PHYSICS

INDEX

MODULE-4

S.No.	CHAPTER NAME	Pg.No.
1.	FLUID MECHANICS	1-82
2.	KINETIC THEORY OF GASES & THERMODYNAMICS	83-158
3.	WAVES ON STRING	159-194
4.	SOUND WAVES	195-234

CHAPTER 1

FLUID MECHANICS

Chapter Contents 01

01.	THEORY	3
02.	EXERCISE (S-1)	47
03.	EXERCISE (O-1)	52
04.	EXERCISE (O-2)	63
05.	EXERCISE (JM)	72
06.	EXERCISE (JA)	75
07.	ANSWER KEY	81

IMPORTANT NOTES

CHAPTER 1

FLUID MECHANICS

KEY CONCEPTS

What Is a Fluid ?

A **fluid,** in contrast to a solid, is a substance that can flow. Fluids conform to the boundaries of any container in which we put them. They do so because a fluid cannot sustain a force that is tangential to its surface. A fluid is a substance that flows because it cannot withstand a shearing stress. It can, however, exert a force in the direction perpendicular to its surface.

FLUID MECHANICS

- FLUID STATICS (Fluid remains stationary wrt container)
- FLUID DYNAMICS
- VISCOUS FLUID

Fluid includes property $\rightarrow$ (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) Pressure (E) Specific gravity.

Assumptions used in fluid mechanics

1. Fluid is incompressible means density remains constant and volume also remains constant.
2. Fluid is non viscous. There is no tangential force between two layers.

DENSITY (ρ)

Mass per unit volume is defined as density. So density at a point of a fluid is represented as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$ Density is a positive scalar quantity.

SI UNIT : kg/m^3 **CGS UNIT :** g/cc **Dimensions :** [ML^{-3}]

RELATIVE DENSITY

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D.)} = \frac{\text{density of given liquid}}{\text{density of pure water at } 4^\circ\text{C}}$$

Relative density or specific gravity is a unitless and dimensionless positive scalar physical quantity.

Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

SPECIFIC GRAVITY

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.

$$\text{Specific gravity} = \frac{\text{specific weight of given liquid}}{\text{specific weight of pure water at } 4^\circ\text{C } (9.81\ \text{kN/m}^3)}$$

$$= \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

Density of a Mixture of substance in the proportion of mass

Let a number of substances of masses M_1, M_2, M_3 etc., and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed together. The total mass of the mixture = $M_1 + M_2 + M_3 + ...$

The total volume = $\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2} + \frac{M_3}{\rho_3} + ...$ therefore, the density of the mixture is

$$\rho = \frac{M_1 + M_2 + M_3 ....}{\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2} + \frac{M_3}{\rho_3} + ...}$$

For two substances the density of the mixture $\rho = \frac{\rho_1\rho_2(M_1 + M_2)}{\rho_1 M_2 + \rho_2 M_1}$

Density of a mixture of substance in the proportion of volume

Suppose that a number of substances of volume V_1, V_2, V_3 etc. and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed. The total mass of the mixture is = $\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...$

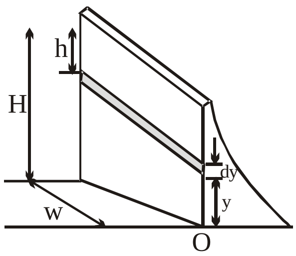
The total volume of the mixture is = $V_1 + V_2 + V_3 + ...$

Therefore, the density of the mixture is $\rho = \frac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + ...}$

Therefore, for two substances we can write $\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$

Example(a) :

Water is filled to a height H behind a dam of width w (fig.). Determine the resultant force exerted by the water on the dam.



Water exerts a force on a dam

Solution :

Let's consider a vertical y axis, starting from the bottom of the dam. Lets consider a thin horizontal strip at a height y above the bottom, such as shown in Fig. We need to consider force due to the pressure of the water only as atmospheric pressure acts on both sides of the dam.

The pressure due to the water at the depth h : $P = \rho gh = \rho g(H - y)$

The force exerted on the shaded strip of area $dA = w dy$:

$dF = P\, dA = \rho g(H - y)\, w\, dy$

Integrate to find the total force on the dam :

$$F = \int P\, dA = \int_0^H \rho g(H - y) w\, dy = 1/2\ \rho g w H^2$$

Example(b) :

In the previous example find the total torque exerted by the water on dam about a horizontal axis through O. Also find the effective line of action of the total force exerted by the water is at a distance 1/3 H above O.

Solution : .

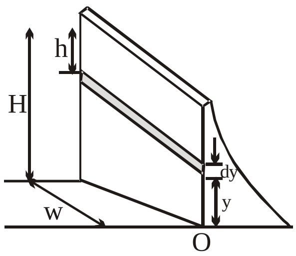
The torque is $\tau = \int d\tau = \int r dF$

From the figure $\tau = \int_0^H y[\rho g(H-y)w dy] = \frac{1}{6}\rho g w H^3$

The total force is given as $\frac{1}{2}\rho g w H^2$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

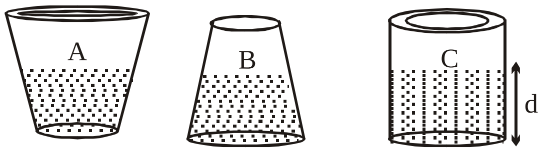
$$\frac{1}{6}\rho g w H^3 = y_{eff}\left[\frac{1}{2}\rho g w H^2\right] \quad \text{and} \quad y_{eff} = \frac{1}{3}H$$



Water exerts a force on a dam

Example(a&b) :

Three vessels having different shapes are as shown in the figure below, they have same base area and the same weight when empty (Fig.). The vessels are filled with mercury to the same level. Neglect the effect of the atmosphere. (a) Which have the largest and which have the smallest pressures at the bottom of the vessel or are they same? (b)Which show the highest weight when weighed on a weighing scale or are they same?



Three differently shaped vessels filled with water to same level.

Solution (a) The mercury at the bottom of each vessel is at the same depth d below the surface. Neglecting the pressure at the surface, the pressures at the bottom must be equal hence:

$$P = \rho g d$$

(b) The weight of each filled vessel is equal to the weight of the vessel itself plus the weight of the mercury inside. The vessels themselves are of equal weights, but vessel A holds more mercury than C, while vessel B holds less mercury than C. Vessel A weighs the most and vessel B weighs the least.

Example(c) : As the mercury exerts the same downward force on the bottom of each vessel, then why does the vessels weigh differently ?

Soln. In vessel C ,forces due to fluid pressure on the sides of the container are horizontal. Forces on any two diametrically opposite points on the walls of the container are equal and opposite; thus, the net force on the container walls is zero. The force on the bottom is

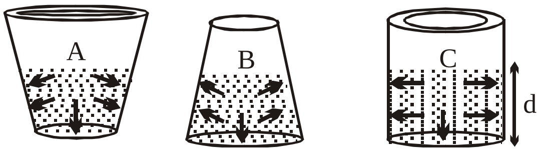
$$F = PA = (\rho g d)(\pi r^2)$$

The volume of water in the cylinder is $V = \pi r^2 d$, so

$$F = \rho g V = (\rho V) g = mg$$

The force on the bottom of vessel C is equal to the weight of the water, as expected. The forces due to fluid pressure on the sides of the containers A and B have vertical components also. Hence the force between the fluid and the base of container will not be equal to the weight of the fluid. These containers support the fluid by exerting an upward force equal in magnitude to the weight of the fluid but some force is being applied by the sidewalls and the remaining by the bottom. Figure shows the forces acting on each container due to the water.

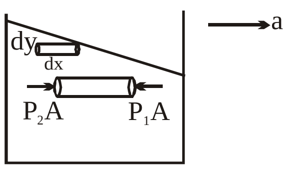
The force on the bottom of vessel A is less than the weight of the mercury in the container, while the force on the bottom of vessel B is greater than the weight of the mercury. In vessel A, the forces on the container walls have downward components as well as horizontal components. The sum of the downward components of the forces on the walls and the downward force on the bottom of the container is equal to the weight of the water. Similarly, the forces on the walls of vessel B have upward components. In each case, the total force on the bottom and sides of the container due to the water is equal to the weight of the water.



Forces exerted on the containers by the water.

Linear Accelerated Motion :

We consider an open container of a liquid that is moving along a straight line with a constant acceleration a as shown in Fig.



Lets consider a small horizontal cylinder of length dx and cross-sectional area A located y below the free surface of the fluid. This cylinder is accelerating in ground frame with acceleration a hence the net horizontal force acting on it should be equal to the product of mass(dm) and acceleration.

$dm = Adx\rho$

$P_2A - P_1A = (Adx\rho)a$

If we say that the right face of the cylinder is y below the free surface of the fluid then the left surface is y + dy below the surface of liquid. Thus

$$P_2 - P_1 = \rho g dy$$

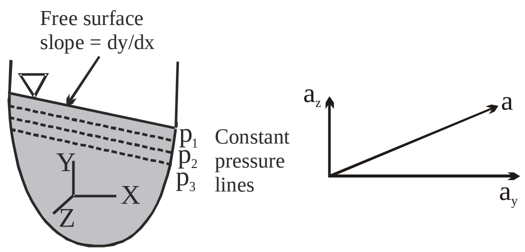
$$\therefore \qquad \frac{dy}{dx} = \frac{a}{g}$$

Since the slope of the free surface is coming out to be constant we can say that it must be straight line.

$$\tan\theta = \frac{a}{g}$$

If the container have acceleration along y also than the slope of this line is given by the relationship.

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y}$$



Along a free surface the pressure is constant, so that for the accelerating mass shown in figure the free surface will be inclined if $a_x \neq 0$. In addition, all lines parallel to the free surface will have same pressure.

For the special circumstance in which $a_x = 0$, $a_y \neq 0$, which corresponds to the mass of fluid accelerating in the vertical direction, Equation indicates that the fluid surface will be horizontal. However, from below equation we see that the pressure variation is not ρgdy, but is given by the equation.

$$dP = \rho(g + a_y)dy$$

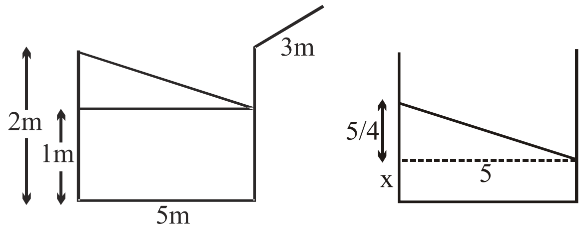
Thus, the pressure on the bottom of a liquid-filled tank which is resting on the floor of an elevator that is accelerating upward will be more than, if the, tank would have been at rest (or moving with a constant velocity). It is to be noted that for a freely falling fluid mass ($a_y = -g$), the pressure variation in all three coordinate directions are zero, which means that the pressure throughout will be same. The pressure throughout a "blob" of a liquid floating in an orbiting space shuttle (a form of free fall) is zero.

Ques : The cross section of a tank kept on a vehicle is shown in Fig.. The rectangular tank is open to the atmosphere. During motion of the vehicle, the tank is subjected to a constant linear acceleration, a = 2.5 m/s^2. How much fluid will be left inside the tank if initially the tank is half filled. The vessel is 5m wide and 2m high.

Ans. If the height of the liquid on the left wall is greater than 2m the fluid will be spilled out.

Now we can find the angle that the fluid will make with the horizontal.

$$\tan\theta = \frac{2.5}{10} = \frac{1}{4}$$



Lets assume that the dimension of tank in the plane perpendicular to the page is d.

From the geometry its easy to see that free surface on RHS will go down and will rise on LHS. Thus if we assume that fluid on RHS has not touched the floor, we will have fluid taking a shape as described in the diagram. The cuboid part will have volume $x \times 5 \times d$, where x is the height above the bottom.

The wedge part will have the volume $\frac{1}{2} \times h \times 5 \times d$ where h can be found as following

$(h/5) = \tan\theta = (1/4)$

Thus total volume will be $\frac{1}{2} \times (5/4) \times 5 \times d + x \times 5 \times d$ and if we assume there is no spilling then it must be equal to the final volume.

$$\frac{1}{2} \times (5/4) \times 5 \times d + x \times 5 \times d = 1 \times 5 \times d$$

solving, we get $x = \frac{3}{8}$

$\therefore$ Total length $\frac{5}{4} + \frac{3}{8} = \frac{10+3}{8} = \frac{13}{8} < 2$

Thus, height is less than 2.

Hence water will not spill.

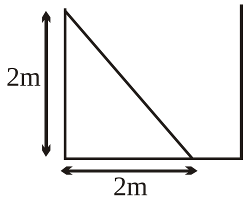
Ex. How much fluid will be left inside the tank if the vehicle accelerates at acceleration, $a = 10\ m/s^2$?

If the height of the liquid on the left wall is greater than 2m the fluid will be spilled out.

Now we can find the angle that the fluid will make with the horizontal.

$$\tan\theta = \frac{10}{10} = 1, \text{ thus} \quad \theta = \frac{\pi}{4}$$

In this case fluid can not remain inside. Fluid having an inclined free surface at 45^0 angle, and covering the bottom of length 5m, will also be 5 m high. This will require the wall to be of 5 m height, which is just 2m for the given vessel. Instead if we think it other way round to keep in contact with the LHS wall, bottom will have to be covered only 2m with the fluid as as shown in the diagram,



Fluid Inside = $(1/2) \times 2 \times 2 \times d\ m^3$

Remain inside = $2d\ m^3$

$\therefore$ Thus volume of fluid gone Outside = $3d\ m^3$

Pascal's Principle

When you squeeze one end of a tube to get toothpaste out the other end, you are watching **Pascal's principle** in action. This principle is also the basis for the Heimlich maneuver, in which a sharp pressure increase properly applied to the abdomen is transmitted to the throat, forcefully ejecting food lodged there. The principle was first stated clearly in 1652 by Blaise Pascal (for whom the unit of pressure is named):

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

Demonstrating Pascal's Principle

Consider the case in which the incompressible fluid is a liquid contained in a tall cylinder, as in Fig. The cylinder is fitted with a piston on which a container of lead shot rests. The atmosphere, container, and shot exert pressure p_{ext} on the piston and thus on the liquid. The pressure p at any point P in the liquid is then

$$p = p_{ext} + \rho_{gh}$$

Let us add a little more lead shot to the container to increase pressure by an amount p_{ext}. The quantities Δp_{ext}, g and h in Eq. are unchanged, so the pressure change at P is

$$\Delta p = \Delta p_{ext}$$

This pressure change is independent of h, so it must hold for all points within the liquid, as Pascal's principle states.

Pascal's Principle and the Hydraulic Lever

Figure shows how Pascal's principle can be made the basis of a hydraulic lever. In operation, let an external force of magnitude F_i be directed downward on the left-hand (or input) piston, whose surface area is A_i . An incompressible liquid in the device then produces an upward force of magnitude F_0 on the right-hand (or output) piston, whose surface area is A_0. To keep the system in equilibrium, there must be a downward force of magnitude F_0 on the output piston from an external load (not shown). The force $\vec{F}_1$ applied on the left and the downward force $\vec{F}_0$ from the load on the right produce a change Δp in the pressure of the liquid that is given by

$$\Delta p = \frac{F_i}{A_i} = \frac{F_0}{A_0}$$

$$F_0 = F_1 \frac{A_0}{A_i}$$

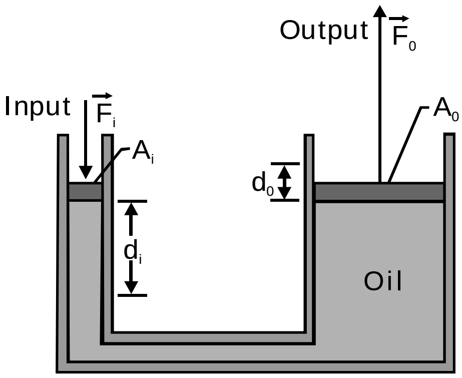


Figure (a)

Equation shows that the output force F_0 on the load must be greater than the input force F_i if $A_0 > A_p$ as is the case in figure.

If we move the input piston downward a distance d_i, the output piston moves upward a distance d_0, such that the same volume V of the incompressible liquid is displaced at both pistons. Then, $V = A_i d_i = A_0 d_0$

which we can write as

$$d_0 = d_i \frac{A_i}{A_0}$$

This shows that, if $A_0 > A_i$ (as in Figure), the output piston moves a smaller distance than the input piston moves.

From Eqs. we can write the output work as

$$W = F_0 d_0 = \left(F_i \frac{A_i}{A_0}\right)\left(d_i \frac{A_i}{A_0}\right) = F_i d_i$$

which shows that the work W done on the input piston by the applied force is equal to the work W done by the output piston in lifting the load placed on it.

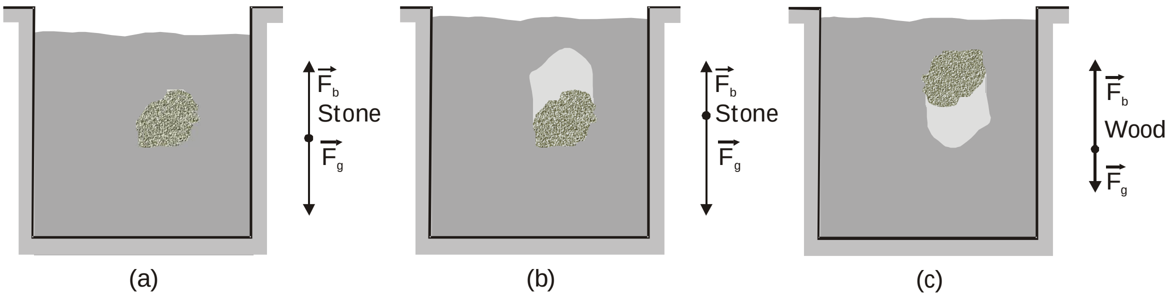
The advantage of a hydraulic lever is this :

With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

The product of force and distance remains unchanged so that the same work is done. However, there is often tremendous advantage in being able to exert the larger force. Most of us, for example, cannot lift an automobile directly but can with a hydraulic jack, even though we have to pump the handle farther than the automobile rises and in a series of small strokes.

Archimedes' Principle

Figure shows a student in a swimming pool, manipulating a very thin plastic sack (of negligible mass) that is filled with water. She finds that the sack and its contained water are in static equilibrium, tending neither to rise nor to sink. The downward gravitational force $\vec{F}_g$ on the contained water must be balanced by a net upward force from the water surrounding the sack.



This net upward force is a **buoyant force** $\vec{F}_b$. It exists because the pressure in the surrounding water increases with depth below the surface. Thus, the pressure near the bottom of the sack is greater than the pressure near the top, which means the forces on the sack due to this pressure are greater in magnitude near the bottom of the sack than near the top. Some of the forces are represented in figure (a) where the space occupied by the sack has been left empty. Note that the force vectors drawn near the bottom of that space (with upward components) have longer lengths than those drawn near the top of the sack (with downward components). If we vectorially add all the forces on the sack from the water, the horizontal components cancel and the vertical components add to yield the upward buoyant force $\vec{F}_b$ on the sack. (Force $\vec{F}_b$ is shown to the right of the pool in Figure (a) Because the sack of water is in static equilibrium, the magnitude $\vec{F}_b$ is equal to the magnitude $m_f g$ of the gravitational force $\vec{F}_g$ on the sack of water: $F_b = m_f g$. (Subscript f refers to fluid, here the water.) In words, the magnitude of the buoyant force is equal to the weight of the water in the sack.

In Fig. (b), we have replaced the sack of water with a stone that exactly fills the hole in Fig. (a). The stone is said to displace the water, meaning that the stone occupies space that would otherwise be occupied by water. We have changed nothing about the shape of the hole, so the forces at the hole's surface must be the same as when the water-filled sack was in place. Thus, the same upward buoyant force that acted on the water-filled sack now acts on the stone; that is, the magnitude $\vec{F}_b$ of the buoyant force is equal to $m_f g$, the weight of the water displaced by the stone.

Unlike the water-filled sack, the stone is not in static equilibrium. The downward gravitational force $\vec{F}_g$ on the stone is greater in magnitude than the upward buoyant force, as is shown in the free-body diagram in Fig. (*b*). The stone thus accelerates downward, sinking to the bottom of the pool. Let us next exactly fill the hole in Fig. (a) with a block of lightweight wood, as in Fig. (c). Again, nothing has changed about the forces at the hole's surface, so the magnitude $\vec{F}_b$ of the buoyant force is still equal to $m_f g$, the weight of the displaced water. Like the stone, the block is not in static equilibrium. However, this time the gravitational force $\vec{F}_g$ is lesser in magnitude than the buoyant force (as shown to the right of the pool), and so the block accelerates upward, rising to the top surface of the water. Our results with the sack, stone, and block apply to all fluids and are summarized in **Archimedes' principle:**

When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body. The buoyant force on a body in a fluid has the magnitude

$$F_b = m_f g \text{ (buoyant force)}$$

where m_f is the mass of the fluid that is displaced by the body.

Floating

When we release a block of lightweight wood just above the water in a pool, the block moves into the water because the gravitational force on it pulls it downward.

As the block displaces more and more water, the magnitude F_b of the upward buoyant force acting on it increases. Eventually, F_b is large enough to equal the magnitude F_g of the downward gravitational force on the block, and the block comes to rest. The block is then in static equilibrium and is said to be floating in the water. In general, When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

We can write this statement as

$$F_b = F_g \text{ (Floating)}$$

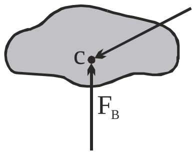
From Eq. we know that $F_b = m_f g$. Thus, When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f\,g$ of the fluid that has been displaced by the body.

We can write this statement as

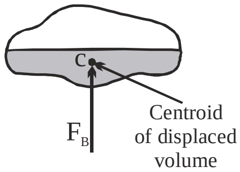
$$F_g = m_f\,g$$

In other words, a floating body displaces its own weight of fluid.

The location of the line of action of the buoyant force can be determined by adding torques of the forces due to pressure forces, with respect to some convenient axis. The buoyant force must pass through the center of mass of the displaced volume, as shown in Fig. (c), as it was in translational and rotational equilibrium. The point through which the buoyant force acts is called the center of buoyancy.



(c)



(d)

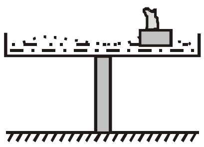
These same results apply to floating bodies which are only partially submerged, as shown in Fig.(d), if the density of the fluid above the liquid surface is very small compared with the liquid in which the body floats. Since the fluid above the surface is usually air, for practial purposes this condition is satisfied.

In the above discussion, the fluid is assumed to have a constant density. If a body is immersed in a fluid in which density varies with depth, such as having multiple layers of fluid, the magnitude of the buoyant force remains equal to the weight of the displaced fluid and the buoyant force passes through the center of mass of the displaced volume.

Q. A wooden block floats vertically in a glass filled with water. How will the level of the water in the glass change if the block is kept in a horizontal position ?

Sol. The level of the water will not change because the quantity of water displaced will remain the same.

Q. A vessel filled with water is placed exactly in middle of a thin wall(fig.). Will the system topple if a small wooden boat carrying some weight is floated in the vessel?



Sol. The system will not topple, since according to Pascal's law the pressure on the bottom of the vessel will be the same everywhere thus the body will still remain in rotational equilibrium

Ex. A homogeneous piece of ice floats in a glass filled with water. How will the level of the water in the glass change when the ice melts ?

Sol. Since the piece of ice floats, the weight of the water displaced by it is equal to the weight of the ice itself or the weight of the water it produces upon melting. For this reason the water formed by the piece of ice will occupy a volume equal to that of the submerged portion, and the level of the water will not change.

Q. A piece of ice is floating in a tub filled with water. How will the level of the water in the tub change when the ice melts ? Consider the following cases :

(1) a stone is frozen in the ice

(2) the ice contains an air bubble

Sol. (1) The volume of the submerged portion of the piece with the stone is greater than the sum of the volumes of the stone and the water produced by the melting ice. Therefore, the level of the water in the glass will drop.

(2) The weight of the displaced water is equal to that of the ice (the weight of the air in the bubble may be neglected). For this reason, as in conceptual eg., the level of the water will not change.

Example :

A vessel with a body floating in it is kept in elevator accelerating downwards with acceleration a such that a < g. Will the body rise or sink further in the vessel?

Sol. The force of bouyancy on the body can be written as $F = \rho V_2(g - a)$, where V_2 is the volume of the submerged portion of the body in the lift. As pressure at a point h below the surface will become $\rho(g - a)h$ instead of ρgh. Applying the Newton's second Law, remembering that the body was accelerating upwards at a.

$$Mg - \rho V_2 (g - a) = Ma$$

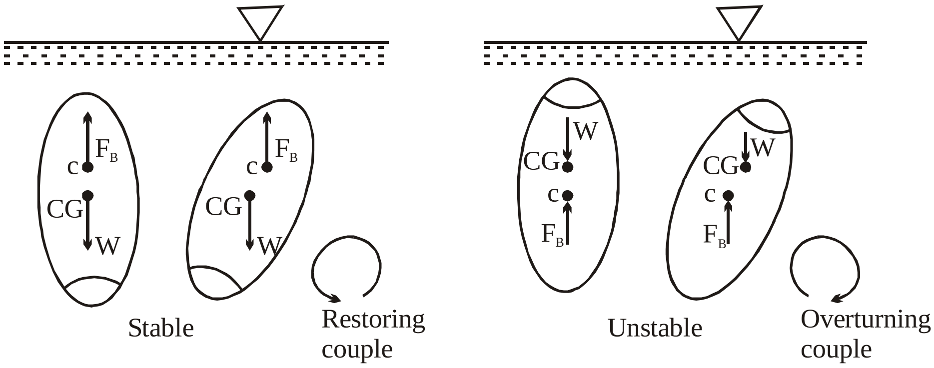
Hence, $V_2 = \frac{M}{\rho}$ thus $V_2 = V$, as in a stationary vessel, $V = \frac{M}{\rho}$. Thus the body does not rise to the surface.

Stability :

The center of buoyancy and center of gravity do not necessarily coincide so the floating or submerged body may not be in stable equilibrium. A small rotation can cause the buoyant force to produce either a restoring or overturning torque. For example, for the completely submerged body shown in Fig., which has a center of gravity below the center of buoyancy, a rotation from its equilibrium position will create a restoring torque by the buoyant force, F_B, which causes the body to rotate back to its original position. Thus, if the center of gravity falls below the center of buoyancy, the body is stable.

However, as shown in Fig., if the center of gravity of the completely submerged body is above the center of buoyancy, the resulting torque formed by the weight and the buoyant force will cause the body to overturn and move to a new equilibrium position. Thus, a completely submerged body with its center of gravity above its center of buoyancy is in an unstable equilibrium position.

For floating bodies the stability problem is more complicated, since as the body rotates the location of the center of buoyancy may change.



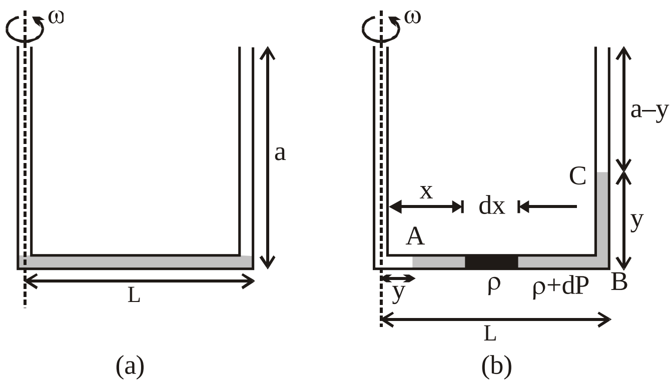
Concept : Length of a horizontal arm of a U-tube is L and ends of both the vertical arms are open to atmospheric pressure P_0. A liquid of density ρ is poured in the tube such that liquid just fills the horizontal part of the tube as shown in figure. Now one end of the opened ends is sealed and the tube is then rotated about a vertical axis passing through the other vertical arm with angular speed ω. If length of each vertical arm is a and in the sealed end liquid rises to a height y, find pressure in the sealed tube during rotation.

The pressure difference across an element of width dx, which is given as

$$dP = dx\rho\omega^2 x$$

Now integrating from A to B, we get

$$P_B - P_A = \int_y^L \rho\omega^2 x\,dx$$



Thus pressure at point C an be given as

$$P_C = P_B - y\rho g$$

and at point A, pressure is atmospheric, thus we have

$$P_C = \frac{\rho\omega^2}{2}\left(L^2 - y^2\right) + P_A - y\rho g$$

Q. A hemisphere of radius R is just submerged is just sinking in water of density ρ. Find the

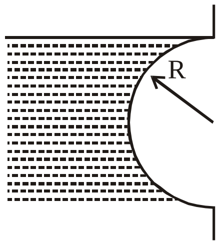
(a) horizontal thrust.

(b) vertical thrust.

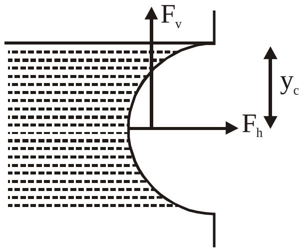
(c) total hydrostatic force.

(d) angle of orientation of total hydrostatic force acting on the hemisphere.

Do not count atmospheric pressure



Sol. (a) Let the horizontal and vertical thrusts on the hemisphere be F_h and F_V respectively



We know that

$$F_h = \rho g y_c A_y$$

where $y_c = R$

and $A_v = \pi R^2$

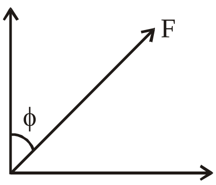
This gives $F_h = \rho g \pi R^3$ (right)

(b) Similarly using the formula $F_v = \rho V g$

Where V = volume of the hemisphere $= \frac{2}{3}\pi R^3$,

we have $= F_V = \frac{2}{3}\rho g \pi R^3$ (up)

(c) Hence the net hydrostatic force on the hemisphere is



$$F = \sqrt{F_h^2 + F_v^2}$$

$$F = \sqrt{(\rho g \pi R^3)^2 + \left(\frac{2}{3}\rho g \pi R^3\right)^2}$$

$$= \frac{\sqrt{13}}{3}\rho g \pi R^3$$

(d) The angle of orientation of the force F is

$$\phi = \tan^{-1}\frac{F_h}{F_v}$$

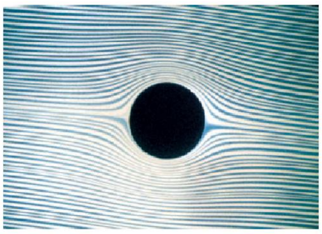
$$= \tan^{-1}\frac{\rho g \pi R^3}{\frac{2}{3}\rho g \pi R^3} = \tan^{-1}\frac{3}{2}$$

FLUID DYMANICS & VISCOSITY

Ideal Fluids in Motion

The motion of real fluids is very complicated and not yet fully understood. Instead, we shall discuss the motion of an **ideal fluid,** which is simpler to handle mathematically and yet provides useful results. Here are four assumptions that we make about our ideal fluid; they all are concerned with flow:

1. **Steady flow** In steady (or laminar) flow, the velocity of the moving fluid at any fixed point does not change with time, either in magnitude or in direction. The gentle flow of water near the center of a quiet stream is steady; the flow in a chain of rapids is not. Figure shows a transition from steady flow to nonsteady (or, nonlaminar or, turbulent) flow for a rising stream of smoke. The speed of the smoke particles increases as they rise and, at a certain critical speed, the flow changes from steady to nonsteady.



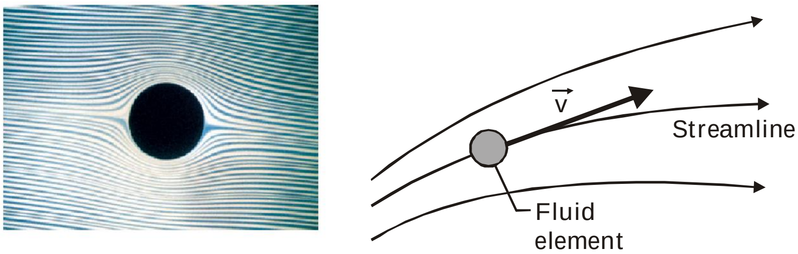
2. **Incompressible flow** We assume, as for fluids at rest, that our ideal fluid is incompressible; that is, its density has a constant, uniform value.

3. **Nonviscous flow** Roughly speaking, the viscosity of a fluid is a measure of how resistive the fluid is to flow. For example, thick honey is more resistive to flow than water, and so honey is said to be more viscous than water. Viscosity is the fluid analog of friction between solids; both are mechanisms by which the kinetic energy of moving objects can be transferred to thermal energy. In the absence of friction, a block could glide at constant speed along a horizontal surface. In the same way, an object moving through a nonviscous fluid would experience no viscous drag force-that is, no resistive force due to viscosity; it could move at constant speed through the fluid.

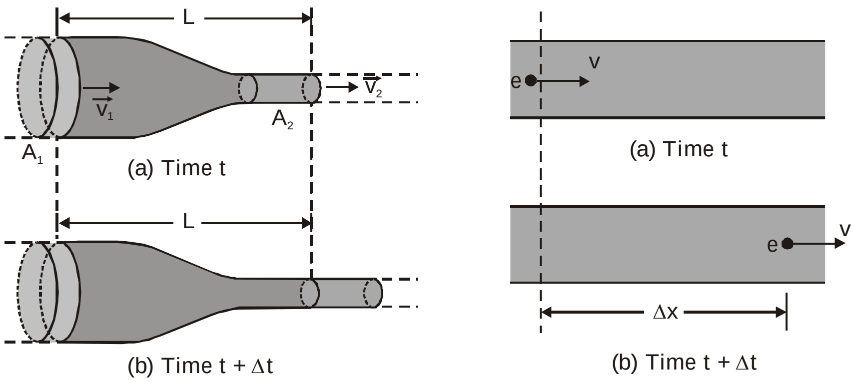
4. **Irrotational flow** : Although it need not concern us further, we also assume that the flow is irrotational. To test for this property, let a tiny grain of dust move with the fluid. Although this test body may (or may not) move in a circular path, in irrotational flow the test body will not rotate about an axis through its own center of mass. For a loose analogy, the motion of a Ferris wheel is rotational; that of its passengers is irrotational. That the velocity of a particle is always tangent to the path taken by the particle. Here the particle is the fluid element, and its velocity is $\vec{v}$ always tangent to a streamline (Figure). For this reason, two streamlines can never intersect; if they did, then an element arriving at their intersection would have two different velocities simultaneously an impossibility.

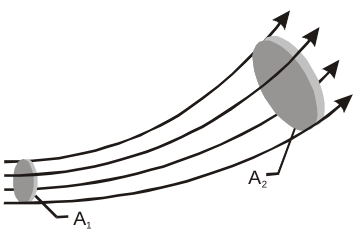
The Equation of Continuity

You may have noticed that you can increase the speed of the water emerging from a garden hose by partially closing the hose opening with your thumb. Apparently the speed v of the water depends on the cross-sectional area A through which the water flows.



Here we wish to derive an expression that relates v and A for the steady flow of an ideal fluid through a tube with varying cross section, like that in Figure. The flow there is toward the right, and the tube segment shown (part of a longer tube) has length L. The fluid has speeds v_1 at the left end of the segment and v_2 at the right end. The tube has cross-sectional areas A_1 at the left end and A_2 at the right end. Suppose that in a time interval Δt a volume ΔV of fluid enters the tube segment at its left end. Then, because the fluid is incompressible, an identical volume ΔV must emerge from the right end of the segment. We can use this common volume ΔV to relate the speeds and areas. To do so, we first consider Fig., which shows a side view of a tube of uniform cross-sectional area A. In Fig.(a), a fluid element e is about to pass through the dashed line drawn across the tube width.The element's speed is v, so during a time interval Δt, the element moves along the tube a distance $\Delta x = v \cdot \Delta t$.The volume ΔV of fluid that has passed through the dashed line in that time interval Δt is





$$\Delta V = A\,\Delta x = Av\,\Delta t.$$

Applying Eq. to both the left and right ends of the tube segment in Fig., we have

$$\Delta V = A_1 v_1\,\Delta t = A_2 v_2\,\Delta t$$

Or, $$A_1 v_1 = A_2 v_2 \text{ (equation of continuity)}$$

This relation between speed and cross-sectional area is called the **equation of continuity** for the flow of an ideal fluid. It tells us that the flow speed increases when we decrease the cross-sectional area through which the fluid flows (as when we partially close off a garden hose with a thumb). Equation applies not only to an actual tube but also to any so-called tube of flow, or imaginary tube whose boundary consists of streamlines. Such a tube acts like a real tube because no fluid element can cross a streamline; thus, all the fluid within a tube of flow must remain within its boundary. Figure shows a tube of flow in which the cross-sectional area increases from area A_1 to area A_2 along the flow direction. From Eq. we know that, with the increase in area, the speed must decrease, as is indicated by the greater spacing between streamlines at the right in Fig. . Similarly, you can see that in Fig. the speed of the flow is greatest just above and just below the cylinder. We can rewrite Eq. as

$R_V = A_v =$ a constant (volume flow rate, equation of continuity), in which R_V is the **volume flow rate** of the fluid (volume past a given point per unit time). Its SI unit is the cubic meter per second (m^3/s). If the density ρ of the fluid is uniform, we can multiply Eq. by that density to get the **mass flow rate** R_m (mass per unit time):

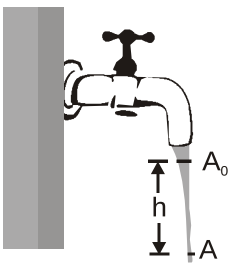
$$R_m = \rho R_V = A_v = \text{a constant (mass flow rate).}$$

The SI unit of mass flow rate is the kilogram per second (kg/s). Equation says that the mass that flows into the tube segment of Fig. each second must be equal to the mass that flows out of that segment each second.

Sample Problem

Figure shows how the stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are $A_0 = 1.2\ cm^2$ and $A = 0.35\ cm^2$. The two levels are separated by a vertical distance h = 45 mm.What is the volume flow rate from the tap?

The volume flow rate through the higher cross section must be the same as that through the lower cross section.



where v_0 and v are the water speeds at the levels corresponding to A_0 and A. From Eq. we can also write, because the water is falling freely with acceleration g,

$$v^2 = v_0^2 - 2gh.$$

Eliminating v between Eqs. and solving for v_0, we obtain

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$v_0 = \sqrt{\frac{(2)\times(9.8\,m/s^2)(0.045\,m)(0.35\,cm^2)^2}{(1.2\,cm^2)^2 - (0.35\,cm^2)^2}}$$

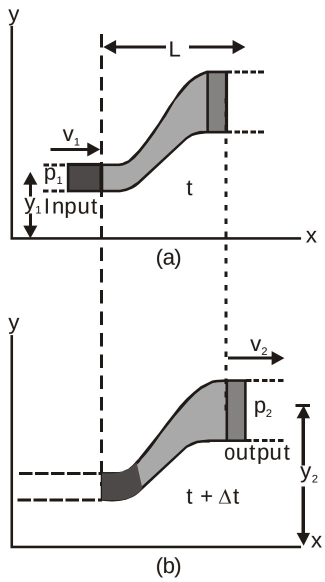
$$v_0 = 0.286 \text{ m/s} = 28.6 \text{ cm/s}.$$

From Eq. , the volume flow rate R_V is then

$$R_V = A_0 v_0 = (1.2 \text{ cm}^2)(28.6 \text{ cm/s})$$
$$= 34 \text{ cm}^3\text{/s. Ans.]}$$

Bernoulli's Equation

Figure represents a tube through which an ideal fluid is flowing at a steady rate. In a time interval Δt, suppose that a volume of fluid ΔV, in Fig. , enters the tube at the left (or, input) end and an identical volume, in Fig. , emerges at the right (or, output) end. The emerging volume must be the same as the entering volume because the fluid is incompressible, with an assumed constant density ρ.



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Let y_1, v_1, and p_1 be the elevation, speed, and pressure of the fluid entering at the left, and y_2, v_2, and p_2 be the corresponding quantities for the fluid emerging at the right. By applying the principle of conservation of energy to the fluid, we shall show that these quantities are related by

In general, the term $\frac{1}{2}\rho v^2$ is called the fluid's **kinetic energy density** (kinetic energy per unit volume).We can also write Eq. as

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant (Bernoulli's equation)}.$$

Equations are equivalent forms of **Bernoulli's equation,** after Daniel Bernoulli, who studied fluid flow in the 1700s.* Like the equation of continuity Eq., Bernoulli's equation is not a new principle but simply the reformulation of a familiar principle in a form more suitable to fluid mechanics. As a check, let us apply Bernoulli's equation to fluids at rest, by putting $v_1 = v_2 = 0$ in Eq. The result is

$$p_2 = p_1 + \rho g(y_1 - y_2)$$

Which is equation.

A major prediction of Bernoulli's equation emerges if we take y to be a constant (y = 0, say) so that the fluid does not change elevation as it flows. Equation then becomes

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

Which tells us that :

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

Put another way, where the streamlines are relatively close together (where the velocity is relatively great), the pressure is relatively low, and conversely. The link between a change in speed and a change in pressure makes sense if you consider a fluid element. When the element nears a narrow region, the higher pressure behind it accelerates it so that it then has a greater speed in the narrow region. When it nears a wide region, the higher pressure ahead of it decelerates it so that it then has a lesser speed in the wide region. Bernoulli's equation is strictly valid only to the extent that the fluid is ideal. If viscous forces are present, thermal energy will be involved. We take no account of this in the derivation that follows.

Proof of Bernoulli's Equation

Let us take as our system the entire volume of the (ideal) fluid shown in Fig. .We shall apply the principle of conservation of energy to this system as it moves from its initial state (Fig. (a)) to its final state (Fig. (b)). The fluid lying between the two vertical planes separated by a distance L in Fig. does not change its properties during this process; we need be concerned only with changes that take place at the input and output ends. First, we apply energy conservation in the form of the work–kinetic energy theorem,

$$W = \Delta K,$$

which tells us that the change in the kinetic energy of our system must equal the net work done on the system. The change in kinetic energy results from the change in speed between the ends of the tube and is

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 = \frac{1}{2}\rho \Delta V(v_2^2 - v_1^2)$$

in which Δm (= $\rho\,\Delta V$) is the mass of the fluid that enters at the input end and leaves at the output end during a small time interval Δt.

The work done on the system arises from two sources.The work W_g done by the gravitational force $(\Delta m\,\vec{g})$ on the fluid of mass Δm during the vertical lift of the mass from the input level to the output level is

$$W_g = \Delta m\, g(y_2 - y_1)$$

$$= -\rho\, g \Delta V(y_2 - y_1)$$

This work is negative because the upward displacement and the downward gravitational force have opposite directions.

Work must also be done on the system (at the input end) to push the entering fluid into the tube and by the system (at the output end) to push forward the fluid that is located ahead of the emerging fluid. In general, the work done by a force of magnitude F, acting on a fluid sample contained in a tube of area A to move the fluid through a distance Δx, is

$$F\Delta x = (pA)(\Delta x) = p(A\Delta x) = p\Delta v$$

The work done on the system is then $p_1\,\Delta V$, and the work done by the system is $-p_2\,\Delta V$. Their sum Wp is

$$Wp = -p_2\,\Delta V - (-p_1\,\Delta V)$$

$$= -(p_2 - p_1)\,\Delta V.$$

The work–kinetic energy theorem of Eq. now becomes

$$W = Wg + Wp = \Delta K.$$

Substituting from Eqs. yields

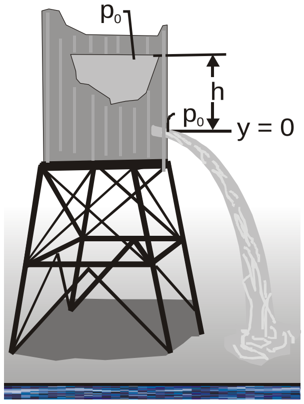
$$-\rho g \Delta V(y_2 - y_1) - \Delta V(p_2 - p_1) = \frac{1}{2}\rho \Delta v(v_2^2 - v_1^2)$$

This, after a slight rearrangement, matches Eq. , which we set out to prove.

Sample Problem : In the old West, a desperado fires a bullet into an open water tank (Fig.), creating a hole a distance h below the water surface.What is the speed v of the water exiting the tank?

Sol. From Eq. $R_V = av = Av_0$ and thus

$$v_0 = \frac{a}{A}v$$



Because a <<A, we see that v_0 <<v. To apply Bernoulli's equation, we take the level of the hole as our reference level for measuring elevations (and thus gravitational potential energy). Noting that the pressure at the top of the tank and at the bullet hole is the atmospheric pressure p_0 (because both places are exposed to the atmosphere), we write Eq. as

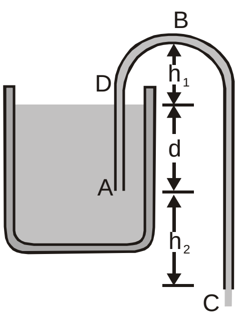
$$p_0 + \frac{1}{2}\rho v_0^2 + \rho gh = p_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

(Here the top of the tank is represented by the left side of the equation and the hole by the right side. The zero on the right indicates that the hole is at our reference level.) Before we solve Eq. for v, we can use our result that $v_0 << v$ to simplify it:We assume that v^2_0 and thus the term $\frac{1}{2}\rho v_0^2$ in Eq. , is negligible relative to the other terms, and we drop it. Solving the remaining equation for v then yields

$v = \sqrt{2gh}$ Ans.

This is the same speed that an object would have when falling a height h from rest.This is because the work done by atmospheric pressure is cancelling out at open surface and the hole.

Sample Problem : shows a siphon, which is a device for removing liquid from a container. Tube ABC must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at A. The liquid has density 1000 kg/m^3 and negligible viscosity. The distances shown are h_1 = 25 cm, d =12 cm, and h_2 = 40 cm. (a) With what speed does the liquid emerge from the tube at C? (b) If the atmospheric pressure is 1.0×10^5 Pa, what is the pressure in the liquid at the topmost point B? (c) Theoretically, what is the greatest possible height h_1 that a siphon can lift water?



Solution: You may have used siphon and you may recollect that lower the exit point of the fluid is, faster the fluid flows out.

We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2}$$

$$\approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{air}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_c = \sqrt{2(9.8\,m/s^2)(0.40m + 0.12m)} = 3.2\ m/s$$

The result confirms our experience.

We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{air}$, Bernoulli's equation becomes

$$p_B = p_C + \rho g(h_C - h_g) = p_{air} - \rho g(h_1 + h_2 + d)$$

$= 1.0 \times 10^5$ Pa $(1.0 \times 10^3$ kg/m $)(9.8$ m/s^2 $)(0.25$ m $+ 0.40$ m $+ 0.12$ m$)$

$= 9.2 \times 10$ Pa.

Since $p_B \geq 0$, we must let $p_{air} - \rho g(h_1 + d + h_2) \geq 0$, which yields

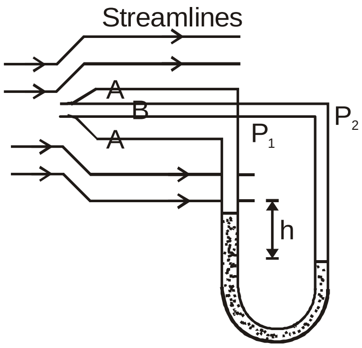
$$h_1 \leq h_{1max} = \frac{p_{air}}{\rho} - d - h_2 \leq \frac{p_{air}}{\rho} = 10.3m$$

Q. In the example just above consider a small hole in the hose at as indicated. When the siphon is used, will water leak out of the hose, or will air leak into the hose and thus causing the siphon to stop ?

Sol. Whether air will leak out of the hose depends on whether the pressure within the hose is less than or greater than atmospheric.

Since the hose diameter is constant, it follows from the continuity equation (AV = constant) that the water velocity in the hose is constant throughout. Also the pressure at the end of the hose is atmospheric so the pressure at all the points above it can be shown to be less than the atmospheric pressure. Thus air will leak into the hose pipe stoping it.

Example: Fig. shows a device called pitot's tube. It measures the velocity of moving fluids. Determine the velocity of the fluid in terms of the density ρ, the density of the fluid in manometer (U-tube) σ and the height 'h'.



figure

Solution.

The difference in the two tubes is that liquid will flow into the tube B with full Kinetic Energy while it will just pass over the tube A without directly entering into it.

This problem is based on the use of Bernoulli's principle, on two different situations.

The fluid inside the right tube must be at rest as the fluid exactly at the end is in contact with the fluid in pitot tube, which is at rest.

The velocity υ_1 is the fluid velocity υ_f. The velocity υ_2 of the fluid at point B is zero and the pressure in the right arm is P_2 (called stagnation pressure).

Thus using Bernoulli's principle $\quad P_1 + \frac{1}{2}\,p\upsilon_1^2 = P_2 + \frac{1}{2}\,p\upsilon_2^2$

We get $\quad P_2 = P_1 + \frac{1}{2}\,p\upsilon_f^2$

On the other hand the openings at point A is not along the flow lines, so we dont need to use Bernoulli's eqn. We can simply say that the pressure just outside the opening is same as that within the pitot tube.

Therefore the pressure at the left arm of the manometer is same as the fluid pressre P_f, i.e., $P_1 = P_f$.

Also $\quad P_2 = P_1 + (\rho - \sigma)\,gh \quad$ ....(3)

Generally σ << ρ, so it is ignored.

Thus $\quad P_2 = P_f + \rho gh \quad$ ....(4)

From eqns. (4) and (3), $\quad \frac{1}{2}\,\sigma\upsilon_f^2 = \rho gh$

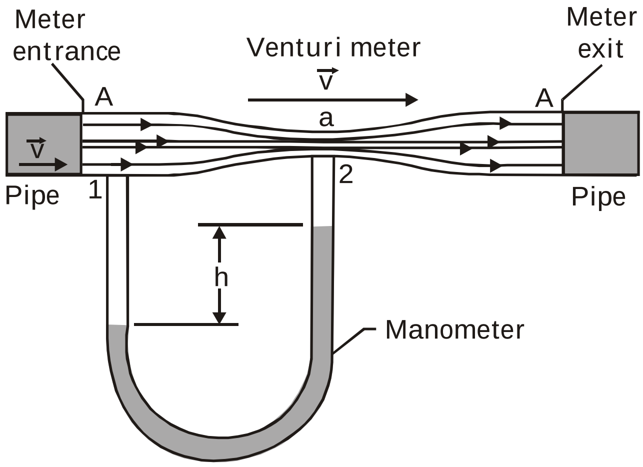
or $\quad \upsilon_f = \sqrt{\frac{2\rho gh}{\sigma}}$

Thus we can see that we have measured the fluid velocity as this was the only difference between the two tubes leading to the pressure difference between the tubes.

Conceptual example:A venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections A venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections of the pipe (Fig.); the cross-sectional area A of the entrance and exit of the meter matches the pipe's cross-sectional area. Between the entrance and exit, the fluid flows from the pipe with speed V and then through a narrow "throat" of cross-sectional area a with speed v. A manometer connects the wider portion of the meter to the narrower portion. The change in the fluid's speed is accompanied by a change Δp in the fluid's pressure, which causes a height difference h of the liquid in the two arms of the manometer. (Here Δp means pressure in the throat minus pressure in the pipe.) (a) By applying Bernoulli's equation and the equation of continuity to points 1 and 2 in Fig. , show that

$$v = \sqrt{\frac{2a^2\Delta p}{\rho(a^2 - A^2)}}$$

where r is the density of the fluid. (b) Suppose that the fluid is fresh water, that the cross-sectional areas are 64 cm^2 in the pipe and 32 cm^2 in the throat, and that the pressure is 55 kPa in the pipe and 41 kPa in the throat.What is the rate of water flow in cubic meters per second?



(a) The continuity equation yields Av = aV, and Bernoulli's equation yields $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho V^2$ where $\Delta p = p_1 - p_2$. The first equation gives V = (A/a)v. We use this to substitute for V in the second equation, and obtain $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho (A/a)^2 v^2$ We solve for v. The result is

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{2a^2\Delta p}{\rho(A^2 - a^2)}}$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32\times10^{-4}m^2)^2(55\times10^3 Pa - 41\times10^3 Pa)}{(1000kg/m^3)((64\times10^{-4}m^2)^2 - (32\times10^{-4}m^2)^2)}} = 30.06 \text{ m/s}$$

Consequently, the flow rate is

$$Av = (64\times10^{-4}m^2)(3.06 \text{ m/s}) = 2.0\times10^{-2} m^3/s.$$

SURFACE TENSION & VISCOSITY

EXPLANATION OF SOME OBSERVED PHENOMENA

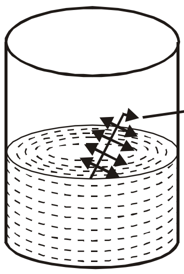
1. Lead balls are spherical in shape.
2. Rain drops and a globule of mercury placed on glass plate are spherical.
3. Hair of a shaving brush/painting brush, when dipped in water spread out, but as soon as it is taken out. Its hair stick together.
4. A greased needle placed gently on the free surface of water in a beaker does not sink.
5. Similarly, insects can walk on the free surface of water without drowning.
6. Bits of Camphor gum move irregularly when placed on water surface.

SURFACE TENSION

Surface Tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface Tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in fig. i.e. Surface tension.

$$(T) = \frac{\text{Total force on either of the imginary line (F)}}{\text{Length of the line } (l)}$$



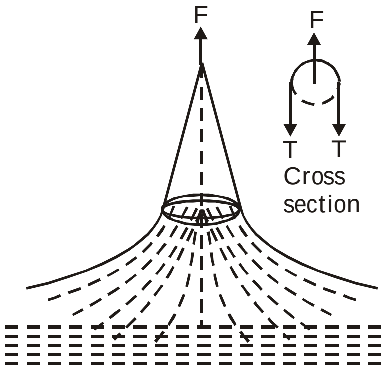
Unit of Surface Tension

In C. G. S. system the unit of surface tension is dyne/cm (dyne cm^{-1}) and SI system its units is Nm^{-1}

Solved Examples

Ex. A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water ? (g = 980 cm/s^2).

Sol.



The ring is in contact with water along its inner and outer circumference ; so when pulled out the total force on it due to surface tension will be

$$F = T(2\pi\, r_1 + 2\pi r_2)$$

So, $T = \dfrac{mg}{2\pi(r_1 + r_2)}$ $[\because F = mg]$

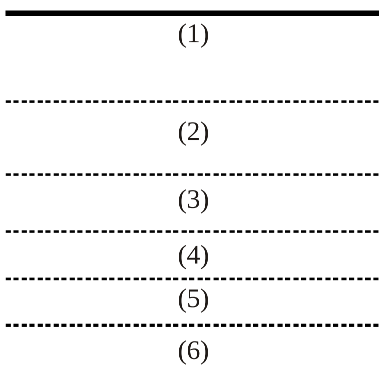
i.e., $T = \dfrac{3.97 \times 980}{3.14 \times (8.5+8.7)}$ = 72.13 dyne/cm

Surface energy

The course of reasoning given below is usually followed to prove that the molecules of the surface layer of a liquid have surplus potential energy. A molecule inside the liquid is acted upon by the forces of attraction from the other molecules which compensate each other on the average. If a molecule is singled out on the surface, the resulting force of attraction from the other molecule is directed into the liquid. For this reason the molecule tends to move into the liquid, and definite work should be done to bring it to the surface. Therefore, each molecule of the surface layer has excess potential energy equal to this work. The average force that acts on any molecule from the side of all the others, however, is always equal to zero if the liquid is in equilibrium. This is why the work done to move the liquid from a depth to the surface should also be zero. What is the origin, in this case, of the surface energy ?

