to Newton's second law which states that net external force on a body equals to time rate of change in its linear momentum.

Angular Impulse Momentum Principle

Rearranging the terms and integrating both the sides obtained form previous equation, we can write

$$\Sigma\int_{t_1}^{t_2}\vec{\tau}_o dt = \vec{L}_{o2} - \vec{L}_{o1}$$

The left hand side of the above equation is the angular impulse of torque of all the external forces in the time interval in the time interval t_1 to t_2.

$$\Sigma\vec{J}_{o,1\to 2} = \vec{L}_{o2} - \vec{L}_{o1}$$

The idea expressed by the above equation is known as angular impulse momentum principle and states that increment in the angular momentum of a body about a point in a time interval equals to the net angular impulse of all the external forces acting on it during the concerned time interval.

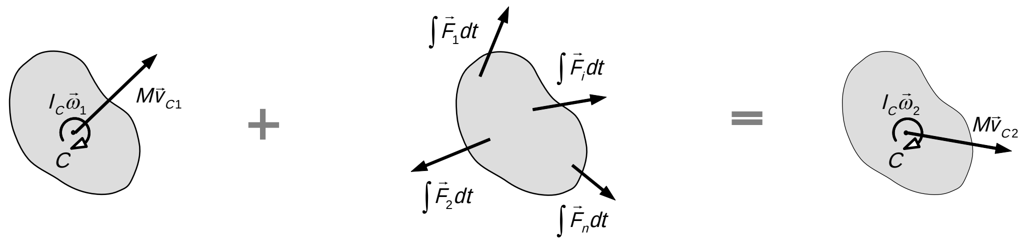
For the ease of application the above equation is rearranged as

$$\vec{L}_{o1} + \sum\vec{J}_{o,1\to 2} = \vec{L}_{o2}$$

Like linear impulse momentum principle, the angular impulse momentum principle provides us solution of problems concerned with change in angular velocity in a time interval or change in angular velocity during very short interval interactions.

Method of Impulse Momentum Principle for Plane motion of a Rigid Body

Linear momentum and angular momentum serve as measures of amount of translation and rotation motion respectively. The external forces acting on a rigid body can change its state of translation as well as rotation motion which is reflected by change in linear as well as angular momentum according to the principles of linear impulse and momentum and angular impulse and momentum.



Linear and angular momenta at the instant t_1 | Impulse of all the forces during time interval t_1 to t_2 | Linear and angular momenta at the instant t_2

In the above figure is shown strategy to apply method of impulse and momentum. Consider a rigid body of mass M in plane motion. Its moment of inertial about the centroidal axis perpendicular to plane of motion is I_C. Let $\vec{v}_{C1}$ and $\vec{\omega}_1$ represent velocity of its mass center and its angular velocity at the beginning of a time interval t_1 to t_2. Under the action of several forces $\vec{F}_1$, $\vec{F}_2$ ……. $\vec{F}_i$ ….. $\vec{F}_n$ during the time interval its mass center velocity and angular velocity become $\vec{v}_{C2}$ and $\vec{\omega}_2$ respectively. The adjacent figure shows strategy representing how to write equations for linear and angular impulse momentum principles.

While applying the principle it becomes simpler to consider translation of the mass center and centroidal rotation separately. Thus in an alternative way we apply linear impulse momentum principle for translation of the mass center and angular impulse momentum principle for centroidal rotation.

Translation of mass center: Linear impulse momentum principle.

$$\vec{p}_1 + \sum \vec{I}_{mp1\to 2} = \vec{p}_2$$

Here $\vec{p}_1 = M\vec{v}_{C1}$ and $\vec{p}_2 = M\vec{v}_{C2}$ represent linear momentums at the beginning and end of the time interval and $\sum \vec{I}_{mp1\to 2}$ stands for impulse of all the external forces during the time interval.

Centroidal rotation: Angular impulse momentum principle.

$$\vec{L}_{C1} + \sum \vec{J}_{C,1\to 2} = \vec{L}_{C2}$$

Here $\vec{L}_{C1} = I_C\vec{\omega}_1$ and $\vec{L}_{C2} = I_C\vec{\omega}_2$ represent angular momentums about the centroidal axis at the beginning and end of the time interval and $\sum \vec{J}_{C,1\to 2}$ stands for angular impulse of all the external forces about the centroidal axis during the time interval.

Conservation of Angular Momentum

If angular impulse of all the external forces about an axis in time interval vanishes, the angular momentum of the system about the same axis in that time interval remain unchanged.

If $\Sigma\int_{t_1}^{t_2} \vec{\tau}_o dt = 0$, we have $\vec{L}_{o1} = \vec{L}_{o2}$

The condition of zero net angular impulse required for conservation of angular momentum can be fulfilled in the following cases.

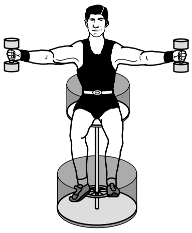
- If no external force acts, the angular impulse about all axes will be zero and hence angular momentum remains conserved about all axes.
- If net torque of all the external forces or torques of each individual force about an axis vanishes the angular momentum about that axes will be conserved.
- If all the external forces are finite in magnitude and the concerned time interval is infinitely small, the angular momentum remain conserved.
- If a system of rigid bodies changes its moment of inertia by changing its configuration due to internal forces only its angular momentum about any axes remains conserved. If we denote the moment of inertias in two configurations by I_1 and I_2 and angular velocities by ω_1 and ω_2, we can write

$$I_1\vec{\omega}_1 = I_2\vec{\omega}_2$$

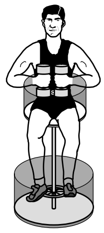
The principle of conservation of angular momentum governs a wide range of physical processes from subatomic to celestial world. The following examples explicate some of these applications.

Student on rotating turntable

The student, the turntable and dumbbells make an isolated system on which no external torque acts, if we ignore friction in the bearing of the turntable and air friction. Initially the student has his arm stretched on rotating turntable. When he pulls dumbbells close to his body, angular velocity increases due to conservation of angular momentum.

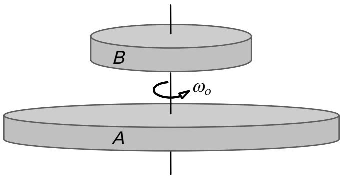


Larger moment of inertia and smaller angular velocity



Smaller moment of inertia and larger angular velocity

Ex. Consider the disk A of moment of inertia I_1 rotating freely in horizontal plane about its axis of symmetry with angular velocity ω_o. Another disk B of moment of inertia I_2 held at rest above the disk A. The axis of symmetry of the disk B coincides with that of the disk A as shown in the figure. The disk B is released to land on the disk A. When sliding stops, what will be the angular velocity of both the disks?



Sol. Both the disks are symmetric about the axis of rotation therefore does not require any external torque to keep the axis stationary. When the disk B lands on A slipping starts. The force of friction provides an internal torque to system of both the disk. It slows down rotation rate of A and increases that of B till both acquire same angular velocity ω.

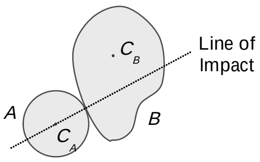
Since there is no external torques on the system of both the disks about the axis of rotation, the total angular momentum of the system remains conserved. The total angular momentum of the system is the sum of angular momentum of both disks. Denoting the angular momentum of the disk A before B lands on it and long after slipping between them stops by symbols $\vec{L}_{A1}$, $\vec{L}_{B1}$, $\vec{L}_{A2}$ and $\vec{L}_{B2}$ respectively, we can express conservation of angular momentum by the following equation.

$$\vec{L}_{A1} + \vec{L}_{B1} = \vec{L}_{A2} + \vec{L}_{B2} \rightarrow \; I_1\omega_o + 0 = I_1\omega + I_2\omega \Rightarrow \omega = \frac{I_1\omega_o}{I_1 + I_2}$$ **Ans.**

Eccentric Impact

In eccentric impact the line of impact which is the common normal drawn at the point of impact does not passes through mass center of at least one of the colliding bodies. It involves change in state of rotation motion of either or both the bodies.

Consider impact of two A and B such that the mass center C_B of B does not lie on the line of impact as shown in figure. If we assume bodies to be frictionless their mutual forces must act along the line of impact. The reaction force of A on B does not passes through the mass center of B as a result state of rotation motion of B changes during the impact.



Problems of Eccentric Impact

Problems of eccentric impact can be divided into two categories. In one category both the bodies under going eccentric impact are free to move. No external force act on either of them. There mutual forces are responsible for change in their momentum and angular momentum. In another category either or both of the bodies are hinged.

Eccentric Impact of bodies free to move

Since no external force acts on the two body system, we can use principle of conservation of linear momentum, principle of conservation of angular momentum about any point and concept of coefficient of restitution.

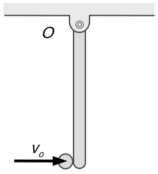
The coefficient of restitution is defined for components of velocities of points of contacts of the bodies along the line of impact.

While applying principle of conservation of angular momentum care must be taken in selecting the point about which we write the equation. The point about which we write angular momentum must be at rest relative to the selected inertial reference frame and as far as possible its location should be selected on line of velocity of the mass center in order to make zero the first term involving moment of momentum of mass center.

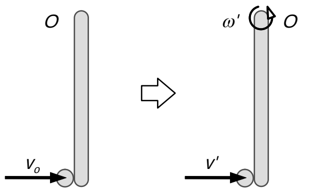
Eccentric Impact of hinged bodies

When either or both of the bodies are hinged the reaction of the hinge during the impact act as external force on the two body system, therefore linear momentum no longer remain conserved and we cannot apply principle of conservation of linear momentum. When both the bodies are hinged we cannot also apply conservation of angular momentum, and we have to use impulse momentum principle on both the bodies separately in addition to making use of coefficient of restitution. But when one of the bodies is hinged and other one is free to move, we can apply conservation of angular momentum about the hinge.

Ex. A uniform rod of mass m and length ℓ is suspended from a fixed support and can rotate freely in the vertical plane. A small ball of mass m moving horizontally with velocity v_o strikes elastically the lower end of the rod as shown in the figure. Find the angular velocity of the rod and velocity of the ball immediately after the impact.



Sol. The rod is hinged and the ball is free to move. External forces acting on the rod ball system are their weights and reaction from the hinge. Weight of the ball as well as the rod are finite and contribute negligible impulse during the impact, but impulse of reaction of the hinge during impact is considerable and cannot be neglected. Obviously linear momentum of the system is not conserved. The angular impulse of the reaction of hinge about the hinge is zero. Therefore angular momentum of the system about the hinge is conserved. Let velocity of the ball after the impact becomes v'_B and angular velocity of the rod becomes ω'.



We denote angular momentum of the ball and the rod about the hinge before the impact by L_{B1} and L_{R1} and after the impact by L_{B2} and L_{R2}.

Applying conservation of angular momentum about the hinge, we have

$$\vec{L}_{B1} + \vec{L}_{R1} = \vec{L}_{B2} + \vec{L}_{R2} \rightarrow \quad mv_o\ell + 0 = mv'_B\ell + I_o\omega'$$

Substituting $\frac{1}{3}M\ell^2$ for I_o, we have

$$3mv'_B + M\ell\omega' = 3mv_o \qquad (1)$$

The velocity of the lower end of the rod before the impact was zero and immediately after the impact it becomes $\ell\omega'$ towards right. Employing these facts we can express the coefficient of restitution according to eq.

$$e = \frac{v'_{Qn} - v'_{Pn}}{v_{pn} - v_{Qn}} \rightarrow \qquad \ell\omega' - v'_B = ev_o \qquad (2)$$

From eq. (1) and (2), we have

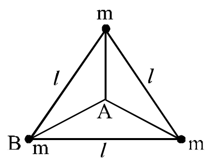
Velocity of the ball immediately after the impact $v'_B = \dfrac{(3m - eM)v_o}{3m + M}$ **Ans.**

Angular velocity of the rod immediately after the impact $\omega' = \dfrac{3(1+e)mv_o}{(3m+M)\ell}$ **Ans.**

EXERCISE (S-1)

Moment of inertia

1. Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown in the figure. What is the ratio of the moment of inertia of the assembly for an axis through B compared with that for an axis through A (centroid). Both the axis are perpendicular to the plane of triangle.



