PHYSICS

INDEX

MODULE-3

CHAPTER 1

SIMPLE HARMONIC MOTION

Chapter Contents 01

IMPORTANT NOTES

CHAPTER 1

SIMPLE HARMONIC MOTION

KEY CONCEPTS

PERIODIC MOTION

- Any motion which repeats itself after regular interval of time is called periodic motion.
- The constant interval of time after which the motion is repeated is called time period.

  Examples : (i) Motion of planets around the sun. (ii) Motion of the pendulum of wall clock.

OSCILLATORY MOTION

- The motion of body is said to be oscillatory if it moves back and forth (to and fro) about a fixed point after regular interval of time. Oscillation of very high frequency & small amplitude is called vibration.
- The fixed point about which the body oscillates is called mean position or equilibrium position.

  Examples : (i) Vibration of the wire of 'Sitar'.

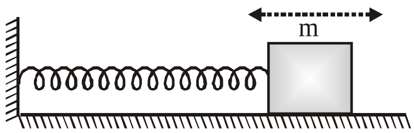
  (ii) Oscillation of the mass suspended from spring.

SIMPLE HARMONIC MOTION (S.H.M.)

Simple harmonic motion is the simplest form of oscillatory motion.

(i) S.H.M. are of two types

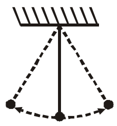
- **Linear S.H.M. :** When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.



Example : Motion of a mass connected to spring.

- **Angular S.H.M.**

  When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.



  Example :- Motion of a bob of simple pendulum.

(ii) Necessary Condition to execute S.H.M.

- **In linear S.H.M. :** The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

  $\therefore \quad F \propto -x \quad \text{or} \quad a \propto -x$

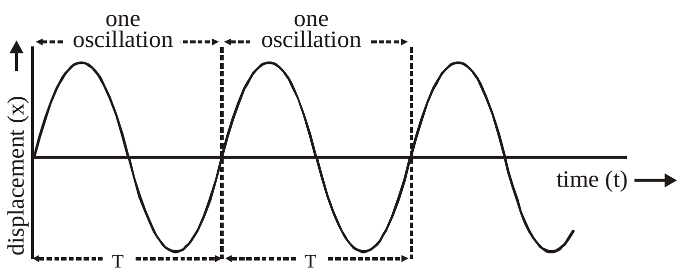
  Negative sign shows that direction of force and acceleration is towards equilibrium position and x is displacement of particle from equilibrium position.

- **In angular S.H.M. :** The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

  $\therefore \quad \tau \propto -\theta \quad$ or $\alpha \propto -\theta$

SOME BASIC TERMS

- **Mean Position :** The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.
- **Restoring Force**
  - The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.
  - Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- **Amplitude :** The maximum (positive or negative) value of displacement of particle from mean position is define as amplitude.
- **Time period (T)**
  - The minimum time after which the particle keeps on repeating its motion is known as time period.
  - The smallest time taken to complete one oscillation or vibration is also define as time period.
  - It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.
- **Oscillation :** When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



- **Frequency (n or f)**
  - The number of oscillations per second is define as frequency.
  - It is given by $n = \frac{1}{T}$, $n = \frac{\omega}{2\pi}$
  - **SI UNIT :** hertz (Hz), 1 hertz = 1 cycle per second (cycle is a number not a dimensional quantity).
  - **Dimensions :** $M^0 L^0 T^{-1}$.

- **Phase** :
  - Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.
  - In the equation $x = A \sin(\omega t + \theta)$, $(\omega t + \theta)$ is the phase of the particle.
  - The phase angle at time $t = 0$ is known as initial phase or epoch.
  - The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.
  - Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2n\pi$ Where $n = 0, 1, 2, 3,....$
  - Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ Where $n = 0, 1, 2, 3,....$
- **Angular frequency (ω)** :

  The rate of change of phase angle of a particle with respect to time is define as its angular frequency.

  SI unit **:** radian/second, *Dimensions* **:** $M^0 L^0 T^{-1}$,

DISPLACEMENT IN S.H.M.

(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.

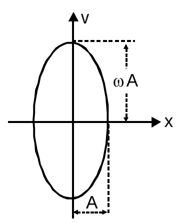
(ii) It can be given by relation $x = A\sin\omega t$ or $x = A\cos\omega t$.

The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

VELOCITY IN SHM

(i) It is define as the time rate of change of the displacement of the particle at the given instant.

(ii) Velocity in S.H.M. is given by $v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) \Rightarrow v = A\omega\cos\omega t$



$$v = \pm A\omega\sqrt{1-\sin^2\omega t} \Rightarrow v = \pm A\omega\sqrt{1-\frac{x^2}{A^2}} = \pm\omega\sqrt{(A^2 - x^2)}\ [\because\ x = A\sin\omega t]$$

Squaring both the sides $v^2 = \omega^2 (A^2 - x^2) \Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2 \Rightarrow \frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.

(iii) The graph between velocity and displacement is shown in figure. If particle oscillates with unit angular frequency ($\omega = 1$) then curve between v and x will be circular.

Note :

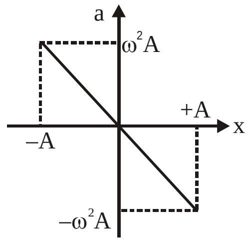
(i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position.

(ii) At mean position (x = 0), velocity is maximum (=$A\omega$) and at extreme position ($x = \pm A$), the velocity of particle executing S.H.M. is zero (minimum).

ACCELERATION IN SHM

(i) It is define as the time rate of change of the velocity of the particle at given instant.

(ii) Acceleration in S.H.M. is given by $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$



$a = -\omega^2 A \sin \omega t \Rightarrow a = -\omega^2 x$

(iii) The graph between acceleration and displacement as shown in figure

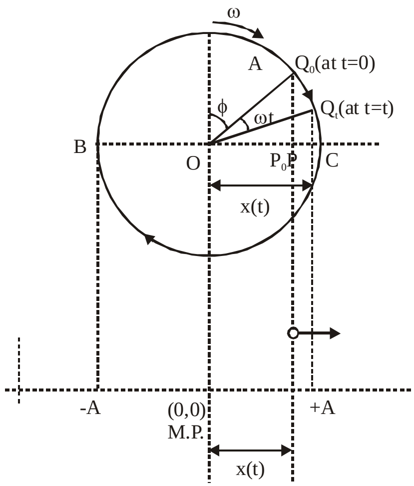
Note

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position (= $\omega^2 A$) and minimum at mean position (= zero)

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle Q, moving on a circle of radius A with constant angular velocity ω. The projection of Q on a diameter BC is P. It is clear from the figure that as Q moves around the circle the projection P executes a simple harmonic motion on the x-axis between B and C. The angle that the radius OQ makes with the +ve vertical in clockwise direction in at t = 0 is equal to phase constant (ϕ).



Let the radius OQ_0 makes an angle ωt with the OQ_t at time t. Then $x(t)=A\sin(\omega t+\phi)$

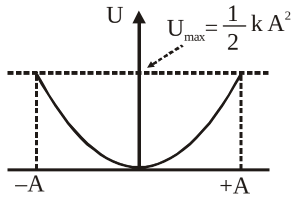
In the above discussion the foot of projection is x-axis so it is called horizontal phasor. Similarly the foot of perpendiuclar on y-axis will also executes SHM of amplitude A and angular frequency ω[$y(t)=A\cos\omega t$]. This is called vertical phasor. The phaser of the two SHM differ by $\pi/2$.

ENERGY OF PARTICLE IN S.H.M.

- **Potential Energy (U or P.E.)**

(i) In terms of displacement

The potential energy is related to force by the relation $F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F dx$

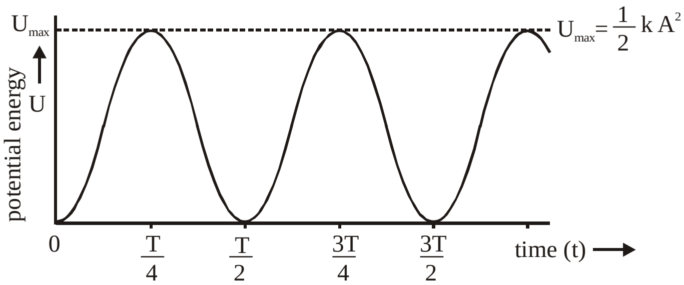


For S.H.M. F = – kx so $\int dU = -\int(-kx)dx = \int kx\,dx \Rightarrow U = \frac{1}{2}kx^2 + C$

At x = 0, $U = U_0 \Rightarrow C = U_0$ So $U = \frac{1}{2}kx^2 + U_0$

Where the potenital energy at equilibrium position = U_0 when $U_0 = 0$ then $U = \frac{1}{2}kx^2$

(ii) In terms of time



Since $x = A\sin(\omega t + \phi)$, $U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero then $U = \frac{1}{2}kA^2\sin^2\omega t = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

Note :

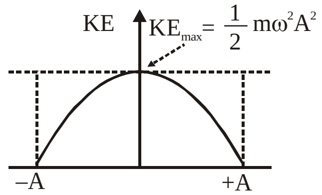
(i) In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position (x = 0) and maximum at extreme position (x = ± A)

(ii) The potential energy is the periodic function of time. For x = A sin (ωt), it is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}...$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}...$

- **Kinetic Energy (K)**

(i) In terms of displacement

If mass of the particle executing S.H.M. is m and its velocity is v then kinetic energy at any instant.



$$K = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k(A^2 - x^2)$$

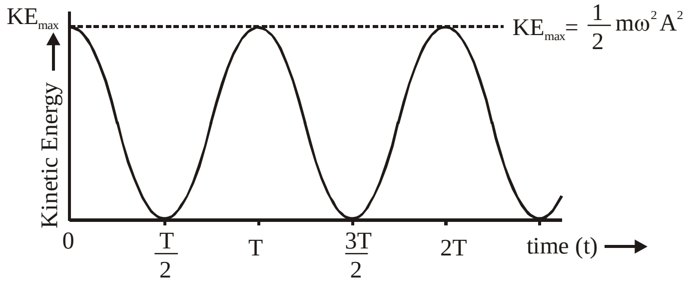
(ii) In terms of time

$$v = A\omega\cos(\omega t + \phi)$$

$$K = \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)$$

If initial phase ϕ is zero

$$K = \frac{1}{2} m\omega^2 A^2 \cos^2\omega t$$



Note :

(i) In S.H.M. the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum ($\frac{1}{2}kA^2$) at mean position ($x = 0$) and minimum (zero) at extreme position ($x = \pm A$)

(ii) The kinetic energy is the periodic function of time. For $x = A \sin(\omega t)$, it is maximum at $t = 0, T, 2T, 3T$.............and minimum at $t = \frac{T}{2}, \frac{3T}{2}, \frac{5T}{2} \ldots$

- **Total energy (E)**

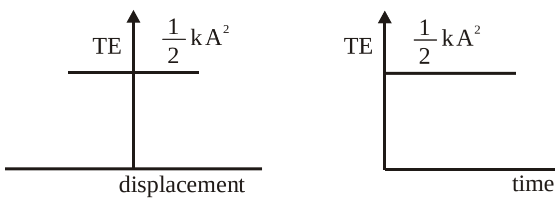
Total energy in S.H.M. is given by ; E = potential energy + kinetic energy = U + K

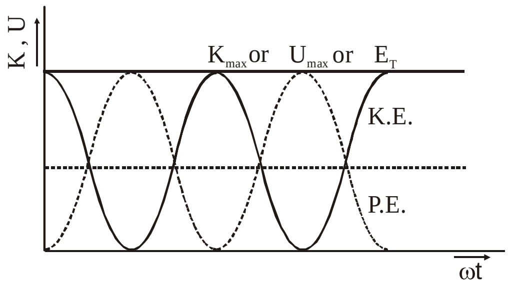
(i) w.r.t. position $E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \Rightarrow E = \frac{1}{2}kA^2 = \text{constant}$

(ii) w.r.t. time

$$E = \frac{1}{2}m\omega^2 A^2 \sin^2\omega t + \frac{1}{2}m\omega^2 A^2 \cos^2\omega t = \frac{1}{2}m\omega^2 A^2 \left(\sin^2\omega t + \cos^2\omega t\right) = \frac{1}{2}m\omega^2 A^2$$

$$= \frac{1}{2}kA^2 = \text{constant}$$





Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum

- **Second's pendulum**

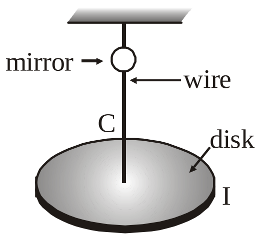
If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

COMPOUND PENDULUM

Any rigid body which is free to oscillate in a vertical plane about a horizontal axis passing through a point, is define compound pendulum

- **Torsional Oscillator :** (Angular SHM)

$$T = 2\pi\sqrt{\frac{I}{C}} \quad \text{where } C = \frac{\eta\pi r^4}{2\ell}$$



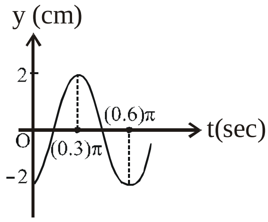
η = modulus of elasticity of the wire ; r = radius of the wire

L = length of the wire ; I = Moment of inertia of the disc

EXERCISE (S-1)

Kinematics of SHM :

1. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is :-



2. The displacement of a body executing SHM is given by $x = A \sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is.

3. A body undergoing SHM about the origin has its equation given by $x = 0.2 \cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

Energy of SHM :

4. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point $x = 0.04$m the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ________.

5. A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3}J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

6. Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by $U = (x^2 - 4x + 3)$. All units are in S.I.

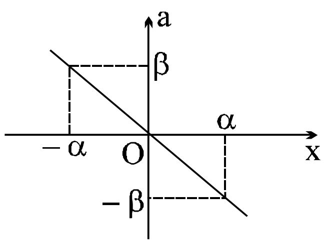
(i) Find the equilibrium position of the body.

(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

Time Period :

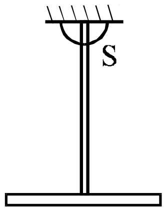
7. The acceleration-displacement (a – x) graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



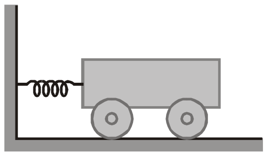
8. A small body of mass m is fixed to the middle of a stretched string of length 2ℓ. In the equilibrium position the string tension is equal to T_0. Find the angular frequency of small oscillations of the body in the transverse direction. The mass of the string is negligible, the gravitational field is absent.

9. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1 : 3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

10. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



11. A cart consists of a body and four wheels on frictionless axles. The body has a mass m. The wheels are uniform disks of mass M and radius R. The cart rolls, without slipping, back and forth on a horizontal plane under the influence of a spring attached to one end of the cart (figure). The spring constant is k. Taking into account the moment of inertia of the wheels, find a formula for the frequency of the back and forth motion of the cart.

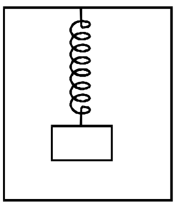


12. A mass M attached to a spring, oscillates with a period of 2 sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hook's Law is obeyed.

Complex situations :

13. A spring mass system is hanging from the ceiling of an elevator in equilibrium. Elongation of spring is l. The elevator suddenly starts accelerating downwards with acceleration g/3, find

(a) the frequency and (b) the amplitude of the resulting SHM.



14. (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$J/rad. The moment of inertia of rigid body is 10 kg m^2 about the axis of rotation.

(b) A simple pendulum of length $l = 0.5$ m is hanging from ceiling of a car. The car is kept on a horizontal plane. The car starts accelerating on the horizontal road with acceleration of 5 m/s^2. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.

15. The motion of a simple pendulum is given by

$$\theta = A\cos\left(\sqrt{\frac{g}{\ell}}\,t\right) \text{ (symbols have their usual meaning)}$$

(a) Find the tension in the string of this pendulum as a function of time assume that $\theta << \ell$.

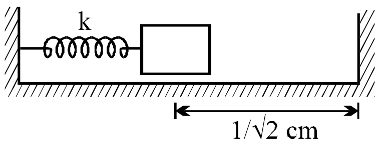
(b) At what time is the tension maximum? What is the value of this maximum tension?

16. A physical pendulum has the shape of a disk of radius R. The pendulum swings about an axis perpendicular to the plane of the disk and at distance ℓ from the center of the disk.

(a) Show that the frequency of the oscillations of this pendulum is $\omega = \sqrt{\dfrac{g\ell}{\frac{1}{2}R^2 + \ell^2}}$

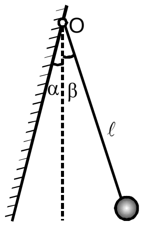
(b) For what value of ℓ is this frequency at a maximum ?

17. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.

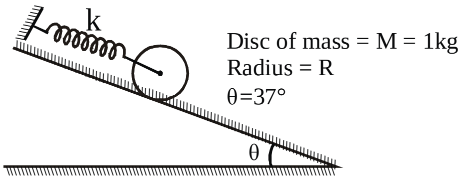


18. A body of mass 1 kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

19. A ball is suspended by a thread of length ℓ at the point O on the wall, forming a small angle α with the vertical as shown in figure. Then the thread with the ball was deviated through a small angle β ($\beta > \alpha$) and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

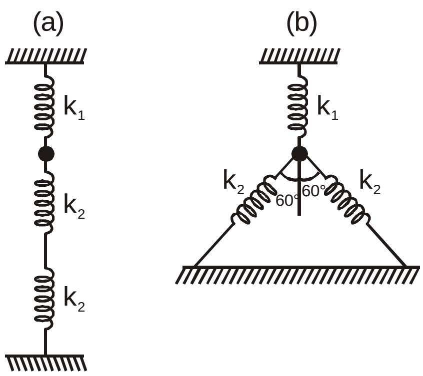


20. A disc of mass m is connected to an ideal spring of force constant 'k'. If disc is released from rest, then what is maximum friction force on disc (in N). Assuming friction is sufficient for pure rolling



21. A particle of mass 5×10^{-5} kg is placed at the lowest point of a smooth parabola having the equation $20x^2 = y$ (x,y in m). Here y is the vertical height. If it is displaced slightly and it moves such that it is constrained to move along the parabola, the angular frequency of oscillation will be, (in rad/s). If your answer is N fill value N/4.

22. Mass m is suspended by ideal massless springs in two different ways, indicated by (a) and (b) in the figure. The mass is displaced upwards by a small amount from equilibrium position and is then released resulting in a SHM of the mass in the vertical direction. We denote the oscillation frequencies associated with the two cases (a) and (b) by f_a and f_b respectively. Find $\frac{f_a}{f_b}$. Given $k_2 = 2k_1$.



23. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[IIT-JEE 2004]**

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Kinematics of SHM :

1. The equation of motion of a particle is $x = a \cos (\alpha t)^2$. The motion is

(A) periodic but not oscillatory. (B) periodic and oscillatory.

(C) oscillatory but not periodic. (D) neither periodic nor oscillatory.

2. **Statement 1** : Position–time equation of a particle moving along x–axis is x=4 + 6 sinωt. Under this situation, motion of particle is not simple harmonic.

and

Statement 2 : $\frac{d^2x}{dt^2}$ for the given equation is not proportional to –x.

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.

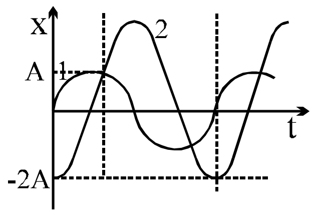
(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

3. A simple harmonic motion having an amplitude A and time period T is represented by the equation : y = 5 sin π(t + 4) m. Then the values of A (in m) and T (in sec) are :

(A) A = 5; T = 2 (B) A = 10 ; T = 1 (C) A = 5 ; T = 1 (D) A = 10 ; T = 2

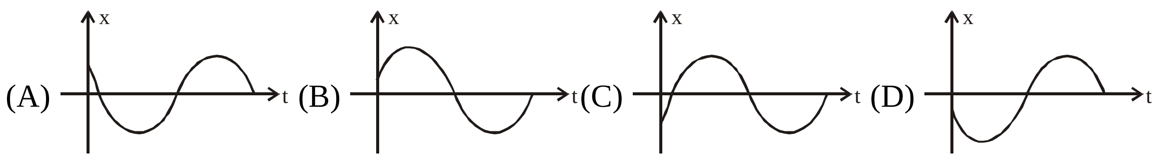
4. The oscillations represented by curve 1 in the graph are expressed by equation x = A sinωt. The equation for the oscillations represented by curve 2 is expressed as:



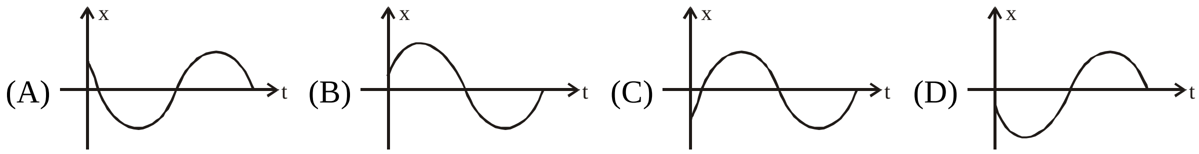
(A) x = 2A sin (ωt – π/2) (B) x = 2A sin (ωt + π/2)

(C) x = – 2A sin (ωt – π/2) (D) x = A sin (ωt – π/2)

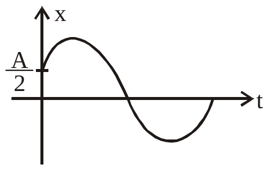
5. A particle performing S.H.M. about mean position x = 0 and at t = 0 it is at position $x = \frac{A}{\sqrt{2}}$ and moving towards the origin. Then which of the following is its possible graph between position (x) and time (t)



6. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $= \frac{A}{\sqrt{2}}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. After $t = \frac{3T}{2}$ what will be the graph of the particle :-



7. Following graph shows a particle performing S.H.M. about mean position x = 0. The equation of particle if $t = \frac{T}{4}$ is taken as starting time is (Notations have usual meanings)



(A) $A\sin\left(\omega t + \frac{2\pi}{3}\right)$ (B) $A\sin\left(\omega t + \frac{\pi}{3}\right)$ (C) $A\sin\left(\omega t + \frac{\pi}{6}\right)$ (D) $A\cos\left(\omega t + \frac{2\pi}{3}\right)$

8. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $\frac{\sqrt{3}A}{2}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. Then what was the position and direction of particle at $t = 0$?

(A) $-\frac{A}{2}$, away from mean position (B) $\frac{A}{2}$, away from mean position

(C) $\frac{A}{2}$, towards mean position (D) $-\frac{A}{2}$, towards mean position

9. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)

(A) 2 (B) 8 (C) 6 (D) 4

10. A bob is attached to a long, light string. The string is deflected by 3° initially with respect to vertical. The length of the string is 1 m . The value of θ at any time t after the bob released can be approximately written as (Use : $g = \pi^2$)

(A) 3° cos πt (B) 3° sin πt (C) $3° \sin(\pi t + \frac{\pi}{6})$ (D) $3° \cos(\pi t + \frac{\pi}{6})$

11. A particle performing SHM is found at its equilibrium at t = 1sec. and it is found to have a speed of 0.25 m/s at t = 2 sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

(A) $\frac{3}{2\pi}$ m (B) $\frac{3}{4\pi}$ m (C) $\frac{6}{\pi}$ m (D) $\frac{3}{8\pi}$

12. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(A) a/T (B) 2a/T (C) 3a/T (D) a/2T

13. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is :-

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

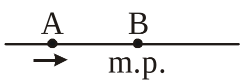
14. Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles

(A) 45° (B) 90° (C) 135° (D) 180°

15. Two pendulums have time periods T and 5T/4. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase:

(A) 5 (B) 4 (C) 11 (D) 9

16. A particle is oscillating simple harmonically with angular frequency ω and amplitude A . It is at a point (A) at certain instant (shown in figure). At this instant it is moving towards mean position (B). It takes time t to reach mean position (B). If time period of oscillation is T, the average speed between A and B is :-



(A) $\frac{A\sin\omega t}{t}$ (B) $\frac{A\cos\omega t}{t}$ (C) $\frac{A\sin\omega t}{T}$ (D) $\frac{A\cos\omega t}{T}$

17. A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in $second^{-1}$) is:

(A) $2\pi\sqrt{3}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2\pi}$ (D) $\frac{1}{2\pi\sqrt{3}}$

18. Speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is given by $v^2 = 108 - 9x^2$ (all quantities in S.I. unit). Then

(A) The motion is uniformly accelerated along the straight line

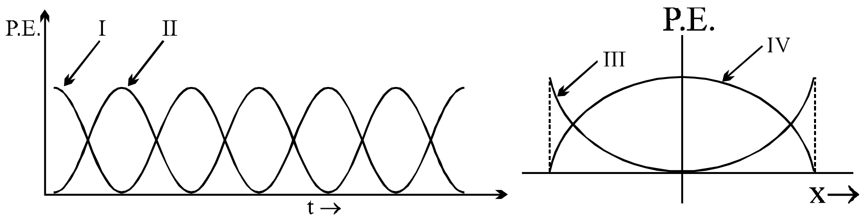
(B) The magnitude of the acceleration at a distance 3 cm from the fixed point is 0.27 m/s^2.

(C) The motion is simple harmonic about x = $\sqrt{12}$ m.

(D) The maximum displacement from the fixed point is 4 cm.

Energy of SHM :

19. A particle is executing SHM according to $x = a \cos \omega t$. Then which of the graphs represents variations of potential energy : **[IIT JEE (Scr) 2003]**



(A) (I) & (III) (B) (II) & (IV) (C) (I) & (IV) (D) (II) & (III)

20. Potential energy of a particle is given as $U(x) = 2x^3 - 9x^2 + 12x$ where U is in joule and x is in metre. If the motion of a particle is S.H.M., then find the approx potential energy of the particle :-

(A) – 36 J (B) 4 J (C) 5 J (D) None of these

21. If the potential energy of a harmonic oscillator of mass 2 kg on its equilibrium position is 5 joules and the total energy is 9 joules. If the amplitude is one meter then period of the oscillator (in sec) is:

(A) 1.5 (B) 3.14 (C) 6.28 (D) 4.67

22. Kinetic energy of a particle executing simple harmonic motion in straight line is pv^2 and potential energy is qx^2, where v is speed at distance x from the mean position. It time period is given by the expression :-

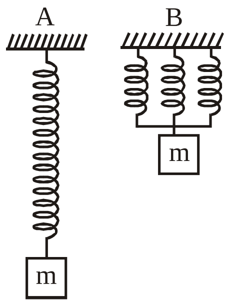
(A) $2\pi\sqrt{\frac{q}{p}}$ (B) $2\pi\sqrt{\frac{p}{q}}$ (C) $2\pi\sqrt{\frac{q}{p+q}}$ (D) $2\pi\sqrt{\frac{p}{p+q}}$

Time Period :

23. The angular frequency of motion whose equation is $4\frac{d^2y}{dt^2} + 9y = 0$ is (y = displacement and t = time)

(A) $\frac{9}{4}$ (B) $\frac{4}{9}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

24. The springs in figure A and B are identical but length of spring in A is three times than the length of each spring in B. The ratio of period T_A/T_B is :-

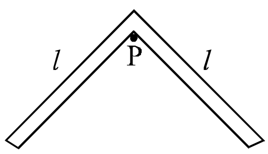


(A) $\sqrt{3}$ (B) 1/3 (C) 3 (D) $1/\sqrt{3}$

25. A rod whose ends are A & B of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9 : 4 at points A and B. Find the distance of the centre of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

26. A system of two identical rods (L-shaped) of mass *m* and length *l* are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations:

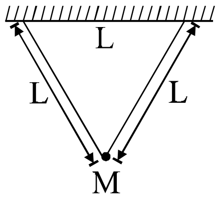


(A) $2\pi\sqrt{\frac{\sqrt{2}l}{3g}}$ (B) $2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$ (C) $2\pi\sqrt{\frac{2l}{3g}}$ (D) $3\pi\sqrt{\frac{l}{3g}}$

27. A ring of diameter 2m oscillates as a compound pendulum about a horizontal axis passing through a point at its rim. It oscillates such that its centre move in a plane which is perpendicular to the plane of the ring. The equivalent length of the simple pendulum is :-

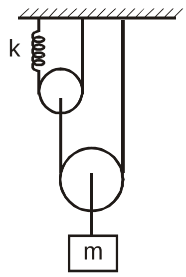
(A) 2m (B) 4m (C) 1.5m (D) 3m

28. A man is swinging on a swing made of 2 ropes of equal length L and in direction perpendicular to the plane of paper. The time period of the small oscillations about the mean position is :-



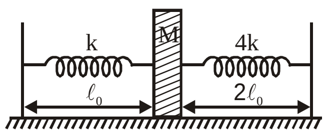
(A) $2\pi\sqrt{\frac{L}{2g}}$ (B) $2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$ (C) $2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$ (D) $\pi\sqrt{\frac{L}{g}}$

29. What is the period of small oscillations of the block of mass m if the springs are ideal and pulleys are massless ?



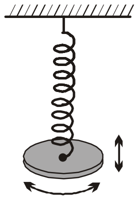
(A) $\frac{\pi}{2}\sqrt{\frac{m}{k}}$ (B) $\frac{\pi}{2}\sqrt{\frac{m}{2k}}$ (C) $\frac{\pi}{2}\sqrt{\frac{2m}{k}}$ (D) $\pi\sqrt{\frac{m}{2k}}$

30. A block of mass M is kept in gravity free space and touches the two springs as shown in the figure. Initially springs are in their natural lengths. Now, the block is shifted ($\ell_0/2$) from the given position in such a way that it compresses a spring and is released. The time-period of oscillation of mass will be:-



(A) $\frac{\pi}{2}\sqrt{\frac{M}{K}}$ (B) $2\pi\sqrt{\frac{m}{5K}}$ (C) $\frac{3\pi}{2}\sqrt{\frac{M}{K}}$ (D) $\pi\sqrt{\frac{M}{2K}}$

31. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system?



(A) $\sqrt{\frac{2c}{k}}$ (B) $\sqrt{\frac{c}{2k}}$ (C) $2\sqrt{\frac{c}{k}}$ (D) $\frac{1}{2}\sqrt{\frac{c}{k}}$

Superposition :

32. The amplitude of the vibrating particle due to superposition of two SHMs,

$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin \omega t$ is :

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

33. Two simple harmonic motions $y_1 = A \sin \omega t$ and $y_2 = A \cos \omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is:

(A) $\frac{1}{2}m\omega^2A^2$ (B) $m\omega^2A^2$ (C) $\frac{1}{4}m\omega^2A^2$ (D) zero

34. The displacement of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$.

(A) The motion is oscillatory but not S.H.M.

(B) The motion is S.H.M. with amplitude $a + b$.

(C) The motion is S.H.M. with amplitude $a^2 + b^2$.

(D) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$

35. Equations $y = 2A \cos^2 \omega t$ and $y = A (\sin \omega t + \sqrt{3} \cos \omega t)$ represent the motion of two particles.

(A) Only one of these is S.H.M. (B) Ratio of maximum speeds is 2 : 1

(C) Ratio of maximum speeds is 1 : 1 (D) Ratio of maximum accelerations is 1 : 4

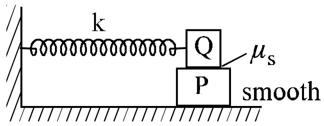
Complex situations :

36. A plank with a small block on top of it is under going vertical SHM. Its period is 2 sec. The minimum amplitude at which the block will separate from plank is :

(A) $\frac{10}{\pi^2}$ (B) $\frac{\pi^2}{10}$ (C) $\frac{20}{\pi^2}$ (D) $\frac{\pi}{10}$

37. A block P of mass m is placed on a frictionless horizontal surface. Another block Q of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between P and Q. The blocks move together performing SHM of amplitude A. The maximum value of the friction force between P and Q is :-

[IIT JEE 2004]

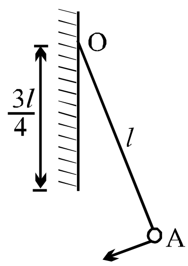


(A) kA (B) $\frac{kA}{2}$ (C) zero (D) $\mu_s mg$

38. Two bodies P & Q of equal mass are suspended from two separate massless springs of force constants k_1 & k_2 respectively. If the maximum velocity of them are equal during their motion, the ratio of amplitude of P to Q is :

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$ (C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

39. A small bob attached to a light inextensible thread of length l has a periodic time T when allowed to vibrate as a simple pendulum. The thread is now suspended from a fixed end O of a vertical rigid rod of length $\frac{3l}{4}$ (as in figure). If now the pendulum performs periodic oscillations in this arrangement, the periodic time will be

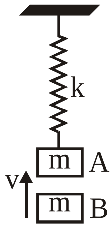


(A) $\frac{3T}{4}$ (B) $\frac{T}{2}$ (C) T (D) 2T

40. Vertical displacement of a plank with a body of mass 'm' on it is varying according to law $y = \sin \omega t + \sqrt{3} \cos \omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after $t = 0$ are given by: (y is positive vertically upwards)

(A) $\sqrt{\frac{g}{2}}, \frac{\sqrt{2}}{6} \frac{\pi}{\sqrt{g}}$ (B) $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\frac{\pi}{g}}$ (C) $\sqrt{\frac{g}{2}}, \frac{\pi}{3}\sqrt{\frac{2}{g}}$ (D) $\sqrt{2g}, \sqrt{\frac{2\pi}{3g}}$

41. Block A is hanging from a vertical spring and is at rest. Block B strikes the block A with velocity v and sticks to it. Then the value of v for which the spring just attains natural length is



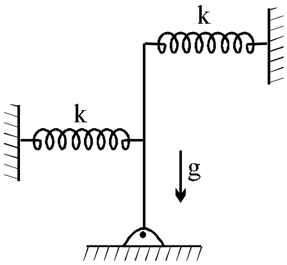
(A) $\sqrt{\frac{60mg^2}{k}}$ (B) $\sqrt{\frac{6mg^2}{k}}$ (C) $\sqrt{\frac{10mg^2}{k}}$ (D) $\sqrt{\frac{8mg^2}{k}}$

42. A particle of mass 10 gm moves in a field where potential energy per unit mass is given by expression $v = 8 \times 10^4 x^2$ erg/gm . If the total energy of the particle is 8×10^7 erg then the relation between x and time t is :[ϕ = constant]

(A) $x = 10 \sin (400\, t + \phi)$ cm (B) $x = \sin (400\, t + \phi)$ m

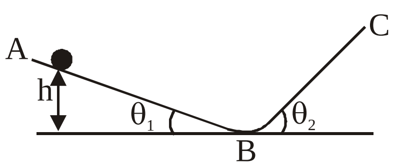
(C) $x = 10 \sin (40\, t + \phi)$ cm (D) $x = 100 \sin (4\, t + \phi)$ m

43. In the figure shown, the spring are connected to the rod at one end and at the midpoint. The rod is hinged at its lower end. Rotational SHM of the rod (Mass m, length L) will occur only if



(A) k > mg/3L (B) k > 2mg/3L (C) k > 2mg/5L (D) k > 0

44. A small glass bead of mass m initially at rest starts from a point at height h above the horizontal and rolls down the inclined plane AB as shown. Then it rises along the inclined plane BC. Assuming no loss of energy, the time period of oscillation of the glass bead is :



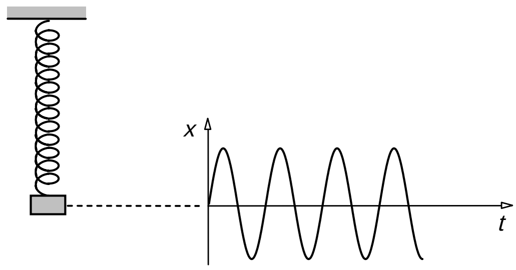
(A) $\sqrt{\frac{8h}{g}}(\sin\theta_1 + \sin\theta_2)$ (B) $2\sqrt{\frac{14h}{5g}}\left(\frac{1}{\sin\theta_1} + \frac{1}{\sin\theta_2}\right)$

(C) $\sqrt{\frac{8h}{g}}\left(\frac{1}{\sin\theta_1} + \frac{1}{\sin\theta_2}\right)$ (D) $\sqrt{\frac{8h}{5g}}\left(\frac{1}{\sin\theta_1} + \frac{1}{\sin\theta_2}\right)$

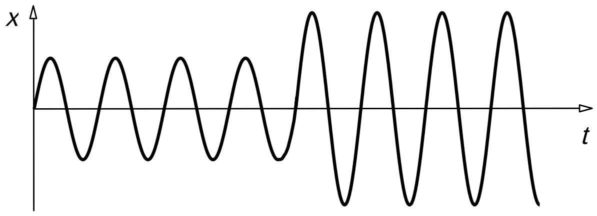
EXERCISE (O-2)

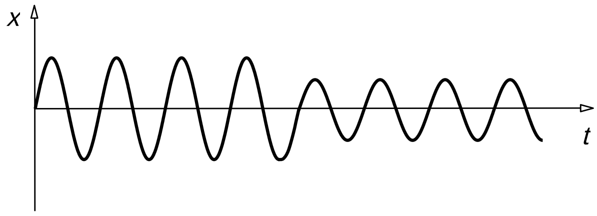
SINGLE CORRECT TYPE QUESTIONS

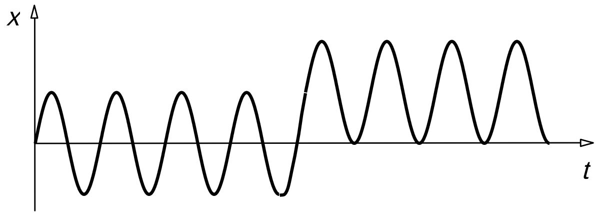
1. In the figure is shown a spring-mass system oscillating in uniform gravity. If we neglect all dissipative forces, it will keep on oscillating endlessly with constant amplitude and frequency. Accompanying graph shows how displacement x of the block from the equilibrium position varies with time t.

