Level - I

Chapter 6

Work, Energy and Power

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. Can KE of a system be increased without applying any external force?

Sol. Yes, by doing work through internal forces, *e.g.*, explosion of a bomb.

2. When is the exchange of energy maximum during an elastic collision?

Sol. When colliding bodies have same mass.

3. Comment on the nature of work done by the stretching force on a spring.

Sol. Positive work.

4. What happens to the PE of a spring as it is compressed or stretched?

Sol. PE of spring increases in both the cases as work is done by us in compression as well as stretching.

5. What is the source of KE of falling rain drops?

Sol. Gravitational PE.

6. Does kinetic energy depend on the direction of motion? Can it be negative?

Sol. No, no

7. Does work done in moving a body depend on how fast or how slow the body is moved?

Sol. No, time is not involved. Time is involved in power.

8. Does KE of a system depend on frame of reference?

Sol. Yes

9. What happens to the PE of an air bubble as it rises in water?

Sol. It decreases as work is done by upthrust on the bubble.

10. Name the smallest and largest practical unit of energy.

Sol. Smallest practical unit is 'electron volt'.

Largest practical unit is 'kilowatt hour'.

Short Answer Type Questions :

11. What happens to the PE of the system as two electrons are brought towards each other? What happens when a proton and an electron are brought nearer?

Sol. Work is done by us in bringing two electrons closer as they repel each other, so, PE increases. PE decreases when a proton and an electron are brought nearer as work is done by the field in bringing them nearer (they attract each other).

12. The dot product of two vectors is zero when the vectors are orthogonal and maximum when they are parallel to each other. Explain.

Sol. $\vec{A}\cdot\vec{B} = AB\cos\theta$

When vectors are orthogonal, θ = 90°, cos90° = 0

$\therefore \quad \vec{A}\cdot\vec{B} = 0$

When vectors are parallel, θ = 0° so $\vec{A}\cdot\vec{B} = AB\cos 0° = AB\,(\text{max.})$

13. Explain scalar product of two vectors and give its geometrical interpretation.

Sol.

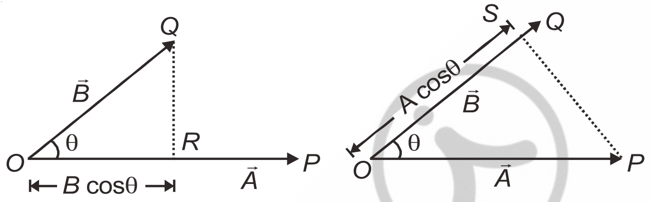


Fig. (a) **Fig. (b)**

The dot product of two vectors $\vec{A}$ and $\vec{B}$, represented by $\vec{A}\cdot\vec{B}$ is a scalar, which is equal to the product of the magnitudes of $\vec{A}$ and $\vec{B}$ and the cosine of the smaller angle between them.

$\vec{A}\cdot\vec{B} = AB\cos\theta$ where θ is the smaller angle between $\vec{A}$ and $\vec{B}$.

$\vec{A}\cdot\vec{B}$ is scalar as A, B and cosθ are scalars.

In Fig. (a), draw $QR \perp OP$, $OR = B\cos\theta$ is the projection of $\vec{B}$ on $\vec{A}$.

In Fig. (b), draw $PS \perp OQ$.

$OS = A\cos\theta$ is the projection of $\vec{A}$ on $\vec{B}$.

$\vec{A}\cdot\vec{B} = A(B\cos\theta) = B(A\cos\theta)$.

$\vec{A}\cdot\vec{B}$ is the product of magnitude of $\vec{A}$ and component of $\vec{B}$ along $\vec{A}$. Alternatively, it is the product of magnitude of $\vec{B}$ and the component of $\vec{A}$ along $\vec{B}$.

14. If $\vec{P}, \vec{Q}$ and $\vec{R}$ are non-zero vectors and $\vec{P}\cdot\vec{Q} = 0$ and $\vec{Q}\cdot\vec{R} = 0$ then find $\vec{P}\cdot\vec{R}$

Sol. $\vec{P}\cdot\vec{Q} = 0 \qquad \therefore\ PQ\cos\theta_1 = 0$ or $\theta_1 = 90°$

$\vec{Q}\cdot\vec{R} = 0 \qquad \therefore\ QR\cos\theta_2 = 0$ or $\theta_2 = 90°$

$\Rightarrow \quad \vec{P}\ \&\ \vec{R}$ are parallel or antiparallel to each other.

$\therefore \quad \vec{P}\cdot\vec{R} = PR\cos 0° = PR$ or, $\vec{P}\cdot\vec{R} = PR\cos 180° = -PR$

15. A body moves from a point $\vec{r}_1 = (2\hat{i} + 3\hat{j})$ to another point $\vec{r}_2 = (3\hat{i} + 2\hat{j})$ due to force $\vec{F} = 5\hat{i} + 5\hat{j}$. Find the work done by the force.

Sol. $\vec{d}$ = displacement = $\vec{r}_2 - \vec{r}_1 = \hat{i} - \hat{j}$

$$W = \vec{F} \cdot \vec{d} = (5\hat{i} + 5\hat{j}) \cdot (\hat{i} - \hat{j})$$

= 5 – 5

= 0

16. A uniform chain of length *l* and mass *m* is lying on a smooth table and one-fourth of its length is hanging vertically over the edge of the table. Find the work needed to be done to pull the hanging part on to the table.

Sol. $h = \frac{l}{8}$ C.G. of the hanging part is at a distance of $\frac{l}{8}$ from the table.

Mass of the hanging part = $\frac{m}{4}$

$\therefore \quad W = \frac{m}{4} g \frac{l}{8} = \frac{mgl}{32}$

17. A man rowing a boat upstream is at rest w.r.t. the shore (i) Comment on the work done by him (ii) Is any work done by him when he stops rowing and moves along with the stream?

Sol. (i) Work = 0 as displacement = 0

(ii) Force of water produces displacement w.r.t. the shore so, work is done by the flowing water.

18. Increase in KE of a car is E_1 when it is accelerated from 15 ms^{-1} to 20 ms^{-1} and increase in KE of a car is E_2 when it is accelerated from 20 ms^{-1} to 25 ms^{-1}. Find $\frac{E_1}{E_2}$.

Sol. $\frac{E_1}{E_2} = \frac{\frac{1}{2}m(20^2 - 15^2)}{\frac{1}{2}m(25^2 - 20^2)} = \frac{35 \times 5}{45 \times 5} = \frac{7}{9}$

19. There are two springs *A* and *B*. Both have spring constant of 100 Nm^{-1}. *A* is compressed by 5 cm. *B* is stretched by 5 cm. What is the difference in PE stored in the two springs?

Sol. $E_P = \frac{1}{2}kx^2$. In both cases, *E* and *x* are same.

$\therefore$ The difference in potential energies is zero.

20. Comment on change in PE

(a) A body is taken against the gravitational force

(b) Air bubble rises up in water

(c) A spring is stretched

Sol. (a) Work is done by us, so PE increases.

(b) Air bubble rises because of upthrust, so PE decreases.

(c) Work is done on the spring, so PE increases.

21. Can a body have momentum without mechanical energy?

Sol. Yes, $E = K + U = 0$

Either both are zero or $K = -U$

Then, KE may or may not be zero.

$$p = \sqrt{2mk}$$

$\therefore$ $p = 0$ only when $K = 0$

$p \neq 0$ when $K = -U$

22. Can a body have momentum when its mechanical energy is negative?

Sol. Yes, when $K > U$ so that U is very negative so that total energy $E = K + U$ is negative. The body has the momentum ($\therefore K \neq 0$) *e.g.*, in an atom, electron has momentum, though its energy is negative.

23. Can a body have mechanical energy without momentum?

Sol. Yes, when $p = 0$, $K = \frac{p^2}{2m} = 0$

But $E = K + U = U$ (potential energy), which may or may not be zero.

24. For a planet moving around the sun, do you think it is a good illustration of the law of conservation of energy?

Sol. The sum of KE and PE is constant at all stages. When the planet is farthest from the sum, it is slowest or it has minimum KE and hence, maximum PE.

25. Justify the statement: fast neutrons can be slowed down by passing them through heavy water.

Sol. There is a large exchange of KE in a collision between fast neutrons and hydrogen nuclei as their masses are almost equal.

26. (a) Two bodies of masses m_1 and m_2 ($m_2 > m_1$) are dropped from the same height. Compare their momentum just before hitting the ground.

(b) Two bodies of masses m_1 and m_2 ($m_2 > m_1$) have same KE. Compare their momentum.