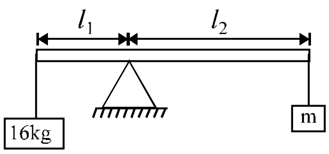
2. Two rods of equal mass m and length l lie along the x axis and y axis with their centres at origin. What is the moment of inertia of the system about the line x = y :

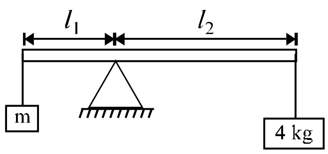
Torque and its calculation

3. A force $\vec{F} = 2\hat{i} + 3\hat{j}$ N is applied to an object that is pivoted about a fixed axle aligned along the z coordinate axis. If the force is applied at the point $\vec{r} = 4\hat{i} + 5\hat{j}$ m, find (a) the magnitude of the net torque about the z axis and (b) the direction of the torque vector τ.

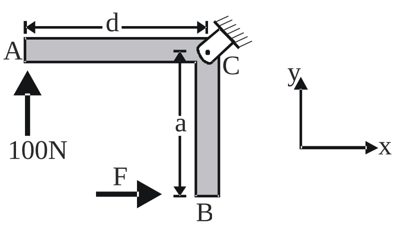
Equilibrium

4. In an experiment with a massless beam balance an unknown mass m is balanced by two known masses of 16kg and 4 kg as shown in figure.Find the value of the unknown mass m.

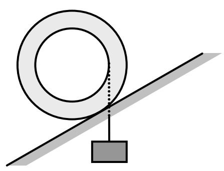




5. Find the force F required to keep the system in equilibrium. The dimensions of the system are d = 0.3 m and a = 0.2 m. Assume the rods to be massless.

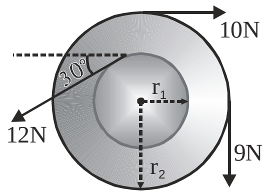


6. A 3.0 kg bobbin consists of a central cylinder of radius 5.0 cm and two end plates each of radius 6.0 cm. It is placed on a slotted incline, where friction is sufficient to prevent sliding. A block of mass 4.5 kg is suspended from a cord wound around the bobbin and passing through the slot under the incline. If the bobbin is in static equilibrium, what is the angle of tilt of the incline?

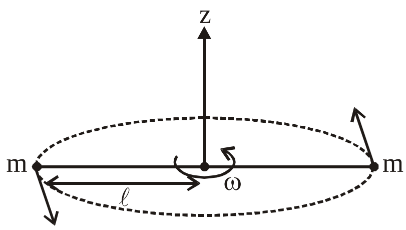


τ = Iα and its calculation

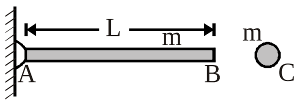
7. In the following figure r_1 and r_2 are 5 cm and 30 cm respectively. If the moment of inertia of the wheel is 5100 kg-m^2 about the axis passing through its centre and perpendicular to the plane of wheel, then what will be its angular acceleration?



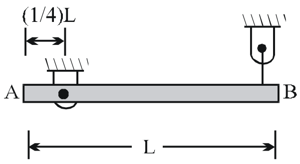
8. A dumbbell consists of two identical particles of mass m connected by a rigid light rod of length 2ℓ. The dumbbell is set spinning with angular speed ω_0 on a surface with a small friction coefficient μ_k. If dumbbell stops in time $t = \dfrac{K\omega_0\ell}{2\mu g}$ where K is a constant, then find the value of K.



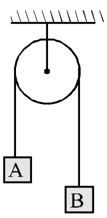
9. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. There is gravity downwards. What is the distance of the point from point B that has the same acceleration as that of ball, immediately after release?



10. A uniform beam of length L and mass m is supported as shown. If the cable at B suddenly breaks, determine; (a) the acceleration of end B. (b) the reaction at the pin support.



11. In the figure, A & B are two blocks of mass 4 kg & 2 kg respectively attached to the two ends of a light string passing over a disc C of mass 40 kg and radius 0.1 m. The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find:

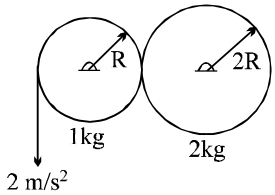


(i) the linear acceleration of mass B .

(ii) the number of revolutions made by the disc at the end of 10sec. from the start.

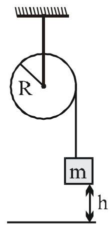
(iii) the tension in the string segment supporting the block A.

12. Two discs A and B touch each other as shown in figure. A rope tightly wound on A is pulled down at 2 m/s². Find the friction force between A and B if slipping is absent

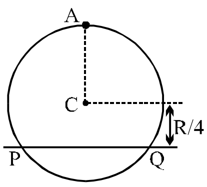


Kinetic energy in pure rotation

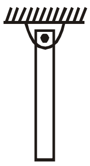
13. A mass m is attached to a pulley through a cord as shown in the figure. The pulley is a solid disk with radius R. The cord does not slip on the disk. The mass is released from rest at a height h from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity ω. Find the mass of the disk.



14. A uniform circular disc has radius R and mass m. A particle also of mass m is fixed at a point A on the edge of the disc as in figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle at it reaches its lowest position.

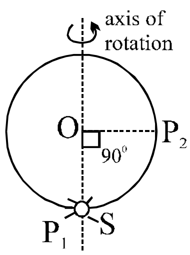


15. A rod of mass m & length ℓ is hinged at it's upper end. It can rotate in vertical plane. It is given angular velocity ω so that it can complete vertical circle. Find (a) ω (b) Tension at centre of rod at initial moment.



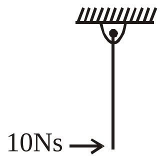
Angular momentum and its conservation

16. A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally reaches the point P_2 (as shown in figure). What is the final angular velocity of the ring?



17. Two men, each of mass 75 kg , stand on the rim of a horizontal large disc, diametrically opposite to each other. The disc has a mass 450 kg and is free to rotate about its axis. Each man simultaneously start along the rim clockwise with the same speed and reaches their original starting points on the disc. Find the angle turned by the disc with respect to the ground in this duration.

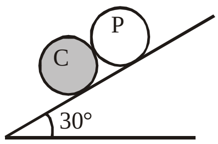
18. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. Find the kinetic energy of rod just after impact.



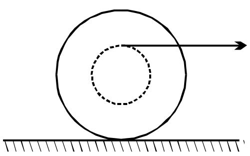
Combined rotation and translation

19. A solid uniform disk of mass m rolls down a fixed inclined plane without slipping with an acceleration a. Find the frictional force on the disk due to surface of the plane :

20. A solid cylinder C and a hollow pipe P of same diameter are in contact when they are released from rest as shown in the figure on a long incline plane. Cylinder C and pipe P roll without slipping. Determine the clear gap (in m) between them after 6 seconds.

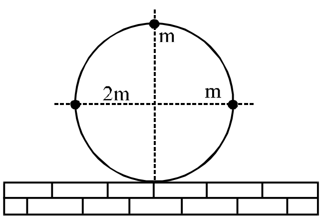


21. A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force = Mg. Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.

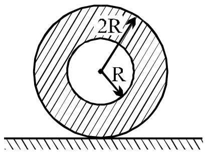


Kinetic energy in rolling

22. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. Find the kinetic energy of the system. (Slipping is absent)

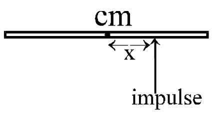


23. A hollow cylinder with inner radius R, outer radius 2R and mass M is rolling with speed of its axis v. Its kinetic energy is :-

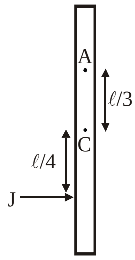


Angular impulse

24. A uniform rod of length l is given an impulse at right angle to its length as shown. Find the distance of instantaneous centre of rotation from the centre of the rod.



25. A uniform rod of mass m and length ℓ is placed in gravity free space and linear impulse J is given to the rod at a distance $x = \ell/4$ from centre 'C' and perpendicular to the rod. Point A is at a distance $\ell/3$ from centre as shown in the figure. Then find



(i) Speed of centre of rod

(ii) Speed of point A

(iii) Speed of upper end of rod

(iv) Speed of lower end of rod

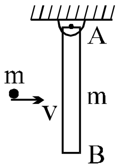
26. A solid sphere of mass m and radius R is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height h = 4R/5 above the centre line. If I is the impulse of the blow then find

(a) the minimum time after which the highest point B will touch the ground

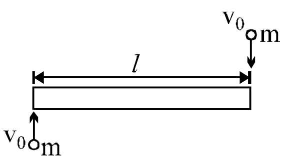
(b) the displacement of the centre of mass during this internal.

Eccentric collision

27. A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle comes to rest after the impact, then find the impulsive reaction at A.



28. On a smooth table two particles of mass m each, travelling with a velocity v_0 in opposite directions, strike the ends of a rigid massless rod of length l, kept perpendicular to their velocity. The particles stick to the rod after the collision. Find the tension in rod during subsequent motion.

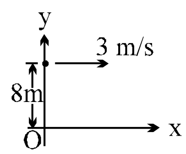


EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

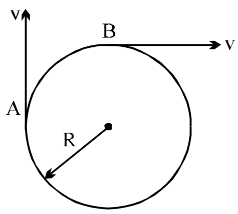
Conecpt of ω, α, v, a

1. A particle starts from the point (0m, 8m) and moves with uniform velocity of $3\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be :



(A) $\frac{8}{289}$ rad/s (B) $\frac{3}{8}$ rad/s (C) $\frac{24}{289}$ rad/s (D) $\frac{8}{17}$ rad/s

2. Two points of a rigid body are moving as shown. The angular velocity of the body is:



(A) $\frac{\upsilon}{2R}$ (B) $\frac{\upsilon}{R}$ (C) $\frac{2\upsilon}{R}$ (D) $\frac{2\upsilon}{3R}$

Moment of inertia

3. Three bodies have equal mass m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B (C) C (D) A and C both

4. For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

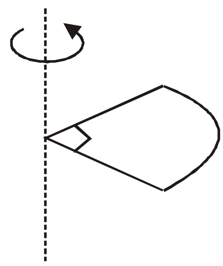
(A) a disc of radius a (B) a ring of radius a

(C) a square lamina of side 2a (D) four rods forming a square of side 2a

5. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

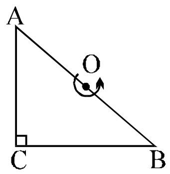
(A) I (B) $I\sin^2\theta$ (C) $I\cos^2\theta$ (D) $I\cos^2(\theta/2)$

6. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is **[IIT-JEE 2000]**



(A) $\frac{1}{2}$ MR^2 (B) $\frac{1}{4}MR^2$ (C) $\frac{1}{8}$ MR^2 (D) $\sqrt{2}$ MR^2

7. Find the moment of inertia of a plate cut in shape of a right angled triangle of mass M, about an axis perpendicular to the plane of the plate and passing through the mid point of side AB. (Side AC = BC = a)

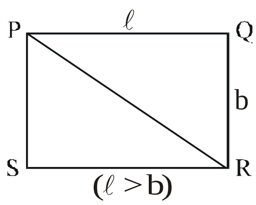


(A) $\frac{Ma^2}{12}$ (B) $\frac{Ma^2}{6}$ (C) $\frac{Ma^2}{3}$ (D) $\frac{2Ma^2}{3}$

8. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is- **[AIEEE - 2003]**

(A) $I_Y = 32\ I_X$ (B) $I_Y = 16I_X$ (C) $I_Y = I_X$ (D) $I_Y = 64\ I_X$

9. Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of PQR about an axis perpendicular to the plane of the plate:



(A) about P = I/2 (B) about R = I/2 (C) about P > I/2 (D) about R > I/2

10. A thin uniform rod of mass M and length L has its moment of inertia I_1 about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2. The ratio of $I_1 : I_2$ will be ________

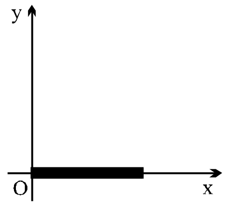
(A) < 1 (B) > 1 (C) = 1 (D) can't be said

11. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that- **[AIEEE - 2004]**

(A) $I_A = I_B$ (B) $I_A > I_B$ (C) $I_A < I_B$ (D) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where d_A and d_B are their densities.