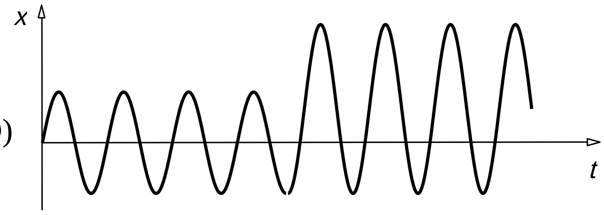


Now at a certain instant $t = t_o$ when the block reaches its lowest position, gravity is switched off by some unknown mechanism. Which of the following graphs would correctly describes the changes taking place due to switching off the gravity?

(A) 

(B) 

(C) 

(D) 

2. Time period of a particle executing SHM is 8 sec. At t = 0 it is at the mean position. The ratio of the distance covered by the particle in the 1st second to the 2nd second is :

(A) $\frac{1}{\sqrt{2}+1}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

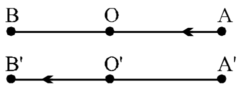
3. A particle executes SHM with time period T and amplitude A. The maximum possible average velocity in time $\frac{T}{4}$ is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$ (C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

4. A particle moves along the x-axis according to : x = A.[1 + sin ωt]. What distance does it travel between t = 0 and t = 2.5π/ω?

(A) 4A (B) 6A (C) 5A (D) None

5. Two particles undergo SHM along parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time



(A) T/8 (B) 3T/8 (C) T/6 (D) 4T/3

6. Two particles are in SHM in a straight line about same equilibrium position. Amplitude A and time period T of both the particles are equal. At time t=0, one particle is at displacement y_1= +A and the other at y_2= –A/2, and they are approaching towards each other. After what time they cross each other ?

(A) T/3 (B) T/4 (C) 5T/6 (D) T/6

7. Two particles A and B perform SHM along the same straight line with the same amplitude 'a', same frequency 'f' and same equilibrium position 'O'. The greatest distance between them is found to be $\frac{3a}{2}$. At some instant of time they have the same displacement from mean position. What is the displacement?

(A) a/2 (B) $a\sqrt{7}/4$ (C) $\sqrt{3}a/2$ (D) $3a/4$

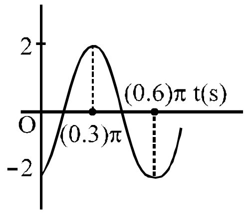
8. A wire frame in the shape of an equilateral triangle is hinged at one vertex so that it can swing freely in a vertical plane, with the plane of the Δ always remaining vertical. The side of the frame is $1/\sqrt{3}$ m. The time period in seconds of small oscillations of the frame will be

(A) $\frac{\pi}{\sqrt{2}}$ (B) $\pi\sqrt{2}$ (C) $\frac{\pi}{\sqrt{6}}$ (D) $\frac{\pi}{\sqrt{5}}$

9. For a particle acceleration is defined as $\vec{a} = \frac{-5x\hat{i}}{|x|}$ for x ≠ 0 and $\vec{a}$ = 0 for x = 0. If the particle is initially at rest at (a, 0), what is period of motion of the particle.

(A) $4\sqrt{2a/5}$ sec. (B) $8\sqrt{2a/5}$ sec. (C) $2\sqrt{2a/5}$ sec. (D) cannot be determined

10. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is



(A) y = 4 cos (0.6t)

(B) $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

(C) $y = 4\sin\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

(D) $y = 2\cos\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

11. The angular frequency of a spring block system is ω_0. This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as a positive direction, choose the wrong statement:

(A) The amplitude of the block is $\frac{v_0}{\omega_0}$

(B) The initial phase of the block is π.

(C) The equation of motion for the block is $\frac{v_0}{\omega_0}\sin\omega_0 t$.

(D) The maximum speed of the block is v_0.

12. Two particles are performing SHM with same angular frequency and amplitudes A and 2A respectively along same straight line with same mean position. They cross each other at position A/2 distance from mean position in opposite direction. The phase between them is :

(A) $\frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (C) $\frac{5\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$

13. A particle executing a simple harmonic motion of period 2s. When it is at its extreme displacement from its mean position, it receives an additional energy equal to what it had in its mean position. Due to this, in its subsequent motion,

(A) its amplitude will change and become equal to $\sqrt{2}$ times its previous amplitude

(B) its periodic time will become doubled i.e. 4s

(C) its potential energy will be decreased

(D) it will continue to execute simple harmonic motion of the same amplitude and period as before receiving the additional energy.

14. A particle is executing SHM of amplitude A, about the mean position x = 0. Which of the following cannot be a possible phase difference between the positions of the particle at x = + $A/2$ and x = − $A/\sqrt{2}$.

(A) 75° (B) 165° (C) 135° (D) 195°

15. A particle free to move along the x-axis has potential energy given by $U(x) = k[1-\exp(-x^2)]$ for $-\infty < x < +\infty$, where k is a positive constant of appropriate dimensions. Then

(A) at point away from the origin, the particle is in unstable equilibrium.

(B) for any finite non-zero value of x, there is a force directed away from the origin.

(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.

(D) for small displacements from x = 0, the motion is simple harmonic.

16. The time period of a bar pendulum when suspended at distances 30 cm and 50 cm from its centre of gravity comes out to be the same. If the mass of the body is 2kg. Find out its moment of inertia about an axis passing through first point.

(A) 0.24 kg-m^2 (B) 0.72 kg-m^2 (C) 0.48 kg-m^2 (D) Data insufficient

MULTIPLE CORRECT TYPE QUESTIONS

17. For a SHM with given angular frequency, two arbitrary initial conditions are necessary and sufficient to determine the motion completely. These initial conditions may be-

(A) Amplitude and initial phase (B) Amplitude and total energy of oscillation

(C) Initial phase and total energy of oscillation (D) Initial position and initial velocity

18. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve.

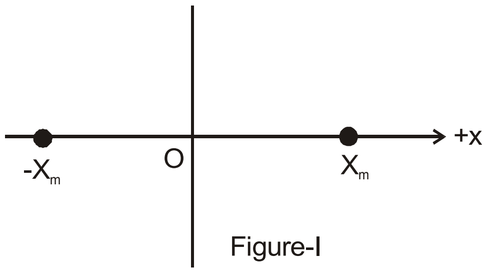


Figure-I

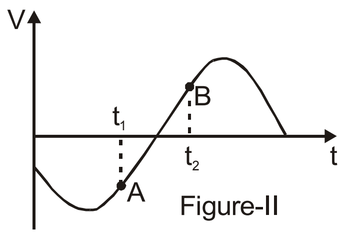


Figure-II

(A) At time t_1 , it is going towards X_m.

(B) At time t_1, its speed is decreasing.

(C) At time t_2, its position lies in between $-X_m$ and O.

(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

19. For a body executing SHM with amplitudes A, time period T, max velocity v_{max} and phase constant zero, which of the following statements are correct?

(A) At $y = (A/2)$, $v > (v_{max} / 2)$ (B) $v = (v_{max}/2)$ for $|y| > (A/2)$

(C) For $t = (T/8)$, $y > (A/2)$ (D) For $y = (A/2)$, $t < (T/8)$

20. The amplitude of a particle executing SHM about O is 10 cm. Then :

(A) When the K.E. is 0.64 of its max. K.E. its displacement is 6cm from O.

(B) When the displacement is 5 cm from O its K.E. is 0.75 of its max.P.E.

(C) Its total energy at any point is equal to its maximum K.E.

(D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement.

21. The position vector of a particle that is moving in space is given by

$$\vec{r} = (1 + 2\cos 2\omega t)\hat{i} + (3\sin^2 \omega t)\hat{j} + (3)\hat{k}$$

in the ground frame. All units are in SI. Choose the correct statement (s) :

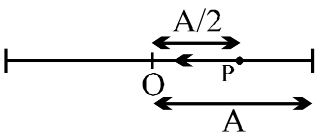
(A) The particle executes SHM in the ground frame about the mean position $\left(1, \frac{3}{2}, 3\right)$

(B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

(C) The amplitude of the SHM of the particle is $\frac{5}{2}$ m.

(D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right)$

22. A particle starts from a point P at a distance of A/2 from the mean position O & travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of motion of particle is :



(A) $x = A\sin\left(\frac{2\pi}{T}t + \frac{\pi}{6}\right)$

(B) $x = A\sin\left(\frac{2\pi}{T}t + \frac{5\pi}{6}\right)$

(C) $x = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{6}\right)$

(D) $x = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{3}\right)$

23. A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to ceiling. The block released from the position, where the spring has length 45 cm.

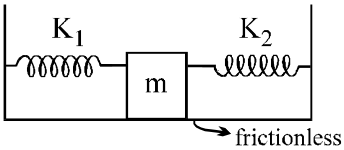
(A) the block will perform SHM of amplitude 5 cm.

(B) the block will have maximum velocity $30\sqrt{5}$ cm/sec.

(C) the block will have maximum acceleration 15 m/s^2

(D) the minimum potential energy of the spring will be zero.

24. Two springs with negligible masses and force constant of $K_1 = 200\ Nm^{-1}$ and $K_2 = 160\ Nm^{-1}$ are attached to the block of mass m = 10 kg as shown in the figure. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time t = 0, a sharp impulse of 50 Ns is given to the block with a hammer.



(A) Period of oscillations for the mass m is $\frac{\pi}{3}$ s.

(B) Maximum velocity of the mass m during its oscillation is 5 ms^{-1}.

(C) Data are insufficient to determine maximum velocity.

(D) Amplitude of oscillation is 0.42 m.

25. A mass of 0.2kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?

(A) In equilibrium, the spring will be stretched by 1cm.

(B) If the mass is raised till the spring is unstretched state and then released, it will go down by 2cm before moving upwards.

(C) The frequency of oscillation will be nearly 5 Hz.

(D) If the system is taken to the moon, the frequency of oscillation will be the same as on the earth.

26. A particle moves in x-y plane according to equation $\vec{S} = \left(2\hat{i} + \hat{j}\right)\cos\omega t$. The motion of particle is:-

(A) On straight line (B) On ellipse (C) Periodic (D) SHM

27. Two particles are in SHM with same amplitude A and same angular frequency ω. At time t = 0, one is at $x = +\frac{A}{2}$ and other is at $x = -\frac{A}{2}$. Both are moving in same direction.

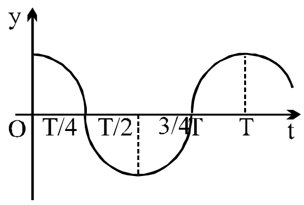
(A) Phase difference between the two particle is $\frac{\pi}{3}$

(B) Phase difference between the two particle is $\frac{2\pi}{3}$

(C) They will collide after time $t = \frac{\pi}{2\omega}$

(D) They will collide after time $t = \frac{3\pi}{\omega}$

28. The displacement-time graph of a particle executing SHM is shown. Which of the following statements is/are true?



(A) The velocity is maximum at t = T/2

(B) The acceleration is maximum at t = T

(C) The force is zero at t = 3T/4

(D) The potential energy equals the oscillation energy at t = T/2.

29. A particle is executing SHM with amplitude A, time period T, maximum acceleration a_o and maximum velocity v_0. Its starts from mean position at t = 0 and at time t, it has the displacement A/2, acceleration a and velocity v then :-

(A) t = T/12 (B) $a = a_o/2$ (C) $v = v_o/2$ (D) t = T/8

30. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then :-

(A) the resultant amplitude is $(1 + \sqrt{2})$ a

(B) the phase of the resultant motion relative to the first is 90°

(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

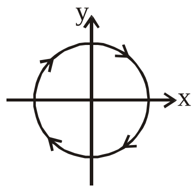
(D) the resulting motion is not simple harmonic

COMPREHENSION TYPE QUESTIONS

Paragraph for question no. 31 and 32

Lissajous figures are produced by superposition of 2 SHM's in mutually perpendicular directions. If both SHM have same frequency, the lissajous figure is simple. It may be a circle, ellipse or a straight line depending on the phase difference between two SHMs. Interesting case occurs when one of the SHM is at double frequency of another. Suppose a body executes SHM vertically with frequency f_0, but horizontally with a frequency $2f_0$ and is initially at A, mean position of vertical as well as horizontal SHM. It can be seen to trace out a figure of 8 in space as shown. Since horizontal SHM has half the time period, it executes two horizontal oscillations by the time it completes a vertical oscillation.

31. A lissajous figure as shown here can be produced by



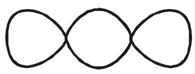
(A) $x = A \sin \omega t$; $y = A \cos \omega t$

(B) $x = A \cos \omega t$; $y = A \sin \omega t$

(C) $x = A \sin \omega t$; $y = A \sin \left(\omega t + \frac{\pi}{4}\right)$

(D) $x = A \sin \omega t$; $y = A \sin \left(\omega t + \frac{3\pi}{4}\right)$

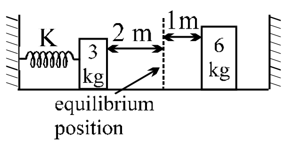
32. For the Lissajous figure shown here, the frequency in vertical direction is _______ times the frequency in horizontal direction :-



(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

Paragraph for Question No. 33 and 34

Two blocks of masses 3 kg and 6 kg rest on on a horizontal frictionless surface. The 3 kg block is attached to a spring with a force constant K = 900 N/m which is compressed 2 m initially from its equilibrium position. When 3 kg mass is released, it strikes the 6 kg mass and the two stick together.



33. The common velocity of the blocks after collision is :-

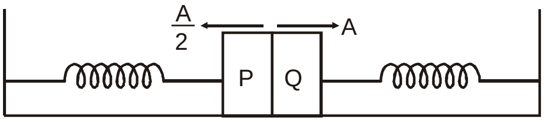
(A) 10 m/s (B) 30 m/s (C) 15 m/s (D) 2 m/s

34. The amplitude of resulting oscillation after the collision is :-

(A) $\frac{1}{\sqrt{2}}$ m (B) $\frac{1}{\sqrt{3}}$ m (C) $\sqrt{2}$ m (D) $\sqrt{3}$ m

Paragraph for Question no. 35 and 36

Two identical blocks P and Q have mass m each. They are attached to two identical springs initially unstretched. Now the left spring (along with P) is compressed by $\frac{A}{2}$ and the right spring (along with Q) is compressed by A. Both the blocks are released simultaneously. They collide perfectly inelastically. Initially time period of both the block was T.



35. The time period of oscillation of combined mass is :

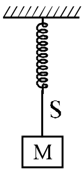
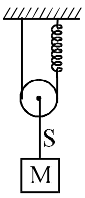
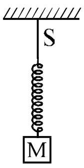
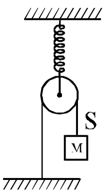
(A) $\frac{T}{\sqrt{2}}$ (B) $\sqrt{2}T$ (C) T (D) $\frac{T}{2}$

36. The amplitude of combined mass is :

(A) $\frac{A}{4}$ (B) $\frac{A}{2}$ (C) $\frac{2A}{3}$ (D) $\frac{3A}{4}$

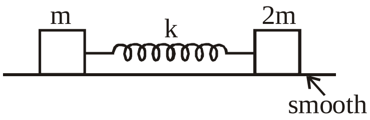
MATRIX MATCH TYPE QUESTION

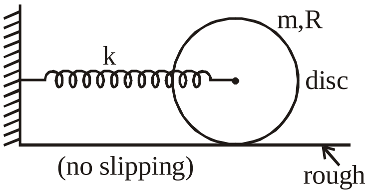
37. In the following four situations, mass M of 1kg is kept in equilibrium. k = 100 N/m in all cases. What speed can be given to mass M vertically so that inextensible string S does not become slack in subsequent motion. Consider that pulley is ideal and string is massless :

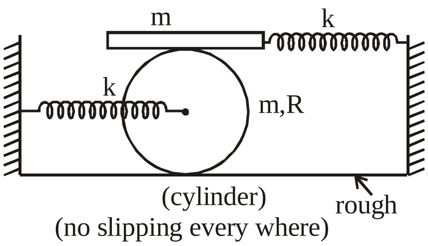
Column-I	Column-II
(A)  (the block is attached to spring by an inextensible thread)	(P) $1\ ms^{-1}$
(B) 	(Q) $0.5\ ms^{-1}$
(C)  (block is attached to the spring)	(R) $0.25\ ms^{-1}$
(D) 	(S) $2\ ms^{-1}$

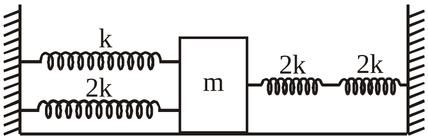
38. In list-I, the systems are performing SHM and in list-II, the time period of SHM is shown then match list-I with list-II.

List-I

(P) 

(Q) 

(R) 

(S) 

List-II

(1) $\left[2\pi\sqrt{\frac{11m}{10k}}\right]$

(2) $2\pi\sqrt{\frac{m}{4k}}$

(3) $2\pi\sqrt{\frac{3m}{2k}}$

(4) $2\pi\sqrt{\frac{2m}{3k}}$

Codes :

	P	Q	R	S
(A)	3	2	1	4
(B)	4	3	2	1
(C)	3	4	2	1
(D)	4	3	1	2

EXERCISE (JM)

1. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :- **[AIEEE-2011]**

(1) $\left(\frac{M}{M+m}\right)^{1/2}$ (2) $\left(\frac{M+m}{M}\right)^{1/2}$ (3) $\frac{M}{M+m}$ (4) $\frac{M+m}{M}$

2. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :-

[AIEEE-2011]

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

3. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :- **[AIEEE-2011]**

(1) $2\pi\sqrt{\frac{\ell\rho}{(\rho-d)g}}$ (2) $2\pi\sqrt{\frac{\ell d}{\rho g}}$

(3) $2\pi\sqrt{\frac{\ell\rho}{dg}}$ (4) $2\pi\sqrt{\frac{\ell d}{(\rho-d)g}}$

4. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE Mains-2014]**

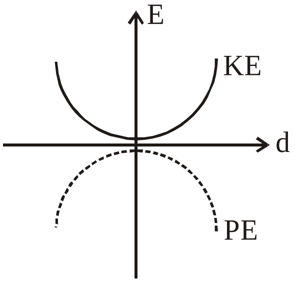
(1) Amplitude of motion is 4a

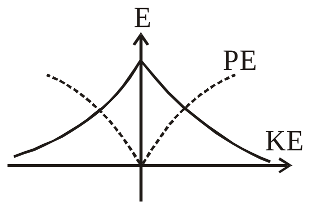
(2) Time period of oscillation is 6τ

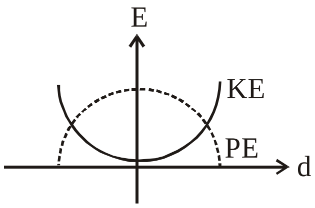
(3) Amplitude of motion is 3a

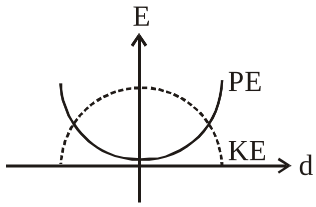
(4) Time period of oscillation is 8τ

5. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale) **[JEE Mains-2015]**

(1) 

(2) 

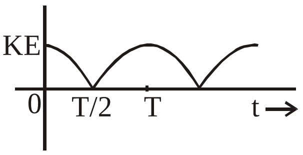
(3) 

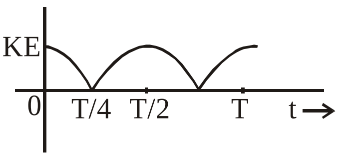
(4) 

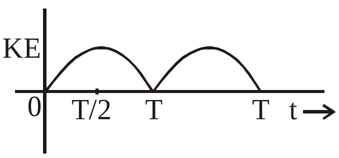
6. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :- **[JEE Mains-2016]**

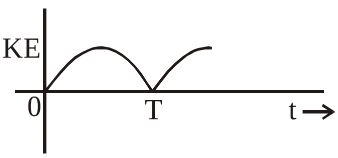
(1) $\frac{7A}{3}$ (2) $\frac{A}{3}\sqrt{41}$ (3) 3A (4) $A\sqrt{3}$

7. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **[JEE Main-2017]**

(1) 

(2) 

(3) 

(4) 

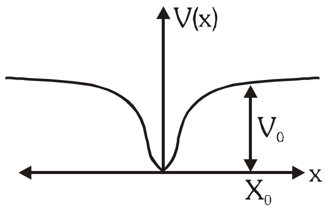
8. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[JEE Main-2018]**

(1) 7.1 N/m (2) 2.2 N/m (3) 5.5 N/m (4) 6.4 N/m

EXERCISE (JA)

Paragraph for Question No. 1 to 3

When a particle of mass m moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x–axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure) **[IIT-JEE 2010]**



1. If the total energy of the particle is E, it will perform periodic motion only if :-

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

2. For periodic motion of small amplitude A, the time period T of this particle is proportional to :-

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

3. The acceleration of this particle for $|x| > X_0$ is :-

(A) proportional to V_0

(B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$

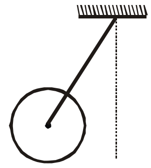
(D) Zero

4. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

[IIT-JEE 2011]

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

5. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotated about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is(are) true? **[IIT-JEE 2011]**



(A) Restoring torque in case A = Restoring torque in case B

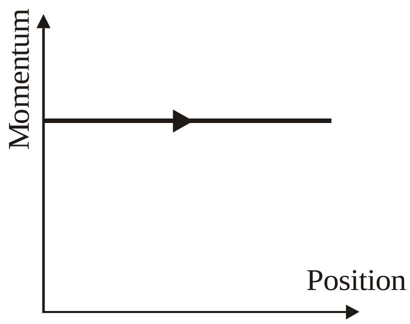
(B) Restoring torque in case A < Restoring torque in case B

(C) Angular frequency for case A> Angular frequency for case B

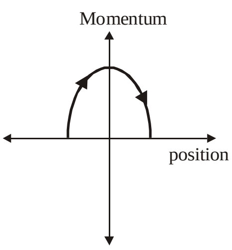
(D) Angular frequency for case A< Angular frequency for case B

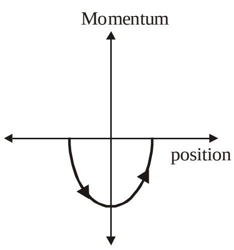
Paragraph for Questions Nos. 6 to 8

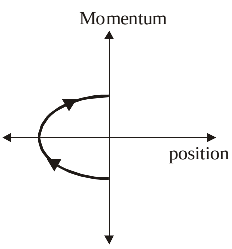
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. **[IIT-JEE 2011]**

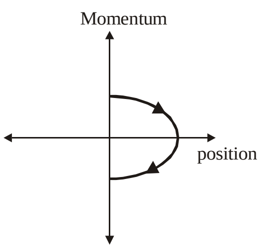


6. The phase space diagram for a ball thrown vertically up from ground is

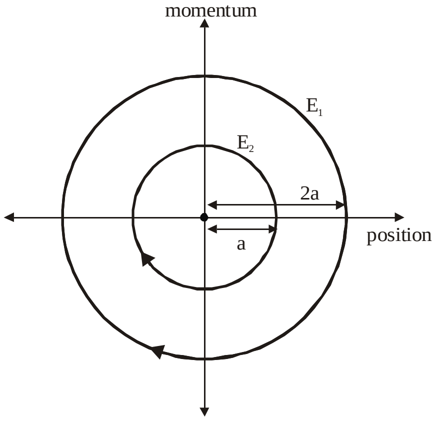
(A)


(B)


(C)


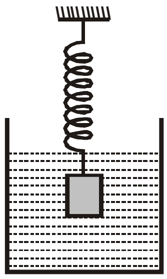
(D)


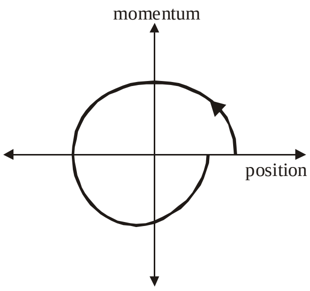
7. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

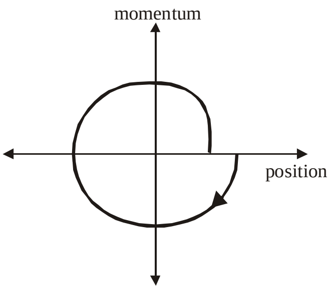


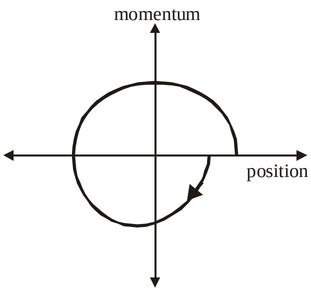
(A) $E_1 = \sqrt{2}E_2$ (B) $E_1=2E_2$ (C) $E_1=4E_2$ (D) $E_1=16E_2$

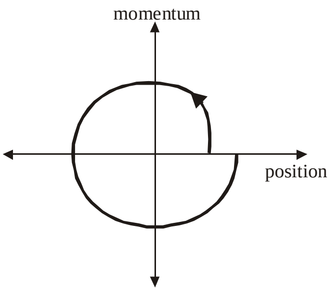
8. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


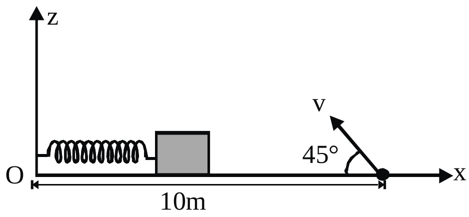
(B)


(C)


(D)


9. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. They system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10m from O. If the pebble hits the block at t = 1s, the value of v is:- (take g = 10 m/s^2)

[IIT-JEE 2012]



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

10. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision :-

[JEE-Advanced-2013]

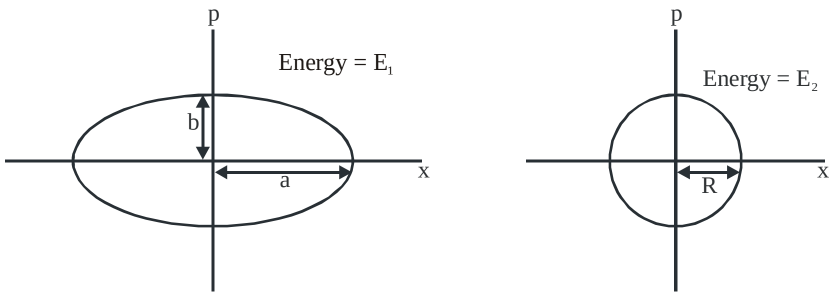
(A) the speed of the particle when it returns to its equilibrium position is u_0

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) the time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b}=n^2$ and $\frac{a}{R}=n$, then the correct equation (s) is (are) **[JEE-Advanced-2015]**



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1}=n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1}=\frac{E_2}{\omega_2}$

12. A particle of unit mass is moving along the x-axis under the influence of a force and its total enegy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-II **[JEE-Advanced-2015]**

Column-I	**Column-II**
(A) $U_1(x)=\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at x = a.
(B) $U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at x = 0.
(C) $U_3(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at x =–a.
(D) $U_4(x)=\frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards x = 0 in the region $\lvert x\rvert < a$.
	(T) The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point x = –a.

13. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M ? **[JEE-Advanced-2016]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

ANSWER KEY

EXERCISE (S-1)

1. Ans. $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$ **2. Ans.** 0.33 sec **3. Ans.** 2 m/s **4. Ans.** 0.06m

5. Ans. $y = 0.1\sin(4t + \pi/4)$ **6. Ans.** (i) $x_0 = 2$m; (ii) $T = \sqrt{2}\,\pi$ sec.; (iii) $2\sqrt{3}$

7. Ans. $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$ **8. Ans.** $\omega = \sqrt{\frac{2T_0}{m\ell}}$ **9. Ans.** $1/\sqrt{3}$ **10. Ans.** $2\pi\sqrt{\frac{17L}{18g}}$

11. Ans. $\omega = \sqrt{\frac{k}{(m+6M)}}$ **12. Ans. M = 1.6 kg** **13. Ans.** (a) $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, (b) $\frac{L}{3}$

14. Ans. (a) 2 sec, (b) $T = \frac{2}{5^{1/4}}$ sec

15. Ans. (a) $T = mg + mgA^2 \sin^2 \sqrt{\frac{g}{\ell}}\,t$ (b) $t = (2n+1)\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$; $n \in I$ $T_{max} = mg + mgA^2$

16. Ans. (b) $\frac{R}{\sqrt{2}}$ **17. Ans.** 100 Nm^{-1} **18. Ans.** $10/\pi$ Hz, $\frac{5\sqrt{37}}{6}$ cm

19. Ans. $T = 2\sqrt{\ell/g}\,[\pi/2 + \sin^{-1}(\alpha/\beta)]$ **20. Ans.** 2 **21. Ans.** 5 **22. Ans.** 1

23. Ans. $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

EXERCISE (O-1)

1. Ans. (C)	**2. Ans. (D)**	**3. Ans. (A)**	**4. Ans. (A)**	**5. Ans. (A)**	**6. Ans. (B)**
7. Ans. (A)	**8. Ans. (A)**	**9. Ans. (B)**	**10. Ans. (A)**	**11. Ans. (A)**	**12. Ans.(C)**
13. Ans. (C)	**14. Ans. (C)**	**15. Ans. (A)**	**16. Ans. (A)**	**17. Ans. (C)**	**18. Ans.(B)**
19. Ans. (A)	**20. Ans. (B)**	**21. Ans. (B)**	**22. Ans. (B)**	**22. Ans. (B)**	**23. Ans.(C)**
24. Ans. (C)	**25. Ans. (D)**	**26. Ans. (B)**	**27. Ans. (C)**	**28. Ans. (B)**	**29. Ans. (A)**
30. Ans. (C)	**31. Ans. (A)**	**32. Ans. (C)**	**33. Ans. (B)**	**34. Ans. (D)**	**35. Ans.(C)**
36. Ans. (A)	**37. Ans. (B)**	**38. Ans. (B)**	**39.Ans. (A)**	**40. Ans. (A)**	**41. Ans. (B)**
42.Ans. (A)	**43.Ans. (C)**	**44.Ans. (B)**			

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (D) 2. Ans. (D) 3. Ans. (D) 4. Ans. (C) 5. Ans. (B) 6. Ans. (D)

7. Ans. (B) 8. Ans. (D) 9. Ans. (A) 10. Ans. (B) 11. Ans. (B) 12. Ans.(A)

13. Ans. (A) 14. Ans. (C) 15. Ans. (D) 16. Ans. (C)

MULTIPLE CORRECT TYPE QUESTIONS

17. Ans. (A,D) 18. Ans. (B, C) 19. Ans. (A,B,C) 20. Ans. (A,B,C) 21. Ans. (A,C,D)

22. Ans. (B,D) 23. Ans. (B,C,D) 24. Ans. (A,B) 25. Ans. (A,B,C,D) 26. Ans. (A,C,D)

27. Ans. (A, C) 28. Ans. (B,C,D) 29. Ans. (A,B) 30. Ans. (A, C)

COMPREHENSION TYPE QUESTIONS

31. Ans. (A) 32. Ans. (A) 33. Ans. (A) 34. Ans. (C) 35. Ans. (C)

36. Ans. (A)

MATRIX MATCH TYPE QUESTION

37. Ans. (A)-P,Q,R ; (B)-Q,R ; (C)-P,Q,R ; (D)-P,Q,R,S 38. Ans. (D)

EXERCISE (JM)

1. Ans. (2) 2. Ans. (4) 3. Ans. (2) 4. Ans. (2)

5. Ans. (4) 6. Ans. (1) 7. Ans. (2) 8. Ans. (1)

EXERCISE (JA)

1. Ans. (C) 2. Ans. (B) 3. Ans. (D) 4. Ans. (B) 5. Ans. (A,D) 6. Ans. (D)

7. Ans. (C) 8. Ans. (B) 9. Ans. (A) 10. Ans. (A,D) 11. Ans. (B,D)

12. Ans. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T 13. Ans. (A,B,D)

CHAPTER 2

GRAVITATION

Chapter Contents 02

IMPORTANT NOTES

CHAPTER 2

GRAVITATION

KEY CONCEPTS

THE DISCOVERY OF THE LAW OF GRAVITATION

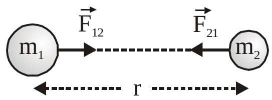
The way the law of universal gravitation was discovered is often considered the paradigm of modern scientific technique. The major steps involved were

- The hypothesis about planetary motion given by **Nicolaus Copernicus (1473–1543)**.
- The careful experimental measurements of the positions of the planets and the Sun by **Tycho Brahe (1546–1601)**.
- Analysis of the data and the formulation of empirical laws by **Johannes Kepler (1571–1630)**.
- The development of a general theory by **Isaac Newton (1642–1727)**.

NEWTON'S LAW OF GRAVITATION

It states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

$F \propto m_1 m_2$ and $F \propto \dfrac{1}{r^2}$ so $F \propto \dfrac{m_1 m_2}{r^2}$



$\therefore\ F = -\dfrac{Gm_1m_2}{r^2}\hat{r}$ [G = Universal gravitational constant]

Note : This formula is only applicable for spherical symmetric masses or point masses.

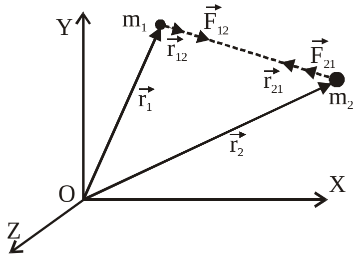
VECTOR FORM OF NEWTON'S LAW OF GRAVITATION :

Let $\vec{r}_{12}$ = Displacement vector from m_1 to m_2

$\vec{r}_{21}$ = Displacement vector from m_2 to m_1

$\vec{F}_{21}$ = Gravitational force exerted on m_2 by m_1

$\vec{F}_{12}$ = Gravitational force exerted on m_1 by m_2



$$\vec{F}_{12} = -\frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21} = -\frac{Gm_1m_2}{r_{21}^3}\vec{r}_{21}$$

Negative sign shows that :

(i) The direction of $\vec{F}_{12}$ is opposite to that $\hat{r}_{21}$

(ii) The gravitational force is attractive in nature

Similarly $\vec{F}_{21} = -\dfrac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$ or $\vec{F}_{21} = -\dfrac{Gm_1m_2}{r_{12}^3}\vec{r}_{12} \Rightarrow \vec{F}_{12} = -\vec{F}_{21}$

The gravitational force between two bodies are equal in magnitude and opposite in direction.