Sol. (a) They hit the ground with same velocity = $\sqrt{2gh}$

$$\therefore \quad \frac{p_2}{p_1} = \frac{m_2}{m_1}$$

(b) $p = \sqrt{2mk}$

$\frac{p_2}{p_1} = \sqrt{\frac{m_2}{m_1}}$ as KE is same.

$m_2 > m_1$ $\quad \therefore p_2 > p_1$

27. A body of mass 60 kg moving with velocity 2 ms^{-1} collides with a body of mass 40 kg moving with velocity 4 ms^{-1}. Calculate the loss of energy assuming perfectly inelastic collision.

Sol. Using conservation of momentum

$60 \times 2 + 40 \times 4 = (60 + 40) \times v$

$v = 2.8\ ms^{-1}$

$$\text{Loss of energy} = \frac{1}{2} \times 60 \times 2 \times 2 + \frac{1}{2} \times 40 \times 4 \times 4 - \frac{1}{2} \times (40 + 60) \times 2.8 \times 2.8$$

$= 120 + 320 - 392$

$= \boxed{48\ J}$

28. A meteorite burns in the atmosphere before reaching Earth's surface. What happens to its momentum? Is momentum conservation principle violated?

Sol. The momentum gets transferred to the air molecules so, momentum conservation principle is not violated.

29. What is meant by mass energy equivalence?

Sol. According to Einstein, energy can be transformed into mass and vice versa, *i.e.*,

$E = mc^2$

where, m = mass that disappears

E = energy that appears

c = velocity of light in vacuum

Conversely, when an amount of energy E is converted into mass, the mass that appears is $m = \frac{E}{c^2}$. According to (modern) quantum physics, mass and energy are not conserved separately, but are conserved as a single entity called 'mass-energy'.

30. A body of mass 10 kg is dropped from a height of 15 m. Assuming that the whole of the energy is converted into heat energy, find the heat energy in calorie. Given g = 10 ms^{-2} and Joule's mechanical equivalent of heat = 4 J cal^{-1}.

Sol. PE = mgh

$= 10 \times 10 \times 15$

$= 1500$ J

$\therefore$ Heat energy = $\frac{1500}{4} = 375$ cal

Long Answer Type Questions :

31. Give examples of dot product of two vectors. Discuss briefly some of the properties of dot product.

Sol. Examples :

(a) Work done is dot product of force vector and displacement vector $W = \vec{F} \cdot \vec{d}$.

(b) Instantaneous power is dot product of force vector and the instantaneous velocity, $P = \vec{F} \cdot \vec{v}$

Properties :

(i) Dot product is commutative

$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$

(ii) It is distributive.

$\vec{A}\cdot(\vec{B}+\vec{C})=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$

(iii) Dot product of a vector with itself gives square of its magnitude.

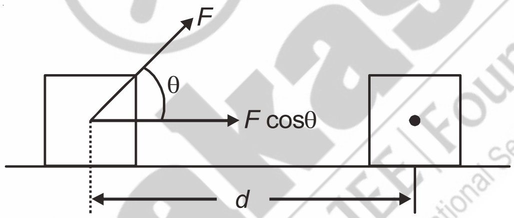
$\vec{A}\cdot\vec{A}=A^2$

(iv) $\vec{A}\cdot(\lambda\vec{B})=\lambda(\vec{A}\cdot\vec{B})$ where λ is a real number

32. Explain the term 'work'. Get an expression for work done by a constant force. What are its dimensions and SI unit?

Sol. The work done by the force is defined as the product of component of the force in the direction of the displacement and the magnitude of this displacement, *i.e.*,

$W = (F\cos\theta)d$ ...(i)



θ is the angle which $\vec{F}$ makes with the positive *X*-direction of displacement. $F\cos\theta$ is the component of force in the direction of displacement. If displacement is in the direction of displacement, $\theta = 0°$ and $W = (F\cos 0°)d = Fd$

Equation (i) can be rewritten as

$W=\vec{F}\cdot\vec{d}$ ...(ii)

Thus, work done by a force is the dot product of force and displacement.

Its dimensions are $[ML^2T^{-2}]$ and SI unit is joule (J).

33. Discuss the absolute and gravitational units of work on SI and CGS systems.

Sol. (i) Absolute unit

(a) **joule :** It is the absolute unit of work in SI system. Work done is said to be one joule, when a force of one newton actually moves a body through a distance of one metre in the direction of applied force.

(b) **erg :** It is the absolute unit of work in CGS system. Work done is said to be one erg, when a force of one dyne actually moves a body through a distance of one cm in direction of applied force.

(ii) Gravitational unit

(a) **kilogram-metre** (kgm) is the gravitational unit of work on SI system. Work done is said to be 1 kgm when a force of 1 kgf moves a body through a distance of 1 m in the direction of the applied force.

(b) **gram-centimeter** (gcm) is the gravitational unit of work on CGS system. Work done is said to be 1 gcm. When a force 1 gf moves a body through a distance of 1 cm in the direction of the applied force.

34. Explain the meaning of KE with examples. Get an expression for KE of a body moving uniformly.

Sol. Kinetic energy is the energy possessed by a body by virtue of its motion *e.g.*, a bullet fired from gun has KE. Sailing ships use KE of wind.

Let m = mass of the body

$d\vec{s}$ = small displacement produced in the direction of force

dW = small amount of work done

$\vec{a}$ = acceleration produced by the force

$$\vec{F} = m\vec{a} = \frac{md\vec{v}}{dt} \qquad \ldots\text{(i)}$$

$$\begin{aligned} dW &= \vec{F} \cdot d\vec{s} \\ &= Fds\cos 0^\circ \\ &= Fds \end{aligned}$$

$$= \left(\frac{mdv}{dt}\right)ds \qquad \text{using (i)}$$

$$= m\left(\frac{ds}{dt}\right)dv$$

$$= mvdv \text{ as } \frac{ds}{dt} = v$$

Total work done by the force in increasing the velocity from zero to v is

$$W = \int_0^v mvdv$$

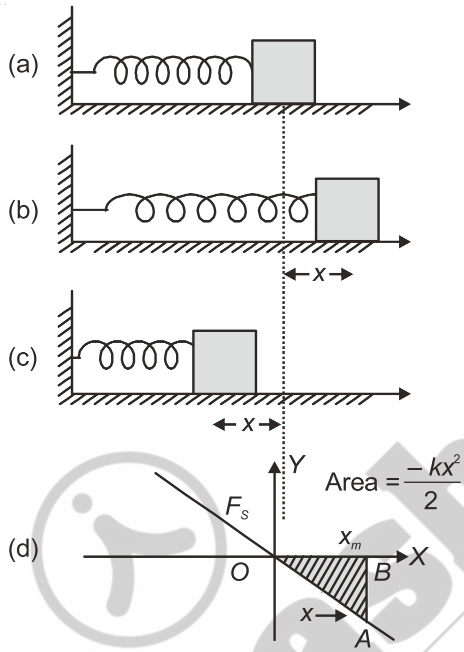
$$= m\int_0^v vdv = m\left[\frac{v^2}{2}\right]_0^v$$

$$W = \frac{1}{2}mv^2$$

$$\therefore \quad \text{KE} = W = \frac{1}{2}mv^2$$

35. What is meant by PE of a spring? Get an expression for it.

Sol. The spring force is an example of a variable force which is conservative Fig.(a) shows a block attached to a spring and resting on a smooth horizontal surface. Consider an ideal spring *i.e.*, the spring force $F_s \propto x$ where x is the displacement of the block from the equilibrium position. Displacement could be either positive Fig.(b) or negative Fig.(c)



$F_S = -kx$

The constant k is called the spring constant. Its unit is Nm^{-1}.

Suppose, we pull the block outward as in Fig. (b) and the extension is x_m, the work done by the spring force is

$$W_S = \int_0^{x_m} F_S dx = -\int_0^{x_m} kxdx = \frac{-kx_m^2}{2} \quad \text{...(i)}$$

This expression. may also be obtained by considering the area of the triangle as in Fig.(d)

PE of a spring is the energy associated with the state of compression or expansion of an elastic spring. The work done, W_S is stored in the spring as potential energy of the spring.

$$\text{PE} = W = -\frac{1}{2}kx_m^2$$

36. Discuss the variation of PE (associated with spring) with distance x. Also, discuss, variations of KE and total energy graphically.

OR

A small object of weight w hangs from a string of length l. A variable horizontal force, F, which starts at zero value and increases gradually is used to pull the object very slowly until the string makes an angle θ with the vertical. Prove that the work done by the force is $wl(1 - \cos\theta)$.

Sol.