Sol. The forces of attraction acting on a molecule in the surface layer from all the other molecules produce a resultant directed downward. The closest neighbours, however, exert a force of repulsion on the molecule which is therefore in equilibrium.

Owing to the forces of attraction and repulsion, the density of the liquid is smaller in the surface layer than inside. Indeed, molecule 1 (figure) is acted upon by the force of repulsion from molecule 2 and the forces of attraction from all the other molecules (3, 4, ...). Molecule 2 is acted upon by the forces of repulsion from 3 and 1 and the forces of attraction from the molecules in the deep layers. As a result, distance 1-2 should be greater than 2-3, etc.



This course of reasoning is quite approximate (thermal motion, etc. is disregarded), but nevertheless it gives a qualitatively correct result.

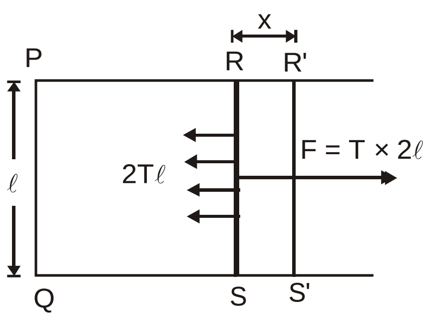
An increase in the surface of the liquid causes new sections of the rarefied surface layer to appear. Here work should be performed against the forces of attraction between the molecules. It is this work that constitutes the surface energy.

We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy. Unit of surface energy is erg cm^{-2} in C.G..S. system and Jm^{-2} in SI system. Dimensional formula of surface energy is $[ML^0T^{-2}]$ surface energy depends on number of surfaces e.g. a liquid drop is having one liquid air surface while bubble is having two liquid air surface.

Relation between surface tension and surface energy

Consider a rectangular frame PQRS of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution. then a soap film is produced in the frame PQRS in fig. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let l be the length of the arm RS, then the force acting on the arm RS towards the film is $F = T \times 2l$ [Since soap film has two surfaces, that is way the length is taken twice.]

$\therefore$ work done, $W = Fx = 2Tlx$



Increase in potential energy of the soap film.

$= EA = 2Elx$ = work done in increasing the area (ΔW)

where E = surface energy of the soap film per unit area.

According the law of conservation of energy, the work done must be equal to the increase in the potential energy.

$$\therefore \quad 2Tlx = 2Elx \quad \text{or} \quad T = E = \frac{\Delta W}{A}$$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

Ex. A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

Sol. If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only one surface, the initial surface area will be $4\pi R^2$ while final area is $n(4\pi r^2)$. So the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So energy expended in the process.

$$W = T\Delta S = 4\pi T\,[nr^2 - R^2] \qquad \text{... (1)}$$

Now since the total volume of n droplets is the same as that of initial drop, i.e.

$$\frac{4}{3}\pi R^3 = n[(4/3)\,\pi r^3] \quad \text{or} \quad r = R/n^{1/3} \qquad \text{... (2)}$$

Putting the value of r from equation (2) in (1)

$$W = 4\pi R^2 T\,(n)^{1/3} - 1$$

Ex. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by

$$\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

where T is the surface tension of water and J is the mechanical equivalent of heat.

Sol. Let n be the number of little droplets.

Since volume will remain constant, hence volume of n little droplets = volume of single drop

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad nr^3 = R^3$$

Decrease in surface area = $n \times 4\pi r^2 - 4\pi R^2$

$$\text{or} \quad \Delta A = 4\pi\,[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy evolved W = T × decrease in surface area = $T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$

Heat produced, $Q = \frac{W}{J} = \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$ But $Q = ms\,d\theta$

where m is the mass of big drop, s is the specific heat of water and $d\theta$ is the rise in temperature.

$$\therefore \quad \frac{4\pi TR^3}{J}\left[\frac{1}{r} - \frac{1}{R}\right] = \text{volume of big drop} \times \text{density of water} \times \text{sp. heat of water} \times d\theta$$

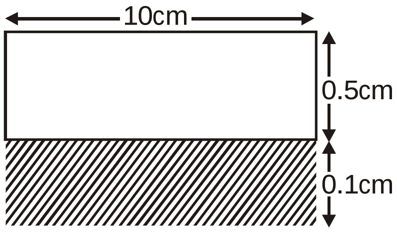
$$\text{or,} \quad \frac{4}{3}\pi R^3 \times 1 \times 1 \times d\theta = \frac{4\pi TR^3}{J}\left(\frac{1}{r} - \frac{1}{R}\right) \quad \text{or,} \quad d\theta = \frac{3T}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Ex. A film of water is formed between two straight parallel wires each 10cm long and at a separation 0.5 cm. Calculate the work required to increase 1mm distance between them. Surface tension of water 72×10^{-3} N/m.

Sol. Here the increase in area is shown by shaded portion in the figure.

Since this a water film, it has two surface, therefore

increase in area, $\Delta S = 2 \times 10 \times 0.1 = 2cm^2$



$\therefore$ Work required to be done

$W = \Delta S \times T$

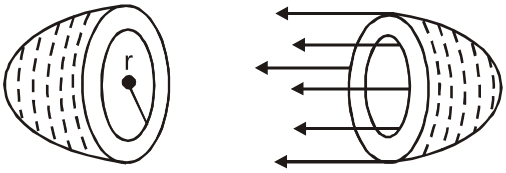
$= 2 \times 10^{-4} \times 72 \times 10^{-3}$

$= 144 \times 10^{-7}$ joule

$= 1.44 \times 10^{-5}$ joule

Excess pressure inside A liquid drop and a bubble

1. Inside a bubble : Consider a soap bubble of radius r. Let p be the pressure inside the bubble and p_a outside. The excess pressure = $p - p_a$. Imagine the bubble broken into two halves, and consider one half of it as shown in fig. Since there are two surface, inner and outer, so the force due to surface tension is



F = surface tension × length = T × 2 (circumference of the bubble) = T × 2 (2T πr) ... (1)

The excess pressure $(p - p_a)$ acts on a cross-sectional area πr^2, so the force due to excess pressure is

$\Rightarrow$ $F = (p - p_a)\pi r^2$ ... (2)

The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium , i.e. $(p - p_a)\pi r^2 = T \times 2(2\pi r)$

or $$(p - p_a) = \frac{4T}{r} = p_{excess}$$

above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

2. Inside the drop : In a drop, there is only one surface and hence excess pressure can be written as

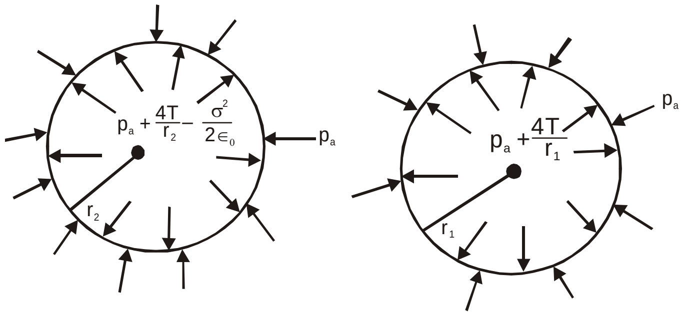
$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

3. Inside air bubble in a liquid :

$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

4. A charged bubble : If bubble is charged, it's radius increases. Bubble has pressure excess due to charge too. Initially pressure inside the bubble

$$= p_a + \frac{4T}{r_1}$$



for charge bubble, pressure inside = $p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\in_0}$, where σ surface is surface charge density.

Taking temperature remains constant, then from Boyle's law

$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^{\ 3} = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\in_0}\right]\frac{4}{3}\pi r_2^{\ 3}$$

From above expression the radius of charged drop may be calculated. It can conclude that radius of charged bubble increases, i.e. $r_2 > r_1$.

Solved example

Ex. A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76cm of mercury.

Sol. The total pressure inside the bubble at depth h_1 is (P atmospheric pressure)

$$= (P + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is = $(P + h_2\rho g) + \frac{2T}{r_2} = P_2$

Now, according to Boyl's Law

$$P_1V_1 = P_2V_2 \text{ where } V_1 = \frac{4}{3}\pi r_1^{\ 3}, \text{ and } V_2 = \frac{4}{3}\pi r_2^{\ 3}$$

Hence we get $\left[(P+h_1\rho g)+\frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^{\ 3} = \left[(P+h_2\rho g)+\frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^{\ 3}$

or, $\left[(P+h_1\rho g)+\frac{2T}{r_1}\right]r_1^{\ 3} = \left[(P+h_2\rho g)+\frac{2T}{r_2}\right]r_2^3$

Given that : h_1 = 100 cm, r_1 = 0.1 mm = 0.01 cm, r_2 = 0.126 cm, T = 567 dyne / cm, P = 76 cm of mercury. Substituting all the values, we get

$$h_2 = 9.48 \text{ cm.}$$

THE FORCE OF COHESION

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Example

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

FORCE OF ADHESION

The force of attraction between molecules of different substance is called adhesion.

Example

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

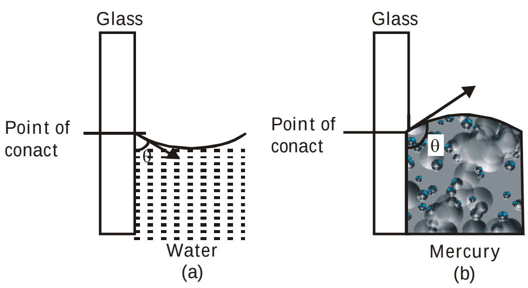
(iii) Large force of adhesion between cement and bricks helps us in construction work.

(iv) Due to force of adhesive, water wets the glass plate.

(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

ANGLE OF CONTACT

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids which wet the wall of the container (say in case of water and glass) have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which don't wet the walls of the container (say in case of mercury and glass) have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.

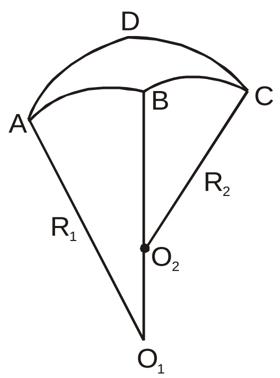


RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE.

Let us consider a small element ABCD (fig.) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature respectively. They are called the 'principal radii of curvature' of the surface. Let p be the excess pressure on the concave side.

then $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. If instead of a liquid surface, we have a liquid film, the above expression will be

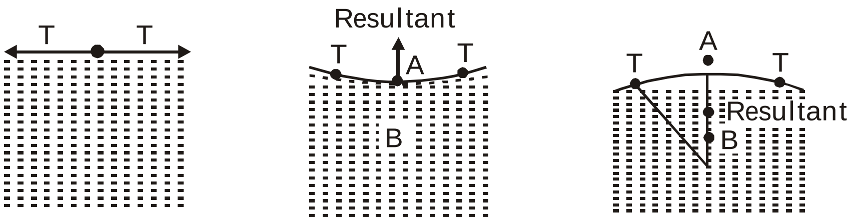
$p = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, because a film has two surface.



EXCESS OF PRESSURE INSIDE A CURVED SURFACE

1. Plane surface : If the surface of the liquid is plane [as shown in fig. (a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore on the liquid surface is normal.

2. Concave surface : If the surface is concave upward [as shown in fig. (b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \frac{2T}{r}$.



(a) Plane Surface (a) Concave surface (c) Convex surface

3. Convex surface : If the surface is convex [as shown fig.(c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension. Hence there is always in excess of pressure on concave side of a curved surface over that on the convex side.

$$p_B - p_A = \frac{2T}{r}$$

Solved Examples

Ex. A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the tube is 0.2m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Sol. Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. Then pressure just below the mensiscus in the wide and narrow tubes respectively are :

$$\left(P_1 - \frac{2T}{r_1}\right) \text{ and } \left(P_2 - \frac{2T}{r_2}\right) \qquad \left[\text{excess pressure} = \frac{2T}{r}\right]$$

Difference in these pressure = $\left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$

$\therefore$ True pressure difference = $P_1 - P_2$

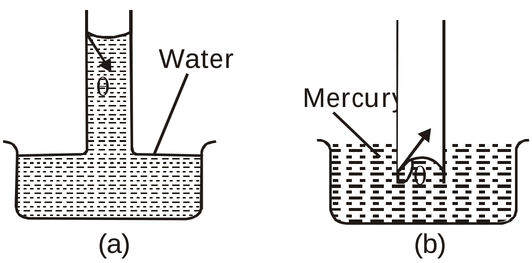
$$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}\left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$$

$$= 1.86 \times 10^3 = 1860 \text{ N/m}^2$$

Capillarity

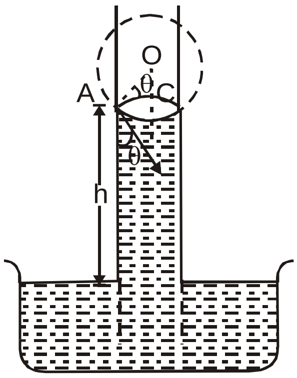
A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in figure (a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in figure (b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.



Particle applications of capillarity

1. The oil in a lamp rises in the wick by capillary action.
2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.
3. Sap and water rise upto the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture in the soil, capillaries must be broken up. This is done by ploughing and leveling the fields.
8. Bricks are porous and behave like capillaries.

Capillary rise (height of a liquid in a capillary tube) ascent formula



consider the liquid which wets the wall of the tube, forms a concave meniscus shown in figure. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure

$(p - p_a)$ is given by $(p - p_a) = \dfrac{2T}{R}$

Where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hρg. Thus in equilibrium state.

Excess pressure = Hydrostatic pressure or $\dfrac{2T}{R} = h\rho g$

Let θ be the angle of contact and r be the radius of the capillary tube shown in the fig.

From ΔOAC, $\dfrac{OC}{OA} = \cos\theta$ or $R = \dfrac{r}{\cos\theta}$ $\Rightarrow h = \dfrac{2T\cos\theta}{r\rho g}$

The expression is called Ascent formula.

Discussion.

(i) For liquids which wet the glass tube or capillary tube, angle of contact $\theta < 90^\circ$. Hence $\cos\theta$ = positive $\Rightarrow$ h = positive. It means that these liquids rise in the capillary tube. Hence, the liquids which wet capillary tube rise in the capillary tube. For example, water milk, kerosene oil, petrol etc.

Solved Examples

Ex. A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0°.

Sol. The surface tension of the liquid is

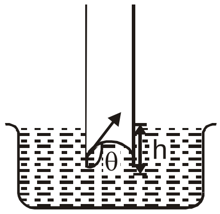
$$T = \frac{rh\rho g}{2} = \frac{(0.025\,cm)(3.0\,cm)(1.5\,gm/cm^3)(980\,cm/sec^2)}{2}$$

= 55 dyne/cm.

Hence excess pressure inside a spherical bubble

$$p = \frac{4T}{R} = \frac{4\times 55\,dyne/cm}{(0.5\,cm)} = 440\ dyne/cm^2.$$

(ii) For liquids which do not wet the glass tube or capillary tube, angle of contact $\theta > 90°$. Hence $\cos\theta$ = negative $\Rightarrow$ h = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

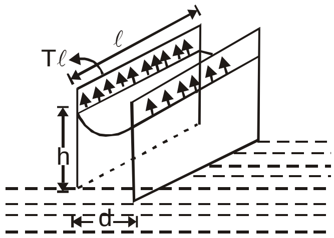


(iii) T, θ, ρ and g are constant and hence $h \propto \frac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. This is called Jurin's Law.

(iv) If two parallel plates with the spacing 'd' are placed in water reservoir, then height of rise

$\Rightarrow$ $3Tl = \rho l h d g$

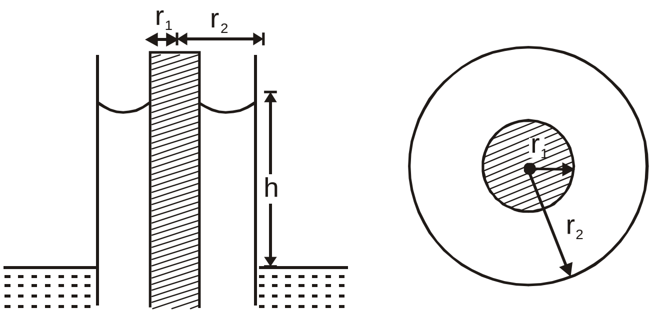
$$h = \frac{2T}{\rho d g}$$



(v) If two concentric tube of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise

$\Rightarrow$ $T\,[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\,\rho g$

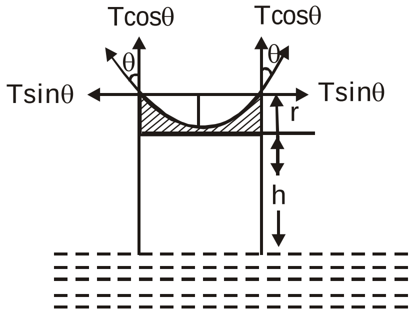
$$h = \frac{2T}{(r_2 - r_1)\rho g}$$



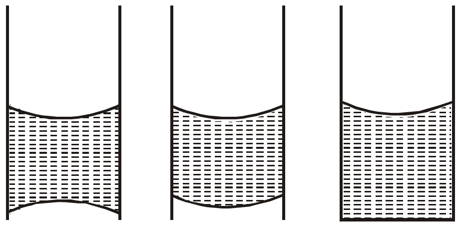
(vi) If weight of the liquid in the menscus is to be consider :

$$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\pi r^2 \times \pi r_1^{\,2} h]\,\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$



(vii) When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T / R_1$ where R_1 = radius of curvature of upper meniscus.



The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.

If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (fig. (a)).

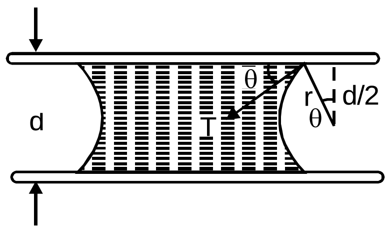
The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

If $p_1 < p_2$ i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig. b).

The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$.

If $p_1 = p_2$, then is no resulting pressure, then $p_1 - p_2 = \frac{2T}{R_2} = 0$ or $R_2 = \infty$ i.e. lower surface will be FLAT. (fig.c).

(viii) **Liquid between two plates :** When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence the pressure inside the film is less than the atmospheric pressure outside it by an amount

p given by $p = T\left(\frac{1}{r} + \frac{1}{r = \infty}\right)$ and we have, $p = \frac{T}{r}$.

If d be the distance between the two plates and θ the angle of contact for water and glass, then,

from the figure, $\cos\theta = \frac{\frac{1}{2}d}{r}$ or $\frac{1}{r} = \frac{2\cos\theta}{d}$.

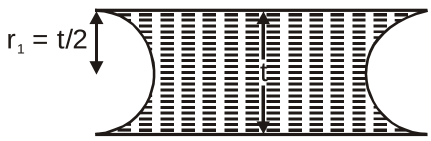
Substituting for $\frac{1}{r}$ in, we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero water and glass, i.e. cos θ = 1. Thus the upper plate is pressed downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$. If A be the area of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by F = resultant pressure × area = $\frac{2TA}{d}$. For very nearly plane surface, d will be very small and hence the pressing force F very large. Therefore it will be difficult to separate the two plates normally.

Solved Examples

Ex. A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in newton.

Sol. Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = 1/2$ and $r_2 = \infty$, then the force required to separate the two glass plates, between which a liquid film is enclosed (figure) is, $F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.



$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}} = 45\text{ N}$$

Ex. A glass plate of length 10cm, breadth 1.54 cm and thickness 0.20 cm weigh 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water 73 dyne per cm, g = 980 cm/sec^2.

Sol. Volume of the portion of the plate immersed in water is

$$10 \times \frac{1}{2}(1.54) \times 0.2 = 1.54 \text{ cm}^3.$$

Therefore, if the density of water is taken as 1, then upthrust

= wt. of the water displaced

= 1.54 × 1 × 980 = 1509.2 dynes.

Now, the total length of the plate in contact with the water surface is 2 (10 + 0.2) = 20.4 cm,

∴ downward pull upon the plate due to surface tension

= 20.4 × 73 = 1489.2 dynes

∴ resultant upthrust

= 1509.2 – 1489.2

$$= 20.0 \text{ dynes} = \frac{20}{980}$$

= 0.0204 gm. wt.

∴ apparent weight of the plate in water

= weight of the plate in air – resultant upthrust

= 8.2 – 0.0204 = 8.1796 gm Ans.

Ex. A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Give : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension os water 73 dyne/cm and g = 980 cm/sec^2.

Sol. Let l be the length of the tube inside water. The forces acting on the tube are :

(i) Upthrust of water acting upward

$$= \pi r^2 l \times 1 \times 980 = \frac{22}{7} \times (0.14)^2\, l \times 980 = 60.368\, l \text{ dyne.}$$

(ii) Weight of the system acting downward

= mg = 0.2 × 980 = 196 dyne.

(iii) Force of surface tension acting downward

$= 2\pi rT$

$$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24 \text{ dyne.}$$

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,

$60.368\, l = 196 + 64.24 = 260.24.$

$$\therefore \quad l = \frac{260.24}{60.368} = 4.31 \text{ cm.}$$

Ex. A glass U-tube is such that the diametre of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 nm^{-1}. Assume that the angle of contact between water and glass is 0°.

Sol. Suppose pressure at the points A, B, C and D be P_A, P_B, P_C and P_0 respectively.

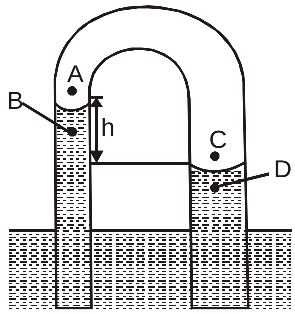
The pressure on the concave side of the liquid surface is greater than that on the other side by 2T/R.

And angle of contact θ is given to be 0°, hence R cos 0° = r or R = r

$\therefore \quad P_A = P_B + 2T / r_1 \quad \text{and} \quad P_C = P_0 + 2T / r_2$

where r_1 and r_2 are the radii of the two limbs

But $\quad P_A = P_C$



$$\therefore \quad P_B + \frac{2T}{r_1} = P_D + \frac{2T}{r_2}$$

$$\text{or} \quad P_D - P_B = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where h is the difference in water levels in the two limbs

$$\text{Now,} \quad h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Given that $\quad T = 0.07\ Nm^{-1}$, $\rho = 1000\ kgm^{-3}$

$$r_1 = \frac{3}{2}\ mm = \frac{3}{20}\ cm = \frac{3}{20 \times 100}\ m = 1.5 \times 10^{-3}\ m,\ r_2 = 3 \times 10^{-3}\ m$$

$$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right) m = 4.76 \times 10^{-3}\ m = 4.76\ mm$$

Ex. Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is $7.3 \times 10^{-2}\ Nm^{-1}$. Take the angle of contact to be zero.

Sol. Given that $r_1 = \frac{3.0}{2} = 1.5$ mm $= 1.5 \times 10^{-3}$ m, $r_2 = \frac{6.0}{2} = 3.0 \times 10^{-3}$ m,

$T = 7.3 \times 10^{-3}\ Nm^{-1}$, $\theta = 0°$, $\rho = 1.0 \times 10^3$ kg m^{-1}, $g = 9.8\ ms^{-2}$

When angle of contact is zero degree, the radius of the meniscus equals radius of bore.

Excess pressure in the first bore, $P_1 = \frac{2T}{r_2} = \frac{2\times 7.3\times 10^{-3}}{1.5\times 10^{-3}} = 97.3$ Pascal

Excess pressure in the second before, $P_2 = \frac{2T}{r^2} = \frac{2\times 7.3\times 10^{-2}}{3\times 10^{-3}} = 48.7$ Pascal

Hence, pressure difference in the two limbs of the tube

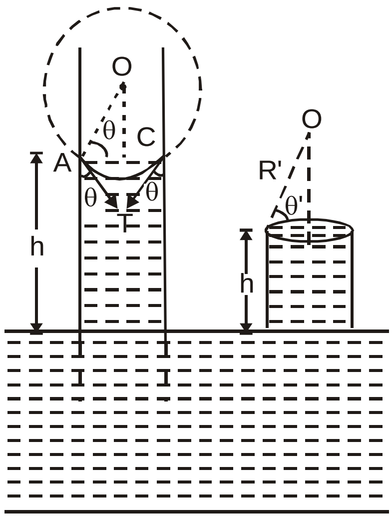
$$\Delta P = P_1 - P_2 = h\rho g$$

or $$h = \frac{P_1 - P_2}{\rho g} = \frac{97.3 - 48.7}{1.0\times 10^3 \times 9.8} = 5.0 \text{ mm.}$$

Capillary rise in a tube of insufficient length

We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \quad \text{or} \quad h\ R = \frac{2T}{\rho g} = \text{constant}$$



When the capillary tube is cut and its length is less then h (i.e. h′). then the liquid rises upto the top of the tube and spreads in such a way that the radius (R′) of the liquid meniscus increases and it becomes more flat so that hR = h′R′ = Constant. Hence the liquid does not overflow.

If $h' < h$ then $R' > R$ or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \quad \cos\theta' < \cos\theta \quad \Rightarrow \quad \theta' > \theta$

Ex. If a 5cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary. (ii) Water will rise up to the upper end of capillary (iii) Water will not overflow out of the upper end of capillary. Explain your answer.

Sol. Given that surface tension of water, T = 75 dyne/cm

Radius $r = \frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm

density $\rho = 1$ gm/cm^3, angle of contact, $\theta = 0°$

Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0°}{0.005 \times 1 \times 981} \text{ cm} = 30.58 \text{ cm}$$

But length of capillary tube, $h' = 5$cm

(i) Because $h > \frac{h'}{2}$ therefore the first possibility does not exist.

(ii) Because the tube is of insufficient length therefore the water will rise upto the upper end of the tube.

(iii) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R′ in such a way that

$$R'h' = Rh \qquad \left[\because hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$$\therefore \quad R' = \frac{Rh}{h'} = \frac{rh}{h'} \qquad \left[\because R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right] = \frac{0.005 \times 30.58}{5} = 0.0306 \text{ cm}$$

Applications of surface tension

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

Effect of temperature and impurities on surface tension

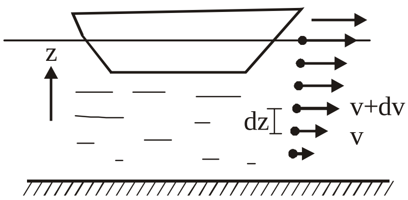
The surface tension of a liquid decreases with the rise in temperature and vice versa. According to Ferguson, $T = T_0\left(1-\frac{\theta}{\theta_C}\right)^n$ where T_0 is surface tension at 0°C, θ is absolute temperature of the liquid, θ_C is the critical temperature and n is a constant varies slightly from liquid and has means value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the while machinery parts get jammed in winter.

The surface tension of a liquid change appreciably with addition of impurities. For example, surface tension of water increases with addition of highly soluble substances like NaCl, $ZnSO_4$ etc. On the other hand surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap etc.

VISCOSITY

When a layer of a fluid slips or tends to slip on another layer in contact, the two layers exert tangential forces on each other. The directions are such that the relative motion between the layers is opposed. this property of a fluid to oppose relative motion between its layers is called viscosity. The forces between the layers opposing relative motion between them are known as the forces of viscosity. Thus, viscosity may be thought of as the internal friction of a fluid in motion.

If a solid surface is kept in contact with a fluid and is moved, forces of viscosity appear between the solid surface and the fluid layer in contact. the fluid in contact is dragged with the solid. If the viscosity is sufficient, the layer moves with the solid and there is no relative slipping. When a boat moves slowly on the water of a calm river, the water in contact with the boat is dragged with it, whereas the water in contact with the bed of the river remains at rest. Velocities of different layers are different. Let v be the velocity of the layer at a distance z from the bed and v +dv be the velocity at a distance z +dz (figure).



Thus, the velocity differs by dv in going through a distance dz perpendicular to it. The quantity dv/dz is called the velocity gradient.

The force of viscosity between two layers of a fluid is proportional to the velocity gradient in the direction perpendicular to the layers. Also the force is proportional to the area of the layer.

Thus, if F is the force exerted by a layer of area A on a layer in contact,

$$F \propto A \text{ and } F \propto dv/dz$$

or, $$F = -\eta A\, dv/dz$$

The negative sign is included as the force is frictional in nature and opposes relative motion. The constant of proportionality η is called the coefficient of viscosity.

The SI unit of viscosity can be easily worked out from equation. It is N-s/m^2. However, the corresponding CGS unit dyne-s/cm^2 is in common use and is called a poise in honour of the French scientist Poiseuille. We have

$$1 \text{ poise} = 0.1 \text{ N-s/m}^2$$

TERMINAL VELOCITY

The viscous force on a solid moving through a fluid is proportional to its velocity. When a solid is dropped in a fluid, the forces acting on it are

(a) weight W acting vertically downward,

(b) the viscous force F acting vertically upward and

(c) the buoyancy force B acting vertically upward.

The weight W and the buoyancy B are constant but the force F is proportional to the velocity v, initially, the velocity and hence the viscous force F is zero and the solid is accelerated due to the force W-B. Because of the acceleration, the velocity increases. Accordingly, the viscous force also increases. At a certain instant the viscous force becomes equal to W-B. the net force then becomes zero and the solid falls with constant velocity. This constant velocity is known as the terminal velocity.

Consider a spherical body falling through a liquid. Suppose the density of the body = ρ, density of the liquid = σ, radius of the sphere = r and the terminal velocity = v_0 . The viscous force is

$$F = 6\pi\eta r v_0$$

the weight $$W = \frac{4}{3}\pi r^3 \rho g$$

and the buoyancy force $$B = \frac{4}{3}\pi r^3 \sigma g$$

We have

$$6\pi\eta r v_0 = W - B = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

or $$v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

Q. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river = 10^{-2} poise.

Sol. The velocity decreases from 2 m/s to zero in 1 m of perpendicular length. Hence, velocity gradient.

$$= dv/dx = 2\ s^{-1}$$

Now, $$\eta = \left|\frac{F/A}{dv/dx}\right|$$

or, $$10^{-3}\ \frac{N-s}{m^2} = \frac{F}{(10m)^2(2s^{-1})}$$

or, $$F = 0.02\ N.$$

Q. The velocity of water in a river is 18 km/hr near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water = 10^{-2} poise.

Sol. The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18\,km/hr}{5m} = 1.0\,s^{-1}$$

The magnitude of the force of viscosity is

$$F = \eta\ A\ \frac{dv}{dx}.$$

The shearing stress is

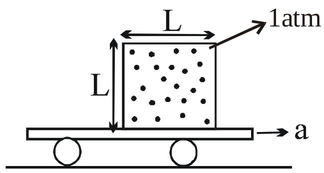
$$F/A = \eta\ \frac{dv}{dx} = (10^{-2}\ \text{poise})\ (1.0\ s^{-1}) = 10^{-3}\ N/m^2$$

EXERCISE (S-1)

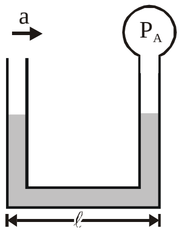
Fluid Statics

1. A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank.

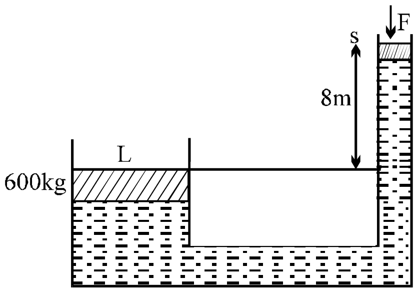
2. A cubical sealed vessel with edge L is placed on a cart, which is moving horizontally with an acceleration 'a' as shown in figure. The cube is filled with an ideal fluid having density ρ. Find the gauge pressure at the centre of the cubical vessel.



3. A liquid of density ρ is filled in a U-tube, whose one end is open & at the other end a bulb is fitted whose pressure is P_A. Now this tube is moved horizontally with acceleration 'a' as shown in the figure. During motion it is found that liquid in both column is at same level at equilibrium. If atmospheric pressure is P_0, then find the value of P_A

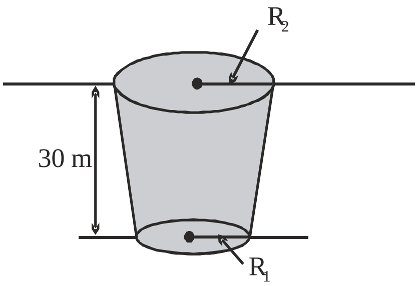


4. For the system shown in the figure, the cylinder on the left at L has a mass of 600 kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil.($\rho = 0.75$ gm/cm^3) Find the force F required to hold the system in equilibrium.

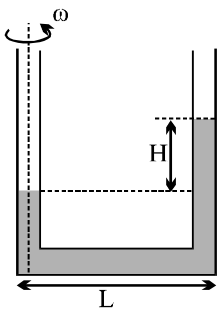


5. Dams at two different locations are needed to form a lake. When the lake is filled, the water level will be at top of both dams. The second dam is twice as high and twice as wide as the first dam. The force of the water on the second dam is how much greater than the force on the first? (Ignore atmospheric pressure since it is pushing on both sides of the dams.)

6. As the drawing illustrates, a pond full of water has the shape of an inverted cone with the tip sliced off and has a depth of 30m. The atmospheric pressure above the pond is 1.0×10^5 Pa. The circular top surface (radius = R_2) and circular bottom surface (radius = R_1) of the pond are both parallel to the ground. The magnitude of the force acting on the top surface by the liquid is the same as the magnitude of the force acting on the bottom surface by the liquid. Obtain $\frac{R_2}{R_1}$.

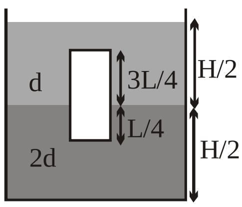


7. A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube d << L. **[IIT-JEE 2005]**



Buoyancy

8. A container of a large uniform cross-sectional area A resting on a horizontal surface holds two immiscible, non viscous and incompressible liquids of densities d and 2d each of height $\frac{H}{2}$ as shown. The lower density liquid is open to atmosphere. A homogeneous solid cylinder of length $L\left(<\frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical to the liquid -liquid interface with length $\frac{L}{4}$ in denser liquid. Find the density of the solid cylinder.

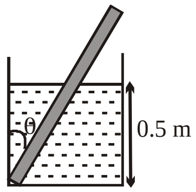


9. In air an object weighs 15N, when immersed completely in water the same object weighs 12N. When immersed in another liquid completely, it weighs 13N. Find

(a) the specific gravity of the object and

(b) the specific gravity of the other liquid.

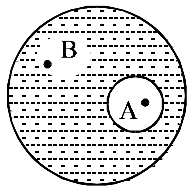
10. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown. The tank is filled with water upto a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ made by the plank in equilibrium position



11. The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. If the atmospheric pressure is H m of mercury & the density of mercury is n times that of lake water. Find the depth of the lake.

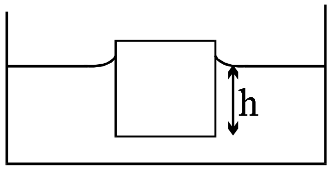
Surface Tension

12. There is an air bubble of radius R inside a drop of water of radius 3R. Find the ratio of gauge pressure at point A to the gauge pressure at point B.



13. Two arms of a U-tube have unequal diameters d_1 = 1.0 mm and d_2 = 1.0 cm. If water (surface tension 7×10^{-2} N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero.

14. A cube with a mass ‘m’ completely wettable by water floats on the surface of water. Each side of the cube is ‘a’. What is the distance h between the lower face of cube and the surface of the water if surface tension is S. Take density of water as ρ_w. Take angle of contact as zero.



Fluid Dynamics

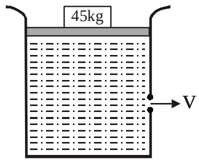
15. A jet of water having velocity = 10 m/s and stream cross-section = 2 cm^2 hits a flat plate perpendicularly, with the water splashing out parallel to plate. Find the force that the plate experiences.

16. A laminar stream is flowing vertically down from a tap of cross-section area 1 cm^2. At a distance 10 cm below the tap, the cross-section area of the stream has reduced to 1/2 cm^2. Find the volumetric flow rate of water from the tap.

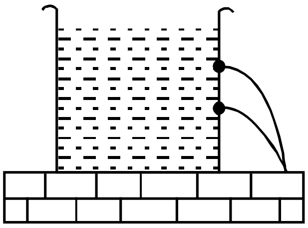
17. Calculate the rate of flow of glycerine of density 1.25×10^3 kg/m^3 through the conical section of a pipe if the radii of its ends are 0.1m & 0.04m and the pressure drop across its length is 10N/m^2.

18. A cylindrical vessel open at the top is 20cm high and 10cm in diameter. A circular hole whose cross-sectional area 1 cm^2 is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate 100 cm^3s^{-1}. Find the height of water in the vessel under steady state.

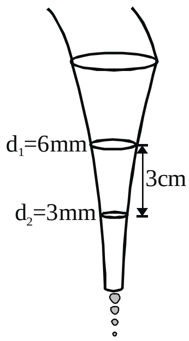
19. A large cylindrical tank of cross-sectional area 1m^2 is filled with water. It has a small hole at a height of 1m from the bottom. A movable piston of mass 5 kg is fitted on the top of the tank such that it can slide in the tank freely. A load of 45 kg is applied on the top of water by piston, as shown in figure. Find the value of v when piston is 7m above the bottom (g = 10 m/s^2)



20. In a cylindrical vessel containing liquid of density ρ, there are two holes in the side walls at heights of h_1 and h_2 respectively such that the range of efflux at the bottom of the vessel is same. Find the height of a hole, for which the range of efflux would be maximum.



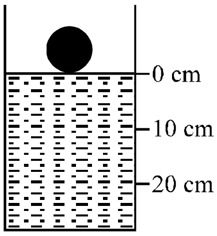
21. The tap in the garden was closed inappropriately resulting in the water flowing freely out of it which forms a downward narrowing beam. The beam of water has a circular cross-section, the diameter of the circle is 6 mm at one point and 3 cm below it is only 3 mm as shown in figure. If the rate of water wasted is ($x \times \pi$) mL/minute then find the value of x. (Neglect the effect of viscosity and surface tension of the flowing water.)



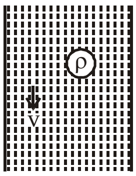
Viscosity

22. A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m^2.

23. A spherical ball of density ρ and radius 0.003m is dropped into a tube containing a viscous fluid filled up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.260 N.m^{-2} and its density $\rho_L = \rho/2$ = 1260 kg.m^{-3}. Assume the ball reaches a terminal speed by the 10 cm mark. Find the time taken by the ball to traverse the distance between the 10 cm and 20 cm mark. (g = acceleration due to gravity = 10 ms^{-2})



24. A liquid of density ρ = 1000 kg/m^3 and coefficient of viscosity η = 0.1 Ns/m^2 is flowing down in vertical pipe of large cross section. A small ball of density ρ_0 = 100 kg/m^3 and r = 5 cm will be at rest in flowing liquid, if velocity of flowing liquid is 10 k m/s. Then find the value of k.



25. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity.

[IIT-JEE 2004]

EXERCISE (O-1)

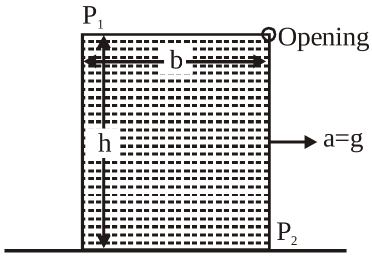
SINGLE CORRECT TYPE QUESTIONS

Fluid Statics

1. A vessel filled with a liquid of density ρ falls vertically downwards with an acceleration a(<g). The guage pressure P at depth h below the free surface of liquid is :

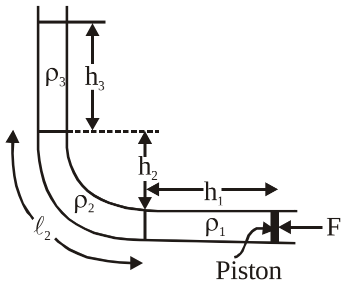
(A) $P = h\rho(g + a)$ (B) $P = h\rho g$ (C) $P = h\rho(g - a)$ (D) $P = h\rho a$

2. A closed rectangular vessel completely filled with a liquid of density ρ moves with an acceleration a = g. The value of the pressure difference $(P_1 - P_2)$ is :



(A) ρgb (B) $\frac{\rho g(b+h)}{2}$ (C) $\rho(ab - gh)$ (D) ρgh

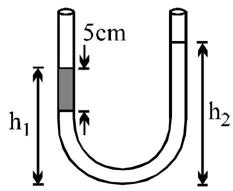
3. A tube is bent into L shape and kept in a vertical plane. If these three liquids are kept in equilibrium by the piston of area A, the value of $\frac{F}{A}$ is :



(A) $(\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)g$ (B) $(\rho_1 h_1 + \rho_2 \ell_2 + \rho_3 h_3)g$

(C) $(\rho_2 h_2 + \rho_3 h_3)g$ (D) $(\rho_2 \ell_2 + \rho_3 h_3)g$

4. An open-ended U-tube of uniform cross-sectional area contains water (density 1.0 gram/centimeter^3) standing initially 20 centimeters from the bottom in each arm. An immiscible liquid of density 4.0 grams/ centimeter^3 is added to one arm until a layer 5 centimeters high forms, as shown in the figure above. What is the ratio h_2/h_1 of the heights of the liquid in the two arms?

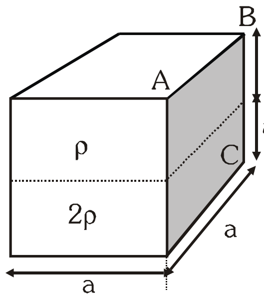


(A) 3/1 (B) 5/2 (C) 2/1 (D) 3/2

5. A tube of length L is filled completely with incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid on the tube at other end is :-

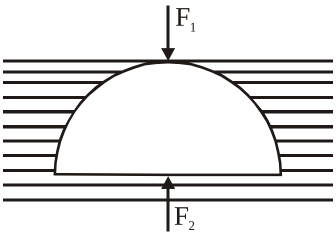
(A) $\frac{M\omega^2L}{2}$ (B) $M\omega^2L$ (C) $\frac{M\omega^2L}{4}$ (D) $\frac{3M\omega^2L}{4}$

6. A cuboid (a × a × 2a) is filled with two immiscible liquids of density 2ρ & ρ as shown in the figure. Neglecting atmospheric pressure, ratio of force on base & side wall of the cuboid is :-



(A) 2 : 3 (B) 1 : 3 (C) 5 : 6 (D) 6 : 5

7. A solid hemisphere is just pressed below the liquid, the value of $\frac{F_1}{F_2}$ is (where F_1 and F_2 are the hydrostatic forces acting on the curved and flat surfaces of the hemisphere) (Neglect atmospheric pressure).



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) none of these

8. A thin walled cylindrical metal vessel of linear coefficient of expansion 10^{-3} °C^{-1} contains benzene of volume expansion coefficient 10^{-3}°C^{-1}. If the vessel and its contents are now heated by 10°C, the pressure due to the liquid at the bottom.

(A) increases by 2% (B) decreases by 1%

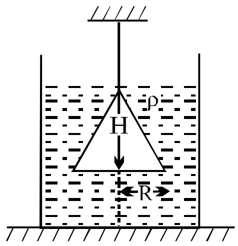
(C) decreases by 2% (D) remains unchanged

9. A cork of density 0.5gcm^{-3} floats on a calm swimming pool. The fraction of the cork's volume which is under water is :-

(A) 0% (B) 25% (C) 10% (D) 50%

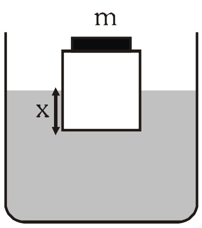
Buoyancy

10. A cone of radius R and height H, is hanging inside a liquid of density ρ by means of a string as shown in the figure. The force, due to the liquid acting on the slant surface of the cone is (Neglect atmosphere pressure)



(A) $\rho\pi gHR^2$ (B) $\pi\rho HR^2$ (C) $\frac{4}{3}\pi\rho gHR^2$ (D) $\frac{2}{3}\pi\rho gHR^2$

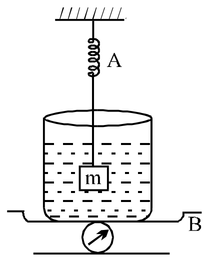
11. A cubical block (a × a × a), with a coin of mass 'm' over it is floating over a liquid of density ρ. In this case x_1 depth of the block is immersed. Now the coin is removed & it is found that x_2 depth is immersed in liquid. Value of $(x_1 – x_2)$ is :-



(A) $\frac{m}{\rho a^2}$ (B) $\frac{\rho a^4}{m}$

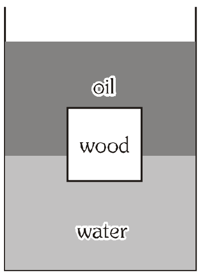
(C) $\frac{m}{2\rho a^2}$ (D) data insufficient

12. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure in this situation:



(A) the balance A will read more than 2 kg
(B) the balance B will read more than 5 kg
(C) the balance A will read less than 2 kg and B will read less than 5 kg
(D) the balances A and B will read 2 kg and 5 kg respectively.

13. A cubical block of wood of side of 10 cm, floats at the interface between oil and water as shown in figure with its lower face 2 cm below the interface. The density of oil is 0.6 g/cm^3. The mass of the block is



(A) 600 g (B) 680 g (C) 800 g (D) 200 g

14. A solid floats with 2/3 of its volume immersed in a liquid and with 3/4 of its volume immersed in another liquid. What fraction of its volume will be immersed if it floats in a homogeneous mixture formed of equal volumes of the liquids?

(A) 6/7 (B) 8/11 (C) 11/16 (D) 12/17

15. A small ball of relative density 0.8 falls into water from a height of 2m. The depth to which the ball will sink is (neglect viscous forces):

(A) 8 m (B) 2 m (C) 6 m (D) 4 m

16. An object of density d_o kept deep inside water of density d_w and released. During the time it moves a vertical distance h with in the water :-

(A) The gravitational potential energy of the water in the vessel increases if $d_0 < d_w$

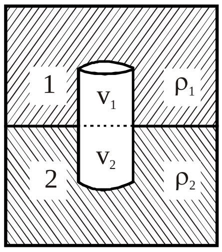
(B) The gravitational potential energy of the water in the vessel decreases if $d_0 < d_w$

(C) The gravitational potential energy of the object increases if $d_0 > d_w$

(D) The gravitational potential energy of the object decreases if $d_0 < d_w$

17. **Statement-1 :** When a body floats such that it's parts are immersed into two immersible liquids then force exerted by liquid-1 is of magnitude $\rho_1 v_1 g$.

Statement-2 : Total Buoyant force = $\rho_1 v_1 g + \rho_2 v_2 g$



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

18. **Statement-1 :** Submarine sailors are advised that they should not allow it to rest on floor of the ocean.

Statement-2 : The force exerted by a liquid on a submerged body may be downwards.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

Bernoulli

19. Water is flowing steadily through a horizontal tube of non uniform cross-section. If the pressure of water is 4×10^4 N/m^2 at a point where cross-section is 0.02 m^2 and velocity of flow is 2 m/s, what is pressure at a point where cross-section reduces to 0.01 m^2.

(A) 1.4×10^4 N/m^2 (B) 3.4×10^4 N/m^2 (C) 2.4×10^{-4} N/m^2 (D) none of these

20. In the case of a fluid, Bernoulli's theorem expresses the application of the principle of conservation of :-

(A) linear momentum (B) energy (C) mass (D) angular momentum

21. Two water pipes P and Q having diameters 2×10^{-2} m and 4×10^{-2}m, respectively, are joined in series with the main supply line of water. The velocity of water flowing in pipe P is :-

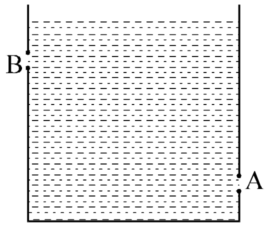
(A) 4 times that of Q (B) 2 times that of Q

(C) 1/2 times of that of Q (D) 1/4 times that of Q

22. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is- **[AIEEE - 2002]**

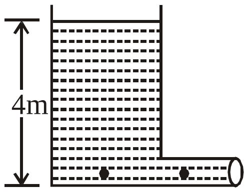
(A) 10 (B) 20 (C) 25.5 (D) 5

23. Two identical holes each of cross-sectional area 10^{-3} m^2 are made on the opposite sides of a tank containing water as shown in the figure. As the water comes out of the holes, the tank will experience a net horizontal force of 20 N. The difference in height between the holes A and B is.



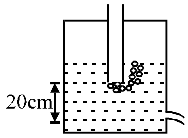
(A) 1 m (B) 0.5 m (C) 2 m (D) 0.25 m

24. A vent tank of large cross-sectional area has a horizontal pipe 0.12 m in diameter at the bottom. This holds a liquid whose density is 1500 kg/m^3 to a height of 4.0 m. Assume the liquid is an ideal fluid in laminar flow. In figure, the velocity with which fluid flows out is **:-**



(A) $2\sqrt{5}$ m/s (B) $\sqrt{5}$ m/s (C) $4\sqrt{5}$ m/s (D) $\sqrt{10}$ m/s

25. A tube is attached as shown in closed vessel containing water. The velocity of water coming out from a small hole is :



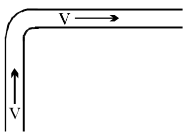
(A) $\sqrt{2}$ m/s

(B) 2 m/s

(C) depends on pressure of air inside vessel

(D) None of these

26. A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is :-



(A) ρVL (B) zero (C) 2ρVL (D) $\sqrt{2}\rho VL$

27. A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to H/η, ($\eta > 1$) and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is :

(A) 2 (B) 3 (C) 4 (D) $2\sqrt{2}$

28. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $\frac{a}{A}$ =0.1 then v^2 is (where v is the velocity of water coming out of the hole) **[IIT-JEE' 2005 (Scr)]**

(A) 48 (B) 51 (C) 50 (D) 51.5

Surface Tension

29. If two soap bubbles of different radii are connected by a tube,

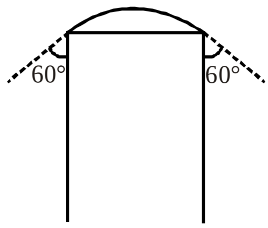
(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(C) air flows from the smaller bubble to the bigger

(D) there is no flow of air.

30. A soap bubble is being blown on a tube of radius 1 cm. The surface tension of the soap solution is 0.05 N/m and the bubble makes an angle of 60° with the tube as shown. The excess of pressure over the atmospheric pressure in the tube is :



(A) 5 Pa (B) 1 Pa (C) 10 Pa (D) 20 Pa

31. When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble :-

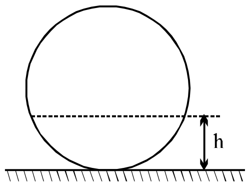
(A) is greater than the pressure outside it (B) is less than the pressure outside it

(C) increases as the bubble moves up (D) remains same as the bubble moves up

32. Two mercury drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if $\frac{1}{\pi}$ is the surface tension (in SI unit), is :

(A) $2^{5/3} r^2$ (B) $4r^2$ (C) $2r^2$ (D) $2^{8/3} r^2$

33. A liquid is filled in a spherical container of radius R till a height h. At this position the liquid surface at the edges is also horizontal. The contact angle is :-



(A) 0 (B) $\cos^{-1}\left(\frac{R-h}{R}\right)$ (C) $\cos^{-1}\left(\frac{h-R}{R}\right)$ (D) $\sin^{-1}\left(\frac{R-h}{R}\right)$

34. An open capillary tube is lowered in a vessel with mercury. The difference between the levels of the mercury in the vessel and in the capillary tube $\Delta h = 4.6$mm. What is the radius of curvature of the mercury meniscus in the capillary tube? Surface tension of mercury is 0.46 N/m, density of mercury is 13.6 gm/cc.

(A) $\frac{1}{340}$ m (B) $\frac{1}{680}$ m

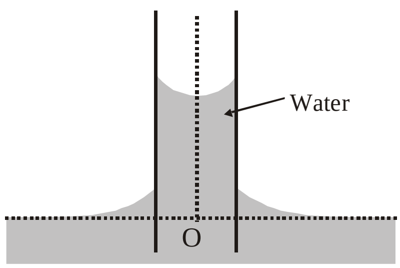
(C) $\frac{1}{1020}$ m (D) Information insufficient

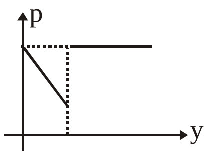
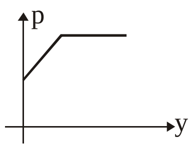
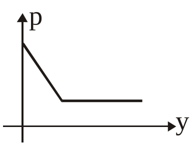
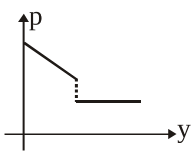
35. A container, whose bottom has round holes with diameter 0.1 mm is filled with water. The maximum height in cm upto which water can be filled without leakage will be what?

Surface tension = 75×10^{-3} N/m and g = 10 m/s^2:

(A) 20 cm (B) 40 cm (C) 30 cm (D) 60 cm

36. Water rises in a capillary as shown. The correct graph of pressure (p) vs height (y) above point O along the axis of capillary is :-



(A)  (B)  (C)  (D) 

37. If two soap bubbles of different radii are connected by a tube- **[AIEEE - 2004]**

(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal

(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged

(C) air flows from the smaller bubble to the bigger bubble

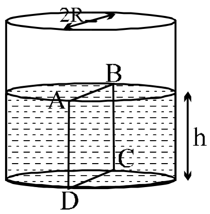
(D) there is no flow of air

38. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be-

(A) 8 cm (B) 10 cm (C) 4 cm (D) 20 cm

[AIEEE - 2005]

39. Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is P_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude **[IIT-JEE 2007]**



(A) $|2P_0Rh + \pi R^2\rho gh - 2RT|$ (B) $|2P_0Rh + R\rho gh^2 - 2RT|$

(C) $|P_0\pi R^2 + R\rho gh^2 - 2RT|$ (D) $|P_0\pi R^2 + R\rho gh^2 + 2RT|$

Viscosity

40. Two drops of same radius are falling through air with steady velocity of v cm/s. If the two drops coalesce, what would be the terminal velocity?

(A) 4 v (B) $(4)^{1/3}$v (C) 2 v (D) 64 v

41. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. the retarding viscous force acting on the spherical ball is- **[AIEEE - 2004]**

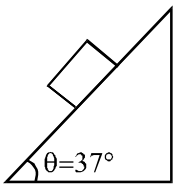
(A) directly proportional to R but inversely proportional to v

(B) directly proportional to both radius R and velocity v

(C) inversely proportional to both radius R and velocity v

(D) inversely proportional to R but directly proportional to velocity v

42. A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be:

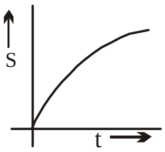


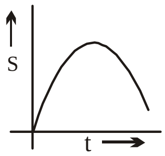
(A) $\frac{3\rho a g t}{5v}$ (B) $\frac{4\rho a g t}{5v}$ (C) $\frac{\rho a g t}{v}$ (D) none of these

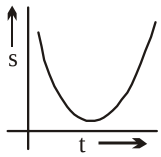
43. There is a 1mm thick layer of glycerine between a flat plate of area 100 cm^2 & a big fixed plate. If the coefficient of viscosity of glycerine is 1.0 kg/m-s then how much force is required to move the plate with a velocity of 7 cm/s?