GRAVITATIONAL CONSTANT "G"

- Gravitational constant is a scalar quantity.
- **Unit :** **S I :** $G = 6.67 \times 10^{-11}$ N–m²/kg²

  CGS : 6.67×10^{-8} dyne–cm²/g²

  Dimensions : $M^{-1}L^{3}T^{-2}$
- Its value is same throughout the universe, G does not depend on the nature and size of the bodies, it also does not depend upon nature of the medium between the bodies.
- Its value was first find out by the scientist **"Henry Cavendish"** with the help of "Torsion Balance" experiment.
- Value of G is small therefore gravitational force is weaker than electrostatic and nuclear forces.

Ex. Two particles of masses 1 kg and 2 kg are placed at a separation of 50 cm. Assuming that the only forces acting on the particles are their mutual gravitation, find the initial acceleration of heavier particle.

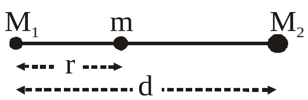
Sol. Force exerted by one particle on another $F = \dfrac{Gm_1m_2}{r^2} = \dfrac{6.67\times10^{-11}\times1\times2}{(0.5)^2} = 5.3\times10^{-10}\,N$

Acceleration of heavier particle = $\dfrac{F}{m_2} = \dfrac{5.3\times10^{-10}}{2} = 2.65\times10^{-10}\,ms^{-2}$

This example shows that gravitation is very weak but only this force keep bind our solar system and also this universe, all galaxies and other interstellar system.

Ex. Two stationary particles of masses M_1 and M_2 are at a distance 'd' apart. A third particle lying on the line joining the particles, experiences no resultant gravitational forces. What is the distance of this particle from M_1.

Sol. The force on m towards M_1 is $F_1 = \dfrac{GM_1m}{r^2}$



The force on m towards M_2 is $F_2 = \dfrac{GM_2m}{(d-r)^2}$

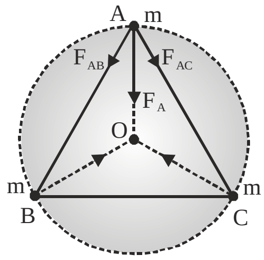
According to question net force on m is zero i.e. $F_1 = F_2$

$$\Rightarrow \frac{GM_1m}{r^2} = \frac{GM_2m}{(d-r)^2} \Rightarrow \left(\frac{d-r}{r}\right)^2 = \frac{M_2}{M_1}$$

$$\Rightarrow \frac{d}{r} - 1 = \frac{\sqrt{M_2}}{\sqrt{M_1}} \Rightarrow r = d\left[\frac{\sqrt{M_1}}{\sqrt{M_1}+\sqrt{M_2}}\right]$$

Ex. Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side 'a'. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining their original separation 'a'. Determine the initial velocity that should be given to each particle and time period of circular motion.

Sol. The resultant force on particle at A due to other two particles i



$$F_A = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\cos 60°} = \sqrt{3}\frac{Gm^2}{a^2} \quad ...(i) \quad \left[\because F_{AB} = F_{AC} = \frac{Gm^2}{a^2}\right]$$

Radius of the circle $r = \frac{2}{3} \times a \sin 60° = \frac{a}{\sqrt{3}}$

If each particle is given a tangential velocity v, so that F acts as the centripetal force,

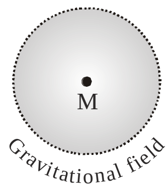
Now $\frac{mv^2}{r} = \sqrt{3}\frac{mv^2}{a}$ ...(ii)

From (i) and (ii) $\sqrt{3}\frac{mv^2}{a} = \frac{Gm^2\sqrt{3}}{a^2} \Rightarrow v = \sqrt{\frac{Gm}{a}}$

Time period $T = \frac{2\pi r}{v} = \frac{2\pi a}{\sqrt{3}}\sqrt{\frac{a}{Gm}} = 2\pi\sqrt{\frac{a^3}{3Gm}}$

- Gravitational forces are central forces as they act along the line joining the centres of two bodies.
- The gravitational forces are conservative forces so work done by gravitational force does not depends upon path and therefore if any particle moves along a closed path under the action of gravitational force then the work done by this force is always zero.
- The total gravitational force on one particle due to number of particles is the resultant of forces of attraction exerted on the given particle due to individual particles i.e. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$ it means the principle of superposition is valid.

GRAVITATIONAL FIELD

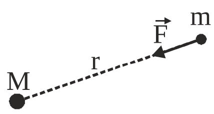


The gravitational field is the space around a mass or an assembly of masses over which it can exert gravitational forces on other masses.

Theoretically speaking, the gravitational field extends up to infinity. However, in actual practice, the gravitational field may become too weak to be measured beyond a particular distance.

Gravitational Field Intensity [g or E_g]

Gravitational force acting per unit mass at any point in the gravitational field is called Gravitational field intensity.



$$g = \frac{GMm}{r^2}\Big/m = \frac{GM}{r^2} \quad \text{Vector form : } \vec{g} = \frac{\vec{F}}{m} \text{ or } \vec{g} = -\frac{GM}{r^2}\hat{r}$$

Gravitational field intensity is a vector quantity having dimension $[LT^{-2}]$ and unit N/kg.

- Since the force between two point masses is having the similar expression as that of force between two point charges, we can write the gravitational field & gravitational potential in the same manner as the electric field & electric potential.

Analogy betwen Electrostatics & Gravitation

(1) Point Charge | **Point Mass**

(a) $E = \dfrac{kQ}{r^2}$ $\qquad$ $g = \dfrac{GM}{r^2}$

(b) $V = \dfrac{kQ}{r}$ $\qquad$ $V_G = \dfrac{-GM}{r}$

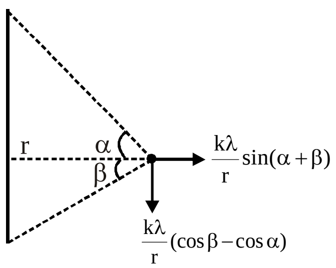
(2) Uniform charged ring | **Ring of uniform mass distribution**

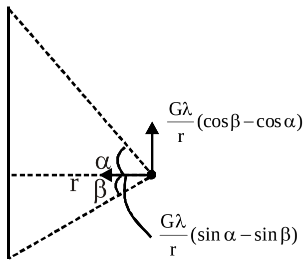
(a) $E = \dfrac{kQx}{(r^2+x^2)^{3/2}}$ on axis $\qquad$ $g = \dfrac{GMx}{(r^2+x^2)^{3/2}}$ on axis

E is max. when $x = \dfrac{r}{\sqrt{2}}$ $\qquad$ g is max. when $x = \dfrac{r}{\sqrt{2}}$

(b) $V = \dfrac{kQ}{\sqrt{r^2+x^2}}$ on axis , $\dfrac{kQ}{r}$ at center $\qquad$ $V_G = \dfrac{-GM}{\sqrt{r^2+x^2}}$ on axis, $\dfrac{-GM}{r}$ at center

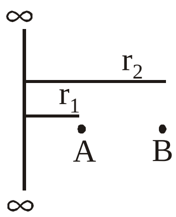
(3) Uniform linear charge | **Uniform linear mass**

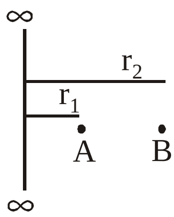




(λ=mass per unit length)

(4) Infinite Linear charge | **Infinite linear mass**





(a) $E = \dfrac{2K\lambda}{r}$ $\qquad$ $g = \dfrac{2G\lambda}{r}$

(b) $V_B - V_A = -2K\lambda \ln\dfrac{r_2}{r_1}$ $\qquad$ $V_B - V_A = 2G\lambda \ln\left(\dfrac{r_2}{r_1}\right)$

(5) Infinite Sheet of charge | **Infinite Sheet of mass**

$E = \dfrac{\sigma}{2\varepsilon_0}$ $\qquad$ $g = \dfrac{\sigma}{2} \times 4\pi G = 2\pi G\sigma$

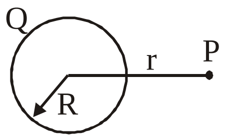
(σ = mass per unit area)

* Notice gravitational force is always attractive and hence gravitational potential is always –ve. (for a repulsive force potential is postive). This can be explained from the sign of W_{ext} in moving the test charge from ∞ to the point under consideration.

** Since $\vec{g}$ points from B towards A potential increases as we move from A to B. Just like electric potencial gravitational potential also increases opposite to field direction.

(6) Uniformly charged hollow shpere
Charge Q, radius R
distance of field point from center r

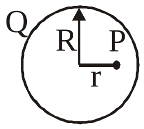
Case I r > R



$E = \dfrac{kQ}{r^2}$

$V = \dfrac{kQ}{r}$

Case II r < R



$E = 0$

$V = \dfrac{kQ}{r}$

Hollow sphere of uniform mass
Mass M , radius R
distance of field point from center r

$g = \dfrac{GM}{r^2}$

$V_G = -\dfrac{GM}{r}$

$g = 0$

$V_G = -\dfrac{GM}{R}$

(7) Electrostatics self energy of uniformly charged thin spherical shell.

$$U = \frac{KQ^2}{2R}$$

Gravitational self energy of uniform thin spherical shell.

$$U = \frac{GM^2}{2R}$$

(8) Uniformly charged solid sphere
mass M, radius R

$E = \dfrac{kQ}{r^2}$, r > R

$\dfrac{kQr}{R^3}$, r < R

$V = \dfrac{kQ}{r}$, r > R

$\dfrac{kQ}{2R^3}\left(3R^3 - r^2\right)$, r > R

Uniformly solid sphere
mass M, radius R

$g = \dfrac{GM}{r^2}$, r > R

$\dfrac{GM}{R^3}r$, r < R

$V_a = -\dfrac{GM}{r}$, r > R

$\dfrac{-GM}{2R^3}\left(3R^3 - r^2\right)$, r > R

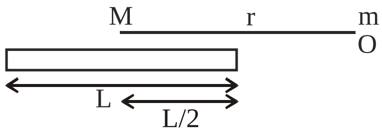
(9) Electrostatics self energy of uniformly charged solid sphere.

$$U = \frac{3}{5}\frac{KQ^2}{R}$$

Gravitational self energy of uniform solid sphere.

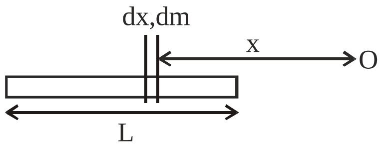
$$U = \frac{3}{5}\frac{GM^2}{R}$$

Ex. Find gravitational force between the point mass & the rod of uniform mass.



Ans. $$\frac{GMm}{\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}$$

Sol. $$dm = \frac{M}{L}dx$$



$$dF = \frac{G(dM)m}{x^2}$$

$$\int_O^F dF = \frac{GMm}{L}\int_{x_1=\left(r-\frac{L}{2}\right)}^{x_2=\left(r+\frac{L}{2}\right)} \frac{dx}{x^2}$$

$$F = \frac{GMm}{L}\left[\frac{1}{x}\right]_{x_2}^{x_1} = \frac{GMm}{L}\frac{L}{\left(r-\frac{L}{2}\right)\left(r+\frac{L}{2}\right)} = \frac{GMm}{\left(r-\frac{L}{2}\right)\left(r+\frac{L}{2}\right)}$$

Ex. A thin rod of mass M and length L is bent in a semicircle as shown in figure.

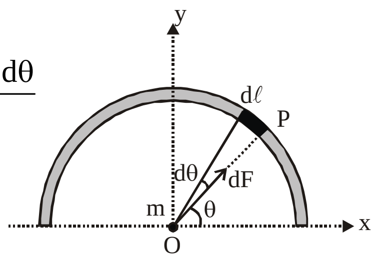
(a) What is its gravitational force (both magnitude and direction) on a particle with mass m at O, the centre of curvature?

(b) What would be the force on m if the rod is, in the form of a complete circle?

Sol. (a) Considering an element of rod of length $d\ell$ as shown in figure and treating it as a point of mass $(M/L)\,d\ell$ situated at a distance R from P, the gravitational force due to this element on the particle will be

$$dF = \frac{Gm(M/L)(Rd\theta)}{R^2} \text{ along OP [as } d\ell = Rd\theta\text{]}$$

So the component of this force along x and y–axis will be



$$dF_x = dF\cos\theta = \frac{GmM\ \cos\theta\ d\theta}{LR}\ ;\ dF_y = dF\sin\theta = \frac{GmM\ \sin\theta\ d\theta}{LR}$$

So that $F_x = \dfrac{GmM}{LR}\displaystyle\int_0^{\pi}\cos\theta\ d\theta = \dfrac{GmM}{LR}[\sin\theta]_0^{\pi} = 0$

and $F_y = \dfrac{GmM}{LR}\displaystyle\int_0^{\pi}\sin\theta\ d\theta = \dfrac{GmM}{LR}[-\cos\theta]_0^{\pi} = \dfrac{2\pi GmM}{L^2}\left[\text{as } R = \dfrac{L}{\pi}\right]$

So $F = \sqrt{F_x^2 + F_y^2} = F_y = \dfrac{2\pi GmM}{L^2}$ [as F_x is zero]

i.e., the resultant force is along the y–axis and has magnitude ($2\pi GmM/L^2$)

(b) If the rod was bent into a complete circle,

$$F_x = \frac{GmM}{LR}\int_0^{2\pi}\cos\theta\, d\theta = 0 \text{ and also } F_y = \frac{GmM}{LR}\int_0^{2\pi}\sin\theta\ d\theta = 0$$

i.e, the resultant force on m at O due to the ring is zero.

Ex. Find ratio of gravitational field on the surface of two planets which are of uniform mass density & have radius R_1 & R_2 if

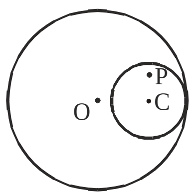
(a) They are of same mass

(b) They are of same density

Ans. (a) $\dfrac{g_1}{g_2} = \dfrac{R_2^2}{R_1^2}$ (b) $\dfrac{g_1}{g_2} = \dfrac{R_1}{R_2}$

Sol. $g = \dfrac{GM}{R^2} = \dfrac{G\frac{4}{3}\pi R^3\rho}{R^2} = \left(\dfrac{4\pi GR}{3}\right)\rho$

Ex. A uniform solid sphere of density ρ and radius R has a spherical cavity of radius r inside it as shown. Find gravitation field at



(a) O

(b) C

(c) P (prove that field inside cavity is uniform)

Ans. (a) $\dfrac{k\left(\frac{4}{3}\pi r^3\rho\right)}{(OC)^2}\hat{CO}$ (b) $\dfrac{4\pi G\rho\overrightarrow{OC}}{3}$ (c) $\dfrac{4}{3}\pi G\rho\overrightarrow{O_1O_2}$

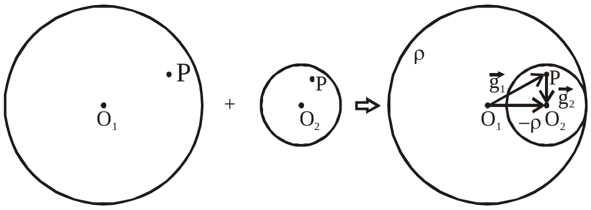
Sol. $\vec{g}$ = gravitational field at any point inside sphere

$$\vec{g}=\frac{GM}{R^3}\vec{r}$$

$$=\frac{G}{R^3}\frac{4}{3}\pi R^3\rho\vec{r}$$

$$\vec{g}=\frac{4}{3}\pi G\rho\,\vec{r}$$

Let the sphere with cavity is formed by superimposing it with a small sphere of density (–ρ) as shown



Resultant field $\vec{g}=\vec{g}_1+\vec{g}_2$

$$=\left(\frac{4}{3}\pi G\rho\right)\overrightarrow{O_1P}+\left(\frac{4}{3}\pi G\rho\right)\overrightarrow{PO_2}$$

$$=\frac{4}{3}\pi G\rho\overrightarrow{O_1O_2} \qquad \left[\overrightarrow{O_1O_2}=\overrightarrow{OC}\right]$$

It is indepndent of position of point inside cavity

At O $\vec{g}=\vec{g}_1+\vec{g}_2$

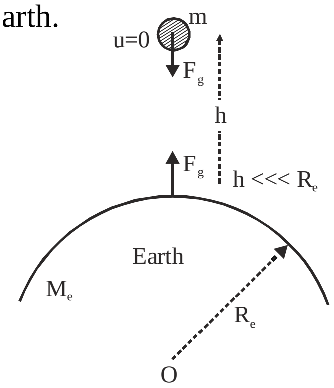
$$=0+\frac{GM}{\left(\overrightarrow{CO}\right)^2}\left(\widehat{CO}\right)$$

$$=\frac{G\frac{4}{3}\pi r^2\rho}{\left(\overrightarrow{CO}\right)^2}\widehat{CO}$$

$$=\frac{\left(\frac{4}{3}\pi Gr^2\rho\right)}{\left(\overrightarrow{CO}\right)^2}\widehat{CO}$$

Acceleration Due to Gravity (g)

Gravitational Force F_g = ma if R_e = Radius of Earth, M_e = Mass of Earth.



then $\dfrac{GM_em}{R_e^{\,2}} = ma_g \Rightarrow a_g = g = \dfrac{GM_e}{R_e^{\,2}}$ $(GM_e = gR_e^{\,2})$ ... (i)

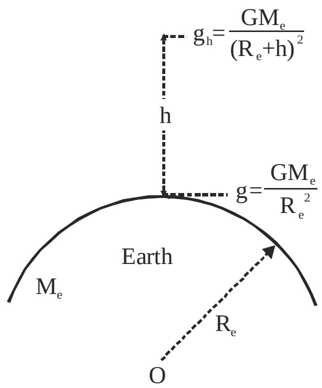
- In form of density $g = \dfrac{GM_e}{R_e^2} = \dfrac{G}{R_e^2} \times \dfrac{4}{3}\pi R_e^{\,3} \times \rho$

$\therefore\ g = \dfrac{4}{3}\pi G R_e \rho$ .... (ii)

If ρ is constant then $g \propto R_e$

Variation in Acceleration due gravity

(a) Due to Altitude (height)



$$\frac{g_h}{g} = \frac{R_e^2}{(R_e + h)^2}$$

By binomial expansion $\left(1+\dfrac{h}{R_e}\right)^{-2} \simeq \left(1-\dfrac{2h}{R_e}\right)$

[If h << R_e, then higher power terms are negligible] $\therefore\ g_h = g\left[1-\dfrac{2h}{R_e}\right]$

Ex. Two equal masses m and m are hung from a balance whose scale pans differ in vertical height by 'h'. Determine the error in weighing in terms of density of the Earth ρ.

Sol. $g_h = g\left[1-\dfrac{2h}{R_e}\right]$, $W_2 - W_1 = mg_2 - mg_1 = 2mg\left[\dfrac{h_1}{R_e} - \dfrac{h_2}{R_e}\right] = 2m\dfrac{GM}{R_e^2} \times \dfrac{h}{R_e}$

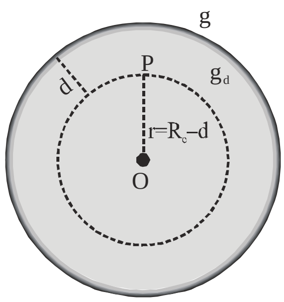
$\left[\because g = \dfrac{GM}{R_e^2}\ \&\ h_1 - h_2 = h\right]$

Error in weighing $= W_2 - W_1 = 2mG\dfrac{4}{3}\pi R_e^3 \rho \dfrac{h}{R_e^3} = \dfrac{8\pi}{3} Gm\rho h$

(b) Due to depth :

Assuming density of Earth remains same throughout. At depth d inside the Earth :

$$g_d = g\left[1 - \frac{d}{R_e}\right] \text{ valid for any depth}$$



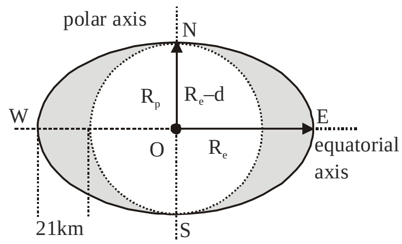
Decrement in g with depth $= \Delta g_d = g - g_d = g - g\left[1 - \frac{d}{R_e}\right]$

$$\therefore \quad \frac{\Delta g_d}{g} = \frac{d}{R_e}$$

Ex. At which depth from Earth surface, acceleration due to gravity is decreased by 1%

Sol. $\frac{\Delta g_d}{g} = \frac{d}{R_e} \Rightarrow \frac{1}{100} = \frac{d}{6400} \quad \therefore \quad d = 64 \text{ km}$

(c) Due to shape of the Earth

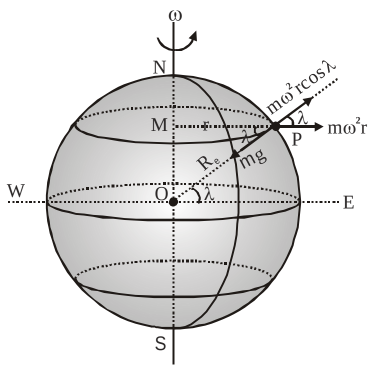


$R_p < R_e$

$\therefore \quad g_e < g_p$

$\Rightarrow$ by putting the values $g_p - g_e = 0.02 \text{ m/s}^2$

(d) Due to Rotation of the Earth



$g' = g - R_e\omega^2 \cos^2\lambda$

If latitude angle $\lambda = 0$. It means at equator. $g'_{min} = g_e = g - R_e\omega^2$

If latitude angle $\lambda = 90°$. It means at poles. $g'_{max.} = g_p = g \Rightarrow g_p > g_e$

Change in "g" only due to rotation $\Delta g_{rot.} = g_p - g_e = 0.03 \text{ m/s}^2$

$\Delta g_{total} = g_p - g_e = (0.05 \text{ m/s}^2) \longrightarrow$ 0.02 m/s^2 (due to shape); 0.03 m/s^2 (due to rotation)

Weightlessness

State of the free fall $\left(\vec{a} = -\frac{GM}{r^2}\vec{r}\right)$ is called state of weightlessness. If a body is in a satellite (which does not produce its own gravity) orbiting the Earth at a height h above its surface then

True weight $= mg_h = \frac{mGM}{(R+h)^2} = \frac{mg}{\left(1+\frac{h}{R}\right)^2}$

Apparent weight $= m(g_h - a)$

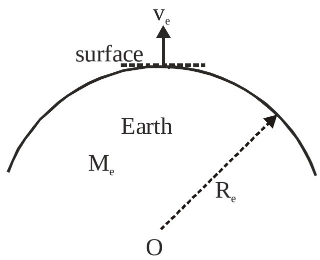
but $a = \frac{v_0^2}{r} = \frac{GM}{r^2} = \frac{GM}{(R+h)^2} = g_h \Rightarrow$ Apparent weight $= m(g_h - g_h) = 0$

Note : The condition of weightlessness can be overcome by creating artificial gravity by rotating the satellite in addition to its revolution.

Escape speed (v_e)

Minimum speed required for an object at Earth's surface so that it just escapes the Earth's gravitational field.

Escape energy



Minimum energy given to a particle in form of kinetic energy so that it can just escape from Earth's gravitational field.

Escape energy $= \frac{GM_e m}{R_e}$ (–ve of PE of Earth's surface)

Escape energy = Kinetic Energy $\Rightarrow \frac{GM_e m}{R_e} = \frac{1}{2}mv_e^2 \Rightarrow v_e = \sqrt{\frac{2GM_e}{R_e}}$

- $v_e = \sqrt{\frac{2GM_e}{R_e}}$ (In form of mass) If M = constant $v_e \propto \frac{1}{\sqrt{R_e}}$
- $v_e = \sqrt{2gR_e}$ (In form of g) If g = constant $v_e \propto \sqrt{R_e}$
- $v_e = R_e\sqrt{\frac{8\pi G.\rho}{3}}$ (In form of density) If ρ = constant $v_e \propto R_e$
- Escape velocity does not depend on mass of body, angle of projection or direction of projection.
  $v_e \propto m^0$ and $v_e \propto \theta°$
- Escape velocity at : Earth surface $v_e = 11.2$ km/s Moon surface $v_e = 2.31$ km/s
- Atmosphere on Moon is absent because root mean square velocity of gas particle is greater then escape velocity. $v_{rms} > v_e$

Ex. A space–ship is launched into a circular orbit close to the Earth's surface. What additional speed should now be imparted to the spaceship so that orbit to overcome the gravitational pull of the Earth.

Sol. Let ΔK be the additional kinetic energy imparted to the spaceship to overcome the gravitation pull

then by energy conservation $-\frac{GMm}{2R} + \Delta K = 0 + 0 \Rightarrow \Delta K = \frac{GMm}{2R}$

Total kinetic energy $= \frac{GMm}{2R} + \Delta K = \frac{GMm}{2R} + \frac{GMm}{2R} = \frac{GMm}{R}$ then $\frac{1}{2}mv_2^2 = \frac{GMm}{R} \Rightarrow v_2 = \sqrt{\frac{2GM}{R}}$

But $v_1 = \sqrt{\frac{GM}{R}}$. So Additional velocity $= v_2 - v_1 = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = \left(\sqrt{2}-1\right)\sqrt{\frac{GM}{R}}$

Ex. Find the minimum speed with which an object should be projected vertically upward from earth's surface to reach a height equal to radius of earth, R_e.

Ans. $\sqrt{\frac{GM}{R_e}}$

Sol. $-\frac{GMm}{R_e} + \frac{1}{2}mv^2 = -\frac{GMm}{2R_e}$

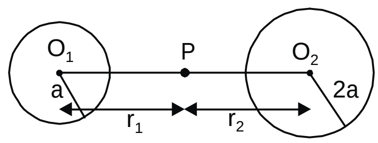
$\therefore\ v = \sqrt{\frac{GM}{R_e}}$

Ex. The distance between the centres of two stars is 10a. The masses of the stars are M and 16M and their radii a and 2a respectively. A body of mass m is fired straight from surface of the larger star towards smaller star.

(a) Find the distance between centre of smaller star and the point of zero gravitational field strength:

Sol. P is the point where field strength is zero.

$\frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2}$



and $r_1 + r_2 = 10a$

So, $r_1 = 2a$, $r_2 = 8a$

(b) The initial minimum speed of the body to reach smaller star is $K\sqrt{\frac{GM}{a}}$. Find the value of K:

Sol. From conservation of mechanical energy.

$$\frac{1}{2}mv_{min}^2 = \text{Potential energy of body at P} - \text{Potential energy of body at larger star}$$

$$= \left[-\frac{GMm}{r_1} - \frac{16GMm}{r_2}\right] - \left[-\frac{GMm}{(10a-2a)} - \frac{16GMm}{2a}\right]$$

$$= \frac{3\sqrt{5}}{2}\left[\sqrt{\frac{GM}{a}}\right]$$

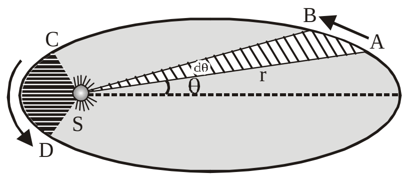
Kepler's Laws

Kepler found important regularities in the motion of the planets. These regularities are known as 'Kepler's three laws of planetary motion'.

(a) **First Law (Law of Orbits) :** All planets move around the Sun in elliptical orbits, having the Sun at one focus of the orbit.

(b) **Second Law (Law of Areas) :** A line joining any planet to the Sun sweeps out equal areas in equal times, that is, the areal speed of the planet remains constant.

According to the second law, when the planet is nearest the Sun, then its speed is maximum and when it is farthest from the Sun, then its speed is minimum. In figure if a planet moves from A to B in a given time–interval, and from C to D in the same time–interval, then the areas ASB and CSD will be equal.



$$\frac{dA}{dt} = \frac{J}{2m} \qquad \text{...(iii)}$$

Now, the areal speed dA/dt of the planet is constant, according to Kepler's second law. Therefore, according to eq. (iii), the angular momentum J of the planet is also constant, that is, the angular momentum of the planet is conserved. Thus, Kepler's second law is equivalent to conservation of angular momentum.

(c) **Third Law : (Law of Periods) :** The square of the period of revolution (time of one complete revolution) of any planet around the Sun is directly proportional to the cube of the semi–major axis of its elliptical orbit.

$T^2 \propto a^3$

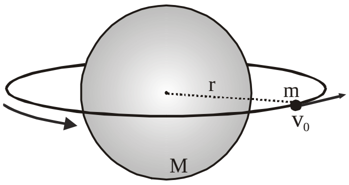
So it is clear through this rule that the farthest planet from the Sun has largest period of revolution. The period of revolution of the closest planet Mercury is 88 days, while that of the farthest dwarf planet Pluto is 248 years.

SATELLITE MOTION

A light body revolving round a heavier body due to gravitational attraction, is called satellite. Earth is a satellite of the Sun while Moon is satellite of Earth.

Orbital velocity (v_0) : A satellite of mass m moving in an orbit of radius r with speed v_0 then required centripetal force is provided by gravitation.

$$F_{cp} = F_g \Rightarrow \frac{mv_0^2}{r} = \frac{GMm}{r^2} \Rightarrow v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R_e + h)}} \quad (r = R_e + h)$$



For a satellite very close to the Earth surface h << R_e $\therefore$ r = R_e

$$v_0 = \sqrt{\frac{GM}{R_e}} = \sqrt{gR_e} = 8 \text{ km/s}$$

- If a body is taken at some height from Earth and given horizontal velocity of magnitude 8 km/sec then the body becomes satellite of Earth.
- v_o depends upon : Mass of planet, Radius of circular orbit of satellite, g (at planet), Density of planet
- If orbital velocity of a near by satellite becomes $\sqrt{2}\,v_o$ (or increased by 41.4%, or K.E. is doubled) then the satellite escapes from gravitational field of Earth.

Time Period of a Satellite $T = \frac{2\pi r}{v_0} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi r^{3/2}}{R\sqrt{g}} \Rightarrow T^2 = \frac{4\pi^2}{GM} r^3 \Rightarrow T^2 \propto r^3 (r = R + h)$

For Geostationary Satellite T = 24 hr, h = 36,000 km $\simeq$ 6 R_e (r $\simeq$ 7 R_e), v_0 = 3.1 km/s

For Near by satellite $v_0 = \sqrt{\frac{GM_e}{R_e}} \simeq 8$ km/s

$$T_{Ns} = 2\pi\sqrt{\frac{R_e}{g}} = 84 \text{ minute} = 1 \text{ hour } 24 \text{ minute} = 1.4 \text{ hr} = 5063 \text{ s}$$

In terms of density $T_{Ns} = \frac{2\pi (R_e)^{1/2}}{(G \times 4/3\pi R_e \times \rho)^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$

Time period of near by satellite only depends upon density of planet.

For Moon h_m = 380,000 km and T_m = 27 days

$$v_{om} = \frac{2\pi(R_e + h)}{T_m} = \frac{2\pi(386400 \times 10^3)}{27 \times 24 \times 60 \times 60} \simeq 1.04 \text{ km/sec.}$$

Energies of a Satellite Kinetic energy $\quad K.E. = \frac{1}{2}mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$

Potential energy $\quad P.E. = -\frac{GMm}{r} = -mv_0^2 = -\frac{L^2}{mr^2}$

Total mechanical energy $T.E. = P.E. + K.E. = -\frac{mv_0^2}{2} = -\frac{GMm}{2r} = -\frac{L^2}{2mr^2}$

Essential Condition's for Satellite Motion

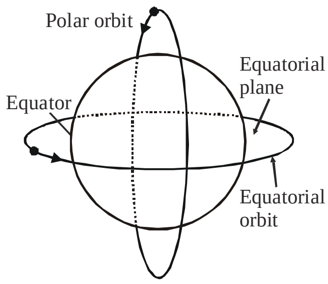
- Centre of satellite's orbit coincide with centre of Earth.
- Plane of orbit of satellite is passing through centre of Earth.

Special Points about Geo–Stationary Satellite

- All three essential conditions for satellite motion should be followed.
- It rotates in equatorial plane.
- Its height from Earth surface is 36000 km. (~$6R_e$)
- Its angular velocity and time period should be same as that of Earth.
- Its rotating direction should be same as that of Earth (West to East).
- Its orbit is called parking orbit and its orbital velocity is 3.1 km./sec.
- Maximum latitude at which message can be recieved by geostationary satellite is $\theta = \cos^{-1}\left(\frac{R_e}{R_e + h}\right)$
- The area of earth's surface covered by geostationary satellite is $S = \Omega R_e^2 = \frac{2\pi h R_e^2}{R_e + h}$

- **Polar Satellite (Sun – synchronous satellite)**

It is that satellite which revolves in polar orbit around Earth. A polar orbit is that orbit whose angle of inclination with equatorial plane of Earth is 90° and a satellite in polar orbit will pass over both the north and south geographic poles once per orbit. Polar satellites are Sun–synchronous satellites. Every location on Earth lies within the observation of polar satellite twice each day. The polar satellites are used for getting the cloud images, atmospheric data, ozone layer in the atmosphere and to detect the ozone hole over Antarctica.



Only the equatorial orbits are stable for a satellite. For any satellite to orbit around in a stable orbit, it must move in such an orbit so that the centre of Earth lies at the centre of the orbit.

BINDING ENERGY

Total mechanical energy (potential + kinetic) of a closed system is negative. The modulus of this total mechanical energy is known as the binding energy of the system. This is the energy due to which system is closed or different parts of the system are bound to each other.

Binding energy of satellite (system)

B.E. $=$ – T.E. B.E. $= \frac{1}{2}mv_0^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$ Hence B.E. = K.E. = – T.E. $= \frac{-\text{P.E.}}{2}$

Work done in Changing the Orbit of Satellite

W = Change in mechanical energy of system but $E = \frac{-GMm}{2r}$ so $W = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Ex. A satellite moves eastwards very near the surface of the Earth in equatorial plane with speed (v_0). Another satellite moves at the same height with the same speed in the equatorial plane but westwards. If R = radius of the Earth and ω be its angular speed of the Earth about its own axis. Then find the approximate difference in the two time period as observed on the Earth.

Sol. $T_{west} = \frac{2\pi R}{v_0 + R\omega}$ and $T_{east} = \frac{2\pi R}{v_0 - R\omega} \Rightarrow \Delta T = T_{east} - T_{west} = 2\pi R\left[\frac{2R\omega}{v_0^2 - R^2\omega^2}\right] = \frac{4\pi\omega R^2}{v_0^2 - R^2\omega^2}$

Ex. An artificial satellite (mass m) of a planet (mass M) revolves in a circular orbit whose radius is n times the radius R of the planet. In the process of motion, the satellite experiences a slight resistance due to cosmic dust. Assuming the force of resistance on satellite to depend on velocity as $F = av^2$ where 'a' is a constant, calculate how long the satellite will stay in the space before it falls onto the planet's surface.

Sol. Air resistance $F = -av^2$, where orbital velocity $v = \sqrt{\frac{GM}{r}}$

r = the distance of the satellite from planet's centre $\Rightarrow F = -\frac{GMa}{r}$

The work done by the resistance force $dW = Fdx = Fvdt = \frac{GMa}{r}\sqrt{\frac{GM}{r}}dt = \frac{(GM)^{3/2}a}{r^{3/2}}dt$ ....(i)

The loss of energy of the satellite = dE $\therefore \frac{dE}{dr} = \frac{d}{dr}\left[-\frac{GM\,m}{2r}\right] = \frac{GMm}{2r^2} \Rightarrow dE = \frac{GMm}{2r^2}dr$ ...(ii)

Since dE = – dW (work energy theorem) $-\frac{GMm}{2r^2}dr = \frac{(GM)^{3/2}}{r^{3/2}}dt$

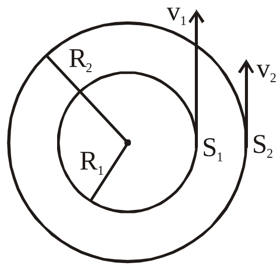
$$\Rightarrow t = -\frac{m}{2a\sqrt{GM}}\int_{nR}^{R}\frac{dr}{\sqrt{r}} = \frac{m\sqrt{R}\left(\sqrt{n}-1\right)}{a\sqrt{GM}} = \left(\sqrt{n}-1\right)\frac{m}{a\sqrt{gR}}$$

Ex. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8h respectively. The radius of the orbit of S_1 is 10^4 km. When S_2 is closest to S_1, find (a) the speed of S_2 relative to S_1 and (b) the angular speed of S_2 as observed by an astronaut in S_1.

Sol. Let the mass of the planet be M, that of S_1 be m_1 and S_2 be m_2. Let the radius of the orbit of S_1 be $R_1(= 10^4$ km) and of S_2 be R_2.