12. The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the xy-plane, about which the moment of inertia of the rod is same as that about O is :-



(A) an ellipse (B) a circle (C) a parabola (D) a straight line

13. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$. The x-coordinate of centre of mass is

(A) x = 2 (B) x = 0 (C) x = 1 (D) x = 3

Equilibrium

14. A weightless rod is acted on by upward parallel forces of 2N and 4N at ends A and B respectively. The total length of the rod is AB = 3m. To keep the rod in equilibrium a force of 6N should act in the following manner:

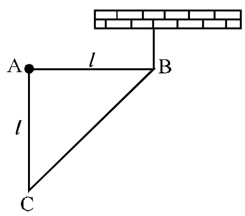
(A) Downwards at any point between A and B.

(B) Downwards at mid point of AB.

(C) Downwards at a point C such that AC = 1m.

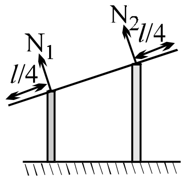
(D) Downwards at a point D such that BD = 1m.

15. A right triangular plate ABC of mass m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A in equilibrium is:



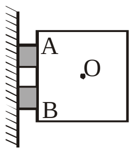
(A) $\frac{mg}{3}$ (B) $\frac{2mg}{3}$ (C) $\frac{mg}{2}$ (D) mg

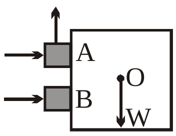
16. A uniform rod of length l is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then :-

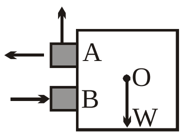


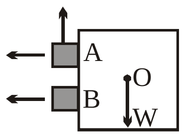
(A) $N_1 > N_2$ (B) $N_1 < N_2$
(C) $N_1 = N_2$ (D) N_1 and N_2 would be in the vertical directions

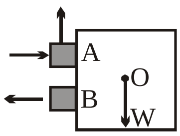
17. A vertical rectangular door with its centre of gravity at O (see figure) is fixed on two hinges A and B along one vertical length side of the door. The entire weight of the door is supported by the hinge A. Then the free body force diagram for the door (the arrows indicate the direction of the forces) is :-



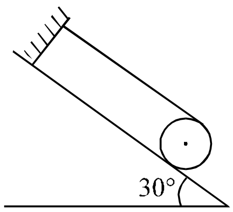
(A) 

(B) 

(C) 

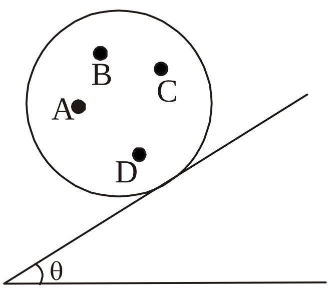
(D) 

18. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration to be in equilibrium is:



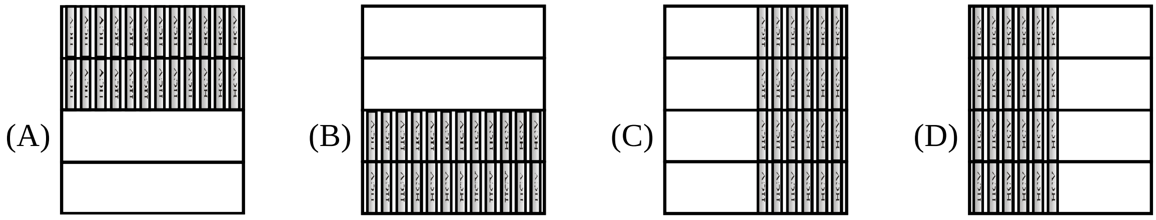
(A) $\frac{1}{3\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2\sqrt{3}}$

19. A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?

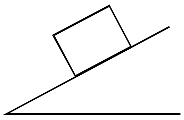


Toppling

20. Same number of books are placed in four book cases as shown. Which bookcase is most likely to topple forward if pulled a little at the top towards right :



21. A homogeneous cubical brick lies motionless on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :



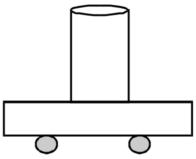
(A) left half

(B) right half

(C) both applies equal pressure

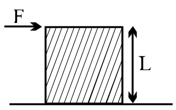
(D) the answer depend upon coefficient of friction

22. A uniform 2 kg cylinder rests on a laboratory cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 cm in diameter and 10 cm in height, which of the following is closest to the maximum acceleration of the cart such that cylinder neither slips nor tips over?



(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2

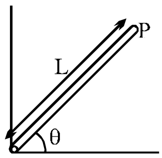
23. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is: **[IIT-JEE'(Scr)'2000]**



(A) infinitesimal (B) mg/4 (C) mg/2 (D) mg(1-μ)

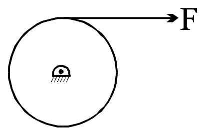
$\tau = I\alpha$

24. A uniform flag pole of length L and mass M is pivoted on the ground with a frictionless hinge. The flag pole makes an angle θ with the horizontal. The moment of inertia of the flag pole about one end is $(1/3)ML^2$. If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the flag pole (labeled P) immediately after it starts falling off would be:



(A) (2/3) gcosθ (B) (2/3) g (C) g (D) (3/2) gcosθ

25. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As time increases,



(A) its angular velocity increases, but force on hinge remains constant

(B) its angular velocity remains same, but force on hinge increases

(C) its angular velocity increases and force on hinge increases

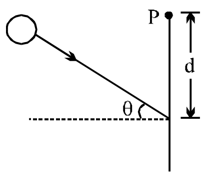
(D) its angular velocity remains same and force on hinge is constant

Angular momentum and its conservation

26. A particle of mass 2 kg located at the position $(\hat{i}+\hat{j})$ m has a velocity $2(+\hat{i}-\hat{j}+\hat{k})$ m/s. Its angular momentum about z-axis in kg-m²/s is:

(A) zero (B) +8 (C) 12 (D) –8

27. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is:



(A) 2 *mvd* (B) 2 *mvd* cosθ (C) 2 *mvd* sin θ (D) zero

28. Two uniform spheres of mass M have radii R and 2R. Each sphere is rotating about a fixed axis through a diameter. The rotational kinetic energies of the spheres are identical. What is the ratio of the magnitude of the angular momenta of these spheres? That is, $\frac{L_{2R}}{L_R} =$

(A) 4 (B) $2\sqrt{2}$ (C) 2 (D) $\sqrt{2}$

29. A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy and what is the work done by the skater?

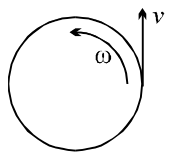
(A) angular momentum remains the same while kinetic energy increases and work done is positive.

(B) angular momentum remains the same while kinetic energy decreases and work done is negative.

(C) both angular momentum and kinetic energy remain the same and work done is zero.

(D) angular momentum increases while kinetic energy remains the same and work done may be positive or negative.

30. A child with mass m is standing at the edge of a disc with moment of inertia I, radius R, and initial angular velocity ω. See figure given below. The child jumps off the edge of the disc with tangential velocity v with respect to the ground. The new angular velocity of the disc is



(A) $\sqrt{\dfrac{I\omega^2 - mv^2}{I}}$

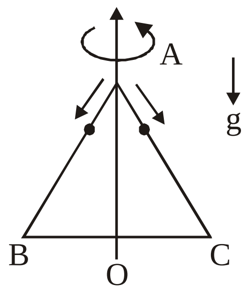
(B) $\sqrt{\dfrac{(I + mR^2)\omega^2 - mv^2}{I}}$

(C) $\dfrac{I\omega - mvR}{I}$

(D) $\dfrac{(I + mR^2)\omega - mvR}{I}$

31. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are

[IIT-JEE 2000]



(A) Angular velocity and total energy (kinetic and potential)

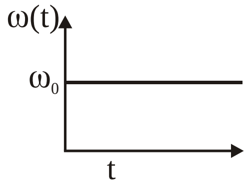
(B) Total angular momentum and total energy

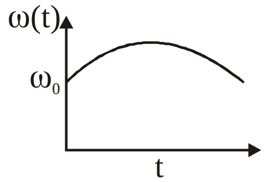
(C) Angular velocity and moment of inertia about the axis of rotation.

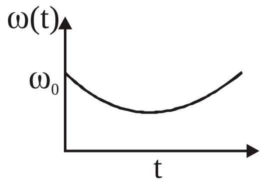
(D) Total angular momentum and moment of inertia about the axis of rotation.

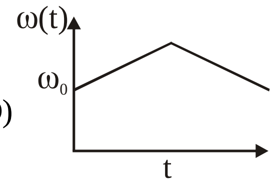
32. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0 . When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$. will vary with time t as

[IIT-JEE 2002]

(A) 

(B) 

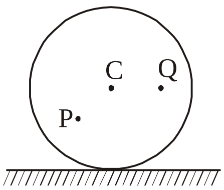
(C) 

(D) 

Combined rotation and translation

Kinematics

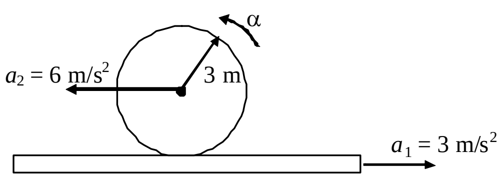
33. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is :- **[IIT-JEE 2004]**



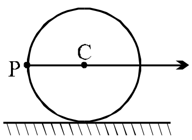
(A) $v_Q > v_C > v_P$
(B) $v_P > v_C > v_Q$
(C) $v_P = v_C, v_Q = v_C/2$
(D) $v_P < v_C > v_Q$

34. In the following figure, a sphere of radius 3 m rolls on a plank. The accelerations of the sphere and the plank are indicated. The value of α is

(A) 3 rad/s^2
(B) 6 rad/s^2
(C) 3 rad/s^2 (opposite to the direction shown in figure)
(D) 1 rad/s^2



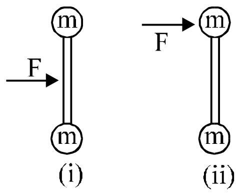
35. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is



(A) zero
(B) 45°
(C) 135°
(D) $\tan^{-1}(1/2)$

Dynamics

36. A force F is applied to a dumbbell for a time interval, t, first as in (i) and then as in (ii). In which case does the dumbbell acquire the greater centre-of-mass speed?



(A) (i)

(B) (ii)

(C) there is no difference

(D) the answer depends on the rotational inertia of the dumbbell

37. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

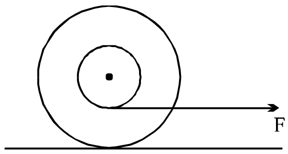
(A) Up the incline while ascending and down the incline while descending **[IIT-JEE 2002]**

(B) Up the incline while ascending as well as descending

(C) down the incline while ascending and up the incline while descending

(D) down the incline while ascending as well as descending

38. Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards

(B*)* Thread winds, spool rotates clockwise and friction acts leftwards

(C) Thread winds, spool moves to the right and friction act rightwards

(D) Thread winds, spool moves to the right and friction does not come into existence.

39. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :

(A) thin pipe
(B) uniform cylinder
(C) uniform sphere
(D) thin spherical shell

40. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere:

(A) must be forward (in direction of v)
(B) must be backward (opposite to v)
(C) cannot be zero
(D) none of the above

Kinetic energy

41. A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal

(A) the hoop has a greater translational speed than the cylinder

(B) the cylinder has a greater translational speed than the hoop

(C) the hoop and the cylinder have the same translational speed

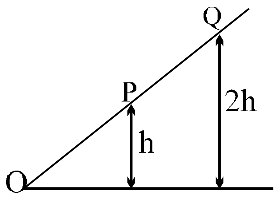
(D) the hoop has a greater rotational speed than the cylinder

42. A ball rolls down an inclined plane as shown in figure. The ball is first released from rest from P and then later from Q. Which of the following statement is/ are correct?