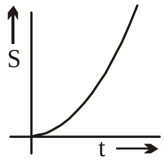
(A) 3.5 N (B) 0.7 N (C) 1.4 N (D) None

44. The displacement of a ball falling from rest in a viscous medium is plotted against time. Choose a possible option

(A) 

(B) 

(C) 

(D) 

45. An air bubble of radius 1cm is found to rise in a cylindrical vessel of large radius at a steady rate of 0.2 cm per second. If the density of the liquid is 1470 kg m^{-3}, then coefficient of viscosity of liquid is approximately equal to

(A) 163 poise (B) 163 centi-poise (C) 140 poise (D) 140 centi-poise

46. **Statement-1 :** The free surface of a liquid at rest with respect to stationary container is always normal to the $\vec{g}$.

Statement-2 : Liquids at rest cannot have shear stress.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

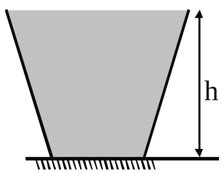
47. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density=10.5 kg/m^3) of the same size in the same liquid. **[AIEEE- 2006]**

(A) 0.4 m/s (B) 0.133 m/s (C) 0.1 m/s (D) 0.2 m/s

MULTIPLE CORRECT TYPE QUESTIONS

Fluid Statics

48. M gm of a liquid of density ρ is filled in a light beaker and kept on a horizontal table as shown in the figure. The height of the liquid in the beaker is h. The beaker is wider on top than at its base and the cross-sectional area of the base is A. Neglect the effect of atmospheric pressure. Now, choose the **CORRECT** statement(s) from the following.



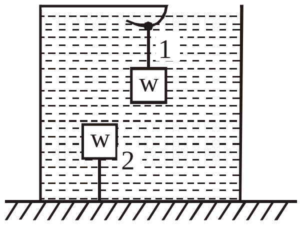
(A) The pressure of liquid at the bottom surface is ρgh.

(B) The normal reaction exerted by the table on the beaker is ρghA.

(C) The pressure of the liquid at the bottom surface is $\frac{Mg}{A}$.

(D) The normal reaction exerted by the table on the beaker is Mg.

49. An iron block and a wooden block are positioned in a vessel containing water as shown in the figure. The iron block (1) hangs from a massless string with a rigid support from the top while the wooden block (2) floats being tied to the bottom through a massless string. If now the vessel starts acceleration towards right.



(A) iron block gets deflected towards right.

(B) wooden block gets deflected towards right.

(C) iron block gets deflected towards left.

(D) wooden block gets deflected towards left.

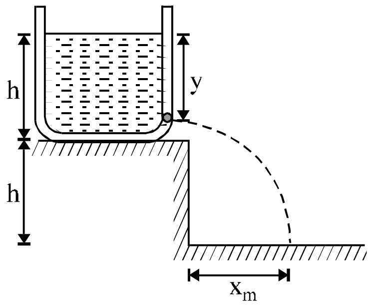
50. A beaker is filled in with water is accelerated a m/s^2 in +x direction. The surface of water shall make an angle

(A) $\tan^{-1}(a/g)$ backwards (B) $\tan^{-1}(a/g)$ forwards

(C) $\cot^{-1}(g/a)$ backwards (D) $\cot^{-1}(g/a)$ forwards

Fluid Dynamics

51. A tank is filled upto a height h with a liquid and is placed on a platform of height h from the ground. To get maximum range x_m a small hole is punched at a distance of y from the free surface of the liquid. Then



(A) $x_m = 2h$ (B) $x_m = 1.5\ h$

(C) $y = h$ (D) $y = 0.75\ h$

MATRIX MATCH TYPE QUESTION

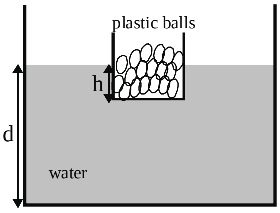
Fluid Statics

52. Bucket A contains only water ; an identical bucket B contains water, but also contains a solid object in the water. Consider the following four situations. Which bucket weighs more ?

Column-I	Column-II
(A) The object floats in bucket B, and the buckets have the same water level	(P) Bucket A
(B) The object floats in bucket B, and the buckets have the same volume of water	(Q) Bucket B
(C) The object sinks completely in bucket B, and the buckets have the same water level	(R) Both buckets have the same weight
(D) The object sinks completely in bucket B, and the buckets have the same volume of water	(S) The answer cannot be determined from the information given.

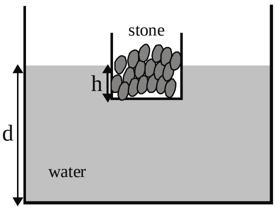
53. Column I shows different system as describe, with some parameter while column-II gives the change in the parameter.

Column-I | **Column-II**

(A) 

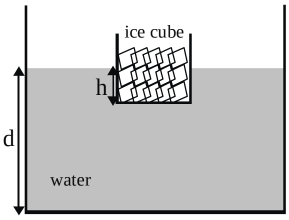
Plastic ball is thrown from the container in the water.
[d is the height of water level while h is the depth to which the container is submerged]

(P) h decreases

(B) 

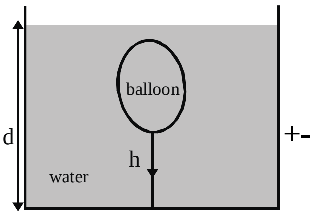
Stone is thrown from the container in the water.
[d is the height of water level while h is the depth to which the container is submerged]

(Q) h remain same

(C) 

Ice melts and remain in the container.
[d is the height of water level while h is the depth to which the container is submerged]

(R) h increases

(D) 

The water is heated slowly.
[d is the height of water level while h is the tension in the string. $\gamma_{air} > \gamma_{water}$]

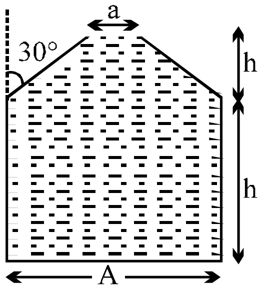
(S) d decreases

(T) d remain same

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. The vessel shown in the figure has two sections. The lower part is a rectangular vessel with area of cross-section A and height h. The upper part is a conical vessel of height h with base area 'A' and top area 'a' and the walls of the vessel are inclined at an angle 30° with the vertical. A liquid of density ρ fills both the sections upto a height 2h. Neglecting atmospheric pressure.



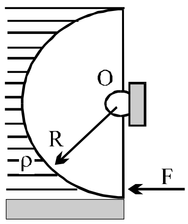
(A) The force F exerted by the liquid on the base of the vessel is $2h\rho g\dfrac{(A+a)}{2}$

(B) the pressure P at the base of the vessel is $2h\rho g\dfrac{A}{a}$

(C) the weight of the liquid W is greater than the force exerted by the liquid on the base

(D) the walls of the vessel exert a downward force (F–W) on the liquid.

2. A light semi cylindrical gate of radius R is pivoted at its mid point O, of the diameter as shown in the figure holding liquid of density ρ. The force F required to prevent the rotation of the gate is equal to



(A) $2\pi R^3\rho g$ (B) $2\rho g R^3 l$ (C) $\dfrac{2R^2 l\rho g}{3}$ (D) none of these

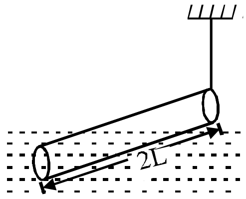
3. A heavy hollow cone of radius R and height h is placed on a horizontal table surface, with its flat base on the table. The whole volume inside the cone is filled with water of density ρ. The circular rim of the cone's base has a watertight seal with the table's surface and the top apex of the cone has a small hole. Neglecting atmospheric pressure find the total upward force exerted by water on the cone is

(A) $(2/3)\pi R^2 h\rho g$ (B) $(1/3)\pi R^2 h\rho g$ (C) $\pi R^2 h\rho g$ (D) None

4. Two cubes of size 1.0 m sides, one of relative density 0.60 and another of relative density = 1.15 are connected by weightless wire and placed in a large tank of water. Under equilibrium the lighter cube will project above the water surface to a height of

(A) 50 cm (B) 25 cm (C) 10 cm (D) zero

5. A slender homogeneous rod of length $2L$ floats partly immersed in water, being supported by a string fastened to one of its ends, as shown. The specific gravity of the rod is 0.75. The length of rod that extends out of water is :



(A) L (B) $\frac{1}{2}$ L (C) $\frac{1}{4}$ L (D) 3 L

6. Mercury is poured into a uniform vertical U-tube and water is poured in above it. The level of water is the same in both arms. A piece of wood is dropped into one arm and some water equal in weight to the piece of wood is added to the other. Then, consider the statements :

(I) The level of mercury will be same in both the arms.

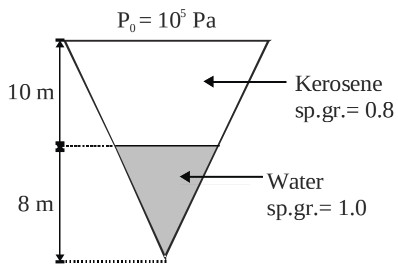
(II) The level of water will be same in both the arms.

(A) Both I and II are true (B) I is true but II is false

(C) I is false but II is true (D) Both I and II are false.

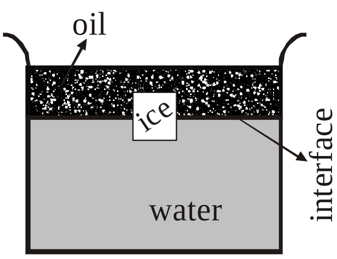
7. Shown in figure, a conical container of half-apex angle 37° filled with certain quantities of kerosene and water. The force exerted by the water on the kerosene is approximately,

(Take atmospheric pressure = 10^5 Pa)



(A) 3×10^7 N (B) 4×10^7 N (C) 2×10^7 N (D) 5×10^7 N

8. An ice cube is floating in water above which a layer of a lighter oil is poured. As the ice melts completely, the level of interface and the upper most level of oil will respectively :-

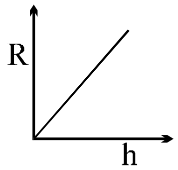


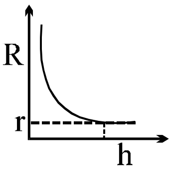
(A) rise and fall (B) fall and rise

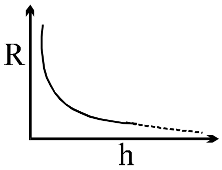
(C) rise and not change (D) not change and fall

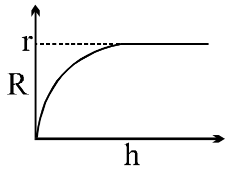
Statics

9. A long capillary tube of radius 'r' is initially just vertically completely immerged inside a liquid of angle of contact 0^0. If the tube is slowly raised then relation between radius of curvature of meniscus inside the capillary tube and displacement (h) of tube can be represented by

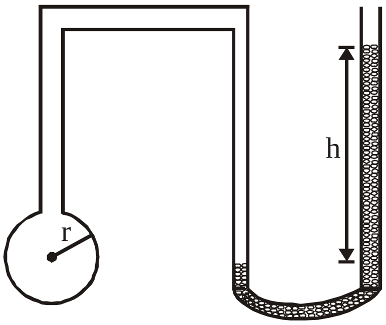
(A) 

(B) 

(C) 

(D) 

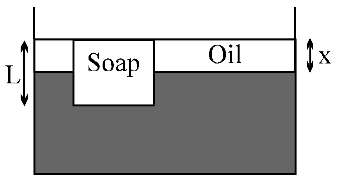
10. If the radius of the Soap-bubble on one side of tube is r and difference in height of liquid of density ρ in manometer is h, then surface tension of liquid used to make the bubble is :-



(A) $T = 2r\rho hg$ (B) $T = \frac{rh\rho g}{4}$ (C) $T = \frac{2\pi rh\rho g}{2}$ (D) $T = \frac{rh\rho g}{2}$

Surface tension

11. A rectangular bar of soap has density 800 kg/m^3 floats in water of density 1000 kg/m^3. Oil of density 300 kg/m^3 is slowly added, forming a layer that does not mix with the water. When the top surface of the oil is at the same level as the top surface of the soap. What is the ratio of the oil layer thickness to the soap's thickness, x/L ?



(A) $\frac{2}{10}$ (B) $\frac{2}{7}$ (C) $\frac{3}{10}$ (D) $\frac{3}{8}$

12. Water coming out of a horizontal tube at a speed ν strikes normally a vertically wall close to the mouth of the tube and falls down vertically after impact. When the speed of water is increased to 2ν.

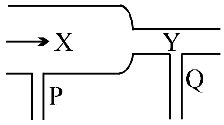
(A) the thrust exerted by the water on the wall will be doubled

(B) the thrust exerted by the water on the wall will be unchanged

(C) the energy lost per second by water striking the wall will also be four times

(D) the energy lost per second by water striking the wall be increased eight times.

13. A steady flow of water passes along a horizontal tube from a wide section X to the narrower section Y, see figure. Manometers are placed at P and Q at the sections. Which of the statements A, B, C, D, is most correct?



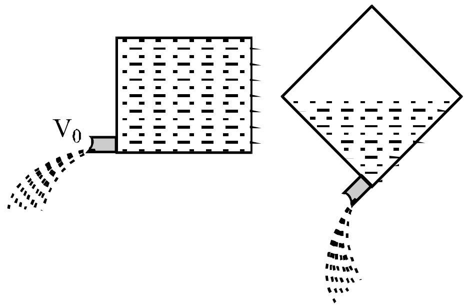
(A) water velocity at X is greater than at Y

(B) the manometer at P shows lower pressure than at Q

(C) kinetic energy per m^3 of water at X = kinetic energy per m^3 at Y

(D) the manometer at P shows greater pressure than at Q

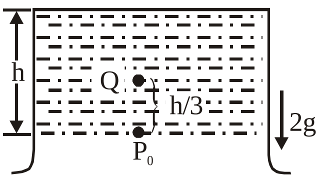
14. A cubical box of wine has a small spout located in one of the bottom corners. When the box is full and placed on a level surface, opening the spout results in a flow of wine with a initial speed of v_0 (see figure). When the box is half empty, someone tilts it at 45° so that the spout is at the lowest point (see figure). When the spout is opened the wine will flow out with a speed of



(A) v_0 (B) $v_0/2$ (C) $v_0/\sqrt{2}$ (D) $v_0/\sqrt[4]{2}$

Bernoulli's

15. An inverted test tube having a liquid of density ρ accelerates down with a = 2g then, $P_Q - P_0$ is:

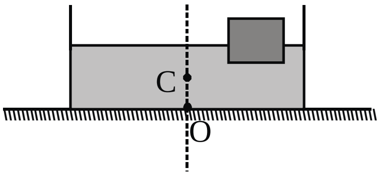


(A) $-\frac{\rho gh}{3}$ (B) $\frac{2\rho gh}{3}$ (C) $\frac{\rho gh}{3}$ (D) indeterminate

16. Some liquid is filled in a cylindrical vessel of radius R. Let F_1 be the force applied by the liquid on the bottom of the cylinder. Now the same liquid is poured into a vessel of uniform square cross-section of side R. Let F_2 be the force applied by the liquid on the bottom of this new vessel. (Neglect atmosphere pressure) Then :

(A) $F_1 = \pi F_2$ (B) $F_1 = \frac{F_2}{\pi}$ (C) $F_1 = \sqrt{\pi}F_2$ (D) $F_1 = F_2$

17. There is water in container with center of mass at C. Now a small wooden piece is place towards right as shown in the figure. After putting the wooden piece.



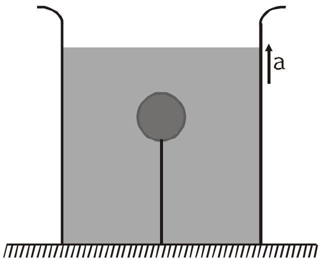
(A) Pressure at base remains same and centre of mass of water and wooden piece will be right of line OC.

(B) Pressure at base remains same and centre of mass of water and wooden piece will be on line OC.

(C) Pressure at base changes and centre of mass of water and wooden piece will be right of line OC.

(D) Pressure at base changes and centre of mass of water and wooden piece will be on line OC.

18. The tension in a string holding a solid block below the surface of a liquid (of density greater than that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension in the string if the system has an upward acceleration a?



(A) $T_0\left(1+\frac{a}{g}\right)$ (B) $T_0\left(1-\frac{a}{g}\right)$ (C) $2T_0\left(1-\frac{a}{g}\right)$ (D) $3T_0\left(1+\frac{2a}{g}\right)$

19. A wooden block floats in water in a sealed container. When the container is at rest, 25% of the block is above the water. Consider the following five situation :

(i) the container is lifted up at constant speed.

(ii) the container is lowered at constant speed.

(iii) the container is lifted up at an increasing speed.

(iv) the container is lowered at a decreasing speed.

(v) the air pressure above the water in the container is increased.

What happens in each situation?

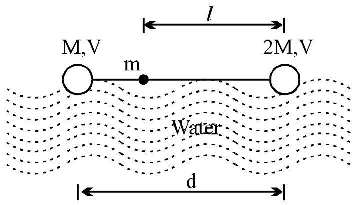
(A) 75% of block is submerged in siutation (i) & (ii) and higher fraction is submerged in situations (iii), (iv) & (v)

(B) 75% of block is submerged in situation (i), (ii) & (v) and higher fraction is submerged in situations (iii), (iv).

(C) 75% of block is submerged in situation (i), (ii), (iii) & (iv) and higher fraction is submerged in situations (v).

(D) 75% of block is submerged in all situation.

20. A dumbbell is placed in water of density ρ. It is observed that by attaching a mass m to the rod, the dumbbell floats with the rod horizontal on the surface of water and each sphere exactly half submerged as shown in the figure. The volume of the mass m is negligible. The value of length l is :-

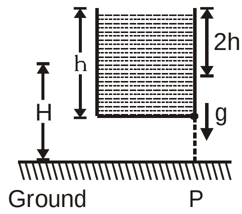


(A) $\frac{d(V\rho-3M)}{2(V\rho-2M)}$ (B) $\frac{d(V\rho-2M)}{2(V\rho-3M)}$ (C) $\frac{d(V\rho+2M)}{2(V\rho-3M)}$ (D) $\frac{d(V\rho-2M)}{2(V\rho+3M)}$

21. Fountains usually seen in gardens are generated by a wide pipe with an enclosure at one end having many small holes. Consider one such fountain which is produced by a pipe of internal diameter 2 cm in which water flows at a rate 3 ms^{-1}. The enclosure has 100 holes each of diameter 0.05 cm. The velocity of water coming out of the holes is (in ms^{-1}):

(A) 0.48 (B) 96 (C) 24 (D) 48

22. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel, as shown in the figure. The water which comes out from the hole at the instant when hole is at height H above the ground, strikes the ground at a distance of x from P. Which of the following is correct for the situation described ?



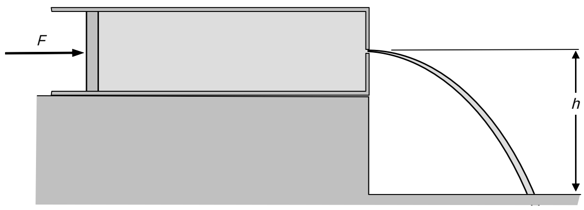
(A) The value of x is $2\sqrt{\frac{2hH}{3}}$

(B) The value of x is $\sqrt{\frac{4hH}{3}}$

(C) The value of x can't be computed from information provided.

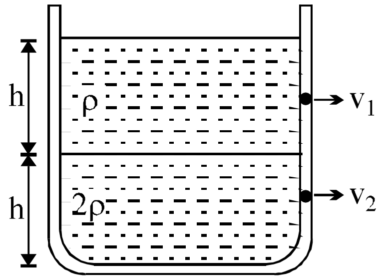
(D) The question is irrelevant as no water comes out from the hole.

23. An ideal liquid of density ρ is filled in a horizontally fixed syringe fitted with piston. There is no friction between the piston and the inner surface of the syringe. Cross-section area of the syringe is A. At one end of the syringe, an orifice of negligible cross-section area is made. When the piston is pushed into the syringe, the liquid comes out of the orifice following parabolic path and falls on the ground. With what speed the liquid strikes the ground? Neglect the air drag :-



(A) $\sqrt{\frac{F+\rho ghA}{\rho A}}$ (B) $\sqrt{\frac{F+2\rho ghA}{\rho A}}$ (C) $\sqrt{\frac{2F+\rho ghA}{\rho A}}$ (D) $\sqrt{\frac{2(F+\rho ghA)}{\rho A}}$

24. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth h/2 and 3h/2 from the surface of lighter liquid. If v_1 and v_2 are the velocities of a flux at these two holes, then v_1/v_2 is :



(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{\sqrt{2}}$

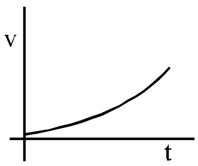
25. A vertical tank, open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross-section of the tank is N times the area of the hole, where N is a large number. Neglect mass of the tank itself. The initial acceleration of the tank is :-

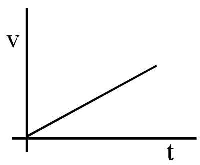
(A) $\frac{g}{2N}$ (B) $\frac{g}{\sqrt{2N}}$ (C) $\frac{g}{N}$ (D) $\frac{g}{2\sqrt{N}}$

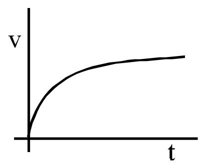
26. A solid metallic sphere of radius r is allowed to fall freely through air. If the frictional resistance due to air is proportional to the cross-sectional area and to the square of the velocity, then the terminal velocity of the sphere is proportional to which of the following?

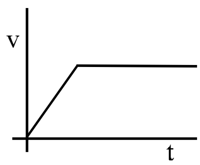
(A) r^2 (B) r (C) $r^{3/2}$ (D) $r^{1/2}$

27. Which of the following graphs best represents the motion of a raindrop?

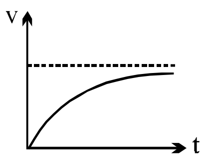
(A) 

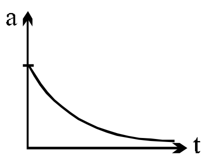
(B) 

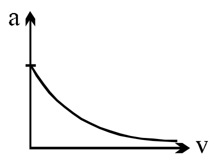
(C) 

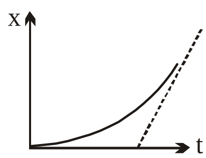
(D) 

28. Which of the following is the ***incorrect*** graph for a sphere falling in a viscous liquid?
(Given at t = 0, velocity v = 0 and displacement x = 0.)

(A) 

(B) 

(C) 

(D) 

MULTIPLE CORRECT TYPE QUESTIONS

29. A cubical block is floating in a liquid with one third of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of g/2 :-

(A) the fraction of volume immersed in the liquid will change.

(B) the buoyancy force on the block will change.

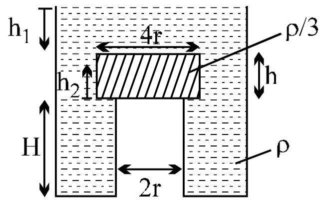
(C) the buoyancy force will increase by 50 percent.

(D) the pressure in the liquid will increased.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question no. 30 to 32

A wooden cylinder of diameter 4r, height h and density $\rho/3$ is kept on a hole of diameter 2r of a tank, filled with water of density ρ as shown in the figure. The height of the base of cylinder from the base of tank is H. **[IIT-JEE 2006]**



30. If level of liquid starts decreasing slowly when the level of liquid is at a height h_1 above the cylinder, the block just starts moving up. Then, value of h_1 is :-

(A) $\frac{2h}{3}$ (B) $\frac{5h}{4}$ (C) $\frac{5h}{3}$ (D) $\frac{5h}{2}$

31. Let the cylinder is prevented from moving up, by applying a force and water level is further decreased. Then, height of water level (h_2 in figure) for which the cylinder remains in original position without application of force is

(A) $\frac{h}{3}$ (B) $\frac{4h}{9}$ (C) $\frac{2h}{3}$ (D) h

32. If height h_2 of water level is further decreased, then

(A) cylinder will not move up and remains at its original position.

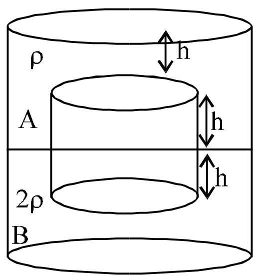
(B) for $h_2 = h/3$, cylinder again starts moving up

(C) for $h_2 = h/4$, cylinder again starts moving up

(D) for $h_2 = h/5$ cylinder again starts moving up

MATRIX MATCH TYPE QUESTION

33. Shown below is a cylinder of radius R floating in vessel containing liquids A and B. Neglecting atmospheric pressure match the quantities mentioned in column-I with corresponding expression in column-II.

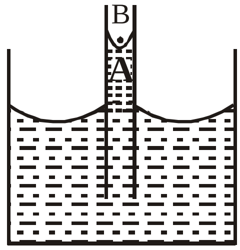


Column-I	Column-II
(A) Net force exerted by liquid A of density ρ on the cylinder.	(P) $9\ \rho g R h^2$
(B) Net force exerted by liquid B of density 2ρ on the cylinder.	(Q) $\pi \rho g R^2 h$
(C) Net force exerted by liquids A and B on the left half of the curved part of cylinder.	(R) $4\pi \rho g R^2 h$
(D) Net force exerted by liquid A and B on the cylinder.	(S) $3\pi \rho g R^2 h$

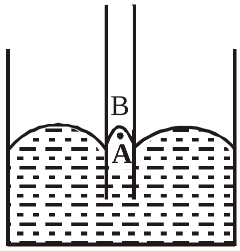
34. Capillary rise and shape of droplets on a plate due to surface tension are shown in column-II. Point A and B are just inside and outside the surface. Match the following

Column-I | **Column-II**

(A) liquid is wetting the solid

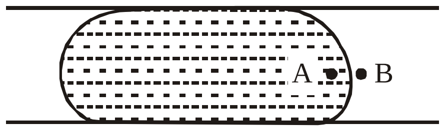
(P) 

(B) liquid is not wetting the solid

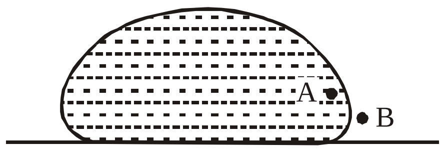
(Q) 

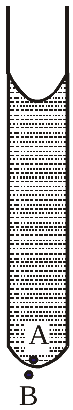
(C) Pressure at A > pressure at B

(R) A liquid drop is pressed between two parallel glass plates.



(D) Pressure at B > Pressure at A

(S) 

(T) 

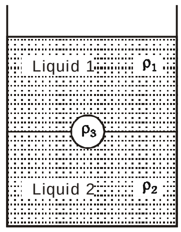
both end open cylindrical pipe

EXERCISE (JM)

1. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2\left(\rho_2 < \rho_1\right)$. Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ (k > 0). Then terminal speed of the ball is **[AIEEE - 2008]**

(1) $\sqrt{\dfrac{Vg\left(\rho_1-\rho_2\right)}{k}}$ (2) $\dfrac{Vg\rho_1}{k}$ (3) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (4) $\dfrac{Vg\left(\rho_1-\rho_2\right)}{k}$

2. A jar is filled with two non-mixing liqudis 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3 **[AIEEE - 2008]**



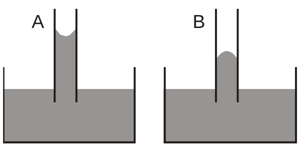
(1) $\rho_3 < \rho_1 < \rho_2$

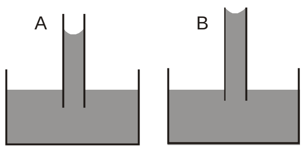
(2) $\rho_1 > \rho_3 > \rho_2$

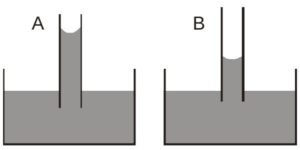
(3) $\rho_1 < \rho_2 < \rho_3$

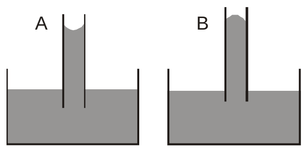
(4) $\rho_1 < \rho_3 < \rho_2$

3. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap -water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[AIEEE - 2008]**

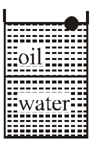
(1) 

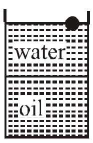
(2) 

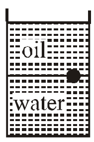
(3) 

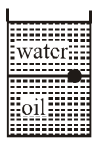
(4) 

4. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? **[AIEEE-2010]**

(1) 

(2) 

(3) 

(4) 

5. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :- **[AIEEE-2011]**

(1) 9.6×10^{-3} m (2) 3.6×10^{-3} m (3) 5.0×10^{-3} m (4) 7.5×10^{-3} m

6. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) :- **[AIEEE-2011]**

(1) 2π mJ (2) $0.4\ \pi$ mJ (3) 4π mJ (4) $0.2\ \pi$ mJ

7. Two mercury drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is : **[AIEEE-2011]**

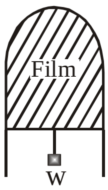
(1) $2^{5/3}\pi r^2 T$ (2) $4\pi r^2 T$ (3) $2\pi r^2 T$ (4) $2^{8/3}\pi r^2 T$

8. If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s) then its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be nearly :- **[AIEEE-2011]**

(1) 1.6×10^{-5} cm s^{-1} (2) 6.25×10^{-4} cm s^{-1}

(3) 6.45×10^{-4} cm s^{-1} (4) 1.5×10^{-5} cm s^{-1}

9. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is :- **[AIEEE-2012]**



(1) 0.025 Nm^{-1} (2) 0.0125 Nm^{-1} (3) 0.1 Nm^{-1} (4) 0.05 Nm^{-1}

10. A uniform cylinder of length L and mass M having cross- sectional area A is suspended, with its length vertical, form a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is : **[AIEEE-2013]**

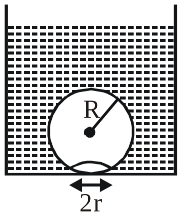
(1) $\frac{Mg}{k}$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (3) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (4) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

(Here k is spring constant)

11. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. **[AIEEE-2013]**

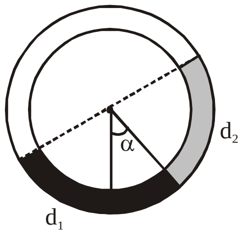
(1) $\frac{\rho L}{T}$ (2) $\sqrt{\frac{T}{\rho L}}$ (3) $\frac{T}{\rho L}$ (4) $\frac{2T}{\rho L}$

12. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R, and the surface tension of water is T, value of r just before bubbles detatch is:- (density of water is ρ_w) **[JEE Mains-2014]**



(1) $R^2\sqrt{\frac{\rho_w g}{T}}$ (2) $R^2\sqrt{\frac{3\rho_w g}{T}}$ (3) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (4) $R^2\sqrt{\frac{\rho_w g}{6T}}$

13. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is : **[JEE Mains-2014]**



(1) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (2) $\frac{1+\sin\alpha}{1-\cos\alpha}$ (3) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (4) $\frac{1+\cos\alpha}{1-\cos\alpha}$

EXERCISE (JA)

1. **Statement-1 :** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

and

Statement-2 : In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **[IIT-JEE 2008]**

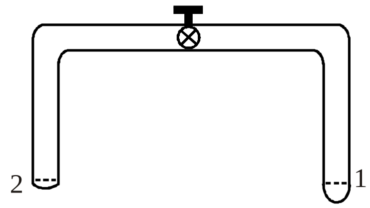
(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

2. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, **[IIT-JEE 2008]**



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles

(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

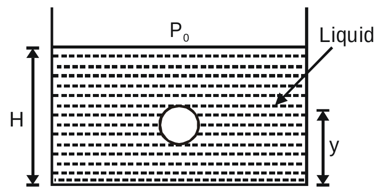
(C) no change occurs

(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

Paragraph for Question Nos. 3 to 5

A small spherical monoatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_ℓ (see figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric

pressure is P_0 (Neglect surface tension). **[IIT-JEE 2008]**



3. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it.

(A) Only the force of gravity

(B) The force due to gravity and the force due to the pressure of the liquid

(C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

(D) The force due to gravity and the force due to viscosity of the liquid.

4. When the gas bubble is at a height y from the bottom, its temperature is :-

(A) $T_0\left(\dfrac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{2/5}$ (B) $T_0\left(\dfrac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{2/5}$

(C) $T_0\left(\dfrac{P_0+\rho_\ell gH}{P_0+\rho_\ell gy}\right)^{3/5}$ (D) $T_0\left(\dfrac{P_0+\rho_\ell g(H-y)}{P_0+\rho_\ell gH}\right)^{3/5}$

5. The buoyancy force acting on the gas bubble is (Assume R is the universal gas constant)

(A) $p_\ell nRgT_0\,\dfrac{(P_0+\rho_\ell gH)^{2/5}}{(P_0+\rho_\ell gy)^{7/5}}$ (B) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{2/5}[P_0+\rho_\ell g(H-y)]^{3/5}}$

(C) $p_\ell nRgT_0\,\dfrac{(P_0+\rho_\ell gH)^{3/5}}{(P_0+\rho_\ell gy)^{8/5}}$ (D) $\dfrac{\rho_\ell nRgT_0}{(P_0+\rho_\ell gH)^{3/5}[P_0+\rho_\ell g(H-y)]^{2/5}}$

6. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.] **[IIT-JEE-2009]**

7. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8N/m^2. The radii of bubbles A and B are 2 cm and 4cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity]

[IIT-JEE-2009]

Paragraph for Questions no. 8 to 10

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper. **[IIT-JEE-2010]**

8. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r <<R) is

(A) 2πrT (B) 2πRT (C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

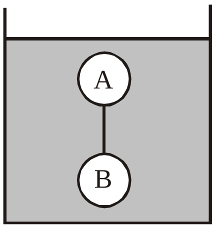
9. If r= 5×10^{-4} m, ρ = 10^3 kgm^{-3}, g= 10 ms^{-2}, T = 0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m (C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

10. After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J (C) 5.4×10^{-6} J (D) 8.1×10^{-6} J

11. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if **[IIT-JEE-2011]**



(A) $d_A < d_F$ (B) $d_B > d_F$ (C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

12. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is **[IIT-JEE-2012]**

(A) more than half-filled if ρ_C is less than 0.5

(B) more than half-filled if ρ_C is more than 1.0

(C) half-filled if ρ_C is more than 0.5

(D) less than half-filled if ρ_c is less than 0.5

13. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **[IIT-JEE-2013]**

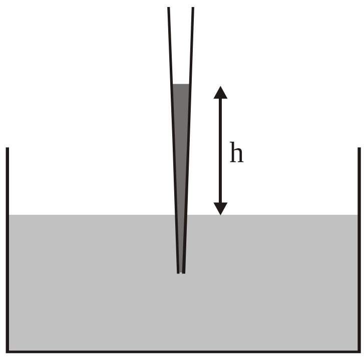
(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

14. A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)

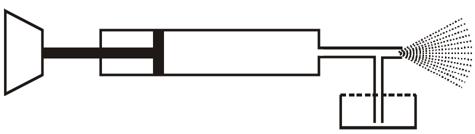


(A) $\frac{2S}{b\rho g}\cos(\theta - \alpha)$ (B) $\frac{2S}{b\rho g}\cos(\theta + \alpha)$ **[JEE Advanced-2014]**

(C) $\frac{2S}{b\rho g}\cos(\theta - \alpha/2)$ (D) $\frac{2S}{b\rho g}\cos(\theta + \alpha/2)$

Paragraph for Questions 15 to 17

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere. **[JEE Advanced-2014]**



15. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

(A) 0.1 ms^{-1} (B) 1 ms^{-1} (C) 2 ms^{-1} (D) 8 ms^{-1}

16. If the density of air is ρ_a and that of the liquid ρ_ℓ, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a \rho_\ell}$ (C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) $\rho\ell$

17. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists. **[JEE Advanced-2014]**

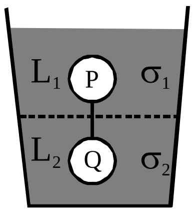
List - I	**List-II**
(P) Lift is accelerating vertically up.	(1) d = 1.2 m
(Q) Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	(2) d > 1.2 m
(R) Lift is moving vertically up with constant speed.	(3) d < 1.2 m
(S) Lift is falling freely.	(4) No water leaks out of the jar

Code :

(A) P-2, Q-3, R-2, S-4 (B) P-2, Q-3, R-1, S-4

(C) P-1, Q-1, R-1, S-4 (D) P-2, Q-3, R-1, S-1

18. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then **[JEE Advanced-2015]**



(A) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (B) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$ (C) $\vec{V}_P . \vec{V}_Q > 0$ (D) $\vec{V}_P . \vec{V}_Q < 0$

19. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is. **[JEE Advanced-2016]**

20. A drop of liquid of radius R = 10^{-2} m having surface tension S = $\frac{0.1}{4\pi}$Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If K = 10^{α} then the value of α is **[JEE Advanced-2017]**

21. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true? **[JEE Advanced-2018]**

(A) For a given material of the capillary tube, h decreases with increase in r

(B) For a given material of the capillary tube, h is independent of σ.

(C) If this experiment is performed in a lift going up with a constant acceleration, then h decreases.,

(D) h is proportional to contact angle θ.

22. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true ? **[JEE Advanced-2018]**

(A) The resistive force of liquid on the plate is inversely proportional to h

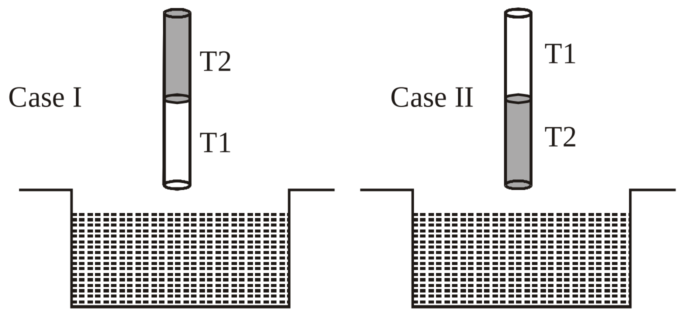
(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with u_0.

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

23. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T1 and T2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is(are) correct ? **[JEE Advanced-2019]**

(Surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2)



(1) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(2) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(3) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

(4) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

ANSWER KEY

EXERCISE (S-1)

1. Ans. 45°, 9600$\sqrt{2}$ Pa	**2. Ans.** $\frac{L}{2}\rho(g+a)$	**3. Ans.** $P_0 - \rho a\ell$	**4. Ans.** 37.5 N
5. Ans. 8	**6. Ans.** 2	**7. Ans.** $H = \frac{L^2\omega^2}{2g}$	**8. Ans.** $\frac{5d}{4}$
9. Ans. (a) 5, (b) 2/3	**10. Ans.** 45°	**11. Ans.** nH	**12. Ans.** 4
13. Ans. 2.52 cm	**14. Ans.** $h = \frac{mg + 4sa}{\rho_w a^2 g}$		**15. Ans.** 20 N
16. Ans. 4.9 litre/min	**17. Ans.** 6.43×10^{-4} m^3/s		**18. Ans.** 5 cm
19. Ans. 11 m/s	**20. Ans.** $\frac{h_2 + h_1}{2}$	**21. Ans.** 108	**22. Ans.** 20.4 m
23. Ans. 5 s	**24. Ans.** 5	**25. Ans.** $\frac{dQ}{dt} \propto r^5$	

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (C)	**2. Ans.** (C)	**3. Ans.** (C)	**4. Ans.** (C)	**5. Ans.** (A)	**6. Ans.** (D)
7. Ans. (C)	**8. Ans.** (C)	**9. Ans.** (D)	**10. Ans.** (D)	**11. Ans.** (A)	**12. Ans.** (B)
13. Ans. (B)	**14. Ans.** (D)	**15. Ans.** (A)	**16. Ans.** (B)	**17. Ans.** (D)	**18. Ans.** (A)
19. Ans. (B)	**20. Ans.** (B)	**21. Ans.** (A)	**22. Ans.** (B)	**23. Ans.** (A)	**24. Ans.** (C)
25. Ans. (B)	**26. Ans.** (D)	**27. Ans.** (C)	**28. Ans.** (C)	**29. Ans.** (C)	**30. Ans.** (C)
31. Ans. (A)	**32. Ans.** (D)	**33. Ans.** (B)	**34. Ans.** (B)	**35. Ans.** (C)	**36. Ans.** (A)
37. Ans. (C)	**38. Ans.** (D)	**39. Ans.** (B)	**40. Ans.** (B)	**41. Ans.** (B)	**42. Ans.** (A)
43. Ans. (B)	**44. Ans.** (D)	**45. Ans.** (A)	**46. Ans.** (A)	**47. Ans.** (C)	

MULTIPLE CORRECT TYPE QUESTIONS

48. Ans. (A,D) **49. Ans.** (B, C) **50. Ans.** (A,C) **51. Ans.** (A,C)

MATRIX MATCH TYPE QUESTION

52. Ans. (A)-R; (B)-Q; (C)-Q; (D)-Q

53. Ans. (A)-P,T; (B)-P,S; (C)-Q,T; (D)-R

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (D)	**2. Ans. (D)**	**3. Ans. (A)**	**4. Ans. (B)**	**5. Ans. (A)**	**6. Ans. (A)**
7. Ans. (C)	**8. Ans. (A)**	**9.Ans. (B)**	**10.Ans.(B)**	**11.Ans.(B)**	**12. Ans. (D)**
13. Ans. (D)	**14. Ans. (D)**	**15. Ans. (C)**	**16. Ans. (D)**	**17. Ans. (D)**	**18. Ans. (A)**
19. Ans. (D)	**20. Ans. (B)**	**21. Ans. (D)**	**22. Ans. (D)**	**23. Ans. (D)**	**24. Ans. (D)**
25. Ans. (C)	**26. Ans. (D)**	**27. Ans. (C)**	**28. Ans. (C)**		

MULTIPLE CORRECT TYPE QUESTIONS

29. Ans. (B, C, D)

COMPREHENSION TYPE QUESTIONS

30. Ans. (C) **31. Ans. (B)** **32. Ans. (A)**

MATRIX MATCH TYPE QUESTION

33. Ans. (A)-Q, (B)-R, (C)-P, (D)-S **34. Ans. (A)-P,T; (B)-Q,R,S; (C)-Q,R,S,T; (D)-P**

EXERCISE (JM)

1. Ans. (1)	**2. Ans. (4)**	**3. Ans. (3)**	**4. Ans. (3)**	**5. Ans. (2)**	**6. Ans. (2)**
7. Ans. (4)	**8. Ans. (2)**	**9. Ans. (1)**	**10. Ans. (3)**	**11. Ans. (4)**	**12. Ans. (3)**
13. Ans. (1)					

EXERCISE (JA)

1. Ans. (A)	**2. Ans. (B)**	**3. Ans. (D)**	**4. Ans. (B)**	**5. Ans. (B)**	**6. Ans. 6**
7. Ans. 6	**8. Ans. (C)**	**9. Ans. (A)**	**10. Ans. (B)**	**11. Ans. (A,B,D)**	
12. Ans. (A)	**13. Ans. (A,D)**	**14. Ans. (D)**	**15. Ans. (C)**	**16. Ans. (A)**	
17. Ans. (C)	**18. Ans. (A,D)**	**19. Ans. 3**	**20. Ans. 6**	**21. Ans. (A,C)**	
22. Ans. (A,C,D)		**23. Ans. (1,3,4)**			

CHAPTER 2

KINETIC THEORY OF GASES & THERMODYNAMICS

Chapter Contents 02

01.	THEORY	85
02.	EXERCISE (S-1)	122
03.	EXERCISE (O-1)	125
04.	EXERCISE (O-2)	134
05.	EXERCISE (JM)	143
06.	EXERCISE (JA)	147
07.	ANSWER KEY	157

IMPORTANT NOTES

CHAPTER 2

KINETIC THEORY OF GASES & THERMODYNAMICS

KINETIC THEORY OF GASES

The properties of the gases are entirely different from those of solid and liquid. In case of gases, thermal expansion is very large as compared to solids and liquids .To state the conditions of a gas, its volume, pressure and temperature must be specified.

Intermolecular force Solid > liquid > real gas > ideal gas (zero)

Potential energy Solid < liquid < real gas < ideal gas (zero)

Internal energy, internal kinetic energy, internal potential energy

At a given temperature for solid, liquid and gas:

(i) Internal kinetic energy : Same for all

(ii) Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE = –ve)

(iii) Internal Energy : Maximum for Ideal gas and Minimum for solid

At a given temperature for rared and compressed gas :

(i) Internal kinetic energy $\rightarrow$ Same

(ii) Internal potential energy $\rightarrow$ $(PE)_{Rared} > (PE)_{compressed}$

(iii) Internal Energy $\rightarrow$ $(U)_{Rared} > (U)_{compressed}$

	N.T.P.	**S.T.P.**
Temperature	0° C = 273.15 K	0.01° C = 273.16K
Pressure	1 atm = 1.01325×10^5 N/m^2 = 1.01325×10^5 pascal	1 atm
Volume	22.4 litre	22.4 litre

IDEAL GAS CONCEPT

- A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.
- Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0K)
- No intermoleculer force act between gas molecules.
- Potential energy of ideal gas is zero so internal energy of ideal gas is perfectly translational K.E. of gas. It is directly proportional to absolute temperature.

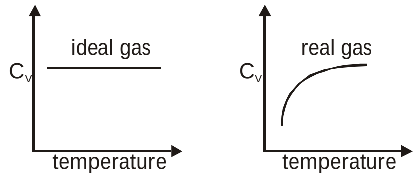
  So, internal energy depends only and only on its temperature.

  $E_{trans} \propto T$

  For a substance $U = U_{KE} + U_{PE}$

  U_{KE} : depends only on T, U_{PE}: depends upon intermolecular forces (Always negative)

- Specific heat of ideal gas is constant quantity and it does not change with temperature
- All real gases behaves as ideal gas at high temperature and low pressure.



- Volume expansion coefficient (α) and pressure expansion coefficient (β) is same for a ideal gas and value of each is $\frac{1}{273}$ per °C $\qquad \alpha = \beta = \frac{1}{273}$ per °C
- Gas molecule have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

EQUATION OF STATE FOR IDEAL GAS

$PV = \mu RT$ where μ = number of moles of gas $\Rightarrow PV = \frac{M}{M_w}RT = \left[\frac{mN}{mN_0}\right]RT = \left[\frac{R}{N_0}\right]N\,T = NkT$

Ex. By increasing temperature of gas by 5° C its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

Sol. $\because$ At constant volume $T \propto P$ $\therefore \frac{\Delta T}{T} \times 100 = \frac{\Delta P}{P} \times 100 = 0.5 \Rightarrow T = \frac{5 \times 100}{0.5} = 1000K$

Ex. Calculate the value of universal gas constant at STP.

Sol. Universal gas constant is given by $R = \frac{PV}{T}$

One mole of all gases at S.T.P. occupy volume V = 22·4 litre = $22{\cdot}4 \times 10^{-3}$ m^3

P = 760 mm of Hg = $760 \times 10^{-3} \times 13{\cdot}6 \times 10^3 \times 9.80$ N m^{-2} $\qquad$ T = 273 K

$\therefore R = \frac{760 \times 10^{-3} \times 13.6 \times 10^3 \times 9.80 \times 22.4 \times 10^{-3}}{273} = 8{\cdot}31$ J mol^{-1} K^{-1}

Ex. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure of 1×10^5 Nm^2. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the masses of the individual gases in the container, assuming them to be ideal. Given : R = 8.314 J/mol/K.

Sol. Let m gram be the mass of neon. Then, the mass of argon is (28 – m)g.

Total number of moles of the mixture, $\quad \mu = \frac{m}{20} + \frac{28-m}{40} = \frac{28+m}{40}$ ...(i)

Now, $\quad \mu = \frac{PV}{RT} = \frac{1 \times 10^5 \times 0.02}{8.314 \times 300} = 0.8$ ...(ii)

By (i) and (ii), $\frac{28+m}{40} = 0.8 \Rightarrow 28 + m = 32 \Rightarrow m = 4$ gram or mass of argon = (28 – 4)g = 24 g

Ex. Calculate the temperature of the Sun if density is 1.4 g cm^{-3}, pressure is 1.4×10^9 atmosphere and average molecular weight of gases in the Sun in 2 g/mole. **[Given R = 8.4 J $mol^{-1}K^{-1}$]**

Sol. $PV = \mu RT \Rightarrow T = \frac{PV}{\mu R}$ ...(i) But $\mu = \frac{M}{M_w}$ and $\rho = \frac{M}{V}$ $\therefore$ $\mu = \frac{\rho V}{M_w}$

From equation (i) $T = \frac{PVM_w}{\rho VR} = \frac{PM_w}{\rho R} = \frac{1.4 \times 10^9 \times 1.01 \times 10^5 \times 2 \times 10^{-3}}{1.4 \times 1000 \times 8.4} = 2.4 \times 10^7$ K

Ex. At the top of a mountain a thermometer reads 7°C and barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

Sol. By gas equation $PV = \frac{M}{M_w}RT \Rightarrow \frac{P}{\rho T} = \frac{R}{M_w}$ $\left[\because \mu = \frac{M}{M_w} \text{ and } \frac{M}{V} = \rho\right]$

Now as M_W and R are same for top and bottom $\left[\frac{P}{\rho T}\right]_T = \left[\frac{P}{\rho T}\right]_B$

So $\frac{\rho_T}{\rho_B} = \frac{P_T}{P_B} \times \frac{T_B}{T_T} = \frac{70}{76} \times \frac{300}{280} = \frac{75}{76} = 0.9868$

Ex. During an experiment an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at temperature T and volume V. What will be the temperature of the gas when it expands to a volume 2V.

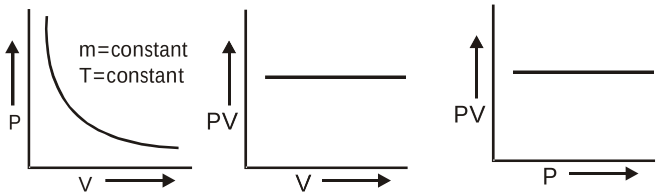
Sol. By gas equation PV = μRT and VP^2 = constant on eliminating P

$$\left[\frac{A}{\sqrt{V}}\right]V = \mu RT \Rightarrow \sqrt{V} = \frac{\mu R}{A}T \quad \therefore \quad \frac{\sqrt{V_1}}{\sqrt{V_2}} = \left[\frac{T_1}{T_2}\right] \Rightarrow \frac{\sqrt{V}}{\sqrt{2V}} = \frac{T}{T'} \quad \Rightarrow \quad T' = \left(\sqrt{2}\right)T$$

GAS LAWS

- **Boyle's Law**

According to it for a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, i.e.,



$V \propto \frac{1}{P}$ if m and T = Constant

Ex. A sample of oxygen with volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

Sol. Temperature is constant, so $P_1 V_1 = P_2 V_2$ $\therefore$ $P_2 = P_1\frac{V_1}{V_2} = 2\left[\frac{500}{400}\right] = 2.5$ atm

Ex. An air bubble doubles in radius on rising from bottom of a lake to its surface. If the atmosphere pressure is equal to that due to a column of 10 m of water, then what will be the depth of the lake. (Assuming that surface tension is negligible) ?

Sol. Given that constant temperature, we use $P_1V_1 = P_2V_2$

$P_2 = (10)$ dg (for water column) $P_1 = (10+h)$ dg (where h=depth of lake)

$V_1 = \frac{4\pi}{3}r^3$, $V_2 = \frac{4\pi}{3}(2r)^3 = 8\left(\frac{4\pi}{3}r^3\right) = 8V_1$ Thus for $P_2V_2 = P_1V_1$,

We have $10\,dg\,(8V_1) = (10 + h)\,dg\,V_1 \Rightarrow 80 = 10 + h \Rightarrow h = 70$ m

Ex. A vessel of volume 8.0×10^{-3} m^3 contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the Pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

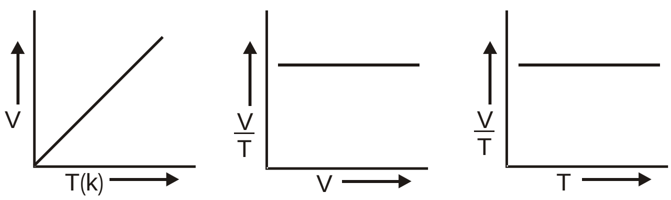
Sol. As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel in given by $n = \frac{PV}{RT}$.

The number of moles in the vessel before the leakage is $n_1 = \frac{P_1V}{RT}$ and that after the leakage is $n_2 = \frac{P_2V}{RT}$.

The amount leaked is $n_1 - n_2 = \frac{(P_1 - P_2)V}{RT} = \frac{(200-125)\times 10^3 \times 8.0 \times 10^{-3}}{8.3 \times 300} = 0.24$ mole

- **Charle's Law**

According to it for a given mass of an ideal gas at constant pressure, volume of a gas is directly proportional to its absolute temperature, i.e. $V \propto T$ if m and P = Constant



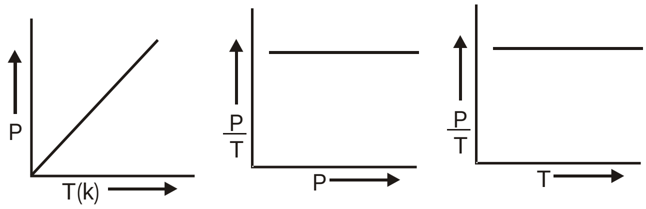
Ex. 1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C, if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person ?

Sol. $T_1 = 273 + 37 = 310$ K; $T_2 = 273 + 23 = 296$ K. Pressure and amount of the gas are kept constant,

So $\frac{V_1}{T_1} = \frac{V_2}{T_2} \therefore V_2 = V_1 \times \frac{T_2}{T_1} = 1500 \times \frac{293}{310} = 1417.74$ ml

Gay–Lussac's Law

According to it, for a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature, i.e., $P \propto T$ if m and V = constant



Ex. A sample of O_2 is at a pressure of 1 atm when the volume is 100 ml and its temperature is 27°C. What will be the temperature of the gas if the pressure becomes 2 atm and volume remains 100 ml.

Sol. $T_1 = 273 + 27 = 300$ K

For constant volume $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow T_2 = T_1 \times \frac{P_2}{P_1} = 300 \times \frac{2}{1}$ = 600 K = 600 – 273 = 327°C

Avogadro's Law

According to it, at same temperature and pressure of equal volumes of all gases contain equal number of molecules, i.e., $N_1 = N_2$ if P,V and T are same.

The kinetic theory of gases

- Rudolph Claussius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions as regards to the motion of molecules and the nature of the gases.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given as are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparison to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speeds is maximum.

Assumptions regarding collision:

- The gas molecules keep on colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collisions.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

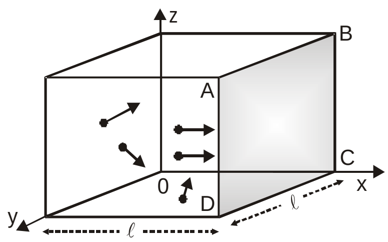
PROPERTIES/ASSUMPTIONS OF IDEAL GAS

- The molecules of a gas are in a state of continuous random motion. They move with all possible velocities in all possible directions. They obey Newton's law of motion.
- Mean momentum = 0; Mean velocity = 0.$< \vec{v} > = 0$; $< v^2 > \neq 0$ (Non zero); $< v^3 > = < v^5 > = 0$
- The average distance travelled by a molecule between two successive collisions is called as mean free path (λ_m) of the molecule.
- The time during which a collision takes place is negligible as compared to time taken by the molecule to cover the mean free path so NTP ratio of time of collision to free time of motion $10^{-8} : 1$.
- When a gas taken into a vessel it is uniformly distributed in entire volume of vessel such that its density, moleculer density, motion of molecules etc. all are identical for all direction, therefore root mean velocity

  $\vec{v}_x^2 = \vec{v}_y^2 = \vec{v}_z^2 \rightarrow$ equal Pressure exerted by the gas in all direction $P_x = P_y = P_z = P \rightarrow$ equal
- All those assumptions can be justified, if number of gas molecules are taken very large

EXPRESSION FOR PRESSURE OF AN IDEAL GAS

Consider an ideal gas enclosed in a cubical vessel of length ℓ. Suppose there are 'N' molecules in a gas which are moving with velocities $\vec{v}_1, \vec{v}_2........\vec{v}_N$.