Let v_1 and v_2 be the linear speeds of S_1 and S_2 with respect to the planet. Figure shows the situation.

As the square of the time period is proportional to the cube of the radius.



$$\left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{8h}{1h}\right)^2 = 64$$

or, $\dfrac{R_2}{R_1} = 4$

or, $R_2 = 4R_1 = 4 \times 10^4$ m.

Now the time period of S_1 is 1 h. So,

$$\frac{2\pi R_1}{v_1} = 1h$$

or, $v_1 = \dfrac{2\pi R_i}{1h} = 2\pi \times 10^4 \text{ km / h}$

similarly, $v_2 = \dfrac{2\pi R_2}{8h} = \pi \times 10^4 \text{ km / h}$

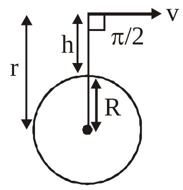
(a) At the closest separation, the are moving in the same direction. Hence the speed of S_2 with respect to S_1 is $|v_2 - v_1| = \pi \times 10^4$ km/h.

(b) As seen from S_1, the satellite S_2 is a distance $R_2 - R_1 = 3 \times 10^4$ km at the closest separation. Also is moving at $\pi \times 10^4$ km/h in a direction perpendicular to the line joining them. Thus, the angular speed of S_2 as observed by S_1 is

$$\omega = \frac{\pi \times 10^4 \text{ km / h}}{3 \times 10^4 \text{ km / h}} = \frac{\pi}{3} \text{ rad / h.}$$

Conditions for different trajectroy

For a body being projected tangentially from above earth's surface, say at a distance r from earth's center, the trajectory would depend on the velocity of projection v.

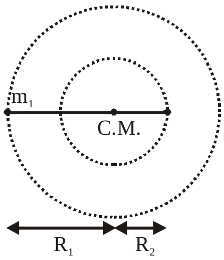


Velocity	Orbit
1. velocity, $v < \sqrt{\frac{GM}{r}\left(\frac{2R}{r+R}\right)}$	Body returns to earth
2. $\sqrt{\frac{GM}{r}} > v > \sqrt{\frac{GM}{r}\left(\frac{2R}{r+R}\right)}$	Body acquires an elliptical orbit with earth as the far-focus w.r.t. the point of projection.
3. Velocity is equal to the critical velocity of the orbit, $v = \sqrt{\frac{GM_e}{r}}$	Circular orbit with radius r
4. Velocity is between the critical and escape velocity of the orbit $\sqrt{\frac{2GM_e}{r}} > v > \sqrt{\frac{GM_e}{r}}$	Body acquires an elliptical orbit with earth as the near focus w.r.t. the point of projection.
5. $v = v_{esc} = \sqrt{\frac{2GM_e}{r}}$	Body just escapes earth's gravity, along a parabolic path.
6. $v > v_{esc} = \sqrt{\frac{2GM_e}{r}}$	Body escape earth's gravity along a hyperbolic path.

Binary star system

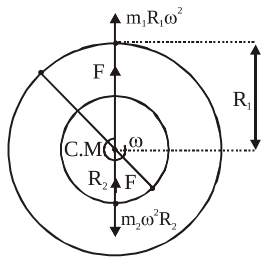
Figure shows two particles moving due to mutually attractive gravitational force about center of mass. Since there is no external force CM of system remains fixed and time period of revolution must be same.

Both bodies have comparable mass and both are moving in circular orbit centre of mass as shown in diagram



$$\omega = \sqrt{\frac{G(m_1+m_2)}{R^3}}$$

Angular momentum of the system about centre of mass.



$$L = \left(\frac{m_1m_2}{m_1+m_2}\right)R^2\omega$$

$$\text{Kinetic energy} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)R^2\omega^2$$

The time period of a simple pendulum of infinite length.

In deriving the formula $T_0 = 2\pi\sqrt{\left(\frac{L}{g}\right)}$ we have assumed that length of the pendulum L is much less than the radius of the earth R so that 'g' always remains vertical. However, if length of pendulum is comparable of the radius of earth, 'g' will not remain vertical but will be directed towards the centre of the earth.

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{1}{g\left[\frac{1}{L}+\frac{1}{R}\right]}} \qquad (< T_0)$$

From this expression it is clear that :

(a) If L << R, (1/L) >> (1/R) so $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$ which is expected.

(b) If L >> R ($\rightarrow \infty$) (1/L) << (1/R) so

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6.4\times10^6}{10}}$$

$= 800 \times 2\pi$ sec $\approx$ 83.8 minute

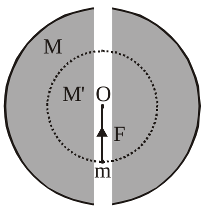
And it is also the maximum time period which an oscillating simple pendulum can have.

(c) If L is comparable to R (say L = R),

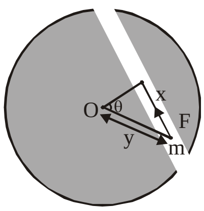
$$T = 2\pi\sqrt{\frac{R}{2g}} \simeq 1\text{hour}.$$

Motion of a ball in a tunnel through the earth :

Case I : If the tunnel is along a diameter and the ball is released from the suface. The ball executes SHM.



(A)



(B)

so that $T = 2\pi\sqrt{\frac{R^3}{GM}}$; $T = 2\pi\sqrt{\left(\frac{R}{g}\right)}$

Which is same as that of a simple pendulum of infinite length and is equal to 84.6 minute.

Case II : If the tunnel is along a chord and ball is released from the suface. The motion is SHM with he same time period.

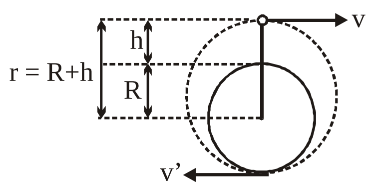
GRAVITATIONAL PRESSURE :

A uniform sphere has a mass M and radius R. The pressure p inside the sphere, caused by gravitational compression, as a function of the distance r from its centre can be found to be $p = \frac{3}{8}\,(1 - r^2/R^2)\,\gamma M^2/\pi R^4$.

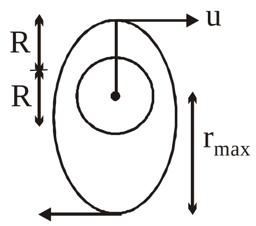
Launching of an artificial satellite around earth

Ex. A satellite is launched tangentially from a height h above earth's surface as shown.

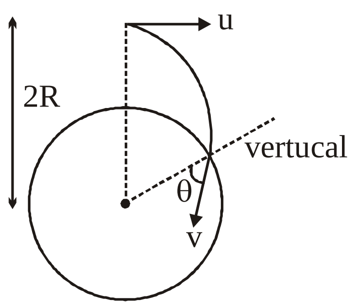
I. Find minimum launch speed 'v' so that it just touches the earth's surface



II. If h = R and satellite is launched tangentially with speed = $\sqrt{\frac{3GM}{5R}}$ find the maximum distance of satellite from earth's center



III. If h = R and satellite is launched tangentially with a speed u = $\sqrt{\frac{GM}{7R}}$. Find the angle w.r.t. vertical at which the satellite will crash on earth's surface.



Sol.

(I) Angular momentum conservation

$$mv(R+h) = mv'R$$

Energy conservation

$$\frac{-GMm}{R+h}+\frac{1}{2}mv^2 = -\frac{GMm}{R}+\frac{1}{2}mv'^2$$

Solving , $v = \sqrt{\frac{2GMR}{r(R+r)}}$

(II) Angular momentum conservation $mu \cdot 2R = mvr_{max}$.

Energy conservation :

$$\frac{-GMm}{2R}+\frac{1}{2}mu^2 = \frac{-GMm}{r_{max}}+\frac{1}{2}mv^2$$

$$\Rightarrow \; GMm\left(\frac{1}{2R}-\frac{1}{r_{max}}\right) = \frac{1}{2}mu^2\left(1-\left(\frac{2R}{r_{max}}\right)^2\right)$$

$$\Rightarrow \; GMm\left(\frac{1}{2R}-\frac{1}{r_{max}}\right) = \frac{3GM}{10R}\left(1-\frac{4R^2}{r_{max}^2}\right)$$

$$\Rightarrow \; 2r_{max}^2 - 10Rr_{max} + 12R^2 = 0$$

$$\Rightarrow \; (r_{max}-2R)(r_{max}-3R) = 0$$

$$\Rightarrow \; r_{max} = 3R$$

(III) Energy conservation

$$\frac{1}{2}mu^2 - \frac{GMm}{2R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \; \frac{1}{2}m(v^2-u^2) = \frac{GMm}{2R}$$

$$v^2 = u^2 + \frac{GM}{R} \Rightarrow v = \sqrt{\frac{8GM}{7R}}$$

Angular momentum conservation

$mu \cdot 2R = mvR\sin\theta$

$$\Rightarrow \sin\theta = \frac{2u}{v}$$

$$\Rightarrow \theta = 45°$$

EXERCISE (S-1)

1. A particle is fired vertically from the surface of the earth with a velocity $k\upsilon_e$, where υ_e is the escape velocity and $k < 1$. Neglecting air resistance and assuming earth's radius as R_e. Calculate the height to which it will rise from the surface of the earth.

2. Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

3. An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:

(i) the initial speed of projection

(ii) the speed at half the maximum height.

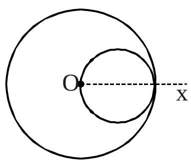
4. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time________.

5. A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5$J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2 \times 10^5$J is equal to ________.

6. A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/ s. Then the time taken for the satellite to reach the earth is ________.

7. A pair of stars rotates about a common center of mass. One of the stars has a mass M which is twice as large as the mass m of the other. Their centres are at a distance d apart, d being large compared to the size of either star. (a) Derive an expression for the period of rotation of the stars about their common centre of mass in terms of d,m, G. (b) Compare the angular momentum of the two stars about their common centre of mass by calculating the ratio L_m/ L_M. (c) Compare the kinetic energies of the two stars by calculating the ratio K_m/K_M.

8. A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_o except that there is a spherical hole of radius $r = R/2$ whose centre is at $x = R/2$ as in fig. (a) Find gravitational field at points on the axis for $|x| > R$ (b) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.

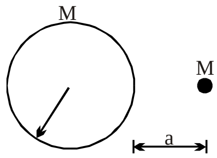


EXERCISE (O-1)

1. If the distance between the centres of Earth and Moon is D and mass of Earth is 81 times that of Moon. At what distance from the centre of Earth gravitational field will be zero?

(A) $\frac{D}{2}$ (B) $\frac{2D}{3}$ (C) $\frac{4D}{5}$ (D) $\frac{9D}{10}$

2. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



(A) Gravitational field and potential both are zero at centre of the shell.

(B) Gravitational field is zero not only inside the shell but at a point outside the shell also.

(C) Inside the shell, gravitational field alone is zero.

(D) Neither gravitational field nor gravitational potential is zero inside the shell.

3. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre

(A) increases

(B) decreases

(C) remains same

(D) during the compression increases then returns at the previous value.

4. Let ω be the angular velocity of the earth's rotation about its axis. Assume that the acceleration due to gravity on the earth's surface has the same value at the equator and the poles in absence of rotation of earth. An object weighed at the equator gives the same reading as a reading taken at a depth d below earth's surface at a pole (d<<R) The value of d is

(A) $\frac{\omega^2R^2}{g}$ (B) $\frac{\omega^2R^2}{2g}$ (C) $\frac{2\omega^2R^2}{g}$ (D) $\frac{\sqrt{Rg}}{g}$

5. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same?

(A) 1/25 (B) 1/5 (C) $1/\sqrt{5}$ (D) 5

6. The mass and diameter of a planet are twice those of earth. What will be the period of oscillation of a pendulum on this planet if it is a seconds pendulum on earth?

(A) $\sqrt{2}$ second (B) $2\sqrt{2}$ seconds (C) $\frac{1}{\sqrt{2}}$ second (D) $\frac{1}{2\sqrt{2}}$ second

7. Two identical satellites are at the heights R and 7R from the Earth's surface. Then which of the following statement is incorrect. (R = radius of the Earth)

(A) Ratio of total energy of both is 5

(B) Ratio of kinetic energy of both is 4

(C) Ratio of potential energy of both 4

(D) Ratio of total energy of both is 4 and ratio of magnitude of potential to kinetic energy is 2

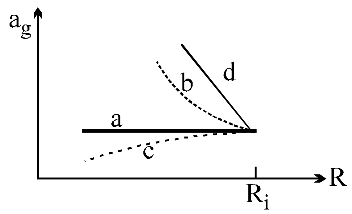
8. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V is

(A) $V_e = 2V$ (B) $V_e = V$ (C) $V_e = V/2$ (D) $V_e = \sqrt{3}\,V$

9. The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(A) v_e (B) $\frac{v_e}{\sqrt{2}}$ (C) $\frac{v_e}{2}$ (D) 0

10. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse?

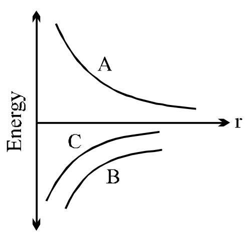


(A) a (B) b (C) c (D) d

11. A satellite of mass m, initially at rest on the earth, is launched into a circular orbit at a height equal to the radius of the earth. The minimum energy required is

(A) $\frac{\sqrt{3}}{4}$mgR (B) $\frac{1}{2}$mgR (C) $\frac{1}{4}$mgR (D) $\frac{3}{4}$mgR

12. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



(A) A shows the kinetic energy, B the total energy and C the potential energy of the system.

(B) C shows the total energy, B the kinetic energy and A the potential energy of the system.

(C) C and A are kinetic and potential energies respectively and B is the total energy of the system.

(D) A and B are kinetic and potential energies and C is the total energy of the system.

13. A satellite of mass 5M orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass 4M. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass 4M is :-

(A) In a circular orbit

(B) unbound

(C) elliptical orbit

(D) data is insufficient to determine the nature of the orbit.

14. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the centre is :-

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$ (C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

15. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change:-

(A) gravitational potential energy (B) angular velocity

(C) linear orbital velocity (D) centripetal acceleration

16. Satellites A and B are orbiting around the earth in orbits of ratio R and 4R respectively. The ratio of their areal velocities is :

(A) 1 : 2 (B) 1 : 4 (C) 1 : 8 (D) 1 : 16

17. The fractional change in the value of free–fall acceleration g for a particle when it is lifted from the surface to an elevation h (h<<R) is

(A) $\frac{h}{R}$ (B) $\frac{2h}{R}$ (C) $-\frac{2h}{R}$ (D) $-\frac{h}{R}$

18. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will- **[AIEEE-2002]**

(A) continue to move in its orbit with same velocity

(B) move tangentially to the original orbit with same velocity

(C) become stationary in its orbit

(D) move towards the earth

19. The time period of a satellite of earth is 5 hours. If the separation between the centre of earth and the satellite is increased to 4 times the previous value, the new time period will become-

[AIEEE-2003]

(A) 10 h (B) 80 h (C) 40 h (D) 20 h

20. A communications Earth satellite

(A) goes round the earth from east to west

(B) can be in the equatorial plane only

(C) can be vertically above any place on the earth

(D) goes round the earth from west to east

21. If a satellite orbits as close to the earth's surface as possible,

(A) its speed is maximum

(B) time period of its rotation is minimum

(C) the total energy of the 'earth plus satellite' system is minimum

(D) the total energy of the 'earth plus satellite'system is maximum

22. For a satellite to orbit around the earth, which of the following must be true?

(A) It must be above the equator at some time

(B) It cannot pass over the poles at any time

(C) Its height above the surface cannot exceed 36,000 km

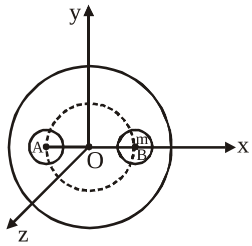
(D) Its period of rotation must be > $2\pi\sqrt{R/g}$ where R is radius of earth

23. In elliptical orbit of a planet, as the planet moves from apogee position to perigee position,

Column–I	**Column–II**
(A) Speed of planet	(P) Remains same
(B) Distance of planet from centre of Sun	(Q) Decreases
(C) Potential energy	(R) Increases
(D) Angular momentum about centre of Sun	(S) Can not say

EXERCISE (O-2)

1. A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (–2, 0, 0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in figure. Then



(A) The gravitational field due to this object at the origin is zero

(B) The gravitational field at the point B (2, 0, 0) is zero

(C) The gravitational potential is the same at all points of circle $y^2 + z^2 = 36$

(D) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$

2. A planet of mass m is in an elliptical orbit about the sun ($m << M_{sun}$) with an orbital period T. If A be the area of orbit, then its angular momentum would be :

(A) $\frac{2mA}{T}$ (B) mAT (C) $\frac{mA}{2T}$ (D) 2mAT

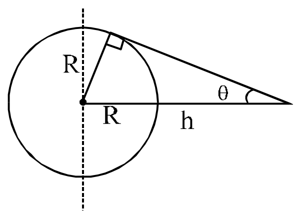
3. A planet revolves about the sun in elliptical orbit. The arial velocity $\left(\frac{dA}{dt}\right)$ of the planet is 4.0×10^{16} m²/s. The least distance between planet and the sun is 2×10^{12} m. Then the maximum speed of the planet in km/s is :

(A) 10 (B) 20 (C) 40 (D) None of these

4. The Sun travels in approximately circular orbit of radius R around the center of the galaxy and completes one revolution in time T. The Earth also revolves around the Sun in time t. Assume orbit of the Earth to be a circle of radius r (r << R) and whole mass of the galaxy centered on its center. By using only these given informations, find an expression for the ratio of the mass of the galaxy to that of the Sun.

(A) $\left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2$ (B) $\left(\frac{R}{r}\right)^3\left(\frac{T}{t}\right)^2$ (C) $\left(\frac{R}{r}\right)^2\left(\frac{t}{T}\right)^3$ (D) $\left(\frac{R}{r}\right)^2\left(\frac{T}{t}\right)^3$

5. A geostationary satellite is at a height h above the surface of earth. If earth radius is R



(A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}(R/R+h)$.

(B) The maximum latitudes on earth upto which the satellite can be used for communication is $\cos^{-1}(R/R+h)$.

(C) The area on earth escaped from this satellite is given as $2\pi R^2(1 + \sin\theta)$

(D) The area on earth escaped from this satellite is given as $2\pi R^2(1 + \cos\theta)$

6. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then

(A) its kinetic energy increases

(B) its kinetic energy decreases

(C) its angular momentum about the earth decreases

(D) its period of revolution around the earth increases

7. Two satellites s_1 & s_2 of equal masses revolve in the same sense around a heavy planet in coplanar circular orbit of radii R & 4R

(A) the ratio of period of revolution of s_1 & s_2 is 1 : 8.

(B) their velocities are in the ratio 2 : 1

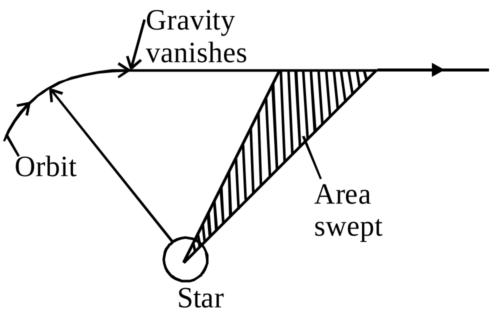
(C) their angular momentum about the planet are in the ratio 2 : 1

(D) the ratio of angular velocities of s_2 w.r.t. s_1 when all three are in the same line is 9 : 5.

8. A double star is a system of two stars of masses m and 2m, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to :

(A) $r^{3/2}$ (B) r (C) $m^{1/2}$ (D) $m^{-1/2}$

9. A planet is orbiting a star when for no apparent reason the star's gravity suddenly vanishes. After which planet moves in a straight line. Mark the **CORRECT** statement(s) :



(A) Newton's first law is obeyed on planet after gravity vanishes

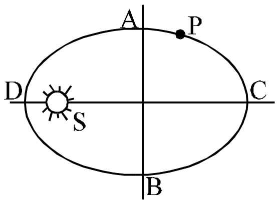
(B) Kepler's law of areas is obeyed only till the planet is in gravity of star

(C) Kepler's law of areas is obeyed even after gravity vanishes

(D) Angular momentum of planet about centre of star is conserved through out its motion

Paragraph for Question No. 10 and 11

Figure shows the orbit of a planet P around the sun S. AB and CD are the minor and major axes of the ellipse.



10. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then

(A) $t_1 = t_2$ (B) $t_1 > t_2$

(C) $t_1 < t_2$ (D) nothing can be concluded

11. If U is the potential energy and K kinetic energy then $|U| > |K|$ at

(A) Only D (B) Only C (C) both D & C (D) neither D nor C

EXERCISE (JM)

1. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :- **[AIEEE - 2011]**

(1) $-\frac{6Gm}{r}$ (2) $-\frac{9Gm}{r}$ (3) zero (4) $-\frac{4Gm}{r}$

2. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is:- **[AIEEE-2011]**

(1) $\sqrt{\frac{Gm}{R}}$ (2) $\sqrt{\frac{Gm}{4R}}$ (3) $\sqrt{\frac{Gm}{3R}}$ (4) $\sqrt{\frac{Gm}{2R}}$

3. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :- **[AIEEE-2012]**

(1) 6.4×10^{10} Joules (2) 6.4×10^{11} Joules (3) 6.4×10^{8} Joules (4) 6.4×10^{9} Joules

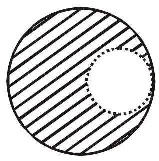
4. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R ? **[JEE-Main 2013]**

(1) $\frac{5GmM}{6R}$ (2) $\frac{2GmM}{3R}$ (3) $\frac{GmM}{2R}$ (4) $\frac{GmM}{3R}$

5. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is : **[JEE-Main 2014]**

(1) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (2) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (3) $\sqrt{\frac{GM}{R}}$ (4) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

6. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is : (G = *gravitational constant*)

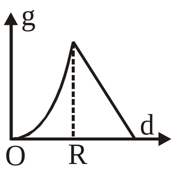


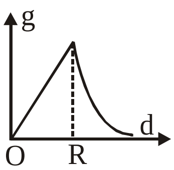
(1) $\frac{-2GM}{3R}$ (2) $\frac{-2GM}{R}$ (3) $\frac{-GM}{2R}$ (4) $\frac{-GM}{R}$

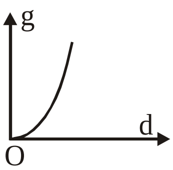
7. A satellite is reolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere). **[JEE-Main 2016]**

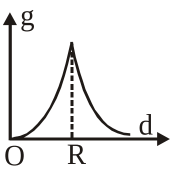
(1) $\sqrt{gR}\left(\sqrt{2}-1\right)$ (2) $\sqrt{2gR}$ (3) $\sqrt{gR}$ (4) $\sqrt{gR/2}$

8. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :- **[JEE-Main 2017]**

(1)


(2)


(3)


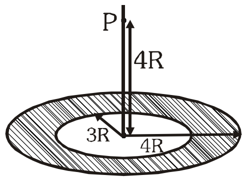
(4)


EXERCISE (JA)

1. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the Earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[IIT-JEE 2010]**

2. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is :- **[IIT-JEE 2010]**

3. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is **[IIT-JEE 2010]**



(A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

4. A satellite is moving with a constant speed *V* in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is :- **[IIT-JEE 2011]**

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

5. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then **[IIT-JEE 2012]**

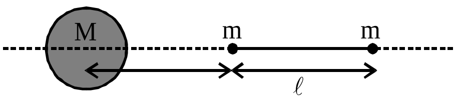
(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = 1/2$

6. A planet of radius $R = \frac{1}{10} \times \text{(radius of Earth)}$ has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2}) **[JEE-Advance 2014]**

(A) 96 N (B) 108 N (C) 120 N (D) 150 N

7. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[JEE-Advance 2015]**

8. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is : **[JEE-Advance 2015]**



9. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is(are) :- **[JEE-Advance 2015]**

(A) P(r = 0) = 0

(B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

(C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$

(D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

10. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is v_e = 11.2 km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the *Sun-Earth system* is closest to (*Ignore the rotation and revolution of the Earth and the presence of any other planet)* **[JEE-Advance 2017]**

(A) v_s = 22 km s^{-1} (B) v_s = 72 km s^{-1} (C) v_s = 42 km s^{-1} (D) v_s = 62 km s^{-1}

11. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[JEE-Advance 2018]**

List–I		**List–II**	
P.	$\frac{v_1}{v_2}$	**1.**	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	**2.**	1
R.	$\frac{K_1}{K_2}$	**3.**	2
S.	$\frac{T_1}{T_2}$	**4.**	8

(A) P $\rightarrow$ 4 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 1 ; S $\rightarrow$ 3
(B) P $\rightarrow$ 3 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1
(C) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 1 ; S $\rightarrow$ 4
(D) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1

12. Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common center with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r) = \rho(r)/m$ is :
[G is universal gravitational constant] **[JEE-Advance 2019]**

(A) $\frac{K}{\pi r^2 m^2 G}$
(B) $\frac{K}{6\pi r^2 m^2 G}$
(C) $\frac{3K}{\pi r^2 m^2 G}$
(D) $\frac{K}{2\pi r^2 m^2 G}$

ANSWER KEY

EXERCISE (S-1)

1. Ans. $\dfrac{R_e k^2}{1-k^2}$

2. Ans. $h = \dfrac{\sqrt{5}-1}{2}R$

3. Ans. *(i)* $\dfrac{4}{3}\sqrt{\dfrac{Gm}{R}}$, *(ii)* $\dfrac{2}{3}\sqrt{\dfrac{2Gm}{5R}}$

4. Ans. 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from east to west

5. Ans. 1×10^5J

6. Ans. $t = \dfrac{GMm}{2C}\left(\dfrac{1}{R_e}-\dfrac{1}{r}\right)$

7. Ans. (a) $T=\dfrac{2\pi d^{3/2}}{\sqrt{3Gm}}$, (b) 2, (c) 2

8. Ans. $\vec{g} = +\dfrac{\pi G\rho_0 R^3}{6}\left[\dfrac{1}{(x-(R/2))^2}-\dfrac{8}{x^2}\right]\hat{i}$, $\vec{g} = -\dfrac{2\pi G\rho_0 R}{3}\hat{i}$

EXERCISE (O-1)

1. Ans. (D)	**2. Ans. (D)**	**3. Ans. (B)**	**4. Ans. (A)**
5. Ans. (B)	**6. Ans. (B)**	**7. Ans. (A)**	**8. Ans. (A)**
9. Ans. (B)	**10. Ans. (B)**	**11. Ans. (D)**	**12. Ans. (D)**
13. Ans. (B)	**14. Ans. (C)**	**15. Ans. (A)**	**16. Ans. (A)**
17. Ans. (C)	**18. Ans. (B)**	**19. Ans. (C)**	**20. Ans. (B,D)**
21. Ans. (A,B,C)	**22. Ans. (A, D)**	**23. Ans. (A)-R, (B)-Q, (C)-Q (D)-P**	

EXERCISE (O-2)

1. Ans. (A,C,D)	**2. Ans. (A)**	**3. Ans. (C)**	**4. Ans. (A)**
5. Ans. (A,B,C)	**6. Ans. (A,C)**	**7. Ans. (A,B,D)**	**8. Ans. (A,D)**
9. Ans. (A,C,D)	**10. Ans. (B)**	**11. Ans. (C)**	

EXERCISE (JM)

1. Ans. (2)	**2. Ans. (2)**	**3. Ans. (1)**	**4. Ans. (1)**
5. Ans. (2)	**6. Ans. (4)**	**7. Ans. (1)**	**8. Ans. (2)**

EXERCISE (JA)

1. Ans. 3	**2. Ans. 6**	**3. Ans. (A)**	**4. Ans. (B)**
5. Ans. (B,D)	**6. Ans. (B)**	**7. Ans. 2**	**8. Ans. 7**
9. Ans. (B,C)	**10. Ans. (C)**	**11. Ans. (B)**	**12. Ans. (D)**

CHAPTER 3

ELASTICITY, THERMAL EXPANSION, CALORIMETRY & HEAT TRANSFER

Chapter 03 Contents

IMPORTANT NOTES

CHAPTER 3

ELASTICITY, THERMAL EXPANSION, CALORIMETRY & HEAT TRANSFER

ELASTICITY

A body is said to be rigid if the relative positions of its constituent particles remains unchanged when external deforming forces are applied to it. The nearest approach to a rigid body is diamond or carborundum.

Actually no body is perfectly rigid and every body can be deformed more or less by the application of suitable forces. All these deformed bodies however regain their original shape or size, when the deforming forces are removed.

The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming forces is called elasticity.

SOME TERMS RELATED TO ELASTICITY :

- **Deforming Force**

  External force which try to change in the length, volume or shape of the body is called deforming force.

- **Perfectly Elastic Body**

  The body which perfectly regains its original form on removing the external deforming force, is defined as a perfectly elastic body. Ex. : quartz – Very nearly a perfect elastic body.

- **Plastic Body**

  (a) The body which does not have the property of opposing the deforming force, is known as a plastic body.

  (b) The bodies which remain in deformed state even after removed of the deforming force are defined as plastic bodies.

- **Internal restoring force**

  When a external force acts at any substance then due to the intermolecular force there is a internal resistance produced into the substance called internal restoring force.

  At equilibrium the numerical value of internal restoring force is equal to the external force.

STRESS

The internal restoring force acting per unit area of cross–section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Area of cross section}} = \frac{F_{\text{internal}}}{A} = \frac{F_{\text{external}}}{A}$$

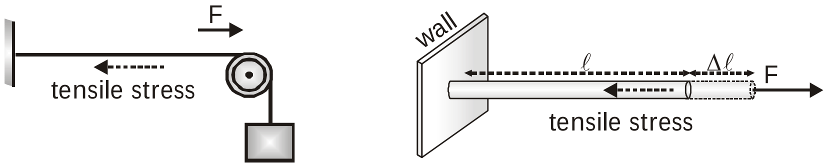
Stress depends on direction of force as well as direction of area of application so it is tensor.

SI UNIT : N–m^{-2} **Dimensions :** $M^1 L^{-1} T^{-2}$

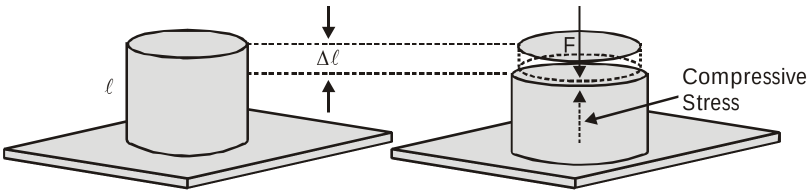
There are three types of stress :-

- **Longitudinal Stress**
  When the stress is normal to the surface of body, then it is known as longitudinal stress. There are two types of longitudinal stress

  (a) Tensile Stress : The longitudinal stress, produced due to increase in length of a body, is defined as tensile stress.

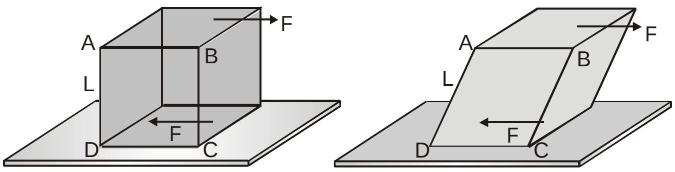


  (b) Compressive Stress : The longitudinal stress, produced due to decrease in length of a body, is defined as compressive stress.



  (c) Volume Stress
  If equal normal forces are applied every one surface of a body, then it undergoes change in volume. The force opposing this change in volume per unit area is defined as volume stress.

  (d) Tangential Stress or Shear Stress
  When the stress is tangential or parallel to the surface of a body then it is known as shear stress. Due to this stress, the shape of the body changes or it gets twisted.



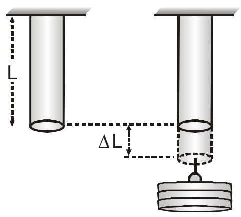
STRAIN

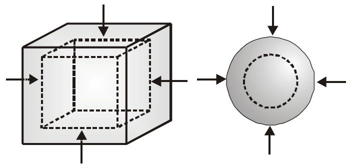
- The ratio of change of any dimension to its original dimension is called strain.

$$\text{Strain} = \frac{\text{change in size of the body}}{\text{original size of the body}}$$

- It is a unitless and dimensionless quantity.
- There are three types of strain : Type of strain depends upon the directions of applied force.

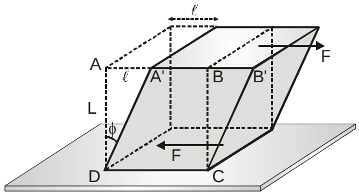
- **Longitudinal strain** $= \dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$





- **Volume strain**$=\dfrac{\text{change in volume of the body}}{\text{original volume of the body}}=\dfrac{\Delta V}{V}$

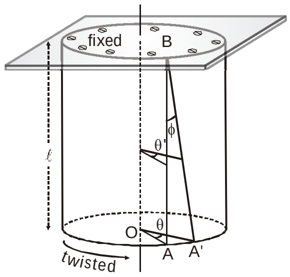
- **Shear strain**



When a deforming force is applied to a body parallel to its surface then its shape (not size) changes. The strain produced in this way is known as shear strain. The strain produced due to change of shape of the body is known as shear strain.

$\tan\phi=\dfrac{\ell}{L}$ or $\phi=\dfrac{\ell}{L}=\dfrac{\text{displacement of upper face}}{\text{distance between two faces}}$

Relation Between angle of twist and Angle of shear



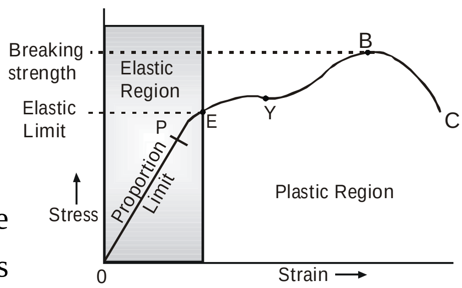
When a cylinder of length 'ℓ' and radius 'r' is fixed at one end and and tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ABA' and angle of twist AOA'. Arc AA' = r θ and Arc AA' = $\ell\phi$ so $r\theta$ =

$\ell\phi\ \Rightarrow\ \phi=\dfrac{r\theta}{\ell}$

where θ = angle of twist, ϕ = angle of shear

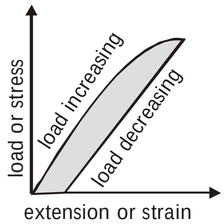
- When a material is under tensile stress restoring force are caused by intermolecular attraction while undercompressive stress, the restoring force are due to intermolecular repulsion.
- If the deforming force is inclined to the surface at an angle θ such that $\theta\neq 0$ and $\theta\neq 90°$ then both tangential and normal stress are developed.
- Linear strain in the direction of force is called **longitudinal** strain while in a direction perpendicular to force **lateral strain**.

STRESS – STRAIN GRAPH



- **Proportion Limit** : The limit in which Hooke's law is valid and stress is directly proportional to strain is called proportion limit.Stress ∝ Strain
- **Elastic limit :** That maximum stress which on removing the deforming force makes the body to recover completely its original state.
- **Yield Point :** The point beyond elastic limit, at which the length of wire starts increasing without increasing stress, is defined as the yield point.

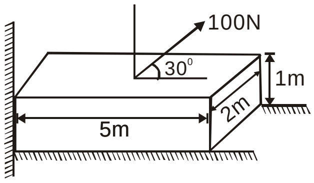
- **Breaking Point :** The position when the strain becomes so large that the wire breaks down at last, is called breaking point. At this position the stress acting in that wire is called breaking stress and strain is called breaking strain.
- **Elastic Hysteresis :** The strain persists even when the stress is removed. This lagging behind of strain is called elastic hysteresis. This is the reason why the values of strain for same stress are different while increasing the load and while decreasing the load.



- **Breaking Stress :** The stress required to cause actual facture of a material is called the breaking stress
  Breaking stress = F/A

- Breaking stress also measures the tensile strength.
- Metals with small plastic deformation are called brittle.
- Metals with large plastic deformation are called ductile.
- Elasticity restoring forces are strictly conservative only when the elastic hysteresis is zero. i.e. the loading and unloading stress – strain curves are identical.

Ex. Find out longitudinal stress and tangential stress on a fixed block.