(i) The ball takes twice as much time to roll from Q to O as it does to roll from P to O.

(ii) The acceleration of the ball at Q is twice as large as the acceleration at P.

(iii) The ball has twice as much K.E. at O when rolling from Q as it does when rolling from P.



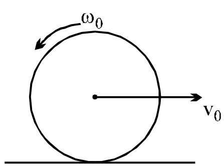
(A) i, ii only (B) ii, iii only (C) i only (D) iii only

43. The moment of inertia of a solid cylinder about its axis is given by $(1/2)MR^2$. If this cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is

(A) 1 : 1 (B) 2 : 2 (C) 1 : 2 (D) 1 : 3

Angular momentum

44. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{r\omega_0}$ is



(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

45. A hollow sphere of radius R and mass m is fully filled with non viscous liquid of mass m. It is rolled down a horizontal plane such that its centre of mass moves with a velocity v. If it purely rolls

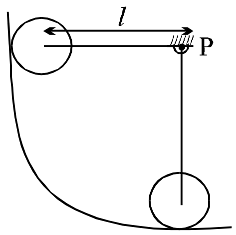
(A) Kinetic energy of the sphere is $\frac{5}{6}mv^2$

(B) Kinetic energy of the sphere is $\frac{4}{5}mv^2$

(C) Angular momentum of the sphere about a fixed point on ground is $\frac{8}{3}mvR$

(D) Angular momentum of the sphere about a fixed point on ground is $\frac{14}{5}mvR$

46. A sphere of mass M and radius R is attached by a light rod of length ℓ to a point P. The sphere rolls without slipping on a circular track as shown. It is released from the horizontal position. The angular momentum of the system about P when the rod becomes vertical is :



(A) $M\sqrt{\frac{10}{7}gl}\,[l+R]$

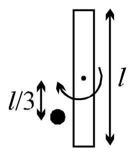
(B) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{2}{5}R\right]$

(C) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{7}{5}R\right]$

(D) $M\sqrt{\frac{10}{7}gl}\left[l-\frac{2}{5}R\right]$

Eccentric collision

47. A uniform rod of length l and mass M rotating about a fixed vertical axis on a smooth horizontal table. It elastically strikes a particle placed at a distance $l/3$ from its axis and stops. Mass of the particle is



(A) 3M

(B) $\frac{3M}{4}$

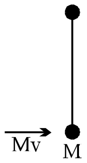
(C) $\frac{3M}{2}$

(D) $\frac{4M}{3}$

48. A mass m is moving at speed v perpendicular to a rod of length d and mass $M = 6m$ which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system right after the collision is

(A) $2v/d$ (B) $2v/(3d)$ (C) v/d (D) $3v/(2d)$

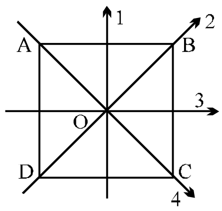
49. Two particles each of mass M are connected by a massless rod of length l. The rod is lying on the smooth sufrace. If one of the particle is given an impulse MV as shown in the figure then angular velocity of the rod would be **[IIT-JEE'(Scr)2003]**



(A) v/l (B) $2v/l$ (C) $v/2l$ (D) None

MULTIPLE CORRECT TYPE QUESTIONS

50. ABCD is a square plate with centre O. The moments of inertia of the plate about the perpendicular axis through O is I and about the axes 1, 2, 3 & 4 are I_1, I_2, I_3 & I_4 respectively. It follows that :



(A) $I_2 = I_3$ (B) $I = I_1 + I_4$ (C) $I = I_2 + I_4$ (D) $I_1 = I_3$

51. A rod of weight w is supported by two parallel knifes at end points A and B and is in equilibrium in a horizontal position. The knives are at a distance 'd' from each other. The centre of mass of the rod is at a distance 'x' from A.

(A) the normal reaction at A is $\frac{wx}{d}$

(B) the normal reaction at A is $\frac{w(d-x)}{d}$

(C) the normal reaction at B is $\frac{wx}{d}$

(D) the normal reaction at B is $\frac{w(d-x)}{d}$

52. A block with a square base measuring 'a' × 'a' and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will

(A) topple before sliding if $\mu > \frac{a}{h}$

(B) topple before sliding if $\mu < \frac{a}{h}$

(C) slide before toppling if $\mu > \frac{a}{h}$

(D) slide before toppling if $\mu < \frac{a}{h}$

53. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.

(A) Angular momentum of the particle about O is increasing .

(B) Torque of the gravitational force on the particle about O is decreasing.

(C) The moment of inertia of the particle about O is decreasing .

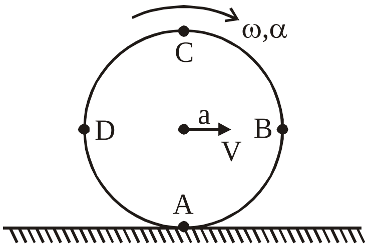
(D) The angular velocity of the particle about O is increasing.

54. A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his

(A) moment of inertia (B) angular momentum

(C) angular velocity (D) rotational kinetic energy

55. A circular disc of radius R rolls without slipping on a rough horizontal surface. At the instant shown its linear velocity is V, linear acceleration a, angular velocity ω and angular acceleration α. Four points A, B, C and D lie on its circumference such that the diameter AC is vertical & BD horizontal then choose the **CORRECT** option(s).



(A) $V_B = \sqrt{V^2 + (R\omega)^2}$ (B) $V_C = V + R\omega$

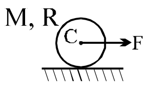
(C) $a_A = \sqrt{(a - R\alpha)^2 + (\omega^2 R)^2}$ (D) $a_D = \sqrt{(a + \omega^2 R)^2 + (R\alpha)^2}$

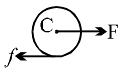
COMPREHENSION TYPE QUESTIONS

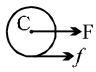
Paragraph for Question No. 56 to 58

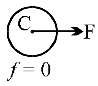
In the following problems, indicate the correct direction of friction force acting on the cylinder, which is pulled on a rough surface by a constant force F.

56. A cylinder of mass M and radius R is pulled horizontally by a force F. The friction force can be given by which of the following diagrams :-



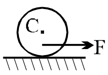
(A) 

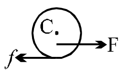
(B) 

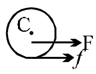
(C) 

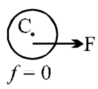
(D) cannot be interpreted

57. A cylinder is pulled horizontally by a force F acting at a point below the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams :-



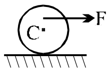
(A) 

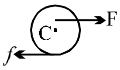
(B) 

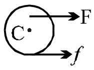
(C) 

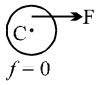
(D) cannot be interpreted

58. A cylinder is pulled horizontally by a force F acting at a point above the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



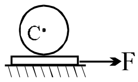
(A) 

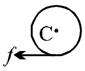
(B) 

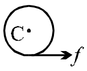
(C) 

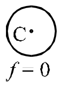
(D) cannot be interpreted

59. A cylinder is placed on a rough plank which in turn is placed on a smooth surface. The plank is pulled with a constant force F. The friction force can be given by which of the following diagrams



(A) 

(B) 

(C) 

(D) canot be interpreted

Paragraph for Question No. 60 to 62

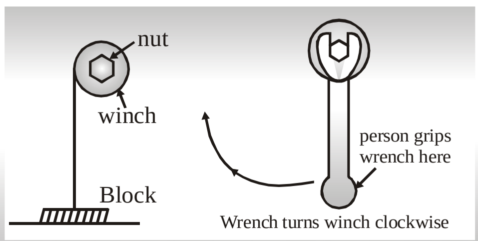
In figure, the winch is mounted on an axle, and the 6-sided nut is welded to the winch. By turning the nut with a wrench, a person can rotate the winch. For instance, turning the nut clockwise lifts the block off the ground, because more and more rope gets wrapped around the winch.

Three students agree that using a longer wrench makes it easier to turn the winch. But they disagree about why. All three students are talking about the case where the winch is used, over a 10 s time interval, to lift the block one metre off the ground.

Student 1 : By using a longer wrench, the person decreases the average force he must exert on the wrench, in order to lift the block one metre in 10 s.

Student 2 : Using a longer wrench reduces the work done by the person as he uses the winch to lift the block 1m in 10s.

Student 3 : Using a longer wrench reduces the power that the person must exert to lift the block 1m in 10s.



60. Student 1 is :-

(A) correct, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(B) correct, because using a longer wrench decreases the torque it must exert on the winch

(C) incorrect, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(D) Incorrect, because using a longer wrench decreases the torque it must exert on the winch.

61. Which of the following is true about student 2 and 3 :-

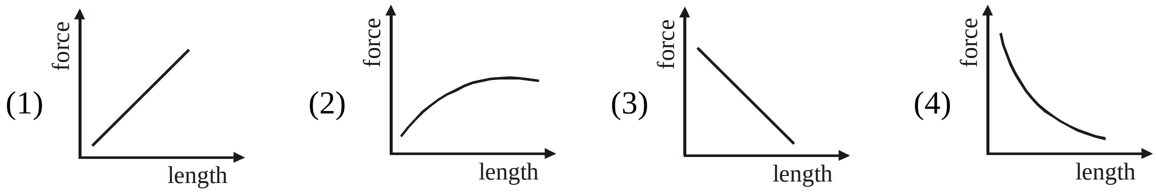
(A) Student 2 and 3 are both correct

(B) Student 2 is correct, but student 3 is incorrect

(C) Student 3 is correct, but student 2 is incorrect

(D) Student 2 and 3 are both incorrect

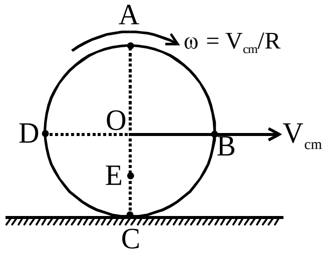
62. If several wrenches all apply the same torque to a nut, which graph best expresses the relationship between the force the person must apply to the wrench, and the length of the wrench :-



(A) 1 (B) 2 (C) 3 (D) 4

MATRIX MATCH TYPE QUESTION

63. Consider a body rolling on a horizontal surface as shown in figure (Symbols have their usual meaning)

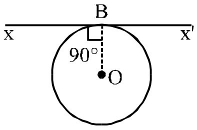


	Column-I		**Column-II**
(A)	Velocity is zero	(P)	Point A
(B)	Speed in maximum	(Q)	Point B
(C)	$0 <$ speed $< v_{cm}$	(R)	Point C
(D)	$1.3v_{cm} <$ speed $< 2\ v_{cm}$	(S)	Point D
		(T)	Point E

EXERCISE (O-2)

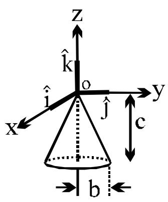
SINGLE CORRECT TYPE QUESTIONS

1. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is : **[IIT-JEE'(Scr)'2000]**



(A) $\rho L^3/8\pi^2$ (B) $\rho L^3/16\pi^2$ (C) $5\rho L^3/16\pi^2$ (D) $3\rho L^3/8\pi^2$

2. A solid cone hangs from a frictionless pivot at the origin O, as shown. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors, and a, b, and c are positive constants, which of the following forces F applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_Z?



(A) $F = a\hat{k}$, P is (0,b,–c) (B) $F = -a\hat{k}$, P is (0,–b,–c)

(C) $F = a\hat{j}$, P is (–b,0,–c) (D) None

3. A heavy seesaw (i.e., not massless) is out of balance. A light girl sits on the end that is tilted downward, and a heavy body sits on the other side so that the seesaw now balances. If they both move forward so that they are one-half their original distance from the pivot point (the fulcrum) what will happen to the seesaw ?