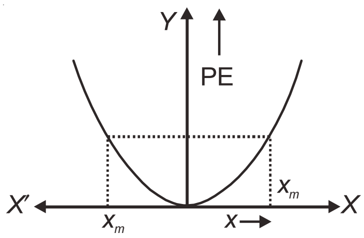
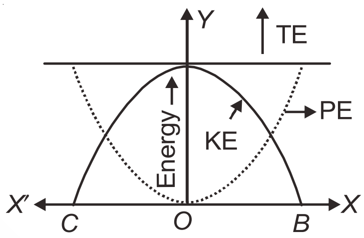


Fig. : Variation of PE with distance x.



At O, x = 0 equilibrium position, PE = $\frac{1}{2}kx_m^2 = 0$

At x_m, & $-\,x_m$ PE = $\frac{1}{2}kx_m^2$ $x_m \rightarrow$ Spring is stretched

$-\,x_m \rightarrow$ Spring is compressed

If we leave the block at B, after stretching the spring, it would move to O due to PE stored in the spring.

At O, the entire PE of the spring is converted into KE of the body. So, the body cannot stop at O. It goes to the position C where $OC = -\,x$.

At C, PE = $-\frac{1}{2}k(-x)^2 = -\frac{1}{2}kx^2$

At C, KE of the body is fully converted into PE. This is repeated. Total energy, TE remains the same in all positions.

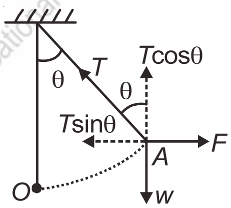
OR

Three forces F, w and tension T act on the body. As the body is in equilibrium, the net horizontal component of all the forces acting on the body = 0

$F = T\sin\theta$ ...(i)

Equating the net vertical component to zero,

$w = T\cos\theta$ ...(ii)



Divide equation (i) by equation (ii)

$$\frac{F}{w} = \tan\theta$$

or $F = w\tan\theta$

Suppose the object swings through arc length x and becomes $x + dx$ when the angular displacement increases from θ to $\theta + d\theta$. Here, $x = l\theta$. $dx = ld\theta$

So, the work done by force F is

$$w = \int \vec{F}\cdot d\vec{x} = \int F\cos\theta dx$$

$$= \int_{\theta}^{\theta} w\tan\theta\cos\theta\, ld\theta$$

$$= wl\int_{0}^{\theta}\sin\theta d\theta$$

$$= wl[-\cos\theta]_0^{\theta}$$

$$\boxed{W = wl(1-\cos\theta)}$$

37. Compare the properties of conservative and non-conservative forces with regard to

(i) PE

(ii) Reversibility of work done

(iii) Dependence on the path

(iv) Work done in a round trip. Give examples

Sol.

Conservative forces	Non-conservative forces
(i) It can always be expressed as negative of the gradient of potential energy *i.e.*, $F = -\dfrac{dU}{dr}$	(i) It has no relation with PE.
(ii) The work done by it is reversible.	(ii) The work done by it is not reversible.
(iii) The work done by it is independent of the path. It depends only on the starting and ending point.	(iii) The work done by it is not independent of the path of the body.
(iv) The work done in a round trip is zero.	(iv) The work done is not necessarily zero.
(v) *e.g.*, gravitational force, elastic force.	(v) *e.g.*, frictional force, viscous force.

38. Consider an object of mass m connected with an ideal spring of spring constant k. Show that the speed and the KE will be maximum at equilibrium position. Get an expression for the maximum speed.

Sol. PE $U(x) = \dfrac{kx^2}{2}$ ...(i)

If the block of mass m is extended to x_m and released from rest, its total mechanical energy at any arbitrary point x, where $-x_m < x < x_m$ is given by

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \text{ (conservation of mechanical energy)}$$

⇒ Speed and the KE will be maximum at equilibrium position, $x = 0$

$$\frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$

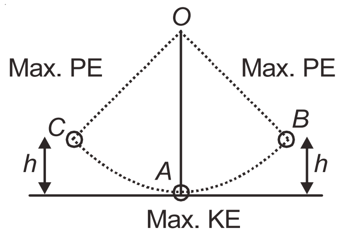
where v_m is the maximum speed.

$$\boxed{v_m = \sqrt{\frac{k}{m}}x_m}$$

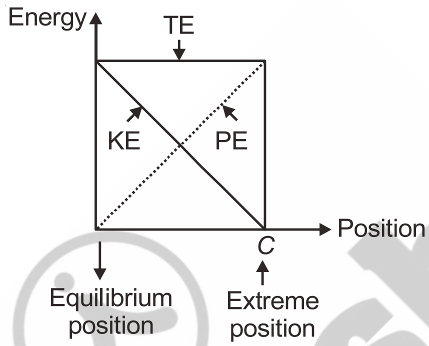
39. Illustrate the conservation of mechanical energy for vibration of a simple pendulum. Explain it with the variation of KE, PE and total energy with the position.

Sol. *OA* is normal position of a simple pendulum. When the bob is displaced to *B*, through height *h*, it is given PE = *mgh*, where *m* is mass of the bob. On releasing the bob at *B*, it moves towards *A*. PE of the bob is being converted into KE. At *A*, entire PE has been converted into KE so, the bob cannot stop at *A*. It reaches

C at the same height h above A due to inertia. Entire KE of the bob gets converted into PE at C. The whole process is repeated. At extreme position B and C the bob is momentarily at rest.



Therefore, KE = 0, the entire energy at B and C is PE. At A, there is no height, so PE = 0. Entire energy at A is KE. Total energy, TE remains constant as indicated in the graph.



40. Explain 'potential energy'. Give its dimensions and SI unit. Give an example of potential energy other than that of gravitational PE.

Sol. The gravitational force on a ball of mass m is mg. The work done by the external agency against the gravitational force is mgh. This work gets stored as potential energy. Gravitational PE of an object, as a function of height h, is denoted by $U(h)$ and it is the negative of work done by the gravitational force in raising the object to that height.

$U(h) = mgh$

Gravitational force $F = -\dfrac{dU(h)}{dh} = -mg$

The –ve sign indicates that the gravitational force is downward.

Mathematically (for simplicity, in one dimension), the potential energy $U(x)$ is defined if the force $F(x)$ can be written as $F(x) = -\dfrac{dU}{dx}$.

$$\Rightarrow \int_{x_i}^{x_f} F(x)dx = -\int_{U_i}^{U_f} dU = U_i - U_f$$

The work done by a conservative force such as gravity depends on the initial and final positions only. Its dimensions are [ML^2T^{-2}].

SI unit is joule (J).

Example of PE other than gravitational PE.

(i) Energy associated with the state of compression or expansion of an elastic spring.

(ii) The energy associated with state of separation of charged particles is called electrical potential energy.

41. Mention some of the different forms of energy and discuss them briefly.

Sol. Different forms of energy

(i) **Heat energy :** It is the energy possessed by a body by virtue of random motion of the molecules of the body. It is also associated with the force of friction, when a body of mass *m* moving on a rough horizontal surface with speed *v* stops over a distance *x*, work done by force of kinetic friction *f* over a distance *x* is (–*fx*). By the work-energy theorem, $\frac{1}{2}mv^2 = fx$. It is often said that KE of the block is lost due to frictional force. Work done by friction is not lost, but it is transferred as heat energy of the system.

(ii) **Chemical energy :** It arises from the fact that the molecules participating in the chemical reaction have different binding energies.

(iii) **Electrical energy :** It is associated with an electric current.

(iv) **Nuclear energy :** In nuclear fusion, effectively four hydrogen nuclei fuse to form a helium nucleus, whose mass < sum of masses of the reactants. This mass difference Δm is the source of energy $(\Delta m)c^2$. In nuclear fission, a heavy nucleus like ${}^{235}_{92}U$ splits into lighter nuclei. Final mass < initial mass and mass difference translates into energy.

42. What do you mean by collision? Discuss two types of collision with their essential characteristics.

Sol. Collision is defined as an isolated event in which two or more colliding bodies exert relatively strong forces on each other for a relatively short time.

Types :

(a) **Elastic collision** is the one in which there is absolutely no loss of KE

Basic characteristics:

(i) Linear momentum is conserved.

(ii) Total energy of system is conserved.

(iii) KE is conserved.

(iv) Forces involved must be conservative forces *e.g.*, collision between atomic and subatomic particles, collision between two ivory balls.

(b) **Inelastic collision** is the one in which there occurs some loss of KE

Basic characteristics :

(i) The linear momentum is conserved.

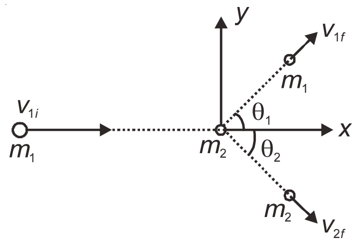
(ii) Total energy is conserved.