Sol. Longitudinal or normal stress $\sigma_1 = \dfrac{100\sin 30^0}{5\times 2} = 5$ N/m^2

Tangential stress $\sigma_2 = \dfrac{100\cos 30^0}{5\times 2} = 5\sqrt{3}\,\text{N}/\text{m}^2$

Ex. The breaking stress of aluminium is 7.5×10^8 dyne cm^{-2}. Find the greatest length of aluminium wire that can hang vertically without breaking. Density of aluminium is 2.7 g cm^{-3}.
Given : g = 980 cm s^{-2}.

Sol. Let ℓ be the greatest length of the wire that can hang vertically without breaking.
Mass of wire m = cross–sectional area (A) × length (ℓ) × density (ρ), Weight of wire = mg = A$\ell\rho$g
This is equal to the maximum force that the wire can withstand.

$\therefore$ Breaking stress $= \dfrac{\ell A\rho g}{A} = \ell\rho g \Rightarrow 7.5 \times 10^8 = \ell \times 2.7 \times 980$

$\Rightarrow \quad \ell = \dfrac{7.5\times 10^8}{2.7\times 980}$ cm $= 2.834 \times 10^5$cm $= 2.834$ km

HOOKE'S LAW

If the deformation is small, the stress in a body is proportional to the corresponding strain, this fact is known as Hooke's Law. Within elastic limit : stress $\propto$ strain $\Rightarrow \dfrac{\text{stress}}{\text{strain}}$ = constant

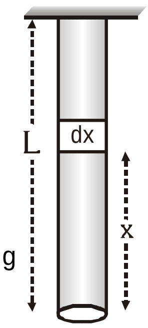
This constant is known as modulus of elasticity or coefficient of elasticity.
The modulus of elasticity depends only on the type of material used. It does not depend upon the value of stress and strain.

YOUNG'S MODULUS OF ELASTICITY 'Y'

- Within elastic limit the ratio of longitudinal stress and longitudinal strain is called Young's modulus of elasticity.
- $Y = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}} = \dfrac{F/A}{\ell/L} = \dfrac{FL}{\ell A}$
- Within elastic limit the force acting upon a unit area of a wire by which the length of a wire becomes double, is equivalent to the Young's modulus of elasticity of material of a wire.
- If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus of elasticity of the material of wire

  $Y = \dfrac{(Mg/\pi r^2)}{(\ell/L)} = \dfrac{MgL}{\pi r^2 \ell}$
- Unit of Y : N/m^2 ● Dimensions of Y : $M^1L^{-1}T^{-2}$

Increment of length due to own weight



Let a rope of mass M and length L is hanged vertically. As the tension of different point on the rope is different. Stress as well as strain will be different at different point.

(i) maximum stress at hanging point (ii) minimum stress at lower point

Consider a dx element of rope at x distance from lower end then tension $T = \left(\dfrac{M}{L}\right) x\, g$

So stress $= \dfrac{T}{A} = \left(\dfrac{M}{L}\right)\dfrac{xg}{A}$

Let increase in length of dx is dy then strain $= \dfrac{dy}{dx}$

So Young modulus of elasticity $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{\frac{M}{L}\frac{xg}{A}}{dy/dx} \Rightarrow \left(\dfrac{M}{L}\right)\dfrac{xg}{A}dx = Y\,dy$

For full length of rope $\dfrac{Mg}{LA}\displaystyle\int_0^L x\,dx = Y\int_0^{\Delta\ell} dy \Rightarrow \dfrac{Mg}{LA}\dfrac{L^2}{2} = Y\Delta\ell \Rightarrow \Delta\ell = \dfrac{MgL}{2AY}$

[Since the stress is varying linearly we may apply average method to evaluate strain.]

BULK MODULUS OF ELASTICITY 'K' or 'B'

- Within elastic limit the ratio of the volume stress and the volume strain is called bulk modulus of elasticity.
- K or B $= \dfrac{\text{volume stress}}{\text{volume strain}} = \dfrac{F/A}{\frac{-\Delta V}{V}} = \dfrac{\Delta P}{\frac{-\Delta V}{V}}$

  The minus sign indicates a decrease in volume with an increase in stress.
- Unit of K : $N\,m^{-2}$ or pascal

- **Bulk modulus of an ideal gas is process dependence.**

  - For isothermal process PV = constant $\Rightarrow$ PdV + VdP = 0 $\Rightarrow$ P=$\frac{-dP}{dV/V}$ So bulk modulus = P

  - For adiabatic process PV^{γ} = constant $\Rightarrow \gamma PV^{\gamma-1}dV + V^{\gamma}dP = 0$

    $\Rightarrow \gamma PdV + VdP = 0 \Rightarrow \gamma P = \frac{-dP}{dV/V}$ So bulk modulus = γP

  - For any polytropic process PV^n = constant

    $\Rightarrow nPV^{n-1}dV + V^n dP = 0 \Rightarrow PdV + VdP = 0 \Rightarrow nP = \frac{-dP}{dV/V}$ So bulk modulus = nP

  - **Compressibility** : The reciprocal of bulk modulus of elasticity is defined as compressibility.

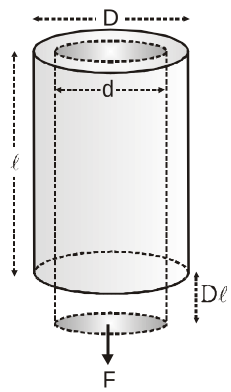
    $$C = \frac{1}{K}$$

MODULUS OF RIGIDITY 'η'

- Within elastic limit the ratio of shearing stress and shearing strain is called modulus of rigidity of a material.

- $\eta = \frac{\text{shearing stress}}{\text{Shearing strain}} = \left(\frac{\frac{F_{\tan gential}}{A}}{\phi}\right) = \frac{F_{\tan gential}}{A\phi}$

  Note : Angle of shear 'ϕ' always taking in radian



Poisson's Ratio (σ)

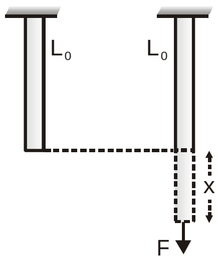
In elastic limit, the ratio of lateral strain and longitudinal strain is called Poisson's ratio.

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\beta}{\alpha};\ \beta = \frac{-\Delta D}{D} = \frac{d-D}{D}\ \&\ \alpha = \frac{\Delta L}{L}$$

WORK DONE IN STRETCHING A WIRE

(Potential energy of a stretched wire)

When a wire is stretched, work is done against the interatomic forces, which is stored in the form of elastic potential energy.



For a wire of length L_o stretched by a distance x, the restoring elastic force is :

$$F = \text{stress} \times \text{area} = Y\left[\frac{x}{L_o}\right]A$$

The work has to be done against the elastic restoring forces in stretching dx

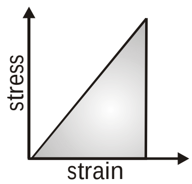
$$dW = Fdx = \frac{YA}{L_o} xdx$$

The total work done in stretching the wire from x = 0 to x = $\Delta\ell$ is, then

$$W = \int_0^{\Delta\ell} \frac{YA}{L_o} xdx = \frac{YA}{L_o}\left[\frac{x^2}{2}\right]_0^{\Delta\ell} = \frac{YA(\Delta\ell)^2}{2L_o}$$

$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{original volume} = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$

- The value of K is maximum for solids and minimum for gases.



- For any ideal rigid body all three elastic modulus are infinite .
- η is the characteristic of solid material only as the fluids do not have fixed shape.
- Potential energy density = area under the stress–strain curve.
- Young's modulus = Slope of the stress–strain curve

Ex. A steel wire of 4.0 m in length is stretched through 2.00 mm. The cross–sectional area of the wire is 2.0 mm^2. If Young's modulus of steel is 2.0×10^{11} N/m^2 find (i) the energy density of wire (ii) the elastic potential energy stored in the wire.

Sol. (i) The energy density of stretched wire $= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2$

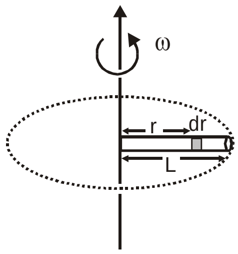
$$= \frac{1}{2} \times 2.0 \times 10^{11} \times \left[\frac{2 \times 10^{-3}}{4}\right]^2 = 2.5 \times 10^4 \text{ J/m}^3$$

(ii) Elastic potential energy = energy density × volume = $2.5 \times 10^4 \times (2.0 \times 10^{-6}) \times 4.0$ J

$= 20 \times 10^{-2} = 0.20$J

Ex. A thin uniform metallic rod of length 0.5 m and radius 0.1 m rotates with an angular velocity 400 rad/s in a horizontal plane about a vertical axis passing through one of its ends. Calculate (a) tension in the rod and (b) the elongation of the rod. The density of material of the rod is 10^4 kg/m^3 and the Young's modulus is 2×10^{11} N/m^2.

Sol. (a) Consider an element of length dr at a distance r from the axis of rotation as shown in figure. The centripetal force acting on this element will be $dT = dmr\omega^2 = (\rho A dr) r\omega^2$. As this force is provided by tension in the rod (due to elasticity), so the tension in the rod at a distance r from the axis of rotation will be due to the centripetal force due to all elements between x = r to x = L



i.e., $T = \int_r^L \rho A\omega^2 r dr = \frac{1}{2}\rho A\omega^2\left[L^2 - r^2\right]$ ...(i)

So here $T = \frac{1}{2} \times 10^4 \times \pi \times 10^{-2} \times (400)^2 \left[\left(\frac{1}{2}\right)^2 - r^2\right] = 8\pi \times 10^6\left[\frac{1}{4} - r^2\right]$ N

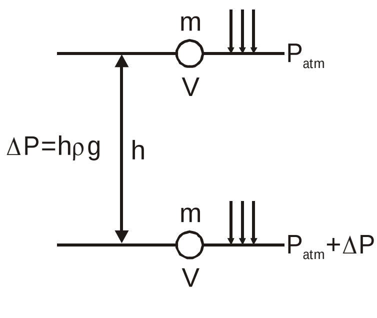
(b) Now if dy is the elongation in the element of length dr at position r then strain

$$\frac{dy}{dr} = \frac{\text{stress}}{Y} = \frac{T}{AY} = \frac{1}{2}\frac{\rho\omega^2}{Y}[L^2 - r^2]\,dr$$

So the elongation of the whole rod

$$\Delta L = \frac{\rho\omega^2}{2Y}\int_0^L \left(L^2 - r^2\right)\,dr = \frac{1}{3}\frac{\rho\omega^2 L^3}{Y} = \frac{1}{3} \times \frac{10^4 \times (400)^2 (0.5)^3}{2 \times 10^{11}} = \frac{1}{3} \times 10^{-3}\,\text{m}$$

Ex. Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility $K = 50 \times 10^{-6}$ /atm.



Sol. $B = \frac{\Delta P}{-\Delta V / V} \Rightarrow \frac{\Delta V}{V} = -\frac{\Delta P}{B}$

We know $P = P_{atm} + h\rho g$ and $m = \rho V$ = constant

$d\rho V + dV\rho = 0 \Rightarrow \frac{d\rho}{\rho} = -\frac{dV}{V}$ i.e. $\frac{\Delta\rho}{\rho} = \frac{\Delta P}{B}$

$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{1}{100} = \frac{h\rho g}{B}$

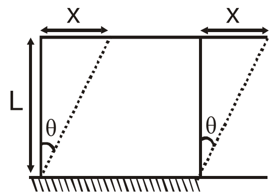
[assuming ρ = constant]; $h\rho g = \frac{B}{100} = \frac{1}{100K}$

$\Rightarrow h\rho g = \frac{1 \times 1 \times 10^5}{100 \times 50 \times 10^{-6}}$

$h = \frac{10^5}{5000 \times 10^{-6} \times 1000 \times 10} = \frac{100 \times 10^3}{50}$ m = 2km

Ex. A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face. Find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4×10^6 N/m^2.

Sol. Stress = $\frac{F}{A} = \eta\frac{x}{L}$



Strain = $\theta = \frac{F}{A\eta} = \frac{1800}{25\times10^{-4}\times2.4\times10^{6}} = \frac{180}{25\times24} = \frac{3}{10}$ = 0.3 radian

as $\frac{x}{L}$ = 0.3 $\Rightarrow$ x = 0.3 × 4 × 10^{-2} = 1.5 × 10^{-2} m = 1.5 mm

EFFECT OF TEMPERATURE ON ELASTICITY

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour.

For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

EFFECT OF IMPURITY ON ELASTICITY

Y is slightly increase by impurity. The inter molecular attraction force inside wire effectively increase by impurity due to this external force can be easily opposed.

TEMPERATURE & DIFFERENT TYPE OF TEMPERATURE SCALES

TEMPERATURE

- Temperature is a macroscopic physical quantity related to our sense of hot and cold.
- The natural flow of heat is from higher temperature to lower temperature, i.e. temperature determines the thermal state of a body whether it can give or receive heat.

DIFFERENT TYPES OF TEMPERATURE SCALES

- The Kelvin temperature scale is also known as thermodynamic scale. The SI unit of temperature is the kelvin and is defined as (1/273.16) of the temperature of the triple point of water. The triple point of water is that point on a P–T diagram where the three phase of water, the solid, the liquid and the gas, can coexist in equilibrium.
- In addition to Kelvin temperature scale, there are other temperature scales also like Celsius, Fahrenheit, Reaumur, Rankine, etc. Temperature on one scale can be converted into other scale by using the following identity

$$\frac{\text{Reading on any scale} - \text{lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{lower fixed point (LFP)}} = \text{constant for all scales}$$

Hence $\dfrac{C - 0°}{100° - 0°} = \dfrac{F - 32°}{212° - 32°} = \dfrac{K - 273.15}{373.15 - 273.15}$

- Different temperature scales :

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	°C	0°C	100°C	100
Fahrenheit	°F	32°F	212°F	180
Kelvin	K	273.15 K	373.15 K	100

Ex. Express a temperature of 60°F in degree celsius and in kelvin.

Sol. By using $\dfrac{C - 0°}{100° - 0°} = \dfrac{F - 32°}{212° - 32°} = \dfrac{K - 273.15}{373.15 - 273.15}$

$\Rightarrow \dfrac{C - 0°}{100° - 0°} = \dfrac{60° - 32°}{212° - 32°} = \dfrac{K - 273.15}{373.15 - 273.15} \Rightarrow C = 15.15°\ C$ and $K = 288.7\ K$

Ex. The temperature of an iron piece is heated from 30° to 90°C. What is the change in its temperature on the fahrenheit scale and on the kelvin scale?

Sol. $\Delta C = 90° - 30° = 60°C$

Temperature difference on Fahrenheit Scale $\Delta F = \frac{9}{5}\Delta C = \frac{9}{5}(60°C) = 108°F$

Temperature difference on Kelvin Scale $\Delta K = \Delta C = 60K$

THERMAL EXPANSION

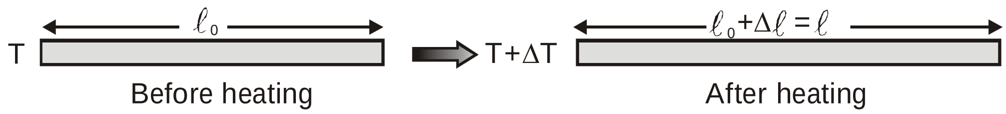
THERMAL EXPANSION

When matter is heated without any change in it's state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration increases and hence energy of atoms increases, hence the average distance between the atom increases. So the matter as a whole expands.

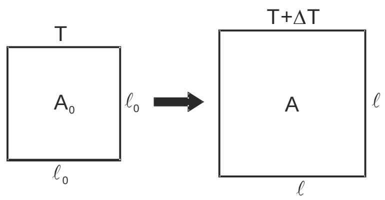
- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (Linear expansion), two dimension (Superficial expansion) and three dimension (Volume expansion) while liquids and gases usually suffers change in volume only.

Linear expansion :

$\ell = \ell_0 (1 + \alpha\Delta\theta) \Rightarrow \Delta\ell = \ell_0\alpha\Delta\theta$



Superficial (areal) expansion :

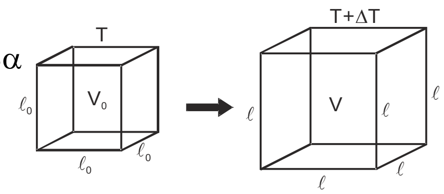


$A = A_0 (1 + \beta\Delta\theta)$

Also $A_0 = \ell_0^2$ and $A = \ell^2$

So $\ell^2 = \ell_0^2(1 + \beta\Delta\theta) = [\ell_0(1 + \alpha\Delta\theta)]^2 \Rightarrow \beta = 2\alpha$

Volume expansion :



$V = V_0 (1 + \gamma\Delta\theta)$ Also $V = \ell^3$ and $V_0=\ell_0^3$ so $\gamma = 3\alpha$

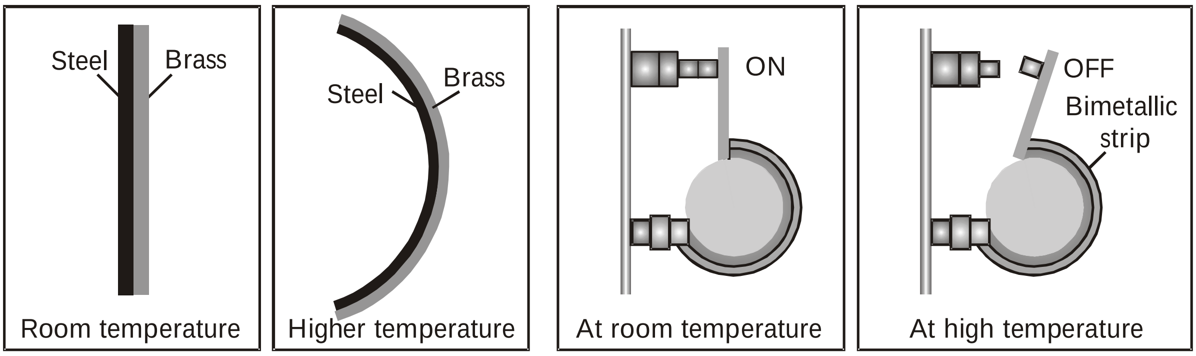
$\Rightarrow 6\alpha = 3\beta = 2\gamma$ or $\alpha : \beta : \gamma = 1 : 2 : 3$

Contraction on heating :

Some rubber like substances contract on heating because transverse vibration of atoms of substance dominate over longitudinal vibration which is responsible for expansion.

Application of thermal Expansion in Solids

(a) Bi–metallic strip : Two strips of equal length but of different materials (different coefficient of linear expansion) when join together, it is called "Bi–metallic strip" and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side.

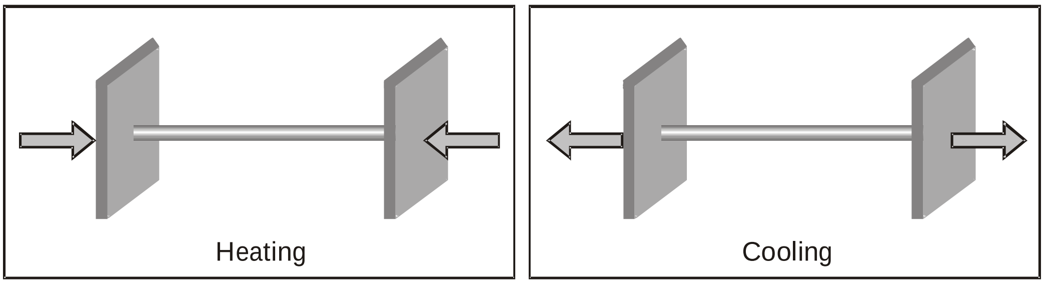


(b) Effect of temperature on the time period of a simple pendulum : A pendulum clock keeps proper time at temperature θ. If temperature is increased to θ' (> θ) then due to linear expansion, length of pendulum increases and hence its time period will increase.

Fractional change in time period $\frac{\Delta T}{T}=\frac{1}{2}\alpha\Delta\theta$ $\left(\because T \propto \sqrt{\ell} \quad \therefore \frac{\Delta T}{T}=\frac{1}{2}\frac{\Delta\ell}{\ell}\right)$

- Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time. Loss of time in a time period $\Delta T=\frac{1}{2}\alpha\,\Delta\theta T$
- The clock will lose time i.e. will become slow if θ' > θ (in summer) and will gain time i.e will become fast if θ' < θ (in winter).
- Since coefficient of linear expansion (α) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.

(c) When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length L is Δθ then :–

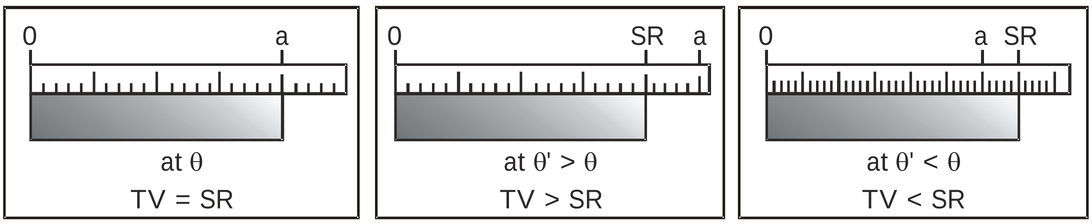


Thermal strain $=\frac{\Delta L}{L}=\alpha\Delta\theta$ $\quad \because \alpha=\frac{\Delta L}{L}\times\frac{1}{\Delta\theta}$ $\quad$ So thermal stress = YαΔθ

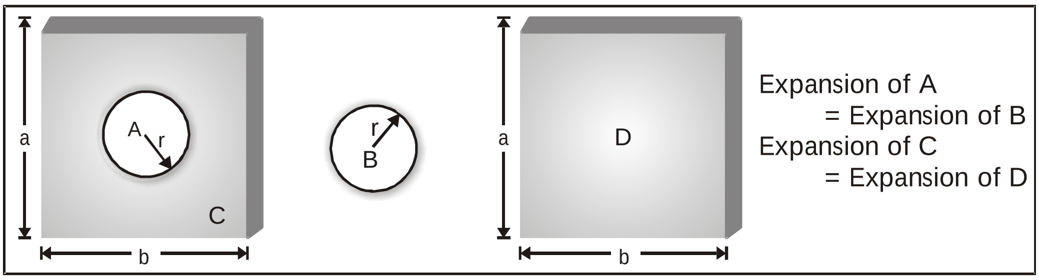
$\because Y=\frac{\text{stress}}{\text{strain}}$

So force on the supports F=YAαΔθ

(d) Error in scale reading due to expansion or contraction: If a scale gives correct reading at temperature θ. At temperature θ'(>θ) due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that,



(e) Expansion of cavity: Thermal expansion of an isotropic object may be imagined as a photographic enlargement.



(f) Some other application

- When rails are laid down on the ground, space is left between the ends of two rails
- The transmission cable are not tightly fixed to the poles
- Test tubes, beakers and cubicles are made of pyrex–glass or silica because they have very low value of coefficient of linear expansion
- The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel
- A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle.

Ex. A steel ruler exactly 20 cm long is graduated to give correct measurements at 20^0C.

$(\alpha_{steel} = 1.2 \times 10^{-5}\ °C^{-1})$

(a) Will it give readings that are too long or too short at lower temperatures?

(b) What will be the actual length of the ruler be when it is used in the desert at a temperature of 40°C?

Sol. (a) If the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20°C, each centimeter division is actually somewhat shorter than 1.0 cm, so the steel ruler gives readings that are too long.

(b) At 40°C, the increases in length of the ruler is

$$\Delta\ell = \ell\alpha\Delta T = (20)\,(1.2 \times 10^{-5})\,(40^0 - 20^0) = 0.48 \times 10^{-2} \text{ cm}$$

$\therefore$ The actual length of the ruler is, $\ell' = \ell + \Delta\ell = 20.0048$ cm

Ex. A second's pendulum clock has a steel wire. The clock is calibrated at 20^0C. How much time does the clock lose or gain in one week when the temperature is increased to 30^0C?

$(\alpha_{steel} = 1.2 \times 10^{-5}\ °C^{-1})$

Sol. The time period of second's pendulum is 2 second. As the temperature increases length time period increases. Clock becomes slow and it loses the time. The change in time period is

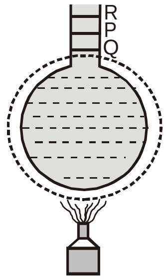
$$\Delta T = \frac{1}{2} T\alpha\Delta\theta = \left(\frac{1}{2}\right)(2)(1.2 \times 10^{-5})\,(30^0 - 20^0) = 1.2 \times 10^{-4} \text{ s}$$

$\therefore$ New Time period is $\quad T' = T + \Delta T = (2 + 1.2 \times 10^{-4}) = 2.00012$ s

$\therefore$ Time lost in one week $\quad \Delta t = \left(\frac{\Delta T}{T'}\right) t = \frac{(1.2 \times 10^{-4})}{(2.00012)} (7 \times 24 \times 3600) = 36.28$ s

Thermal Expansion in Liquids

- Liquids do not have linear and superficial expansion but these only have volume expansion.
- Since liquids are always to be heated along with a vessel which contains them so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid.



PQ $\rightarrow$ represents expansion of vessel

QR $\rightarrow$ represents the real expansion of liquid.

- The actual increase in the volume of the liquid

  = The apparent increase in the volume of liquid + the increase in the volume of the vessel.
- Liquids have two coefficients of volume expansion.

(i) Co–efficient of apparent expansion (γ_a)

- It is due to apparent (that appears to be, but in not) increase in the volume of liquid if expansion of vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

(ii) Co–efficient of real expansion (γ_r)

- It is due to the actual increase in volume of liquid due to heating.

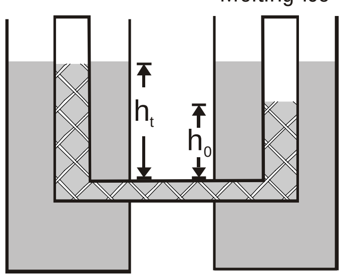
$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Initial volume} \times \Delta\theta} = \frac{(\Delta V)}{V \times \Delta\theta}$$

- Also coefficient of expansion of flask $\gamma_{\text{Vessel}} = \frac{(\Delta V)_{\text{Vessel}}}{V \times \Delta\theta}$
- $\gamma_{\text{Real}} = \gamma_{\text{Apparent}} + \gamma_{\text{Vessel}}$
- Change (apparent change) in volume in liquid relative to vessel is

  $\Delta V_{\text{app}} = V(\gamma_{\text{Real}} - \gamma_{\text{Vessel}})\,\Delta\theta = V(\gamma_r - 3\alpha)\Delta\theta$

  α = Coefficient of linear expansion of the vessel.

Ex. In figure shown, left arm of a U–tube is immersed in a hot water bath at temperature t°C, and right arm is immersed in a bath of melting ice; the height of manometric liquid in respective columns is h_t and h_0. Determine the coefficient of expansion of the liquid.



Sol. The liquid is in hydrostatic equilibrium $\Rightarrow \rho_t g h_t = \rho_0 g h_0$

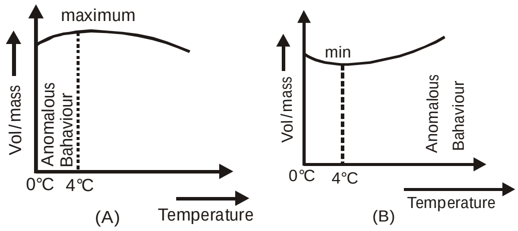
Where, ρ_t is density of liquid in hot bath, ρ_0 is density of liquid in cold bath.

Volumes of a given mass M of liquid at temperatures t and 0^0C

are related by $V_t = V_0(1+\gamma t)$ Since $\rho_t V_t = \rho_0 V_0 \Rightarrow \rho_t = \frac{\rho_0 V_0}{V_t} = \frac{\rho_0}{(1+\gamma t)}$

Since $h_t = \frac{\rho_0 h_0}{\rho_t} = h_0(1+\gamma t)$ which on solving for γ, yields $\gamma = \frac{(h_t - h_0)}{h_0 t}$

Anomalous expansion of water



Generally matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e. γ is negative. This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.

This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature increases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches 0°C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4°C. At 4°C, density of water is maximum while its specific volume is minimum.

Ex. The difference between lengths of a certain brass rod and of a steel rod is claimed to be constant at all temperatures. Is this possible ?

Sol. If L_B and L_S are the lengths of brass and steel rods respectively at a given temperature, then the lengths of the rods when temperature is changed by θ °C.

$L'_B = L_B(1 + \alpha_B \Delta\theta)$ and $L'_S = L_S(1 + \alpha_B \Delta\theta)$ So that $L'_B = L'_S(L_B - L_S) + (L_B \alpha_B - L_S\alpha_S)\ \Delta\theta$

So $(L'_B - L'_S)$ will be equal to $(L_B - L_S)$ at all temperatures if , $L_B\alpha_B - L_S\alpha_S = 0$ [as $\Delta\theta \neq 0$]

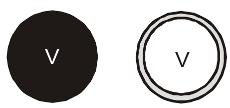
or $\frac{L_B}{L_S} = \frac{\alpha_S}{\alpha_B}$

i.e., the difference in the lengths of the two rods will be independent of temperature if the lengths are in the inverse ratio of their coefficients of linear expansion.

Ex. There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if

(a) they are heated to the same temperature, (b) same heat is given to them ?

Sol. (a) As thermal expansion of isotropic solids is similar to true photographic enlargement,



expansion of a cavity is same as if it had been a solid body of the same material

i.e. $\Delta V = V\gamma\ \Delta\theta$

As here V, γ and $\Delta\theta$ are same for both solid and hollow spheres treated (cavity) ;

so the expansion of both will be equal.

(b) If same heat is given to the two spheres due to lesser mass, rise in temperature of hollow sphere will be more [as $\Delta\theta = \frac{Q}{mc}$] and hence its expansion will be more [as $\Delta V = V\gamma\Delta\theta$].

CALORIMETRY

HEAT

When a hot body is put in contact with a cold one, the former gets colder and the latter warmer. From this observation it is natural to conclude that a certain quantity of heat has passed from the hot body to the cold one. Heat is a form of energy.

Heat is felt by its effects. Some of the effects of heat are :

(a) Change in the degree of hotness (b) Expansion in length, surface area and volume

(c) Change in state of a substance (d) Change in the resistance of a conductor

(e) Thermo e.m.f. effect

SI UNIT : J (joule) Also measured in the unit calorie.

- **Calorie**

  It is defined as the amount of heat required to raise the temperature of 1 g water by 1°C.

- **International calorie**

  International calorie is the amount of heat required to raise the temperature of 1g water from 14.5 °C to 15.5 °C rise of temperature.

MECHANICAL EQUIVALENT OF HEAT

According to Joule, work may be converted into heat and vice–versa. The ratio of work done to heat produced is always constant. $\frac{W}{H}$ = constant (J) $\Rightarrow W = J H$

W must be in joule, irrespective of nature of energy or work and H must be in calorie.

J is called mechanical equivalent of heat. It is not a physical quantity but simply a conversion factor.

It converts unit of work into that of heat and vice–versa.

J = 4.18 joule/cal or 4.18×10^3 joule per kilo–cal. For rough calculations we take J = 4.2 joule/cal

SPECIFIC HEAT (s or c)

It is the amount of energy required to raise the temperature of unit mass of that substance by 1°C (or 1K) is called specific heat. It is represented by s or c.

If the temperature of a substance of mass m changes from T to T + dT when it exchanges an amount of heat dQ with its surroundings then its specific heat is $c = \frac{1}{m}\frac{dQ}{dT}$

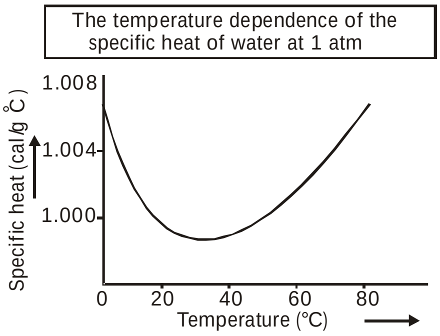
The specific heat depends on the pressure, volume and temperature of the substance.

For liquids and solids, specific heat measurements are most often made at a constant pressure as functions of temperature, because constant pressure is quite easy to produce experimentally.

SI UNIT : joule/kg–K **CGS UNIT :** cal/g –°C

Specific heat of water : c_{water} = 1 cal/g–°C = 1 cal/g–K = 1 kcal/kg–K = 4200 joule/kg–K

When a substance does not undergo a change of state (i.e., liquid remains liquid or solid remains solid), then the amount of heat required to raise the temperature of mass m of thesubstance by an amount $\Delta\theta$ is $Q = ms\Delta\theta$.



The temperature dependence of the specific heat of water at 1 atmospheric pressure is shown in figure. Its variation is less than 1% over the interval from 0 to 100°C. Such a small variation is typical for most solids and liquids, so their specific heats can generally be taken to be constant over fairly large temperature ranges.

- There are many processes possible to give heat to a gas.

  A specific heat can be associated to each such process which depends on the nature of process.
- Value of specific heats can vary from zero (0) to infinity.
- Generally two types of specific heat are mentioned for a gas –

  (a) Specific heat at constant volume (C_V) (b) Specific heat at constant pressure (C_P)
- These specific heats can be molar or gram.

MOLAR HEAT CAPACITY

The amount of energy needed to raise the temperature of one mole of a substance by 1°C (or 1K) is called molar heat capacity. The molar heat capacity is the product of molecular weight and specific heat i.e.,

Molar heat capacity C = Molecular weight (M) × Specific heat(c) $\Rightarrow C = \frac{1}{\mu}\left(\frac{dQ}{dT}\right)$

If the molecular mass of the substance is M and the mass of the substance is m then number of moles of the substance $\mu = \frac{m}{M} \Rightarrow C = \frac{M}{m}\left(\frac{dQ}{dT}\right)$ **SI UNIT :** J/mol–K

THERMAL CAPACITY

The quantity of heat required to raise the temperature of the whole of that substance through 1°C is called thermal capacity. The thermal capacity of mass m of the whole of substance of specific heat s is = ms

Thermal capacity = mass × specific heat

Thermal capacity depends on property of material of the body and mass of the body.

SI UNIT : cal/°C or cal/K, **Dimensions :** $ML^2T^{-2}K^{-1}$

WATER EQUIVALENT OF A BODY

As the specific heat of water is unity so the thermal capacity of a body (ms) represents its water equivalent also.

- Mass of water having the same thermal capacity as the body is called the water equivalent of the body
- The water equivalent of a body is the amount of water that absorbs or gives out the same amount of heat as is done by the body when heated or cooled through 1°C.
  Water equivalent= mass of body × specific heat of the material $\Rightarrow$ (w = ms).

LATENT HEAT OR HIDDEN HEAT

When state of a body changes, change of state takes place at constant temperature [melting point or boiling point] and heat released or absorbed is $Q = mL$ where L is latent heat. Heat is absorbed if solid converts into liquid (at melting point) or liquid converts into vapours (at boiling point) and heat is released if liquid converts into solid or vapours converts into liquid.

- **Latent heat of fusion**
  It is the quantity of heat (in kilocalories) required to change its 1 kg mass from solid to liquid state at its melting point. Latent heat of fusion for ice : 80 kcal/kg = 80 cal /g.
- **Latent heat of vaporization**
  The quantity of heat required to change its 1 kg mass from liquid to vapour state at its boiling point. Latent heat of vaporisation for water : 536 kcal/kg = 536 cal/g

CHANGE OF STATE

- **Melting**
  Conversion of solid into liquid state at constant temperature is known as melting.
- **Boiling**
  Evaporation within the whole mass of the liquid is called boiling. Boiling takes place at a constant temperature known as boiling point. A liquid boils when the saturated vapour pressure on its surface is equal to atmospheric pressure. Boiling point reduces on decreasing pressure.
- **Evaporation**
  Conversion of liquid into vapours at all temperatures is called evaporation. It is a surface phenomenon. Greater the temperature, faster is the evaporation. Smaller the boiling point of liquid, more rapid is the evaporation. Smaller the humidity, more is the evaporation. Evaporation increases on decreasing pressure that is why evaporation is faster in vacuum.
  - **Heat of evaporation**
    Heat required to change unit mass of liquid into vapour at a given temperature is called heat of evaporation at that temperature.
- **Sublimation**
  Direct conversion of solid in to vapour state is called sublimation.
  - **Heat of sublimation**
    Heat required to change unit mass of solid directly into vapours at a given temperature is called heat of sublimation at that temperature.
  - Camphor and ammonium chloride sublimates on heating in normal conditions.
  - A block of ice sublimates into vapours on the surface of moon because of very–very low pressure on its surface

- **Condensation**

  The process of conversion from gaseous or vapour state to liquid state is known as condensation. These materials again get converted to vapour or gaseous state on heating.