(A) The side the body is sitting on will tilt downward

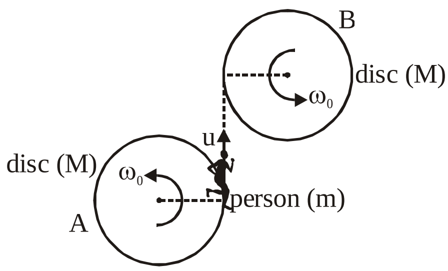
(B) The side the girl is sitting on will once again tilt downward

(C) Nothing ; the seesaw will still be balanced

(D) It is impossible to say without knowing the masses and the distances

4. Two identical discs each of mass M(= 4m) and radius R are rotating in opposite sense with equal angular speed ω_0 about vertical axes passing through their centres, (as shown in figure). A person of mass m sitting on circumference of disc A jumps with a tangential relative velocity u (after jumping) w.r.t one rotating disc (A) and lands on other disc (B) also tangential. Now the second disc (B) comes to a stop. Find the relative velocity u.

(A) $u = \frac{\omega_0 R}{2}$

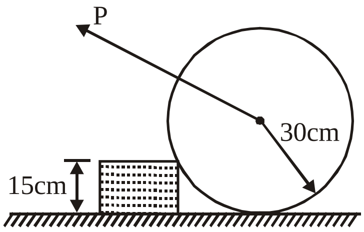


(B) $u = \omega_0 R$

(C) $u = \frac{3\omega_0 R}{2}$

(D) $u = 2\omega_0 R$

5. In the given figure a uniform wheel of radius 30cm rests against a rigid rectangular block 15cm high. The wheel weighs 1000 N. The minimum pull *P* through the center which will turn the wheel over the block is :-



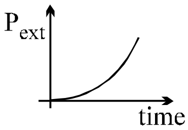
(A) $500\sqrt{3}$ N (B) $1000\sqrt{3}$ N

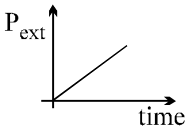
(C) 1000 N (D) $400\sqrt{3}$ N

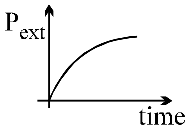
6. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse J is applied to the end B, perpendicular to the rod in the horizontal direction. Speed of particle P at a distance $\frac{l}{6}$ from the centre towards A of the rod after time $t = \frac{\pi m l}{12J}$ is

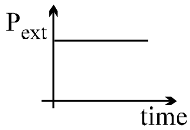
(A) $2\frac{J}{m}$ (B) $\frac{J}{\sqrt{2}m}$ (C) $\frac{J}{m}$ (D) $\sqrt{2}\frac{J}{m}$

7. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :

(A) 

(B) 

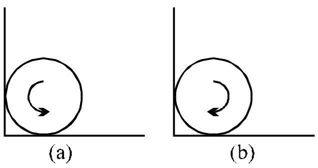
(C) 

(D) 

8. A straight rod of length L is released on a frictionless horizontal floor in a vertical position. As it falls + slips, the distance of a point on the rod from the lower end, which follows a quarter circular locus is :-

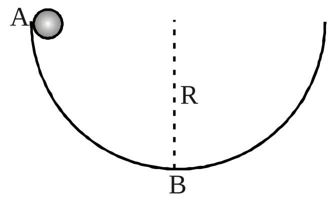
(A) L/2 (B) L/4 (C) L/8 (D) None

9. A sphere is placed rotating with its centre initially at rest in a corner as shown in figure (a) & (b). Coefficient of friction between all surfaces and the sphere is 1/3. Find the ratio of the frictional force $\frac{f_a}{f_b}$ by ground in situations (a) & (b).



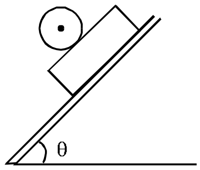
(A) 1 (B) $\frac{9}{10}$ (C) $\frac{10}{9}$ (D) none

10. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R (>> r) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.



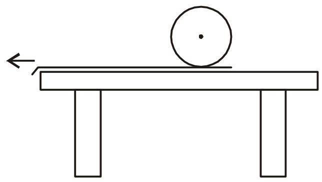
(A) $\frac{10}{7}$mg (B) $\frac{17}{7}$ mg (C) $\frac{5}{7}$ mg (D) $\frac{7}{5}$mg

11. A plank of mass M is placed over smooth inclined plane and a sphere is also placed over the plank. Friction is sufficient between sphere and plank. If plank and sphere are released from rest, the frictional force on sphere is:



(A) up the plane (B) down the plane (C) horizontal (D) zero

12. A thin table cloth covers a horizontal table and a uniform body of round shape lies on top of it. The table cloth is pulled from under the body, and friction causes the body to slide and rotate. What is the body's final motion on the table? (Assume that the table is so large that the body does not fall off it.)



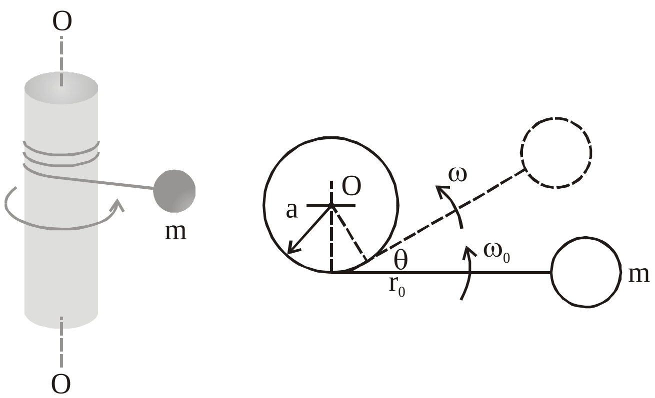
(A) Body will finally roll towards left

(B) Body will finally roll towards right

(C) Body will finally come to rest

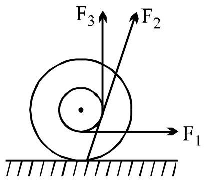
(D) Any of the above is possible depending on shape of body

13. The small particle of mass m is given an initial high velocity in the horizontal plane and winds its cord around the fixed vertical shaft of radius 1m. All motion occurs essentially in the horizontal plane. If the angular velocity of the cord is 0.8 rad/s when the distance from the particle to the tangential point is 5m, determine the angular velocity ω (in rad/s) of the cord after it has turned through an angle 1 rad.



(A) 1 (B) 2 (C) 4 (D) 6

14. A yo-yo is resting on a perfectly rough horizontal table. Forces F_1, F_2 and F_3 are applied separately as shown. The correct statement is :-



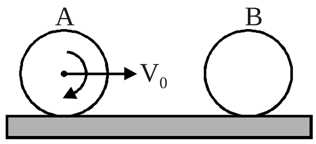
(A) when F_3 is applied the centre of mass will move to the right.

(B) when F_2 is applied the centre of mass will move to the left.

(C) when F_1 is applied the centre of mass will move to the right.

(D) when F_2 is applied the centre of mass will move to the right.

15. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is:



(A) 1 : 1 (B) 2 : 3 (C) 3 : 2 (D) None

16. A wheel of radius r rolling on a straight line, the velocity of its centre being v. At a certain instant the point of contact of the wheel with the grounds is M and N is the highest point on the wheel (diametrically opposite to M). The incorrect statement is:

(A) The velocity of any point P of the wheel is proportional to MP.

(B) Points of the wheel moving with velocity greater than v form a larger area of the wheel than points moving with velocity less than v.

(C) The point of contact M is instantaneously at rest.

(D) The velocities of any two parts of the wheel which are equidistant from centre are equal.

17. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor. At a certain moment, the speed of the end in contact with the horizontal floor is v and the ladder makes an angle $\alpha = 30°$ with the horizontal. Then the speed of the ladder's center must be :-

(A) $2v/\sqrt{3}$ (B) v/2 (C) v (D) None

18. In the previous question, if dv/dt = 0, then the angular acceleration of the ladder when $\alpha = 45°$ is:-

(A) $2v^2/L^2$ (B) $v^2/2L^2$ (C) $\sqrt{2}[v^2/L^2]$ (D) None

19. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b/2?

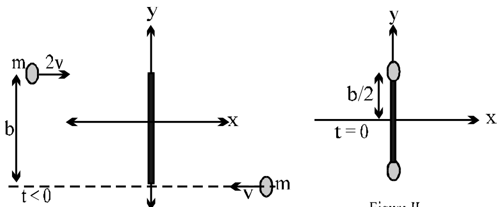


Figure I Figure II

(A) $x = 2vt, y = b/2$

(B) $x = vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(C) $x = 0.5vt + 0.5b\sin(3vt/b), y = 0.5b\cos(3vt/b)$

(D) $x = 0.5vt + 0.5b\sin(6vt/b), y = 0.5b\cos(6vt/b)$

MULTIPLE CORRECT TYPE QUESTIONS

20. A rigid object is rotating in a counterclockwise sense around a fixed axis. If the rigid object rotates through more than 180° but less than 360°, which of the following pairs of quantities can represent an initial angular position and a final angular position of the rigid object. Which of the sets can only occur.

(A) 3 rad, 6 rad (B) – 1 rad, 1 rad (C) 1 rad, 5 rad (D) –1 rad, 2.5 rad

21. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

(A) 2, if their lines of action pass through the centre of mass of the body.

(B) 3, if their lines of action are not parallel.

(C) 3, if their lines of action are parallel.

(D) 4, if their lines of action are parallel and all the forces have the same magnitude.

22. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}\times\vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that

(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time

(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time

(C) the magnitude of $\vec{L}$ does not change with time

(D) $\vec{L}$ does not change with time

23. A block of mass m moves on a horizontal rough surface with initial velocity v. The height of the centre of mass of the block is h from the surface. Consider a point A on the surface.

(A) angular momentum about A is mvh initially

(B) the velocity of the block decreases at time passes.

(C) torque of the forces acting on block is zero about A

(D) angular mometum is not conserved about A.

24. If a cylinder is rolling down the incline with sliding.

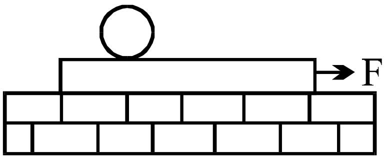
(A) after some time it may start pure rolling

(B) after sometime it will start pure rolling

(C) it may be possible that it will never start pure rolling

(D) none of these

25. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is correct.



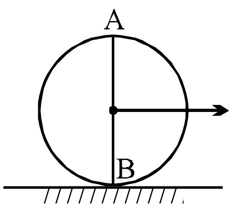
(A) Acceleration of the centre of sphere is less than that of the plank.

(B) Work done by friction acting on the sphere is equal to its total kinetic energy.

(C) Total kinetic energy of the system is equal to work done by the force F

(D) None of the above

26. A uniform disc is rolling on a horizontal surface. At a certain instant B is the point of contact and A is at height 2R from ground, where R is radius of disc.



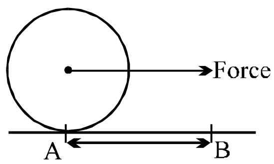
(A) The magnitude of the angular momentum of the disc about B is thrice that about A.

(B) The angular momentum of the disc about A is anticlockwise.

(C) The angular momentum of the disc about B is clockwise

(D) The angular momentum of the disc about A is equal to that about B.

27. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB = s. The disc moves from A to B in time T. To the right of B,



(A) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged

(B) linear acceleration of the disc will increase

(C) the disc will make one rotation in time T/2

(D) the disc will cover a distance greater than s in further time T.

28. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is ν.

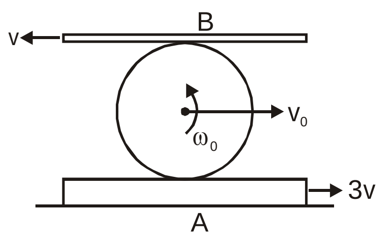
(A) $0 \le \nu \le 2u$

(B) $\nu = u$, if CP is horizontal

(C) $\nu = u$, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C.