(iii) KE is not conserved.

(iv) Some or all of the forces involved may be non-conservative.

e.g., mud thrown on a wall sticks to the wall, when an arrow gets stuck in a target and the two move together.

43. Consider the collision shown in Fig. between two billiard balls with equal masses. The player wants to 'sink' the target ball in a corner pocket, which is at an angle θ_2 = 30°. Assume an elastic collision. Neglect friction and rotational motion. Find θ_1



Sol. Using momentum conservations

$$\vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$$

$$v_{1i}^2 = (\vec{v}_{1f} + \vec{v}_{2f}) \cdot (\vec{v}_{1f} + \vec{v}_{2f})$$

$$= v_{1f}^2 + v_{2f}^2 + 2\vec{v}_{1f} \cdot \vec{v}_{2f}$$

$$= v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f}\cos(\theta_1 + 30°) \quad \ldots(i)$$

As the collision is elastic and $m_1 = m_2$ and kinetic energy before collision = Kinetic energy after collision

$$v_{1i}^2 = v_{1f}^2 + v_{2f}^2 \quad \ldots(ii)$$

From (i) & (ii), we get

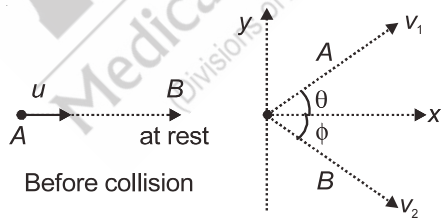
$\cos(\theta_1 + 30°) = 0$

or $\theta_1 + 30° = 90°$

$\Rightarrow \boxed{\theta_1 = 60°}$

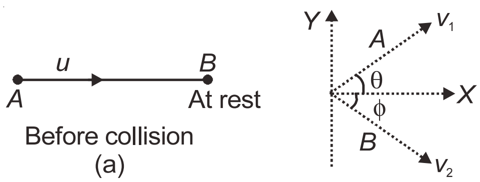
When two equal masses undergo a glancing elastic collision with one of them at rest, after the collision, they will move at right angle to each other.

44. Discuss elastic collision in two dimensions briefly.



After collision

Sol. Consider two particles A and B moving in one plane.



After collision
(b)

Fig.(a) shows a particle *A*, of mass m_1 moving along *x*-axis in *XY*-plane with an initial speed *u*. When it collides with *B* of mass m_2 at rest, A moves in a direction inclined at an angle θ, with its initial direction and *B* moves in a direction inclined at an angle ϕ with its initial direction of *A*.

$m_1u_1 + 0 = m_1v_1\cos\theta + m_2v_2\cos\phi$ ...(i) Law of conservation of linear momentum *X*-component

and $0 = m_1v_1\sin\theta - m_2v_2\sin\phi$ ...(ii) *Y*-component

As it is perfectly elastic collision,

Total KE before collision = Total KE after collision

$$\frac{1}{2}m_1u^2 + 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iii)}$$

or $m_1u^2 = m_1v_1^2 + m_2v_2^2$

If value of any one of the four unknowns *i.e.*, v_1, v_2, θ or ϕ is given, the remaining three can be calculated using equations (i), (ii) & (iii).

45. For Q.44, take $m_1 = m_2$. Get an equation involving θ and ϕ.

OR

Consider a perfectly inelastic collision in one dimension. Show that some KE is always lost in it.

Sol. Eq. (i), (ii), (iii) in solution 44 reduce to

$$u = v_1\cos\theta + v_2\cos\phi \quad \text{...(iv)}$$

$$0 = v_1\sin\theta - v_2\sin\phi \quad \text{...(v)}$$

$$u^2 = v_1^2 + v_2^2 \quad \text{...(vi)}$$

From Eq. (iv) & (vi)

$$(v_1\cos\theta + v_2\cos\phi)^2 = v_1^2 + v_2^2$$

$$v_1^2\cos^2\theta + v_2^2\cos^2\phi + 2v_1v_2\cos\theta\cos\phi = v_1^2 + v_2^2$$

$$2v_1v_2\cos\theta\cos\phi = v_1^2(1-\cos^2\theta) + v_2^2(1-\cos^2\phi)$$

$$= v_1^2\sin^2\theta + v_2^2\sin^2\phi \quad \text{...(vii)}$$

Using Eq. (v), we can rewrite Eq. (vii) as

$$2v_1v_2\cos\theta\cos\phi = 2v_1^2\sin^2\theta$$

$$\cos\theta = \left(\frac{v_1}{v_2}\right)\frac{\sin^2\theta}{\cos\phi} \quad \text{...(viii)}$$

$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

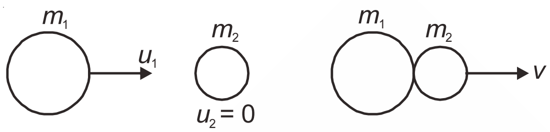
$$= \left(\frac{v_1}{v_2}\right)\frac{\sin^2\theta}{\cos\phi}\cos\phi - \frac{v_1}{v_2}\sin^2\theta \qquad \text{(From Eq. (v) \& (viii))}$$

$$= \left(\frac{v_1}{v_2}\right)\sin^2\theta - \frac{v_1}{v_2}\sin^2\theta$$

$$= 0$$

$\Rightarrow \boxed{\theta + \phi = 90^\circ}$

OR



Consider a perfectly inelastic collision between two bodies of masses m_1 and m_2. Body of mass m_1 moving with velocity u_1 hits body of mass m_2 which is at rest. After the collision, the two bodies move together with common velocity v.

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$ (Conservation of linear momentum)

$m_1u_1 = (m_1 + m_2)v$ as $u_2 = 0$

$$v = \frac{m_1u_1}{m_1 + m_2} \qquad \text{...(i)}$$

Total KE before collision, $E_1 = \frac{1}{2}m_1u_1^2$

Total KE after collision, $E_2 = \frac{1}{2}(m_1 + m_2)v^2$

$$= \frac{1}{2}(m_1 + m_2)\left(\frac{m_1u_1}{m_1 + m_2}\right)^2 \text{ using (i)}$$

$$= \frac{m_1^2u_1^2}{2(m_1 + m_2)}$$

Loss of KE = $E_1 - E_2$

$$= \frac{1}{2}m_1u_1^2 - \frac{m_1^2u_1^2}{2(m_1 + m_2)}$$

$$= \frac{m_1^2u_1^2 + m_1m_2u_1^2 - m_1^2u_1^2}{2(m_1 + m_2)}$$

$$= \frac{m_1m_2u_1^2}{2(m_1 + m_2)} \qquad \text{...(ii)}$$

Which is positive therefore some KE is always lost in an inelastic collision.

SECTION - B

Model Test Paper

Very Short Answer Type Questions :

1. Does KE depend upon the direction of motion?

Sol. No.

2. Is work done by a non-conservative force always negative. Comment.

Sol. No, *e.g.*, as long as the body does not start moving, work done by non-conservative force like friction is zero. Work done by friction is positive when friction causes motion.

3. Can PE of a body be negative?

Sol. Yes, when forces involved are attractive.

4. Does the work done in raising a body depend upon how fast it is raised?

Sol. No, the work done is independent of time.

5. What is the work done by centripetal force?

Sol. Zero as centripetal force is always perpendicular to displacement.

6. How does PE of a spring change when it is compressed or stretched?

Sol. PE increases in both the cases as work is done by us in compression as well as stretching.

7. A light and a heavy body have same KE, which body has more momentum?

Sol. Heavier body has larger momentum because $p = \sqrt{2mk}$ $\quad p \propto \sqrt{m}$

8. In which type of collision, elastic or inelastic, the momentum is conserved?

Sol. Momentum is conserved in both types of collisions.

9. In which type of collision, elastic or inelastic, KE is conserved?

Sol. KE is conserved only in elastic collision.

10. Two balls of same mass moving in opposite direction with same speed collide head-on with each other. Assuming a perfectly elastic collision, predict the outcome.

Sol. The balls rebound from each other with same speed.

Short Answer Type Questions :

11. (a) Can KE of a system be changed without changing its momentum?

(b) Can momentum of a system be changed without changing its KE?

Sol. (a) Yes, *e.g.*, when a bomb explodes, linear momentum is conserved, but KE changes.

(b) Yes, *e.g.*, in uniform circular motion, KE remains unchanged, but linear momentum changes because of change in direction of motion.

12. The momentum of a body is increased by 50%. Find the percentage change in KE

Sol. When momentum is increased by 50%, velocity becomes $\frac{3}{2}$ times $\Rightarrow$ KE becomes $\frac{9}{4}$ times *i.e.*,

$\frac{9}{4}\times 100 = 225\%$.