PHASE OF A SUBSTANCE

The phase of a substance is defined as its form which is homogeneous, physically distinct and mechanically separable from the other forms of that substance.

Phase diagram

- A phase diagram is a graph in which pressure (P) is represented along the y–axis and temperature (T) is represented along the x–axis.
- **Characteristics of Phase diagram**

  (i) Different phases of a substances can be shown on a phase diagram.

  (ii) A region on the phase diagram represents a single phase of the substance, a curve represents equilibrium between two phases and a common point represents equilibrium between three phases.

  (iii) A phase diagram helps to determine the condition under which the different phases are in equilibrium.

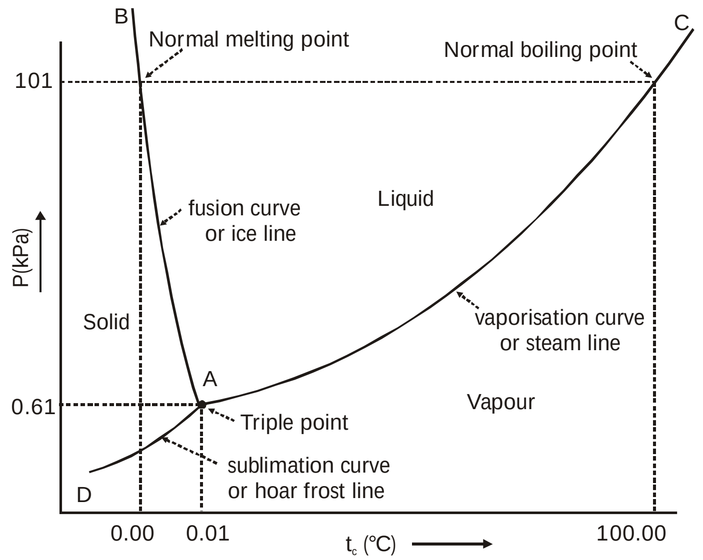
  (iv) A phase diagram is useful for finding a convenient way in which a desired change of phase can be produced.

PHASE DIAGRAM FOR WATER

The phase diagram for water consists of three curves AB, AC and AD meeting each other at the point A, these curves divide the phase diagram into three regions.

Phase diagram for water
(Scale are not linear)

At the triple point solid, liquid and vapour phases are all in equilibrium



Region to the left of the curve AB and above the curve AD represents the solid phase of water (ice). The region to the right of the curve AB and above the curve AC represents the liquid phase of water. The region below the curves AC and AD represents the gaseous phase of water (i.e. water vapour). A curve on the phase diagram represents the boundary between two phases of the substance.

Along any curve the two phases can coexist in equilibrium

- Along curve AB, ice and water can remain in equilibrium.This curve is called fusion curve or ice line. This curve shows that the melting point of ice decreases with increase in pressure.
- Along the curve AC, water and water vapour can remain in equilibrium. The curve is called vaporisation curve or steam line. *The curve shows that the boiling point of water increases with increase in pressure.*
- Along the curve AD, ice and water vapour can remain in equilibrium.
  This curve is called sublimation curve or hoar frost line.

TRIPLE POINT OF WATER

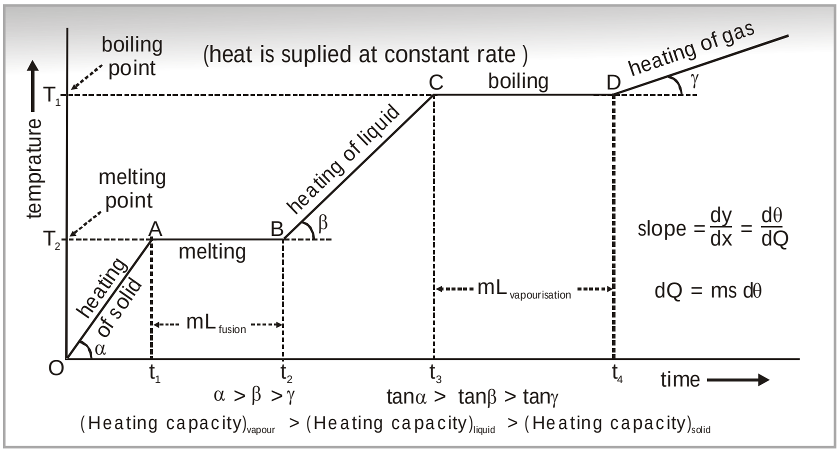
The three curves in the phase diagram of water meet at a single point A, which is called the triple point of water. The triple point of water represents the co–existance of all the three phases of water ice water and water vapour in equilibrium. The pressure corresponding to triple point of water is 6.03×10^{-3} atmosphere or 4.58 mm of Hg and temperature corresponding to it is 273.16K.

- **Significance of triple point of water**
  Triple point of water represents a unique condition and it is used to define the absolute temperature. While making Kelvin's absolute scale, upper fixed point is 273.16 K and lower fixed point is 0 K.

  One kelvin of temperature is fraction $\dfrac{1}{273.16}$ of the temperature of triple point of water.

HEATING CURVE

If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and time as shown in figure is called heating curve.



- **In the region OA**
  Rate of heat supply P is constant and temperature of solid is changing with time

  So, $Q = mc_S \Delta T \Rightarrow P\,\Delta t = mc_S\,\Delta T\ [\because Q = P\,\Delta t]\ \therefore \dfrac{\Delta T}{\Delta t}$ = The slope of temperature–time curve so

  specific heat of solid $c_S \propto \dfrac{1}{\text{slope of line OA}}$ specific heat (or thermal capacity) is inversely proportional to the slope of temperature–time curve.

- **In the region AB**

  Temperature is constant, so it represents change of state, i.e., melting of solid with melting point T_1. At point A melting starts and at point B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion then

  $Q = mL_F \Rightarrow L_F = \frac{P(t_2 - t_1)}{m}$ [as $Q = P(t_2 - t_1)$] $\Rightarrow L_F \propto$ length of line AB

  i.e., Latent heat of fusion is proportional to the length of line of zero slope.

  [In this region specific heat $\propto \frac{1}{\tan 0°} = \infty$]

- **In the region BC**

  Temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, $c_L \propto \frac{1}{\text{slope of line BC}}$

- **In the region CD**

  Temperature in constant, so it represents change of state, i.e., liquid is boiling with boiling point T_2. At C all substance is in liquid state while at D is vapour state and between C and D partly liquid and partly gas. The length of line CD is proportional to latent heat of vaporisation, i.e., $L_V \propto$ Length of line CD.

  [In this region specific heat $\propto \frac{1}{\tan 0°} = \infty$]

  The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

LAW OF MIXTURES

- When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature released heat while body at lower temperature absorbs it, so that Heat lost = Heat gained. Principle of calorimetry represents the law of conservation of heat energy.
- Temperature of mixture (T) is always $\geq$ lower temperature (T_L) and $\leq$ higher temperature (T_H), $T_L \leq T \leq T_H$

  The temperature of mixture can never be lesser than lower temperature (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Further more usually rise in temperature of one body is not equal to the fall temperature of the other body though heat gained by one body is equal to the heat lost by the other.

Ex. 5g ice at 0°C is mixed with 5g of steam at 100°C . What is the final temperature?

Sol. Heat required by ice to raise its temperature to 100°C,

$Q_1 = m_1L_1 + m_1c_1\Delta\theta_1 = 5 \times 80 + 5 \times 1 \times 100 = 400 + 500 + 900 = 1800$ cal

Heat given by steam when condensed $Q_2 = m_2L_2 = 5 \times 536 = 2680$ cal

As $Q_2 > Q_1$. This means that whole steam is not even condensed.

Hence temperature of mixture will remain at 100°C.

Ex. A calorimeter of heat capacity 100 J/K is at room temperature of 30°C. 100 g of water at 40°C of specific heat 4200 J/kg–K is poured into the calorimeter. What is the temperature of water in calorimeter?

Sol. Let the temperature of water in calorimeter is t. Then heat lost by water = heat gained by calorimeter

$(0.1) \times 4200 \times (40 - t) = 100\,(t - 30) \Rightarrow 42 \times 40 - 42t = 10t - 300 \Rightarrow t = 38.07°C$

Ex. Find the quantity of heat required to convert 40 g of ice at –20°C into water at 20°C.

Given $L_{ice} = 0.336 \times 10^6$ J/kg.

Specific heat of ice = 2100 J/kg–K, specific heat of water = 4200 J/kg–K

Sol. Heat required to raise the temperature of ice from –20°C to 0°C = 0.04 × 2100 × 20 = 1680 J

Heat required to convert the ice into water at 0°C = mL = $0.04 \times 0.336 \times 10^6 = 13440$ J

Heat required to heat water from 0°C to 20°C = 0.04 × 4200 × 20 = 3360 J

Total heat required = 1680 + 13440 + 3360 = 18480 J

Ex. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensed? Latent heat of steam = 536 cal/g.

Sol. Heat required by (calorimeter + water)

$$Q = (m_1c_1 + m_2c_2)\,\Delta\theta = (0.02 + 1.1 \times 1)\,(80 - 15) = 72.8 \text{ kcal}$$

If m is mass of steam condensed, then heat given by steam

$$Q = mL + mc\,\Delta\theta = m \times 536 + m \times 1 \times (100 - 80) = 556\,m \quad \therefore\ 556\,m = 72.8$$

$\therefore$ Mass of steam condensed $m = \dfrac{72.8}{556} = 0.130$ kg

MODE OF HEAT TRANSFER

Heat is a form of energy which transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any one of the following modes :

- **Conduction**

  The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.

- **Convection**

  The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant is fluids (liquids and gases).

- **Radiation**

  Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the inter–venning medium.

Conduction	Convection	Radiation
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer with out any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solid body (in mercury also)	Heat transfer in fluids (Liquid + gas)	All
Slow process	Slow process	Fast process (3×10^8 m/sec)
Irregular path	Irregular path	Straight line (like light)

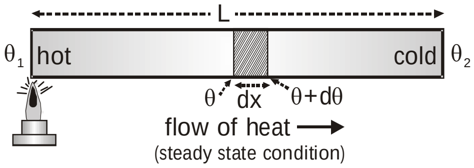
THERMAL CONDUCTION

The process by which heat is transferred from hot part to cold part of a body through the transfer of energy from one particle to another particle of the body without the actual movement of the particles from their equilibrium positions is called conduction. The process of conduction only in solid body (except Hg) Heat transfer by conduction from one part of body to another continues till their temperatures become equal.

Steady state : When temperature of the each cross–section of the bar becomes constant through different for different cross–sections is called steady state.

- **Equation of thermal conduction**

  Rate of heat flow $\frac{dQ}{dt} = -KA\frac{d\theta}{dx}$ $\quad \frac{dQ}{dt} = \frac{KA}{L}(\theta_1 - \theta_2)$



  Cross section area = A ; Length = L

  Thermal conductivity of material = K

- **Thermal (temperature) gradient**

  The decrease in temperature with distance from hot end of the rod is known as temperature gradient or in the direction of heat energy flow, the rate of fall in temperature w.r.t. distance is called as temperature gradient. It is denoted by –dT/dx

- **Thermal conductivity (K) :** It's depends on nature of material.
  - **SI UNIT :** $J\ s^{-1}\ m^{-1}\ K^{-1}$ **Dimensions :** $M^1 L^1 T^{-3} \theta^{-1}$ $K \begin{cases} \text{For Ag maximum is (410 W/mK)} \\ \text{For Freon minimum is 12 (0.008 W/mK)} \end{cases}$
  - For an ideal or perfect conductor of heat the value of $K = \infty$
  - For an ideal or perfect bad conductor or insulator the value of $K = 0$
  - For cooking the food, low specific heat and high conductivity utensils are most suitable.

APPLICATION OF THERMAL CONDUCTION

- In winter, the iron chairs appear to be colder than the wooden chairs.
- Cooking utensils are made of aluminium and brass whereas their handles are made of wood.
- Ice is covered in gunny bags to prevent melting of ice.
- We feel warm in woollen clothes and fur coat.
- Two thin blankets are warmer than a single blanket of double the thickness.
- Birds often swell their feathers in winter.
- A new quilt is warmer than old one.

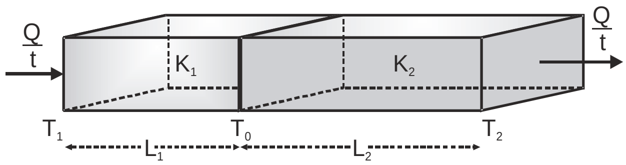
THERMAL RESISTANCE (R)

The thermal resistance of a body is a measure of its opposition of the flow of heat through it. $R = \frac{L}{KA}$

- **Heat flow through slabs in series**

$$R_{eq} = R_1 + R_2$$

$$\frac{L_1 + L_2}{K_{eq} A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$



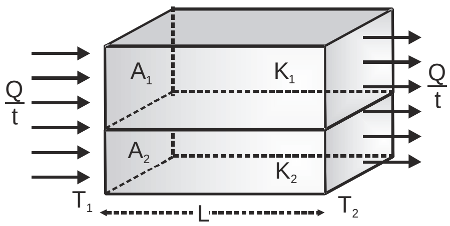
Equivalent thermal conductivity of the system is

$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{\Sigma L_i}{\Sigma \frac{L_i}{K_i}}$$

equivalent to

- **Heat flow through slabs in parallel**

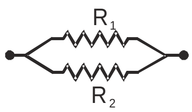
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2},\ R = \frac{L}{KA};\ \frac{K_{eq}}{L}(A_1 + A_2) = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L}$$



Equivalent thermal conductivity

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2} = \frac{\Sigma K_i A_i}{\Sigma A_i}$$

equivaleent to



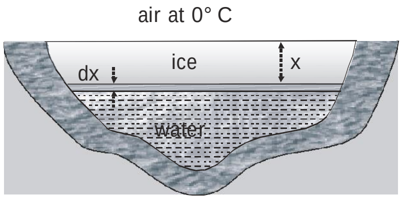
GROWTH OF ICE ON LAKES

In winter atmospheric temperature falls below 0°C and water in the lake start freezing.

Let at time t thickness of ice on the surface of the lake = x and air temperature = $-\theta°$ C

The temperature of water in contact with the lower surface of ice = 0°C

Let area of the lake = A



Heat escaping through ice in time dt is $dQ = KA\frac{[0-(-\theta)]}{x}dt$

Due to escape of this heat increasing extra thickness of ice = dx

Mass of this extra thickness of ice is m = ρV = ρ A.dx

dQ = mL = (ρ A.dx) L

$\therefore \quad KA\frac{\theta}{x}dt = (\rho\ A.dx)\ L \Rightarrow \quad dt = \frac{\rho L}{K\theta}x\,dx$

So time taken by ice to grow a thickness x is $t = \frac{\rho L}{K\theta}\int_0^x x\,dx = \frac{1}{2}\frac{\rho L}{K\theta}x^2$

So time taken by ice to grow from thickness x_1 to thickness x_2 is

$$t = t_2 - t_1 = \frac{1}{2}\frac{\rho L}{KT}(x_2^2 - x_1^2) \quad \text{and} \quad t \propto (x_2^2 - x_1^2)$$

Time taken to double and triple the thickness ratio $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$

So $t_1 : t_2 : t_3 :: 1 : 4 : 9$

Ex. One end of a brass rod 2m long and having 1 cm radius is maintained at 250°C. When a steady state is reached, the rate of heat flow across any cross–section is 0.5 cal s^{-1}. What is the temperature of the other end K = 0.26 cal s^{-1} cm^{-1} °C^{-1}.

Sol. $\frac{Q}{t} = 0.5$ cal s^{-1}; r = 1 cm $\quad \therefore$ Area A = $\pi r^2 = 3.142 \times 1$ cm^2 = 3.142 cm^2

L = Length of rod = 2m = 200 cm, T_1 = 250°C, T_2 = ?

We know $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$ or $(T_1 - T_2) = \frac{Q}{t} \times \frac{\Delta x}{kA} = \frac{0.5 \times 200}{0.26\,C^{-1} \times 3.142}$ =122.4°C

$\therefore \quad T_2$ = 250°C – 122.4°C = 127.6°C

Ex. Steam at 373 K is passed through a tube of radius 10 cm and length 2 m. The thickness of the tube is 5 mm and thermal conductivity of the material is 390 W m^{-1} K^{-1}, calculate the heat lost per second. The outside temp. is 0°C.

Sol. Using the relation $Q = \frac{KA(T_1 - T_2)t}{L}$

Here, heat is lost through the cylindrical surface of the tube.

A = 2πr (radius of the tube) (length of the tube) = 2π × 0.1 × 2 = 0.4 πm^2

K = 390 W m^{-1} K^{-1}

T_1 = 373 K, $\quad T_2$ = 0°C = 273 K, $\quad$ L = 5 mm = 0.005 m $\quad$ and $\quad$ t = 1 s

$$\therefore Q = \frac{390 \times 0.4\pi \times (373 - 273) \times 1}{0.005} = \frac{390 \times 0.4\pi \times 100}{0.005} = 98 \times 10^5 \text{ J.}$$

Ex. The thermal conductivity of brick is 1.7 W m^{-1} K^{-1}, and that of cement is 2.9 W m^{-1} K^{-1}. What thickness of cement will have same insulation as the brick of thickness 20 cm.

Sol. Since $Q = \dfrac{KA(T_1 - T_2)t}{L}$. For same insulation by the brick and cement Q, A ($T_1 - T_2$) and t do not change. Hence, $\dfrac{K}{L}$ remain constant. If K_1 and K_2 be the thermal conductivities of brick and cement respectively and L_1 and L_2 be the required thickness then $\dfrac{K_1}{L_1} = \dfrac{K_2}{L_2}$ or $\dfrac{1.7}{20} = \dfrac{2.9}{L_2}$

$\therefore L_2 = \dfrac{2.9}{1.7} \times 20 = 34.12$ cm

Ex. Two vessels of different material are identical in size and wall–thickness. They are filled with equal quantities of ice at 0°C. If the ice melts completely, in 10 and 25 minutes respectively then compare the coefficients of thermal conductivity of the materials of the vessels.

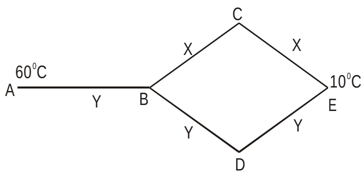
Sol. Let K_1 and K_2 be the coefficients of thermal conductivity of the materials, and t_1 and t_2 be the time in which ice melts in the two vessels. Since both the vessels are identical, so A and x in both the cases is same.

Now, $Q = \dfrac{K_1A(\theta_1 - \theta_2)t_1}{L} = \dfrac{K_2A(\theta_1 - \theta_2)t_2}{L} \quad \Rightarrow \dfrac{K_1}{K_2} = \dfrac{t_2}{t_1} = \dfrac{25 \text{ min}}{10 \text{ min}} = \dfrac{5}{2}$

Ex. Three rods of material X and three rods of material Y are connected as shown in figure. All the rods are identical in length and cross–sectional area. If the end A is maintained at 60^{0}C and the junction E at at 10^{0}C, calculate the temp. of the junctions B,C,D. The thermal conductivity of X is 0.92 CGS units and that of Y is 0.46 CGS units.

Sol. $R_X \propto \dfrac{1}{K_X}$, $R_Y \propto \dfrac{1}{K_Y} \Rightarrow \dfrac{R_X}{R_Y} = \dfrac{K_Y}{K_X} = \dfrac{0.46}{0.92} = \dfrac{1}{2}$ Let $R_X = R \therefore R_Y = 2R$

The total resistance $\Sigma R = R_Y$ + effective resistance in the bridge



$\Sigma R = 2R + \dfrac{2R \times 4R}{2R + 4R} = 2R + \dfrac{4}{3}R = \dfrac{10}{3}R$ & $\because \Delta\theta = \ell \times R$

Further $\quad I_{BCE}(2R) = I_{BDE}(4R)$ and $I_{BCE} + I_{BDE} = I \Rightarrow I_{BCE} = \dfrac{2}{3}I$ and $I_{BDE} = \dfrac{1}{3}I$

For A and B $\quad \theta_A - \theta_B = 60^0 - \theta_B \Rightarrow 60 - \theta_B = 2R \times I$ ...(i)

For B and C $\quad \theta_B - \theta_C = \dfrac{2}{3}(I \times R)$ ....(ii) $\theta_C - \theta_E = \dfrac{2}{3} \times R \times I$

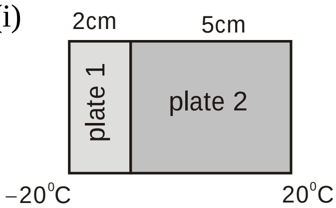
For A and E $\quad \theta_A - \theta_E = 60 - 10 = 50 \Rightarrow \dfrac{10}{3}(R \times I) = 50$ ....(iii) $\therefore R \times I = 15$

$\therefore \theta_A - \theta_B - 2 \times 15 = 30$, $\theta_B = 60 - 30 = 30^0$C, $\theta_B - \theta_C = \left(\dfrac{2}{3}\right) \times 15 = 10$

$\therefore \theta_C = 30 - 10 = 20^0C$ Obviously, $\theta_C = \theta_D = 20^0C$

Ex. Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is –20°C and that of the external surface of the second plate is 20°C. What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.

Sol. Rate of flow of heat in the plates is $\frac{Q}{t} = \frac{K_1A(\theta_1 - \theta)}{L_1} = \frac{K_2A(\theta - \theta_2)}{L_2}$ ...(i)



(i) Here θ_1 = –20°C, θ_2 = 20°C,
L_1 = 2 cm = 0.02 m, L_2 = 5 cm = 0.05 m and $K_1 = K_2 = K$

$\therefore$ equation (i) becomes $\frac{KA(-20-\theta)}{0.02} = \frac{KA(\theta - 20)}{0.05}$

$\therefore$ 5(–20–θ) = 2(θ – 20) ⇒ –100 – 5θ = 2θ – 40 ⇒ 7θ = –60 ⇒ θ = –8.6°C

(ii) $\frac{K_1}{K_2} = \frac{2}{5}$ or $K_1 = \frac{2}{5}K_2$

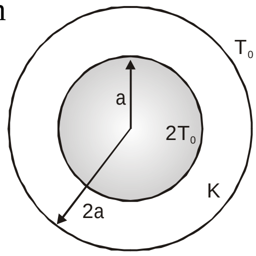
$\therefore$ from equation (i) $\frac{2/5\,K_2A(-20-\theta)}{0.02} = \frac{K_2A(\theta-20)}{0.05}$ –20 – θ = θ – 20 or –2θ = 0 ∴θ = 0°C

Ex. An ice box used for keeping eatables cold has a total wall area of 1 metre^2 and a wall thickness of 5.0 cm. The thermal conductivity of the ice box is K = 0.01 joule/metre–°C. It is filled with ice at 0°C along with eatables on a day when the temperature is 30°C. The latent heat of fusion of ice is 334 × 10^3 jule/kg. Calculate the amount of ice melted in one day.

Sol. $\frac{dQ}{dt} = \frac{KA}{L}d\theta = \frac{0.01 \times 1}{0.05} \times 30 = 6$ joule/s So $\frac{dQ}{dt} \times 86400 = 6 \times 86400$

Q = mL (L – latent heat), $m = \frac{Q}{L} = \frac{6 \times 86400}{334 \times 10^3} = 1.552$ kg

Ex. A hollow spherical ball of inner radius a and outer radius 2a is made of a uniform material of constant thermal conductivity K. The temperature within the ball is maintained at $2T_0$ and outside the ball it is T_0. Find, (a) the rate at which heat flows out of the ball in the steady state, (b) the temperature at r = 3a/2, where r is radial distance from the centre of shell. Assume steady state condition.



Sol. In the steady state, the net outward thermal current is constant, and does not depend on the radial position.

Thermal current, $C_1 = \left(\frac{dQ}{dt}\right) = -K.(4\pi r^2)\frac{dT}{dr} \Rightarrow \frac{dT}{dr} = -\frac{C_1}{4\pi K}\frac{1}{r^2} + C_2$

At r=a, T = $2T_0$ and r =2a, T=T_0 $\Rightarrow T = \frac{2a}{r}T_0$ (a) $\frac{dQ}{dt} = 8\pi a K T_0$ (b) $T(r = 3a/2) = 4T_0/3$

Thermal Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. When a body is heated and placed in vacuum, it loses heat even when there is no medium surrounding it. The heat can not go out from the body by the process of conduction or convection since both of these process require the presence of a material medium between source and surrounding objects. The process by which heat is lost in this case is called radiation. This does not require the presence of any material medium.

It is by radiation that the heat from the Sun reaches the Earth. Radiation has the following properties:

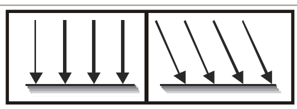
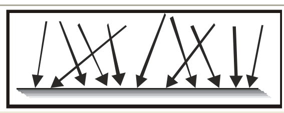
(a) Radiant energy travels in straight lines and when some object is placed in the path, it's shadow is formed at the direction.

(b) It can travel through vacuum.

(c) Intensity of radiation follows the law of inverse square.

All these and many other properties establish that heat radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency. The wavelength of heat radiation is larger than that of visible light.

Types of thermal Radiation :– Two types of thermal radiation.

Plane Radiation	Diffuse Radiation
	
Radiations which are incident on a surface at certain angle	Incident on the surface at all angles

- When radiation passes through any medium then radiations slightly absorbed by medium according to its absorptive power so temperature of medium slightly increases.
- Heat radiation are always obtained in infra–red region of electromagnetic wave spectrum so they are called Infra red rays.

BASIC FUNDAMENTAL DEFINITIONS

- **Energy Density (u)**

  The radiation energy of whole wavelength (0 to ∞) present in unit volume at any point in space is defined as energy density. **S I UNIT :** J/m^3

- **Spectral energy density (u_λ) :** Energy density per unit spectral region. $u = \int_0^\infty u_\lambda d\lambda$

  SI UNIT : J/m^3 Å

- **Absorptive power or absorptive coefficient 'a' :** The ratio of amount of radiation absorbed by a surface (Q_a) to the amount of radiation incident (Q) upon it is defined as the coefficient of absorption $a = \frac{Q_a}{Q}$ .It is unitless

- **Spectral absorptive power (a_λ)** $a_\lambda = \dfrac{Qa_\lambda}{Q_\lambda}$: Also called monochromatic absorptive coefficient

  At a given wavelength $a = \int_0^\infty a_\lambda d\lambda$. For ideal black body a_λ and $a = 1$, a and a_λ are unitless

- **Emissive power (e)** : The amount of heat radiation emitted by unit area of the surface in one second at a particular temperature. **SI UNIT :** J/m^2s

- **Spectral Emmisive power (e_λ)** : The amount of heat radiation emitted by unit area of the body in one second in unit spectral region at a given wavelength. Emissive power or total emissive power

  $e = \int_0^\infty e_\lambda \, d\lambda$

  SI UNIT : W/m^2 Å

EMISSIVITY (e)

- **Absolute emissivity or emissivity :** Radiation energy given out by a unit surface area of a body in unit time corresponding to unit temperature difference w.r.t. the surroundings is called Emissivity.

  SI UNIT : W/m^2 °K

- **Relative emissivity (e_r) :** $e_r = \dfrac{Q_{GB}}{Q_{IBB}} = \dfrac{e_{GB}}{E_{IBB}} = \dfrac{\text{emitted radiation by gray body}}{\text{emitted radiation by ideal black body}}$

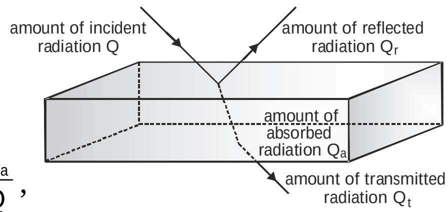
  GB = gray or general body, IBB = Ideal black body

  (i) No unit (ii) For ideal black body $e_r = 1$ (iii) range $0 < e_r < 1$

SPECTRAL, EMISSIVE, ABSORPTIVE AND TRANSMITTIVE POWER OF A GIVEN BODY SURFACE

Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts. (a) Reflection (b) Absorption (c) Transmission

- **From energy conservation**



  $Q = Q_r + Q_a + Q_t \Rightarrow \dfrac{Q_r}{Q} + \dfrac{Q_a}{Q} + \dfrac{Q_t}{Q} = 1 \Rightarrow \quad r + a + t = 1$

  Reflective Coefficient $r = \dfrac{Q_r}{Q}$, Absorptive Coefficient $a = \dfrac{Q_a}{Q}$,

  Transmittive Coefficient $t = \dfrac{Q_t}{Q}$

  $r = 1$ and $a = 0$, $t = 0$ $\Rightarrow$ Perfect reflector

  $a = 1$ and $r = 0$, $t = 0$ $\Rightarrow$ Ideal absorber (ideal black body)

  $t = 1$ and $a = 0$, $r = 0$ $\Rightarrow$ Perfect transmitter

  Reflection power (r) $= \left[\dfrac{Q_r}{Q} \times 100\right]\%$, Absorption power (a) $= \left[\dfrac{Q_a}{Q} \times 100\right]\%$

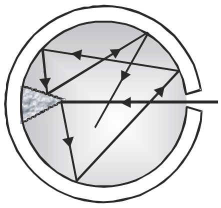
  Transmission power (t) $= \left[\dfrac{Q_t}{Q} \times 100\right]\%$

Ex. Total radiations incident on body = 400 J, 20% radiation reflected and 120 J absorbs. Then find out % of transmittive power

Sol. $Q = Q_t + Q_r + Q_a \Rightarrow 400 = 80 + 120 + Q_t \Rightarrow Q_t = 200$. So % of transmittive power is 50%

IDEAL BLACK BODY

- For a body surface which absorbs all incident thermal radiations at low temperature irrespective of their wave length and emitted out all these absorbed radiations at high temperature assumed to be an ideal black body surface.
- The identical parameters of an ideal black body is given by
  $a = a_\lambda = 1$ and $r = 0 = t$, $e_r = 1$



Ferry's ideal black body

- The nature of emitted radiations from surface of ideal black body only depends on its temperature
- The radiations emitted from surface of ideal black body called as either full or white radiations.
- At low temperature surface of ideal black body is a perfect absorber and at a high temperature it proves to be a good emitter.
- An ideal black body need not be black colour (eg. Sun)

PREVOST'S THEORY OF HEAT ENERGY EXCHANGE

According to Prevost at every possible temperature (Not absolute temperature) there is a continuous heat energy exchange between a body and its surrounding and this exchange carry on for infinite time.

The relation between temperature difference of body with its surrounding decides whether the body experience cooling effect or heating effect.

When a cold body is placed in the hot surrounding : The body radiates less energy and absorbs more energy from the surrounding, therefore the temperature of body increases.

When a hot body placed in cooler surrounding : The body radiates more energy and absorb less energy from the surroundings. Therefore temperature of body decreases.

When the temperature of a body is equal to the temperature of the surrounding

The energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body, therefore its temperature remains constant.

- At absolute zero temperature (0 kelvin) all atoms of a given substance remains in ground state, so, at this temperature emission of radiation from any substance is impossible, so Prevost's heat energy exchange theory does not applied at this temperature, so it is called **limited temperature** of prevosts theory.
- With the help of Prevost's theory rate of cooling of any body w.r.t. its surroundings can be worked out (applied to Stefen Boltzman law, Newton's law of cooling.)

KIRCHHOFF'S LAW

At a given temperature for all bodies the ratio of their spectral emissive power (e_λ) to spectral absorptive power (a_λ) is constant and this constant is equal to spectral emissive power (E_λ) of the ideal black body at same temperature

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant} \qquad \left[\frac{e_\lambda}{a_\lambda}\right]_1 = \left[\frac{e_\lambda}{a_\lambda}\right]_2 = \text{constant} \quad e_\lambda \propto a_\lambda$$

Good absorbers are good emitters and bad absorbers are bad emitters

- For a constant temperature the spectral emmisive power of an ideal black body is a constant parameter
- The practical confirmation of Kirchhoff's law carried out by Rishi apparatus and the main base of this apparatus is a Lessilie container.
- The main conclusion predicted from Kirchhof's law can be expressed as

Good absorber	$\rightleftharpoons$	Good emitter
Bad absorber	$\rightleftharpoons$	Bad emitter
(at Low temperature)		(at high temperature)

APPLICATIONS OF KIRCHOFF LAW

- **In deserts days are hot and nights cold**

  Sand is rough and black, so it is a good absorber and hence in deserts, days (When radiation from Sun is incident on sand) will be very hot. Now in accordance with Kirchhoff's Law, good absorber is a good emitter.

  So nights (when send emits radiation) will be cold.

STEFAN'S LAW

The amount of radiation emitted per second per unit area by a black body is directly proportional to the fourth power of its absolute temperature.

Amount of radiation emitted $E \propto T^4$ where T = temperature of ideal black body (in K)

$E = \sigma T^4$ This law is true for only ideal black body

SI Unit : E = watt/m² σ = Stefen's constant = 5.67×10^{-8} watt /m² K⁴

Dimensions of σ : $M^1 L^0 T^{-3} \theta^{-4}$

Total radiation energy emitted out by surface of area A in time t : If T is constant.

Ideal black body $Q_{IBB} = \sigma A T^4 t$ and for any other body $Q_{GB} = e_r \sigma A T^4 t$

Rate of emission of radiation

When Temperature of surrounding T_0 (Let $T_0 < T$)

Rate of emission of radiation from ideal black body surface $E_1 = \sigma T^4$

Rate of absorption of radiation from surrounding $E_2 = \sigma T_0^4$

Net rate of loss of radiation from ideal black body surface is $E = E_1 - E_2 = \sigma T^4 - \sigma T_0^4 = \sigma (T^4 - T_0^4)$

Net loss of radiation energy from entire surface area in time t is $Q_{IBB} = \sigma A (T^4 - T_0^4) t$

For any other body $Q_{GB} = e_r A\sigma (T^4 - T_0^4) t$

If in time dt the net heat energy loss for ideal black body is dQ and because of this its temperature falls by $d\theta$

Rate of loss of heat $R_H = \frac{dQ}{dt} = \sigma A(T^4 - T_0^4)$

It is also equal to emitted power or radiation emitted per second

Rate of fall in temperature (Rate of cooling) $R_F = \frac{d\theta}{dt} = \frac{\sigma A}{ms\,J}(T^4 - T_0^4)\left[\because \frac{dQ}{dt} = ms\,J\frac{d\theta}{dt}\right]$

Note :

(i) If all of T, T_0, m, s, V, ρ, are same for different shape body then R_F and R_H will be maximum in the flat surface.

(ii) If a solid and hollow sphere are taken with all the parameters same then hollow will cool down at fast rate.

(iii) Rate of temperature fall , $R_F \propto \frac{1}{s} \propto \frac{d\theta}{dt}$ so dt $\propto$ s. If condition in specific heat is $\Rightarrow s_1 > s_2 > s_3$

If all cooled same temperature i.e. temperature fall is also identical for all then required time $t \propto s \therefore t_1 > t_2 > t_3$

- **When a body cools by radiation the cooling depends on :**

(i) Nature of radiating surface : greater the emissivity (e_r), faster will be the cooling.

(ii) Area of radiating surface : greater the area of radiating surface, faster will be the cooling.

(iii) Mass of radiating body : greater the mass of radiating body slower will be the cooling.

(iv) Specific heat of radiating body : greater the specific heat of radiating body slower will be the cooling.

(v) Temperature of radiating body : greater the temperature of radiating body faster will be the cooling.