(D) $\nu = \sqrt{2}u$, if CP is horizontal

29. The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then



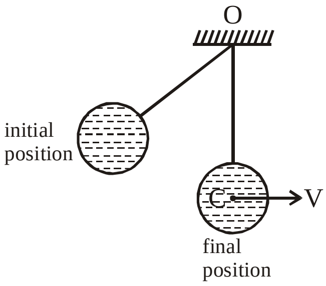
(A) linear velocity $v_0 = v$

(B) angular velocity of disc is $\frac{3v}{2r}$

(C) angular velocity of disc is $\frac{2v}{r}$

(D) None of these

30. A massles rod has a massless hollow sphere attached to it. This sphere can be fully filled either with a liquid (non viscous) or with a solid (rigidly fitted into sphere) of same mass. System is released from rest from initial position (as shown). When it reaches final position which of the following is/are true for the system.



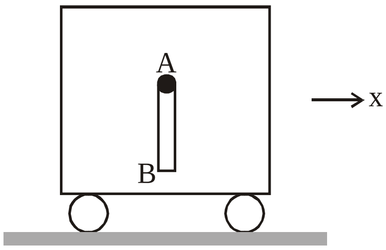
(A) Kinetic energy in case of liquid will be more than in case of solid.

(B) Velocity of centre (V) in case of liquid will be more than in case of solid.

(C) About C angular momentum in case of liquid will be more than in case of solid.

(D) About C angular momentum in case of liquid will be less than in case of solid.

31. A uniform rod AB of length ℓ and mass M hangs from point A at which it is freely hinged in a car moving with velocity v_0. The rod can rotate in vertical plane about the axis at A. If the car suddenly stops,



(A) The angular speed ω with which the rod starts rotating is $\frac{3v_0}{2\ell}$

(B) The minimum value of v_0 so that the rod completes the rotation $\sqrt{\frac{8}{3}g\ell}$

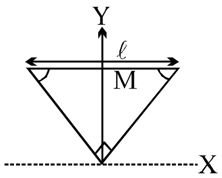
(C) Loss of energy during the process $\frac{1}{8}Mv_0^2$

(D) There is no loss of energy

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 32 to 35

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X–axis



32. The moment of inertia of the plate about the z-axis is :-

(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{24}$ (C) $\frac{ML^2}{6}$ (D) none of these

33. The moment of inertia of the plate about the x-axis is :-

(A) $\frac{ML^2}{8}$ (B) $\frac{ML^2}{32}$ (C) $\frac{ML^2}{24}$ (D) $\frac{ML^2}{6}$

34. The moment of inertia of the plate about its base parallel to the x-axis is :-

(A) $\frac{ML^2}{18}$ (B) $\frac{ML^2}{36}$ (C) $\frac{ML^2}{24}$ (D) none of these

35. The moment of inertia of the plate about the y-axis is :-

(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{8}$ (C) $\frac{ML^2}{24}$ (D) none of these

Paragraph for Question on 36 and 37

In the treatment of moments of inertia, introductory textbooks often present two theorems, generally called the parallel axis theorem and the perpendicular axis theorem. There is another theorem of this same genre, which is not usually included, but which is interesting and useful. It is

$$I_x + I_y + I_z = 2\sum_i m_i r_i^2$$

Here, I_x, I_y and I_z are the moments of inertia about three mutually perpendicular intersecting axes, m_1 is the mass of the i^{th} particle and r_i is the distance from the intersection. The proof is simple: Taking the three axes as coordinate axes, we have :

$$I_x + I_y + I_z$$

$$= \sum_i m_i\left(y_i^2 + z_i^2\right) + \sum_i m_i\left(z_i^2 + x_i^2\right) + \sum_i m_i\left(x_i^2 + y_i^2\right)$$

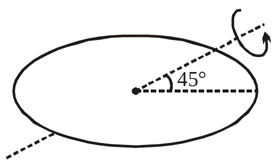
$$= 2\sum_i m_i\left(x_i^2 + y_i^2 + z_i^2\right) = 2\sum_i m_i r_i^2$$

One important application is the calculation of the moment of inertia I_d of a uniform thin-walled spherical shell, of mass M and radius R, about a diameter. Taking the centre as the origin of coordinates, we have $I_x = I_y = I_z = I_d$, and $r_i = R$. The theorem gives $3I_d = 2\Sigma_i m_i R^2 = 2(\Sigma_i m_i)R^2 = 2MR^2$, where $I_d = 2MR^2/3$.

36. Consider a solid cube of mass m and side L. What will be the value of $\Sigma m_i r_i^2$ for this body when the point of intersection of axes is the centre of the cube :

(A) $\frac{ML^2}{2}$ (B) $\frac{ML^2}{4}$ (C) $\frac{ML^2}{3}$ (D) $\frac{ML^2}{6}$

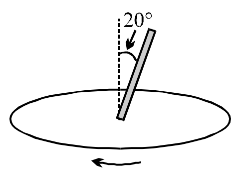
37. Find the moment of inertia of ring of mass m and radius R about an axis passing through its centre and making an angle 45° with its plane :



(A) $\frac{MR^2}{4}$ (B) $\frac{MR^2}{2}$ (C) $\frac{3}{4}MR^2$ (D) MR^2

Paragraph for Question No. 38 and 39

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



38. What is the direction of the rod's angular momentum vector (calculated about its lower end)?

(A) vertically downwards (B) down at 20° to the horizontal

(C) up at 20° to the horizontal (D) vertically upwards

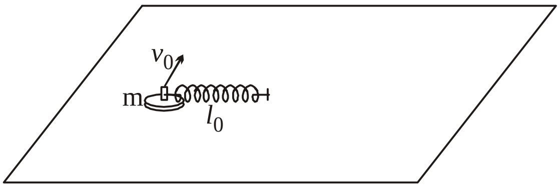
39. Is there a torque acting on it, and if so in what direction?

(A) yes, vertically (B) yes, horizontally

(C) yes at 20° to the horizontal (D) no

Paragraph for Question No. 40 to 43

A spring having initial unstretched length ℓ_0 is lying on a smooth table. It's one end is fixed and the other one is fastened to a small particle of mass m. The particle is imparted an initial speed v_0 horizontally in a direction perpendicular to the spring. In the course of the motion in horizontal plane, the maximum elongation of the spring is $\Delta\ell = \ell_0/10$.
(Given : m = 0.1 kg, ℓ_0 = 1m, v_0 = 11 m/s).



40. In the course of motion, which of the quantities relating to spring block system are conserved ?

(A) kinetic energy
(B) momentum
(C) angular momentum
(D) potential energy

41. Which of the following is correct about initial situation and situation at maximum elogation ?

(A) the orientiation of spring in both positions should be perpendicular to each other.
(B) the velocity at maximum extension should be zero
(C) the velocity at maximum extension as well as at initial position should be perpendicular to spring.
(D) Acceeration should be zero at the maximum extension as well as at initial position.

42. **Student-A :** at maximum extension $mv'(\ell_0 + \Delta\ell) = mv_0\ell_0$.

Student-B : at maximum extension $k\Delta\ell = \dfrac{mv'^2}{(\ell_0+\Delta\ell)}$. **Student-C :** $\dfrac{1}{2}mv_0^2 = \dfrac{1}{2}mv'^2 + \dfrac{1}{2}k\Delta\ell^2$.

where v' is velocity at instant of maximum extension :

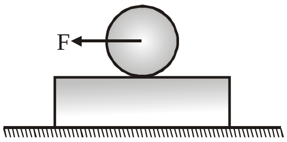
(A) Only Student-A and B are correct
(B) Only Student-A and C are correct
(C) Only Student-B and C are correct
(D) All are correct

43. The value of spring constant (in N/m) is :

(A) 100
(B) 210
(C) 420
(D) 105

Paragraph for Question No. 44 and 45

A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between disc and plank to prevent slipping. A force F is applied at the centre of the disc.



44. Acceleration of the plank is :-

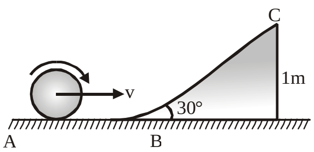
(A) $\frac{F}{2m}$ (B) $\frac{3F}{4m}$ (C) $\frac{F}{4m}$ (D) $\frac{3F}{2m}$

45. Force of friction between the disc and the plank is :-

(A) $\frac{F}{2}$ (B) $\frac{F}{4}$ (C) $\frac{F}{3}$ (D) $\frac{2F}{3}$

Paragraph for Question No. 46 and 47

A small sphere of mass 1 kg is rolling without slipping on a rough stationary base with linear speed $v = \sqrt{\frac{200}{7}}$ m/s. It leaves the inclined plane at point C.



46. Find its linear speed at point C :-

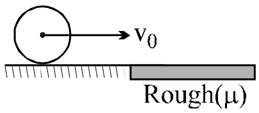
(A) $\sqrt{\frac{100}{7}}$ m/s (B) $\sqrt{\frac{50}{7}}$ m/s (C) $\sqrt{\frac{100}{35}}$ m/s (D) $\sqrt{\frac{200}{35}}$ m/s

47. Find ratio of rotational and translational kinetic energy of the sphere when it strikes the ground after leaving from point C :-

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$

Paragraph for Question No. 48 to 51

A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in figure.



48. Choose the correct statement(s)

(A) As the ring enters on the rough surface, the limiting friction force acts on it

(B) The direction of friction is opposite to the direction of motion

(C) The friction force accelerates the ring in the clockwise sense about its centre of mass

(D) As the ring enters on the rough surface it starts rolling

49. Choose the correct statement(s)

(A) The momentum of the ring is conserved

(B) The angular momentum of the ring is conserved about its centre of mass

(C) The angular momentum of the ring conserved about any point on the horizontal surface in line of friction.

(D) The mechanical energy of the ring is conserved

50. Choose the correct statement(s)

(A) The ring starts its rolling motion when the centre of mas stationary

(B) The ring starts rolling motion when the point of contact becomes stationary

(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$

(D) The rolling velocity is $\frac{v_0}{2}$

51. Choose the correct alternative(s) :-

(A) The linear distance moved by the centre of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$

(B) The net work done by friction force is $-\frac{3}{8}mv_0^2$

(C) The loss is kinetic energy of the ring is $\frac{mv_0^2}{4}$

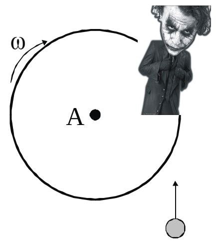
(D) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

MATRIX MATCH TYPE QUESTION

52. Column-I depicts various situations where some sudden events are taking place. Column-II describes changes in various parameters of systems immediately after the events taking place in column-I.

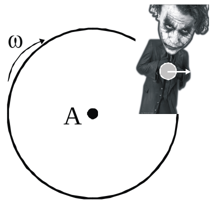
Column-I

(A) Joker is standing on revolving platform and batman throws the ball and joker catches the ball while it was moving horizontally.



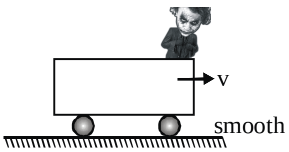
Joker, ball and platform is system.

(B) Joker throws the ball horizontally and perpendicular to his motion while standing on the revolving platform.



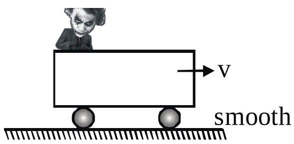
Joker, ball and platform is system.

(C) Joker jumps horizontally towards right from the cart which is moving at speed v on smooth horizontal floor.



Joker and cart is the system

(D) Joker drops himself vertically from the moving cart with no horizontal velocity relative to cart.



Joker and cart is the system

Column-II

(P) Linear momentum remains conserved.

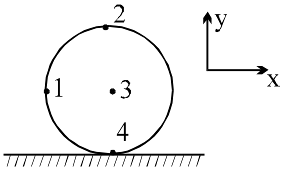
(Q) Mechanical energy is conserved

(R) Mechanical energy increases.

(S) Mechanical energy decreases.