If we consider any single molecule than its instantaneous velocity $\vec{v}$ can be expressed as $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Due to random motion of the molecule $v_x = v_y = v_z$

$$|v| = v_x\sqrt{3} = v_y\sqrt{3} = v_z\sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Suppose a molecule of mass m is moving with a velocity v_x towards the face ABCD. It strikes the face of the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision $\Delta p = - mv_x - mv_x = - 2\,mv_x$

Momentum transferred to the wall of the vessel per molecule per collision $\Delta p = 2\,mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is 2ℓ.

So, the time between two successive collision is $\Delta t = \dfrac{2\ell}{v_x}$

Number of collision per sec per molecule is $f_c = \dfrac{v_x}{2\ell} = \dfrac{\text{molecule velocity}}{\text{mean free path}}$, $f_c = \dfrac{v_{rms}}{\lambda_m}$ or, $f_c = \dfrac{v_m}{\lambda_m}$

Hence momentum transferred in the wall per second by the molecule is = force on the wall

$$\text{(force) } F = (2\,mv_x)\frac{v_x}{2\ell} = \frac{mv_x^2}{\ell} = \frac{mv^2}{3\ell}$$

Pressure exerted by gas molecule $P = \dfrac{F}{A} = \dfrac{1}{3}\dfrac{mv^2}{\ell \times A} \quad \Rightarrow P = \dfrac{1}{3}\dfrac{mv^2}{V} \quad [\because A \times \ell = V]$

Pressure exerted by gas $P = \sum \dfrac{1}{3}\dfrac{mv^2}{V} = \sum \dfrac{1}{3}\dfrac{mv^2}{V} \times \dfrac{N}{N} = \dfrac{1}{3}\dfrac{mN}{V}\dfrac{\sum v^2}{N} = \dfrac{1}{3}\dfrac{mN}{V}v_{rms}^2$

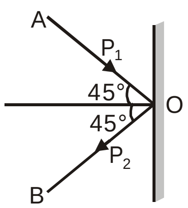
$$v_{rms}^2 = \frac{3PV}{M} = \frac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M_w}},\ P = \frac{1}{3}\frac{M}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$$

- Average number of molecules for each wall $= \dfrac{N}{6}$. No. of molecules along each axis $= \dfrac{N}{3}$ $(N_x = N_y = N_z)$
- $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \dfrac{\bar{v}_{rms}^2}{3}$ Root mean square velocity along any axis for gas molecule is

$$(v_{rms})_x = (v_{rms})_y = (v_{rms})_z = \frac{v_{rms}}{\sqrt{3}}$$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

Ex. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} molecules are colliding per second on a stationary wall of area 2 cm^2 at an angle of 45° to the normal to the wall and reflected elastically with a speed 10^3 m/s. Find the pressure exerted on the wall will be (in N/m^2)



Sol. As the impact is elastic $\therefore |\vec{p}_1| = |\vec{p}_2| = p = mv = 3.32 \times 10^{-24}$ kg m/s

The change in momentum along the normal $\Delta p = |\vec{p}_2 - \vec{p}_1| = 2p\cos 45° = \sqrt{2}p$

If f is the collision frequency then force applied on the wall $F = \dfrac{\Delta p}{\Delta t} = \Delta p \times f = \sqrt{2}pf$

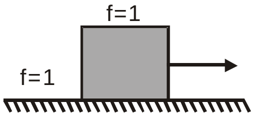
$\therefore$ Pressure $P = \dfrac{F}{A} = \dfrac{\sqrt{2}pf}{A} = \dfrac{\sqrt{2} \times 3.32 \times 10^{-24} \times 10^{23}}{2 \times 10^{-4}} = 2.347 \times 10^3\ N/m^2$

DEGREE OF FREEDOM (f)

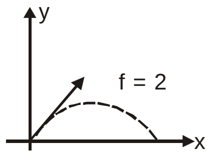
- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.

For example

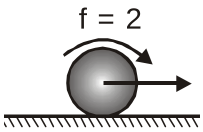
(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one translational degree of freedom.



(b) The projectile has two degrees of freedom becomes it is confined to move in a plane and so it has two translational degrees of freedom.



(c) The sphere has two degrees of freedom one rotational and another translational.Similarly a particle free to move in space will have three translational degrees of freedom.



Note : In pure rolling sphere has one degree of freedom as KE = $\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = \frac{7}{10}mv^2$

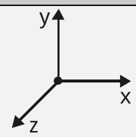
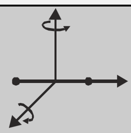
- The degrees of freedom are of three types :

(a) **Translational Degree of freedom :** Maximum three degree of freedom are there corresponding to translational motion.

(b) **Rotational Degree of freedom** : The number of degrees of freedom in this case depends on the structure of the molecule.

(c) **Vibrational Degree of freedom :** It is exhibited at high temperatures.

Degree of freedom for different gases according to atomicity of gas at low temperature

Atomicity of gas	Translational	Rotational	Total	
Monoatomic Ex. Ar, Ne, Ideal gas etc	3	0	3	
Diatomic Ex. O_2, Cl_2, N_2 etc.	3	2	5	

At high temperatures a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

Ex. Calculate the total number of degrees of freedom possessed by the molecules in one cm^3 of H_2 gas at NTP.

Sol. 22400 cm^3 of every gas constains 6.02×10^{23} molecules.

$\therefore$ Number of molecules in 1 cm^3 of H_2 gas = $\frac{6.02\times10^{23}}{22400}$ = 0.26875×10^{20}

Number of degrees of freedom of a H_2 gas molecule = 5

$\therefore$ Total number of degrees of freedom of $0.26875 \times 10^{20} \times 5 = 1.34375 \times 10^{20}$.

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$

For one molecule of gas

Energy related with each degree of freedom = $\frac{1}{2}kT$

Energy related with all degree of freedom = $\frac{f}{2}kT$ $\because \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{\overline{v_{rms}^2}}{3} \Rightarrow \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So energy related with one degree of freedom = $\frac{1}{2}m\frac{v_{rms}^2}{3} = \frac{3}{2}\frac{kT}{3} = \frac{1}{2}kT$

Ex. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root–mean–square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take R = $\frac{25}{3}$ J/mol–K and k = 1.38 × 10^{-23} J/K.

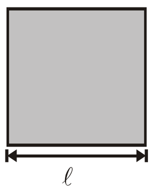
(a) Evaluate the temperature of the gas.

(b) Evaluate the average kinetic energy per atom.

(c) Evaluate the total mass of helium gas in the box.

Sol. Volume of the box = 1m^3, Pressure of the gas = 100 N/m^2. Let T be the temperature of the gas

(a) Time between two consecutive collisions with one wall = $\frac{1}{500}$ sec



This time should be equal to $\frac{2l}{v_{rms}}$, where ℓ is the side of the cube.

$$2\ell v_{rms} = \frac{1}{500} \Rightarrow v_{rms} = 1000 \text{ m/s} \therefore \sqrt{\frac{3RT}{M}} = 1000 \Rightarrow T = \frac{(1000)^2 M}{3R} = \frac{(10)^6(3\times10^{-3})}{3\left(\frac{25}{3}\right)} = 160K$$

(b) Average kinetic energy per atom = $\frac{3}{2}kT = \frac{3}{2}$ [(1.38 × 10^{-23}) = 160] J = 3.312 ×10^{-21} J

(c) From PV = nRT = $\frac{m}{M}$RT, Mass of helium gas in the box m = $\frac{PVM}{RT}$

Substituting the values, $m = \frac{(100)(1)(4\times10^{-3})}{\left(\frac{25}{3}\right)(160)}$ = 3.0 × 10^{-4} kg

DIFFERENT K.E. OF GAS (INTERNAL ENERGY)

- **Translatory kinetic energy (E_T)** $E_T = \frac{1}{2}Mv_{rms}^2 = \frac{3}{2}PV$

  Kinetic energy of volume V is = $\frac{1}{2}Mv_{rms}^2$ Note : Total internal energy of ideal gas is kinetic

- **Energy per unit volume or energy density (E_V)**

  $E_V = \frac{\text{Total energy}}{\text{Volume}} = \frac{E}{V}$; $E_V = \frac{1}{2}\left[\frac{M}{V}\right]v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2 \because P = \frac{2}{3}\left[\frac{1}{2}\rho v_{rms}^2\right] \therefore E_V = \frac{3}{2}P$

- **Molar K.E. or Mean Molar K.E. (E)**

  $E = \frac{1}{2}M_w v_{rms}^2$ for N_0 molecules or M_w (gram) $E = \frac{3}{2}RT = \frac{3}{2}N_0 kT$

- **Molecular kinetic energy or mean molecular K.E. ($\bar{E}$)**

  $E = \frac{1}{2}M_w v_{rms}^2$, $\bar{E} = \frac{E}{N_0} = \frac{3}{2}\frac{RT}{N_0} \Rightarrow \bar{E} = \frac{3}{2}kT$

GOLDEN KEY POINTS

- Except 0 K, at any temperature T , $E > E_m > \bar{E}$
- At a common temperature, for all ideal gas

  E and $\bar{E}$ are same while E_m is different and depends upon nature of gas (M_w or m)
- For thermal equilibrium of gases, temperature of each gas is same and this temperature called as temperature of mixture (T_m) which can be find out on basis of conservation of energy (All gases are of same atomicity). $T_m = \frac{\sum NT}{\sum N} = \frac{N_1T_1 + N_2T_2 + \ldots\ldots\ldots + N_nT_n}{N_1 + N_2 \ldots\ldots N_n}$

- **1 mole gas :** Mean kinetic energy = $\frac{3}{2}RT$; Total kinetic energy = $\frac{f}{2}RT$

  1 molecule of gases : Mean kinetic energy = $\frac{3}{2}kT$; Total kinetic energy = $\frac{f}{2}kT$

  $f \rightarrow$ Degree of freedom

Ex. Two ideal gases at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. Find the temperature of the mixture.

Sol. Total energy of molecules of first gas = $\frac{3}{2}n_1kT_1$, Total energy of molecules of second gas = $\frac{3}{2}n_2kT_2$

Let temperature of mixture be T then total energy of molecules of mixture = $\frac{3}{2}k(n_1 + n_2)T$

$\therefore \frac{3}{2}(n_1 + n_2)kT = \frac{3}{2}k(n_1T_1 + n_2T_2) \Rightarrow T = \frac{n_1T_1 + n_2T_2}{(n_1 + n_2)}$

Ex. The first excited state of hydrogen atom is 10.2 eV above its ground state. What temperature is needed to excite hydrogen atoms to first excited level.

Sol. K.E. of the hydrogen atom $\frac{3}{2}kT = 10.2\text{ eV} = 10.2 \times \left(1.6 \times 10^{-19}\right)$ J

$$\Rightarrow T = \frac{2}{3} \times \frac{10.2 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = 7.88 \times 10^{4}\text{ K}$$

DIFFERENT SPEEDS OF GAS MOLECULES

- **Average velocity**

Because molecules are in random motion in all possible direction in all possible velocity. Therefore, the average velocity of the gas in molecules in container is zero. $< \vec{v} > = \frac{\vec{v}_1 + \vec{v}_2 + ...\vec{v}_N}{N} = 0$

rms speed of molecules $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$

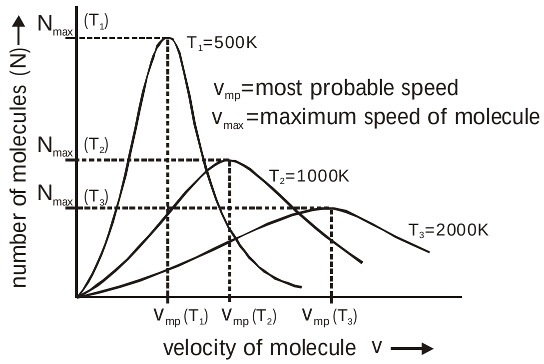
Mean speed of molecules : By maxwell's velocity distribution law v_M or $<|\vec{v}|> = v_{mean}$

$$<|\vec{v}|> = v_{mean} = \frac{|\vec{v}_1| + |\vec{v}_2| + ...|\vec{v}_n|}{N} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8kT}{\pi m}} = 1.59\sqrt{\frac{kT}{m}}$$

Most probable speed of molecules (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most probable speed (v_{mp}) $\quad v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{\frac{kT}{m}}$

MAXWELL'S LAW OF DISTRIBUTION OF VELOCITIES



GOLDEN KEY POINTS

- At any given temperature graph drawn in between molecular velocity and number of molecules is known as velocity distribution curve.
- The velocities of molecules of a gas are in between zero and infinity $(0 - \infty)$
- With the increase in the temperature, the most probable velocity and maximum molecule velocity both increases.
- The number of molecules within certain velocity range is constant although the velocity of molecule changes continuously at particular temperature.
- The area enclosed between the $(N - v)$ curve and the velocity axis presents the total number of molecules.

  On the basis of velocity distribution Maxwell established gives the law of equipartition of energy for gases of any temperature.

Ex. The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate

(i) average speed and (ii) rms speed (iii) most probable speed.

Sol. (i) average speed, $v_{av} = \frac{0+2+3+4+4+4+5+5+6+9}{10} = \frac{42}{10} = 4{\cdot}2\ ms^{-1}$

(ii) rms speed, $v_{rms} = \left[\frac{(0)^2+(2)^2+(3)^2+(4)^2+(4)^2+(4)^2+(5)^2+(5)^2+(6)^2+(9)^2}{10}\right]^{1/2} = \left[\frac{228}{10}\right]^{1/2} = 4.77\ ms^{-1}$

(iii) most probable speed $v_{mp} = 4$ m/s

Ex. At what temperature, will the root mean square velocity of hydrogen be double of its value at S.T.P., pressure remaining constant ?

Sol. Let v_1 be the r.m.s. velocity at S.T.P. and v_2 be the r.m.s. velocity at unknown temperature T_2.

$$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2} \quad \text{or} \quad T_2 = T_1\left[\frac{v_2}{v_1}\right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092\ K = (1092 - 273) = 819°C$$

Ex. Calculate rms velocity of oxygen molecule at 27°C

Sol. Temperature, T = 27° C $\Rightarrow$ 273 + 27 = 300 K,

Molecular weight of oxygen = 32×10^{-3} kg and $R = 8{\cdot}31\ J\ mol^{-1}\ K^{-1}$

rms velocity is $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8{\cdot}31 \times 300}{32 \times 10^{-3}}} = 483{\cdot}5\ ms^{-1}$

Ex. Calculate the kinetic energy of a gram moelcule of argon at 127°C.

Sol. Temperature, T = 127°C = 273 + 127 = 400 K, R = 8.31 J/mol K

K.E. per gram molecule of argon = $\frac{3}{2}RT = \frac{3}{2} \times 8.31 \times 400 = 4986$ J

THERMODYNAMICS

Branch of physics which deals with the inter–conversion between heat energy and any other form of energy is known as thermodynamics. In this branch of physics we deals with the processes involving heat, work and internal energy. In this branch of science the conversion of heat into mechanical work and vice versa is studied.

- **Thermodynamical System**

  The system which can be represented in of pressure (P), volume (V) and temperature (T), is known thermodynamic system. A specified portion of matter consisting of one or more substances on which the effects of variables such as temperature, volume and pressure are to be studied, is called a system. e.g. A gas enclosed in a cylinder fitted with a piston is a system.

- **Surroundings**

  Anything outside the system, which exchanges energy with the system and which tends to change the properties of the system is called its surroundings.

- **Heterogeneous System**

  A system which is not uniform throughout is said to be heterogeneous. e.g. A system consisting of two or more immiscible liquids.

- **Homogeneous System**

  A system is said to be homogeneous if it is completely uniform throughout. e.g. Pure solid or liquid.

- **Isolated System**

  A system in which there can be no exchange of matter and energy with the surroundings is said to be an isolated system.

- **Universe**

  The system and its surroundings are together known as the universe.

- **Thermodynamic variables of the system**

  (i) Composition (μ) (ii) Temperature (T) (iii) Volume (V) (iv) Pressure (P)

- **Thermodynamic state**

  The state of a system can be described completely by composition, temperature, volume and pressure.

  If a system is homogeneous and has definite mass and composition, then the state of the system can be described by the remaining three variables namely temperature, pressure and volume. These variables are interrelated by equation $PV = \mu RT$. The thermodynamic state of the system is its condition as identified by two independent thermodynamic variables (P, V or P, T or V, T).

- **Zeroth law of thermodynamics**

  If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduce the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are the same.

  In measuring the temperature of a body, it is important that the thermometer be in the thermal equilibrium with the body whose temperature is to be measured.

- **Thermal equilibrium**

  Thermal equilibrium is a situation in which two objects in thermal contact cease to exchange energy by the process of heat. Heat is the transfer of energy from one object to another object as a result of a difference in temperature between them.

- **Internal Energy**

  Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy (U_k) and that due to molecular configuration is called internal potential energy (U_p). $dU = dU_k + dU_p$

  If there no intermolecular forces, then $dU_p = 0$ and $dU = dU_k = m\ c_v\ dT$

  c_v = Specific heat at constant volume and dT = Infinitesimal change in temperature

  m = Mass of system M = Molecular weight

  Molar heat capacity $C_v = Mc_v$ For μ–moles of ideal gas $dU = \mu C_v dT = \frac{m}{M} C_v dT$

  Internal energy in the absence of inter–molecular forces is simply the function of temperature and state only, it is independent of path followed. $\Delta U = U_f - U_i$

  U_i = Internal energies in initial state and U_f = Internal energies in final state

- **Thermodynamic Processes**

  In the thermodynamic process pressure, volume, temperature and entropy of the system change with time.

  Thermodynamic process is said to take place if change occurs in the state of a thermodynamic system.

- **Sign convention used for the study of thermodynamic processes**

Heat gained by a system	Positive
Heat lost by a system	Negative
The work done by a system	Positive
Work done on the system	Negative
Increase in the internal energy of system	Positive
Decrease in the internal energy of system	Negative

- **Indicator Diagram or P–V Diagram**

In the equation of state of a gas $PV = \mu RT$

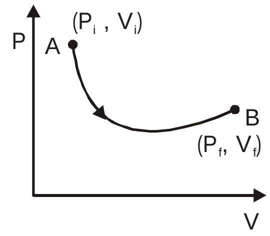
Two thermodynamic variables are sufficient to describe the behavior of a thermodynamic system.

If any two of the three variables P, V and T are known then the third can be calculated.

P–V diagram is a graph between the volume V and the pressure P of the system.

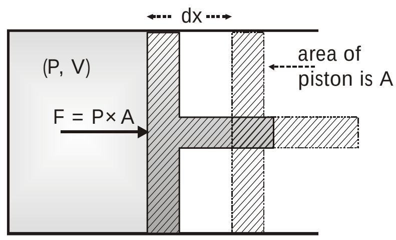
The volume is plotted against X–axis while the pressure is plotted against Y–axis.

The point A represents the initial stage of the system. Initial pressure of the system is P_i and initial volume of the system V_i .



The point B represents the final state of the system. P_f and V_f are the final pressure and final volume respectively of the system. The points between A and B represent the intermediate states of the system. With the help of the indicator diagram we calculate the amount of work done by the gas or on the gas during expansion or compression.

WORK DONE BY THERMODYNAMIC SYSTEM



One of the simple example of a thermodynamic system is a gas in a cylinder with a movable piston.

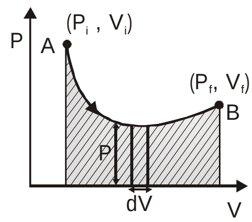
- **If the gas expands against the piston**

Gas exerts a force on the piston and displace it through a distance and does work on the piston.

- **If the piston compresses the gas**

When piston moved inward, work is done on the gas.

- **The work associated with volume changes**



If pressure of gas on the piston = P.

Then the force on the piston due to gas is F = PA

When the piston is pushed outward an infinitesimal distance dx, the work done by the gas is $dW = F \times dx = PA\, dx$

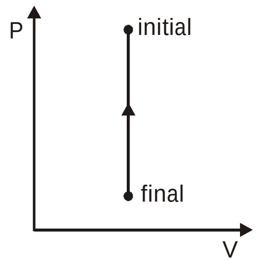
The change in volume of the gas is $dV = Adx$, $\therefore\ dW = PdV$

For a finite change in volume from V_i to V_f, this equation is then integrated between V_i to V_f to find the net work done $W = \int_{V_i}^{V_f} dW = \int_{V_i}^{V_f} PdV$

Hence the work done by a gas is equal to the area under P–V graph.

Following different cases are possible.

(i) Volume is constant



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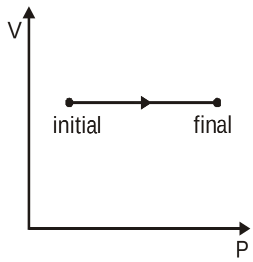
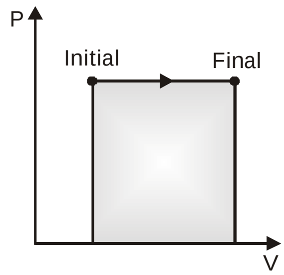


V = constant and $W_{AB} = 0$

(ii) Volume is increasing



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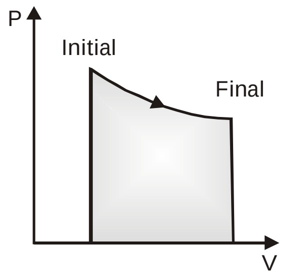
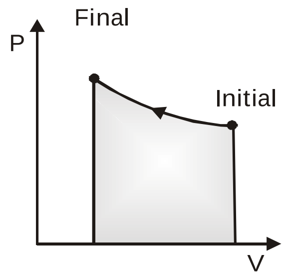


V is increasing $\qquad W_{AB} > 0 \qquad W_{AB}$ = Shaded area

(iii) Volume is decreasing



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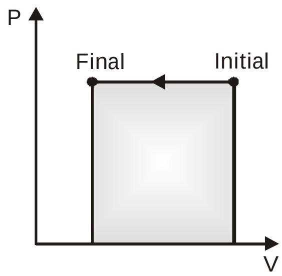
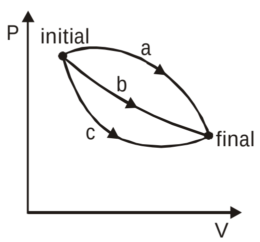


V is decreasing $\qquad W_{AB} < 0 \qquad W_{AB}$ = – Shaded area

FIRST LAW OF THERMODYNAMICS



Let a gas in a cylinder with a moveable piston changes from an initial equilibrium state to a final equilibrium state.

System change its state through path **'a'** :

The heat absorbed by the system in this process = δQ_a

The work done by the system = δW_a

- **Again for path 'b' :**

  Heat absorbed by the system = δQ_b , Work done by the system = δW_b.

  It is experimental fact that the $\delta Q_a - \delta W_a = \delta Q_b - \delta W_b$

  Both δQ and δW depend on the thermodynamic path taken between two equilibrium states, but difference $(\delta Q - \delta W)$ does not depends on path in between two definite states of the system.

  So, there is a function (internal energy) of the thermodynamic coordinates (P, V and T) whose final value (U_f) minus its initial value (U_i) equals the change $\delta Q - \delta W$ in the process.

- **$dU = \delta Q - \delta W$.** This is the first law of thermodynamics.

  Heat supplied to the system and work done by the system are path dependent so they are denoted by δQ and δW respectively. Change in internal energy $\Delta U = U_f - U_i$ does not depends on path it depends only on initial and final positions of the system. So, it is denoted by dU (or ΔU)

- **First Law of Thermodynamics**

  If some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system. $\delta Q = dU + \delta W$ or $Q = W + \Delta U$

  * This law is applicable to every process in nature

  * The first law of thermodynamics introduces the concept of internal energy.

  * The first law of thermodynamics is based on the law of conservation of energy.

  * δQ, dU and δW must be expressed in the same units (either in units of work or in units of heat).

  * This law is applicable to all the three phases of matter, i.e., solid, liquid and gas.

  * dU is a characteristic of the state of a system, it may be any type of internal energy–translational kinetic energy, rotational kinetic energy, binding energy etc.

- **Limitations of first law of thermodynamics** :

  It does not explain the direction of heat flow and it does not explain how much amount of heat given will be converted into work.

- **Significance of the first law of thermodynamics** :

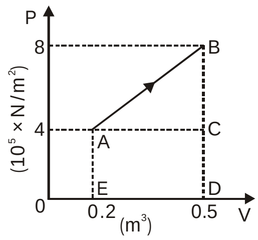
  The first law of thermodynamics tells us that it is impossible to get work from any machine without giving it an equivalent amount of energy.

Ex. The pressure in monoatomic gas increases linearly from 4×10^5 Nm^{-2} to $8 \times 10^{+5}$ Nm^{-2} when its volume increases from 0.2m^3 to 0.5 m^3. Calculate.

(i) Work done by the gas, (ii) Increase in the internal energy,

(iii) Amount of heat supplied, (iv) Molar heat capacity of the gas R = 8.31 J mol^{-1} K^{-1}

Sol. $P_1 = 4 \times 10^5$ Nm^{-2} $P_2 = 8 \times 10^{+5}$ Nm^{-2}, $V_1 = 0.2$ m^3, $V_2 = 0.5$ m^3



(i) Work done by the gas = Area under P–V graph (Area ABCDEA)

$$= \frac{1}{2}(AE + BD) \times AC = \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5\text{–}0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5 \text{ J}$$

(ii) Increase in internal energy $\Delta U = C_V(T_2 - T_1) = \frac{C_V}{R} R(T_2 - T_1) = \frac{C_V}{R}(P_2V_2 - P_1V_1)$

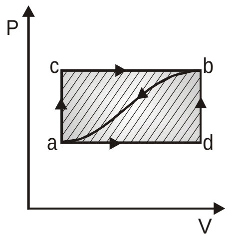
For monoatomic gas

$$C_V = \frac{3}{2}R \quad \therefore \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)] = \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5 \text{ J}$$

(iii) $Q = \Delta U + W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5$ J

(iv) $$C = \frac{Q}{\eta \Delta T} = \frac{QR}{\eta R \Delta T} = \frac{QR}{\eta(P_2V_2 - P_1V_1)} = \frac{6.6 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5} = 17.14 \text{ J/mole K}$$

Ex. When a system is taken from state a to state b, in figure along the path $a \to c \to b$, 60 J of heat flow into the system, and 30 J of work is done :



(i) How much heat flows into the system along the path $a \to d \to b$ if the work is 10 J.

(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and how much?

(iii) If, $U_a = 0$ and $U_d = 22$ J, find the heat absorbed in the process $a \to d$ and $d \to b$.

Sol. For the path a, c, b, $\Delta U = Q - W = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path a, d, b, $Q = \Delta U + W = 30 + 10 = 40$ J

(ii) Along the curved path b, a, $Q = (U_a - U_b) + W = (-30) + (-20) = -50$ J, heat flows out the system

(iii) $Q_{ad} = 32$ J; $Q_{db} = 8$ J

ISOMETRIC OR ISOCHORIC PROCESS

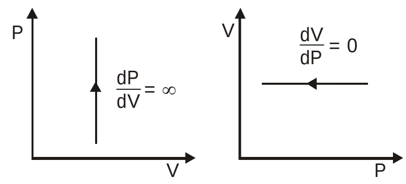
- Isochoric process is a thermodynamic process that takes place at constant volume of the system, but pressure and temperature varies for change in state of the system.

  Equation of state P = constant × T (P and T are variable, V is constant)

  Work done In this process volume remains constant $\Delta V = 0$ or $dV = 0 \Rightarrow W = \int_{V_i}^{V_f} PdV = 0$

  Form of first Law $Q = \Delta U$

  It means whole of the heat supplied is utilized for change in internal energy of the system. $Q = \Delta U = \mu C_v \Delta T$



  Slope of the P–V curve $\frac{dP}{dV} = \infty$

Specific heat at constant volume (C_V)

The quantity of heat required to raise the temperature of 1 gram mole gas through 1 °C at constant volume is equal to the specific heat at constant volume.

- A gas enclosed in a cylinder having rigid walls and a fixed piston. When heat is added to the gas, there would be no change in the volume of the gas.
- When a substance melts, the change in volume is negligibly small. So, this may be regarded as a nearly isochoric process.
- Heating process in pressure cooker is an example of isometric process.

Ex. An ideal gas has a specific heat at constant pressure $C_P = \frac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300K and a pressure of 1.6×10^6 Nm^{-2}. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas.

Sol. $C_V = C_P - R = \frac{5R}{2} - R = \frac{3R}{2}$, $\Delta V = 0$, $T_1 = 300$ K, $V = 0.0083$ m^3, $P_1 = 1.6 \times 10^6 Nm^{-2}$

From first law of thermodynamics $Q = \Delta U + P\Delta V \Rightarrow \Delta U = Q = 2.49 \times 10^4$ J

From gas equation $n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3}$

$\therefore \Delta U = nC_V\Delta T \Rightarrow \Delta T = \frac{\Delta U}{nC_v} = \frac{2.49 \times 10^4 \times 6}{3 \times 8.3 \times 16} = 375$ K

Final temperature = 300 + 375 = 675K

According to pressure law $P \propto T \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = \frac{T_2}{T_1} \times P_1 = \frac{1.6 \times 10^6 \times 675}{300} = 3.6 \times 10^6$ Nm^{-2}

Ex. 5 moles of oxygen is heated at constant volume from 10°C to 20°C. What will be change in the internal energy of the gas? The gram molecular specific heat of oxygen at constant pressure. C_P = 8 cal/mole and R = 8.36 joule/mole °C.

Sol. $\because C_V = C_P - R = 8 - 2 = 6$ cal/mole °C

$\therefore$ Heat absorbed by 5 moles of oxygen at constant volume

$$Q = nC_V \Delta T = 5 \times 6\,(20 - 10) = 30 \times 10 = 300 \text{ cal}$$

At constant volume $\Delta V = 0$. $\quad \therefore \Delta W = 0$

$\therefore$ From first law of thermodynamics $Q = \Delta U + W \Rightarrow 300 = \Delta U + 0 \Rightarrow \Delta U = 300$ cal

ISOBARIC PROCESS

Isobaric process is a thermodynamic process that takes place at constant pressure, but volume and temperature varies for change in state of the system.

- **Equation of state** $V = \text{constant} \times T$ or $V \propto T$
- **Work done** In this process pressure remains constant $\Delta P = 0$

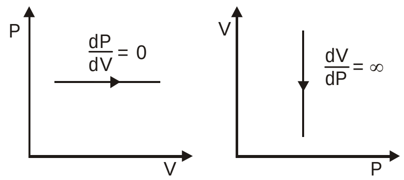
  Work done $W = \int_{V_i}^{V_f} P\,dV = P(V_f - V_i)$
- **Form of first Law** $Q = \Delta U + P\,(V_f - V_i)$

  $\mu C_p\,dT = \mu C_v\,dT + P(V_f - V_i)$

It is clear that heat supplied to the system is utilized for :

(i) Increasing internal energy and

(ii) Work done against the surrounding atmosphere.



- **Slope of the PV curve :** $\left(\frac{dP}{dV}\right)_{\text{isobaric}} = 0$
- **Specific heat at constant pressure (C_P)**

  The quantity of heat required to raise the temperature of 1 gram mole gas through 1°C at constant pressure is equal to the specific heat. Heating of water at atmospheric pressure.
- Melting of solids and boiling of liquids at atmospheric pressure.

Ex. At normal pressure and 0°C temperature the volume of 1 kg of ice is reduced by 91 cm^3 on melting. Latent heat of melting of ice is 3.4×10^5 J/kg. Calculate the change in the internal energy when 2kg of ice melts at normal pressure and 0°C. ($P = 1.01 \times 10^5\ Nm^{-2}$)

Sol. Heat energy absorbed by 2 kg of ice for melting $Q = mL = 2 \times 3.4 \times 10^5 = 6.8 \times 10^5 J$

Change in volume of 2 kg of ice $= 2 \times 91 = 182\ cm^3 = 182 \times 10^{-6}\ m^3$

$\therefore W = P\Delta V = 1.01 \times 10^5 \times (-182 \times 10^{-6}) = -18.4$ J

Since, work is done on ice so work W is taken –ve. Now from first law of thermodynamics

$$Q = \Delta U + W \Rightarrow \Delta U = Q - W = 6.8 \times 10^5 - (-18.4) = (6.8 \times 10^5 + 18.4)J$$

Ex. What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise the temperature by 45°C at constant pressure. Molecular mass of $N_2 = 28$, $R = 8.3\ J\ mol^{-1}\ K^{-1}$.

Sol. Here $m = 2 \times 10^{-2}$ kg, $\Rightarrow n = \frac{m}{M} = \frac{2\times10^{-2}}{28\times10^{-3}} = \frac{5}{7}$ & $C_P = \frac{7}{2}R$

$\therefore Q = nC_p\Delta T = \frac{5}{7}\times\frac{7}{2}\times 8.3\times 45 = 933.75$ J

ISOTHERMAL PROCESS

In this process pressure and volume of system change but temperature remains constant.

In an isothermal process, the exchange of heat between the system and the surroundings is allowed.

Isothermal process is carried out by either supplying heat to the substance or by extracting heat from it.

A process has to be extremely slow to be isothermal.

- **Equation of state**

$$P\,V = \text{constant } (\mu RT) \quad [T \text{ is constant}]$$

- **Work Done**

Consider μ moles of an ideal gas, enclosed in a cylinder, at absolute temperature T, fitted with a frictionless piston. Suppose that gas undergoes an isothermal expansion from the initial state (P_1, V_1) to the final state (P_2, V_2).

$\therefore$ Work done : $W = \int_{V_1}^{V_2} \frac{\mu RT}{V} dV = \mu RT \int_{V_1}^{V_2} \frac{dV}{V} = \mu RT \left[\log_e V\right]_{V_1}^{V_2}$

$= \mu RT\,[\log_e V_2 - \log_e V_1] = \mu RT \log_e \left[\frac{V_2}{V_1}\right]$

$\Rightarrow W = 2.303 \mu RT \log_{10}\left[\frac{P_1}{P_2}\right] \quad [\because P_1V_1 = P_2V_2]$

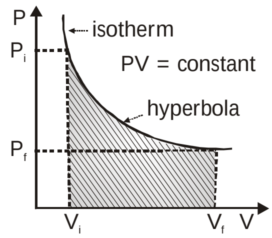
Form of First Law

There is no change in temperature and internal energy of the system depends on temperature only

So $\qquad \Delta U = 0,\ Q = 2.303\ \mu RT \log_{10}\left[\frac{V_2}{V_1}\right]$

It is clear that Whole of the heat energy supplied to the system is utilized by the system in doing external work. There is no change in the internal energy of the system.

Slope of the isothermal curve



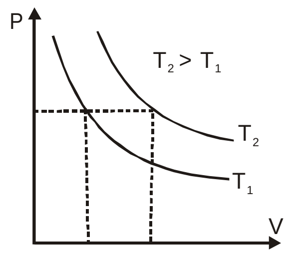
For an isothermal process, PV = constant

Differentiating, $PdV + VdP = 0 \quad \Rightarrow VdP = -PdV \Rightarrow \dfrac{dP}{dV} = -\dfrac{P}{V}$

Slope of isothermal curve, $\left[\dfrac{dP}{dV}\right]_{isothermal} = -\dfrac{P}{V}$

For a given system :

- The product of the pressure and volume of a given mass of a perfect gas remains constant in an isothermal process.
- Boyle's law is obeyed in an isothermal process.
- A graph between pressure and volume of a given mass of a gas at constant temperature is known as isotherm or isothermal of the gas.
- Two isotherms for a given gas at two different temperatures T_1 and T_2 are shown in figure.
- The curves drawn for the same gas at different temperatures are mutually parallel and do not cut each other.
- If two isotherms intersect each other at a single point we get same value of P and V at intersection point.



at const P $V \propto T$ so $T_2 > T_1$

- $PV = \mu RT_1$ for temperature T_1 and $PV = \mu RT_2$ for temperature T_2. It means $T_1 = T_2$ which is not possible.
- An ideal gas enclosed in a conducting cylinder fitted with a conducting piston. Let the gas be allowed to expand very–very slowly.
- This shall cause a very slow cooling of the gas, but heat will be conducted into the cylinder from the surrounding. Hence the temperature of the gas remains constant. If the gas is compressed very–very slowly, heat will be produced, but this heat will be conducted to the surroundings and the temperature of the gas shall remain constant.
- The temperature of a substance remains constant during melting. So, the melting process is an isothermal process.
- Boiling is an isothermal process, when a liquid boils, its temperature remains constant.
- If sudden changes are executed in a vessel of infinite conductivity then they will be isothermal.

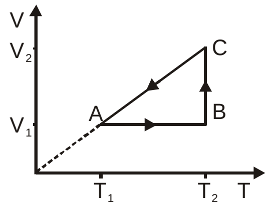
Ex. Two moles of a gas at 127°C expand isothermally until its volume is doubled. Calculate the amount of work done.

Sol. $n = 2$, $T = 127 + 273 = 400K$, $\frac{V_2}{V_1} = 2$

From formula $W = 2.3026\, nRT \log_{10} \frac{V_2}{V_1} = 2.3026 \times 2 \times 8.3 \times 400 \times \log_{10} 2$

$= 2.3026 \times 2 \times 8.3 \times 400 \times 0.3010 \approx 4.6 \times 10^3 J$

Ex. Figure shows a process ABCA performed on an ideal gas.



Find the net heat given to the system during the process.

Sol. Since the process is cyclic, hence the change in internal energy is zero.

The heat given to the system is then equal to the work done by it.

The work done in part AB is $W_1 = 0$ (the volume remains constant). The part BC represents an isothermal process so that the work done by the gas during this part is $W_2 = nRT_2\, \ell n \frac{V_2}{V_1}$

During the part CA $V \propto T$ So, V/T is constant and hence, $P = \frac{nRT}{V}$ is constant

The work done by the gas during the part CA is $W_3 = P(V_1 - V_2) = nRT_1 - nRT_2 = -nR(T_2 - T_1)$.

The net work done by the gas in the process ABCA is $W = W_1 + W_2 + W_3 = nR\left[T_2 \ell n \frac{V_2}{V_1} - (T_2 - T_1)\right]$

The same amount of heat is given to the gas.

ADIABATIC PROCESS

It is that thermodynamic process in which pressure, volume and temperature of the system change but there is no exchange of heat between the system and the surroundings.

A sudden and quick process will be adiabatic since there is no sufficient time available for exchange of heat so process adiabatic.

Equation of state : $PV = \mu RT$

Equation for adiabatic process PV^γ = constant

Work done

Let initial state of system is (P_1, V_1, T_1) and after adiabatic change final state of system is (P_2, V_2, T_2) then we can write $P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K$ (here K is const.)

So $$W = \int_{V_1}^{V_2} PdV = K\int_{V_1}^{V_2} V^{-\gamma} dV = K\left(\frac{V^{-\gamma+1}}{-\gamma+1}\right)_{V_1}^{V_2} = \frac{K}{(-\gamma+1)}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right] \quad \left(\because\ K = P_1V_1^{\gamma} = P_2V_2^{\gamma}\right)$$

$$\Rightarrow W = \frac{1}{(\gamma-1)}\left[P_1V_1^{\gamma}V_1^{-\gamma}.V_1 - P_2V_2^{\gamma}V_2^{-\gamma}.V_2\right] = \frac{1}{(\gamma-1)}\ [P_1V_1 - P_2V_2]$$

$$\Rightarrow W = \frac{\mu R}{(\gamma-1)}(T_1 - T_2) \quad (\because\ PV = \mu RT)$$

Form of first law : $dU = -\delta W$

It means the work done by an ideal gas during adiabatic expansion (or compression) is on the cost of change in internal energy proportional to the fall (or rise) in the temperature of the gas.

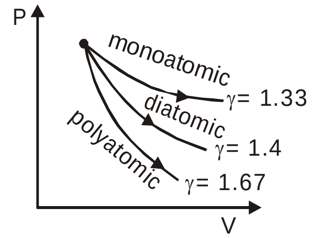
If the gas expands adiabatically, work is done by the gas. So, W_{adia} is positive.

The gas cools during adiabatic expansion and $T_1 > T_2$.

If the gas is compressed adiabatically, work is done on the gas. So, W_{adia} is negative.

The gas heats up during adiabatic compression and $T_1 < T_2$.

Slope of the adiabatic curve



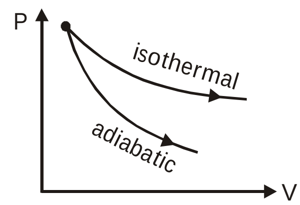
For an adiabatic process, PV^{γ} = constant

Differentiating, $P^{\gamma} V^{\gamma-1} dV + V^{\gamma} dP = 0$

$$\Rightarrow V^{\gamma} dP = -\gamma PV^{\gamma-1} dV \Rightarrow \frac{dP}{dV} = -\frac{\gamma PV^{\gamma-1}}{V^{\gamma}} = -\gamma\frac{P}{V} = \gamma\left(-\frac{P}{V}\right)$$

Slope of adiabatic curve, $$\left[\frac{dP}{dV}\right]_{adiabatic} = -\frac{\gamma P}{V}$$

Slope of adiabatic is greater than the slope of isotherm



$$\left[\frac{dP}{dV}\right]_{adia} = \gamma\left[-\frac{P}{V}\right] = \gamma\left[\frac{dP}{dV}\right]_{iso} \Rightarrow \frac{\text{slope of adiabatic changes}}{\text{slope of isothermal changes}} = \gamma$$

Since γ is always greater than one so an adiabatic is steeper than an isotherm

Examples of adiabatic process

- A gas enclosed in a thermally insulated cylinder fitted with a non–conducting piston. If the gas is compressed suddenly by moving the piston downwards, some heat is produced. This heat cannot escape the cylinder. Consequently, there will be an increase in the temperature of the gas.
- If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.
- Bursting of a cycle tube.
- Propagation of sound waves in a gas.
- In diesel engines burning of diesel without spark plug is done due to adiabatic compression of diesel vapour and air mixture

Ex. Why it is cooler at the top of a mountain than at sea level?

Sol. Pressure decreases with height. Therefore if hot air rises, it suffers adiabatic expansion.

From first law of thermodynamics $\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W\ [\because \Delta Q = 0]$

This causes a decrease in internal energy and hence a fall of temperature.

Ex. $2m^3$ volume of a gas at a pressure of $4 \times 10^5\ Nm^{-2}$ is compressed adiabatically so that its volume becomes $0.5m^3$. Find the new pressure. Compare this with the pressure that would result if the compression was isothermal. Calculate work done in each process. $\gamma = 1.4$

Sol. $V_1 = 2m^3$, $P_1 = 4 \times 10^5\ Nm^{-2}$, $V_2 = 0.5m^3$

In adiabatic process $P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow P_2 = 4 \times 10^5\left[\frac{2}{0.5}\right]^{1.4} = 4 \times 10^5\ (4)^{1.4} = 2.8 \times 10^6\ Nm^{-2}$

In isothermal process $P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2} = \frac{4 \times 10^5 \times 2}{0.5} = 1.6 \times 10^6\ Nm^{-2}$.

Now work done in adiabatic process $W = \frac{P_2V_2 - P_1V_1}{\gamma - 1} = \frac{(2.8 \times 10^6 \times 0.5) - (4 \times 10^5 \times 2)}{1.4 - 1} = 1.48 \times 10^6$ J.

Work done in isothermal process $W = 2.3026RT \log \frac{V_2}{V_1} = 2.3026P_1V_1 \log \frac{V_2}{V_1}$

$$= 2.3026 \times 4 \times 10^5 \times 2 \times \log\left[\frac{0.5}{2.0}\right] = 2.3026 \times 4 \times 10^5 \times 2\log\left(\frac{1}{4}\right) = -1.1 \times 10^6 \text{ J}$$

Ex. Two samples of a gas initially at same temperature and pressure are compressed from a volume V to $\frac{V}{2}$. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater ?

Sol. Let initial volume, $V_1 = V$ and pressure, $P_1 = P$, final volume, $V_2 = \frac{V}{2}$ and final pressure, $P_2 = ?$

For isothermal compression $P_2V_2 = P_1V_1$ or $P_2 = \frac{P_1V_1}{V_2} = \frac{PV}{\frac{V}{2}} = 2P$

For adiabatic compression $P_2' = P_1\left[\frac{V_1}{V_2}\right]^{\gamma} \Rightarrow P_2' = P\left[\frac{V}{V/2}\right]^{\gamma} = 2^{\gamma}P$

$\Rightarrow P_2' = 2^{\gamma}P \quad \gamma > 1 \therefore 2^{\gamma} > 2$ and $P_2' > P_2$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Ex. Calculate the work done when 1 mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litre respectively. The final volume of the gas is 2 liters. Molar specific heat of the gas at constant volume is $\frac{3R}{2}$ $[(3)^{5/3} = 6.19]$

Sol. For an adiabatic change PV^{γ} = constant $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

As molar specific heat of gas at constant volume $C_v = \frac{3}{2}R$

$$C_P = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R \Rightarrow \gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$$

$$\therefore P_2 = \left[\frac{V_1}{V_2}\right]^{\gamma} P_1 = \left[\frac{6}{2}\right]^{5/3} \times 10^5 = (3)^{5/3} \times 10^5 = 6.19 \times 10^5 \text{ N/m}^2$$

Work done $W = \frac{1}{1-\gamma}[P_2V_2 - P_1V_1] = \frac{1}{1-(5/3)}[6.19 \times 10^5 \times 2 \times 10^{-3} - 10^{-5} \times 6 \times 10^{-3}]$

$$= -\left[\frac{2 \times 10^2 \times 3}{2}(6.19 - 3)\right] = -3 \times 10^2 \times 3.19 = -957 \text{ joules}$$

– ive sign shows external work done on the gas

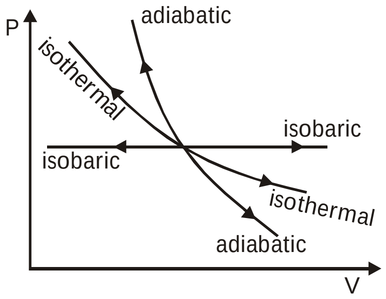
Ex. A motor tyre pumped to a pressure of 3 atm. It suddenly bursts. Calculate the fall in temperature due to adiabatic expansion. The temperature of air before expansion is 27°C. Given γ=1.4.

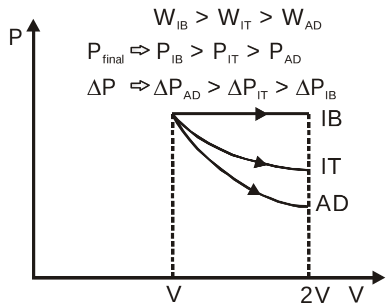
Sol. We know that $T_2^{\gamma}P_2^{1-\gamma} = T_1^{\gamma}P_1^{1-\gamma} \Rightarrow \left[\frac{T_2}{T_1}\right]^{\gamma} = \left[\frac{P_1}{P_2}\right]^{1-\gamma} \Rightarrow \left[\frac{T_2}{300}\right]^{1.4} = \left[\frac{3}{1}\right]^{1-1.4}$

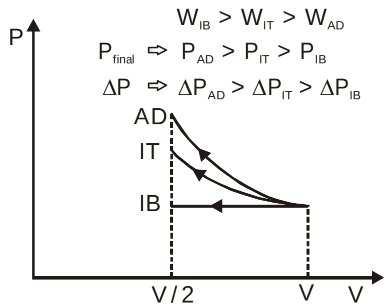
$\Rightarrow \left[\frac{T_2}{300}\right]^{1.4} = \left[\frac{1}{3}\right]^{0.4} \Rightarrow T_2 = 219.2\text{ K} \Rightarrow T_1 - T_2 = (300 - 219.2)\text{ K} = 80.8\text{ K}$

GOLDEN KEY POINTS

- When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.



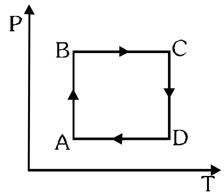




First Law of Thermodynamics Applied to Different Processes

Process	Q	ΔU	W
Cyclic	W	0	Area of the closed curve
Isochoric	ΔU	$\mu C_V\Delta T$ (μ mole of gas)	0
Isothermal	W	0	$\mu RT\log_e\left[\frac{V_f}{V_i}\right] = \mu RT\log_e\left[\frac{P_i}{P_f}\right]$
Adiabatic	0	–W	$\frac{\mu R(T_f - T_i)}{1-\gamma}$
Isobaric	$\mu C_P\Delta T$	$\mu C_V\Delta T$	$P(V_f - V_i) = \mu R(T_f - T_i)$

Ex. Plot P – V , V – T graph corresponding to the P–T graph for an ideal gas shown in figure. Explain your answers.

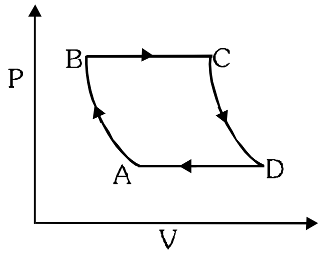


Sol. For process AB $\quad T$ = constant so $P \propto \frac{1}{V}$

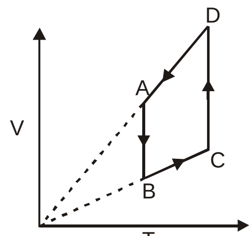
For process BC $\quad P$ = constant so $V \propto T$

For process CD $\quad T$ = constant so $V \propto \frac{1}{P}$

For process DA $\quad P$ = constant so $V \propto T$



(P - V curve)

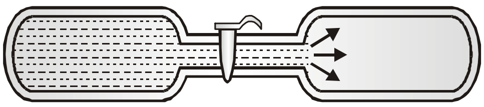


(V - T curve)

FREE EXPANSION

Take a thermally insulated bottle with ideal gas at some temperature T_1 and, by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated. If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.

Experimentally, we find that this process of free expansion does not change the temperature of the gas – when the gas attains equilibrium and stops flowing, the final temperature of both bottles are equal to the initial temperature T_1.



free expansion of a gas

This process is called a free expansion.

The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free–expansion process.

The process is adiabatic because of the insulation, So Q = 0.

No part of the surroundings moves so the system does no work on its surroundings.

- **For ideal gas** $(\delta W)_{ext.}$ = Work done against external atmosphere

$= P\,dV = 0$ (because P = 0)

$(\delta W)_{int.}$ = Work done against internal molucular forces = 0

$\delta Q = dU + \delta W \Rightarrow 0 = dU + 0$

The internal energy does not change dU = 0 So U and T are constant.

The initial and final states of this gas have the same internal energy.

Which implies that the internal energy of an ideal gas does not depend on the volume at all.

The free–expansion process has led us to the following conclusion :

The internal energy U(T) of an ideal gas depends only on the temperature.

- **For real gas**

In free expansion of real gases, measurements show that the temperature changes slightly in a free expansion. Which implies that the internal energy of a real gas depends on the volume also.

$\delta Q = 0 \quad (\delta W)_{ext.} = 0 \; (\because P = 0)$

$(\delta W)_{int.} \neq 0$ (Intermolucular forces are present in real gases)

$\delta Q = dU + \delta W \Rightarrow 0 = dU + (\delta W)_{int.} \Rightarrow dU = -(\delta W)_{int.}$

$\Rightarrow$ U decreases. So T decreases.

RELATION BETWEEN DEGREE OF FREEDOM AND SPECIFIC HEAT OF GAS

Energy related with each degree of freedom = $\frac{1}{2}kT$, Energy related with all degree of freedom = $\frac{f}{2}kT$

Internal energy of one mole of ideal gas (total K.E.) $U = \frac{f}{2}RT$ for Isometric process (volume constant)

$\delta W = 0$

By first law of thermodynamics $\delta Q = \delta W + dU \Rightarrow C_V\, dT = dU \Rightarrow C_V = \frac{dU}{dT}$

$C_V = \frac{dU}{dT} = \frac{f}{2}R = \frac{R}{\gamma - 1}$. $C_P = C_V + R = \left[\frac{f}{2}+1\right]R = \frac{\gamma R}{\gamma - 1}$ and $\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

$C_V = \frac{R}{\gamma - 1}$, $C_P = \frac{\gamma R}{\gamma - 1}$ and $\gamma = 1 + \frac{2}{f}$

General expression for C (C_P or C_V) in the process PV^x = constant $C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$

For isobaric process P = constant so $x = 0$ $\therefore$ $C = C_P = \frac{R}{\gamma - 1} + R = C_V + R$

For isothermal process, PV = constant so $x = 1$ $\therefore$ $C = \infty$

For adiabatic process PV^γ = constant so $x = \gamma$ $\therefore$ $C = 0$

Values of f, U, C_V, C_P and γ for different gases are shown in table below.

Atomicity of gas	f	C_V	C_P	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.4$
Triatomic and Triatomic linear	7	$\frac{7}{2}R$	$\frac{9}{2}R$	$\frac{9}{7} = 1.28$
Poly atomic Triangular Non-linear	6	$\frac{6}{2}R = 3R$	$\frac{8}{2}R = 4R$	$\frac{4}{3} = 1.33$

- $1 < \gamma < 2$
- If atomicity of gases is same U, C_P, C_V and γ is same for gas mixture.
- If in a gas mixture gases are of different atomicity, then for gas mixture γ changes according to following condition. Diatomic $\gamma_1 \le$ mixture $\le \gamma_2$ mono atomic where $\gamma_1 < \gamma_2$
- If 'f' is the degree of freedom per molecule for a gas, then

  Total energy of each molecule $= \frac{fkT}{2}$

  Total energy per mole of gas = $N_0 \frac{f}{2}kT = \frac{f}{2}RT$

- According to kinetic theory of gases, the molecule are not interacting with each other. So potential energy is zero and internal energy of gas molecules is only their kinetic energy.

- For 'μ' mole of a gas : Internal energy at temperature T is $U = \frac{\mu f R T}{2} = \mu C_V T$

- Change in internal energy is given by $dU = \frac{\mu f R}{2}(dT) = \mu C_V dT$

  This change is process independent.

C_P is greater than C_V

If a gas is heated at constant volume, the gas does no work against external pressure. In this case, the whole of the heat energy supplied to the gas is spend in raising the temperature of the gas.

If a gas is heated at constant pressure, its volume increases. In this case, heat energy is required for the following two purpose :

(i) To increase the volume of the gas against external pressure.

(ii) To increase the temperature of 1 mole of gas through 1 K.

Thus, more heat energy is required to raise the temperature of 1 mole of gas through 1 K when it is heated at constant pressure than when it is heated at constant volume. $\therefore\ C_P > C_V$

The difference between C_P and C_V is equal to thermal equivalent of the work done by the gas in expanding against external pressure.

Mayer's formula : $C_P - C_V = R$

$\because$ At constant pressure $dQ = \mu C_P dT$, $dU = \mu C_V dT$ & $dW = PdV = \mu RdT$

Now from first law of thermodynamics $dQ = dW + dU$

$\Rightarrow \mu C_P dT = \mu RdT + \mu C_V dT \Rightarrow C_P = R + C_V \Rightarrow C_P - C_V = R$

Ex. Calculate the difference between two specific heats of 1 g of helium gas at NTP. Molecular weight of helium = 4 and J = 4.186 × 10^7 erg cal^{-1}.