Percent increase in KE = 225 – 100 = $\boxed{125\%}$.

13. If $\vec{P}\cdot\vec{Q} = \vec{P}\cdot\vec{R}$ can we conclude that $\vec{Q} = \vec{R}$?

Sol. $\vec{P}\cdot\vec{Q} = \vec{P}\cdot\vec{R}$

$PQ\cos\theta_1 = PR\cos\theta_2$

Where θ_1 is smaller angle between $\vec{P}$ & $\vec{Q}$

and θ_2 is smaller angle between $\vec{P}$ & $\vec{R}$

$Q\cos\theta_1 = R\cos\theta_2$

If $\theta_1 = \theta_2$, then $Q = R$ or $\vec{Q} = \vec{R}$

If $\theta_1 \neq \theta_2$, then $Q \neq R$ or $\vec{Q} \neq \vec{R}$

14. A particle moves from position $\vec{r}_1 = (2\hat{i} + 3\hat{j} - 4\hat{k})$m to position $\vec{r}_2 = (7\hat{i} + 8\hat{j} + \hat{k})$m under the action of a force $\vec{F} = (2\hat{i} + 2\hat{j} - 2\hat{k})$ newton. Calculate the work done

Sol. $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$= (7\hat{i} + 8\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k})$

$= 5\hat{i} + 5\hat{j} + 5\hat{k}$

Work, $W = \vec{F}\cdot\vec{r}_{12}$

$= (5\hat{i} + 5\hat{j} + 5\hat{k})\cdot(2\hat{i} + 2\hat{j} - 2\hat{k})$

= 10 + 10 – 10

= $\boxed{10 \text{ J}}$

15. Can a body have energy without momentum?

Sol. Yes, when $p = 0$, $k = \frac{p^2}{2m} = 0$

But $E = K + V = V$ (Potential energy), which may or may not be zero.

16. Two springs P and Q are identical but P is harder than Q ($K_P > K_Q$). On which spring, more work will be done, if

    (i) They are stretched by same force?

    (ii) They are stretched through same distance?

Sol. (i) $W = \dfrac{F^2}{2K}$

$\therefore$ $K_P > K_Q$

$\therefore$ $W_P < W_Q$

(ii) $W = \dfrac{1}{2}kx^2$

$k_P > k_Q$

$\therefore$ $W_P > W_Q$

More work will be done on P.

Short Answer Type Questions :

17. What do you mean by zero work? State the conditions for zero work. Give one example.

Sol. W = Work = $\vec{F}\cdot\vec{d} = (F\cos\theta)d$

Where θ is the angle between force $\vec{F}$ and displacement $\vec{d}$.

Work = 0 when F = 0 or d = 0 or θ = 90°

i.e., (i) No force is applied.

(ii) There is no displacement.

(iii) Force is applied in a direction perpendicular to the direction of displacement.

e.g., work done by gravitational force of sun on a planet moving around it in a circular orbit is zero as centripetal force (gravitational force) acts in a direction perpendicular to the direction of displacement.

18. Define work. State its dimensions, CGS & SI unit.

Sol. The work done by a force is defined as the product of component of the force in the direction of the displacement and the magnitude of this displacement.

W = ($F\cos\theta$) = $\vec{F}\cdot\vec{d}$

where θ is the smaller angle between the force $\vec{F}$ and displacement $\vec{d}$.

Its dimensions are [ML^2T^{-2}].

Its CGS unit is erg. Its SI unit is joule (J).

19. A bullet of mass 100 g moving with a velocity of 300 ms^{-1} strikes a wall and goes out of the other side with a velocity of 100 ms^{-1}. Calculate work done in passing through the wall.

Sol. Work done in passing through the wall

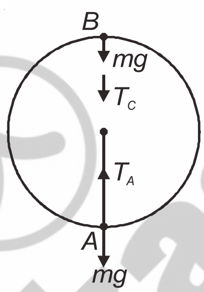
= Loss in KE of bullet

$= \frac{1}{2}m(v_1^2 - v_2^2)$

$= \frac{1}{2}\times 100\times 10^{-3}\times(300^2 - 100^2)$

$= \frac{1}{2}\times\frac{1}{10}\times 400\times 200 = \boxed{4\times 10^3\,\text{J}}$

20. A bob of mass m is suspended by a light string of length l. It moves with horizontal velocity v at the lowest point A such that it completes a semicircular trajectory in the vertical plane with the string becoming slack only on reaching the top-most point B. Show that minimum speed at A should be $\sqrt{5\,gl}$.



Sol. PE of the system = 0 at A

At A, $E = \frac{1}{2}mv^2$ ...(i)

$T_A - mg = \frac{mv^2}{l}$

At B, $T_C = 0$

At C, $E = \frac{1}{2}mv_C^2 + 2mgl$ ...(ii)

$mg = \frac{mv_C^2}{l}$ ...(iii)

Where v_C is speed at C.

From (ii) & (iii) $E = \frac{5}{2}mgl$

Equating this to the energy at A.

$\frac{5}{2}mgl = \frac{m}{2}v^2$

$\boxed{v = \sqrt{5gl}}$

21. What is meant by mass-energy equivalence?

Sol. According to Einstein, energy can be transformed into mass and vice versa. The mass-energy equivalence relation as put forth by Einstein is

$$E = mc^2$$

Where m = mass that disappears

E = energy that appears

c = velocity of light in vacuum.

According to (modern) Quantum Physics, mass and energy are not conserved separately, but are conserved as a single entity called 'mass-energy'.

22. A ball of mass 0.1 kg makes an elastic head-on collision with a ball of unknown mass that is initially at rest. If 0.1 kg ball rebounds at one-third of its original speed, what is the mass of the other ball?

Sol. m_1 = 0.1 kg, m_2 = ?, u_2 = 0, Let $u_1 = u$.

$$v_1 = -\frac{u}{3}$$

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

$$\Rightarrow \quad -\frac{u}{3} = \frac{(0.1 - m_2)u}{0.1 + m_2}$$

$$\Rightarrow \quad -\frac{1}{3} = \frac{0.1 - m_2}{0.1 + m_2}$$

$$\Rightarrow \quad -0.3 + 3m_2 = 0.1 + m_2$$

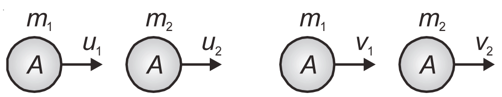
$$\Rightarrow \quad 2m_2 = 0.4$$

$$\Rightarrow \quad m_2 = \boxed{0.2\text{ kg}}$$

Long Answer Type Questions :

23. Consider two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 along the same straight line with $u_1 > u_2$. Find their velocities after the collision, assuming an elastic collision.

Sol.



Before collision After collision

Let the velocities of A and B after the collision be v_1 and v_2 respectively.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i) (Applying law of conservation of linear momentum)

$m_2(v_2 - u_2) = m_1(u_1 - v_1)$ ...(ii)

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$ (Applying law of conservation of KE.)

$$\frac{1}{2}m_2(v_2^2 - u_2^2) = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

$$m_2(v_2^2 - u_2^2) = m_1(u_1^2 - v_1^2) \quad \ldots\text{(iii)}$$

Divide equation (iii) by equation (ii)

$$\frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)} = \frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)}$$

$$v_2 + u_2 = u_1 + v_1$$

$$v_2 - v_1 = u_1 - u_2 \quad \ldots\text{(iv)}$$

In one-dimensional elastic collision.

Relative velocity of separation after collision = Relative velocity of approach before collision

$$\frac{v_2 - v_1}{u_1 - u_2} = 1 = e$$

From (iii),

$$v_2 = u_1 - u_2 + v_1 \quad \ldots\text{(v)}$$

Put this value in (i)

$$m_1v_1 + m_2(u_1 - u_2 + v_1) = m_1u_1 + m_2u_2$$

$$m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1 = m_1u_1 + m_2u_2$$

$$v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2$$

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2} \quad \ldots\text{(vi)}$$

Put this value in (v)

$$v_2 = u_1 - u_2 + \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

$$= u_1\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right) + u_2\left(\frac{2m_2}{m_1 + m_2} - 1\right)$$

$$= u_1\left(\frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2}\right) + u_2\left(\frac{2m_2 - m_1 - m_2}{m_1 + m_2}\right)$$

$$\boxed{v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2}} \quad \ldots\text{(vii)}$$

24. State and explain work-energy principle.

Sol. Time rate of change of KE is

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$= m\frac{dv}{dt}v$$

$= Fv$ (From Newton's IInd law)

$$= F\frac{dx}{dt}$$

$dk = Fdx$

Integrating from the initial position (x_i) to final position (x_f)

$$\int_{K_i}^{K_f} dk = \int_{x_i}^{x_f} Fdx$$

Where K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f,

$$K_f - K_i = \int_{x_i}^{x_f} Fdx$$

$\boxed{K_f - K_i = W}$ as $\int_{x_i}^{x_f} Fdx = W$ (work done)

Thus work-energy theorem is proved for a variable force. Work done by net force in displacing a body is equal to change in kinetic energy of the body.