(T) v or ω changes

53. A rigid cylinder is kept on a smooth horizontal surface as shown. If **Column-I** indicates velocities of various points (3-centre of cylinder, 2- top point, 4-bottom point, 1- on the level of 3 at the rim) on it shown, choose correct state of motion from **Column-II**.



Column-I		**Column-II**	
(A)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_2 = 2\hat{i}$	(P)	Pure rotation about centre
(B)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_3 = -\hat{i}$	(Q)	Rolling without slipping to left
(C)	$\vec{v}_2 = \hat{i}, \vec{v}_3 = 0$	(R)	Rolling without slipping to right
(D)	$\vec{v}_4 = 0, \vec{v}_1 = -\hat{i} - \hat{j}$	(S)	Not possible

EXERCISE (JM)

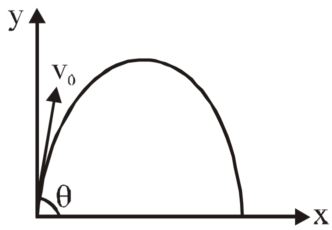
1. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of:-

[AIEEE - 2009]

(1) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (2) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (3) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (4) $\frac{1}{6}\frac{l\omega}{g}$

2. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is:

Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. **[AIEEE-2010]**



(1) $\frac{1}{2}mg\, v_0\, t^2 \cos\theta\, \hat{i}$ (2) $-mg\, v_0\, t^2 \cos\theta\, \hat{j}$

(3) $mg\, v_0\, t \cos\theta\, \hat{k}$ (4) $-\frac{1}{2}mg\, v_0\, t^2 \cos\theta\, \hat{k}$

3. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10kg m^2, the number of rotations made by the pulley before its direction of motion it reversed, is :- **[AIEEE-2011]**

(1) more than 6 but less than 9 (2) more than 9

(3) less than 3 (4) more than 3 but less than 6

4. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. Euring the fjourney of the insect, then angular speed of the disc :-

[AIEEE-2011]

(1) continuously increases (2) first increases and then decreases

(3) remains unchanged (4) continuously decreses

5. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :- **[AIEEE-2011]**

(1) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ (2) zero (3) $\frac{mv^3}{\sqrt{2}g}$ (4) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

6. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE Mains-2013]**

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

7. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension : **[JEE Mains-2014]**

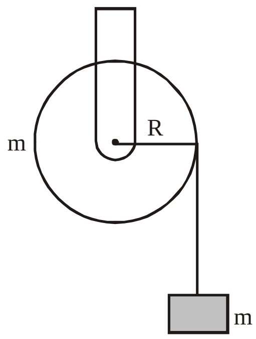
(1) Angular momentum changes in direction but not in magnitude

(2) Angular momentum changes both in direction and magnitude

(3) Angular momentum is conserved

(4) Angular momentum changes in magnitude but not in direction.

8. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE Mains-2014]**

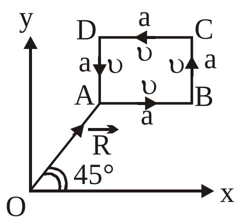


(1) $\frac{5g}{6}$ (2) g (3) $\frac{2g}{3}$ (4) $\frac{g}{2}$

9. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:- **[JEE Mains-2015]**

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$ (3) $\frac{MR^2}{32\sqrt{2}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

10. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure : **[JEE Mains-2016]**



Which of the following statement is false for the angular momentum $\vec{L}$ about the origin ?

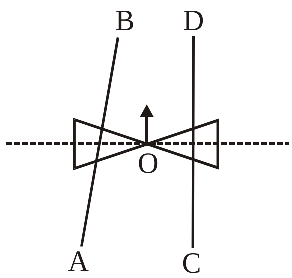
(1) $\vec{L} = \frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A

(2) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B

(3) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(4) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

11. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :- **[JEE Mains-2016]**



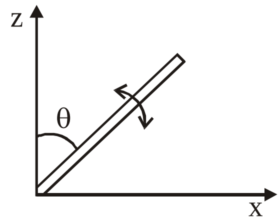
(1) turn left and right alternately. (2) turen left.

(3) turn right. (4) go straight.

12. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum? **[JEE Main-2017]**

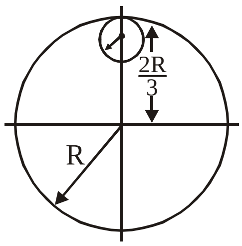
(1) 1 (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

13. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is : **[JEE Main-2017]**



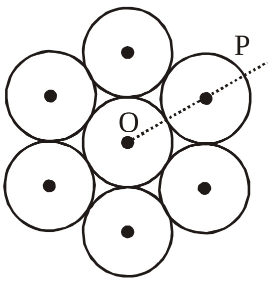
(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{3\ell}\sin\theta$

14. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE Main-2018]**



(1) $\frac{40}{9}MR^2$ (2) 10 MR^2 (3) $\frac{37}{9}MR^2$ (4) 4 MR^2

15. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[JEE Main-2018]**



(1) $\frac{55}{2}$ MR^2 (2) $\frac{73}{2}$ MR^2 (3) $\frac{181}{2}$ MR^2 (4) $\frac{19}{2}$ MR^2

EXERCISE (JA)

1. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then :- **[IIT-JEE 2009]**

(A) at θ =30°, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

2. If the resultant of the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[IIT-JEE 2009]**

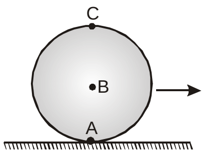
(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

3. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then **[IIT-JEE 2009]**



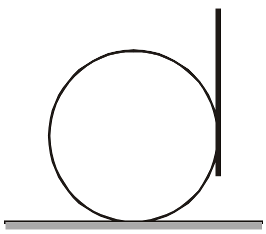
(A) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$

(B) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

(C) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$

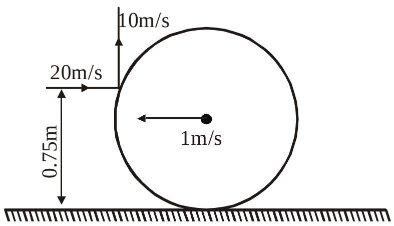
(D) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

4. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[IIT-JEE 2011]**



5. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg-m^2, then N is **[IIT-JEE 2011]**

6. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[IIT-JEE 2011]**



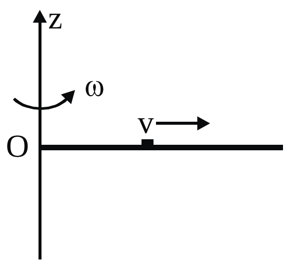
(A) the ring has pure rotation about its stationary CM

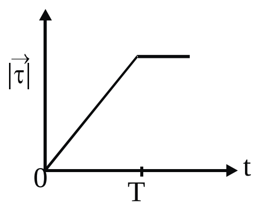
(B) the ring comes to a complete stop

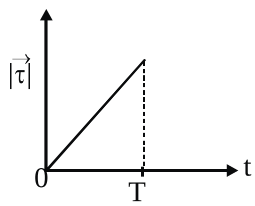
(C) friction between the ring and the ground is to the left

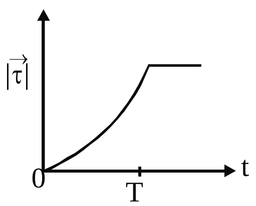
(D) there is no friction between the ring and the ground

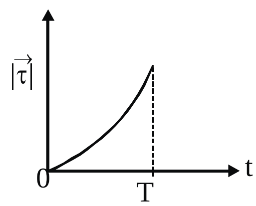
7. A thin uniform rod, pivoted at OP, is rotating in the horizontal plane with constant angular speed ω. as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. If reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\tau|)$ on the system about O, as a function of time is best represented by which plot? **[IIT-JEE 2012]**



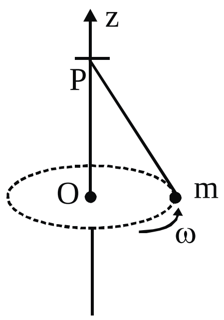
(A) 

(B) 

(C) 

(D) 

8. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



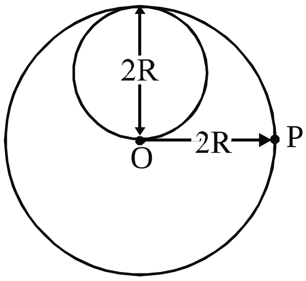
(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

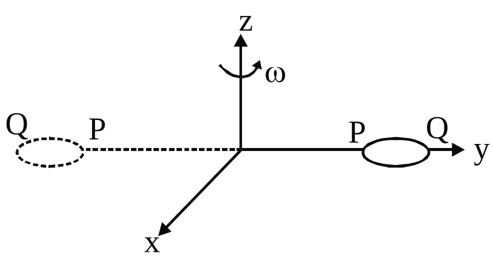
(D) $|\vec{L}_O|$ and $|\vec{L}_P|$ both do not vary with time **[IIT-JEE 2012]**

9. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes assing through O and P is I_0 and I_p respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is **[IIT-JEE 2012]**

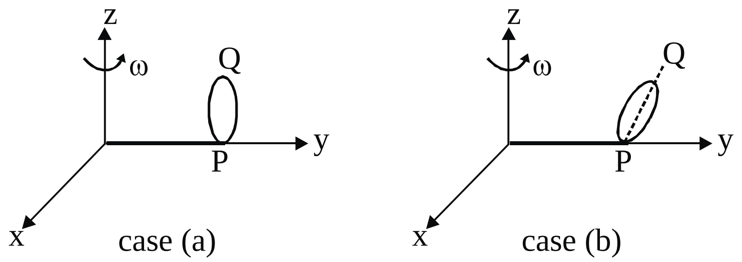


Paragraph for Questions 10 and 11

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure : case (A) the disc with its face vertical and parallel to x-z plane; Case (B) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis. **[IIT-JEE 2012]**



10. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2\omega}$ for both the cases.

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2\omega}$ for case (b).

(D) It is ω for both the cases.

11. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

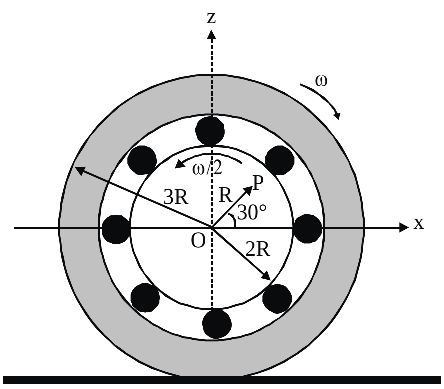
(A) It is vertical for both the cases (a) and (b).

(B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(D) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

12. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearing. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **[IIT-JEE 2012]**



(A) the point O has a linear velocity $3R\omega\hat{i}$

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

13. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders P and Q reach the ground at the same time. **[IIT-JEE 2012]**

(B) Cylinder P has larger acceleration than cylinder Q.