Sol. Gas constant for 1 g of helium, $r = \frac{R}{M_w} = \frac{PV}{T \times M_w} = \frac{76 \times 13.6 \times 981 \times 22400}{273 \times 4} = 2.08 \times 10^7$ erg g^{-1} K^{-1}

$C_P - C_V = \frac{r}{J} = \frac{2.08 \times 10^7}{4.186 \times 10^7} = 0.5$ cal g^{-1} K^{-1}

Ex. Calculate the molar specific heat at constant volume. Given : specific heat of hydrogen at constant pressure is 6.85 cal mol^{-1} K^{-1} and density of hydrogen = 0.0899 g cm^{-3}. One mole of gas = 2.016 g, J = 4.2 × 10^7 erg cal^{-1} and 1 atmosphere = 10^{6} dyne cm^{-2}.

Sol. Since the density of hydrogen is 0.0899 g cm^{-3} therefore volume occupied by 0.0899 g of hydrogen at NTP is 1000 cm^3. So, volume of 1 mole (2.016 g) of gas, $V = \frac{1000}{0.0899} \times 2.016\ cm^3$

$$C_P - C_V = \frac{R}{J} = \frac{PV}{T \times J} = \frac{10^6 \times 1000 \times 2.016}{0.0899 \times 273 \times 4.2 \times 10^7} = 1.96\ cal\ mol^{-1}\ K^{-1}$$

$$\therefore C_V = C_P - 1.96 = (6.85 - 1.96) = 4.89\ cal\ mol^{-1}\ K^{-1}$$

Ex. The specific heat of argon at constant volume is 0.075 kcal/kg K. Calculate its atomic weight, [R = 2cal/mol K]

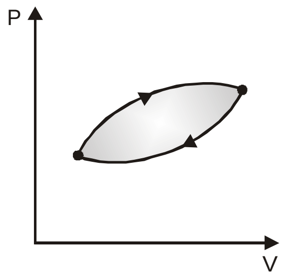
Sol. As argon is monoatomic, its molar specific heat at constant volume will be

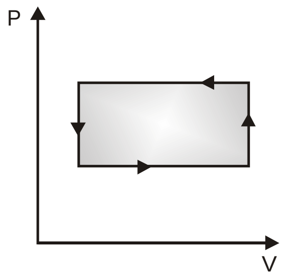
$$C_V = \frac{3}{2}R = \frac{3}{2} \times 2 = 3\ \text{cal/mol K},\ C_V = M_w c_V \text{ and } c_V = 0.075\ \text{cal/g K}$$

So $3 = M_w \times 0.075 \Rightarrow M_w = \frac{3}{0.075} = 40$ gram/mole

Cyclic process

Cyclic process is that thermodynamic process in which the system returns to its initial stage after undergoing a series of changes. A typical cyclic process is represented on PV diagram as shown in figure.

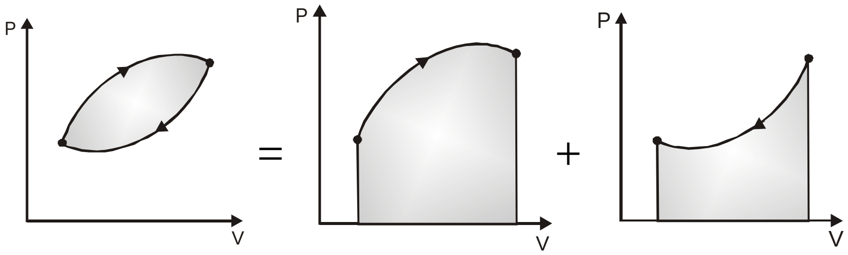




$W_{\text{clockwise cycle}}$ = + Shaded area

$W_{\text{anticlockwise cycle}}$ = – Shaded area

WORK DONE IN CLOCKWISE CYCLE



$W_{AB\ I}$ Positive $\qquad\qquad$ $W_{AB\ II}$ Negative

$W_{cyclic} = W_{AB\ I}$(positive) + $W_{AB\ II}$ (negative) = 

Similarly work done in anticlockwise cyclic process is negative.

8. CARNOT CYCLE

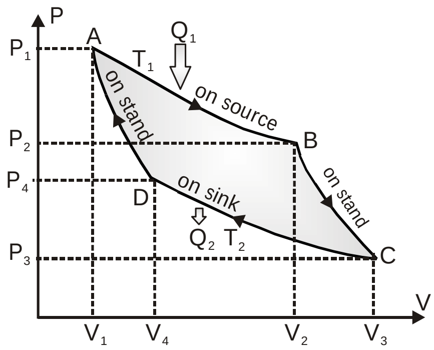
Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession :

(i) isothermal expansion, $\qquad A \rightarrow B$

(ii) adiabatic expansion, $\qquad B \rightarrow C$

(iii) isothermal compression $\qquad C \rightarrow D$

(iv) adiabatic compression. $\qquad D \rightarrow A$



Main parts of Carnot's engine are

Source of heat

It is a hot body of very large heat capacity kept at a constant high temperature T_1.

Mechanical arrangements and working substance

It is a cylinder whose walls are perfectly non-conducting and its base is perfectly conducting fitted with non-conducting piston. This piston move without any friction. Ideal gas enclosed in cylinder as a working substance.

Heat sink

It is a cold body at low temperature T_2. It is a body of large heat capacity.

Stand

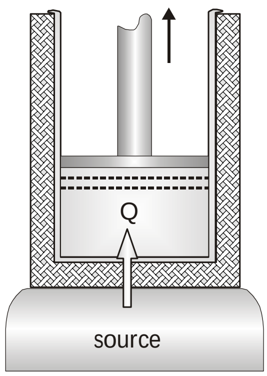
It is of two types : conducting and nonconducting.

Working

Through a set of reversible processes the working substance is taken back to initial condition to get maximum work from this type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicator diagram.

Isothermal expansion A $\rightarrow$ B



Initially the cyclinder is taken to be in thermal equilibrium with the high temperature T_1, this is initial state of working substance denoted by point A (P_1, V_1, T_1).

After that the piston is allowed to move outward slowly. With the movement of the piston the process is very slow so that it is isothermal.

Heat from reservoir flows through the base of cylinder into the gas so temperature of the gas remains T_1.

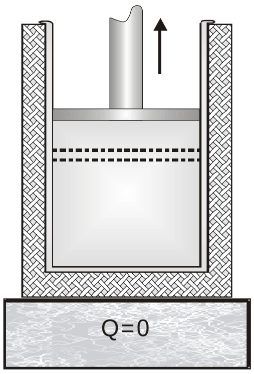
Gas expand and receive heat Q_1 from source and gets state B(P_2, V_2, T_1)

This heat input Q_1 to the gas from path A to B is utilized for doing work W_1.

By path A to B the heat input to the gas = the work done against the external pressure.

$$W_1 = Q_1 = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{\mu RT_1}{V} dV = \mu RT_1 \, \ell n \frac{V_2}{V_1}$$

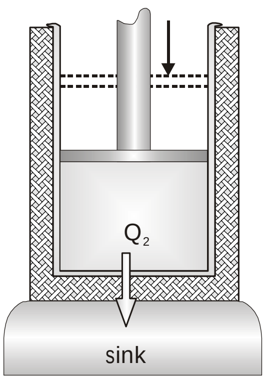
Adiabatic expansion B $\rightarrow$ C



Now cylinder is put in contact with a non-conducting stand and piston is allowed to move outward,because no heat can enter in or leave out so the expansion of gas is adiabatic. The tempeature falls to T_2K and gas describes the adiabatic process from B to **point C** (P_3, V_3, T_2) during this expansion more work is done (W_2) at the expense of the internal energy.

Work done in adiabatic path BC is $W_2 = \frac{\mu R}{\gamma - 1}(T_1 - T_2)$

Isothermal compression C $\rightarrow$ D



Now the gas cylinder is placed in contact with sink at temperature T_2. The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed from C to **point D**. (P_4, V_4, T_2)

The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over this path.

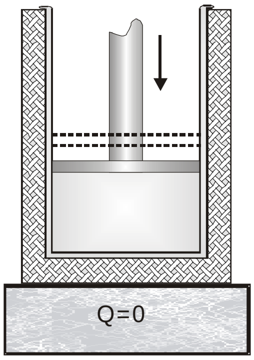
Amount of work done on gas W_3 = amount of heat rejected to the sink

$$Q_2 = W_3 = \mu RT_2 \ell n \left(\frac{V_4}{V_3}\right) \Rightarrow Q_2 = \mu RT_2 \ell n \left(\frac{V_4}{V_3}\right)$$

Adiabatic compression D → A

The cylinder is removed from the sink and is put in contact with insulating stand now piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Now work is done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1).

No heat exchanges occur over the adiabatic path.



Work done on the system $W_4 = \frac{\mu R}{\gamma - 1}(T_2 - T_1)$

This cycle of operations is called a Carnot cycle.

In first two steps work is done by engine W_1 and W_2 are positive

In last two steps work is done on gas W_3 and W_4 are negative

The work done in complete cylce W = the area of the closed part of the P–V cycle.

$$W = W_1 + W_2 + W_3 + W_4$$

$$\therefore \quad W = \mu RT_1 \ell n\frac{V_2}{V_1} + \frac{\mu R}{\gamma - 1}(T_1 - T_2) + \mu RT_2 \ell n\frac{V_4}{V_3} + \frac{\mu R}{\gamma - 1}(T_2 - T_1) = \mu RT_1 \ell n\frac{V_2}{V_1} + \mu RT_2 \ell n\frac{V_4}{V_3}$$

Efficiency of Carnot Engine, $\eta = \frac{W}{Q_1} = \dfrac{\mu RT_1 \ell n\frac{V_2}{V_1} + \mu RT_2 \ell n\frac{V_4}{V_3}}{\mu RT_1 \ell n\frac{V_2}{V_1}}$

B to C and D to A are adiabatic paths

so $T_1 V_2^{(\gamma-1)} = T_2 V_3^{(\gamma-1)}$ and $T_1 V_1^{(\gamma-1)} = T_2 V_4^{(\gamma-1)} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\eta = \frac{T_1 - T_2}{T_1} \times 100\% \Rightarrow \eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

The efficiency for the Carnot engine is the best that can be obtained for any heat engine.

The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink $T_2 = 0\,K$ and $T_1 = \infty$ which is impossible.

CARNOT THEOREM

No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R) working between same hot and cold reservoirs. $\eta_R > \eta_I \Rightarrow 1 - \frac{T_2}{T_1} > 1 - \frac{Q_2}{Q_1}$

9. SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work :

(i) A device called engine with a working substance is essential.

(ii) The engine must work in a reversible cyclic process.

(iii) The engine must operate between two temperatures. It will absorb heat from a hot body (called source), and converts a part of it into useful work and reject the rest to a cold body (called sink).

There are two conventional statements of second law :

Kelvin-Planck Statement

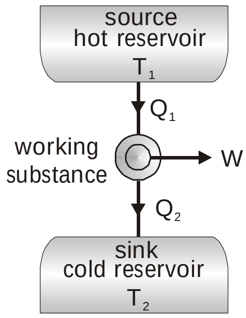
It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

Clausius Statement

It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.

10. HEAT ENGINE

Heat engine is a device which converts heat into work.



Three parts of a heat engine:

(i) Source of high temperature reservoir at temperature T_1

(ii) Sink or low temperature reservoir at temperature T_2

(iii) Working substance.

In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to the sink.

Efficiency of heat engine $\eta = \dfrac{\text{work done (W)}}{\text{heat taken from source } (Q_1)}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

Ex. A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

Sol. W = 1200 J, T_1 = 800 K, T_2 = 400 K

$$\therefore \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow 1 - \frac{400}{800} = \frac{1200}{Q_1} \Rightarrow 0.5 = \frac{1200}{Q_1}$$

Heat energy supplied by source $Q_1 = \frac{1200}{0.5}$ = 2400 joule per cycle

Ex. The temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine are 1500°C and 500°C respectively. Which of the following : increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?

Sol. T_1 = 1500°C = 1500 + 273 = 1773 K and T_2 = 500°C = 500 + 273 = 773 K.

The efficiency of a carnot's engine $\eta = 1 - \frac{T_2}{T_1}$

When the temperature of the source is increased by 100°C, keeping T_2 unchanged, the new temperature of the source is T'_1 = 1500 + 100 = 1600°C = 1873 K. The efficiency becomes

$$\eta' = 1 - \frac{T_2}{T_1'} = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by 100°C, keeping T_1 unchanged, the new temperature of the sink is T'_2 = 500 – 100 = 400°C = 673 K. The efficiency now becomes

$$\eta'' = 1 - \frac{T'_2}{T_1} = 1 - \frac{673}{1773} = 0.62$$

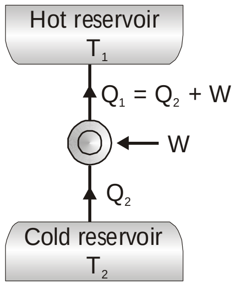
Since η'' is greater than η', decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperatue of the source by 100°C.

Ex. A heat engine operates between a cold reservoir at temperature T_2 = 300 K and a hot reservoir at temperature T_1. It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of hot reservoir?

Sol. Work done by the engine in a cycle is W = 200 – 120 = 80 J. $\eta = \frac{W}{Q} = \frac{80}{200} = 0.4$

From carnot's Theorem $0.4 \le 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1}$ or $\frac{300}{T_1} \le 0.6$ or $T_1 \ge \frac{300}{0.6}$ or $T_1 \ge 500$

11. REFRIGERATOR



It is inverse of heat engine. It extracts heat (Q_2) from a cold reservoir, External work W is done on it and rejects heat (Q_1) to hot reservoir. The coefficient of performance of a refrigerator.

$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For Carnot reversible refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\therefore\ \beta = \frac{Q_2}{W} = \frac{1}{\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]} \Rightarrow \beta = \frac{T_2}{T_1 - T_2}$$

Ex. A carnot engine works as a refrigerator between 250 K and 300 K. If it receives 750 cal of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

Sol. $T_1 = 300$ K $\quad T_2 = 250$ K $\quad Q_2 = 750 \quad Q_1 = ?$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \qquad Q_1 = \frac{300}{250} \times 750 = 900 \text{ cal}$$

Ex. The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

Sol. $T_2 = 260$ K, $T_1 = 315$ K, W = 1 joule

Coefficient of performance of Carnot refrigerator $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore\ \frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55} \Rightarrow Q_2 = \frac{260}{55} = 4.7 \text{ J}$$

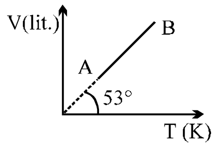
Ex. A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

Sol. Coefficient of performance (COP) $= \frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} = \frac{273}{27}$

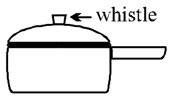
$$W = \frac{Q_2}{COP} = \frac{mL}{COP} = \frac{100 \times 3.4 \times 10^5}{273/27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6 \text{ J}$$

EXERCISE (S-1)

1. V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.



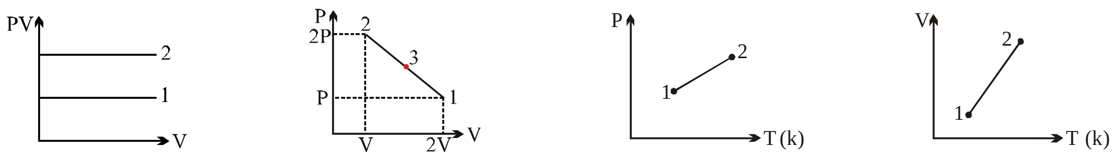
2. An empty pressure cooker of volume 10 litres contains air at atmospheric pressure 10^5 Pa and temperature of 27°C. It contains a whistle which has area of 0.1 cm^2 and weight of 100 g. What should be the temperature of air inside so that the whistle is just lifted up?



3. Examine the following plots and predict whether in

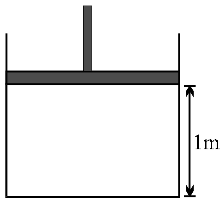
(i) $P_1 < P_2$ and $T_1 > T_2$, in (ii) $T_1 = T_2 < T_3$, in

(iii) $V_1 > V_2$, in (iv) $P_1 > P_2$ or otherwise.



4. Find the molecular mass of a gas if the specific heats of the gas are C_P=0.2 cal/gm°C and C_V = 0.15 cal/gm°C. [Take R = 2 cal/mole°C]

5. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is1 m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas. You can leave the answer in fraction. **[JEE' 2004]**



6. A mixture of 4 gm helium and 28 gm of nitrogen is enclosed in a vessel of constant volume 300 K. Find the quantity of heat absorbed by the mixture to doubled the root mean velocity of its molecules. (R = Universal gas constant)

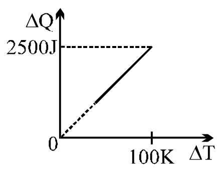
7. An insulated container containing monoatomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **[JEE 2003]**

8. If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature change. When the temperature of the gas changes by 300 K, determine the change is the internal energy of the gas.

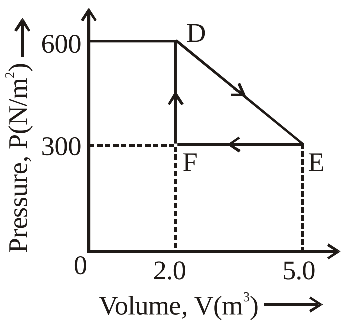
9. Ideal diatomic gas is taken through a process $\Delta Q = 2\Delta U$. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

10. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume ?

11. One mole of a gas mixture is heated under constant pressure, and heat required ΔQ is plotted against temperature difference acquired. Find the value of γ for mixture.

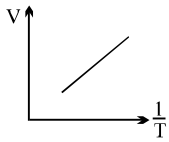


12. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in figure. Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F



13. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case ? (Take 1 cal = 4.19 J)

14. One mole of an ideal monoatomic gas undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.



15. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following :

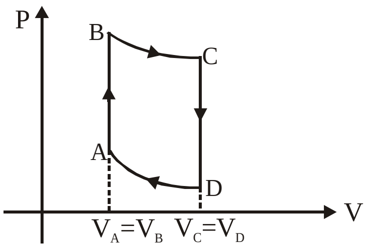
(a) What is the final pressure of the gas in A and B ?

(b) What is the change in internal energy of the gas ?

(c) What is the change in the temperature of the gas ?

(d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface ?

16. A cycle followed by an engine (made of one mole of perfect gas in a cylinder with a piston) is shown in figure.



A to B : volume constant

B to C : adiabatic

C to D : volume constant

D to A : adiabatic

$V_C = V_D = 2V_A = 2V_B$

(a) In which part of the cycle heat is supplied to the engine from outside?

(b) In which part of the cycle heat is being given to the surrounding by the engine?

(c) What is the work done by the engine in one cycle? Write your answer in term of P_A, P_B, V_A

(d) What is the efficiency of the engine?

$[\gamma = \frac{5}{3}$ for the gas], ($C_V = \frac{3}{2}R$ for one mole)

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

KTG

1. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3×10^5 pascal

(A) 2.4×10^{23} (B) 3×10^{23} (C) 6×10^{23} (D) 4.8×10^{23}

2. An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3}$ = constant. The temperature inside the balloon is

(A) increasing (B) decreasing (C) constant (D) can't be said

3. At a temperature T K, the pressure of 4.0g argon in a bulb is p. The bulb is put in a bath having temperature higher by 50K than the first one. 0.8g of argon gas had to be removed to maintained original pressure. The temperature T is equal to

(A) 510 K (B) 200 K (C) 100 K (D) 73 K

4. 28 gm of N_2 gas is contained in a flask at a pressure of 10 atm and at a temperature of 57°. It is found that due to leakage in the flask, the pressure is reduced to half and the temperature reduced to 27°C. The quantity of N_2 gas that leaked out is :-

(A) 11/20 gm (B) 20/11 gm (C) 5/63 gm (D) 63/5 gm

5. A container X has volume double that of container Y and both are connected by a thin tube. Both contains same ideal gas. The temperature of X is 200K and that of Y is 400K. If mass of gas in X is m then in Y it will be:

(A) m/8 (B) m/6 (C) m/4 (D) m/2

6. When 2 gms of a gas are introduced into an evacuated flask kept at 25^0C the pressure is found to be one atmosphere. If 3 gms of another gas added to the same flask the pressure becomes 1.5 atmospheres. The ratio of the molecular weights of these gases will be :-

(A) 1 : 3 (B) 3 : 1 (C) 2 : 3 (D) 3 : 2

7. A rigid tank contains 35 kg of nitrogen at 6 atm. Sufficient quantity of oxygen is supplied to increase the pressure to 9 atm, while the temperature remains constant. Amount of oxygen supplied to the tank is :

(A) 5 kg (B) 10 kg (C) 20 kg (D) 40 kg

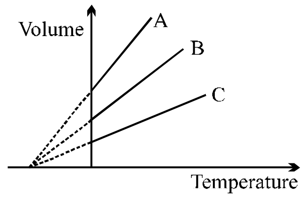
8. During an experiment an ideal gas obeys an addition equation of state P^2V = constant. The initial temperature and pressure of gas are T and V respectively. When it expands to volume 2V, then its temperature will be:

(A) T (B) $\sqrt{2}\ T$ (C) 2 T (D) $2\sqrt{2}\ T$

9. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 2T has a pressure of

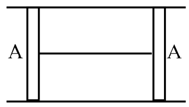
(A) P/8 (B) P (C) 2P (D) 8P

10. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line B. Then the expansion of the same ideal gas of mass 2 m at a pressure 2P is given by the straight line



(A) C (B) A (C) B (D) none

11. A cylindrical tube of cross-sectional area A has two air tight frictionless pistons at its two ends. The pistons are tied with a straight piece of metallic wire. The tube contains a gas at atmospheric pressure P_0 and temperature T_0. If temperature of the gas is doubled then the tension in the wire is



(A) $4\ P_0\ A$ (B) $P_0A/2$ (C) $P_0\ A$ (D) $2\ P_0\ A$

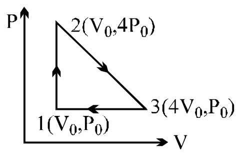
12. One mole of an ideal diatomic gas is taken through the cycle as shown in the figure.

$1 \rightarrow 2$: isochoric process

$2 \rightarrow 3$: straight line on P-V diagram

$3 \rightarrow 1$: isobaric process

The average molecular speed of the gas in the states 1, 2 and 3 are in the ratio



(A) 1 : 2 : 2 (B) $1 : \sqrt{2} : \sqrt{2}$ (C) 1 : 1 : 1 (D) 1 : 2 : 4

13. One mole of an ideal gas at STP is heated in an insulated closed container until the average speed of its molecules is doubled. Its pressure would therefore increase by factor.

(A) 1.5 (B) $\sqrt{2}$ (C) 2 (D) 4

14. Three particles have speeds of 2u, 10u and 11u. Which of the following statements is correct?

(A) The r.m.s. speed exceeds the mean speed by about u.

(B) The mean speed exceeds the r.m.s. speed by about u.

(C) The r.m.s. speed equals the mean speed.

(D) The r.m.s. speed exceeds the mean speed by more than 2u.

15. One mole of an ideal gas is contained with in a cylinder by a frictionless piston and is initially at temperature T. The pressure of the gas is kept constant while it is heated and its volume doubles. If R is molar gas constant, the work done by the gas in increasing its volume is :-

(A) RT $ln2$ (B) 1/2 RT (C) RT (D) 3/2 RT

16. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :-

(A) 4 RT (B) 15 RT (C) 9 RT (D) 11 RT

First Law of Thermodynamics :

17. In thermodynamic process pressure of a fixed mass of gas is changed in such a manner that the gas releases 30 joule of heat and 18 joule of work was done on the gas. If the initial internal energy of the gas was 60 joule, then, the final internal energy will be :

(A) 32 joule (B) 48 joule (C) 72 joule (D) 96 joule

18. Two monoatomic ideal gas at temperature T_1 and T_2 are mixed. There is no loss of energy. If the masses of molecules of the two gases are m_1 and m_2 and number of their molecules are n_1 and n_2 respectively. The temperature of the mixture will be :

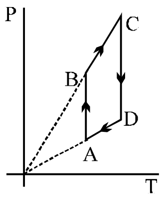
(A) $\frac{T_1+T_2}{n_1+n_2}$ (B) $\frac{T_1}{n_1}+\frac{T_2}{n_2}$ (C) $\frac{n_2T_1+n_1T_2}{n_1+n_2}$ (D) $\frac{n_1T_1+n_2T_2}{n_1+n_2}$

19. An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then **[JEE' 2004 (Scr)]**

(A) $P_3 > P_1$, $W > 0$ (B) $P_3 < P_1$, $W < 0$

(C) $P_3 > P_1$, $W < 0$ (D) $P_3 = P_1$, $W = 0$

20. Pressure versus temperature graph of an ideal gas is shown in figure



(A) During the process AB work done by the gas is positive

(B) During the process CD work done by the gas is negative

(C) During the process BC internal energy of the gas is increasing

(D) None

21. A polyatomic gas with six degrees of freedom does 25J of work when it is expanded at constant pressure. The heat given to the gas is :-

(A) 100J (B) 150J (C) 200J (D) 250J

22. A reversible adiabatic path on a P-V diagram for an ideal gas passes through state A where $P = 0.7\times10^5$ N/m^{-2} and $V = 0.0049$ m^3. The ratio of specific heat of the gas is 1.4. The slope of path at A is :

(A) 2.0×10^7 Nm^{-5} (B) 1.0×10^7 Nm^{-5} (C) -2.0×10^7 Nm^{-5} (D) -1.0×10^7 Nm^{-5}

23. The adiabatic Bulk modulus of a diatomic gas at atmospheric pressure is

(A) 0 Nm^{-2} (B) 1 Nm^{-2} (C) 1.4×10^4 Nm^{-2} (D) 1.4×10^5 Nm^{-2}

24. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :

(A) 2P/3 (B) P (C) 3P/2 (D) 2P

25. One mole of an ideal gas at temperature T_1 expends according to the law $\frac{P}{V^2}$ = a (constant).

The work done by the gas till temperature of gas becomes T_2 is :

(A) $\frac{1}{2}R(T_2 - T_1)$ (B) $\frac{1}{3}R(T_2 - T_1)$ (C) $\frac{1}{4}R(T_2 - T_1)$ (D) $\frac{1}{5}R(T_2 - T_1)$

26. The first law of thermodynamics can be written as $\Delta U = \Delta Q + \Delta W$ for an ideal gas. Which of the following statements is correct?

(A) ΔU is always zero when no heat enters or leaves the gas

(B) ΔW is the work done by the gas in this written law.

(C) ΔU is zero when heat is supplied and the temperature stays constant

(D) $\Delta Q = -\Delta W$ when the temperature increases very slowly.

27. 2 moles of a monoatomic gas are expanded to double its initial volume, through a process P/V = constant. If its initial temperature is 300 K, then which of the following is not true.

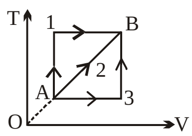
(A) ΔT = 900 K (B) ΔQ = 3200 R (C) ΔQ = 3600 R (D) W = 900 R

28. A student records ΔQ, ΔU & ΔW for a thermodynamic cycle $A \to B \to C \to A$. Certain entries are missing. Find correct entry in following options.

	AB	BC	CA
ΔW	40J		30J
ΔU		50J	
ΔQ	150J	10J	

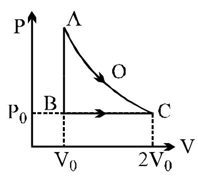
(A) W_{BC} = – 70 J (B) ΔQ_{CA} = 130 J (C) ΔU_{AB} = 190 J (D) ΔU_{CA} = – 160 J

29. A given mass of a gas expands from a state A to the state B by three paths 1, 2 and 3 as shown in T-V indicator diagram. If W_1, W_2 and W_3 respectively be the work done by the gas along the three paths, then



(A) $W_1 > W_2 > W_3$ (B) $W_1 < W_2 < W_3$ (C) $W_1 = W_2 = W_3$ (D) $W_1 < W_2, W_1 > W_3$

30. An ideal gas is taken from point A to point C on P-V diagram through two process AOC and ABC as shown in the figure. Process AOC is isothermal



(A) Process AOC requires more heat than process ABC.

(B) Process ABC requires more heat than process AOC.

(C) Both process AOC & ABC require same amount of heat.

(D) Data is insufficient for comparison of heat requirement for the two processes.

31. Monoatomic, diatomic and triatomic gases whose initial volume and pressure are same, are compressed till their volume becomes half the initial volume.

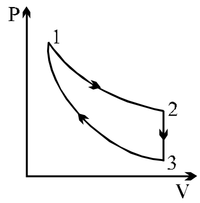
(A) If the compression is adiabatic then monoatomic gas will have maximum final pressure.

(B) If the compression is adiabatic then triatomic gas will have maximum final pressure.

(C) If the compression is adiabatic then their final pressure will be same.

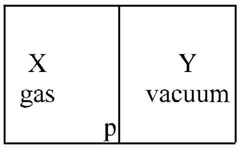
(D) If the compression is isothermal then their final pressure will be different.

32. Three processes form a thermodynamic cycle as shown on P-V diagram for an ideal gas. Process $1 \rightarrow 2$ takes place at constant temperature (300K). Process $2 \rightarrow 3$ takes place at constant volume. During this process 40J of heat leaves the system. Process $3 \rightarrow 1$ is adiabatic and temperature T_3 is 275K. Work done by the gas during the process $3 \rightarrow 1$ is :-



(A) – 40J (B) – 20J (C) +40J (D) +20J

33. A closed container is fully insulated from outside. One half of it is filled with an ideal gas X separated by a plate P from the other half Y which contains a vacuum as shown in figure. When P is removed, X moves into Y. Which of the following statements is correct?



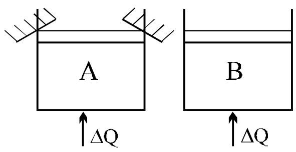
(A) No work is done by X (B) X decreases in temperature

(C) X increases in internal energy (D) X doubles in pressure

34. Water is heated in an open pan where the air pressure is 10^{+5} Pa. The water remains a liquid, which expands by a small amount as it is heated. Determine the ratio of the heat absorbed by the water to the work done by water. (γ for water = 10^{-3}/C, S = 1 cal/gm°C)

(A) 4.2×10^3 (B) 4.2×10^5 (C) 4.2×10^2 (D) 4.2×10^4

35. Two identical vessels A & B contain equal amount of ideal monoatomic gas. The piston of A is fixed but that of B is free. Same amount of heat is absorbed by A & B. If B's internal energy increases by 100 J the change in internal energy of A is :-



(A) 100 J (B) $\frac{500}{3}$ J (C) 250 J (D) none of these

36. An ideal gas expands from volume V_1 to V_2. This may be achieved by either of the three processes: isobaric, isothermal and adiabatic. Let ΔU be the change in internal energy of the gas, Q be the quantity of heat added to the system and W be the work done by the gas. Identify which of the following statements is false for ΔU?

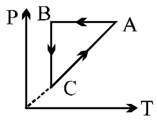
(A) ΔU is least under adiabatic process.

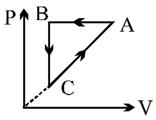
(B) ΔU is greatest under adiabatic process.

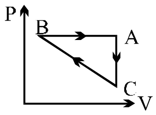
(C) ΔU is greatest under the isobaric process.

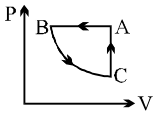
(D) ΔU in isothermal process lies in-between the values obtained under isobaric and adiabatic processes.

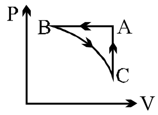
37. A cyclic process ABCA is shown in PT diagram. When presented on PV, it would



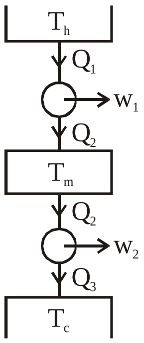
(A) 

(B) 

(C) 

(D) 

38. Suppose that two heat engines are connected in series, such that the heat exhaust of the first engine is used as the heat input of the second engine as shown in figure. The efficiencies of the engines are η_1 and η_2, respectively. The net efficiency of the combination is given by



(A) $\eta_{net} = \eta_2 + (1 - \eta_1)\eta_2$

(B) $\eta_{net} = \dfrac{\eta_1}{(1-\eta_1)\eta_2}$

(C) $\eta_{net} = \eta_1 + (1 - \eta_1)\eta_2$

(D) $\eta_{net} = \dfrac{1-\eta_1}{(1-\eta_2)\eta_2}$

39. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is- **[AIEEE - 2003]**

(A) 4/3 (B) 2 (C) 5/3 (D) 3/2

40. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of- **[AIEEE - 2003]**

(A) second law of thermodynamics
(B) conservation of momentum
(C) conservation of mass
(D) first law of thermodynamics

41. A carnot engine takes 3×10^6 cal of heat from a reservoir at 627 °C and gives it to a sink at 27 °C. The work done by the engine is- **[AIEEE - 2003]**

(A) 4.2×10^6 J
(B) 8.4×10^6 J
(C) 16.8×10^6 J
(D) zero

MULTIPLE CORRECT TYPE QUESTIONS

42. Two gases have the same initial pressure, volume and temperature. They expand to the same final volume, one adiabatically and the other isothermally

(A) The final temperature is greater for the isothermal process
(B) The final pressure is greater for the isothermal process
(C) The work done by the gas is greater for the isothermal process
(D) All the above options are incorrect

43. On the P–T graph of an ideal gas,

(A) adiabatic process will be a straight line
(B) isochoric process will be a straight line passing through the origin
(C) adiabatic curve will have a positive slope
(D) the slope of adiabatic curve will decrease with increase in T

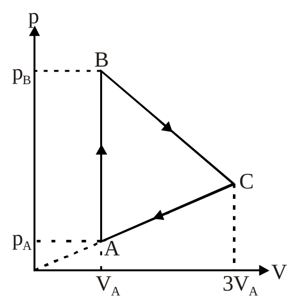
44. During the melting of a slab of ice at 273 K at atmospheric pressure

(A) Positive work is done by the ice-water system on the atmosphere
(B) Positive work is done on the ice-water system by the atmosphere
(C) The internal energy of the ice-water increases
(D) The internal energy of ice-water system decreases

COMPREHENSION TYPE QUESTIONS

Paragraph for question nos. 45 and 46

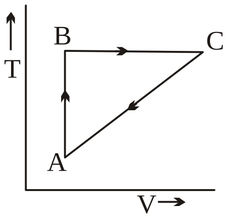
A sample of ideal gas is taken through the cyclic process shown in the figure. The temperature of the gas at state A is T_A = 200 K. At states B and C the temperature of the gas is the same.

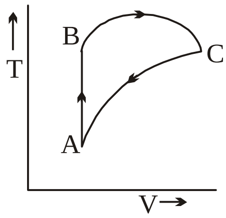


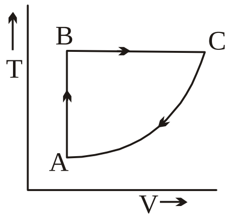
45. Net work done by the gas in the process is

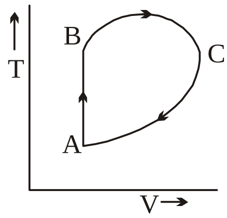
(A) $2P_AV_A$
(B) $4P_AV_A$
(C) $6P_AV_A$
(D) $8P_AV_A$

46. Which of the following graphs best represent the cyclic process in T-V diagram.

(A) 

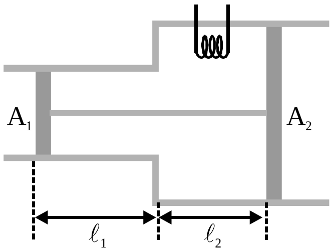
(B) 

(C) 

(D) 

Paragraph for Question Nos. 47 to 49

The walls of the two connecting cylinders shown in the figure are adiabatic (thermally insulating). The cross-sectional areas of the parts are $A_1$and A_2. There is a well-fitting but freely moving, thermally insulating piston in each cylinder, at a distance $\ell_1 = \ell_2 = \ell$ from the point where the cross-sectional area changes. The pistons are connected to each other by a thin and rigid rod. The enclosed volume contains air. The temperature and air pressure are T_0 and p_0 both inside and outside. The heater filament inside is operated for time t at a power of P.



47. Mark the **CORRECT** statement :-

(A) Process performed is adiabatic
(B) Piston shifts towards left
(C) Process performed is polytropic
(D) Piston shifts towards right

48. What is change in temperature (Q is heat energy supplied, v_0 is initial volume) :-

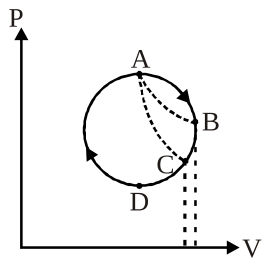
(A) $\Delta T = \frac{2}{5}\frac{QT_0}{p_0V_0}$ (B) $\Delta T = \frac{2}{7}\frac{QT_0}{p_0V_0}$ (C) $\Delta T = \frac{5}{7}\frac{QT_0}{p_0V_0}$ (D) $\Delta T = \frac{QT_0}{p_0V_0}$

49. What is displacement (x) of piston till equilibrium is reached :-

(A) $x = \frac{2Pt}{(A_2 - A_1)7p_0}$ (B) $x = \frac{2Pt}{(A_2 - A_1)5p_0}$ (C) $x = \frac{2Pt}{(A_2 - A_1)3p_0}$ (D) $x = \frac{2Pt}{(A_2 - A_1)9p_0}$

MATRIX MATCH TYPE QUESTIONS

50. For an ideal gas a process PV diagram is a circle. An adiabatic from A passes through C. An isotherm from A passes through B. We take a part of the circular cyclic process. Comment on the sign of the quantity of column-I.



Column-I		Column-II	
(A)	Heat given to the gas in going from A to C along circle	(P)	Positive
(B)	Heat given to the gas in going from B to C along circle	(Q)	Negative
(C)	Heat given to the gas in going from C to D along circle	(R)	Zero
		(S)	can't be said

EXERCISE (O-2)

SINGLE CORRECT TYPE QUESTIONS

1. An open and wide glass tube is immersed vertically in mercury in such a way that length 0.05 m extends above mercury level. The open end of the tube is closed and the tube is raised further by 0.43m. The length of air column above mercury level in the tube will be :

Take P_{atm} = 76 cm of mercury

(A) 0.215 m (B) 0.2 m (C) 0.1 m (D) 0.4 m

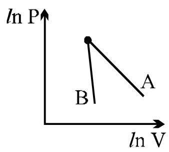
2. A barometer tube, containing mercury, is lowered in a vessel containing mercury until only 50 cm of the tube is above the level of mercury in the vessel. If the atmospheric pressure is 75 cm of mercury, what is the pressure at the top of the tube ?

(A) 33.3 kPa (B) 66.7 kPa (C) 3.33 MPa (D) 6.67 MPa

3. At temperature T, N molecules of gas A each having mass m and at the same temperature 2N molecules of gas B each having mass 2m are filled in a container. The mean square velocity of molecules of gas B is v^2 and mean square of x component of velocity of molecules of gas A is w^2. The ratio of w^2/v^2 is :

(A) 1 (B) 2 (C) 1/3 (D) 2/3

4. The figure, shows the graph of logarithmic reading of pressure and volume for two ideal gases A and B undergoing adiabatic process. From figure it can be concluded that



(A) gas B is diatomic (B) gas A and B both are diatomic

(C) gas A is monoatomic (D) gas B is monoatomic & gas A is diatomic

5. When unit mass of water boils to become steam at 100^0C, it absorbs Q amount of heat. The densities of water and steam at 100^0C are ρ_1 and ρ_2 respectively and the atmospheric pressure is p_0. The increase in internal energy of the water is

(A) Q (B) $Q + p_0\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$ (C) $Q + p_0\left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right)$ (D) $Q - p_0\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

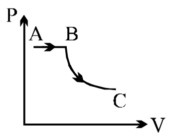
6. A cylinder made of perfectly non conducting material closed at both ends is divided into two equal parts by a heat proof piston. Both parts of the cylinder contain the same masses of a gas at a temperature t_0 = 27° C and pressure P_0 = 1 atm. Now if the gas in one of the parts is slowly heated to t = 57°C while the temperature of first part is maintained at t_0 the distance moved by the piston from the middle of the cylinder will be (length of the cylinder = 84 cm)

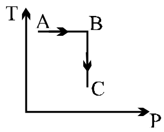
(A) 3 cm (B) 5 cm (C) 2 cm (D) 1 cm

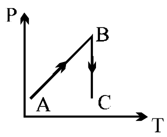
7. 1 kg of a gas does 20 kJ of work and receives 16 kJ of heat when it is expanded between two states. A second kind of expansion can be found between the initial and final state which requires a heat input of 9 kJ. The work done by the gas in the second expansion is :

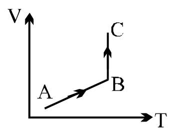
(A) 32 kJ (B) 5 kJ (C) – 4 kJ (D) 13 kJ

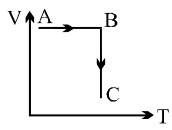
8. A process is shown in the diagram. Which of the following curves may represent the same process?



(A) 

(B) 

(C) 

(D) 

9. An ideal gas expands in such a way that PV^2 = constant throughout the process.

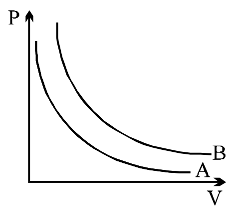
(A) The graph of the process of T-V diagram is a parabola.

(B) The graph of the process of T-V diagram is a straight line.

(C) Such an expansion is possible only with heating.

(D) Such an expansion is possible only with cooling.

10. Figure shows the pressure P versus volume V graphs for two different gas sample at a given temperature. M_A and M_B are masses of two samples, n_A and n_B are numbers of moles. Which of the following ***must be incorrect.***



(A) $M_A > M_B$ (B) $M_A < M_B$ (C) $n_A > n_B$ (D) $n_A < n_B$

11. A vertical cylinder with heat-conducting walls is closed at the bottom and is fitted with a smooth light piston. It contains one mole of an ideal gas. The temperature of the gas is always equal to the surrounding's temperature, T_0. The piston is moved up slowly to increase the volume of the gas to η times. Which of the following is incorrect?

(A) Work done by the gas is $RT_0 \ln \eta$.

(B) Work done against the atmosphere is $RT_0(\eta - 1)$.

(C) There is no change in the internal energy of the gas.

(D) The final pressure of the gas is $\frac{1}{(\eta-1)}$ times its initial pressure.

12. A gas is enclosed in a vessel at a constant temperature at a pressure of 5 atmosphere and volume 4 litre. Due to a leakage in the vessel, after some time, the pressure is reduced to 4 atmosphere. As a result, the

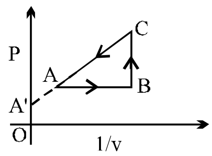
(A) volume of the gas decreased by 20%

(B) average K.E. of gas molecule decreases by 20%

(C) 20% of the gas escaped due to the leakage

(D) 25% of the gas escaped due to the leakage

13. An enclosed ideal gas is taken through a cycle as shown in the figure. Then



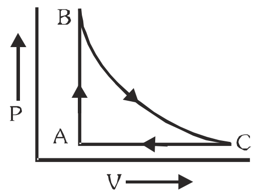
(A) Along AB, temperature decreases while along BC temperature increases

(B) Along AB, temperature increases while along BC the temperature decreases.

(C) Along CA work is done by the gas and the internal energy remains constant.

(D) Along CA work is done on the gas and internal energy of the gas increases.

14. One mole of monoatomic ideal gas undergoes a cyclic process ABCA as shown in figure. Process BC is adiabatic. The temperatures at A, B and C are 300, 600 and 450K respectively. Choose the correct statement(s).



(A) In process CA change in internal energy is 225R.

(B) In process AB change in internal energy is –150R.

(C) In process BC change in internal energy is –225R.

(D) Change in internal energy during the whole cyclic process is +150R.

MULTIPLE CORRECT TYPE QUESTIONS

15. Let v_{av}, v_{rms} and v_p respectively denote mean speed, root mean square speed and the most probable speed of the molecule in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m then :

(A) no molecule can have speed greater than $\sqrt{2}\, v_{rms}$

(B) no molecule can have speed less than $v_p/\sqrt{2}$

(C) $v_p < v_{av} < v_{rms}$

(D) the average kinetic energy of a molecule is $3/4\, m v_p^2$

16. A gas expands such that its initial and final temperature are equal. Also, the process followed by the gas traces a straight line on the P-V diagram :

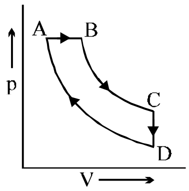
(A) The temperature of the gas remains constant throughout.

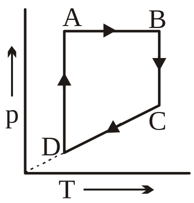
(B) The temperature of the gas first increases and then decreases

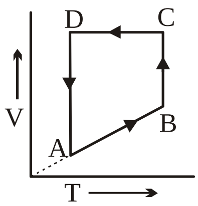
(C) The temperature of the gas first decreases and then increases

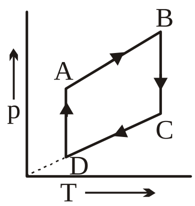
(D) The straight line has a negative slope.

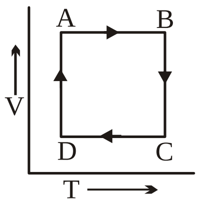
17. A cyclic process ABCD is shown in the p-V diagram. Which of the following curves represents the same process if BC & DA are isothermal processes



(A) 

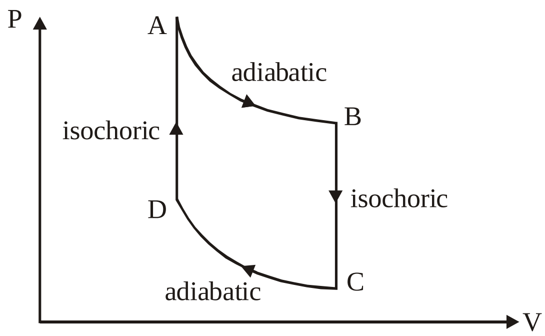
(B) 

(C) 

(D) 

18. Two moles of helium gas is taken through the cycle ABCDA as shown in the figure.

If $T_A = 1000$ K, $2P_A = 3P_B = 6P_C$.



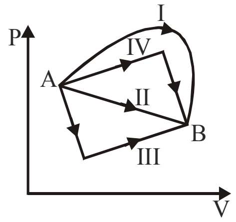
(A) work done by the gas in the process A to B is 3741 J.

(B) heat lost by the gas in the process B to C is 10600 J.

(C) temperature T_D is 2000 K.

(D) none of these

19. Figure shows the P-V diagram of an ideal gas undergoing a change of state from A to B. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state.



(A) Change in internal energy is same in IV and III cases, but not in I and II.

(B) Change in internal energy is same in all the four cases.

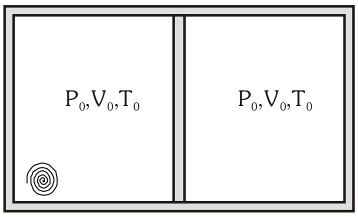
(C) Work done is maximum in case I

(D) Work done is minimum in case II.

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 20 to 22

One mole of a monoatomic ideal gas occupies two chambers of a cylinder partitioned by means of a movable piston. The walls of the cylinder as well as the piston are thermal insulators. Initially equal amounts of gas fill both the chambers at (P_0, V_0, T_0). A coil is burnt in the left chamber which absorbs heat and expands, pushing the piston to the right. The gas on the right chamber is compressed until to pressure becomes 32 P_0.



20. The final volume of left chamber is :-

(A) $\frac{V_0}{8}$ (B) $\frac{15}{8}V_0$ (C) $\frac{7}{8}V_0$ (D) $\frac{9}{8}V_0$

21. The work done on the gas in the right chamber is :-

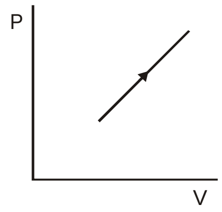
(A) $\frac{9}{2}P_0V_0$ (B) $-\frac{9}{2}P_0V_0$ (C) $\frac{13}{2}P_0V_0$ (D) $\frac{17}{2}P_0V_0$

22. The change in internal energy of the gas in the left chamber is :-

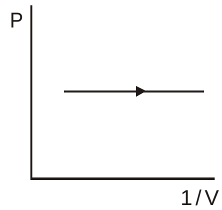
(A) $\frac{186}{4}RT_0$ (B) $\frac{177}{4}RT_0$ (C) $\frac{59}{2}RT_0$ (D) $\frac{131}{4}RT_0$

MATRIX MATCH TYPE QUESTION

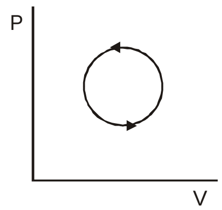
23. **Column I** **(Pressure volume graph)** | **Column II** **(W is work done by gas, Q heat supply to gas)**

(A) 

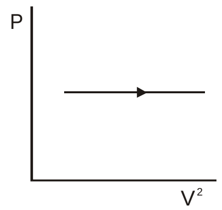
(P) $W > 0$

(B) 

(Q) $W < 0$

(C) 

(R) $Q > 0$

(D) 

(S) $Q < 0$

SUPPLEMENT FOR JEE-MAINS

1. Two engines A and B have their sources at 400 K and 350 K and sinks at 350 K and 300 K respectively. Which engine is more efficient and by how much ?

2. A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

3. A carnot engine works as a refrigrator between 250 K and 300 K. If it receives 750 cal of heat from the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

4. The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

5. A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

6. The coefficient of performance of a refrigerator working between 10° C and 20° C is :–

(A) 1 (B) 28.3 (C) 29.3 (D) 3.53

7. A refrigerator freezes 5 kg of water at 0°C into ice at 0°C in a time interval of 20 minutes. Assume that room temperature is 20°C. Calculate the minimum power needed to accomplish it ?

(A) 24.4 Watt (B) 0.1025 Watt (C) 0.0244 Watt (D) 102.5 Watt

8. A carnot engine whose source temperature is at 400 K takes 100 kcal of heat at this temperature in each cycle and gives 70 kcal to the sink. Calculate (i) the temperature of the sink and (ii) the efficiency of the engine.

(A) 280 K, 30 % (B) 270 K, 30% (C) 280 K, 10 % (D) None of these

9. An ionization gauge installed in the artificial satellite showed that 1 cm^3 of the atmosphere contained about a thousand million particles of gas at a height of 300 km from the Earth's surface. Find the mean free path of the gas particles at this height. Take the diameter of the particles equal to 2×10^{-10} m.

(A) $\ell = 5.6$ km (B) $\ell = 2.6$ km (C) $\ell = 8.4$ km (D) $\ell = 1.1$ km

10. What maximum number of molecules of a gas should be contained in 1 cm^3 of a spherical vessels with a diameter of 15 cm so that the molecules do not collide with each other? The diameter of a gas molecule is 3×10^{-8} cm.

(A) 1.7×10^{18}/CC (B) 1.7×10^{16}/CC (C) 2.5×10^{19}/CC (D) 1.1×10^{14}/CC

11. An ideal refrigerator operates according to the reverse Carnot cycle and transmits heat from a cold source with water at a temperature of 0°C to a boiler with water at a temperature of 100 °C. What amount of water must be frozen in the cooler to convert 1 kg of water into vapour in the boiler?

(A) 4.94 kg (B) 3.24 kg (C) 5.63 kg (D) 2.12 kg

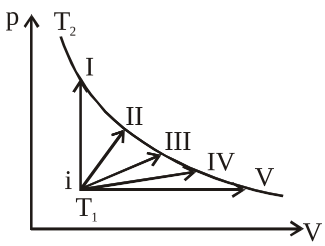
12. Oxygen (10 grammes) is heated from t_1 = 50°C to t_2 = 150°C. Find the change in entropy if the oxygen is heated: (1) isochorically, (2) isobarically.

(A) $\Delta S_V = 1.76$ J/K; $\Delta S_P = 2.46$ J/K
(B) $\Delta S_V = \Delta S_P = 1.76$ J/K
(C) $\Delta S_V = \Delta S_P = 2.46$ J/K
(D) $\Delta S_V = 2.46$ J/K; $\Delta S_P = 1.76$ J/K

13. An ideal gas expands into a vacuum in a rigid vessel. As a result there is :

(A) a change in entropy
(B) an increase of pressure
(C) a change in temperature
(D) a decrease of internal energy
(E) a change in phase

14. An ideal gas is to taken reversibly from state i, at temperature T_1, to any of the other states labelled, I, II, III, IV and V on the p-V diagram below. All are at the same temperature T_2. Rank the five processes according to the change in entropy of the gas, least to greatest :



(A) I, II, III, IV, V
(B) V, IV, III, II, I
(C) I, then II, III, IV, and V tied
(D) I, II, III and IV tied, then V
(E) I and V tied, then II, III, IV

15. According to the second law of thermodynamics:

(A) Heat energy cannot be completely converted to work
(B) Work cannot be completely converted to heat energy
(C) For all cyclic processes we have dQ/T < 0
(D) The reason all heat engine efficiencies are less than 100% is friction, which is unavoidable
(E) All of the above are true

16. A reversible refrigerator operates between a low temperature reservoir at T_C and a high temperature reservoir at T_H. Its coefficient of performance is given by :

(A) $(T_H - T_C)/T_C$ (B) $T_C/(T_H - T_C)$ (C) $(T_H - T_C)/T_H$ (D) $T_H/(T_H - T_C)$

(E) $T_H(T_H + T_C)$

17. An inventor claims to have a heat engine that has an efficiency of 40% when it operates between a high temperature reservoir of 150°C and a low temperature reservoir of 30°C. This engine:

(A) Must violate the zeroth law of thermodynamics

(B) Must violate the first law of thermodynamics

(C) Must violate the second law of thermodynamics

(D) Must violate the third law of thermodynamics

(E) Does not necessarily violate any of the laws of thermodynamics

18. A carnot heat engine runs between a cold reservoir at temperature T_C and a hot reservoir at temperature T_H. You want to increase its efficiency. Of the following, which change results in the greatest increase in efficiency? The value of ΔT is the same for all changes.

(A) Raise the temperature of the hot reservoir by ΔT

(B) Raise the temperature of the cold reservoir by ΔT

(C) Lower the temperature of the hot reservoir by ΔT

(D) Lower the temperature of the cold reservoir by ΔT

(E) Lower the temperature of the hot reservoir by $1/2\Delta T$ and raise the temperature of the cold reservoir by $1/2\Delta T$.

19. A Carnot engine takes 3×10^6 cal of heat from reservoir at 627° and gives it to a sink at 27°C. Then work done by the engine is :-

(A) 4.2×10^6 J (B) 8.4×10^6 J (C) 16.8×10^6 J (D) zero

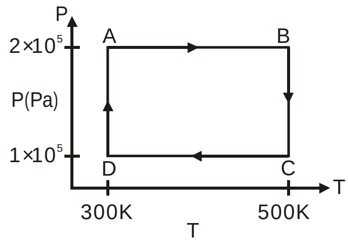
EXERCISE (JM)

1. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion ? **[AIEEE-2009]**

(1) 6×10^4 J (2) 7×10^4 J (3) 3×10^4 J (4) 5×10^4 J

Directions : Question number 2, 3 and 4 are based on the following paragraph.

Two moles of helium gas are taken over the cycle ABCDA, as shown in the P–T diagram.