❑ ❑ ❑

Solutions (Set-2)

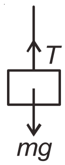
Objective Type Questions

(Work)

1. A string is used to pull a block of mass m vertically up by a distance h at a constant acceleration $\frac{g}{3}$. The work done by the tension in the string is

(1) $\frac{2}{3}mgh$ (2) $\frac{-mgh}{3}$ (3) mgh (4) $\frac{4}{3}mgh$

Sol. Answer (4)



$T - mg = ma$

$T = m(g + a)$

$= \frac{4}{3}mg$

Work (w) = $T.h$

$= \frac{4}{3}mgh$

2. A body constrained to move in z direction is subjected to a force given by $\vec{F} = (3\hat{i} - 10\hat{j} + 5\hat{k})\text{N}$. What is the work done by this force in moving the body through a distance of 5 m along z-axis?

(1) 15 J (2) – 15 J (3) –50 J (4) 25 J

Sol. Answer (4)

$W = \left(3\hat{i} - 10\hat{j} + 5\hat{k}\right).5\hat{k}$

= 25 J

3. If 250 J of work is done in sliding a 5 kg block up an inclined plane of height 4 m. Work done against friction is (g = 10 ms^{-2})

(1) 50 J (2) 100 J (3) 200 J (4) Zero

Sol. Answer (1)

$W_{Total} = W_{friction} + W_{gravity}$

$-250 = W_f - 50(4)$

$W_f = -50$ J

4. A man carries a load on his head through a distance of 5 m. The maximum amount of work is done when he

(1) Moves it over an inclined plane (2) Moves it over a horizontal surface

(3) Lifts it vertically upwards (4) None of these

Sol. Answer (3)

Displacement is maximum while moving it vertically upwards.

5. A body moves a distance of 10 m along a straight line under the action of a force 5 N. If the work done is 25 joule, the angle which the force makes with the direction of motion of body is

(1) 0° (2) 30° (3) 60° (4) 90°

Sol. Answer (3)

$\vec{F}\,.\,\vec{S} = 25$

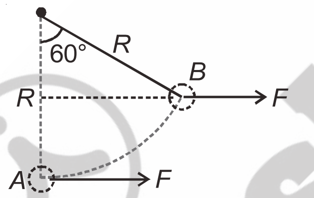
$FS\cos\theta = 25$

$(5)\,(10)\cos\theta = 25$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$

6. A block of mass *m* is pulled along a circular arc by means of a constant horizontal force *F* as shown. Work done by this force in pulling the block from *A* to *B* is

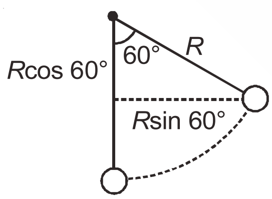


(1) $\frac{FR}{2}$ (2) FR (3) $\frac{\sqrt{3}}{2}FR$ (4) mgR

Sol. Answer (3)

Work = Force × (Displacement in the direction of force)

$= F(R\sin 60°) = \frac{\sqrt{3}}{2}FR$



7. A particle is displaced from a position $(2\hat{i} - \hat{j} + \hat{k})$ metre to another position $(3\hat{i} + 2\hat{j} - 2\hat{k})$ metre under the action of force $(2\hat{i} + \hat{j} - \hat{k})$ N. Work done by the force is

(1) 8 J (2) 10 J (3) 12 J (4) 36 J

Sol. Answer (1)

$W = \vec{F}\,.\,\vec{S}$

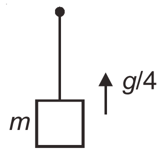
Displacement vector $(\vec{S})$ = $(3i + 2j - 2k) - (2i - j + k)$

$= i + 3\hat{j} - 3\hat{k}$

W = $(2i + j - k)\,.\,(i + 3j - 3k)$

$= 2 + 3 + 3 = 8J$

8. A string is used to pull a block of mass m vertically up by a distance h at a constant acceleration $\frac{g}{4}$. The work done by the tension in the string is



(1) $+\frac{3mgh}{4}$ (2) $-\frac{mgh}{4}$ (3) $+\frac{5}{4}mgh$ (4) $+mgh$

Sol. Answer (3)

$T - mg = ma$

$$T = m(g+a) \Rightarrow T = m\left(g+\frac{g}{4}\right) = \frac{5}{4}mg$$

$$W = T.h = \frac{5}{4}mgh$$

9. Work done by frictional force

(1) Is always negative

(2) Is always positive

(3) Is zero

(4) May be positive, negative or zero

Sol. Answer (4)

Frictional force can act in the direction of displacement, opposite to it and sometimes not let the body move. So the work can be positive, negative or zero.

(Kinetic Energy)

10. Two bodies of masses m_1 and m_2 have same kinetic energy. The ratio of their momentum is

(1) $\sqrt{\frac{m_2}{m_1}}$ (2) $\sqrt{\frac{m_1}{m_2}}$ (3) $\frac{m_1^2}{m_2^2}$ (4) $\frac{m_2^2}{m_1^2}$

Sol. Answer (2)

$$\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$$

11. Two bodies of masses m_1 and m_2 have same momentum. The ratio of their KE is

(1) $\sqrt{\frac{m_2}{m_1}}$ (2) $\sqrt{\frac{m_1}{m_2}}$ (3) $\frac{m_1}{m_2}$ (4) $\frac{m_2}{m_1}$

Sol. Answer (4)

$$\sqrt{2m_1k_1} = \sqrt{2m_2k_2}$$

$$\frac{k_1}{k_2} = \frac{m_2}{m_1}$$

12. KE of a body is increased by 44%. What is the percent increase in the momentum?

(1) 10% (2) 20% (3) 30% (4) 44%

Sol. Answer (2)

$$1.44K = \frac{(P')^2}{2m}$$

$P' = 1.2\ P$

$$\frac{P'-P}{P}\times 100 = \frac{1.2P-P}{P} = 20\%$$

13. When momentum of a body increases by 200%, its KE increases by

(1) 200% (2) 300% (3) 400% (4) 800%

Sol. Answer (4)

$P' = 3P$

$$K' = \frac{(3P)^2}{2m} = \frac{9P^2}{2m}$$

$$\frac{K'-K}{K}\times 100 = \frac{\frac{9P^2}{2m}-\frac{P^2}{2m}}{\frac{P^2}{2m}}\times 100 = 800\%$$

14. Two bodies of masses m_1 and m_2 are moving with same kinetic energy. If P_1 and P_2 are their respective momentum, the ratio $\frac{P_1}{P_2}$ is equal to

(1) $\frac{m_1}{m_2}$ (2) $\sqrt{\frac{m_2}{m_1}}$ (3) $\sqrt{\frac{m_1}{m_2}}$ (4) $\frac{m_1^{\ 2}}{m_2^{\ 2}}$

Sol. Answer (3)

$$\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$$

(Work Done by a Variable Force)

15. A particle moves along x-axis from $x = 0$ to $x = 5$ metre under the influence of a force $F = 7 - 2x + 3x^2$. The work done in the process is

(1) 70 (2) 135 (3) 270 (4) 35

Sol. Answer (2)

$$W = \int_0^5 \vec{F}.dx = \int_0^5 \left(7-2x+3x^2\right)dx$$

$$= [7x - x^2 + x^3]_0^5 = 135$$

(Notion of Work and Kinetic Energy : The Work-Energy Theorem)

16. Under the action of a force, a 2 kg body moves such that its position x as a function of time t is given by $x = \frac{t^2}{3}$, where x is in metre and t in second. The work done by the force in first two seconds is

(1) 1600 J (2) 160 J

(3) 16 J (4) $\frac{16}{9}$ J

Sol. Answer (4)

$$x = \frac{t^2}{3} \quad \Rightarrow \quad v = \frac{2t}{3}$$

$$W = \Delta\text{K.E.} = \frac{1}{2}(2)\left[\left(\frac{4}{3}\right)^2 - 0\right] = \frac{16}{9}\text{J}$$

17. A particle of mass 2 kg travels along a straight line with velocity $v = a\sqrt{x}$, where a is a constant. The work done by net force during the displacement of particle from x = 0 to x = 4 m is

(1) a^2 (2) $2a^2$

(3) $4a^2$ (4) $\sqrt{2}\,a^2$

Sol. Answer (3)

$$v = a\sqrt{x}$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2}(2)\left[\left(a\sqrt{4}\right)^2 - 0\right]$$

$$= 4a^2$$

18. The position x of a particle moving along x-axis at time (t) is given by the equation $t = \sqrt{x} + 2$, where x is in metres and t in seconds. Find the work done by the force in first four seconds.