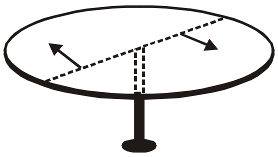
(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

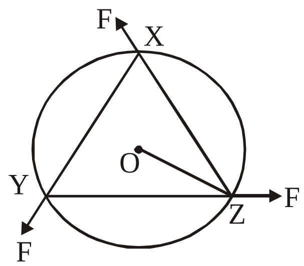
14. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[IIT-JEE 2013]**

15. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

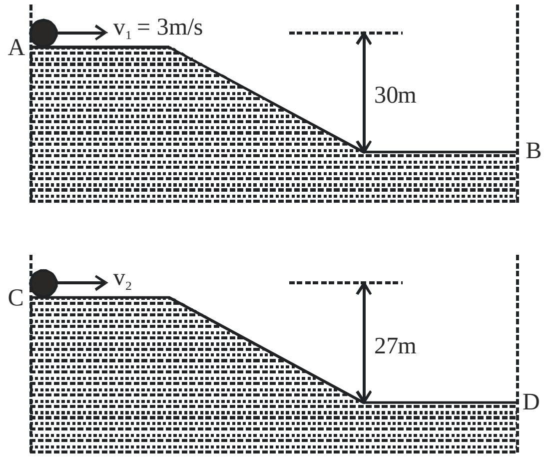
[JEE Advanced-2014]



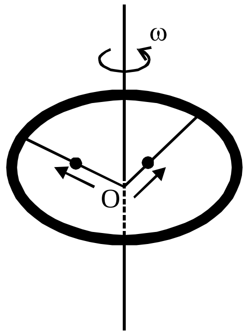
16. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is **[JEE Advanced-2014]**



17. Two identical uniform discs roll without slipping on two different, surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$m/s, then v_2 in m/s is ($g = 10$ m/s^2) **[JEE Advanced-2015]**



18. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$ R from O. At this instant the distance of the other mass from O is : **[JEE Advanced-2015]**



(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

19. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the indivisual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is. **[JEE Advanced-2015]**

20. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are: ($g = 10\ ms^{-2}$) **[JEE Advanced-2016]**

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} N$

(B) $\frac{h}{\ell} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3} N$

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3} N$

(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} N$

21. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, where $\alpha = \frac{10}{3} ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? **[JEE Advanced-2016]**

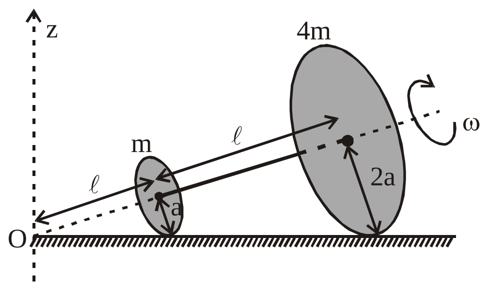
(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\left(\frac{5}{3}\right)\hat{k}$ Nms

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right)\hat{k}$ Nm

22. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, right rod of length $\ell = \sqrt{24}a$ throught their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true ? **[JEE Advanced-2016]**



(A) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(B) The magnitude of the z-component of $\vec{L}$ is $55\ ma^2\omega$

(C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$

(D) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

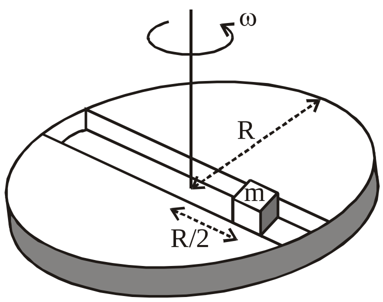
Paragraph for Question No. 23 and 24

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{\upsilon}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot. **[JEE Advanced-2016]**



23. The distance r of the block at time t is :

(A) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$

(B) $\frac{R}{2}\cos 2\omega t$

(C) $\frac{R}{2}\cos \omega t$

(D) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

24. The net reaction of the disc on the block is :

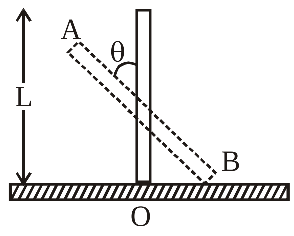
(A) $-m\omega^2 R \cos \omega t\hat{j} - mg\hat{k}$

(B) $m\omega^2 R \sin \omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

(D) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

25. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct ? **[JEE Advanced-2017]**



(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

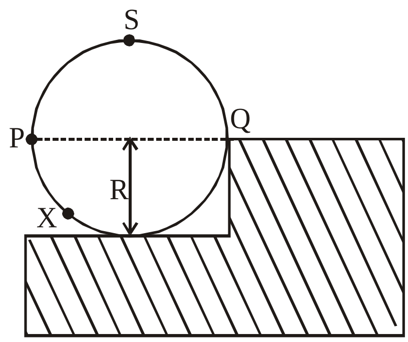
(B) The midpoint of the bar will fall vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(D) The trajectory of the point A is a parabola

26. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct ?

[JEE Advanced-2017]



(A) If the force is applied normal to the circumference at point X then τ is constant

(B) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(C) If the force is applied normal to the circumference at point P then τ is zero

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

Paragraph for Question no. 27 & 28

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g. **[JEE Advanced-2017]**

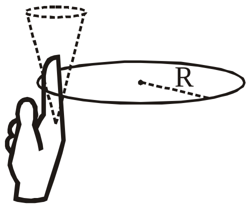


Figure 1

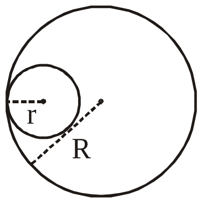


Figure 2

27. The total kinetic energy of the ring is :-

(A) $M\omega_0^2R^2$ (B) $M\omega_0^2(R-r)^2$ (C) $\frac{1}{2}M\omega_0^2(R-r)^2$ (D) $\frac{3}{2}M\omega_0^2(R-r)^2$

28. The minimum value of ω_0 below which the ring will drop down is :-

(A) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$ (C) $\sqrt{\frac{2g}{\mu(R-r)}}$ (D) $\sqrt{\frac{2g}{2\mu(R-r)}}$

29. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ?

[JEE Advanced-2018]

(A) $v=\sqrt{\frac{k}{2m}}R$ (B) $v=\sqrt{\frac{k}{m}}R$ (C) $L=\sqrt{mk}R^2$ (D) $L=\sqrt{\frac{mk}{2}}R^2$

30. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where α = 1.0 Ns^{-1} and β = 1.0 N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true? **[JEE Advanced-2018]**

(A) $|\vec{\tau}| = \frac{1}{3}\text{Nm}$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}\text{m}$

31. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}\text{s}$, then the height of the top of the inclined plane, in meters, is ______. Take g = 10 ms^{-2}.

[JEE Advanced-2018]

32. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? [g is the acceleration due to gravity] **[JEE Advanced-2019]**

(1) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(2) The angular acceleration of the rod will be $\frac{2g}{L}$

(3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(4) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. 2

2. Ans. $\frac{ml^2}{12}$

3. Ans. 2.00 N·m, (b) $\hat{k}$

4. Ans. 8 kg

5. Ans. $150(+\hat{i})$

6. Ans. 30°

7. Ans. 10^{-3} rad/s^2 clockwise

8. Ans. 2

9. Ans. $\frac{L}{3}$

10. Ans. (a) $\frac{9g}{7}\downarrow$ (b) $\frac{4mg}{7}\uparrow$

11. Ans. (i) 10/13m/s^2, (ii) 5000/26π,(iii) 480/13N

12. Ans. 2N

13. Ans. $M = 2m\left(\frac{2gh}{R^2\omega^2} - 1\right)$

14. Ans. $\sqrt{5gR}$

15. Ans. (a) $\omega = \sqrt{\frac{6g}{\ell}}$, (b) $T = \frac{11mg}{4}$

16. Ans. $\frac{\omega}{3}$

17. Ans. $\frac{4\pi}{5}$

18. Ans. 75 J

19. Ans. 1/2 ma

20. Ans. 15

21. Ans. 16 m/s^2

22. Ans. $6\,mv_0^2$

23. Ans. $\frac{13}{16}$ Mv^2

24. Ans. $\frac{l^2}{12x}$

25. Ans. (i) $\frac{J}{m}$ (ii) zero (iii) $\frac{J}{2m}$ (iv) $\frac{5}{2}\frac{J}{m}$

26. Ans. (a) $t = \frac{\pi Rm}{2I}$; (b) $s = \frac{\pi R}{2}$

27. Ans. $\frac{mv}{4}$

28. Ans. $\frac{2mv_0^2}{l}$

EXERCISE (O-1)

1. Ans. (C) **2. Ans. (B)** **3. Ans. (B)** **4. Ans. (D)** **5. Ans. (A)** **6. Ans. (A)**
7. Ans. (B) **8. Ans. (D)** **9. Ans. (C)** **10. Ans. (A)** **11. Ans. (C)** **12. Ans. (B)**
13. Ans. (D) **14. Ans. (D)** **15. Ans. (B)** **16. Ans. (C)** **17. Ans. (B)** **18. Ans. (D)**
19. Ans. (C) **20. Ans. (C)** **21. Ans. (A)** **22. Ans. (B)** **23. Ans. (C)** **24. Ans. (D)**
25. Ans. (A) **26. Ans. (D)** **27. Ans. (B)** **28. Ans. (C)** **29. Ans. (A)** **30. Ans. (D)**
31. Ans. (B) **32. Ans. (B)** **33. Ans. (A)** **34. Ans. (A)** **35. Ans. (B)** **36. Ans. (C)**
37. Ans. (B) **38. Ans. (B)** **39. Ans. (A)** **40. Ans. (D)** **41. Ans. (B)** **42. Ans. (D)**
43. Ans. (C) **44. Ans. (A)** **45. Ans. (C)** **46. Ans. (D)** **47. Ans. (B)** **48. Ans. (B)**
49. Ans. (A) **50. Ans. (A,B,C,D)** **51. Ans. (B,C)** **52. Ans. (A,D)** **53.Ans. (A,C,D)**
54. Ans. (A,C,D) **55. Ans. (A,B,C,D)** **56. Ans. (A)** **57. Ans. (A)**
58. Ans. (D) **59. Ans. (B)** **60. Ans. (A)** **61. Ans. (D)** **62. Ans. (D)**
63. Ans. (A)→(R); (B)→(P); (C)→(T); (D)→(Q,S)

EXERCISE (O-2)

1. Ans. (D) 2. Ans. (C) 3. Ans. (B) 4. Ans. (C) 5. Ans. (A) 6. Ans. (D)
7. Ans. (B) 8. Ans. (B) 9. Ans. (B) 10. Ans. (B) 11. Ans. (D) 12. Ans. (C)
13. Ans. (A) 14. Ans. (C) 15. Ans. (C) 16. Ans. (D) 17. Ans. (C) 18. Ans. (A)
19. Ans. (C) 20. Ans. (C,D) 21. Ans. (B,C,D) 22. Ans. (A,B,C) 23.Ans. (A,B,D)
24. Ans. (A,C) 25. Ans. (A,B,C) 26. Ans. (A,B,C) 27.Ans. (B,C,D)
28. Ans. (A,C,D) 29. Ans. (A,C) 30. Ans. (B,D) 31. Ans. (A,B,C)
32. Ans. (C) 33. Ans. (A) 34. Ans. (C) 35. Ans. (C) 36. Ans. (B) 37. Ans. (C)
38. Ans. (B) 39. Ans. (B) 40. Ans. (C) 41. Ans. (C) 42. Ans. (B) 43. Ans. (B)
44. Ans. (C) 45. Ans. (B) 46. Ans. (A) 47. Ans. (C) 48. Ans. (A,B,C)
49. Ans. (C) 50. Ans. (B,C,D) 51. Ans. (A,C,D)
52. Ans. (A) ST (B) R (C) PRT (D) PQ 53. Ans. (A)-R, (B)-S, (C)-P, (D)-Q

EXERCISE (JM)

1. Ans. (2) 2. Ans. (4) 3. Ans. (4) 4. Ans. (2) 5. Ans. (4) 6. Ans. (3)
7. Ans. (1) 8. Ans. (4) 9. Ans. (1) 10. Ans. (1 or 3) 11. Ans. (2) 12. Ans. (3)
13. Ans. (3) 14. Ans. (4) 15. Ans. (3)

EXERCISE (JA)

1. Ans. (B) 2. Ans. (A) 3. Ans. (B,C) 4. Ans. 4 5. Ans. 9 6. Ans.(A C or C)
7. Ans. (B) 8. Ans. (C, D) 9. Ans. 3 10. Ans. (D) 11. Ans. (A) 12. Ans. (A,B)
13. Ans. (D) 14. Ans. 8 15. Ans. 4 16. Ans. 2 17. Ans. 7 18. Ans. (C, D)
19. Ans. 6 20. Ans. (D) 21. Ans. (A, B, D) 22. Ans. (D) 23. Ans. (D)
24. Ans. (C) 25. Ans. (A), (B), (C) 26. Ans. (B,C or C)
27. Ans. (Bonus) 28. Ans. (B) 29. Ans. (B,C) 30. Ans. (A,C)
31. Ans. 0.75 [0.74, 0.76] 32. Ans. (1,3,4)