Ex. The operating temperature of a tungesten filament in an incandescent lamp is 2000 K and its emissivity is 0.3. Find the surface area of the filament of a 25 watt lamp. Stefan's constant $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}

Sol. $\because$ Rate of emission = wattage of the lamp

$\therefore W = Ae\sigma T^4 \Rightarrow A = \frac{W}{e\sigma T^4} = \frac{25}{0.3 \times 5.67 \times 10^{-8} \times (200)^4} = 0.918\ m^2$

NEWTON'S LAW OF COOLING

Rate of loss of heat $\left(\frac{dQ}{dt}\right)$ is directly proportional to excess of temperature of the body over that of surrounding. [(when $(\theta - \theta_0) \not> 35°C$] $\frac{dQ}{dt} \propto (\theta - \theta_0) \Rightarrow \frac{dQ}{dt} = ms\frac{d\theta}{dt}$

θ = temperature of body [in °C], θ_o = temperature of surrounding, $\theta - \theta_0$ = excess of temperature ($\theta > \theta_0$)

If the temperature of body decrease $d\theta$ in time dt then rate of fall of temperature $-\frac{d\theta}{dt} \propto (\theta - \theta_0)$

Where negative sign indictates that the rate of cooling is decreasing with time.

Excess of temperature

If the temperature of body decreases from θ_1 to θ_2 and temperature of surroundings is θ_0 then

average excess of temperature = $\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right] \Rightarrow \left[\frac{\theta_1-\theta_2}{t}\right] = -K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$

Ex. If a liquid takes 30 seconds in cooling of 80°C to 70°C and 70 seconds in cooling 60°C to 50°C, then find the room temperature.

Sol. $\frac{\theta_1-\theta_2}{t} = K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$

In first case, $\frac{80-70}{30} = K\left(\frac{80+70}{2}-\theta_0\right)$ $\quad \frac{1}{3} = K(75-\theta_0)$ ...(i)

In second case , $\frac{60-50}{70} = K\left[\frac{60+50}{2}-\theta_0\right]$ $\quad \frac{1}{7} = K(55-\theta_0)$ ... (ii)

Equation (i) divide by equation (ii) $\frac{7}{3} = \frac{(75-\theta_0)}{(55-\theta_0)} \Rightarrow 385-7\theta_0 = 225-3\theta_0 \Rightarrow \theta_0 = \frac{160}{4} = 40°C$

Limitations of Newton's Law

- Temperature difference should not exceed 35° C, $(\theta-\theta_0) \ngtr 35°$ C
- Loss of heat should only be by radiation.
- This law is an extended form of Stefan–Boltzman's law.

For Heating, Newton's law of heating $\frac{\theta_1-\theta_2}{t} = +H\left[\theta_0-\frac{\theta_1+\theta_2}{2}\right]$ H heating constant.

Derivation of Newton's law from Steafen's Boltzman law

$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}(T^4-T_0^4)$ $\quad \left\{\begin{matrix} T-T_0 = \Delta T \\ T = T_0+\Delta T\end{matrix}\right\}$

$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}(T^4-T_0^4)$ $\quad T-T_0 = \Delta T\,]\ \Delta T <<< T_0$

$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}\left[(T_0+\Delta T)^4-T_0^4\right]$ $\quad$ If x <<< 1 then $(1+x)^n = 1+nx$

$$\frac{d\theta}{dt} = \frac{\sigma A}{msJ}\left[T_0^4(1+\frac{\Delta T}{T_0})^4-T_0^4\right] = \frac{\sigma A}{msJ}T_0^4\left[(1+\frac{\Delta T}{T_0})^4-1\right] = \frac{\sigma A}{msJ}T_0^4\left[1+4\frac{\Delta T}{T_0}-1\right]$$

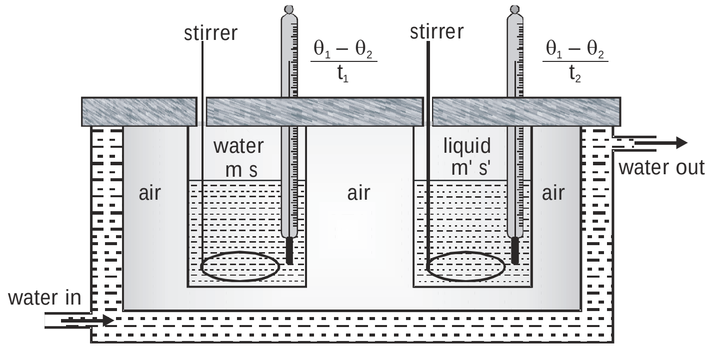
$\frac{d\theta}{dt} = \left[4\frac{\sigma A}{msJ}T_0^3\right]\Delta T \Rightarrow \frac{d\theta}{dt} = K\,\Delta T$ $\quad$ constant $K = \frac{4\sigma A T_0^3}{msJ}$

Newton's law of cooling $\frac{d\theta}{dt} \propto \Delta T$ (for small temperature difference)

APPLICATION OF NEWTON'S LAW OF COOLING

- **To find out specific heat of a given liquid**

  If for the two given liquids their volume, radiating surface area, nature of surface, initial temperature are allowed to cool down in a common environments then rate of loss of heat of these liquids are equal .

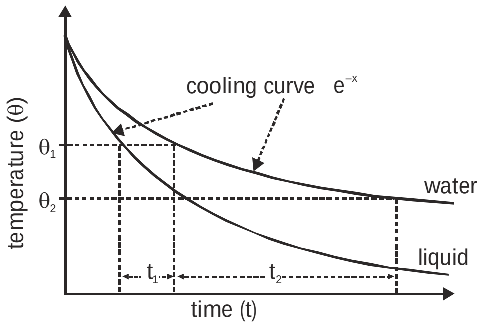


fall of temperature from $\theta_1 \to \theta_2$ in time t_1 for water and t_2 for liquid

$$\because \left[\frac{dQ}{dt}\right]_{Water} = \left[\frac{dQ}{dt}\right]_{Liquid} \quad \therefore (ms+w)\left[\frac{\theta_1-\theta_2}{t_1}\right] = (m's'+w)\left[\frac{\theta_1-\theta_2}{t_2}\right] \Rightarrow \frac{ms+w}{t_1} = \frac{m's'+w}{t_2}$$

where w = water equivalent of calorimeter.

Cooling curve :



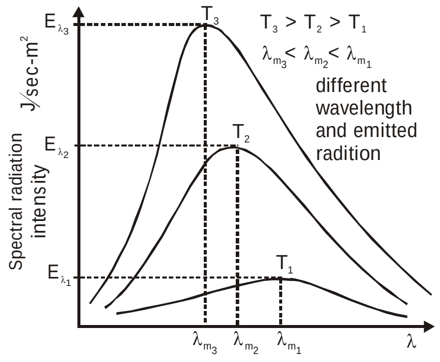
Ex. When a calorimeter contains 40g of water at 50°C, then the temperature falls to 45°C in 10 minutes. The same calorimeter contains 100g of water at 50°C, it takes 20 minutes for the temperature to become 45°C. Find the water equivalent of the calorimeter.

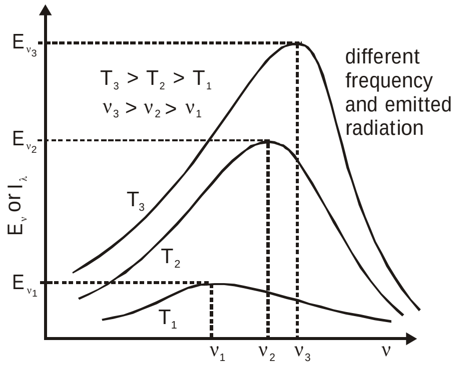
Sol. $\frac{m_1s_1+W}{t_1} = \frac{m_2s_2+W}{t_2}$ where W is the water equivalent

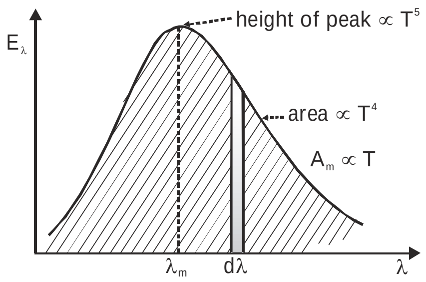
$$\Rightarrow \frac{40\times1+W}{10} = \frac{100\times1+W}{20} \Rightarrow 80 + 2W = 100 + W \Rightarrow W = 20 \text{ g}$$

SPECTRAL ENERGY DISTRIBUTION CURVE OF BLACK BODY RADIATIONS

Practically given by : Lumers and Pringshem   Mathematically given by : Plank







spectral energy distribution curve (E_λ–λ)

(i) $\lambda_m \propto \dfrac{1}{T}$

(ii) $E_{\lambda_m} \propto T^5$

(iii) Area $\int_0^\infty E_\lambda \, d\lambda = E = \sigma T^4 \quad \dfrac{A_1}{A_2} = \left[\dfrac{T_1}{T_2}\right]^4$

- Spectral energy distribution curves are continuous. At any temperature in between possible wavelength $(0 - \infty)$ radiation emitted but for different wavelength quantity of radiations are different.
- As the wave length increases, the amount of radiation emitted first increase, becomes maximum and then decreases.
- At a particular temperature the area enclosed between the spectral energy curve shows the spectral emissive power of the body. Area $= \int_0^\infty E_\lambda d\lambda = E = \sigma T^4$

WEIN'S DISPLACEMENT LAW

The wavelength corresponding to maximum emission of radiation decrease with increasing temperature $\left[\lambda_m \propto \dfrac{1}{T}\right]$. This is known as Wein's displacement law. $\lambda_m T = b$ where b Wein's constant $= 2.89 \times 10^{-3}$ mK.

Dimensions of b : $= M^0 L^1 T^0 \theta^1$

Relation between frequency and temperature $\nu_m = \dfrac{c}{b} T$

Ex. The temperature of furnace is 2000°C, in its spectrum the maximum intensity is obtained at about 4000Å, If the maximum intensity is at 2000Å calculate the temperature of the furnace in °C.

Sol. by using $\lambda_m T = b$, $4000\ (2000+273) = 2000(T) \Rightarrow T = 4546K$

The temperature of furnace = 4546 – 273 = 4273 °C

SOLAR CONSTANT 'S'

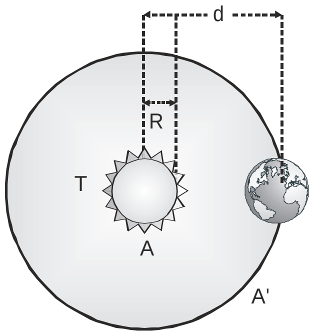
The Sun emits radiant energy continuously in space of which an in significant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant.

The solar constant S is taken to be 1340 watts/m^2 or 1.937 Cal/cm^2–minute

- **Temperature of the Sun**

Let R be the radius of the Sun and 'd' be the radius of Earth's orbit around the Sun. Let E be the energy emitted by the Sun per second per unit area. The total energy emitted by the Sun in one second $= E.A = E \times 4\pi R^2$. (This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e., on a sphere of surface area $4\pi d^2$)

So, The energy falling per unit area of Earth $= \dfrac{4\pi R^2 \times E}{4\pi d^2} = \dfrac{E\,R^2}{d^2}$



$R = 7\times 10^8 m$, $d = 1.5 \times 10^{11} m$, $s = 5.7 \times 10^{-8}\ W\ m^{-2}\ K^{-4}$

Solar constant $S = \dfrac{E\,R^2}{d^2}$

By Stefan's Law $E = \sigma T^4$

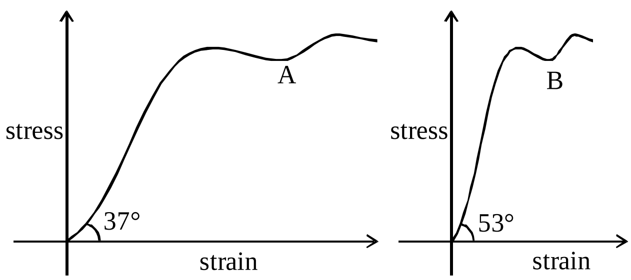
$$S = \frac{\sigma T^4 R^2}{d^2} \Rightarrow T = \left[\frac{S \times d^2}{\sigma \times R^2}\right]^{\frac{1}{4}} = \left[\frac{1340 \times (1.5 \times 10^{11})^2}{5.7 \times 10^{-8} \times (7 \times 10^8)^2}\right]^{\frac{1}{4}} = 5732\ K$$

EXERCISE (S-1)

Elasticity

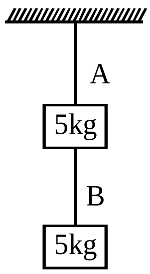
1. A steel wire of length 4.5 m and a copper wire of length 3.5 m are stretched same amount under a given load. If ratio of youngs modulli of steel to that of copper is $\frac{12}{7}$, then what is the ratio of cross sectional area of steel wire to copper wire ?

2. Diagram shows stress-strain graph for two material A & B. The graphs are drawn to scale.



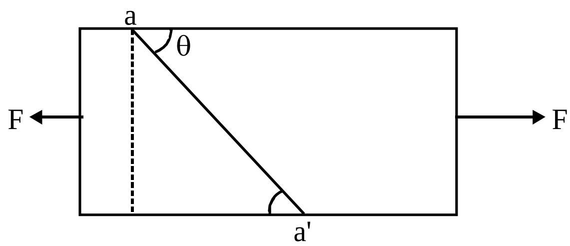
The ratio of young modulli of material A to material B is.

3. Two identical wires A & B of same material are loaded as shown in figure. If the elongation in wire B is 1.5 mm, what is the elongation in A. (Mass of A & B can be neglected)



4. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by l. Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

5. Consider a long steel bar under a tensile stress due to forces $\vec{F}$ acting at the edges along the length of the bar (Fig.). Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?



(a) For what angle is the tensile stress a maximum?

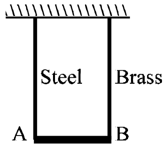
(b) For what angle is the shearing stress a maximum?

6. A light rigid bar AB is suspended horizontally from two vertical wires, one of steel and one of brass, as shown in figure. Each wire is 2.00 m long. The diameter of the steel wire is 0.60 mm and the length of the bar AB is 0.20 m. When a mass of 10 kg is suspended from the centre of AB bar remains horizontal.

(i) What is the tension in each wire?

(ii) Calculate the extension of the steel wire and the energy stored in it.

(iii) Calculate the diameter of the brass wire.



(iv) If the brass wire were replaced by another brass wire of diameter 1 mm, where should the mass be suspended so that AB would remain horizontal? The Young modulus for steel = 2.0×10^{11} Pa, the Young modulus for brass = 1.0×10^{11} Pa.

Calorimetry

7. One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some times, say 5-10 minutes before he could take bath. Now he had two options: (i) fill the remaining bucket completely by cold water and then attend to the work, (ii) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer ? Explain.

8. An aluminium container of mass 100 gm contains 200 gm of ice at – 20°C. Heat is added to the system at the rate of 100 cal/s. Find the temperature of the system after 4 minutes (specific heat of ice = 0.5 and L = 80 cal/gm, specific heat of A*l* = 0.2 cal/gm/°C)

9. A hot liquid contained in a container of negligible heat capacity loses temperature at rate 3 K/min, just before it begins to solidify. The temperature remains constant for 30 min. Find the ratio of specific heat capacity of liquid to specific latent heat of fusion is in K^{-1} (given that rate of losing heat is constant).

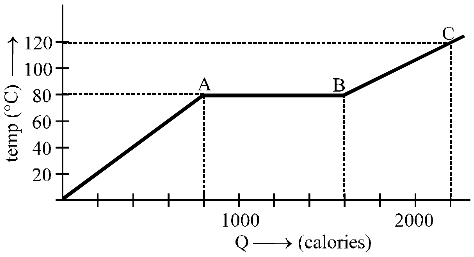
10. Two 50 gm ice cubes are dropped into 250 gm of water into a glass. If the water was initially at a temperature of 25°C and the temperature of ice –15°C. Find the final temperature of water. (specific heat of ice = 0.5 cal/gm/°C and L = 80 cal/gm). Find final amount of water and ice.

11. A flow calorimeter is used to measure the specific heat of a liquid. Heat is added at a known rate to a stream of the liquid as it passes through the calorimeter at a known rate. Then a measurement of the resulting temperature difference between the inflow and the outflow points of the liquid stream enables us to compute the specific heat of the liquid. A liquid of density 0.2 g/cm^3 flows through a calorimeter at the rate of 10 cm^3/s. Heat is added by means of a 250-W electric heating coil, and a temperature difference of 25°C is established in steady-state conditions between the inflow and the outflow points. Find the specific heat of the liquid.

12. Two identical calorimeter A and B contain equal quantity of water at 20°C. A 5 gm piece of metal X of specific heat 0.2 cal g^{-1} $(C°)^{-1}$ is dropped into A and a 5 gm piece of metal Y into B. The equilibrium temperature in A is 22°C and in B 23°C. The initial temperature of both the metals is 40°C. Find the specific heat of metal Y in cal g^{-1} $(C°)^{-1}$.

13. The temperature of 100 gm of water is to be raised from 24° C to 90° C by adding steam to it. Calculate the mass of the steam required for this purpose.

14. A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph



(i) the mass of the substance ;

(ii) the specific latent heat of the melting process, and

(iii) the specific heat of the substance in the liquid state.

Thermal expansion

15. If two rods of length L and 2 L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3 L, determine the average coefficient of linear expansion of the composite rod.

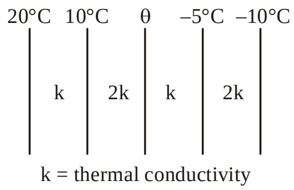
16. A clock pendulum made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate where average temperature is 30°C, approximately. How much fast or slow will the clock run in 10^6 sec. ($\alpha_{invar}=1\times10^{-6}$/°C)

17. An iron bar (Young's modulus = 10^{11} N/m^2 , $\alpha = 10^{-6}$ /°C) 1 m long and 10^{-3} m^2 in area is heated from 0°C to 100°C without being allowed to bend or expand. Find the compressive force developed inside the bar.

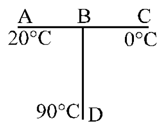
Conduction

18. A thin walled metal tank of surface area 5m^2 is filled with water and contains an immersion heater dissipating 1 kW. The tank is covered with 4 cm thick layer of insulation whose thermal conductivity is 0.2 W/m/K. The outer face of the insulation is 25°C. Find the temperature of the tank in the steady state

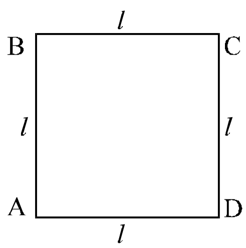
19. The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under steady state condition, find the value of temperature θ.



20. Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at 20°C, 90°C and 0°C. Find the ratio of length BD and BC if there is no heat flow in AB.



21. In the square frame of side l of metallic rods, the corners A and C are maintained at T_1 and T_2 respectively. The rate of heat flow from A to C is ω. If A and D are instead maintained T_1 & T_2 respectively. Find the total rate of heat flow.



22. One end of copper rod of uniform cross-section and of length 1.5 meters is in contact with melting ice and the other end with boiling water. At what point along its length should a temperature of 200°C be maintained, so that in steady state, the mass of ice melting is equal to that of steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings.

Radiation

23. Two spheres of same radius R have their densities in the ratio 8 : 1 and the ratio of their specific heats are 1 : 4. If by radiation their rates of fall of temperature are same, then find the ratio of their rates of losing heat.

24. A solid receives heat by radiation over its surface at the rate of 4 kW. The heat convection rate from the surface of solid to the surrounding is 5.2 kW, and heat is generated at a rate of 1.7 kW over the volume of the solid. The rate of change of the average temperature of the solid is $0.5°Cs^{-1}$. Find the heat capacity of the solid.

25. A solid copper cube and sphere, both of same mass & emissivity are heated to same initial temperature and kept under identical conditions. What is the ratio of their initial rate of fall of temperature?

26. A vessel containing 100 gm water at 0°C is suspended in the middle of a room. In 15 minutes the temperature of the water rises by 2°C. When an equal amount of ice is placed in the vessel, it melts in 10 hours. Calculate the specific heat of fusion of ice.

27. The maximum in the energy distribution spectrum of the sun is at 4753 Å and its temperature is 6050K. What will be the temperature of the star whose energy distribution shows a maximum at 9506 Å.

28. A pan filled with hot food cools from 50.1 °C to 49.9 °C in 5 sec. How long will it take to cool from 40.1 °C to 39.9°C if room temperature is 30°C ?

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Elasticity

1. The maximum load a wire can withstand without breaking, when its length is reduced to half of its original length, will

(A) be double. (B) be half. (C) be four times. (D) remain same.

2. The temperature of a wire is doubled. The Young's modulus of elasticity

(A) will also double. (B) will become four times.

(C) will remain same. (D) will decrease.

3. A spring is stretched by applying a load to its free end. The strain produced in the spring is

(A) volumetric. (B) shear.

(C) longitudinal and shear. (D) longitudinal.

4. Overall changes in volume and radii of a uniform cylindrical steel wire are 0.2% and 0.002% respectively when subjected to some suitable force. Longitudinal tensile stress acting on the wire is :- ($Y = 2.0 \times 10^{11}\ Nm^{-2}$)

(A) $3.2 \times 10^9\ Nm^{-2}$ (B) $3.2 \times 10^7\ Nm^{-2}$ (C) $3.6 \times 10^9\ Nm^{-2}$ (D) $3.9 \times 10^8\ Nm^{-2}$

5. A solid sphere of radius R made of material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. When a mass m is placed on the piston to compress the liquid, the fractional change in the radius of the sphere $\delta R/R$ is

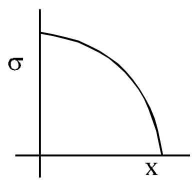
(A) mg/AK (B) mg/3AK (C) mg/A (D) mg/3AR

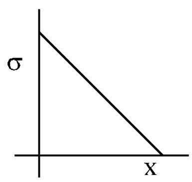
6. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower ends. The weight stretches the wire by 1mm. Then the elastic energy stored in the wire is-

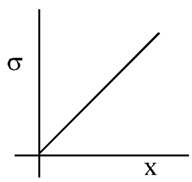
[AIEEE - 2003]

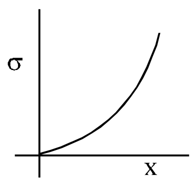
(A) 0.2 J (B) 10 J (C) 20 J (D) 0.1 J

7. A uniform rod rotating in gravity free region with certain constant angular velocity. The variation of tensile stress with distance x from axis of rotation is best represented by which of the following graphs.

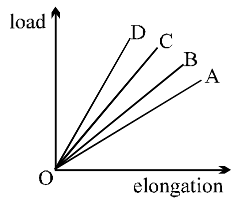
(A) 

(B) 

(C) 

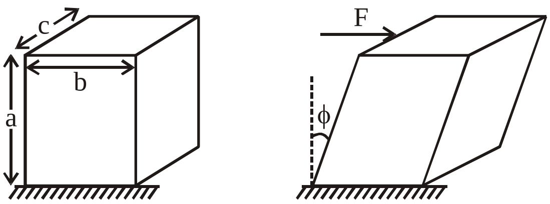
(D) 

8. The load versus strain graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line :-



(A) OB (B) OA (C) OD (D) OC

9. A cuboidal block of sides a, b & c is fixed on ground. The top is pushed by a horizontal force F as shown. The angle ϕ by which the block deforms is : (η is modulus of rigidity)



(A) $\frac{F}{ab\eta}$ (B) $\frac{F}{ac\eta}$ (C) $\frac{F}{bc\eta}$ (D) $\frac{F}{\sqrt{b^2+c^2}\,\eta}$

Calorimetry

10. Heat is associated with

(A) kinetic energy of random motion of molecules.

(B) kinetic energy of orderly motion of molecules.

(C) total kinetic energy of random and orderly motion of molecules.

(D) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other.

11. Equal amount of heat energy are transferred into equal mass of ethyl alcohol and water sample. The rise in temperature of water sample is 25°C. The temperature rise of ethyl alcohol will be. (Specific heat of ethyl alcohol is one half of the specific heat of water).

(A) 12.5°C (B) 25°C

(C) 50°C (D) It depends on the rate of energy transfer.

12. A block of mass 2.5 kg is heated to temperature of 500°C and placed on a large ice block. What is the maximum amount of ice that can melt (approx.). Specific heat for the body = 0.1 Cal/gm°C.

(A) 1 kg (B) 1.5 kg (C) 2 kg (D) 2.5 kg

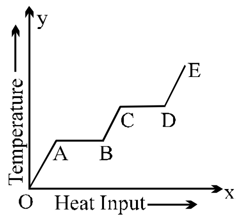
13. 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

(A) 6200 cal (B) 7200 cal (C) 13600 cal (D) 8200 cal

14. A continuous flow water heater (geyser) has an electrical power rating = 2 kW and efficienty of conversion of electrical power into heat = 80%. If water is flowing through the device at the rate of 100 cc/sec, and the inlet temperature is 10°C, the outlet temperature will be

(A) 12.2°C (B) 13.8°C (C) 20°C (D) 16.5°C

15. A solid material is supplied with heat at a constant rate. The temperature of material is changing with heat input as shown in the figure. What does slope DE represents?



(A) latent heat of liquid

(B) latent heat of vapour

(C) heat capacity of vapour

(D) inverse of heat capacity of vapour

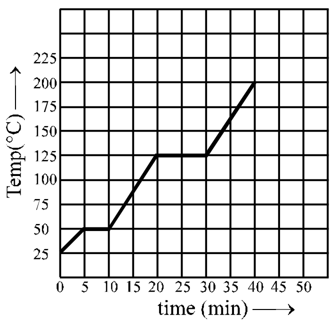
16. A block of ice with mass *m* falls into a lake. After impact, a mass of ice *m*/5 melts. Both the block of ice and the lake have a temperature of 0°C. If L represents the heat of fusion, the minimum distance the ice fell before striking the surface is

(A) $\frac{L}{5g}$ (B) $\frac{5L}{g}$ (C) $\frac{gL}{5m}$ (D) $\frac{mL}{5g}$

17. The specific heat of a metal at low temperatures varies according to $S = aT^3$ where a is a constant and T is the absolute temperature. The heat energy needed to raise unit mass of the metal from T = 1 K to T = 2 K is :-

(A) 3 a (B) $\frac{15a}{4}$ (C) $\frac{2a}{3}$ (D) $\frac{12a}{5}$

18. The graph shown in the figure represent change in the temperature of 5 kg of a substance as it abosrbs heat at a constant rate of 42 kJ min^{-1}. The latent heat of vapourazation of the substance is :



(A) 630 kJ kg^{-1}

(B) 126 kJ kg^{-1}

(C) 84 kJ kg^{-1}

(D) 12.6 kJ kg^{-1}

19. The density of a material A is 1500 kg/m^3 and that of another material B is 2000 kg/m^3. It is found that the heat capacity of 8 volumes of A is equal to heat capacity of 12 volumes of B. The ratio of specific heats of A and B will be

(A) 1 : 2 (B) 3 : 1 (C) 3 : 2 (D) 2 : 1

20. 2 kg ice at – 20°C is mixed with 5 kg water at 20°C. Then final amount of water in the mixture would be; Given specific heat of ice = 0.5cal/g°C, specific heat of water = 1 cal/g°C, Latent heat of fusion of ice = 80 cal/g. **[JEE' (Scr) 2003]**

(A) 6 kg (B) 5 kg (C) 4 kg (D) 2 kg

21. Some steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C so that the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensing. (in kg)

(A) 0.130 (B) 0.065 (C) 0.260 (D) 0.135

22. Find the amount of heat supplied to decrease the volume of an ice water mixture by 1 cm^3 without any change in temperature. ($\rho_{ice} = 0.9\ \rho_{water}$, L_{ice} = 80 cal/gm).

(A) 360 cal (B) 500 cal (C) 720 cal (D) none of these

Thermal expansion

23. The radius of a metal sphere at room temperature T is R, and the coefficient of linear expansion of the metal is α. The sphere is heated a little by a temperature ΔT so that its new temperature is $T + \Delta T$. The increase in the volume of the sphere is approximately

(A) $2\pi R\ \alpha\ \Delta T$ (B) $\pi R^2\ \alpha\ \Delta T$ (C) $4\pi R^3\ \alpha\ \Delta T/3$ (D) $4\pi R^3\ \alpha\ \Delta T$

24. A hole is made in a metal plate, when the temperature of metal is raised then the diameter of the hole will :-

(A) Decrease

(B) Increase

(C) Remain same

(D) Answer depends upon the initial temperature of the metal

25. A rod of length 2m rests on smooth horizontal floor. If the rod is heated from 0°C to 20°C. Find the longitudinal strain developed? ($\alpha = 5 \times 10^{-5}$/°C)

(A) 10^{-3} (B) 2×10^{-3} (C) Zero (D) None

26. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:

(A) 25 cm (B) <25 cm (C) >25 cm (D) can not say

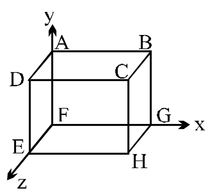
27. The bulk modulus of copper is 1.4×10^{11} Pa and the coefficient of linear expansion is $1.7 \times 10^{-5}\,(C°)^{-1}$. What hydrostatic pressure is necessary to prevent a copper block from expanding when its temperature is increased from 20°C to 30°C?

(A) 6.0×10^5 Pa (B) 7.1×10^7 Pa (C) 5.2×10^6 Pa (D) 40 atm

28. A thin copper wire of length L increase in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2?

(A) 1% (B) 2% (C) 3% (D) 4%

29. A cuboid ABCDEFGH is anisotropic with $\alpha_x = 1 \times 10^{-5}$ /°C, $\alpha_y = 2 \times 10^{-5}$ /°C, $\alpha_z = 3 \times 10^{-5}$ /°C. Coefficient of superficial expansion of faces can be



(A) $\beta_{ABCD} = 5 \times 10^{-5}$ /°C (B) $\beta_{BCGH} = 4 \times 10^{-5}$ /°C

(C) $\beta_{CDEH} = 3 \times 10^{-5}$ /°C (D) $\beta_{EFGH} = 2 \times 10^{-5}$ /°C

30. The coefficient of apparent expansion of a liquid in a copper vessel is C and in a silver vessel is S. The coefficient of volume expansion of copper is γ_c. What is the coefficient of linear expansion of silver?

(A) $\frac{(C+\gamma_c+S)}{3}$ (B) $\frac{(C-\gamma_c+S)}{3}$ (C) $\frac{(C+\gamma_c-S)}{3}$ (D) $\frac{(C-\gamma_c-S)}{3}$

31. Two rods one of aluminium of length l_1 having coefficient of linear expansion α_a, and other steel of length l_2 having coefficient of linear expansion α_s are joined end to end. The expansion in both the rods is same on variation of temperature. Then the value of $\frac{l_1}{l_1+l_2}$ is **[JEE' (Scr) 2003]**

(A) $\frac{\alpha_s}{\alpha_a+\alpha_s}$ (B) $\frac{\alpha_s}{\alpha_a-\alpha_s}$ (C) $\frac{\alpha_a+\alpha_s}{\alpha_s}$ (D) None of these

32. An open vessel is filled completely with oil which has same coefficient of volume expansion as that of the vessel. On heating both oil and vessel,

(A) the vessel can contain more volume and more mass of oil

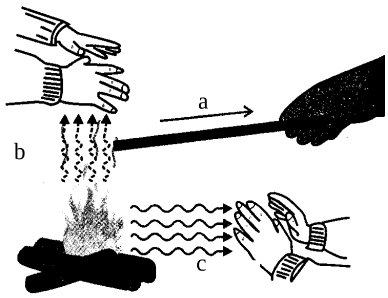
(B) the vessel can contain same volume and same mass of oil

(C) the vessel can contain same volume but more mass of oil

(D) the vessel can contain more volume but same mass of oil

Conduction

33. Diagram shows a heat source 'S' and three position of heat recover (hand). The main made of heat transfer is given as 'a', 'b' & 'c'. Choose the correct matching :-



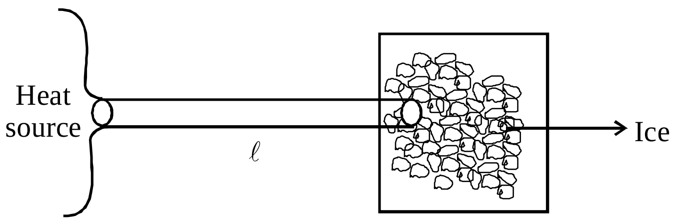
(A) a – conduction ; b – convection ; c – radiation

(B) b – conduction ; a – convection ; c – radiation

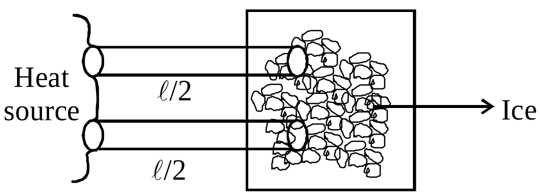
(C) a – conduction ; c – convection ; b – radiation

(D) c – conduction ; b – radiation ; a – convection

34. A rod of length 'ℓ' and cross-section 'A' is used to melt a piece of ice as shown.



Now if the rod broken into two equal parts and is arranged as shown.



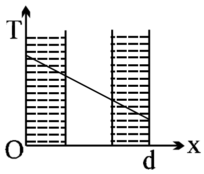
Time taken to melt ice in second use becomes.

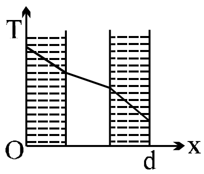
(A) Half (B) One-forth (C) Twice (D) Four times

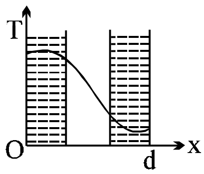
35. One end of a 2.35m long and 2.0cm radius aluminium rod ($K = 235\ W.m^{-1}K^{-1}$) is held at 20^0C. The other end of the rod is in contact with a block of ice at its melting point. The rate in $kg.s^{-1}$ at which ice melts is [Take latent heat of fusion for ice as $\frac{10}{3}\times10^5 J.kg^{-1}$]

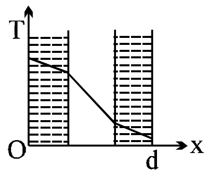
(A) $48\pi \times 10^{-6}$ (B) $24\pi \times 10^{-6}$ (C) $2.4\pi \times 10^{-6}$ (D) $4.8\pi \times 10^{-6}$

36. The wall with a cavity consists of two layers of brick separated by a layer of air. All three layers have the same thickness and the thermal conductivity of the brick is much greater than that of air. The left layer is at a higher temperature than the right layer and steady state condition exists. Which of the following graphs predicts correctly the variation of temperature T with distance d inside the cavity?

(A) 

(B) 

(C) 

(D) 

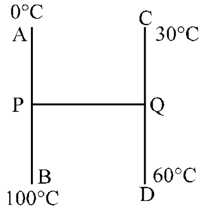
37. A wall has two layer A and B each made of different material, both the layers have the same thickness. The thermal conductivity of the material A is twice that of B. Under thermal equilibrium the temperature difference across the wall B is 36°C. The temperature difference across the wall A is

(A) 6°C (B) 12°C (C) 18°C (D) 72°C

38. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0° C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of melting of ice in the two cases respectively. The ratio q_2/q_1 is **[JEE' 2004 (Scr.)]**

(A) 1/2 (B) 2/1 (C) 4/1 (D) 1/4

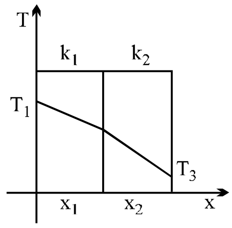
39. Three identical rods AB, CD and PQ are joined as shown. P and Q are mid points of AB and CD respectively. Ends A, B, C and D are maintained at 0°C, 100°C, 30°C and 60°C respectively. The direction of heat flow in PQ is



(A) from P to Q (B) from Q to P

(C) heat does not flow in PQ (D) data not sufficient

40. The temperature drop through each layer of a two layer furnace wall is shown in figure. Assume that the external temperature T_1 and T_3 are maintained constant and $T_1 > T_3$. If the thickness of the layers x_1 and x_2 are the same, which of the following statements are correct.