2. Assuming the gas to be ideal the work done on the gas in taking it from A to B is :- **[AIEEE-2009]**

(1) 400 R (2) 500 R (3) 200 R (4) 300

3. The work done on the gas in taking it from D to A is :- **[AIEEE-2009]**

(1) –690 R (2) +690 R (3) –414 R (4) +414 R

4. The net work done on the gas in the cycle ABCDA is :- **[AIEEE-2009]**

(1) 1076 R (2) 1904 R (3) Zero (4) 276 R

5. A diatomic ideal gas is used in a carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is :- **[AIEEE-2010]**

(1) 0.25 (2) 0.5 (3) 0.75 (4) 0.99

6. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :- **[AIEEE-2011]**

(1) $\frac{\gamma M v^2}{2R}$ (2) $\frac{(\gamma-1)}{2R} M v^2$ (3) $\frac{(\gamma-1)}{2(\gamma+1)R} M v^2$ (4) $\frac{(\gamma-1)}{2\gamma R} M v^2$

7. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively:- **[AIEEE-2011]**

(1) 330 K and 268 K (2) 310 K and 248 K (3) 372 K and 310 K (4) 372 K and 330 K

8. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2, and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, then final temperature of the mixture is :- **[AIEEE-2011]**

(1) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (2) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (3) $\frac{T_1+T_2+T_3}{3}$ (4) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

9. The specific heat capacity of a metal at low temperautre (T) is given as $C_p(kJk^{-1}kg^{-1}) = 32\left(\frac{T}{400}\right)^3$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is:- **[AIEEE-2011]**

(1) equal to 0.002 kJ (2) greater than 0.148 kJ

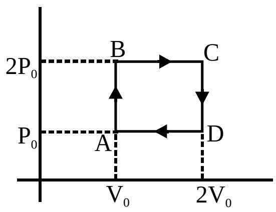
(3) between 0.148 kJ and 0.028 kJ (4) less than 0.028 kJ

10. A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be :- **[AIEEE-2011]**

(1) $\frac{P}{2}, T$ (2) $\frac{P}{2}, \frac{T}{2}$ (3) P, T (4) $P, \frac{T}{2}$

11. Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) :-

[AIEEE-2012]



(1) 12.5% (2) 15.4% (3) 9.1% (4) 10.5%

12. A Carnot engine, whose efficiency is 40% takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :- **[AIEEE-2012]**

(1) 600 K

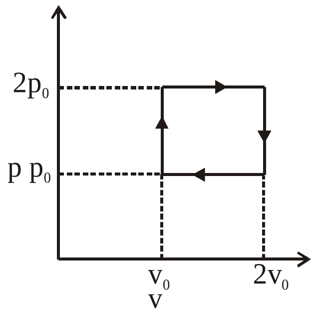
(2) efficiency of Carnot engine cannot be made larger than 50%

(3) 1200 K

(4) 750 K

13. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is :

[JEE-Mains-2013]



(1) p_0v_0

(2) $\left(\frac{13}{2}\right)p_0v_0$

(3) $\left(\frac{11}{2}\right)p_0v_0$

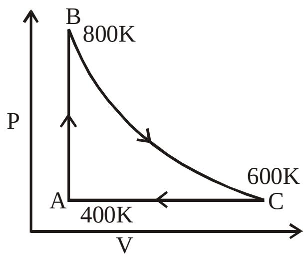
(4) $4p_0v_0$

14. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open of the tube is then closed and sealed and the tube is raised vertically up by addition 46 cm. What will be length of the air column above mercury in the tube now ?

(Atmospheric pressure = 76 cm of Hg) **[JEE-Mains-2014]**

(1) 38 cm (2) 6 cm (3) 16 cm (4) 22 cm

15. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement :



(1) The change in internal energy in the process AB is –350 R.

(2) The change in internal energy in the process BC is –500R

(3) The change in internal energy in whole cyclic process is 250 R.

(4) The change in internal energy in the process CA is 700 R. **[JEE-Mains-2014]**

16. A solid body of constant heat capacity 1 J°/C is being heated by keeping it in contact with reservoirs in two ways - **[JEE-Mains-2015]**

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is -

(1) ln2, 2ln2 (2) 2ln2, 8ln2 (3) ln2, 4ln2 (4) ln2, ln2

17. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is -

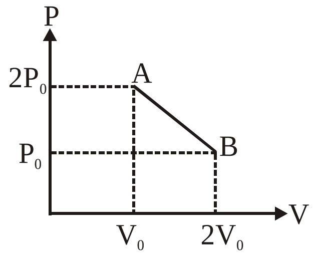
[JEE-Mains-2015]

(1) $T \propto \frac{1}{R}$ (2) $T \propto \frac{1}{R^3}$ (3) $T \propto e^{-R}$ (4) $T \propto e^{-3R}$

18. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is :- $\left(\gamma = \frac{C_p}{C_v}\right)$ **[JEE-Mains-2015]**

(1) $\frac{\gamma+1}{2}$ (2) $\frac{\gamma-1}{2}$ (3) $\frac{3\gamma+5}{6}$ (4) $\frac{3\gamma-5}{6}$

19. 'n' moles of an ideal gas undergoes a process A $\rightarrow$ B as shown in the figure. The maximum temperature of the gas during the process will be : **[JEE-Mains-2016]**



(1) $\frac{9P_0V_0}{nR}$ (2) $\frac{9P_0V_0}{4nR}$ (3) $\frac{3P_0V_0}{2nR}$ (4) $\frac{9P_0V_0}{2nR}$

20. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) :- **[JEE-Mains-2016]**

(1) $n=\frac{C-C_V}{C-C_P}$ (2) $n=\frac{C_P}{C_V}$ (3) $n=\frac{C-C_P}{C-C_V}$ (4) $n=\frac{C_P-C}{C-C_V}$

21. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be :- **[JEE-Main 2017]**

(1) 2.5×10^{25} (2) -2.5×10^{25} (3) -1.61×10^{23} (4) 1.38×10^{23}

22. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v$ = a for hydrogen gas
$C_p - C_v$ = b for nitrogen gas
The correct relation between a and b is : **[JEE-Main 2017]**

(1) a = 14 b (2) a = 28 b (3) $a = \frac{1}{14}b$ (4) a = b

23. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) –2.7 kJ (2) (a) 189 K (b) –2.7 kJ **[JEE-Main 2018]**
(3) (a) 195 K (b) 2.7 kJ (4) (a) 189 K (b) 2.7 kJ

EXERCISE (JA)

1. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **[JEE-2009]**

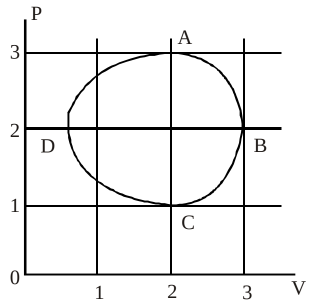
(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_p / C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p \cdot C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

2. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semicircle and CDA is half of an ellipse. Then, **[JEE-2009]**



(A) the process during the path $A \rightarrow B$ is isothermal

(B) heat flows out of the gas during the path $B \rightarrow C \rightarrow D$

(C) work done during the path $A \rightarrow B \rightarrow C$ is zero

(D) positive work is done by the gas in the cycle ABCDA

3. A real gas behaves like an ideal gas if its

(A) pressure and temperature are both high

(B) pressure and temperature are both low

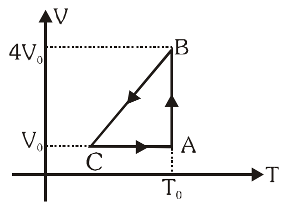
(C) pressure is high and temperature is low

(D) pressure is low and temperature is high

4. One mole of an ideal gas in initial state A undergoes 1 a cyclic process ABCA, as shown in the figure. its pressure at A is P_0. Choose the correct option(s) from the following. **[JEE-2010]**

(A) Internal energies at A and B are the same

(B) Work done by the gas in process AB is P_0V_0 ln4



(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

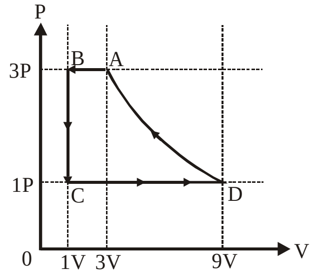
5. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. In the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is **[JEE-2010]**

6. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is **[JEE-2011]**

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

7. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in P-V diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**. **[JEE-2011]**

Column I	**Column II**
(A) Process A→B	(P) Internal energy decreases.
(B) Process B→C	(Q) Internal energy increases.
(C) Process C→D	(R) Heat is lost.
(D) Process D→A	(S) Heat is gained.
	(T) Work is done on the gas.



8. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(helium)}{v_{rms}(\arg on)}\right)$ is **[JEE-2012]**

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

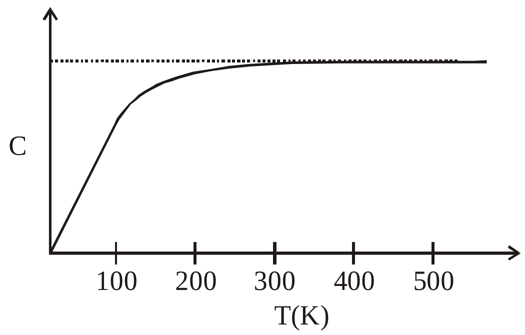
9. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to required no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (Take : R = 8.31 J/mol.K) **[JEE-2012]**

(A) 62 J (B) 104 J (C) 124 J (D) 208 J

10. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is :- **[JEE-2013]**

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

11. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation :- **[JEE-2013]**



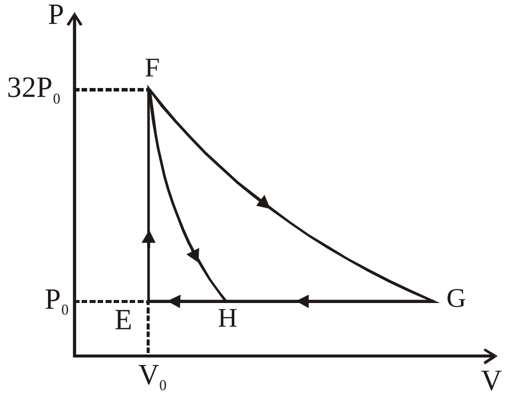
(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.

(B) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.

(C) there is no change in the rate of heat absorption in the range 400–500 K

(D) the rate of heat absorption increases in the range 200–300 K

12. One mole of a monoatomic ideal gas is taken along two cyclic processes E→F→G→E and E→F→H→E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. **[JEE-2013]**



Match the paths in List I with the magnitudes of the work done in the List II and select the correct answer using the codes given blow the lists.

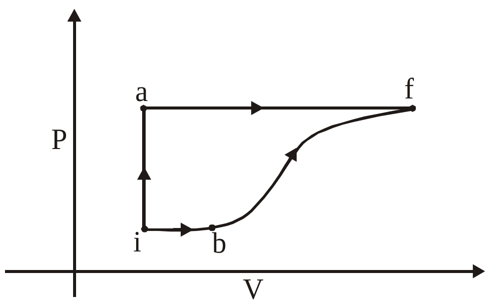
List I		**List II**	
P.	$G \to E$	1.	$160\ P_0V_0 \ln 2$
Q.	$G \to H$	2.	$36\ P_0V_0$
R.	$F \to H$	3.	$24\ P_0V_0$
S.	$F \to G$	4.	$31\ P_0V_0$

Codes :

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

13. A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio Q_{bf}/Q_{ib} is.

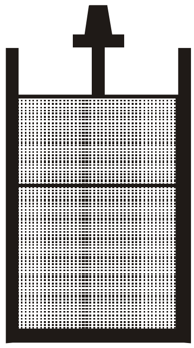
[JEE-Advance-2014]



Paragraph for Questions 14 and 15

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monoatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monoatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.

[JEE-Advance-2014]



14. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(A) 550 K (B) 525 K (C) 513 K (D) 490 K

15. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

16. A container of fixed volume has a mixutre of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) :-

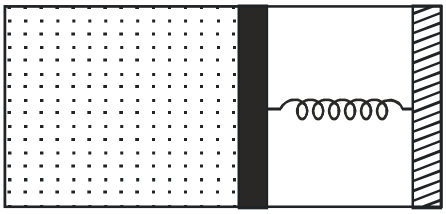
(A) The average energy per mole of the gas mixture is 2RT. **[JEE-Advance-2015]**

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

17. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) :- **[JEE-Advance-2015]**



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

18. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = \left(\frac{1}{32}\right) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumes V_f. The amount of heat supplied to the system in the two step process is approximately

[JEE-Advance 2016]

(A) 112 J (B) 294 J (C) 588 J (D) 813 J

19. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true ? **[JEE-Advance 2017]**

(A) The resistive force experienced by the plate is proportional to v

(B) The pressure difference between the leading and trailing faces of the plate is proportional to uv.

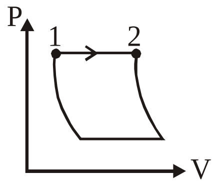
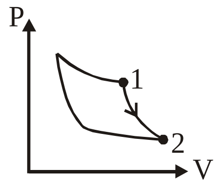
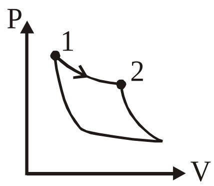
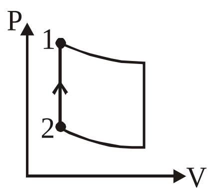
(C) The plate will continue to move with constant non-zero acceleration, at all times

(D) At a later time the external force F balances the resistive force.

Answer Q.20, Q.21 and Q.22 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamics process in different ways as shown in the corresponding P–V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamics processes. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

[JEE-Advance 2017]

Column-1	Column-2	Column-3
(I) $W_{1\to2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to2} = -nRT \ln \frac{V_2}{V_1}$	(iv) Adiabatic	(S) 

20. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?

(A) (II) (iv) (R) (B) (II) (iii) (P) (C) (II) (iii) (S) (D) (III) (iii) (P)

21. Which one of the following options is the correct combination ?

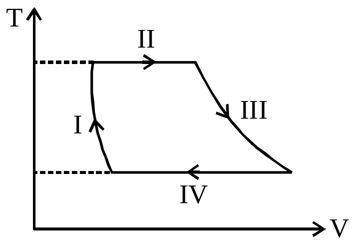
(A) (III) (ii) (S) (B) (II) (iv) (R) (C) (II) (iv) (P) (D) (IV) (ii) (S)

22. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ?

(A) (III) (iv) (R) (B) (I) (ii) (Q) (C) (IV) (ii) (R) (D) (I) (iv) (Q)

23. One mole of a monoatomic ideal gas undergoes a cyclic process as shown in the figure (whre V is the volume and T is the temperature). Which of the statements below is (are) true ?

[JEE-Advance 2018]

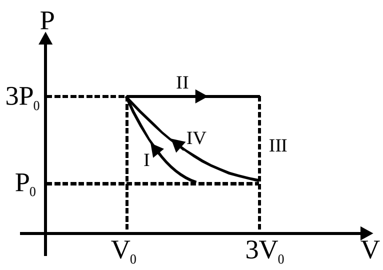


(A) Process I is an isochoric process
(B) In process II, gas absorbs heat
(C) In process IV, gas releases heat
(D) Processes I and II are not isobaric

24. One mole of a monoatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is............ .

[JEE-Advance 2018]

25. One mole of a monoatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II. **[JEE-Advance 2018]**

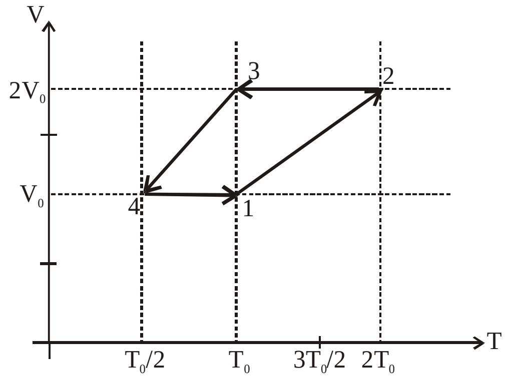


List–I		List–II	
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	No heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is 6 P_0V_0

(A) $P \rightarrow 4$; $Q \rightarrow 3$; $R \rightarrow 1$; $S \rightarrow 2$
(B) $P \rightarrow 1$; $Q \rightarrow 3$; $R \rightarrow 2$; $S \rightarrow 4$
(C) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 1$; $S \rightarrow 2$
(D) $P \rightarrow 3$; $Q \rightarrow 4$; $R \rightarrow 2$; $S \rightarrow 1$

26. One mole of a monoatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are :

[R is the gas constant] **[JEE-Advance 2019]**



(1) Work done in this thermodynamic cycle ($1\to2\to3\to4\to1$) is $|W| = \frac{1}{2}RT_0$

(2) The ratio of heat transfer during processes $1\to2$ and $2\to3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(3) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(4) The ratio of heat transfer during processes $1\to2$ and $3\to4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$

27. A mixture of ideal gas containing 5 moles of monoatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0 and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,

(Give $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant) **[JEE-Advance 2019]**

(1) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$

(2) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

(3) The work $|W|$ done during the process is $13RT_0$

(4) Adiabatic constant of the gas mixture is 1.6

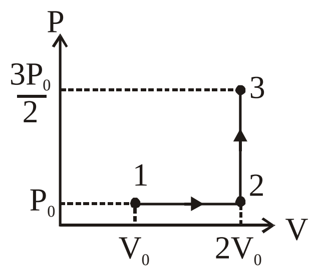
28. Answer the following by appropriately matching the lists based on the information given in the paragraph. [JEE-Advance 2019]

In a thermodynamics process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monoatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	List-II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3+\ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process carried out on one mole of monoatomic ideal gas is as shown in figure in the PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



(1) I $\to$ Q, II $\to$ R, III $\to$ P, IV $\to$ U
(2) I $\to$ S, II $\to$ R, III $\to$ Q, IV $\to$ T
(3) I $\to$ Q, II $\to$ R, III $\to$ S, IV $\to$ U
(4) I $\to$ Q, II $\to$ S, III $\to$ R, IV $\to$ U

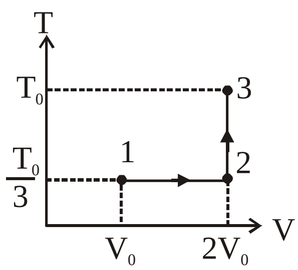
29. Answer the following by appropriately matching the lists based on the information given in the paragraph. [JEE-Advance 2019]

In a thermodynamic process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monoatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	List-II
(I) Work done by the system in process $1 \rightarrow 2 \rightarrow 3$	(P) $\frac{1}{3}RT_0 \ln 2$
(II) Change in internal energy in process $1 \rightarrow 2 \rightarrow 3$	(Q) $\frac{1}{3}RT_0$
(III) Heat absorbed by the system in process $1 \rightarrow 2 \rightarrow 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \rightarrow 2$	(S) $\frac{4}{3}RT_0$
	(T) $\frac{1}{3}RT_0(3+\ln 2)$
	(U) $\frac{5}{6}RT_0$

If the process on one mole of monoatomic ideal gas is an shown is as shown in the TV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is



(1) $I \rightarrow S, II \rightarrow T, III \rightarrow Q, IV \rightarrow U$
(2) $I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow S$
(3) $I \rightarrow P, II \rightarrow T, III \rightarrow Q, IV \rightarrow T$
(4) $I \rightarrow P, II \rightarrow R, III \rightarrow T, IV \rightarrow P$

ANSWER KEY

EXERCISE (S-1)

1. Ans. 1.25×10^4 N/m^2 **2. Ans.** 327 °C

3. Ans. (i) $P_1 < P_2$, $T_1 < T_2$; (ii) $T_1 = T_2 < T_3$; (iii) $V_2 > V_1$; (iv) $P_1 > P_2$ **4. Ans.** 40

5. Ans. (a) 4/3 m, (b) $T_3 = 400\left(\frac{4}{3}\right)^{0.4}$ K **6. Ans.** 3600 R **7. Ans.** $\Delta T = \frac{mv_0^2}{3R}$

8. Ans. 12600 J **9. Ans.** 5R **10. Ans.** 2.64 **11. Ans.** 1.5

12. Ans. 450 J **13. Ans.** 16.9 J **14. Ans.** R/2

15. Ans. (a) 0.5 atm (b) zero (c) zero (d) no

16. Ans. (a) A to B (b) C to D

(c) $W_{AB} = \frac{1}{1-\gamma}(2^{1-\gamma}-1)(P_B - P_A)V_A = \frac{3}{2}\left(1-\left(\frac{1}{2}\right)^{2/3}\right)(P_B - P_A)V_A$

(d) $\left[1-\left(\frac{1}{2}\right)^{2/3}\right]$

EXERCISE (O-1)

1. Ans. (A)	**2. Ans. (A)**	**3. Ans. (B)**	**4. Ans. (D)**	**5. Ans. (C)**	**6. Ans. (A)**
7. Ans. (C)	**8. Ans. (B)**	**9. Ans. (C)**	**10. Ans. (C)**	**11. Ans. (C)**	**12. Ans. (A)**
13. Ans. (D)	**14. Ans. (A)**	**15. Ans. (C)**	**16. Ans. (D)**	**17. Ans. (B)**	**18. Ans. (D)**
19. Ans. (C)	**20. Ans. (C)**	**21. Ans. (A)**	**22. Ans. (C)**	**23. Ans. (D)**	**24. Ans. (B)**
25. Ans. (B)	**26. Ans. (C)**	**27. Ans. (B)**	**28. Ans. (D)**	**29. Ans. (A)**	**30. Ans. (A)**
31. Ans. (A)	**32. Ans. (A)**	**33. Ans. (A)**	**34. Ans. (D)**	**35. Ans. (B)**	**36. Ans. (B)**
37. Ans. (C)	**38. Ans. (C)**	**39. Ans. (D)**	**40. Ans. (A)**	**41. Ans. (B)**	**42.Ans. (A,B,C)**
43. Ans. (B,C)		**44. Ans. (B,C)**	**45. Ans. (D)**	**46. Ans. (D)**	**47. Ans. (C, D)**
48. Ans. (B)		**49. Ans. (A)**	**50. Ans. (A) P (B) Q (C) Q**		

EXERCISE (O-2)

1. Ans. (C)	**2. Ans. (A)**	**3. Ans. (D)**	**4. Ans. (D)**	**5. Ans. (B)**	**6. Ans. (C)**
7. Ans. (D)	**8. Ans. (C)**	**9. Ans. (D)**	**10. Ans. (C)**	**11. Ans. (D)**	**12. Ans. (C)**
13. Ans. (A)	**14. Ans. (C)**	**15. Ans. (C,D)**	**16. Ans. (B,D)**	**17. Ans. (A,B)**	**18. Ans. (A,B)**
19. Ans. (B,C)		**20. Ans. (B)**	**21. Ans. (A)**	**22. Ans. (B)**	

23. Ans. (A) $\rightarrow$ (P,R) ; (B) $\rightarrow$ (Q, S) ; (C) $\rightarrow$ (Q, S) ; (D) $\rightarrow$ (P, R)

SUPPLEMENT FOR JEE-MAINS

1. Ans. B & 1.8% more **2. Ans. 2400 joule per cycle** **3. Ans. 900 cal** **4. Ans. 5.7 J**

5. Ans. 3.36×10^6 J **6. Ans. (B)** **7. Ans. (D)** **8. Ans. (A)** **9. Ans. (A)**

10. Ans. (A) **11. Ans. (A)** **12. Ans. (A)** **13. Ans. (A)** **14. Ans. (A)**

15. Ans. (A) **16. Ans. (B)** **17. Ans. (C)** **18. Ans. (D)** **19. Ans. (B)**

EXERCISE (JM)

1. Ans. (4) **2. Ans. (1)** **3. Ans. (4)** **4. Ans. (4)** **5. Ans. (3)** **6. Ans. (2)**

7. Ans. (3) **8. Ans. (4)** **9. Ans. (2)** **10. Ans. (1)** **11. Ans. (2)** **12. Ans. (4)**

13. Ans. (2) **14. Ans. (3)** **15. Ans. (2)** **16. Ans. (Bonus)** **17. Ans. (1)**

18. Ans. (1) **19. Ans. (2)** **20. Ans. (3)** **21. Ans. (2)** **22. Ans. (1)** **23. Ans. (2)**

EXERCISE (JA)

1. Ans. (BD) **2. Ans. (B,D)** **3. Ans. (D)** **4. Ans. (A,B,C,D)** **5. Ans. 4**

6. Ans. (A) **7. Ans. (A) $\rightarrow$ (P,R,T) ; (B) $\rightarrow$ (P,R) ; (C) $\rightarrow$ (Q,S) ; (D) $\rightarrow$ (R,T)**

8. Ans. (D) **9. Ans. (D)** **10. Ans. (D)** **11. Ans. (A, B, C, D) or (B, C, D)**

12. Ans. (A) **13. Ans. 2** **14. Ans. (D)** **15. Ans. (D)** **16. Ans. (A,B,D)**

17. Ans. (A,B,C) **18. Ans. (C)** **19. Ans. (A,B,D)** **20. Ans. (B)**

21. Ans. (A) **22. Ans. (D)** **23. Ans. (B,C,D)** **24. Ans. 900 [899.95, 900.05]**

25. Ans. (C) **26. Ans. (1,2)** **27. Ans. (1,3,4)** **28. Ans. (3)** **29. Ans. (4)**

CHAPTER 3

WAVES ON STRING

Chapter Contents 03

01.	THEORY	161
02.	EXERCISE (S-1)	178
03.	EXERCISE (O-1)	180
04.	EXERCISE (O-2)	186
05.	EXERCISE (JM)	191
06.	EXERCISE (JA)	192
07.	ANSWER KEY	194

IMPORTANT NOTES

CHAPTER 3

WAVES ON STRING

KEY CONCEPTS

INTRODUCTION OF WAVES

What is wave motion ?

- When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place)
- There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

Ex. You (Kota) want to communicate your friend (Delhi)

Write a letter | Use telephone

1st option involves the concept of particle & the second choice involves the concept of wave.

Ex. When you say "Namaste" to your friend no material particle is ejected from your lips to fall on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transferred to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

Note :- In the above example air itself does not move.

A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

Few examples of waves :

The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES

Wave classification According to

- Necessity of medium
  - (i) Elastic or mechanical wave
  - (ii) Electro magnetic wave (E.M. wave)
- Propagation of energy
  - (i) Progressive
  - (ii) Sationary
- Dimension
  - (i) One dimensional
  - (ii) Two dimensional
  - (iii) Three dimentional
- Vibration of particle
  - (i) Transverse
  - (ii) Longitudinal

1. ***Based on medium necessity*** :- A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**.

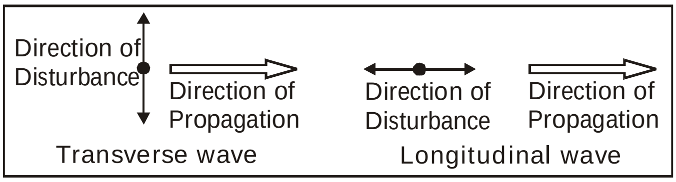
   Example : Sound waves in water, seismic waves in earth's crust.

2. **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with fixed velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

3. ***Based on direction of propagation*** :- Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.

4. ***Based on the motion of particles of medium*** :

   Waves are of two types on the basis of motion of particles of the medium.
   (i) Longitudinal waves
   (ii) Transverse waves



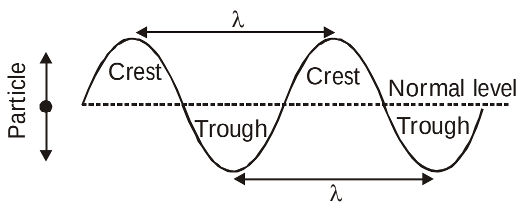
   In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

TRANSVERSE WAVE MOTION

Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.

$\Rightarrow$ Mechanical transverse waves are generated only in solids & surface of liquid.

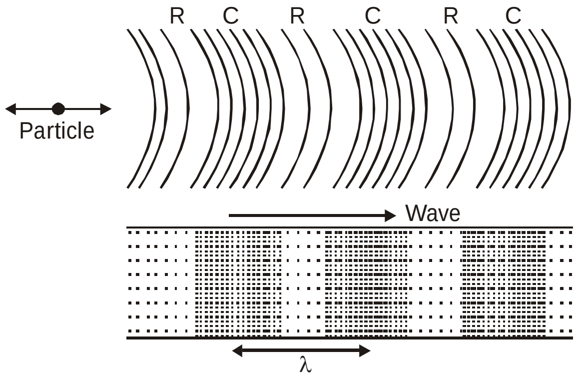


In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.

A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION



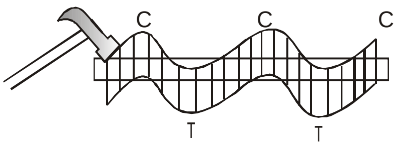
In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

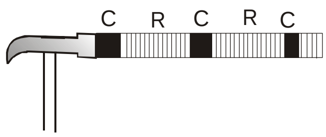
- The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation.
- In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



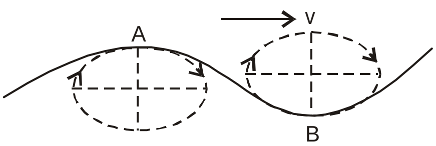
Transverse-wave
(A)



Longitudinal-wave
(B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds [$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane [Fig.]



Ripple

CHARACTERISTICS OF WAVE MOTION

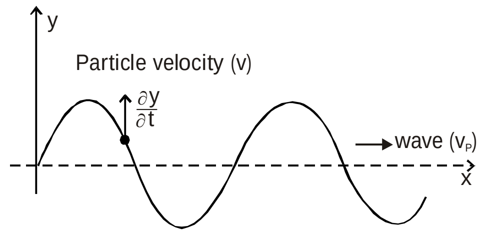
Some of the important characteristics of wave motion are as follows :

- In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

SOME IMPORTANT TERMS CONNECTED WITH WAVE MOTION

- **Wavelength (λ) [length of one wave]**
  Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.
- **Frequency (n)** :Number of vibrations (Number of complete wavelengths) complete by a particle in one second.
- **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$
- **Phase :** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation y = A sin (ωt – kx); (ωt – kx) = phase.
- **Angular wave number (k) :** It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number ($\vec{v}$) :** It is defined as $\vec{v} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration :** In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae what we have read in SHM apply to the particles here also. For example, maximum particle velocity is $\pm A\omega$ at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between + $A\omega$ and – $A\omega$) the wave velocity is constant for given characteristics of the medium.

- **Particle velocity in wave motion :**
The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



- **Wave velocity :** The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.

- **Relation between particle velocity and wave velocity :**

Wave equation :- y = A sin (ωt - kx), Particle velocity $v=\frac{\partial y}{\partial t}$ = Aω cos (ωt - kx).

Wave velocity = $v_P=\frac{\lambda}{T}=\lambda\frac{\omega}{2\pi}=\frac{\omega}{k}$, $\frac{\partial y}{\partial x}$ = - Ak cos (ωt–kx) =-$\frac{A}{\omega}$ωk cos (ωk - kx) = -$\frac{1}{v_p}\frac{\partial y}{\partial t}$

$$\Rightarrow \frac{\partial y}{\partial x}=-\frac{1}{v_P}\frac{\partial y}{\partial t}$$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

- **Differential equation of harmonic progressive waves :**

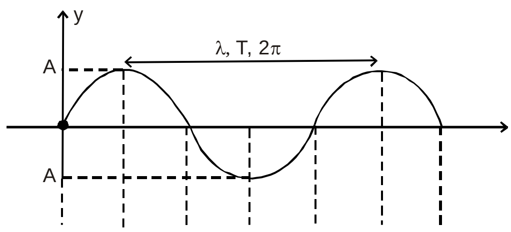
$$\frac{\partial^2 y}{\partial t^2}=-A\omega^2\sin(\omega t-kx)\Rightarrow\frac{\partial^2 y}{\partial x^2}=-Ak^2\sin(\omega t-kx)\Rightarrow\frac{\partial^2 y}{\partial x^2}=\frac{1}{v_P^2}\frac{\partial^2 y}{\partial t^2}$$

- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave :**
The acceleration of the particle is the second particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P=\frac{\partial^2 y(x,t)}{\partial t^2}=\omega^2 A\sin(kx-\omega t)=-\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, a_p = –w2 (displacement)

- **Relation between Phase difference,**
Path difference & Time difference



Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3}{2}\lambda$
Time-period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$$\Rightarrow\frac{\Delta\phi}{2\pi}=\frac{\Delta\lambda}{\lambda}=\frac{\Delta T}{T}\Rightarrow \text{Path difference} = \left(\frac{\lambda}{2\pi}\right)\text{Phase difference}$$

Ex. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60^o out of phase.

Sol. We know that for a wave v = f λ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60^o = (\pi/180) \times 60 = (\pi/3)$ rad,

so path difference $\Delta x = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

THE GENERAL EQUATION OF WAVE MOTION

Some physical quantity (say y) is made to oscillate at one place and these oscillations of y propagate to other places. The y may be,

(i) displacement of particles from their mean position in case of transverse wave in a rope or longitudinal sound wave in a gas.

(ii) pressure difference (dP) or density difference (dρ) in case of sound wave or

(iii) electric and magnetic fields in case of electromagnetic waves.

The oscillations of y may or may not be simple harmonic in nature. Consider one-dimensional wave travelling along x-axis. In this case y is a function of x and t. i.e. y = f(x, t) But only those function of x & t, represent a wave motion which satisfy the differential equation.

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \qquad \text{...(i)}$$

The general solution of this equation is of the form y (x, t) = f (ax ± bt) ...(ii)

Thus, any function of x and t and which satisfies equation (i) or which can be written as equation (ii) represents a wave. The only condition is that it should be finite everywhere and at all times,

Further, if these conditions are satisfied, then speed of wave (v) is given by $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{b}{a}$

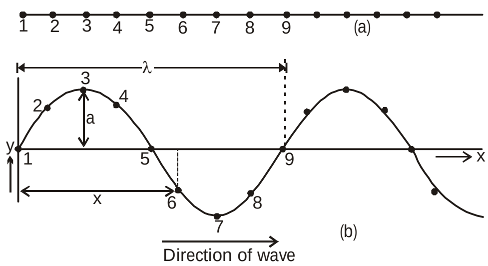
Ex. Which of the following functions represent a travelling wave ?

(a) $(x - vt)^2$ (b) $\ell n(x + vt)$ (c) $e^{-(x-vt)^2}$ (d) $\frac{1}{x + vt}$

Sol. Although all the four functions are written in the form f (ax $\pm$ bt), only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Equation of a Plane Progressive Wave

If, on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'. Suppose, a simple harmonic progressive wave is propagating in a medium along the positive direction of the x-axis (from left to right). In fig. (a) are shown the equilibrium positions of the particles 1, 2, 3 ....



When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) are shown the instantaneous positions of these particles at a particular instant. The curve joining these positions represents the wave. Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then $y = a \sin \omega t$...(i)

where a is the amplitude of oscillation and $\omega = 2\pi$ n, where n is the frequency. As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time t – (x/v). The displacement of particle 1 at time t – (x/v) can be the particle 6, distant x from the origin (particle 1), at time t is given by

$y = a \sin \omega \left(t - \frac{x}{v}\right)$ But $\omega = 2\pi n$, $y = a \sin (\omega t - kx) \left(k = \frac{\omega}{v}\right)$ ...(ii)

$y = a \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right]$ Also $k = \frac{2\pi}{\lambda}$ ...(iii) $y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$ ...(iv)

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equations, instead of minus sign there will be plus sign. For example, equation (iv) will be of the following form : $y = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)$.

If ϕ be the phase difference between the above wave travelling along the +x direction and an other wave, then the equation of that wave will be

$$y = a \sin \left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \pm \phi\right\}$$

Ex. The equation of a wave is, $y(x,t) = 0.05 \sin\left[\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right] m$

Find : (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Sol. : (a) The equation may be rewritten as, $y(x,t) = 0.05 \sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right) m$

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A \sin(kx - \omega t + \phi)$ we have, wave number $k = \frac{2\pi}{\lambda} = 5\pi \text{ rad/m}$ $\therefore \lambda = 0.4m$

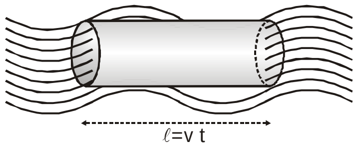
The angular frequency is, $\omega = 2\pi f = 20\pi \text{ rad/s}$ $\therefore$ f = 10Hz

The wave velocity is, $v = f\lambda = \frac{\omega}{k} = 4 ms^{-1}$ in + x direction

(b) The particle velocity and acceleration are, $v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right)$ =2.22m/s

$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right)$ =140 m/s^2

INTENSITY OF WAVE



The amount of energy flowing per unit area and per unit time is called the intensity of wave. It is represented by I. Its units are J/m^{2}s or watt/metre^2. $I = 2\pi^2 f^2 A^2 \rho v$ i.e. $I \propto A^2$ and $I \propto A^2$. If P is the power of an isotropic point source, then intensity at a distance r is given by,

$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2} \text{ (for a point source)}$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r\ell} \text{ or } I \propto \frac{1}{r} \text{ (for a line source) As, } I \propto A^2$$

Therefore, $A \propto \frac{1}{r}$ (for a point source) and $A \propto \frac{1}{\sqrt{r}}$ (for a line source)

SUPERPOSITION PRINCIPLE

Two or more waves can propagate in the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle.

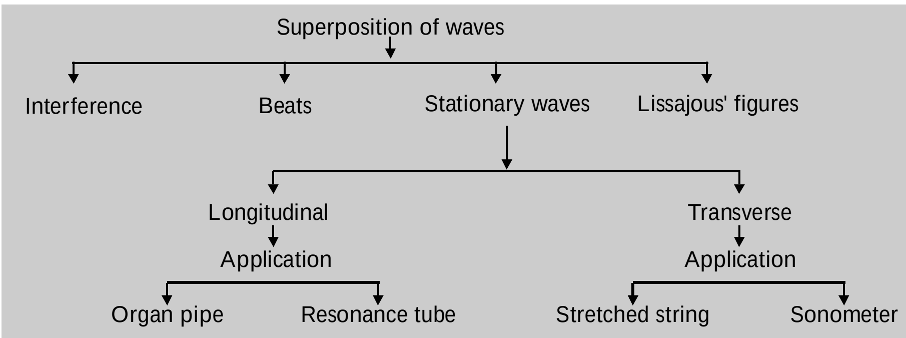
> The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

- **Interference :** Superposition of two waves having equal frequency and nearly equal amplitude.
- **Beats :** Superposition of two waves of nearly equal frequency in same direction.
- **Stationary waves :** Superposition of equal wave from opposite direction.
- **Lissajous' figure :** Superposition of perpendicular waves.



INTERFERENCE OF WAVES :

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves $y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition $y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

where $A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$ and $\tan\delta = \dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

As intensity $I \propto A^2$ so $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$

- **Constructive interference (maximum intensity) :**

  Phase difference $\phi = 2n\pi$ or path difference $= n\lambda$ where n = 0, 1, 2, 3, ...

  $\Rightarrow A_{max} = A_1 + A_2$ and $I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}$

- **Destructive interference (minimum intensity) :**

  Phase difference $\phi = (2n+1)\pi$, or path difference $= (2n-1)\dfrac{\lambda}{2}$ where n = 0, 1, 2, 3, ...

  $\Rightarrow A_{min} = A_1 - A_2$ and $I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}$

KEY POINTS

- Maximum and minimum intensities in any interference wave form.

$$\frac{I_{Max}}{I_{Min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

- Average intensity of interference wave form % & < I >or $I_{av} = \dfrac{I_{max} + I_{min}}{2} = I_1 + I_2$

  if $a = a_1 = a_2$ and $I_1 = I_2 = I$ then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$

- Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or

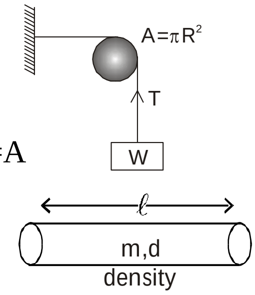
  Degree of visibility (Light Wave) $f = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$

  In condition of perfect interference degree of interference pattern is maximum $f_{max} = 1$ or 100%

- Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{max} = 4I$ and $I_{min} = 0$

  For perfect destructive interference we have a maximum contrast in interference wave form.

VELOCITY OF TRANSVERSE WAVE



Mass of per unit length $m = \dfrac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$, where d = Density of matter

Velocity of transverse wave in any wire $v = \sqrt{\dfrac{T}{m}}$ or $\sqrt{\dfrac{T}{\pi r^2 d}} = \sqrt{\dfrac{T}{Ad}}$ $\because \pi r^2 = A$

• If m is constant then, $v \propto \sqrt{T}$ it is called tension law.

• If tension is constant then $v \propto \sqrt{\dfrac{1}{m}}$ ← it is called law of mass

• If T is constant & take wire of different radius for same material then $v \propto \dfrac{1}{r}$ ← it is called law of radius

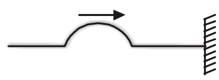
• If T is constant & take wire of same radius for different material. Then $v \propto \sqrt{\dfrac{1}{d}}$ ← law of density

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall, the element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transmits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down, so the resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.

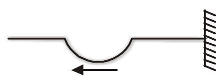
Equation of wave propagating in +ve x-axis

Incident wave $y_1 = a \sin(\omega t - kx)$



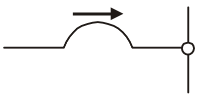
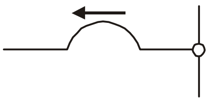
Reflected wave $y_2 = a \sin(\omega t + kx + \pi)$

or $y_2 = -a \sin(\omega t + kx)$



REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up. However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulse i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one. The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin(\omega t - kx)$  Reflected wave $y_2 = a \sin(\omega t + kx)$ 

STATIONARY WAVES

* **Definition :** The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave. Formation of stationary wave is possible only and only in bounded medium.

ANALYTICAL METHOD FOR STATIONARY WAVES

- **From rigid end** : We know equation for progressive wave in positive x-direction
  $y_1 = a \sin(\omega t - kx)$
  After reflection from rigid end $y_2 = a \sin(\omega t + kx + \pi) = -a \sin(\omega t + kx)$
  By principle of super position. $y = y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + kx) = -2a \sin kx \cos \omega t$
  This is equation of stationary wave reflected from rigid end

Amplitude $= 2a \sin kx$ **Velocity of particle** $v_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t$

Strain $\frac{dy}{dx} = -2ak \cos kx \cos \omega t$ **Elasticity** $E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$ **Change in pressure** $dp = E\frac{dy}{dx}$

- **Node** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A = 0$, $V_{pa} = 0$, strain $\rightarrow$ max **Change in pressure** $\rightarrow$ max

- **Antinode** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ .... $A \rightarrow$ max, $-V_{pa} \rightarrow$ max. strain = 0 **Change in pressure** = 0

- **From free end** : we know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from free end $y_2 = a \sin(\omega t + kx)$
  By Principle of superposition $y = y_1 + y_2 = a \sin(\omega t - kx) + a \sin(\omega t + kx) = 2\ a \sin \omega t \cos kx$

  Amplitude = $2a \cos kx$, **Velocity of particle** = $v_{Pa} = \frac{dy}{dt} = 2a\,\omega \cos \omega t \cos kx$

  Strain $\frac{dy}{dx} = -2ak \sin \omega t \sin kx$ **Change in pressure** $dp = E\frac{dy}{dx}$

- **Antinode :** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A \to$ Max, $V_{pa} = \frac{dy}{dt} \to$ max. Strain = 0, dp = 0

- **Node :** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... $A = 0$, $V_{pa} = \frac{dy}{dt} = 0$, strain $\to$ max, $dp \to$ max

PROPERTIES OF STATIONARY WAVES

The stationary waves are formed due to the superposition of two identical simple harmonic waves travelling in opposite direction with the same speed.

Important characteristics of stationary waves are:–

(i) Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.

(ii) In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.

(iii) All the particles except at the nodes vibrate simple harmonically with the same period.

(iv) The distance between any two successive nodes or antinodes is $\lambda/2$.

(v) The amplitude of vibration gradually increases from zero to maximum value from node to antinode.

(vi) All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180°

(vii) All points of the medium pass through their mean position simultaneously twice in each period.

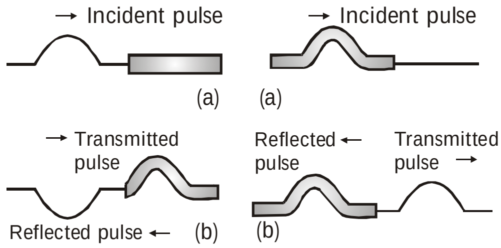
(viii) Velocity of the particles while crossing mean position varies from maximum at antinodes to zero at nodes.

(ix) In a stationary wave the medium is splited into segments and each segment is vibrating up and down as a whole.

(x) In longitudinal stationary waves, condensation (compression) and refraction do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(xi) These waves do not transfer energy in the medium. Transmission of energy is not possible in a stationary wave.

TRANSMISSION OF WAVES



We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, same part of it is reflected and inverted and same part of it is transmitted to the heavier string.

As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected. When a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string. In the previous section, we found that the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

KEY POINTS

Phenomenon of reflection and transmission of waves obeys the laws of reflection and refraction.

The frequency of these wave remains constant i.e. does not change. $\omega_i = \omega_r = \omega_t = \omega$

From rarer to denser medium $y_i = a_i \sin(\omega t - k_1x)$ $y_r = -a_i \sin(\omega t + k_1x)$ $y_t = a_t \sin(\omega t - k_2x)$

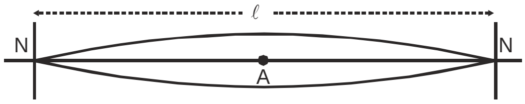
From denser to rarer medium $y_i = a_i \sin(\omega t - k_1x)$ $y_r = a_i \sin(\omega t + k_1x)$ $y_t = a_t \sin(\omega t - k_2x)$

STATIONARY WAVE ARE OF TWO TYPES :

(i) Transverse st. wave (stretched string) **(ii)** Longitudinal st. wave (organ pipes)

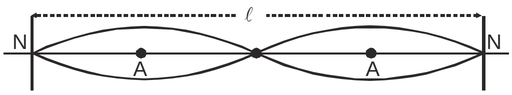
(i) Transverse Stationary wave

First Harmonic



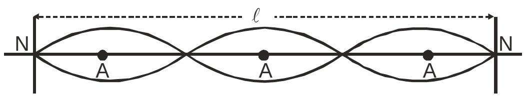
$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$

Second Harmonic



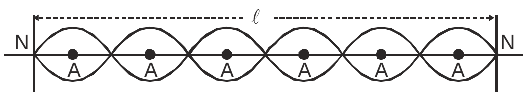
$\frac{2\lambda}{2} = \ell \Rightarrow \lambda = \ell$

Third Harmonic



$\frac{3\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{3}$

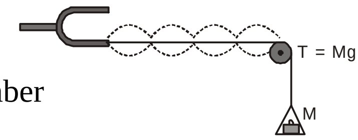
p th harmonic



$\frac{p\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{p}$

- **Law of length :** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e, $n \propto \frac{1}{\ell}$ (T and m are constant)
- **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (ℓ and m are constant)
- **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{m}}$ (ℓ and T are constant)
- **Melde's experiment :** In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded. There are two arrangements to vibrate the tied fork with thread.

Transverse arrangement :

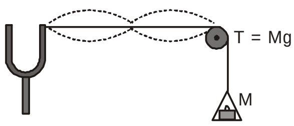


Case 1. In a vibrating string of fixed length, the product of number of loops and square root of tension are constant or p $\sqrt{T}$ = constant.

Case 2. When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal nodes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3. If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread only makes node at the midpoint when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal arrangement :

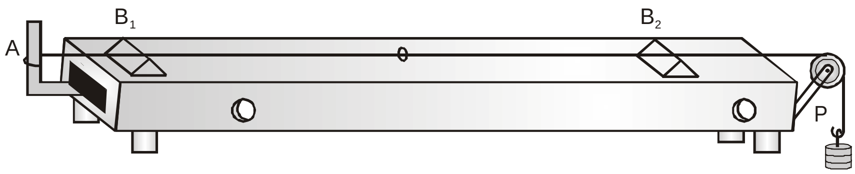


The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork.

Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

SONOMETER :

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ℓ, then the frequency of vibration is

$$n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire. There are three laws of vibration of a wire.

COMPARISON OF PROGRESSIVE AND STATIONARY WAVES

Progressive waves	Stationary waves
1. These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2. These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3. The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180°
4. No particle of medium is Permanently at rest.	Particles at nodes are permanently at rest.
5. All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6. All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement

WORKED OUT EXAMPLES

Ex.1 A wave is propagating along x-axis. The displacement of particles of the medium in z-direction at t = 0 is given by: $z = \exp[-(x+2)^2]$, where 'x' is in meters. At t = 1s, the same wave disturbance is given by: $z = \exp[-(2-x)^2]$. Then, the wave propagation velocity is

(A) 4 m/s in + x direction (B) 4 m/s in –x direction

(C) 2 m/s in + x direction (D) 2 m/s in – x direction

Ans. **(D)**

Ex.2 A transverse wave is propagating along +x direction. At t = 2 sec, the particle at x = 4m is at y = 2 mm. With the passage of time its y coordinate increases and reaches to a maximum of 4 mm. The wave equation may be (using ω and k with their usual meanings)

(A) $y = 4\sin(\omega(t+2)+k(x-2)+\frac{\pi}{6})$ (B) $y = 4\sin(\omega(t+2)+k(x)+\frac{\pi}{6})$

(C) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{5\pi}{6})$ (D) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{\pi}{6})$

Ans. **(D)**

Ex.3 $Y(x,t) = 0.05 / [(4x+2t)^2+5]$ represents a moving wave pulse, where x and y are in meters and t is in seconds. Then which statement(s) are **CORRECT**:

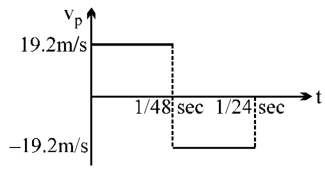
(A) Pulse is moving in – x direction (B) Wave speed is 0.5 m/s

(C) Maximum particle displacement is 1 cm (D) It is a symmetric pulse

Ans. **(A,B,C,D)**

Ex.4 A symmetrical triangular pulse of maximum height 0.4 m and total length 1 m is moving in the positive x-direction on a string on which the wave speed is 24 m/s. At t = 0 the pulse is entirely located between x = 0 and x = 1 m. Draw a graph of the transverse velocity of particle of string versus time at x = +1m.

Ans.



Ex.5 A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[IIT-JEE 2005]**

Ans. $y = (10\text{ cm})\sin\left(30t \pm \frac{3}{2}x + f\right)$

Ex.6 A non-uniform rope of mass M and length L has a variable linear mass density given by μ=kx where x is the distance from one end of the wire and k is a constant.

(a) Show that $M = kL^2/2$

(b) Show that the time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$ where F (constant)is the tension in the wire.

Ex.7 One end of a long string of linear mass density 10^{-2} kg m^{-1} is connected to an electrically driven tuning fork of frequency 150 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves from this end have negligible amplitude. At t = 0, the left end (fork end) of the string is at x = 0 has a transverse displacement of 2.5 cm and is moving along positive y-direction. The amplitude of the wave is 5 cm. Write down the transverse displacement y (in cm) as function of x (in m) and t (in sec) that describes the wave on the string.

Ans. $y = 5\sin\left\{\pi(300t - x) + \frac{\pi}{6}\right\}$

Ex.8 If the tension in a stretched string fixed at both ends is changed by 20%, the fundamental frequency is found to increase by 15Hz. then the

(A) original frequency is 157 Hz

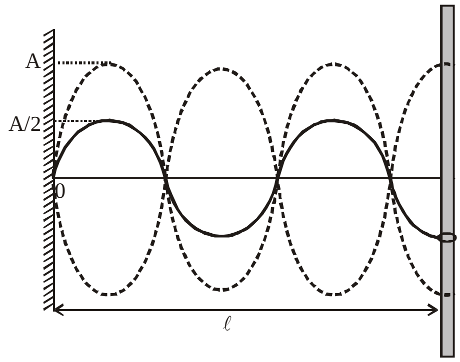
(B) velocity of propagation of the transverse wave along the string changes by 5%

(C) velocity of propagation of the transverse wave along the string changes by 10%.

(D) fundamental wave length on the string does not change.

Ans. **(A,C,D)**

Ex.9 Here given snap shot at $t = \frac{T}{12}$ of a standing wave. Then the equations of the wave will be when particles are moving towards their extreme and when particles are moving towards the mean position respectively $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$.



(A) y = A sin kx sin ωt, $y = A\sin\left(\omega t + \frac{2\pi}{3}\right)\sin kx$

(B) y = A sin kx cos ωt, $y = A\sin\left(\omega t + \frac{2\pi}{3}\right)\sin kx$

(C) y = A cos kx cos ωt, $y = A\cos\left(\omega t + \frac{2\pi}{3}\right)\sin kx$

(D) y = A cos kx sin ωt, $y = A\cos\left(\omega t + \frac{2\pi}{3}\right)\cos kx$

Ans. **(A)**

Ex.10 A standing wave of time period T is set up in a string clamped between two rigid supports. At t = 0 antinode is at its maximum displacement 2A.

(A) The energy density of a node is equal to energy density of an antinode for the first time at t = T/4.

(B) The energy density of node and antinode becomes equal after T/2 second.

(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

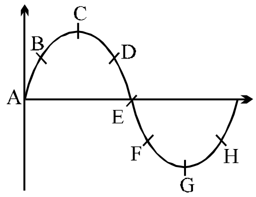
(D) The displacement of the particle at node is zero

Ans. **(C,D)**

EXERCISE (S-1)

Wave equation

1. A transverse wave is travelling along a string from left to right. The fig. represents the shape of the string (snap-shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity.

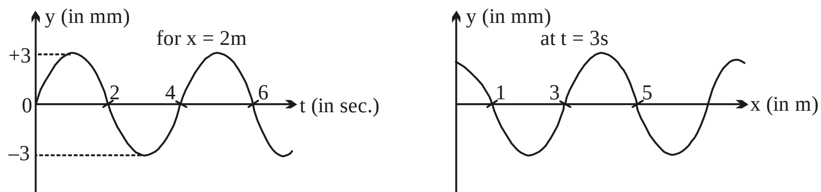


2. A sinusoidal wave propagates along a string. In figure (a) and (b) 'y' represents displacement of particle from the mean position. 'x' & 't' have usual meanings. Find :

(a) wavelength, frequency and speed of the wave.

(b) maximum velocity and maximum acceleration of the particles

(c) the magnitude of slope of the string for x = 2 m at t = 4 sec.



(a) (b)

3. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and plate is allowed to fall freely. 8 complete oscillations are counted when the plate falls through 10 cm. What is the frequency of the tuning fork ?

4. A long uniform string of mass density 0.1 kg/m is stretched with a force of 40 N. One end of the string (x = 0) is oscillated transversely (sinusoidally) with an amplitude of 0.02 m and a period of 0.1 sec, so that travelling waves in the +x direction are set up.

(a) What is the velocity of the waves?

(b) What is their wavelength?

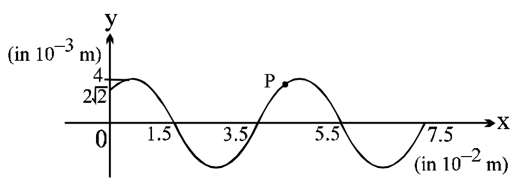
(c) If at the driving end (x = 0) the displacement (y) at t = 0 is 0.01 m with dy/dt negative, what is the equation of the travelling waves?