(1) Zero (2) 2 J

(3) 4 J (4) 8 J

Sol. Answer (1)

$$x = (t - 2)^2$$

$$\frac{dx}{dt} = v = 2(t-2)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{m}{2}\left[4^2 - 4^2\right] = 0$$

19. KE acquired by a mass m in travelling a certain distance d, starting from rest, under the action of a constant force F is

(1) Directly proportional to $\sqrt{m}$

(2) Directly proportional to m

(3) Directly proportional to $\frac{1}{m}$

(4) None of these

Sol. Answer (4)

$F.d = \Delta K$

$F.d = K_f$

K is independent of mass here.

20. A simple pendulum with bob of mass m and length x is held in position at an angle θ_1 and then angle θ_2 with the vertical. When released from these positions, speeds with which it passes the lowest positions are v_1 & v_2 respectively. Then, $\frac{v_1}{v_2}$ is

(1) $\frac{1-\cos\theta_1}{1-\cos\theta_2}$

(2) $\sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_2}}$

(3) $\sqrt{\frac{2gx(1-\cos\theta_1)}{1-\cos\theta_2}}$

(4) $\sqrt{\frac{1-\cos\theta_1}{2gx(1-\cos\theta_2)}}$

Sol. Answer (2)

$U_i + k_i = U_f + k_f$ (Mechanical energy conservation)

$$mgl\left(1-\cos\theta\right) = \frac{1}{2}mv^2$$

$$\frac{v_1^2}{v_2^2} = \frac{1-\cos\theta_1}{1-\cos\theta_2}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_2}}$$

21. The total work done on a particle is equal to the change in its kinetic energy. This is applicable

(1) Always

(2) Only if the conservative forces are acting on it

(3) Only in inertial frames

(4) Only when pseudo forces are absent

Sol. Answer (1)

$W = \Delta k$ is always applicable

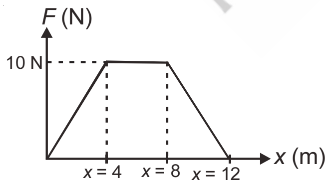
22. A particle of mass 0.1 kg is subjected to a force which varies with distance as shown. If it starts its journey from rest at $x = 0$, then its velocity at $x = 12$ m is

(1) 0 m/s

(2) $20\sqrt{2}$ m/s

(3) $20\sqrt{3}$ m/s

(4) 40 m/s



Sol. Answer (4)

Total work done = Area under $F - x$ curve

= ΔK.E.

$$\frac{1}{2}(4)(10) + \frac{1}{2}(4)10 + 40 = \Delta K$$

$$80\ J = \frac{1}{2}(0.1)v^2 \Rightarrow v = 40 \text{ m/s}$$

23. An unloaded bus can be stopped by applying brakes on straight road after covering a distance x. Suppose, the passenger add 50% of its weight as the load and the braking force remains unchanged, how far will the bus go after the application of the brakes? (Velocity of bus in both case is same)

(1) Zero (2) $1.5x$ (3) $2x$ (4) $2.5x$

Sol. Answer (2)

$$F.x = \frac{1}{2}mv^2$$

$$F.x^1 = \frac{1}{2}(1.5m)v^2 \Rightarrow x^1 = 1.5\ x$$

(The Concept of Potential Energy)

24. Potential energy is defined

(1) Only in conservative fields

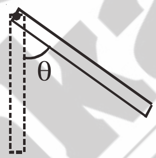
(2) As the negative of work done by conservative forces

(3) As the negative of workdone by external forces when $\Delta K = 0$

(4) All of these

Sol. Answer (1)

25. A stick of mass m and length l is pivoted at one end and is displaced through an angle θ. The increase in potential energy is



(1) $mg\ \frac{l}{2}(1-\cos\theta)$ (2) $mg\ \frac{l}{2}(1+\cos\theta)$

(3) $mg\ \frac{l}{2}(1-\sin\theta)$ (4) $mg\ \frac{l}{2}(1+\sin\theta)$

Sol. Answer (1)

Using mechanical energy conservation, $U = \frac{mgl}{2}(1-\cos\theta)$

26. A spring with spring constants k when compressed by 1 cm, the potential energy stored is U. If it is further compressed by 3 cm, then change in its potential energy is

(1) $3U$ (2) $9U$ (3) $8U$ (4) $15U$

Sol. Answer (4)

$$U = \frac{1}{2}k(1)^2 = \frac{k}{2}$$

$$U^1 = \frac{1}{2}k(4)^2 = \frac{1}{2}k(16) = 16U$$

$$\Delta U = U^1 - U = 16U - U = 15U$$

27. Two springs have force constant K_1 and K_2 ($K_1 > K_2$). Each spring is extended by same force F. It their elastic potential energy are E_1 and E_2 then $\frac{E_1}{E_2}$ is

(1) $\frac{K_1}{K_2}$ (2) $\frac{K_2}{K_1}$ (3) $\sqrt{\frac{K_1}{K_2}}$ (4) $\sqrt{\frac{K_2}{K_1}}$

Sol. Answer (2)

$$x = \frac{F}{K}$$

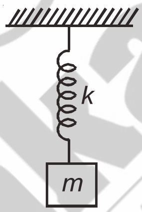
$$U = \frac{1}{2}Kx^2 = \frac{1}{2}K\left(\frac{F}{K}\right)^2$$

$$U = \frac{F^2}{2K}$$

$$U \propto \frac{1}{K}$$

$$\frac{U_1}{U_2} = \frac{K_2}{K_1}$$

28. Initially mass m is held such that spring is in relaxed condition. If mass m is suddenly released, maximum elongation in spring will be



(1) $\frac{mg}{k}$ (2) $\frac{2mg}{k}$ (3) $\frac{mg}{2k}$ (4) $\frac{mg}{4k}$

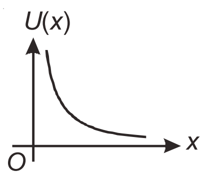
Sol. Answer (2)

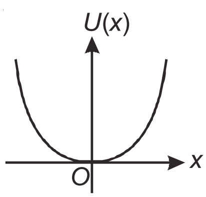
$E_i = E_f$

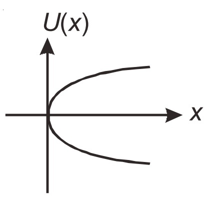
$$\Rightarrow 0 = -mgx + \frac{1}{2}kx^2$$

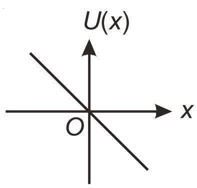
$$x = \frac{2mg}{k}$$

29. On a particle placed at origin a variable force $F = -ax$ (where a is a positive constant) is applied. If $U(0) = 0$, the graph between potential energy of particle $U(x)$ and x is best represented by

(1) 

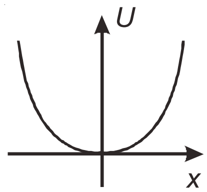
(2) 

(3) 

(4) 

Sol. Answer (2)

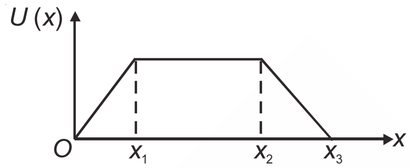
$$F = -ax = \frac{-dU}{dx}$$

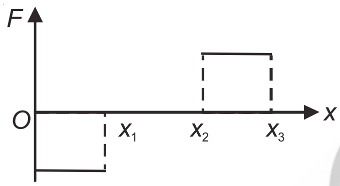


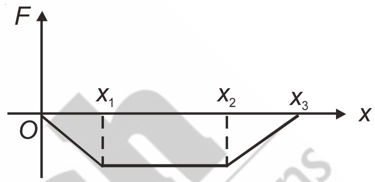
Integrating both sides

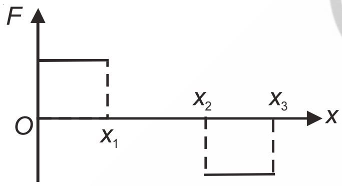
$$U = \frac{ax^2}{2}$$

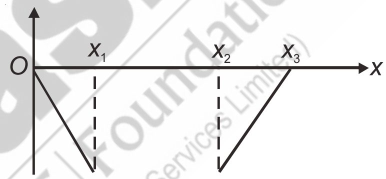
30. The variation of potential energy U of a system is shown in figure. The force acting on the system is best represented by