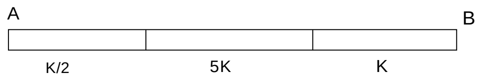
(A) $k_1 > k_2$

(B) $k_1 < k_2$

(C) $k_1 = k_2$ but heat flow through material (1) is larger then through (2)

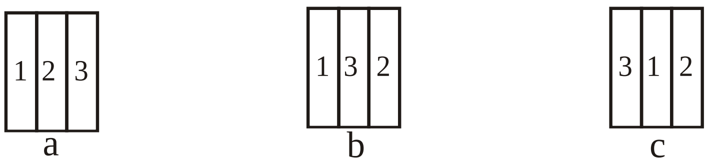
(D) $k_1 = k_2$ but heat flow through material (1) is less than that through (2)

41. A composite rod made of three rods of equal length and cross-section as shown in the fig. The thermal conductivities of the materials of the rods are K/2, 5K and K respectively. The end A and end B are at constant temperatures. All heat entering the end A goes out of the end B, there being no loss of heat from the sides of the bar. The effective thermal conductivity of the bar is



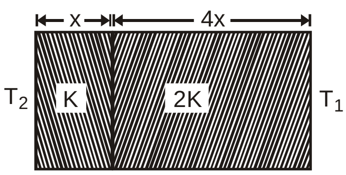
(A) 15K/16 (B) 6K/13 (C) 5K/16 (D) 2K/13.

42. Figure shows three different arrangements of materials 1, 2 and 3 to form a wall. Thermal conductivities are $k_1 > k_2 > k_3$. The left side of the wall is 20°C higher than the right side. Temperature difference ΔT across the material 1 has following relation in three cases :



(A) $\Delta T_a > \Delta T_b > \Delta T_c$ (B) $\Delta T_a = \Delta T_b = \Delta T_c$ (C) $\Delta T_a = \Delta T_b > \Delta T_c$ (D) $\Delta T_a = \Delta T_b < \Delta T_c$

43. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively are T_2 and $T_1(T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right)f$, with f equals to- **[AIEEE - 2004]**



(A) 1 (B) 1/2 (C) 2/3 (D) 1/3

Radiation

44. A black metal foil is warmed by radiation from a small sphere at temperature 'T' and at a distance 'd'. It is found that the power received by the foil is P. If both the temperature and distance are doubled, the power received by the foil will be :

(A) 16 P (B) 4 P (C) 2 P (D) P

45. The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be :-

(A) $\frac{E}{16}$ (B) $\frac{E}{4}$ (C) $\frac{E}{8}$ (D) 0

46. The power radiated by a black body is P and it radiates maximum energy around the wavelength λ_0. If the temperature of the black body is now changed so that it radiates maximum energy around wavelength $3/4\lambda_0$, the power radiated by it will increase by a factor of :-

(A) 4/3 (B) 16/9 (C) 64/27 (D) 256/81

47. Spheres P and Q are uniformly constructed from the same material which is a good conductor of heat and the radius of Q is thrice the radius of P. The rate of fall of temperature of P is x times that of Q when both are at the same surface temperature. The value of x is :

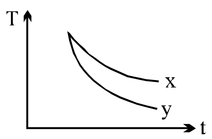
(A) 1/4 (B) 1/3 (C) 3 (D) 4

48. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is-

(A) 1 : 1 (B) 16 : 1 (C) 4 : 1 (D) 1 : 9

49. If emissivity of bodies X and Y are e_x and e_y and absorptive power are A_x and A_y then

[JEE' (Scr) 2003]



(A) $e_y > e_x$; $A_y > A_x$ (B) $e_y < e_x$; $A_y < A_x$ (C) $e_y > e_x$; $A_y < A_x$ (D) $e_y = e_x$; $A_y = A_x$

50. Three discs A, B, and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively. The power radiated by them are Q_A, Q_B and Q_C respectively.

[JEE' 2004 (Scr.)]

(A) Q_A is maximum (B) Q_B is maximum (C) Q_C is maximum (D) $Q_A = Q_B = Q_C$

51. A black body calorimeter filled with hot water cools from 60°C to 50°C in 4 min and 40°C to 30°C in 8 min. The approximate temperature of surrounding is :

(A) 10°C (B) 15°C (C) 20°C (D) 25°C

52. A system S receives heat continuously from an electrical heater of power 10W. The temperature of S becomes constant at 50°C when the surrounding temperature is 20°C. After the heater is switched off, S cools from 35.1°C to 34.9°C in 1 minute. The heat capacity of S is

(A) 100J/°C (B) 300J/°C (C) 750J/°C (D) 1500J/°C

53. If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously, will be- **[AIEEE - 2004]**

(A) 4 (B) 16 (C) 32 (D) 64

MULTIPLE CORRECT TYPE QUESTIONS

Elasticity

54. A wire is suspended from the ceiling and stretched under the action of a weight F suspended from its other end. The force exerted by the ceiling on it is equal and opposite to the weight.

(A) Tensile stress at any cross section A of the wire is F/A.

(B) Tensile stress at any cross section is zero.

(C) Tensile stress at any cross section A of the wire is $2F/A$.

(D) Tension at any cross section A of the wire is F.

55. A copper and a steel wire of the same diameter are connected end to end. A deforming force F is applied to this composite wire which causes a total elongation of 1cm. The two wires will have

(A) the same stress. (B) different stress. (C) the same strain. (D) different strain.

56. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is l.

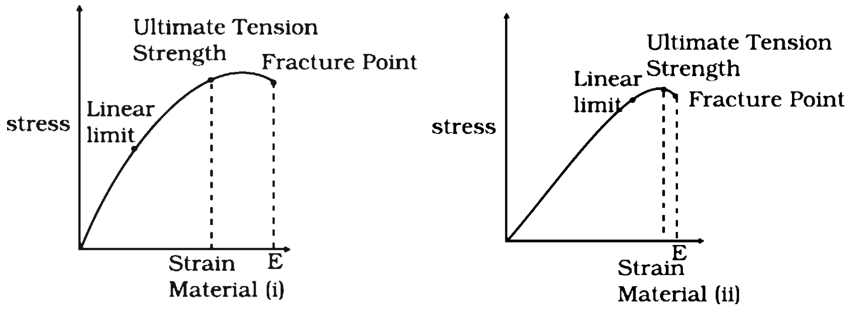
(A) Loss in gravitational potential energy of M is Mgl

(B) The elastic potential energy stored in the wire is Mgl

(C) The elastic potential energy stored in the wire is 1/2 Mgl

(D) Heat produced is 1/2 Mgl.

57. The stress-strain graphs for two materials are shown in figure (assume same scale).



(A) Material (ii) is more elastic than material (i) and hence material (ii) is more brittle.

(B) Material (i) and (ii) have the same elasticity and the same brittleness.

(C) Material (ii) is elastic over a larger region of strain as compared to (i).

(D) Material (ii) is more brittle than material (i).

58. A composite rod consists of a steel rod of length 25 cm and area 2A and a copper rod of length 50cm and area A. The composite rod is subjected to an axial load F. If the Young's modulus of steel and copper are in the ratio 2 : 1.

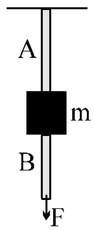
(A) the extension produced in copper rod will be more .

(B) the extension in copper and steel parts will be in the ratio 2 : 1.

(C) the stress applied to the copper rod will be more.

(D) no extension will be produced in the steel rod.

59. The wires A and B shown in the figure are made of the same material and have radii r_A and r_B respectively. The block between them has a mass m. When the force F is mg/3, one of the wires breaks.



(A) A breaks if $r_A = r_B$

(B) A breaks if $r_A < 2r_B$

(C) Either A or B may break if $r_A = 2r_B$

(D) The lengths of A and B must be known to predict which wire will break

Calorimetry

60. Mark the **CORRECT** options:

(A) A system X is in thermal equilibrium with Y but not with Z. System Y and Z may be in thermal equilibrium with each other.

(B) A system X is in thermal equilibrium with Y but not with Z. Systems Y and Z are not in thermal equilibrium with each other.

(C) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z must be in thermal equilibrium with each other.

(D) A system X is neither in thermal equilibrium with Y nor with Z. The system Y and Z may be in thermal equilibrium with each other.

61. 50 gm ice at – 10°C is mixed with 20 gm steam at 100°C. When the mixture finally reaches its steady state inside a calorimeter of water equivalent 1.5 gm then : [Assume calorimeter was initially at 0°C, Take latent heat of vaporization of water = 540 cal/gm, Latent heat of fusion of water = 80 cal/gm, specific heat capacity of water = 1 cal/gm-°C, specific heat capacity of ice = 0.5 cal/gm°C]

(A) Mass of water remaining is : 67.4 gm

(B) Mass of steam remaining is : 2.6 gm

(C) Mass of water remaining is : 67.87 gm

(D) Mass of steam remaining is : 2.13 gm

Thermal expansion

62. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.

(A) Percentage rise in the area of a face is 0.4 %

(B) Percentage rise in the thickness is 0.4 %

(C) Percentage rise in the volume is 0.6 %

(D) Coefficient of linear expansion of copper is $0.25 \times 10^{-4}\ C^{\circ\,-1}$.

Radiation

63. Two metallic sphere A and B are made of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.

(A) The ratio of heat loss of A to that of B is $2^{4/3}$.

(B) The ratio of heat loss of A to that of B is $2^{2/3}$.

(C) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3}$.

(D) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3}$.

64. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies radiate energy at the same rate. The wavelength λ_B, corresponding to the maximum spectral radiancy in the radiation from B, is shifted from the wavelength corresponding to the maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K,

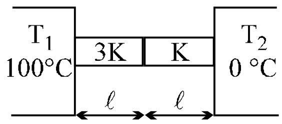
(A) the temperature of B is 1934 K
(B) λ_B =1.5 μm
(C) the temperature of B is 11604 K
(D) the temperature of B is 2901 K

COMPREHENSION TYPE QUESTIONS

Conduction

Paragraph for Question No. 65 to 67

Two rods A and B of same cross-sectional are A and length *l* connected in series between a source (T_1 = 100°C) and a sink (T_2 = 0°C) as shown in figure. The rod is laterally insulated



65. The ratio of the thermal resistance of the rods is

(A) $\frac{R_A}{R_B} = \frac{1}{3}$ (B) $\frac{R_A}{R_B} = 3$ (C) $\frac{R_A}{R_B} = \frac{3}{4}$ (D) $\frac{4}{3}$

66. If T_A and T_B are the temperature drops across the rod A and B, then

(A) $\frac{T_A}{T_B} = \frac{3}{1}$ (B) $\frac{T_A}{T_B} = \frac{1}{3}$ (C) $\frac{T_A}{T_B} = \frac{3}{4}$ (D) $\frac{T_A}{T_B} = \frac{4}{3}$

67. If G_A and G_B are the temperature gradients across the rod A and B, then

(A) $\frac{G_A}{G_B} = \frac{3}{1}$ (B) $\frac{G_A}{G_B} = \frac{1}{3}$ (C) $\frac{G_A}{G_B} = \frac{3}{4}$ (D) $\frac{G_A}{G_B} = \frac{4}{3}$

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. A cylindrical wire of radius 1 mm, length 1 m, Young's modulus = 2×10^{11} N/m^2, poisson's ratio $\mu = \pi/10$ is stretched by a force of 100 N. Its radius will become

(A) 0.99998 mm (B) 0.99999 mm (C) 0.99997 mm (D) 0.99995 mm

2. A thermally insulated vessel contains some water at 0^0C. The vessel is connected to a vacuum pump to pump out water vapour. This results in some water getting frozen. It is given Latent heat of vaporization of water at 0°C = 21×10^5 J/kg and latent heat of freezing of water = 3.36×10^5 J/kg. The maximum percentage amount of water that will be solidified in this manner will be :-

(A) 86.2% (B) 33.6% (C) 21% (D) 24.36%

3. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted the temperature of the flask and contents is 40°C. When a further 80g of ice has been added and has all metled, the temperature of the whole is 10°C. Calculate the specific latent heat of fusion of ice. [Take S_w =1 cal /gm °C.]

(A) 3.8×10^5 J/ kg (B) 1.2×10^5 J/ kg (C) 2.4×10^5 J/ kg (D) 3.0×10^5 J/ kg

4. The coefficient of linear expansion of copper is 17×10^{-6} (°C)$^{-1}$. A copper statue is 93 m tall on the summer morning of temperature 25°C. What is maximum order of increase in magnitude of the height in statue (maximum temperature of day is 45°C)

(A) 0.1 mm (B) 1 mm (C) 10 mm (D) 100 mm

5. The coefficients of thermal expansion of steel and a metal X are respectively 12×10^{-6} and 2×10^{-6} per°C. At 40°C, the side of a cube of metal X was measured using a steel vernier callipers. The reading was 100 mm. Assuming that the calibration of the vernier was done at 0°C, then the actual length of the side of the cube at 0°C will be

(A) > 100 mm (B) < 100 mm
(C) = 100 mm (D) data insufficient to conclude

6. The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0. The coefficient of linear expansion of glass is α_g per °C and the cubical expansion of mercury γ_m per °C. If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C.

(A) $\dfrac{V_0T(\gamma_m+3\alpha_g)}{A_0(1+2\alpha_gT)}$ (B) $\dfrac{V_0T(\gamma_m-3\alpha_g)}{A_0(1+2\alpha_gT)}$ (C) $\dfrac{V_0T(\gamma_m+2\alpha_g)}{A_0(1+3\alpha_gT)}$ (D) $\dfrac{V_0T(\gamma_m-2\alpha_g)}{A_0(1+3\alpha_gT)}$

7. A rod of length 2m at 0°C and having expansion coefficient $\alpha = (3x + 2) \times 10^{-6}$ °C^{-1} where x is the distance (in cm) from one end of rod. The length of rod at 20°C is :

(A) 2.124 m (B) 3.24 m (C) 2.0120 m (D) 3.124 m

8. A liquid is given some heat.

Statement A : Some liquid evaporates. **Statement B** : The liquid starts boiling.

(A) A implies B and B implies A
(B) B implies A but, A does not imply B
(C) A implies B but B does not imply A
(D) Neither A implies B nor B implies A

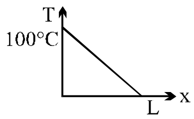
9. A long solid cylinder is radiating power. It is remoulded into a number of smaller cylinders, each of which has the same length as original cylinder. Each small cylinder has the same temperature as the original cylinder. The total radiant power emitted by the pieces is twice that emitted by the original cylinder. How many smaller cylinders are there ? Neglect the energy emitted by the flat faces of cylinder.

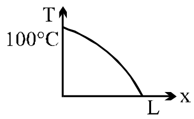
(A) 3 (B) 4 (C) 5 (D) 6

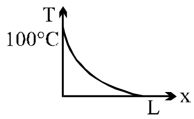
10. Four rods of same material with different radii r and length ℓ are used to connect two reservoirs of heat at different temperatures. Which one will conduct most heat ?

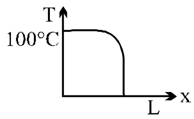
(A) r =2cm, ℓ=0.5m
(B) r = 2cm, ℓ = 2m
(C) r =0.5cm, ℓ =0.5m
(D) r = 1 cm, ℓ = 1 m

11. A rod of length L and uniform cross-sectional area has varying thermal conductivity which changes linearly from 2K at end A to K at the other end B. The ends A and B of the rod are maintained at constant temperature 100°C and 0°C, respectively. At steady state, the graph of temperature : T = T(x) where x = distance from end A will be

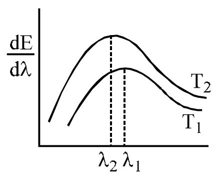
(A) 

(B) 

(C) 

(D) 

12. The spectral emissive power E_λ for a body at temperature T_1 is plotted against the wavelength and area under the curve is found to be A. At a different temperature T_2 the area is found to be 9A. Then λ_1/λ_2 =



(A) 3 (B) 1/3 (C) $1/\sqrt{3}$ (D) $\sqrt{3}$

13. 'Gulab Jamuns' (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice big (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following:

(A) Both size gulab jamuns will get heated in the same time.
(B) Smaller gulab jamuns are heated before bigger ones.
(C) Smaller pizzas are heated before bigger ones.
(D) Bigger pizzas are heated before smaller ones.

14. An experiment is perfomed to measure the specific heat of copper. A lump of copper is heated in an oven, then dropped into a beaker of water. To calculate the specific heat of copper, the experimenter must know or measure the value of all of the quantities below EXCEPT the

(A) heat capacity of water and beaker

(B) original temperature of the copper and the water

(C) final (equilibrium) temperature of the copper and the water

(D) time taken to achieve equilibrium, after the copper is dropped into the water

15. One end of a conducting rod is maintained at temperature 50°C and at the other end, ice is melting at 0°C. The rate of melting of ice is doubled if:

(A) the temperature is made 200°C and the area of cross-section of the rod is doubled

(B) the temperature is made 100°C and length of rod is made four times

(C) area of cross-section of rod is halved and length is doubled

(D) the temperature is made 100°C and the area of cross-section of rod and length both are doubled.

16. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wien constant b = 2.88×10^6 nm K. Then

(A) $U_1 = 0$ (B) $U_3 = 0$ (C) $U_1 > U_2$ (D) $U_2 > U_1$

MULTIPLE CORRECT TYPE QUESTIONS

17. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficient of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is

(A) proportional at ΔT

(B) inversely proportional to ΔT

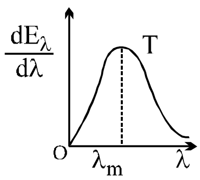
(C) proportional to $|\alpha_B - \alpha_C|$

(D) inversely proportional to $|\alpha_B - \alpha_C|$

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 18 and 19

The figure shows a radiant energy spectrum graph for a black body at a temperature T.



18. Choose the **CORRECT** statement(s)

(A) The radiant energy is not equally distributed among all the possible wavelengths

(B) For a particular wavelength the spectral intensity is maximum

(C) The area under the curve is equal to the total rate at which heat is radiated by the body at that temperature

(D) None of these

19. If the temperature of the body is raised to a higher temperature T', then choose the correct statement(s)

(A) The intensity of radiation for every wavelength increases

(B) The maximum intensity occurs at a shorter wavelength

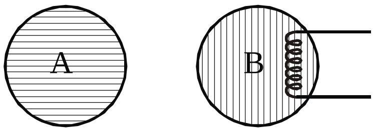
(C) The area under the graph increases

(D) The area under the graph is proportional to the fourth power of temperature

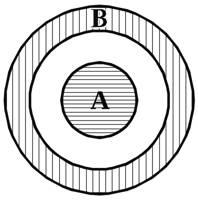
MATRIX MATCH TYPE QUESTION

20. A & B are two black bodies of radii r_A and r_B respectively, placed in surrounding of temperature T_0. At steady state the temperature of A & B is T_A & T_B respectively.

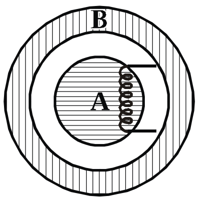
Column I

(A) 

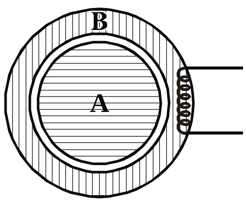
- A & B are solid sphere
- $r_A = r_B$
- Body 'B' is being heated by a heater of constant power 'P'

(B) 

- B is thin spherical shell
- A is a solid sphere
- $r_A < r_B$

(C) 

- B is thin spherical shell
- A is a solid sphere
- $r_A < r_B$
- Body A is being heated by a heater of constant power 'P'

(D) 

- B is thin spherical shell
- A is a solid sphere
- $r_A \approx r_B$
- Body B is being heated by a heater of constant power 'P'

Column II

(P) $T_A = T_B$

(Q) $T_A < T_B$

(R) Heat received by A is more than heat radiated by it at steady state.

(S) Radiation spectrum of A & B is distinguishable

(T) Steady state can't be achieved

21. A sample 'A' of liquid water and a sample B of ice of equal mass are kept in 2 nearby containers so that they can exchange heat with each other but are thermally insulated from the surroundings. The graphs in column-II show the sketch of temperature T of samples versus time t. Match with appropriate description in column-I.

Column I

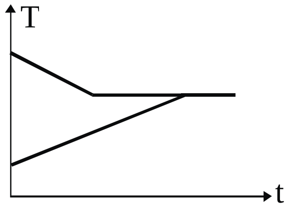
(A) Equilibrium temperature is above melting point of ice.

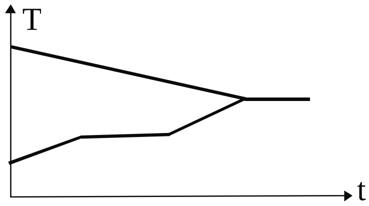
(B) At least some of water freezes.

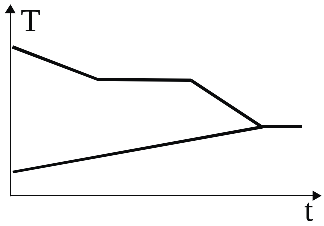
(C) At least some of ice melts.

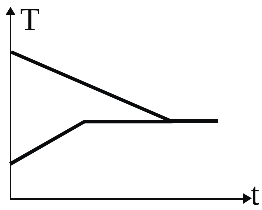
(D) Equilibrium temperature is below freezing point of water

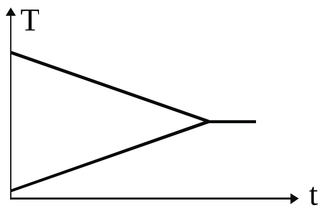
Column II

(P) 

(Q) 

(R) 

(S) 

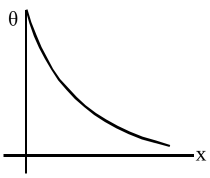
(T) 

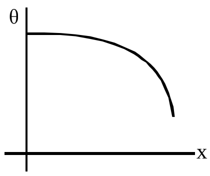
EXERCISE (JM)

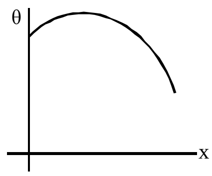
1. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area 3A. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount ? **[AIEEE-2009]**

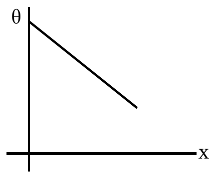
(1) 6F (2) 9F (3) F (4) 4F

2. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures ? **[AIEEE - 2009]**

(1) 

(2) 

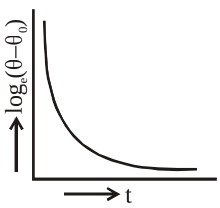
(3) 

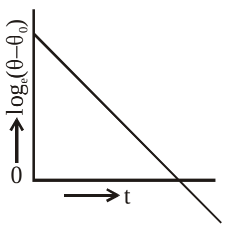
(4) 

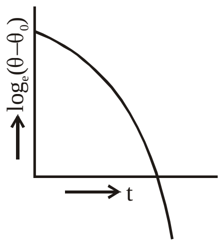
3. 100 g of water is heated from 30°C to 50°C Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) :- **[AIEEE - 2011]**

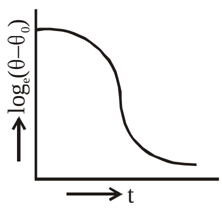
(1) 84 kJ (2) 2.1 kJ (3) 4.2 kJ (4) 8.4 kJ

4. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :- **[AIEEE 2012]**

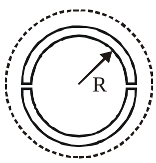
(1) 

(2) 

(3) 

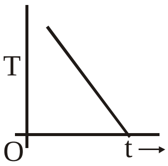
(4) 

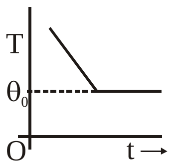
5. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and Length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is : **[AIEEE 2012]**

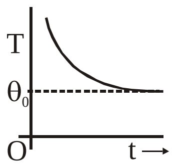


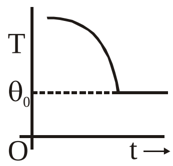
(1) $2SY\alpha\Delta T$ (2) $2\pi SY\alpha\Delta T$ (3) $SY\alpha\Delta T$ (4) $\pi SY\alpha\Delta T$

6. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closed to: **[JEE-Main- 2013]**

(1) 

(2) 

(3) 

(4) 

7. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is :

(For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

[JEE-Main-2014]

(1) 2.2×10^7 Pa (2) 2.2×10^6 Pa (3) 2.2×10^8 Pa (4) 2.2×10^9 Pa

8. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is : **[JEE-Main-2014]**

(1) 4.8 cal/s (2) 6.0 cal/s (3) 1.2 cal/s (4) 2.4 cal/s

9. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :- (g = gravitational acceleration) **[JEE-Main-2015]**

(1) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (2) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$ (3) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (4) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

10. A pendulume clock loses 12s a day if the temperature is 40°C and gains 4s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coeffecient of linear expansion (α) of the metal of the pendulum shaft are respectively :- **[JEE-Main-2016]**

(1) 55°C ; $\alpha = 1.85 \times 10^{-2}$ / °C (2) 25°C ; $\alpha = 1.85 \times 10^{-5}$ / °C

(3) 60°C ; $\alpha = 1.85 \times 10^{-4}$ / °C (4) 30°C ; $\alpha = 1.85 \times 10^{-3}$ / °C

11. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by : (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C) **[JEE-Main-2017]**

(1) 1250°C (2) 825°C (3) 800°C (4) 885° C

12. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of : **[JEE-Main-2017]**

(1) 81 (2) $\frac{1}{81}$ (3) 9 (4) $\frac{1}{9}$

13. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by : **[JEE-Main-2017]**

(1) $\frac{3\alpha}{PK}$ (2) $3PK\alpha$ (3) $\frac{P}{3\alpha K}$ (4) $\frac{P}{\alpha K}$

14. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is : **[JEE-Main-2018]**

(1) $\frac{Ka}{3mg}$ (2) $\frac{mg}{3Ka}$ (3) $\frac{mg}{Ka}$ (4) $\frac{Ka}{mg}$

EXERCISE (JA)

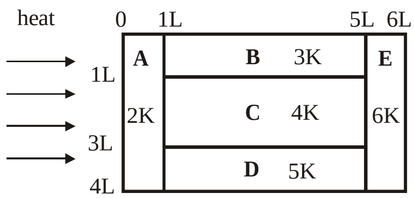
1. A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ.[Neglect any heat loss to the surrounding] **[JEE 2009]**

2. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering then to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? **[JEE 2010]**

3. A piece of ice (heat capacity = 2100 J kg^{-1} $°C^{-1}$ and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice–water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is **[JEE 2010]**

4. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross–sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is **[IIT-JEE 2010]**

5. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state **[JEE 2011]**



(A) heat flow through A and E slabs are same

(B) heat flow through slab E is maximum

(C) temperature difference across slab E is smallest

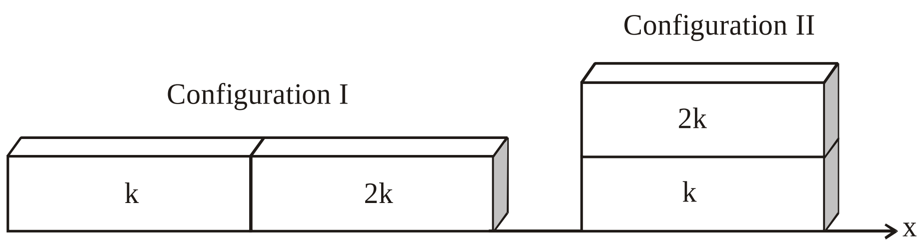
(D) heat flow through C= heat flow through B + Heat flow through D

6. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40° to 30° C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L>> diameter of the wire. Then the value of 'm' in kg is nearly **[JEE 2011]**

7. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is **[JEE 2012]**

(A) $\left(\frac{65}{2}\right)^{1/4} T$ (B) $\left(\frac{97}{4}\right)^{1/4} T$ (C) $\left(\frac{97}{2}\right)^{1/4} T$ (D) $(97)^{1/4} T$

8. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity k and the other 2k. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is :- **[JEE-Advance-2013]**



(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

9. One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is :- **[JEE-Advance-2013]**

(A) 0.25 (B) 0.50 (C) 2.00 (D) 4.00

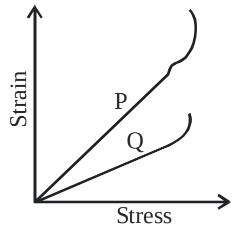
10. Parallel rays of light of intensity I = 912 Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to :- **[JEE-Advance-2014]**

(A) 330 K (B) 660 K (C) 990 K (D) 1550 K

11. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is. **[JEE-Advance-2015]**

12. In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statement(s) is (are):-

[JEE-Advance-2015]



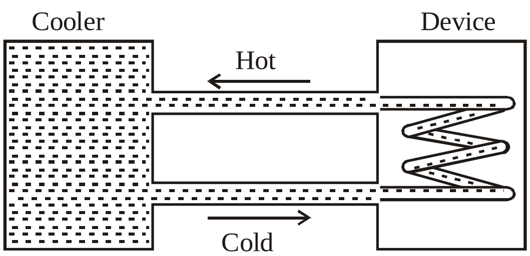
(A) P has more tensile strength than Q

(B) P is more ductile than Q

(C) P is more brittle than Q

(D) The Young's modulus of P is more than that of Q

13. A water cooler of storage capacity 120 litres can cool water at constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is : **[JEE-Advance-2016]**



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

14. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2(P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487 °C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C ? **[JEE-Advance-2016]**

15. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400°C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\,K^{-1}$, the change in length of the wire PQ is

[JEE-Advance-2016]

(A) 0.78 mm (B) 0.90 mm (C) 1.56 mm (D) 2.34 mm

16. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of σT_0^4 = 460 Wm^{-2} (where σ is the Stefan-Boltzmann constant). Which of the following options is/are **correct** ? **[JEE-Advance-2017]**

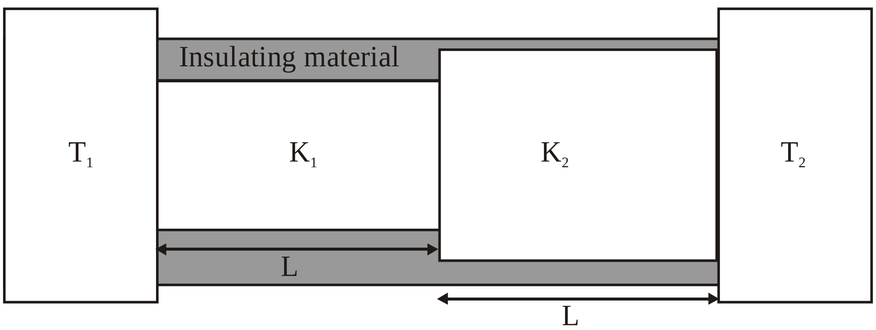
(A) The amount of energy radiated by the body in 1 second is close to 60 Joules

(B) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(D) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

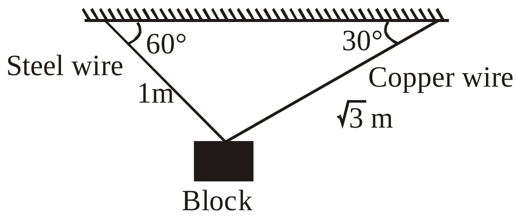
17. Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures T_1 = 300 K and T_2 = 100 K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then K_1/K_2 = __________. **[JEE-Advance-2018]**



18. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is $(\Delta\ell_C)$ and elongation in steel wire is $(\Delta\ell_S)$, then the ratio $\frac{\Delta\ell_C}{\Delta\ell_S}$ is ______.

[Young's modulus for copper and steel are 1×10^{11} N/m^2 and 2×10^{11} N/m^2 respectively]

[JEE-Advance-2019]



19. A liquid at 30°C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be ______ °C.

[Neglect the heat exchange with surrounding] **[JEE-Advance-2019]**

ANSWER KEY

EXERCISE (S-1)

1. Ans. 0.75 **2. Ans.** $\frac{9}{16}$ **3. Ans.** 3mm **4. Ans.** ℓ'

5. Ans. (a) $\theta = \frac{\pi}{2}$ (b) $\theta = \pi/4$.

6. Ans. (i) 50 N, (ii) 0.045 J, 1.8×10^{-3} m (iii) 8.4×10^{-4} m, (iv) x = 0.12 m **7. Ans.** The first one

8. Ans. 25.5°C **9. Ans.** 1/90 **10. Ans.** 0 °C, 125/4 g ice, 1275/4 g water

11. Ans. 5000 J/°C kg **12. Ans.** 27/85 **13. Ans.** 12 gm

14. Ans. (i) 0.02kg, (ii) 40,000calkg^{-1}, (iii) 750calkg^{-1}K^{-1} **15. Ans.** $5\alpha/3$

16. Ans. 5 sec slow **17. Ans.** 10, 000 N **18. Ans.** 65°C **19. Ans.** 5°C

20. Ans. 7/2 **21. Ans.** $(4/3)\omega$ **22. Ans.** 10.34 cm **23. Ans.** 2:1

24. Ans. 1000 J (C°)$^{-1}$ **25. Ans.** $\left(\frac{6}{\pi}\right)^{1/3}$ **26. Ans.** 80 kcal/kg **27. Ans.** 3025 K

28. Ans. 10 sec

EXERCISE (O-1)

1. Ans. (D)	**2. Ans. (D)**	**3. Ans. (C)**	**4. Ans. (D)**	**5. Ans. (B)**	**6. Ans. (D)**
7. Ans. (A)	**8. Ans. (C)**	**9. Ans. (C)**	**10. Ans. (A)**	**11. Ans. (C)**	**12. Ans. (B)**
13. Ans. (D)	**14. Ans. (B)**	**15. Ans. (D)**	**16. Ans. (A)**	**17. Ans. (B)**	**18.Ans. (C)**
19. Ans. (D)	**20. Ans. (A)**	**21. Ans. (A)**	**22. Ans. (C)**	**23. Ans. (D)**	**24. Ans. (B)**
25. Ans. (C)	**26. Ans. (B)**	**27. Ans. (B)**	**28. Ans. (B)**	**29. Ans. (C)**	**30. Ans. (C)**
31. Ans. (A)	**32. Ans. (D)**	**33. Ans. (A)**	**34. Ans. (B)**	**35. Ans. (C)**	**36.Ans. (D)**
37. Ans. (C)	**38. Ans. (D)**	**39. Ans. (A)**	**40. Ans. (A)**	**41. Ans. (A)**	**42. Ans. (B)**
43. Ans. (D)	**44. Ans. (B)**	**45. Ans. (A)**	**46. Ans. (D)**	**47. Ans. (C)**	**48. Ans. (A)**
49. Ans. (A)	**50. Ans. (B)**	**51. Ans. (B)**	**52. Ans. (D)**	**53. Ans. (D)**	

54.Ans. (A,D)	**55. Ans. (A,D)**	**56. Ans. (A,C,D)**	**57. Ans. (C,D)**
58. Ans. (A,C)	**59. Ans. (A,B,C)**	**60. Ans. (B,D)**	**61. Ans. (A,B)**
62. Ans. (A,C,D)	**63. Ans. (A,C)**	**64. Ans. (A,B)**	**65. Ans. (A)**
66. Ans. (B)	**67. Ans. (B)**		

EXERCISE (O-2)

1. Ans. (D)	2. Ans. (A)	3. Ans. (A)	4. Ans. (C)	5. Ans. (A)	6. Ans. (B)
7. Ans. (C)	8. Ans. (B)	9. Ans. (B)	10. Ans. (A)	11. Ans. (B)	12.Ans. (D)
13. Ans. (B)	14. Ans. (D)	15. Ans. (D)	16. Ans. (D)	17. Ans. (B,D)	

18. Ans. (A,B)

19. Ans. (A,B,C,D)

20. Ans. A - Q,S; B - P; C - S; D - P

21. Ans. A - Q; B - P,R; C - Q,S; D - R

EXERCISE (JM)

1. Ans. (2)	2. Ans. (4)	3. Ans. (4)	4. Ans. (2)	5. Ans. (1)	6. Ans. (3)
7. Ans. (3)	8. Ans. (1)	9. Ans. (3)	10. Ans. (2)	11. Ans. (4)	12. Ans. (3)
13. Ans. (3)	14. Ans. (2)				

EXERCISE (JA)

1. Ans. 9	2. Ans. 9	3. Ans. 8	4. Ans. 4	5. Ans. (ACD or ABCD)	
6. Ans. 3	7. Ans. (C)	8. Ans. (A)	9. Ans. (C)	10. Ans. (A)	11. Ans. 2
12. Ans. (A,B)	13. Ans. (B)	14. Ans. 9	15. Ans. (A)	16. Ans. (C)	

17. Ans. 4 [3.99, 4.01]

18. Ans. (2.00)

19. Ans. (270.00)

IMPORTANT NOTES