5. The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with x-axis at P is 6°.

(a) Find the direction in which the wave is moving

(b) the equation of the wave

(c) the total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm/m.



Velocity of wave

6. The extension in a string, obeying Hooke's law is x. The speed of wave in the stretched string is v. If the extension in the string is increased to 1.5 x find the new speed of wave.

7. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. Ignore gravity. Find the time required for a transverse wave to travel from one end of the rope to the other.

8. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible. Find the speed of transverse waves in this wire at 10°C.

Given : Young modulus of copper = 1.3×10^{11} N/m^2.

Coefficient of linear expansion of copper = 1.7×10^{-5} °C^{-1}.

Density of copper = 9×10^3 kg/m^3. **[IIT-1979]**

Energy of wave

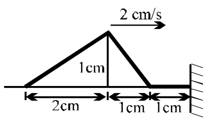
9. A steel wire has a mass of 5g/m and is under tension 450 N. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength.

Superposition of waves

10. The figure shown a triangular pulse on a rope at t = 0. It is approaching a fixed end at 2 cm/s

(a) Draw the pulse at t = 2 sec.

(b) The particle speed on the leading edge at the instant depicted is _____.



11. A 40 cm long wire having a mass 3.2 gm and area of cross-section 1 mm^2 is stretched between the supports 40.05 cm apart. In its fundamental mode it vibrate with a frequency 1000/64 Hz. Find the young's modulus of the wire.

12. A plane wave given by equation $y = 0.04 \sin(0.5\pi x - 100\pi t)$, where x and y are in meter and t in sec is incident normally on a boundary between two media beyond which wave speed becomes doubled. State boundary condition and find the equation of the reflected and transmitted waves. Take x = 0 as the boundary between two media.

13. A string between x = 0 and x = l vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[IIT-JEE 2003(Scr)]**

EXERCISE (O-1)

Wave equation

1. The function of x and t that does not represent a progressive wave is :-

(A) $y = 2\sin(4t - 3x)$ (B) $y = e^{(4+(4t-3x))}$ (C) $y = [4t - 3x]^{-1}$ (D) $y = [4t-3x]$

2. At x = 0 particle oscillate by law $y = \dfrac{3}{2t^2+1}$. If wave is propagating along –ve x axis with velocity 2m/s. Find equation of wave

(A) $y = \dfrac{3}{2\left(t-\dfrac{x}{2}\right)^2+1}$ (B) $y = \dfrac{3}{2\left(t+\dfrac{x}{2}\right)^2+1}$

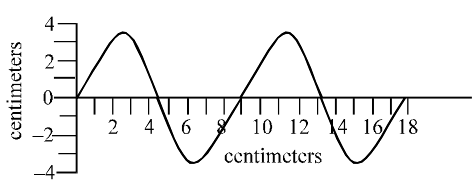
(C) $y = \dfrac{3}{2\left(t-\dfrac{z}{2}\right)^2+1}$ (D) $y = \dfrac{3}{2\left(t+\dfrac{z}{2}\right)^2+1}$

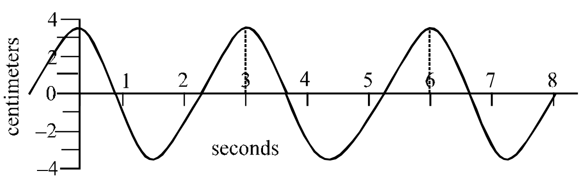
3. The shape of a wave propagating in the positive x or negative x- direction is given $y = \dfrac{1}{\sqrt{1+x^2}}$ at t = 0 and $y = \dfrac{1}{\sqrt{2-2x+x^2}}$ at t = 1s where x and y are in meters. The shape the wave disturbance does not change during propagation. Find the velocity of the wave.

(A) 1 m/s in positive x direction (B) 1 m/s in negative x direction

(C) $\dfrac{1}{2}$ m/s in positive x direction (D) $\dfrac{1}{2}$ m/s in negative x direction

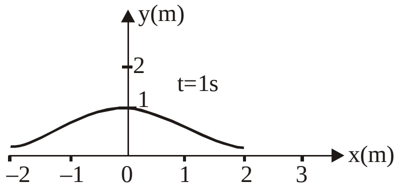
4. A transverse wave is travelling along a horizontal string. The first picture shows the shape of the string at an instant of time. This picture is superimposed on a coordinate system to help you make any necessary measurements. The second picture is a graph of the vertical displacement of one point along the string as a function of time. How far does this wave travel along the string in one second?

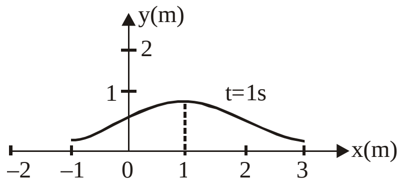


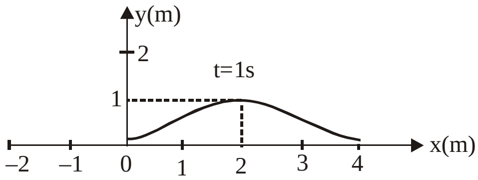


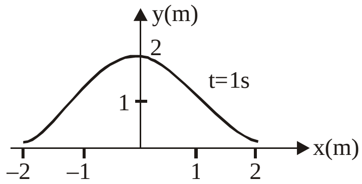
(A) 0.3 cm (B) 3.0 cm (C) 9.0 cm (D) 27 cm

5. A wave pulse is given by the equation $y = f(x, t) = A \exp(-B(x-vt)^2)$. Given $A = 1.0$ m, $B = 1.0\ m^{-2}$ and $v = +2.0$ m/s, which of the following graph shows the correct wave profile at the instant $t = 1$ s?

(A) 

(B) 

(C) 

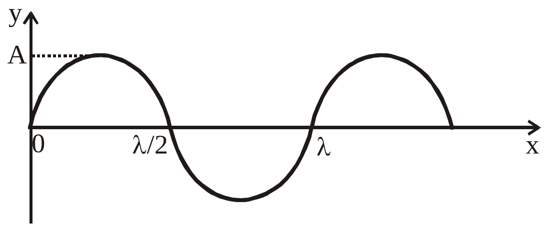
(D) 

6. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment $t = T/6$ (T is the time period). Assume that the source was at mean position at $t = 0$. The wavelength of the running wave is

(A) 0.96 m (B) 0.48 m (C) 0.24 m (D) 0.12 m

7. Here given snap shot of a progressive wave at $t = 0$ with time period = T. Then the equation of the wave if wave is going in +ve x-direction and if wave is going in –ve x-direction will be respectively.

$$\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$$



(A) $y = A \sin(kx + \omega t)$, $y = A \sin(kx - \omega t)$ (B) $y = A \cos(kx + \omega t)$, $y = A \cos(kx - \omega t)$

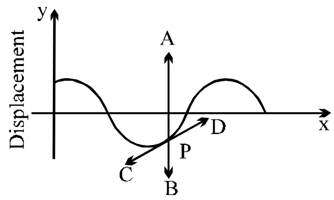
(C) $y = A \sin(\omega t - kx)$, $y = A \sin(\omega t + kx)$ (D) $y = A \sin(kx - \omega t)$, $y = A \sin(kx + \omega t)$

8. A sinusoidal wave travelling in the positive direction of x on a stretched string has amplitude 2.0 cm, wavelength 1 m and wave velocity 5.0 m/s. At $x = 0$ and $t = 0$, it is given that displacement $y = 0$ and $\frac{\partial y}{\partial t} < 0$. Express the wave function correctly in the form $y = f(x, t)$:-

(A) $y = (0.02\text{ m}) \sin 2\pi(x-5t)$ (B) $y = (0.02\text{ cm}) \cos 2\pi(x-5t)$

(C) $y = (0.02\text{ m}) \sin 2\pi\left(x - 5t + \frac{1}{4}\right)$ (D) $y = (0.02\text{ cm}) \cos 2\pi\left(x - 5t + \frac{1}{4}\right)$

9. The figure below shows a snap photograph of a simple harmonic progressive wave, progressing in the negative X-axis, at a given instant. The direction of the velocity of the particle at the stage P on the figure is best represented by the arrow.

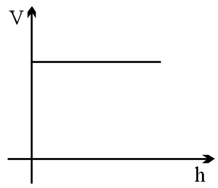
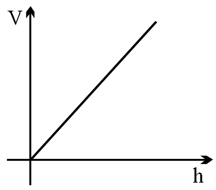
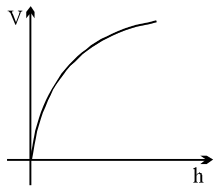
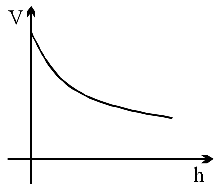


(A) $\vec{PA}$ (B) $\vec{PB}$

(C) $\vec{PC}$ (D) $\vec{PD}$

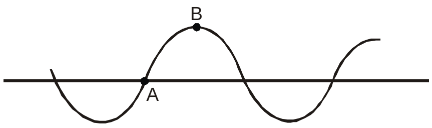
Velocity of wave

10. A uniform rope having some mass hanges vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A) 
(B) 
(C) 
(D) 

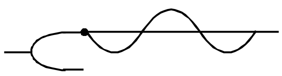
Energy of wave

11. A progressive wave is travelling in a string as shown. Then which of the following statement about KE and potential energy of the elements A and B is true?



(A) For point A : kinetic energy is maximum and potential energy is min.
(B) For point B : kinetic energy is minimum and potential energy is min.
(C) For point A : kinetic energy is minimum and potential energy is max.
(D) For point B : kinetic energy is minimum and potential energy is max.

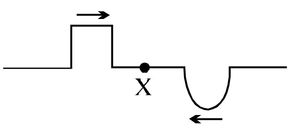
12. The prong of a electrically operated tuning fork is connected to a long string of $\mu = 1$ kg/m and tension 25N. The maximum velocity of the prong is 1 cm/s, then the average power needed to drive the prong is:

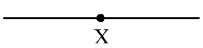
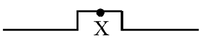
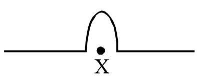
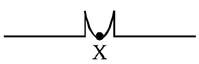


(A) 5×10^{-4} W (B) 2.5×10^{-4} W (C) 1×10^{-4} W (D) 10^{-3} W

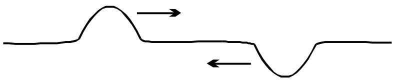
Superposition of waves

13. The diagram below shows two pulses traveling towards each other in a uniform medium with same speed. Pulses in the figure are at the same distance from X and has same height & width. Which diagram best represents the medium when the pulses meet at point X ?



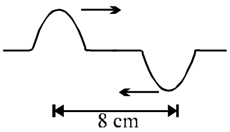
(A) 
(B) 
(C) 
(D) 

14. Two symmetric, identical pulses of opposite amplitude travel along a stretched string in opposite directions as shown in the figure below. Which one of the following statements most fully describes the situation ?



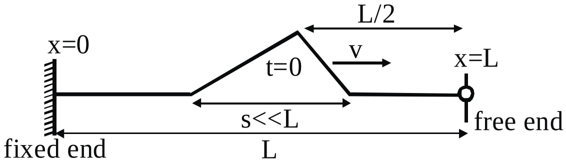
(A) There is an instant when the string is straight
(B) When the two pulses interfere completely, the energy of the wave is zero
(C) There is a point on the string that does not move up or down
(D) Both A and C
(E) Both A and B

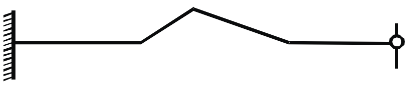
15. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be :- **[IIT-JEE 2001(Scr)]**

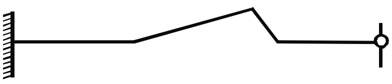


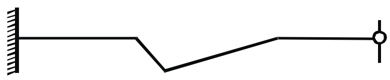
(A) zero
(B) purely kinetic
(C) purely potential
(D) partly kinetic and partly potential

16. A small pulse travelling with speed v in a string is shown at t = 0, moving towards free end. Which of these is not **CORRECTLY** matched.



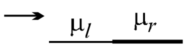
(i) $t = \dfrac{L}{v}$ (P) 

(ii) $t = \dfrac{2L}{v}$ (Q) 

(iii) $t = \dfrac{3L}{v}$ (R) 

(A) (i)
(B) (ii)
(C) (iii)
(D) None of these

17. A string consists of two parts attached at x = 0. The right part of the string (x > 0) has mass μ_r per unit length and the left part of the string (x < 0) has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string ?



(A) 1
(B) $\dfrac{2}{1+\sqrt{\mu_l/\mu_r}}$
(C) $\dfrac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$
(D) $\dfrac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

18. A wave travels on a light string .The equation of the wave is Y= Asin(kx–wt+30°). It is reflected from a heavy string tied to an end of the light string at x = 0.If 64% of the incident energy is reflected the equation of the reflected wave is

(A)$Y = 0.8A\sin(kx-wt+30^0+180^0)$ (B)$Y = 0.8A\sin(kx+wt+30^0+180^0)$

(C)$Y = 0.8A\sin(kx+wt-30^0)$ (D)$Y = 0.8A\sin(kx+wt+30^0)$

19. A wave is represented by the equation $y = 10\sin 2\pi(100t-0.02x) + 10\sin 2\pi(100t+0.02x)$. The maximum amplitude and loop length are respectively

(A) 20 units and 30 units (B) 20 units and 25 units

(C) 30 units and 20 units (D) 25 units and 20 units

20. A wave represented by the equation $y = A\cos(kx-\omega t)$ is superimposed with another wave to form a statioary wave such that the point x = 0 is a node. The equation of the other wave is:

(A) $-A\sin(kx+\omega t)$ (B) $-A\cos(kx+\omega t)$ (C) $A\sin(kx+\omega t)$ (D) $A\cos(kx+\omega t)$

21. Five waveforms moving with equal speeds on the x-axis

$y_1 = 8\sin(\omega t + kx)$; $y_2 = 6\sin(\omega t + \frac{\pi}{2} + kx)$; $y_3 = 4\sin(\omega t + \pi + kx)$; $y_4 = 2\sin(\omega t + \frac{3\pi}{2} + kx)$;

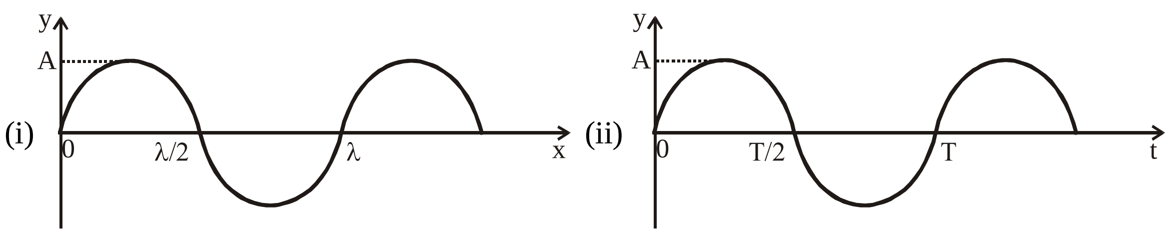
$y_5 = 4\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$ are superimposed on each other. The resulting wave is :

(A) $8\sqrt{2}\cos kx\sin(\omega t + \frac{\pi}{4})$ (B) $8\sqrt{2}\sin(\omega t - kx + \frac{\pi}{4})$

(C) $8\sqrt{2}\sin kx\cos(\omega t + \frac{\pi}{4})$ (D) $8\sin(\omega t + kx)$

22. Here given figure (i) shows snap shot at t = T/4 and figure (ii) shows motion of particle at x = λ/4.

Then the possible equations of the wave will be $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$:-



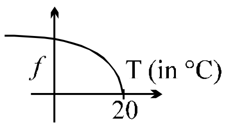
(A) $y = A\sin\left(\omega t + kx - \frac{\pi}{2}\right)$

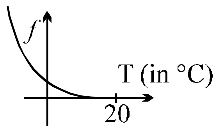
(B) $y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right)$

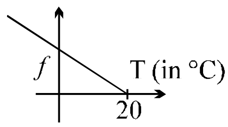
(C) Both $y = A\cos\left(\omega t + kx - \frac{\pi}{2}\right)$ and $y = A\cos\left(\omega t - kx + \frac{\pi}{2}\right)$

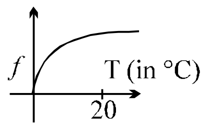
(D) Both $y = A\sin\left(\omega t + kx - \frac{\pi}{2}\right)$ and $y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right)$

23. A metal wire is clamped between two vertical walls. At 20 °C the unstrained length of the wire is exactly equal to the separation between walls. If the temperature of the wire is decreased the graph between fundamental frequency (f) and temperature (T) of the wire is

(A) 

(B) 

(C) 

(D) 

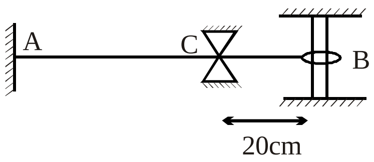
24. What is the fractional change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

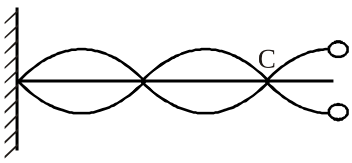
(A) 1/4 (B) 1/2 (C) 2/3 (D) 3/4 (E) 2/1

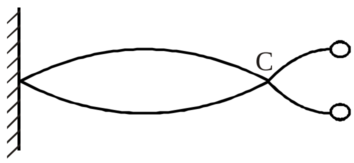
25. A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be

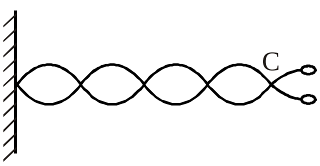
(A) zero everywhere
(B) dependent on the location along the string
(C) non zero everywhere
(D) non-zero and in the same direction everywhere

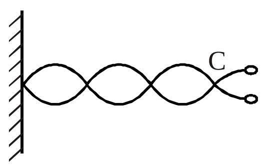
26. A 1m long wire having tension T is fixed at A and free at B. The point C, 20 cm from B is constrained to be stationary. What is shape of string for fundamental mode?



(A) 

(B) 

(C) 

(D) 

27. The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment, the displacement of the wire is $y_1 = A\sin(\pi x/L)\sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then **[JEE 2001 (Scr)]**

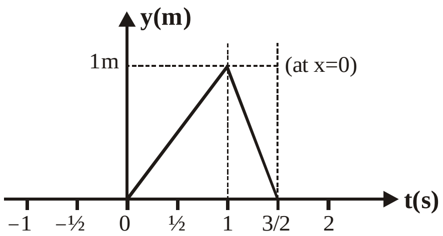
(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

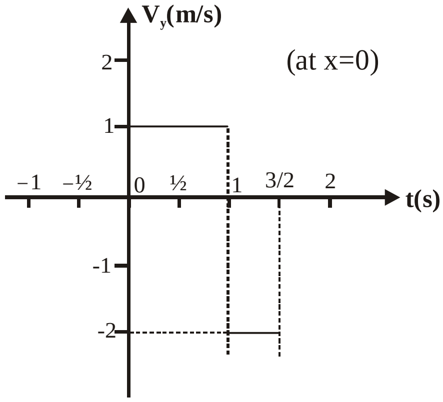
EXERCISE (O-2)

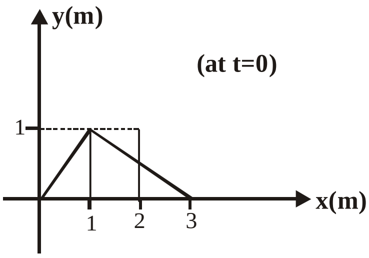
1. Consider a hypothetical wave pulse (at time t = 0) given by the following (y, x in meter)

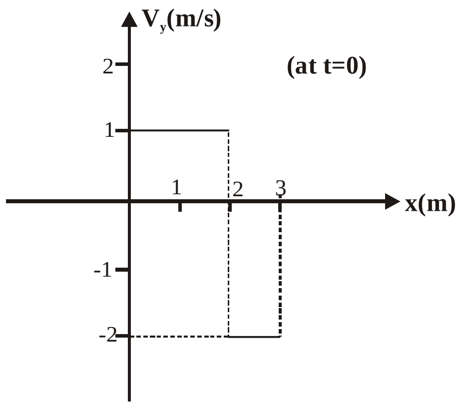
$$y = 0,\ x < 0$$
$$y = x/2,\ 2 > x \geq 0$$
$$y = 3 - x,\ 3 \geq x \geq 2$$
$$y = 0,\ x > 3$$

The pulse travels leftwards (negative x direction) at speed v = 2 m/s. Which of the following plots are correct? [V_y is the velocity of particle]

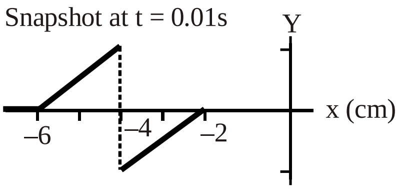
(A) 

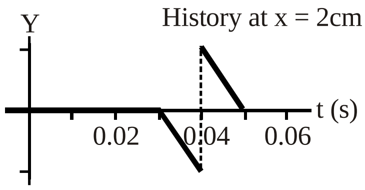
(B) 

(C) 

(D) 

2. Figure shows a snapshot graph and a history graph for a wave pulse on a stretched string. They describe the same wave from two perspectives.





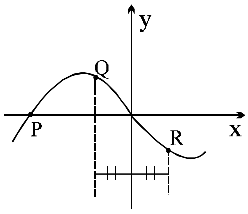
(A) the wave is travelling in positive x-direction

(B) the wave is travelling in negative x-direction

(C) the speed of the wave is 2 m/s.

(D) the peak is located at x = –6 cm at t = 0.

3. At a certain moment, the photograph of a string on which a harmonic wave is travelling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.



(A) v_P is upwards (B) $v_Q = -v_R$ (C) $|v_P| > |v_Q| = |v_R|$ (D) $v_Q = v_R$

4. An string has resonant frequencies given by 1001 Hz and 2639 Hz.

(A) If the string is fixed at one end only, 910 Hz can be a resonance frequency.

(B) If the string is fixed at one end only, 1911 Hz can be a resonance frequency.

(C) If the string is fixed at both the ends, 364 Hz can be one of the resonant frequency.

(D) 1001 Hz is definitely not the fundamental frequency of the string.

5. In a travelling one dimensional mechanical sinusoidal, wave

(A) potential energy and kinetic energy of an element become maximum simultaneously.

(B) all particles oscillate with the same frequency and the same amplitude

(C) all particles may come to rest simultaneously

(D) we can find two particles, in a length equal to half of a wavelength, which have the same non zero acceleration simultaneously.

Paragraph for Question Nos. 6 to 8

A wave represented by equation $y = 2(\text{mm}) \sin[4\pi(\text{sec}^{-1})t - 2\pi(\text{m}^{-1})x]$ is superimposed with another wave $y = 2\ (\text{mm}) \sin[4\pi(\text{sec}^{-1})t + 2\pi(\text{m}^{-1})x + \pi/3]$ on a tight string.

6. Phase difference between two particles which are located at $x_1 = 1/7$ and $x_2 = 5/12$ is :-

(A) 0 (B) $\frac{5\pi}{6}$ (C) π (D) $\frac{5\pi}{3}$

7. Which of the following is not a location of antinode?

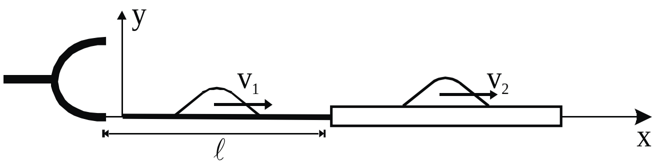
(A) $\frac{2}{3}$ (B) $\frac{11}{12}$ (C) $\frac{5}{12}$ (D) $\frac{17}{12}$

8. The location having maximum potential energy is

(A) 1/7 (B) 1/6 (C) 5/12 (D) 23/12

Paragraph for Question Nos. 9 to 11

A harmonic oscillator at x = 0, oscillates with a frequency $\frac{\omega}{2\pi}$ and amplitude a. It is generating waves at end of a thin string in which velocity of wave is v_1 and which is connected to another heavier string in which velocity of wave is v_2 as shown, length of first string is ℓ.



9. If harmonic oscillator oscillates by an equation y = a sinωt. The equation of incident wave in first string is

(A) $y = a\sin\omega\left(t - \frac{x}{v_1}\right)$

(B) $y = a\sin\omega\left(t + \frac{x}{v_1}\right)$

(C) $y = a\sin\left[\omega\left(t - \frac{x}{v_1}\right) + \pi\right]$

(D) $y = a\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$

10. Equation of transmitted wave in second string if its amplitude is a_t is

(A) $y = a_t\sin\omega\left(t - \frac{x}{v_2}\right)$

(B) $y = a_t\sin\omega\left(t - \frac{\ell}{v_1}\right)$

(C) $y = a_t\sin\omega\left(t - \frac{\ell}{v_1} - \frac{x-\ell}{v_2}\right)$

(D) $y = a_t\sin\omega\left(t - \frac{x}{v_2}\right)$

11. Equation of reflected wave, if it is reflecting at the joint and amplitude of reflected wave is a_R

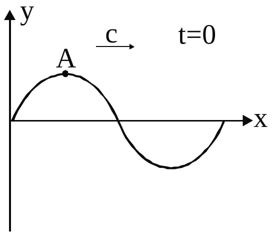
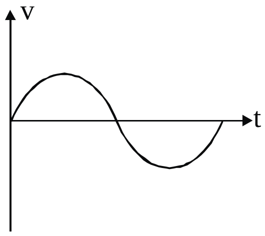
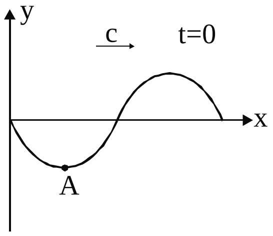
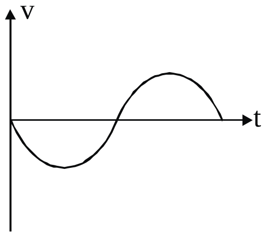
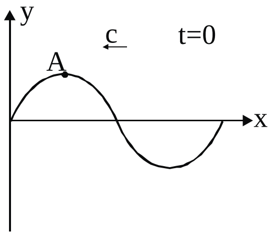
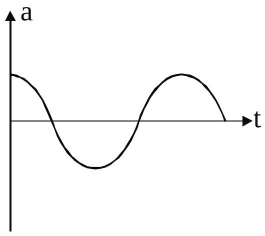
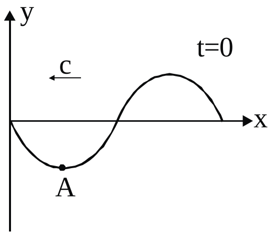
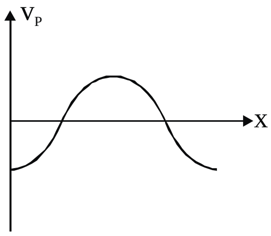
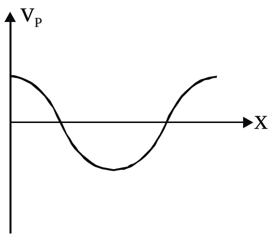
(A) $y = a_R\sin\omega\left(t - \frac{x}{v_2}\right)$

(B) $y = a_R\sin\left[\omega\left(t - \frac{\ell}{v_1} - \frac{\ell - x}{v_1}\right) + \pi\right]$

(C) $y = a_R\sin\left[\omega\left(t + \frac{x}{v_1}\right) + \pi\right]$

(D) $y = a_R\sin\left[\omega\left(t + \frac{2\ell + x}{v_1}\right) + \pi\right]$

12. In column-I transverse waves travelling on a string at t = 0 is shown. Wave velocity is indicated by '*c*'. Column-II describes variation of different parameters for particle *A* or for all the particles.

Column-I		**Column-II**	
(A)		(P)	 For particle A
(B)		(Q)	 For particle A
(C)		(R)	 For particle A
(D)		(S)	 At t = 0 for all the particles
		(T)	 At t = 0 for all the particles

13. In a string a standing wave is set up whose equation is given as $y = 2A \sin kx \cos \omega t$. The mass per unit length of the string is μ.

Column-I	**Column-II**
(A) at $t = 0$	(P) Total energy per unit length at $x = 0$ is $2\mu A^2\omega^2$.
(B) at $t = \frac{T}{8}$	(Q) Total energy per unit length at $x = \lambda/4$ is $2\mu A^2\omega^2$.
(C) at $t = \frac{T}{4}$	(R) Total energy per unit length at $x = \lambda$ is $2\mu A^2\omega^2$.
(D) at $t = \frac{T}{2}$	(S) power transmitted through a point at $x = \lambda$ is 0.
	(T) power transmitted through a point at $x = \lambda/4$ is 0.

EXERCISE (JM)

1. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is : **[AIEEE - 2010]**

(1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N

2. The transverse displacement y(x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}xt\right)}$.

This represents a :- **[AIEEE - 2011]**

(1) standing wave of frequency $\sqrt{b}$

(2) standing wave of frequency $\frac{1}{\sqrt{b}}$

(3) wave moving in +x directionwith speed $\sqrt{\frac{a}{b}}$

(4) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

3. A travelling wave represented by y = A sin(ωt–kx) is superimposed on another wave represented by y = A sin (ωt + kx). The resultant is :- **[AIEEE-2011]**

(1) A standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2$

(2) A wave travelling along + x direction

(3) A wave travelling along –x direction

(4) A standing wave having nodes at $x = \frac{n\lambda}{2}$; n = 0, 1, 2

4. A sonometer wire of length 1.5m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively ? **[JEE-Main-2013]**

(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz

5. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :-
(take g = 10 ms^{-2}) **[JEE-Main-2016]**

(1) $\sqrt{2}$ s (2) $2\pi\sqrt{2}$ s (3) 2s (4) $2\sqrt{2}$ s

6. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ? **[JEE-Main-2018]**

(1) 2.5 kHz (2) 10 kHz (3) 7.5 kHz (4) 5 kHz

EXERCISE (JA)

1. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01m) \sin [(62.8\ m^{-1})x]\cos [(628\ s^{-1})t]$. Assuming $\pi = 3.14$, the correct statement(s) is (are) **[JEE-Advance-2013]**

(A) The number of nodes is 5.

(B) The length of the string is 0.25 m.

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.

(D) The fundamental frequency is 100 Hz.

2. One end of a taut string of length 3m along the x–axis is fixed at x = 0. The speed of the waves in the string is $100\ ms^{-1}$. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform (s) of these stationary waves is(are) :- **[JEE-Advance-2014]**

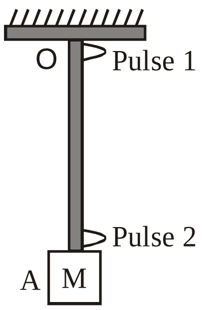
(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

3. A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_0 is produced at point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are **correct** ? **[JEE-Advance-2017]**



(A) The time $T_{AO} = T_{OA}$

(B) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the midpoint of rope

(C) The wavelength of Pulse 1 becomes longer when it reaches point A

(D) The velocity of any pulse along the rope is independent of its frequency and wavelength.

4. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be, **[JEE-Advance-2019]**

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

(1) I→P, II→R, III→S, IV→Q
(2) I→P, II→Q, III→T, IV→S
(3) I→Q, II→S, III→R, IV→P
(4) I→Q, II→P, III→R, IV→T

5. **Answer the following by appropriately matching the lists based on the information given in the paragraph.**

A musical instrument is made using four different metal strings, 1,2,3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

[JEE-Advance-2019]

List-I	**List-II**
(I) String-1(μ)	(P) 1
(II) String-2 (2μ)	(Q) 1/2
(III) String-3 (3μ)	(R) $1/\sqrt{2}$
(IV) String-4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

The length of the string 1,2,3 and 4 are kept fixed at $L_0, \frac{3L_0}{2}, \frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. Strings 1,2,3 and 4 are vibrated at their 1^{st}, 3^{rd}, 5^{th} and 14^{th} harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be.

(1) I→P, II→Q, III→T, IV→U
(2) I→T, II→Q, III→R, IV→U
(3) I→P, II→Q, III→R, IV→T
(4) I→P, II→R, III→T, IV→U

ANSWER KEY

EXERCISE (S-1)

1. Ans. (a) DEF, (b) ABH, (c) CG, (D) AE

2. Ans. (a) $\lambda = 4\text{m}$, $f = \frac{1}{4}$ Hz, 1 m/s, (b) $\frac{3\pi}{2}$ mm/s, $\frac{3\pi^2}{4}$ mm/s^2 , (c) $\frac{3\pi}{2} \times 10^{-3}$

3. Ans. $40\sqrt{2}$

4. Ans. (a) 20 m/s; (b) 2 m; (c) $y(x,t) = 0.02\sin(\pi x - 20\pi t + \pi/6)]$

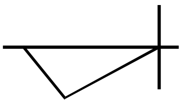
5. Ans. (a) negative x ; (b) $y = 4\times10^{-3} \sin 100\pi\left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2 \times 10^{-5}$ J

6. Ans. 1.22 v

7. Ans. $\frac{\pi}{\sqrt{2\omega}}$

8. Ans. 70 m/s

9. Ans. 10.8×10^4W

10. Ans. (a)  , (b) 2 cm/s

11. Ans. 1×10^9 Nm^2

12. Ans. $A_t = \frac{4}{3}A_i$, $A_r = \frac{1}{3}A_i$, $y_r = -\frac{0.04}{3}\sin(0.5\,\pi x + 100\,\pi t)$; $y_t = +\frac{0.16}{3}\sin(0.25\pi x - 100\,\pi t)$

13. Ans. $E = \frac{A^2\pi^2 T}{4l}$

EXERCISE (O-1)

1. Ans. (C)	**2. Ans. (B)**	**3. Ans. (A)**	**4. Ans. (B)**	**5. Ans. (C)**	**6. Ans. (B)**
7. Ans. (D)	**8. Ans. (A)**	**9. Ans. (A)**	**10. Ans. (C)**	**11. Ans. (B)**	**12. Ans. (B)**
13. Ans. (D)	**14. Ans. (D)**	**15. Ans. (B)**	**16. Ans. (B)**	**17. Ans. (C)**	**18. Ans. (C)**
19. Ans. (B)	**20. Ans. (B)**	**21. Ans. (A)**	**22. Ans. (D)**	**23. Ans. (A)**	**24. Ans. (D)**
25. Ans. (B)	**26. Ans. (A)**	**27. Ans. (C)**			

EXERCISE (O-2)

1. Ans. (A,B,D) **2. Ans. (A,C,D)** **3. Ans. (C,D)** **4. Ans. (B,C,D)**

5. Ans. (A,B,D) **6. Ans. (C)** **7. Ans. (A)** **8. Ans. (B)** **9. Ans. (A)**

10. Ans. (C) **11. Ans. (B)**

12. Ans. (A) – (Q,S) ; (B) – (P,R,T) ; (C) – (Q,T) ; (D) – (P,R,S)

13. Ans. (A) – (P,R,S,T) ; (B) – (S,T) ; (C) – (Q,S,T) ; (D) – (P,R,S,T)

EXERCISE (JM)

1. Ans. (1)	**2. Ans. (4)**	**3. Ans. (1)**	**4. Ans. (2)**	**5. Ans. (4)**	**6. Ans. (4)**

EXERCISE (JA)

1. Ans. (B,C) **2. Ans. (A,C,D)** **3. Ans. (A,D)** **4. Ans. (1)** **5. Ans. (1)**

CHAPTER 4

SOUND WAVES

Chapter Contents 04

01.	THEORY	197
02.	EXERCISE (S-1)	215
03.	EXERCISE (O-1)	217
04.	EXERCISE (O-2)	224
05.	EXERCISE (JM)	230
06.	EXERCISE (JA)	231
07.	ANSWER KEY	234

IMPORTANT NOTES

CHAPTER 4

SOUND WAVES

KEY CONCEPTS

Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker.

Sound waves are the most common example of longitudinal waves. They travel through any material waves. They travel through any material medium with a speed that depends on the properties of the medium. As the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidally, the pressure variations are also sinusoidal sound waves is very similar to that of sinusoidal string waves.

EQUATION OF SOUND WAVES

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between two successive compressions (or two successive rarefactions) equals the wavelength λ. As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave. If s(x, t) is the position of a small element relative to its equilibrium position, We can express this harmonic position function as

$$s(x, t) = s_{max} \cos(kx - \omega t)$$

Where s_{max} is the maximum position of the element relative to equilibrium. This is often called the displacement amplitude of the wave. The parameter k is the wave number and ω is the angular frequency of the piston . Note that the displacement of the element is along s, in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston in figure. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus , the pressure variation in the gas is

$$\Delta P = -B\frac{\Delta V}{V_i}$$

The element has a thickness Δx in the horizontal direction and a cross-sectional area A, so its volume is $V_i = A\Delta x$. The change in volume ΔV accompanying the pressure change is equal to $A\Delta s$, where Δs is the difference between the value of s at $x + \Delta x$ and the value of s at x. Hence, we can express ΔP as

$$\Delta P = -B\frac{\Delta V}{V_i} = -B\frac{A\Delta s}{A\Delta x} = -B\frac{\Delta s}{\Delta x}$$

As Δx approaches zero , the ratio $\Delta s/\Delta x$ becomes $\partial s/\partial x$ (The partial derivative indicates that we are interested in the variation of s with position at a fixed time.) Therefore,

$$\Delta P = -B\frac{\partial s}{\partial x}$$

If the position function is the simple sinusoidal function given by Equation , we find that

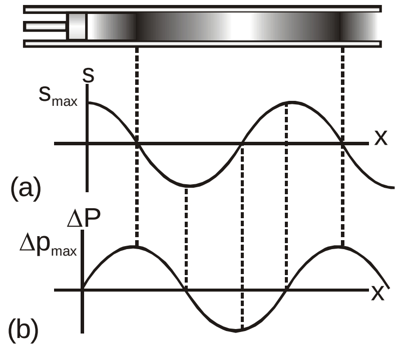


Figure : (a) Displacement amplitude and (b) pressure amplitude versus position for a sinusoidal longitudinal wave

$$\Delta P = -B\frac{\partial}{\partial x}[s_{max}\cos(kx-\omega t)] = Bs_{max}\,k\sin(kx-\omega t)$$

$\Delta P = \Delta P_{max}\sin(kx-\omega t)$

Thus we can describe sound waves either in terms of excess pressure (equation 1.1) or in terms of the longitudinal displacement suffered by the particles of the medium.

If $s = s_0 \sin\omega(t - x/v)$ represents a sound wave where

s = displacement of medium particle from its mean position at x, then it can be proved that

$P = P_0 \sin\{w(t - x/v) + \pi/2\}$ ... (3.2)

represents that same sound wave where, P is excess pressure at position x, over and above the average atmospheric pressure and the pressure amplitude P_0 is given by

$P_0 = \dfrac{B\omega s_0}{v} = BKs_0$ ... (3.3)

(B = Bulk modulus of the medium, K = angular wave number)

Note from equation (3.1) and (3.2) that the displacement of a medium particle and excess pressure at any position are out of phase by $\dfrac{\pi}{2}$. Hence a displacement maxima corresponds to a pressure minima and vice-versa.

Ex. The equation of a sound wave in air is given by P = 0.2 sin [3000 t – 9x], where all variables are in S.I. units.

(a) Find the frequency, wavelength and the speed of sound wave in air.

(b) If the equilibrium pressure of air is 1.0×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point?

Sol. (a) Comparing with the standard form of a travelling wave

$P = P_0 \sin[\omega(t - x/v)]$

we see that $\omega = 3000\ s^{-1}$. The frequency is

$$t = \frac{\omega}{2\pi} = \frac{3000}{2\pi}\ \text{Hz}$$

Also from the same comparison, $\omega/v = 9.0\ m^{-1}$.

$$\text{or,}\quad v = \frac{\omega}{9.0\,m^{-1}} = \frac{3000\,s^{-1}}{9.0\,m^{-1}} = \frac{1000}{3}\ m/s^{-1}$$

$$\text{The wavelength is } \lambda = \frac{v}{f} = \frac{1000/3m/s}{3000/2\pi\,Hz} = \frac{2\pi}{9}\ m$$

(b) The pressure amplitude is $P_0 = 0.02$ N/m^2. Hence, the maximum and minimum pressures at a point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)$ N/m^2.

Ex. A sound wave of wavelength 40cm travels in air. If the difference between the maximum and minimum pressure at a given point is 4.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m^2.

Sol. The pressure amplitude is

$$p_0 = \frac{4.0 \times 10^{-3}\ \mathrm{N/m^2}}{2} = 2 \times 10^{-3}\ \mathrm{N/m^2}$$

The displacement amplitude s_0 is given by

$$p_0 = Bks_0$$

or $$s_0 = \frac{p_0}{Bk} = \frac{p_0\lambda}{2\pi B}$$

$$= \frac{2\times10^{-3}\ \mathrm{N/m^2} \times (40 \times 10^{-2}\ \mathrm{m})}{2 \times \pi \times 14 \times 10^4\ \mathrm{N/m^2}} = \frac{200}{7\pi}\ \text{Å}$$

$$= 13.2\ \text{Å}$$

SPEED OF SOUND WAVES :

Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}} \qquad (4.1)$$

where Y = young's modulus of elasticity and ρ = density

Velocity of sound wave in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}} \qquad (4.2)$$

where, ρ = density of the medium and B = Bulk modulus of the medium given by,

$$B = -V\frac{dP}{dV} \qquad (4.3)$$

Newton's formula : Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

$$PV = \text{constant}$$

$$\Rightarrow \quad \frac{dP}{dV} = \frac{-P}{V} \quad \text{and} \quad \text{hence } B = P$$

and thus he obtained for velocity of sound in a gas.

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}} \quad \text{where M = molar mass}$$

Laplace's correction : Later Laplace established that propagation of sound in a gas is not an isothermal but an adiabatic process and hence PV′ = constant

$$\Rightarrow \quad \frac{dP}{dV} = -\gamma\frac{P}{V}$$

where, $B = -V\dfrac{dP}{dV} = \gamma P$

and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \qquad ...(4.4)$$

FACTORS AFFECTING SPEED OF SOUND IN ATMOSPHERE

(a) Effect of temperature : as temperature (T) increases velocity (v) increases. For small change in temperature above room temperature v increases linearly by 0.6 m/s for every 1°C rise in temp.

(b) Effect of humidity : With increase in humidity density decreases. This is because the molar mass of water vapour is less than the molar mass of air.

(c) Effect of pressure :

The speed of sound in a gas is given by $v = \sqrt{\dfrac{\gamma P}{P}} = \sqrt{\dfrac{\gamma RT}{M}}$

So at constant temperature, if P changes then ρ also changes in such a way that P/ρ remains constant. Hence pressure does not have any effect on velocity of sound as long as temperature is constant.

Ex. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Sol. $v = \sqrt{\dfrac{\gamma RT}{M}}$

since temperature, T is constant

$$\therefore \quad \frac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4 \Rightarrow v(H_2) = 4 \times 450 = 1800 \text{ m/s}$$

Aliter : The speed of sound in a gas is given by $u = \sqrt{\dfrac{\gamma P}{\rho}}$. At STP, 22.4 litres of oxygen has a mass of 32g whereas the same volume of hydrogen has a mass of 2g. Thus, the density of oxygen is 16times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases.

$$\frac{f_{(hydrogen)}}{f_{(oxygen)}} = \sqrt{\frac{\rho_{(oxygen)}}{\rho_{(hydrogen)}}}$$

or $\qquad f_{(hydrogen)} = 4f_{(oxygen)} = 4 \times 450 \text{ m/s} = 1800 \text{ m/s}$

INTENSITY OF PERIODIC SOUND WAVES

Like any other progressive wave, sound waves also carry energy from one point of space to the other. This energy can be used to work, for example, forcing the eardrums to vibrate or in the extreme case of a sonic boom created by supersonic jet, can even cause glass panes of windows to crack.

The amount of energy carried per unit by a wave is called its power and power per unit area held perpendicular a sound wave travelling along positive x-axis described by the equation.

We define the intensity I of a wave, or the power per unit area , to be the rate at which the energy being transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave :

$$I = \frac{P}{A}$$

In the present case, therefore, the intensity is

$$I = \frac{P}{A} = \frac{1}{2}\rho A v(\omega s_{max})^2$$

Thus, we see that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency (as in the case of a periodic string wave). this can also be written in terms of the pressure amplitude ΔP_{max} ; in this case, we use Equation to obtain

$$I = \frac{\Delta P_{max}^2}{2\rho v}$$

Ex. The pressure amplitude in a sound wave from a radio receiver is 4.0×10^{-3} N/m^2 and the intensity at a point is 10^{-6} W/m^2. If by fuming the "Volume" knob the pressure amplitude is increased to 6×10^{-3} N/m^2, evaluate the intensity.

Sol. The intensity is proportional to the square of the pressure amplitude.

Thus, $\frac{I'}{I} = \left(\frac{p_0'}{p_0}\right)^2$ or $I' = \left(\frac{p_0'}{p_0}\right)^2 I = \left(\frac{p_0'}{p_0}\right)^2 \times 10^{-6}$ W/m^2 = 2.25×10^{-16} W/m^2.

APPEARANCE OF SOUND TO HUMAN EAR

Pitch and Frequency

Frequency as we have discussed till now is an objective property measured its units of Hz and which can be assigned a unique value. However a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called Pitch. A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound.

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice is of low pitch, a male voice has higher pitch and a female voice has (generally) still higher pitch. This sensation primarily depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa. The dominant frequency of a buffalo voice is smaller than that of a male voice which in turn is smaller than that of a female voice.

Loudness and Intensity

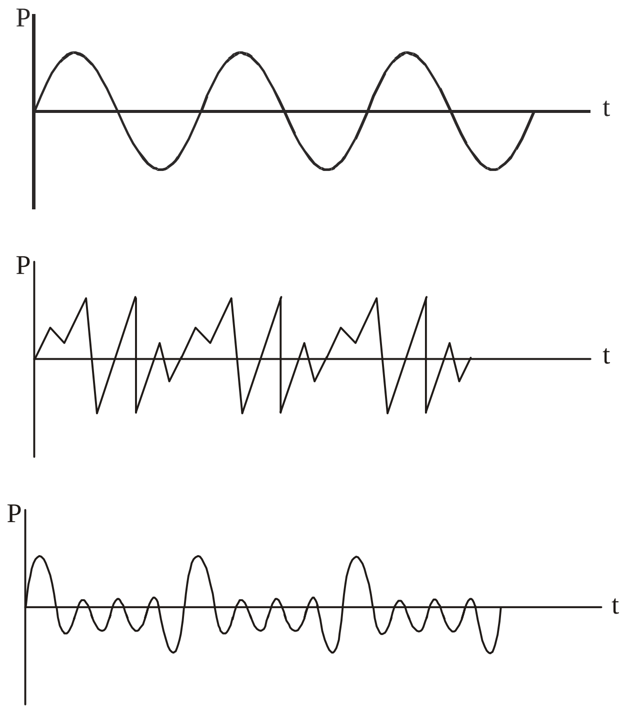
The loudness that we sense is related to the intensity of sound through it is not directly proportional to it. Our perception of loudness is better correlated with the sound level measured in decibels (abbreviated and dB) and defined as follows.

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right),$$

where I is the intensity of the sound and I_0 is a constant reference intensity 10^{-12} W/m^2. The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$, the sound level $\beta = 0$. Table shows the approximate sound levels of some of the sounds commonly encountered.

Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.



We differentiate between the sound from a table and that from a mridang by saying that they have different quality. A musical sound has certain well-defined frequencies which have considerable amplitude. These frequencies are generally harmonics of a fundamental frequency. Such a sound is particularly pleasant to the ear. On the other hand, a noise has frequencies that do not bear well-defined relationship among themselves.

Ex. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

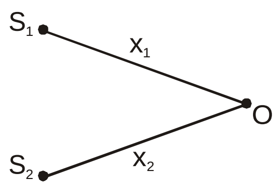
Sol. $\beta_1 = 10 \log \dfrac{I_1}{I_0}$

$\beta_2 = 10 \log \dfrac{I_2}{I_0} \Rightarrow \beta_2 - \beta_1 = 10 \log \dfrac{I_2}{I_1}$ or $10 = 10 \log_{10}\left(\dfrac{I_2}{I_1}\right)$

$\Rightarrow \dfrac{I_2}{I_1} = 10^1 = 10$

for point source $I \propto \dfrac{1}{r^2} \Rightarrow \dfrac{r_1}{r_2} = \sqrt{\dfrac{I_2}{I_1}} = \sqrt{10}$

INTERFERENCE OF SOUND WAVES :



If $p_1 = p_{m1} \sin(\omega t - kx_1 + \theta_1)$

and $p_2 = p_{m2} \sin(\omega t - kx_2 + \theta_1)$

resultant excess pressure at point O is

$p = p_1 + p_2$

$\Rightarrow p = p_0 \sin(\omega t - kx + \theta)$

where, $p_0 = \sqrt{p_{m_1}^2 + p_{m_2}^2 + 2p_{m_1}p_{m_2}\cos\phi}$, $\phi = |\,k(x_1 - x_2) + (\theta_1 - \theta_2)\,|$ ... (6.1)

(i) for constructive interference

$\phi = 2n\pi \qquad \Rightarrow \qquad p_0 = p_{m1} + p_{m2}$

(ii) for destructive interference

$\phi = (2n + 1)\pi \qquad \Rightarrow \qquad p_0 = |\,p_{m1} - p_{m2}\,|$

If ϕ is only due to path difference, then $\phi = \dfrac{2\pi}{\lambda}\Delta x$, and

Condition for constructive interference : $\Delta x = n\lambda$, $\qquad n = 0, \pm 1, \pm 2$

Condition for destructive interference : $\Delta x = (2n + 1)\dfrac{\lambda}{2}$, $\qquad n = 0 \pm 1, \pm 2$

from equation (6.1)

$P_0^{\,2} = P_{m_1}^2 + P_{m_2} + 2P_{m_1} P_{m_2} \cos\phi$

Since intensity, $I \propto$ (Pressure amplitude)^2,

we have, for resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_2 I_2}\cos\phi$ ... (6.2)

If $I_1 = I_2 = I_0$

$I = 2I_0(1 + \cos\phi)$

$\Rightarrow I = 4I_0 \cos^2\dfrac{\phi}{2}$ ... (6.3)

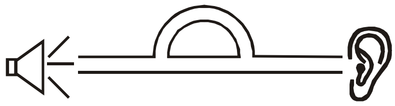
Hence in this case,

for constructive interference : $\phi = 0\ 2\pi, 4\pi$ ..... and $I_{max} = 4I_0$

and for destructive interference : $\phi = \pi, 3\pi$ .... and $I_{min} = 0$

Coherence : Two sources are said to be coherent if the phase difference between them does not change with time. In this case their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

Ex. Figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 10.0 cm. The frequency of the sound source can be varied from 1 to 10 kHz. Find the frequencies at which the ear perceives maximum intensity. The speed of sound in air = 342 m/s.



Sol. The sound wave bifurcates at the junction of the straight and the semicircular parts. The wave through the straight part travels a distance $s_1 = 2 \times 10$ cm and the wave through the curved part travels a distance $s_2 = \pi\ 10$ cm $= 31.4$ cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore.

$$\Delta s = s_2 - s_1 = 31.4 \text{ cm} - 20 \text{ cm} = 11.4 \text{ cm}$$

The wavelength of either wave is $\frac{v}{\upsilon} = \frac{330\ m/s}{\upsilon}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

$$\text{or , } \Delta p = n \cdot \frac{v}{\upsilon} \quad \Rightarrow \quad \upsilon = \frac{n \cdot v}{\Delta p} \Rightarrow \frac{n \cdot 342}{(0.114)} = 3000\ n$$

Thus, the frequencies within the specified range which cause maximum of intensity are

3000×1 $\quad$ 3000×2 $\quad$ and $\quad$ 3000×3 Hz

LONGITUDINAL STANDING WAVES :

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering wave are given by

$p_1 = p_0 \sin(\omega t - kx)$

and $p_2 = p_0 \sin(\omega t + kx + \phi)$

then the equation of the resultant standing wave would be given by

$$p + p_1 + p_2 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad p = p_0' \sin\left(\omega t + \frac{\phi}{2}\right) \qquad \text{... (8.1)}$$

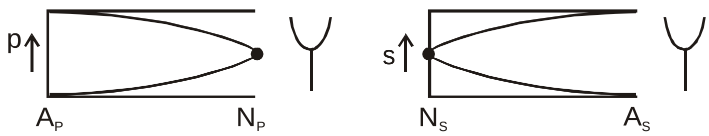
VIBRATION OF AIR COLUMNS

Standing waves can be set up in air-columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequency of air columns. In such a case the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air column. To determine the natural frequency of the air column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air-column.

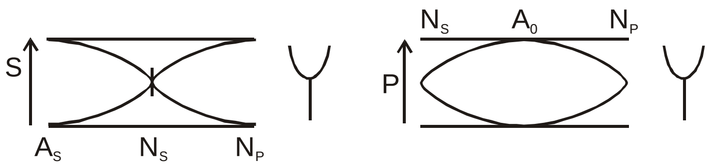
In reality antinodes do not occurs exactly at the open end but a little distance outside. However if diameter of tube is small compared to its length, this end correction can be neglected.

Closed organ pipe

(In the diagram, A_P = Pressure antinode, A_S = displacement antinode, N_P = pressure node, N_s = displacement node.)

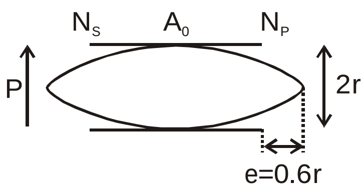


Open organ pipe :



End correction : As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by

$$e = 0.6\,r$$



where r = radius of the organ pipe.

with end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)} \text{ and}$$

$$f_0 = \frac{v}{2(\ell + 1.2r)} \qquad \text{......... (9.5)}$$

Ex. A tuning fork is vibrating at frequency 200 Hz. When another tuning fork is sounded simultaneously, 6 beats per second are heard. When some mass is added to the tuning fork of 200 Hz, beat frequency decreases. Find the frequency of the other tuning fork.

Sol. $|f - 200| = 6$

$\Rightarrow$ f = 194 or 206

when 1st tuning fork is loaded its frequency decreases and so does beat frequency

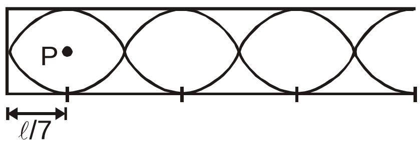
$\Rightarrow$ 200 > f

$\Rightarrow$ f = 194 Hz

Ex. A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Sol. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$ or $\frac{\lambda}{4} = \frac{\ell}{7}$

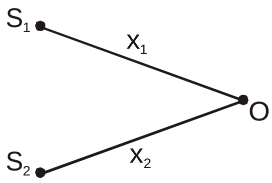


Hence the amplitude at P at a distance $\frac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point.

BEATS

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,



$p_1 = p_0 \sin(2\pi f_1 t - kx_1 + \theta_1)$

$p_2 = p_0 \sin(2\pi f_2 t - kx_2 + \theta_2)$

By principle of superposition

$p = p_1 + p_2$

$= 2\pi_0 \cos\{\pi(f_1 - f_2)t + \frac{\theta_1+\theta_2}{2}\} \sin\{\pi(f_1 + f_2)t + \frac{\theta_1+\theta_2}{2}\}$

i.e., the resultant sound at point O has frequency $\left(\frac{f_1+f_2}{2}\right)$ while pressure amplitude p'_0 (t) variex with time as

$$p_0(t) = 2p_0 \cos\left\{\pi(f_1 - f_2)t + \frac{\phi_1+\phi_2}{2}\right\}$$

Hence pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1-f_2}{2}\right)$.

Hence sound intensity will vary with a frequency $f_1 - f_2$.

This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$f_B = f_1 - f_2$

$$T_B = \frac{1}{f_1 - f_2}$$

IMPORTANT POINTS :

(i) The frequency $|f_1 - f_2|$ should be less than 16 Hz, for it to be audible.

(ii) Beat phenomenon can be used for determining an unknown frequency by sounding it together with a sound of known frequency.

Ex. Two strings X and Y of a sitar produces a beat of frequency 4Hz. When the tension of string Y is slightly increased, the beat frequency is found to be 2Hz. If the frequency of X is 300 Hz, then the original frequency of Y was.

Ans. 296 Hz

DOPPLER EFFECT

We can express the general relationship for the observed frequency when a source is moving and an observer is at rest as equation , with the same sign convention applied to v_s as was applied to v_0 : a positive value is substituted for v_s when the source moves toward the observer and a negative value is substituted when the source moves away from the observer.

Finally, we find the following general relationship for the observed frequency :

$$f' = \left(\frac{v + v_0}{v - v_s}\right) f$$

In this expression, the signs for the values substituted for v_0 and v_s depend on the direction of the velocity. A positive value is used for motion of the observer or the source toward the other, and a negative sign for motion of one away from the other.

A convenient rule concerning signs for you to remember when working with all Doppler effect problems is as follows :

The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency.

Ex. A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward one another. The second submarine is moving at 9.00 m/s.

(A) What frequency is detected by an observer riding on sub B as the subs approach each other?

(B) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?

Sol. (A) We use equation to find the Doppler - shifted frequency. As the two submarines approach each other, the observer in sub B hears the frequency

$$f' = \left(\frac{v + v_0}{v - v_s}\right) f = \left(\frac{1533 m/s + (+9.00 m/s)}{1533 m/s - (+8.00 m/s)}\right) (1400 \text{ Hz}) = 1416 \text{ Hz}$$

(B) As the two submarines recede from each other, the observer in sub B hears the frequency

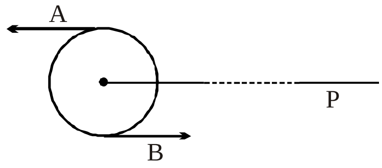
$$f' = \left(\frac{v + v_0}{v - v_s}\right) f$$

$$= \left(\frac{1533 m/s + (-9.00 m/s)}{1533 m/s - (-8.00 m/s)}\right) (1400 \text{ Hz}) = 1385 \text{ Hz}$$

Ex. A whistle of frequency 540 Hz is moving in a circle of radius 2m at a constant angular speed of 15rad/s. What are the lowest and height frequencies heard by a listener standing at rest, a long distance away from the centre of the circle ? (velocity of sound in air is 330 m/s ft/sec.)

Sol. The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$$v_s = \omega R$$



where, R is radius of the circle

At points A and B, the velocity v_s of whistle is parallel to line OP, i.e. with respect to observer at P, whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP) Observer, therefore, listens maximum frequency when source is at B moving towards observer.

$$f_{max} = f\frac{v}{v - v_s} = 540 \times \frac{330}{330-30} = 540 \times \frac{330}{300} = 594 \text{ Hz}$$

where, v is speed of sound in air. Similarly, observer listens minimum frequency when source is at A, moving away from observer.

$$f_{min} = \frac{f\ v}{v + v_s} = 540 \times \frac{330}{360} = 495 \text{ Hz}$$

1. Vibrating air columns :

(i) In a pipe of length L closed at one end, the fundamental note has a frequency $f_1 = \frac{v}{4L}$, where v is the velocity of sound in air.

(ii) The first overtone $f_2 = \frac{v}{L} = 2f_1$

2. Propagation of sound in solids :

(i) The velocity of propagation of a longitudinal wave in a rod of Young's modulus Y and density ρ is given by $v = \sqrt{\frac{Y}{\rho}}$

(ii) In a sonometer wire of length L and mass per unit length m under tension T vibrating in n loops $f_n = \frac{n}{2L}\sqrt{\frac{T}{m}}$

(iii) Propagation of sound in gases

Laplace formula $v = \sqrt{\frac{\gamma P}{\rho}}$ where γ is the ratio of specific heats, P is the pressure and ρ is the density. $\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$

3. Doppler Effect :

(i) When a source of sound moves with a velocity v_s in a certain direction, the wavelength decreases in front of the source and increases behind the source.

$$\lambda' \text{ (in front)} = \frac{v - v_s}{f_s} \; ; \; f' = \frac{v}{\lambda'} = \frac{v}{v - v_s} f_s$$

$$\lambda'' \text{(behind)} = \frac{v + v_s}{f_s} \; ; \; f'' = \frac{v}{\lambda''} = \frac{v}{v + v_s} f_s$$

Here v is the velocity of sound in air.

(ii) The apparent frequency $= \dfrac{v - v_0}{v} f_s$

(a) When the source is moving towards the observer and the observer is moving away from the source, the apparent frequency

$$f_a = \frac{v - v_0}{v - v_s} f_s$$

$$v_0 \longleftarrow o \qquad v_s \longleftarrow s$$

(b) When the source and the observer are moving towards each other.

$$f_a = \frac{v + v_0}{v - v_s} f_s$$

$$o \longrightarrow v_0 \qquad v_s \longleftarrow s$$

(c) When the source and observer are moving away from each other,

$$f_a = \frac{v - v_0}{v + v_s} f_s$$

$$v_0 \longleftarrow o \qquad s \longrightarrow v_s$$

(d) When the source is moving away from the observer and the observer is moving towards the source

$$f_a = \frac{v + v_0}{v + v_s} f_s$$

$$o \longrightarrow v_0 \qquad s \longrightarrow v_s$$

Here all velocities are relation to the medium.

4. Loudness of sound :

The loudness level B of sound is expressed in decibels, $B = 10 \log \dfrac{I}{I_0}$ where I is the intensity, I_0 is a reference intensity.

5. Beats :

When two tuning forks of close but different frequencies f_1 and f_2 are vibrating simultaneously at nearby places, a listener observes a fluctuation in the intensity of sound, called beats. The number of beats heard per second is $f_1 - f_2$.

WORKED OUT EXAMPLES

Ex.1 It is found that an increase in pressure of 100 kPa causes a certain volume of water to decrease by 5×10^{-3} percent of its original volume. Then the speed of sound in the water is about (density of water 10^3 kg/m^3)

(A) 330 m/s (B) 1414 m/s (C) 1732 m/s (D) 2500 m/s

Ans. **(B)**

Sol. Bulk modulus $\beta = -\dfrac{\Delta P}{\frac{\Delta v}{v}} = \dfrac{100\times10^3}{\frac{5\times10^{-3}}{100}} = 2\times10^9$

speed $v = \sqrt{\dfrac{\beta}{\rho}} = \sqrt{\dfrac{2\times10^9}{10^3}} \approx 1414 \text{ m/s}$

Ex.2 A sound wave is travelling in a uniform pipe with gas of adiabatic exponent γ. If u is the particle velocity at any point in medium and c is the wave velocity, then relative change in pressure $\dfrac{dP}{P}$ while wave passes through this point is :-

(A) $\dfrac{u}{\gamma c}$ (B) $\gamma\sqrt{\dfrac{u}{c}}$ (C) $\gamma\dfrac{u}{c}$ (D) $\dfrac{u^2}{\gamma c^2}$

Ans. **(C)**

Sol. $\Delta P = -B\left(\dfrac{dv}{v}\right)$

$\Delta P = -B\left(\dfrac{\delta y}{\delta x}\right)$

$\Delta P = -\gamma P\dfrac{\delta y}{\delta x}$

$\dfrac{\Delta P}{P} = \gamma\left(\dfrac{u}{c}\right)$

Ex.3 A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 meters and 25 meters respectively from the source. The ratio of the amplitudes of the waves at P and Q is :-

(A) 5 : 3 (B) 3 : 5 (C) 25 : 9 (D) 625 : 81

Ans. **(C)**

Sol. $I \propto A^2$, $I \propto \dfrac{1}{r^2}$, $\dfrac{A_1}{A_2} = \dfrac{r_2}{r_1}$

Ex.4 In a resonance tube experiment, an 80 cm air column is in resonance with a turning fork in first overtone. Which equation can represent correct pressure variation in the air column (x = 0 is the top point of the tube, neglect end correction, speed of sound = 320 m/sec) :-

(A) $A \sin \frac{15\pi}{8} x \cos 600\pi t$ (B) $A \cos \frac{15\pi}{8} x \sin 600\, \pi t$

(C) $A \cos \frac{15\pi}{8} x \sin 300\, \pi t$ (D) $A \sin \frac{15\pi}{8} x \sin 300\, \pi t$

Ans. **(A)**

Sol. $\frac{\omega}{k} = 320$

Ex.5 The displacement of the medium in a sound wave is given by the equation ; $y_1 = A \cos (ax + bt)$ where A, a & b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave.

(a) what are the wavelength & frequency of the incident wave.

(b) write the equation for the reflected wave.

(c) in the resultant wave formed after reflection, find the maximum & minimum values of the particle speeds in the medium.

Ans. (a) $2\,\pi/a$, $b/2\pi$, (b) $y_2 = \pm\, 0.8\, A \cos (ax - bt)$, (c) max.=1.8 bA, min. = 0,

Sol. (a) $\omega = b$ & $k = a$

$$f = \frac{2\pi}{\omega} \;\&\; \lambda = \frac{2\pi}{k}$$

(b) $I \propto A^2$

So $A_r = 0.8\, A$

(c) $(A_{net})_{max} = A + 0.8\, A = 1.8\, A$

Ex.6 An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. what is the percentage increase in the apparent frequency ? **[AIEEE - 2005]**

(1) zero (2) 0.5% (3) 5% (4) 20%

Ans. **(4)**

Sol. $$\frac{f_{app}}{f} = \left(\frac{v + \frac{v}{5}}{v}\right) = \frac{6}{5}$$

Ex.7 The equation of a longitudinal standing wave due to superposition of the progressive waves produced by two sources of sound is s = – 20 sin 10 πx sin 100 πt where s is the displacement from mean position measured in mm ; x is in meters and t in seconds. The specific gravity of the medium is 10^{-3}. Find

(a) wavelength, frequency and velocity of the progressive waves.

(b) Bulk modulus of the medium and the pressure amplitude of the progressive waves.

(c) minimum distance between pressure antinode and the displacement antinode.

Ans. (a) 1/5 m, 50 Hz, 10 m/s; (b) 100 Pa, 10π Pa, (c) 1/20 m

Sol. $k = 10\pi$

$\omega = 100\ \pi$

$\because\ \lambda = \frac{1}{5}$ & f = 50 Hz

$v = \lambda f = 10$ m/s

$B = \rho v^2$

$P_0 = BKs$

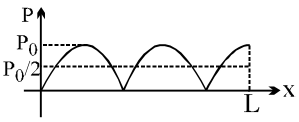
Minimum distance between pressure antinode and displacment antinode = $\frac{\lambda}{4}$

Ex.8 The air column in a pipe closed at one end and open to atmosphere at the other end is made to vibrate in its fifth harmonic by a tuning fork of frequency 470 Hz. The length of air column is $\frac{15}{16}$ m. Neglect end correction. Let p_0 denote the maximum gauge pressure at the closed end

(a) Find the speed of sound in air.

(b) Draw the graph of pressure amplitude vs distance from the open end of the tube.

(c) Find the points where the maximum gauge pressure is $\frac{p_0}{2}$.

Ans. (a) 352.5 m/s, (b) 

, (c) $\frac{l}{15}, \frac{5l}{15}, \frac{7l}{15}, \frac{11l}{15}, \frac{13l}{15}$

Sol. $\frac{5\lambda}{4} = \frac{15}{16}$

$\lambda = 0.75$ m

f = 470

$v = \lambda f = 352.5$ m/s

$$P = P_0 \sin\left(\frac{2\pi}{\lambda}x\right)$$

where $\lambda = \frac{4\ell}{5}$

$$P = P_0 \sin\left(\frac{5\pi}{2\ell}x\right)$$

Ex.9 A metal rod of length $l = 100$ cm is clamped at two points. Distance of each clamp from nearer end is a = 30cm. If density and Young's modulus of elasticity of rod material are $\rho = 9000$ kg m^{-3} and Y = 144 GPa respectively, calculate minimum and next higher frequency of natural longitudinal oscillations of the rod.

Ans. 10kHz, 30kHz

Sol. $v = \sqrt{\frac{Y}{\rho}} = 4 \times 10^3$ m/s

$\lambda = 0.4$ m

$f_0 = \frac{v}{\lambda} = 10\text{kHz}$

Ex.10 When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2 ? **[AIEEE - 2005]**

(1) 200 Hz (2) 202 Hz (3) 196 Hz (4) 204 Hz

Ans. **(3)**

Sol. $f_1 - f_2 = 4$ or $f_2 - f_1 = 4$

But according to question

$f_1 - f_2 = 4$

So $f_2 = 196$

Ex.11 A whistling train approaches a junction. An observer standing at junction observers the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed of sound = 300 m/s). **[JEE 2005]**

Ans. $V_s = 30$ m/s

Sol. $\left(\frac{v}{v - v_S}\right) f_0 = 2.2 \times 10^3$

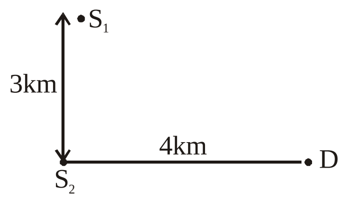
$\left(\frac{v}{v + v_S}\right) f_0 = 1.8 \times 10^3$

Dividing both we get

$\frac{v + v_S}{v - v_S} = \frac{11}{9}$

$v_S = 30$

Ex.12 Two point sound source S_1 and S_2 are both have the same power and send out sound waves in the same phase. The wavelength of both the waves is $\frac{48}{5}$ m. The intensity due to S_2 alone at D is 25 W/m^2. The resultant intensity at D is :



(A) 59 W/m^2 (B) 61 W/m^2 (C) 65 W/m^2 (D) None of these

Ans. **(B)**

Sol. $I \propto \frac{1}{r^2}$

Intensity due to S_1 alone at D = 16 W/m^2

Phase difference = $\frac{2\pi}{\left(\frac{48}{5}\right)} \times 1000 = \frac{625}{3}\pi$

$I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$= 25 + 16 + 2\,(5)\,(4)\left(\frac{1}{2}\right)$

Ex.13 Spherical sound waves are emitted uniformly in all directions from a point source. The variation in sound level SL as a function of distance r(> 0) from the source can be written as :- (where a and b are positive constant)

(A) $SL = -b \log r^a$ (B) $SL = a - b \log r$ (C) $SL = a - b(\log r)^2$ (D) $SL = a - \frac{b}{r^2}$

Ans. **(B)**

Sol. $I \propto \frac{1}{r^2}$

$SL = 20 \log_{10} = \frac{I}{I_0}$

EXERCISE (S-1)

Sound basics

1. Find the intensity of sound wave whose frequency is 250 Hz. The displacement amplitude of particles of the medium at this position is 1×10^{-8} m. The density of the medium is 1 kg/m^3, bulk modulus of elasticity of the medium is 400 N/m^2.

2. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is *c*. Find the velocity of sound in the gas.

3. The loudness level at a distance R from a long linear source of sound is found to be 40dB. At this point, the amplitude of oscillations of air molecules is 0.01 cm. Then find the loudness level & amplitude at a point located at a distance '10R' from the source.

Superposition of sound

4. The first overtone of a pipe closed at one end resonates with the third harmonic of a string fixed at its ends. The ratio of the speed of sound to the speed of transverse wave travelling on the string is 2 :1. Find the ratio of the length of pipe to the length of string.

5. In a resonance-column experiment, a long tube, open at the top, is clamped vertically. By a separate device, water level inside the tube can be moved up or down. The section of the tube from the open end to the water level act as a closed organ pipe. A vibrating tuning fork is held above the open end, first and the second resonances occur when the water level is 24.1 cm and 74.1 cm respectively below the open end. Find the diameter of the tube.

6. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length, 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air.

7. An open organ pipe filled with air has a fundamental frequency 500Hz. The first harmonic of another organ pipe closed at one end and filled with carbon dioxide has the same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each pipe. Assume that the velocity of sound in air and in carbondioxide to be 330 and 264 m/s respectively.

8. A steel rod having a length of 1 m is fastened at its middle. Assuming young's modulus to be 2×10^{11} Pa, and density to be 8 gm/cm^3 find the fundamental frequency of the longitudinal vibration and frequency of first overtone.

9. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. **[JEE 2002]**

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

10. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320m/sec. Estimate the diameter of the tube. **[IIT-1980]**

Beats

11. A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning fork.

12. A, B and C are three tuning forks. Frequency of A is 350Hz. Beats produced by A and B are 5 per second and by B and C are 4 per second. When a wax is put on A beat frequency between A and B is 2Hz and between A and C is 6Hz. Then, find the frequency of B and C respectively.

13. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? **[IIT-1981]**

14. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork (assume v_{sound} = 340 m/s) **[IIT-1986]**

Doppler effect

15. Two tuning forks A and B lying on opposite sides of observer 'O' and of natural frequency 85 Hz move with velocity 10 m/s relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him. A wind with a speed 10 m/s is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s]

16. A car is moving towards a huge wall with a speed = c/10 , where c = speed of sound in still air. A wind is also blowing parallel to the velocity of the car in the same direction and with the same speed. If the car sounds a horn of frequency f, then what is the frequency of the reflected sound of the horn heard by driver of the car?

17. A plane sound wave of frequency f_0 and wavelength λ_0 travels horizontally toward the right. It strikes and is reflected from a large, rigid, vertical plane surface, perpendicular to the direction of propagation of the wave and moving towards the left with a speed v.

(a) How many positive wave crests strike the surface in a time interval t ?

(b) At the end of this time interval, how far to the left of the surface is the wave that was reflected at the beginning of the time interval ?

(c) What is the wavelength of the reflected waves, in terms of λ_0 ?

(d) What is the frequency, in terms of f_0 ?

(e) A listener is at rest at the left of the moving surface. Describe the sensation of sound that he hears as a result of the combined effect of the incident and reflected wave trains.

18. A bus is moving towards a huge wall with a velocity of 5 ms^{-1}. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (speed of sound in air = 342 ms^{-1}) **[IIT-1994]**

EXERCISE (O-1)

Sound basics

1. A sound wave has a wavelength of 3.0 m. The distance from a compression center to the adjacent rarefaction center is :-

(A) 0.75 m (B) 1.5 m

(C) 3.0 m (D) need to know wave speed

2. You are listening to an "A" note played on a violin string. Let the subscript "s" refer to the violin string and "a" refer to the air. Then :-

(A) $f_s = f_a$ but $\lambda_s \neq \lambda_a$ (B) $f_s = f_a$ and $\lambda_s = \lambda_a$ (C) $\lambda_s = \lambda_a$ but $f_s \neq f_a$ (D) $\lambda_s \neq \lambda_a$ and $f_s \neq f_a$

3. The fig.(i) shows the graphical representation of the air molecules in a tube of air (length = L) at atmospheric pressure on the absolute pressure P(x) graph. Which one of the following pictures corresponds to the absolute pressure P(x) graph of fig. (ii).

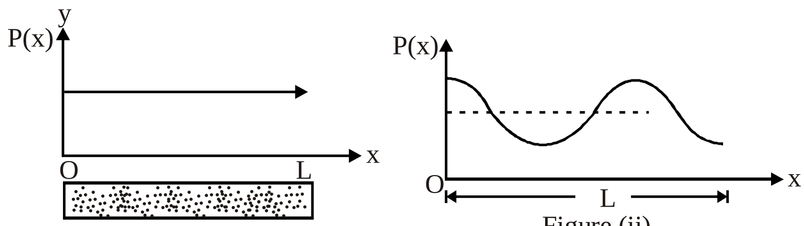
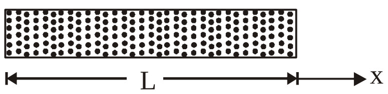
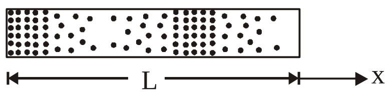
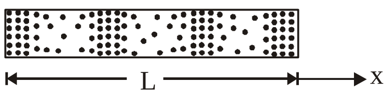
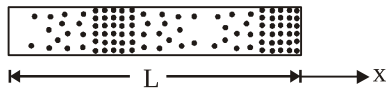


Figure (i) Figure (ii)

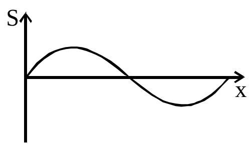
(A) 

(B) 

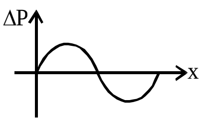
(C) 

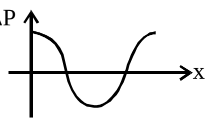
(D) 

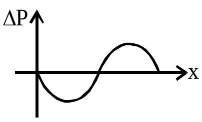
4. If a sound wave is travelling and snap shot at t = 0 is as shown in figure.

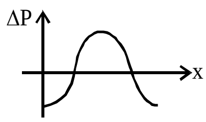


Choose snapshot of pressure variation.

(i) 

(ii) 

(iii) 

(iv) 

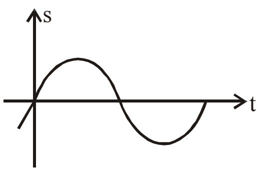
(A) For wave travelling towards right or left (i) is correct.

(B) For wave travelling towards right graph (iv) and for wave travelling towards left graph (iv) is correct.

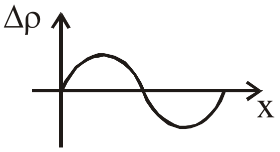
(C) For wave travelling towards right graph (i) and for wave travelling towards left graph (iii) is correct

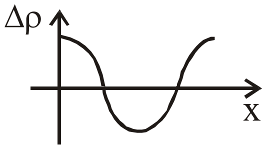
(D) For wave travelling towards right or left (ii) is correct.

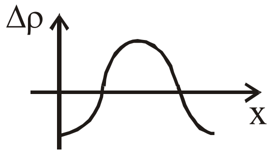
5. A sound waves is travelling towards right and its s-t graph is as shown for x = 0.

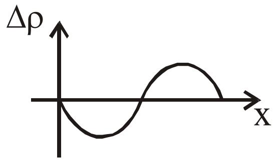


What will be the variation in density vs x graph at t = T/4 :-

(A) 

(B) 

(C) 

(D) 

6. A point source of sound is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at x = –8 m and x = +2.0 m.
A third listener is positioned along the positive y-axis. What is her y-coordinate (in m) if the same wave front reaches her at the same instant as it does the first two listeners ?

(A) 4 (B) 3 (C) 2 (D) 5

7. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate container kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by **[JEE 2000 (Scr)]**

(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$ (C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$

8. A firecracker exploding on the surface of a lake is heard as two sounds a time interval *t* apart by a man on a boat close to water surface. Sound travels with a speed *u* in water and a speed *v* in air. The distance from the exploding firecracker to the boat is

(A) $\frac{uvt}{u+v}$ (B) $\frac{t(u+v)}{uv}$ (C) $\frac{t(u-v)}{uv}$ (D) $\frac{uvt}{u-v}$

9. The speed of longitudinal wave is 100 times the speed of transverse wave in a taut brass wire. If the Young's modulus of brass is 1.0×10^{11} N/m^2, the stress in wire is :-

(A) 1.0×10^7 N/m^2 (B) 1.0×10^6 N/m^2 (C) 1.0×10^5 N/m^2 (D) 1.0×10^8 N/m^2

10. The equations of S.H.M. of medium particle due to sound waves propagating in a medium are given by $s_1 = 2 \sin(200\pi t)$ and $s_2 = 5 \sin(150\pi t)$. The ratio of average intensities of sound at these points is:

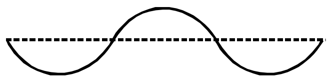
(A) 4 : 25 (B) 9 : 100 (C) 8 : 15 (D) 64 : 225

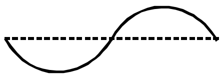
11. A is singing a note and at the same time B is also singing a note with 1/8th the frequency of A. The energies of the two sounds are equal. The displacement amplitude of the note of B is:

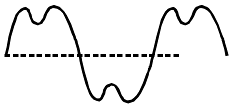
(A) same as that of A (B) twice that of A

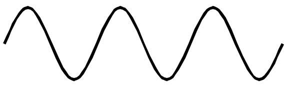
(C) four times that of A (D) eight times that of A

12. A microphone is connected to an oscilloscope. The diagram shows the trace on the screen when the microphone receives a pure note. Which trace can be obtained when a musical instrument produces a note of the same pitch but of a different quality ?



(A) 

(B) 

(C) 

(D) 

13. Choose **correct** statement?

(A) Two different acoustic musical instruments can not have same loudness

(B) Two different acoustic musical instruments can not have same pitch

(C) Two different acoustic musical instruments can not have same quality

(D) Two different acoustic musical instruments can have more than two characteristics same

14. A plane transverse wave is propagating in a direction making an angle of 30° with positive x-axis in the x-y plane. Find phase difference between points (0, 0, 0) and (1, 1, 1). Wavelength of the wave is 1m :-

(A) 2π rad (B) $\left(\sqrt{3}+1\right)\pi$ rad (C) $\left(\sqrt{2}+1\right)\pi$ rad (D) None

15. Which of the following is the equation of a spherical wave :-

(A) $S = S_0 \sin (Kx - \omega t)$ (B) $S = S_0 \cos (Kx - \omega t)$

(C) $S = (S_0/x) \sin (\omega t - Kx)$ (D) $S = (S_0/x^2) \sin(\omega t - Kx)$

16. A note is produced when you blow air across the top of a test tube. Two students were asked about the effect of blowing harder.

Student-A : The pitch of sound would increase.

Student-B : The intensity of sound would increase

(A) A is correct, B is wrong (B) B is correct, A is wrong

(C) both are correct (D) both are wrong

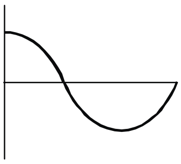
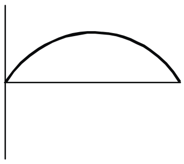
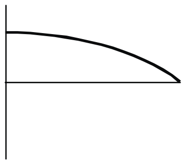
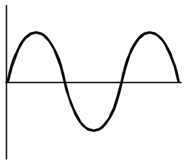
17. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of-

[AIEEE - 2007]

(A) 1000 (B) 10000 (C) 10 (D) 100

Standing waves

18. Which of the figures, shows the pressure difference from regular atmospheric pressure for an organ pipe of length L closed at one end, corresponds to the 1st overtone for the pipe?

(A)  (B)  (C)  (D) 

19. In an organ pipe whose one end is at x = 0, the pressure is expressed by $p = p_0 \cos\frac{3\pi x}{2} \sin 300\pi t$ where x is in meter and t in sec. The organ pipe can be

(A) closed at one end, open at another with length = 0.5m

(B) open at both ends, length = 1m

(C) closed at both ends, length = 2m

(D) closed at one end, open at another with length = $\frac{2}{3}$m

20. If l_1 and l_2 are the lengths of air column for the first and second resonance when a tuning fork of frequency n is sounded on a resonance tube, then the distance of the displacement antinode from the top end of the resonance tube is:

(A) $2(l_2 - l_1)$ (B) $\frac{1}{2}(2l_1 - l_2)$ (C) $\frac{l_2 - 3l_1}{2}$ (D) $\frac{l_2 - l_1}{2}$

21. A student is experimenting with resonance tube apparatus in Physics lab to find the speed of sound at room temperature. He got resonating lengths of air column as 17 cm and 51 cm, using tuning fork of frequency 512 Hz. Find speed of sound at room temperature and specify, whether the side water reservoir was moved upward or downward to obtain the second resonance (51 cm) ?

(A) 348 m/s, downwards (B) 348 m/s, upwards

(C) 332 m/s, downwards (D) 332 m/s, upwards

Interference

22. The ratio of maximum to minimum intensity due to superposition of two waves is $\frac{49}{9}$. Then the ratio of the intensity of component waves is :-

(A) $\frac{25}{4}$ (B) $\frac{16}{25}$ (C) $\frac{4}{49}$ (D) $\frac{9}{49}$

23. Two waves of sound having intensities I and 4I interfere to produce interference pattern. The phase difference between the waves is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(A) 2I (B) 4I (C) 5I (D) 7I

24. Three coherent waves of equal frequencies having amplitude 10 μm, 4μm and 7 μm respectively, arrive at a given point with successive phase difference of π/2. The amplitude of the resulting wave in μm is given by

(A) 5 (B) 6 (C) 3 (D) 4

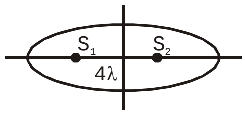
25. The ratio of intensities between two coherent soud sources is 4 : 1. The differenmce of loudness in dB between maximum and minimum intensities when they interfere in space is:

(A) 10 log 2 (B) 20 log 3 (C) 10 log 3 (D) 20 log 2

26. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is:

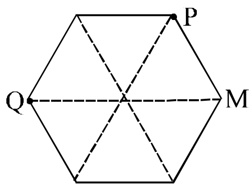
(A) 10/9 cm (B) 1 cm (C) 1/2 cm (D) 5/9 cm

27. S_1, S_2 are two coherent sources of sound located along x – axis separated by 4λ where λ is wavelength of sound emitted by them. Number of maxima located on the elliptical boundary around it will be :



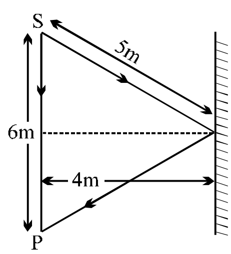
(A) 16 (B) 12 (C) 8 (D) 4

28. Two sound source emitting sound of wavelength 1 m are located at points P and Q as shown in figure. All sides of the polygon are equal and of length 1m. The intensity of sound at M due to both the individual sources is I_0. What will be the intensity of sound at point M when both the sources are on.



(A) $4I_0$ (B) $2I_0$ (C) I_0 (D) $(1/2)I_0$

29. A person standing at a distance of 6 m from a source of sound receives sound wave in two ways, one directly from the source and other after reflection from a rigid boundary as shown in the figure. The maximum wavelength for which, the person will receive maximum sound intensity, is



(A) 4 m (B) $\frac{16}{3}$ m (C) 2 m (D) $\frac{8}{3}$ m

30. Beats are heard when the A strings of two violins are played. The beat frequency decreases as the tension in the A string of violin 1 is slowly increased. Which of the following statement is correct ?

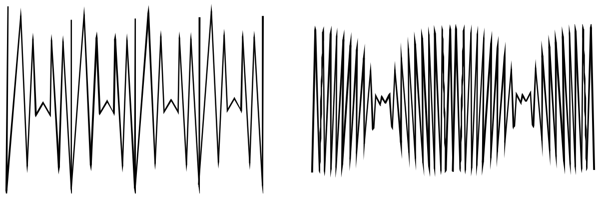
(A) the fundamental frequency of the A string in violin 1 is less than that for violin 2

(B) the fundamental frequency of the A string in violin 1 is greater than that for violin 2

(C) the fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the linear mass densities of the two strings.

(D) None of these

31. Two waves with similar frequencies are added. The resulting waveform oscillates with the average frequency and with an oscillating amplitude that changes with a frequency equal to the difference between the original frequencies. These oscillations in the amplitude are known as beats. The traces show the resulting waveforms that occur when two different pairs of waves are added. Graph is for the same time interval in both cases, which of the following statements is TRUE?



(A) On average, the waves on the left had higher frequencies, but the difference between frequencies less

(B) On average, the waves on the right had higher frequencies, but the difference between frequencies less

(C) On average, the waves on the left had higher frequencies, but the difference between frequencies more

(D) On average, the waves on the right had higher frequencies, but the difference between frequencies more

Doppler effect

32. A source when at rest in a medium produces waves with a velocity v and a wavelength of λ. If the source is set in motion with a velocity v_s what would be the wavelengths produced directly in front of the source?

(A) $\lambda\left(1-\frac{v_s}{v}\right)$ (B) $\lambda\left(1+\frac{v_s}{v}\right)$ (C) $\lambda\left(1+\frac{v}{v_s}\right)$ (D) $\frac{\lambda v}{v+v_s}$

33. A source of sound S having frequency f. Wind is blowing from source to observer O with velocity u. If speed of sound with respect to air is C, the wavelength of sound detected by O is:

(A) $\frac{C+u}{f}$ (B) $\frac{C-u}{f}$ (C) $\frac{C(C+u)}{(C-u)f}$ (D) $\frac{C}{f}$

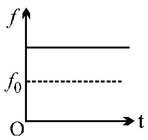
34. A train moving towards a hill at a speed of 72 km / hr sounds a whistle of frequency 500 Hz. A wind is blowing from the hill at a speed of 36 km / hr. If the speed of sound in air is 340 m/s, the frequency heard by a man on the hill is

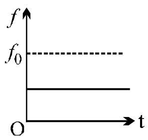
(A) 532.25 Hz. (B) 565.0 Hz. (C) 516.1 Hz. (D) none of the above.

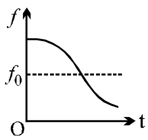
35. A source is moving with constant speed v_s = 20 m/sec towards a stationary observer due east of source. Wind is blowing at the speed of 20 m/sec due to 60° north of east. The source is generating of frequency 500 Hz. Then frequency registered by observer is:
[Speed of sound in still air = 330 m/sec.]

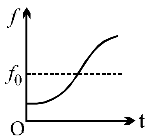
(A) 500 Hz (B) 532 Hz (C) 531 Hz (D) 530 Hz

36. Source and observer both start moving simultaneously from origin, one along x-axis and the other along y-axis with speed of source = twice the speed of observer. The graph between the apparent frequency observed by observer f and time t would approximately be :

(A) 

(B) 

(C) 

(D) 

37. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is :-

(A) 242/252 (B) 2 (C) 5/6 (D) 11/6

38. A police van moving with velocity 22 m/s and emitting sound of frequency 176 Hz, follows a motor cycle in turn is moving towards a stationary car and away from the police van. The stationary car is emitting frequency 165 Hz. If motorcyclist does not hear any beats then his velocity is (v_s = 330 m/s) **[JEE 2003 (Scr)]**

(A) 22 m/s (B) 24 m/s (C) 20 m/s (D) 18 m/s

EXERCISE (O-2)

1. Two tuning forks of frequency 250 Hz and 256 Hz produce beats. If a maximum of intensity is observed just now, after how much time the minimum is observed at the same place ?

(A) $\frac{1}{18}$ sec (B) $\frac{1}{4}$ sec. (C) $\frac{1}{3}$ sec. (D) $\frac{1}{12}$ sec.

2. The particle displacement of a travelling longitudinal wave is represented by S = S (x, t). The midpoints of a compression zone and an adjacent rarefaction zone are represented by the letter 'C' and 'R'. Which of the following is true?

(A) $|\partial S / \partial x|_C = |\partial S / \partial x|_R$

(B) $|\partial S / \partial t|_C = |\partial S / \partial t|_R = 0$

(C) $(\text{pressure})_C - (\text{pressure})_R = 2\, |\partial S / \partial x|_C$ x Bulk modulus of air.

(D) Particles of air are stationary mid-way between 'C' and 'R'.

3. Which of the following statements are wrong about the velocity of sound in air:

(A) decreases with increases in temperature (B) increases with decrease in temperature

(C) decreases as humidity increases (D) independent of density of air.

4. A car moves towards a hill with speed v_c. It blows a horn of frequency f which is heard by an observer following the car with speed v_0. The speed of sound in air is v.

(A) the wavelength of sound reaching the hill is $\frac{v}{f}$

(B) the wavelength of sound reaching the hill is $\frac{v - v_c}{f}$

(C) the beat frequency observed by the observer is $\left(\frac{v + v_o}{v - v_c}\right)f$

(D) the beat frequency observed by the observer is $\frac{2v_c(v + v_o)f}{v^2 - v_c^2}$

5. Three coherent source kept along the same line produce intensity I_0 each at point P on this line. When S_1 & S_2 are switched on simultaneously, intensity at point P is $2I_0$. When S_2 and S_3 are switched on simultaneously, intensity at point P is $2I_0$. Then

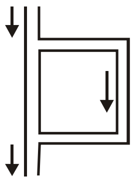
(A) When S_1 and S_3 are switched on simultaneously, intensity at point P can be $2I_0$

(B) When S_1 and S_3 are switched on simultaneously, intensity at point P can be 0

(C) When all 3 sources are switched on simultaneously, intensity at point P can be I_0

(D) When all 3 sources are switched on simultaneously, intensity at point P can be $3I_0$

6. A sound consists of four frequencies $\rightarrow$ 300 Hz, 900 Hz, 2400 Hz and 4500 Hz. A sound 'filter' is made by passing this sound through a bifurcated pipe as shown. The sound waves have to travel a distance of 50 cm more in the right branch-pipe than in the straight pipe. The speed of sound in air is 300 m/s. Then, which of the following frequencies will be almost completely muffled or "silenced" at the outlet ?



(A) 300 Hz (B) 900 Hz (C) 2400 Hz (D) 4500 Hz

Paragraph for Question No. 7 to 9

A metallic rod of length 1m has one end free and other end rigidly clamped. Longitudinal stationary waves are set up in the rod in such a way that there are total six antinodes present along the rod. The amplitude of an antinode is 4×10^{-6} m. Young's modulus and density of the rod are 6.4×10^{10} N/m^2 and 4×10^3 Kg/m^3 respectively. Consider the free end to be at origin and at t=0 particles at free end are at positive extreme.

7. The equation describing displacements of particles about their mean positions is

(A) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\cos\left(22\pi\times10^{3}t\right)$ (B) $s = 4\times10^{-6}\cos\left(\frac{11\pi}{2}x\right)\sin\left(22\pi\times10^{3}t\right)$

(C) $s = 4\times10^{-6}\cos(5\pi x)\cos\left(20\pi\times10^{3}t\right)$ (D) $s = 4\times10^{-6}\cos(5\pi x)\sin\left(20\pi\times10^{3}t\right)$

8. The equation describing stress developed in the rod is

(A) $140.8\pi\times10^{4}\cos\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^{3}t\right)$

(B) $140.8\pi\times10^{4}\sin\left(\frac{11}{2}\pi x+\pi\right)\cos\left(22\pi\times10^{3}t\right)$

(C) $128\pi\times10^{4}\cos(5\pi x+\pi)\cos\left(20\pi\times10^{3}t\right)$

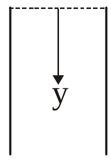
(D) $128\pi\times10^{4}\sin(5\pi x+\pi)\cos\left(20\pi\times10^{3}t\right)$

9. The magnitude of strain at midpoint of the rod at t= 1 sec is

(A) $11\sqrt{3}\pi\times10^{-6}$ (B) $11\sqrt{2}\pi\times10^{-6}$ (C) $10\sqrt{3}\pi\times10^{-6}$ (D) $10\sqrt{2}\pi\times10^{-6}$

Paragraph for Question No. 10 to 12

In an organ pipe (may be closed or open) of length 1m standing wave is setup, whose equation for longitudinal displacement is given by $\xi = (0.1 \text{ mm}) \cos \frac{2\pi}{0.8}(y) \cos (400) t$ where y is measured from the top of the tube in meters and t in second.



10. The upper end and the lower ends of the tube are respectively:

(A) open – closed (B) closed – open (C) open – open (D) closed – closed

11. The air column is vibrating in

(A) First overtone (B) Second overtone (C) Third harmonic (D) Fundamental node

12. Equation of the standing wave in terms of excess pressure is

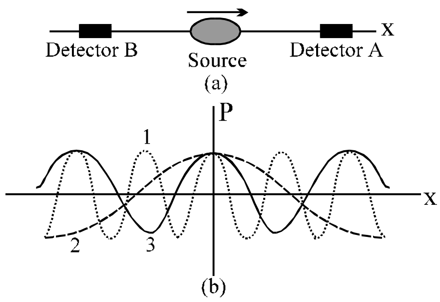
(Bulk modulus of air $B = 5 \times 10^5 \text{ N/m}^2$)

(A) $P_{ex} = (125\,\pi \text{N/m}^2) \sin \frac{2\pi}{0.8}(y) \cos (400\,t)$ (B) $P_{ex} = (125\,\pi \text{N/m}^2) \cos \frac{2\pi}{0.8}(y) \sin (400\,t)$

(C) $P_{ex} = (225\,\pi \text{N/m}^2) \sin \frac{2\pi}{0.8}(y) \cos (200\,t)$ (D) $P_{ex} = (225\,\pi \text{N/m}^2) \cos \frac{2\pi}{0.8}(y) \sin (200\,t)$

Paragraph for Question No. 13 to 16

A source emitting a sound wave at a certain frequency moves with constant speed along an x-axis figure (a). The source moves directly towards a stationary detector A and directly away from another stationary detector B. The superimposed three plots of figure (b) indicate the pressure function P(x) of the sound wave as measured by detector A, by detector B, and by someone (c) in the rest frame of the source.



13. Which of the following plot corresponds to the measurement done by detector A?

(A) 1 (B) 2

(C) 3 (D) These plots are not possible

14. The plot corresponding to the measurement done by detector B is

(A) 1 (B) 2

(C) 3 (D) These plots are not possible

15. The plot corresponding to the measurement done by the detector C is

(A) 1 (B) 2

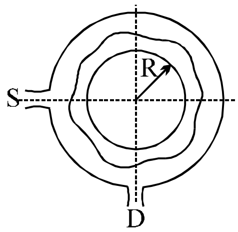
(C) 3 (D) These plots are not possible

16. Now the source stops and begins to move along y-axis with same speed, the plot which corresponds to the measurement of B now is

(A) 1 (B) 2 (C) 3 (D) none of these

Paragraph for Question No. 17 to 21

A narrow tube is bent in the form of a circle of radius R, as shown in the figure. Two small holes S and D are made in the tube at the positions right angle to each other. A source placed at S generated a wave of intensity I_0 which is equally divided into two parts : One part travels along the longer path, while the other travels along the shorter path. Both the part waves meet at the point D where a detector is placed



17. If a maxima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by :-

(A) πR (B) $\frac{\pi R}{2}$ (C) $\frac{\pi R}{4}$ (D) $\frac{2\pi R}{3}$

18. If the minima is formed at the detector then, the magnitude of wavelength λ of the wave produced is given by :-

(A) $2\pi R$ (B) $\frac{3\pi R}{2}$ (C) $\frac{2\pi R}{3}$ (D) $\frac{2\pi R}{5}$

19. The maximum intensity produced at D is given by :-

(A) $4I_0$ (B) $2I_0$ (C) I_0 (D) $3I_0$

20. The maximum value of λ to produce a maxima at D is given by :-

(A) πR (B) $2\pi R$ (C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

21. The maximum value of λ to produce a minima at D is given by :-

(A) πR (B) $2\pi R$ (C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

Paragraph for Question Nos. 22 to 24

Two waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling in a pipe placed along x-axis. **[JEE 2006]**

22. Find the number of times intensity is maximum in time interval of 1 sec.

(A) 4 (B) 6 (C) 8 (D) 10

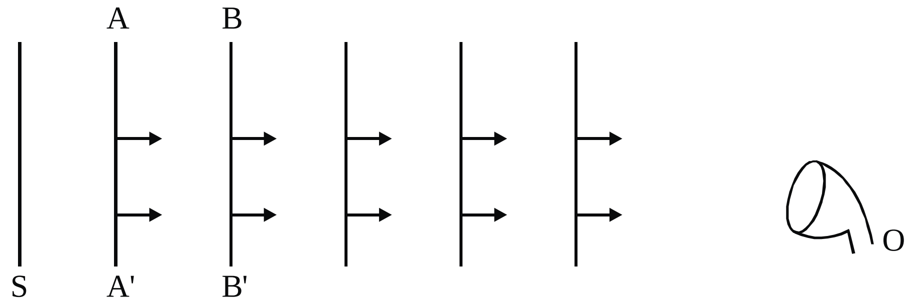
23. Find wave velocity of louder sound

(A) 100 m/s (B) 192 m/s (C) 200 m/s (D) 96 m/s

24. At x = 0 how many times the value of $y_1 + y_2$ is zero in one second?

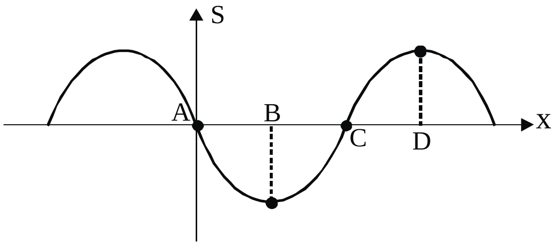
(A) 100 (B) 46 (C) 192 (D) 96

25. Consider a large plane diaphragm 'S' emitting sound and a detector 'O'. The diagram shows plane wavefronts for the sound wave travelling in air towards right when source, observer and medium are at rest. AA' and BB' are fixed imaginary planes. Column-I describes about the motion of source, observer or medium and column-II describes various effects. Match them correctly.



Column-I	**Column-II**
(A) Source starts moving towards right	(P) Distance between any two wavefronts will increase.
(B) Air starts moving towards right	(Q) Distance between any two wavefronts will decrease.
(C) Observer and source both move towards left with same speed.	(R) The time needed by sound to move from plane AA' to BB' will increase.
(D) Source and medium (air) both move towards right with same speed.	(S) The time needed by sound to move from plane AA' to BB' will decrease.
	(T) Frequency received by observer increases.

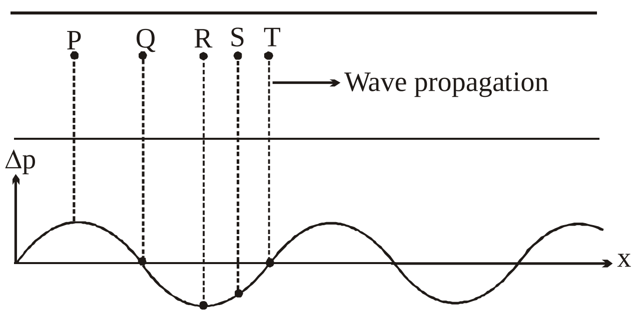
26. Figure shows a graph of particle displacement function of x at t = 0 for a longitudinal wave travelling in positive x-direction in a gas. A,B,C,D denote position of particles in space.



Column-I		**Column II**	
(A)	point A	(P)	Particle velocity is in direction of wave propagation
(B)	point B	(Q)	Maximum magnitude of strain
(C)	point C	(R)	Excess pressure is zero
(D)	point D	(S)	Maximum density
		(T)	Maximum magnitude of excess pressure

27. Sound is travelling in a long tube towards right and the graph of excess pressure variation versus position (at some instant) is given below.

Match velocities in column-I with column-II. P,Q,R,S,T are medium particles inside the tube.



Column-I		**Column-II**	
(A)	velocity is towards right	(P)	P
(B)	velocity is towards left	(Q)	Q
(C)	velocity is zero	(R)	R
(D)	Speed is maximum	(S)	S
		(T)	T

EXERCISE (JM)

1. **Statement-1 :** Two longitudinal waves given by equations : $y_1(x, t) = 2a \sin(\omega t - kx)$ and $y_2(x, t) = a \sin(2\omega t - 2kx)$ will have equal intensity.

Statement-1: Intensity of waves of given frequency in same medium is proportional to square of amplitude only. **[AIEEE - 2011]**

(1) Statement-1 is false, statement-2 is true

(2) Statement-1 is ture, statement-2 is false

(3) Statement-1 is ture, statement-2 true; statement-2 is the correct explanation of statement-1

(4) Statement-1 is true, statement-2 is true; statement -2 is not correct explanation of statement-1.

2. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[JEE Main - 2014]**

(1) 6 (2) 4 (3) 12 (4) 8

3. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :- **[JEE Main - 2015]**

(1) 18% (2) 24% (3) 6% (4) 12%

4. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now :- **[JEE-Main-2016]**

(1) f (2) $\frac{f}{2}$ (3) $\frac{3f}{4}$ (4) 2f

5. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1}) **[JEE Main - 2017]**

(1) 17.3 GHz (2) 15.3 GHz (3) 10.1 GHz (4) 12.1 GHz

EXERCISE (JA)

1. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is **[IIT-JEE 2010]**

2. **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves. **[IIT-JEE 2011]**

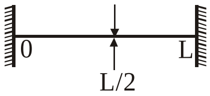
Column I

(A) Pipe closed at one end

(B) Pipe open at both ends

(C) Stretched wire clamped at both ends

(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse Waves

(r) λ_f=L

(s) λ_f = 2L

(t) λ_f = 4L

3. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is **[JEE 2011]**

(A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

4. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe. **[JEE 2012]**

(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(B) a low -pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(C) a low pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

5. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance column tube. When the first resonance occurs, the reading of the water level in the column is :- **[JEE 2012]**

(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

6. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(**Useful information :** $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

[JEE Advanced-2014]

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$ (B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

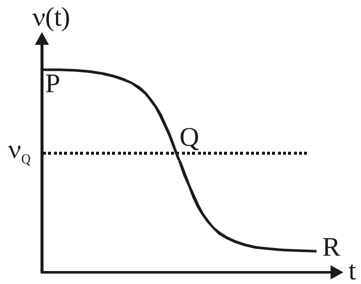
(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$ (D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

7. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is.

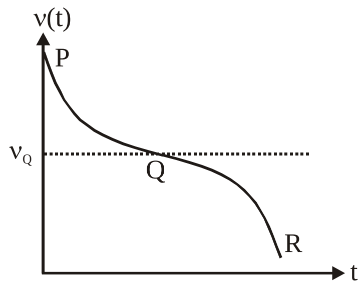
[JEE-Advance-2015]

8. Two loudspeakers M and N are located 20m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is(are) true regarding the sound heard by the person ? **[JEE Advanced 2016]**

(A) The plot below represents schematically the variation of beat frequency with time



(B) The plot below represents schematically the variations of beat frequency with time



(C) The rate of change in beat frequency is maximum when the car passes through Q

(D) $\nu_P + \nu_R = 2\nu_Q$

9. A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz ? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received). **[JEE Advanced 2017]**

10. Two men are walking along a horizontal straight line in the same direction. The man in front walks at a speed 1.0 ms^{-1} and the man behind walks at a speed 2.0 ms^{-1}. A third man is standing at a height 12m above the same horizontal line such that all three men are in a vertical plane. The two walking men are blowing identical whistles which emit a sound of frequency 1430 Hz. The speed of sound in air is 330 ms^{-1}. At the instant, when the moving men are 10 m apart, the stationary man is equidistant from them. The frequency of beats in Hz, heard by the stationary man at this instant, is ______ **[JEE Advanced 2018]**

11. In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true ? **[JEE Advanced 2018]**

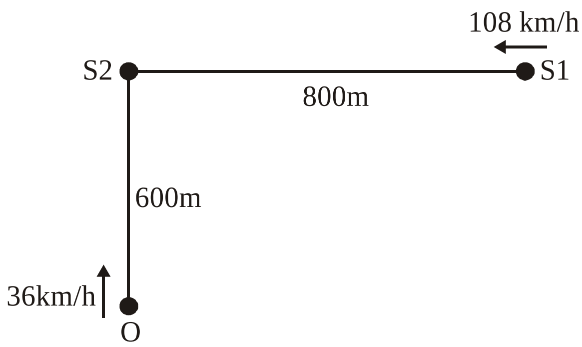
(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 cm

(C) The wavelength of the sound wave is 66.4 cm

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic

12. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ____.
[Speed of the sound = 330 m/s] **[JEE Advanced 2019]**



ANSWER KEY

EXERCISE (S-1)

1. Ans. $\frac{\pi^2 \times 10^{-9}}{4}$ W/m^2 **2. Ans.** 2c/3 **3. Ans.** 30 dB, $10\sqrt{10}$ µm

4. Ans. 1 : 1 **5. Ans.** 3 cm **6. Ans.** 336 m/s

7. Ans. 33 cm and 13.2 cm **8. Ans.** 2.5 kHz, 7.5 kHz **9. Ans.** (a) 2.116, (b) $\frac{3}{4}$

10. Ans. 3.33 cm, 163 Hz **11. Ans.** 1% **12. Ans.** 345, 341 or 349 Hz

13. Ans. 8 **14. Ans.** 1.5 m/s **15. Ans. 5** **16. Ans.** 11f / 9

17. Ans. (a) $\left(\frac{v+\lambda_0 f_0}{\lambda_0}\right)t$ (b) $(\lambda_0 f_0 - v)t$ (c) $\lambda_0\left(\frac{\lambda_0 f_0 - v}{\lambda_0 f_0 + v}\right)$ (d) $f_0\left(\frac{\lambda_0 f_0 + v}{\lambda_0 f_0 - v}\right)$ (e) $\frac{2vf_0}{\lambda_0 f_0 - v}$

18. Ans. 6 Hz

EXERCISE (O-1)

1. Ans. (B)	**2. Ans. (A)**	**3. Ans. (B)**	**4. Ans. (B)**	**5. Ans. (A)**	**6. Ans. (A)**
7. Ans. (B)	**8. Ans. (D)**	**9. Ans. (A)**	**10. Ans. (D)**	**11. Ans. (D)**	**12. Ans. (B)**
13. Ans. (C)	**14. Ans. (B)**	**15. Ans. (C)**	**16. Ans. (B)**	**17. Ans. (D)**	**18. Ans.(A)**
19. Ans. (C)	**20. Ans. (C)**	**21. Ans. (A)**	**22. Ans. (A)**	**23. Ans. (B)**	**24. Ans. (A)**
25. Ans. (B)	**26. Ans. (B)**	**27. Ans. (A)**	**28. Ans. (A)**	**29. Ans. (A)**	**30. Ans. (A)**
31. Ans. (B)	**32. Ans. (A)**	**33. Ans. (A)**	**34. Ans. (A)**	**35. Ans. (C)**	**36. Ans. (B)**
37. Ans. (B)	**38. Ans. (A)**				

EXERCISE (O-2)

1. Ans. (B,D)	**2. Ans. (A,C,D)**	**3. Ans. (A,B,C,D)**	**4. Ans. (B,D)**
5. Ans. (B, C)	**6. Ans. (A, B, D)**	**7. Ans. (A)**	**8. Ans. (B)**
9. Ans. (B)	**10. Ans. (A)**	**11. Ans. (B)**	**12. Ans. (A)**
13. Ans. (A)	**14. Ans. (B)**	**15. Ans. (C)**	**16. Ans. (D)**
17. Ans. (A,B,C)	**18. Ans. (A,C,D)**	**19. Ans. (B)**	**20. Ans. (A)**
21. Ans. (B)	**22. Ans. (A)**	**23. Ans. (C)**	**24. Ans. (A)**

25. Ans. (A) (Q,T) ; (B) (P,S) ; (C) (P) (D) (S,T)

26. Ans. (A) (P,Q,S,T); (B) (R); (C) (Q,T); (D) (R)

27. Ans. (A) (P); (B) (R,S); (C) (Q,T); (D) (P,R)

EXERCISE (JM)

1. Ans. (4) **2. Ans. (1)** **3. Ans. (4)** **4. Ans. (1)** **5. Ans. (1)**

EXERCISE (JA)

1. Ans. 5 **2. Ans. (A) p,t (B) p,s (C) q,s (D) q,r** **3. Ans. (A)**

4. Ans. (B,D) **5. Ans. (B)** **6. Ans. (D)** **7. Ans. 3**

8. Ans. (A, C, D) **9. Ans. 6** **10. Ans. 5 [4.99, 5.01]**

11. Ans. (A,B,C) **12. Ans. (8.12 to 8.13)**