(1)


(2)


(3)


(4)


Sol. Answer (1)

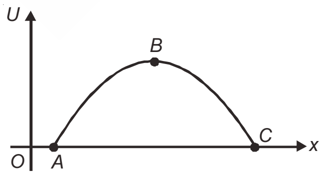
$F = \frac{-dU}{dx} \Rightarrow$ Slope of U–x curve will represent force

from $0 \to x_1$ Slope is positive and non zero

from $x_1 \to x_2$ Slope is zero

from $x_2 \to x_3$ Slope is negative and non zero

31. The variation of potential energy U of a body moving along x-axis varies with its position (x) as shown in figure



The body is in equilibrium state at

(1) A (2) B (3) C (4) Both A & C

Sol. Answer (2)

at B, $\frac{dU}{dx} = 0$ (Slope of $U - x$ curve)

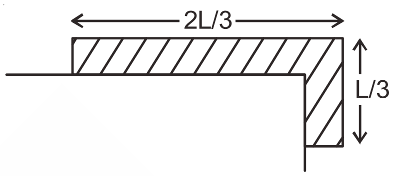
$\Rightarrow F = 0$ at B, So its a position of equilibrium

(The Conservation of Mechanical Energy)

32. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the minimum work required to pull the hanging part of the chain on the table is

(1) MgL (2) $\frac{MgL}{3}$ (3) $\frac{MgL}{9}$ (4) $\frac{MgL}{18}$

Sol. Answer (4)



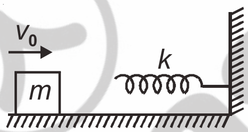
Mass of $\frac{L}{3}$ part will be $\frac{M}{3}$

Centre of mass of $\frac{L}{3}$ part is $\frac{L}{6}$ below the table

So total displacement of C.M. to bring it on the table

$$W = \frac{M}{3}g\left(\frac{L}{6}\right) = \frac{MgL}{18}$$

33. A block of mass m moving with velocity v_0 on a smooth horizontal surface hits the spring of constant k as shown. The maximum compression in spring is



(1) $\sqrt{\frac{2m}{k}}\cdot v_0$ (2) $\sqrt{\frac{m}{k}}\cdot v_0$ (3) $\sqrt{\frac{m}{2k}}\cdot v_0$ (4) $\frac{m}{2k}\cdot v_0$

Sol. Answer (2)

$E_i = E_f$

$$\frac{1}{2}m v_0^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{m}{k}}v_0$$

34. A particle of mass 200 g is moving in a circle of radius 2 m. The particle is just 'looping the loop'. The speed of the particle and the tension in the string at highest point of the circular path are (g = 10 ms^{-2})

(1) 4 ms^{-1}, 5 N (2) 4.47 ms^{-1}, zero (3) 2.47 ms^{-1}, zero (4) 1 ms^{-1}, zero

Sol. Answer (2)

$v = \sqrt{gL} = 4.47$ m/s

$T = 0$

35. A particle of mass 200 g, is whirled into a vertical circle of radius 80 cm using a massless string. The speed of the particle when the string makes an angle of 60° with the vertical line is 1.5 ms^{-1}. The tension in the string at this position is

(1) 1 N (2) 1.56 N (3) 2 N (4) 3 N

Sol. Answer (2)

$$T - mg\cos\theta = \frac{mv^2}{R}$$

θ = 60°

Solving this T = 1.56 N

36. A stone of mass 1 kg is tied with a string and it is whirled in a vertical circle of radius 1 m. If tension at the highest point is 14 N, then velocity at lowest point will be

(1) 3 m/s (2) 4 m/s (3) 6 m/s (4) 8 m/s

Sol. Answer (4)

$$T + mg = \frac{mv^2}{R} \text{ (at the highest point)}$$

$$14 = \frac{1(v^2)}{R(1)} + 10$$

$v^2 = 4 \Rightarrow v = 2$ m/s

Using mechanical energy conservation

$$\frac{1}{2}(1)u^2 = \frac{1}{2}(1)(2^2) + 1(10)(2)$$

$u^2 = 64 \Rightarrow u = 8$ m/s

(Power)

37. The power of water pump is 4 kW. If $g = 10$ ms^{-2}, the amount of water it can raise in 1 minute to a height of 20 m is

(1) 100 litre (2) 1000 litre

(3) 1200 litre (4) 2000 litre

Sol. Answer (3)

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{mgh}{t}$$

$$\frac{m(10)(20)}{60} = 4000 \Rightarrow m = 1200 \text{ litre}$$

38. A particle moves with the velocity $\vec{v} = (5\hat{i} + 2\hat{j} - \hat{k})\text{ms}^{-1}$ under the influence of a constant force, $\vec{F} = (2\hat{i} + 5\hat{j} - 10\hat{k})$ N. The instantaneous power applied is

(1) 5 W (2) 10 W (3) 20 W (4) 30 W

Sol. Answer (4)

$$P = \vec{F}\,.\,\vec{V}$$

$= (2i + 5j - 10k)\,.\,(5i + 2j - k)$

$= 10 + 10 + 10 = 30$ W

39. A body is projected from ground obliquely. During downward motion, power delivered by gravity to it

(1) Increases
(2) Decreases
(3) Remains constant
(4) First decreases and then becomes constant

Sol. Answer (1)

Power = $Fv\cos\theta$ $\quad\theta < 90°$

Velocity of particle will increase

So power will increase as F is constant

40. The blades of a wind mill sweep out a circle of area A. If wind flows with velocity v perpendicular to blades of wind mill and its density is ρ, then the mechanical power received by wind mill is

(1) $\frac{\rho A v^3}{2}$
(2) $\frac{\rho A v^2}{2}$
(3) $\rho A v^2$
(4) $2\rho A v^2$

Sol. Answer (1)

$$P = \frac{dk}{dt} = \frac{d}{dt}\left[\frac{1}{2}mv^2\right]$$

$$= \frac{1}{2}v^2\frac{dm}{dt} = \frac{PAv^3}{2}$$

41. A body of mass m accelerates uniformly from rest to velocity v_1 in time interval T_1. The instantaneous power delivered to the body as a function of time t is

(1) $\frac{mv_1^2}{T_1^2}t$
(2) $\frac{mv_1}{T_1^2}t$
(3) $\left(\frac{mv_1}{T_1}\right)^2 t$
(4) $\frac{mv_1^2}{T_1}t^2$

Sol. Answer (1)

$v_1 = u + at_1$ $\quad\Rightarrow a = \frac{v_1}{t_1}$

again $v = u + at$

$$\Rightarrow v = 0 + \left(\frac{v_1}{T_1}\right)t$$

$$F = ma = \frac{mv_1}{T_1} \quad \Rightarrow P = \frac{mv_1}{T_1}\left(\frac{v_1}{T_1}t\right)$$

42. The power of a pump, which can pump 500 kg of water to height 100 m in 10 s is

(1) 75 kW
(2) 25 kW
(3) 50 kW
(4) 500 kW

Sol. Answer (3)

$$P = \frac{500(1)(100)}{10} = 50{,}000 = 50\text{ kW}$$

43. A pump is used to pump a liquid of density ρ continuously through a pipe of cross section area A. If liquid is flowing with speed V, then average power of pump is

(1) $\frac{1}{3}\rho AV^2$
(2) $\frac{1}{2}\rho AV^2$
(3) $2\rho AV^2$
(4) $\frac{1}{2}\rho AV^3$

Sol. Answer (4)

$$P_{avg} = \frac{v^2}{2}\frac{dm}{dt} = \frac{1}{2}\rho A v^3$$

44. From a water fall, water is pouring down at the rate of 100 kg/s, on the blades of a turbine. If the height of the fall is 100 m, the power delivered to the turbine is approximately equal to

(1) 100 kW (2) 10 kW (3) 1 kW (4) 100 W

Sol. Answer (1)

$$P_{avg} = \frac{W}{t} = \frac{mgh}{t}$$

$$= \left(\frac{m}{t}\right)gh$$

$$= 100 \times 10 \times 100 = 100 \text{ kW}$$

(Collision)

45. A U–238 nucleus originally at rest, decays by emitting an α-particle, say with a velocity of v m/s. The recoil velocity (in ms^{-1}) of the residual nucleus is

(1) $\frac{4v}{238}$ (2) $-\frac{4v}{238}$ (3) $\frac{v}{4}$ (4) $-\frac{4v}{234}$

Sol. Answer (4)

Using momentum conservation, $0 = 4v + 234\, v' \Rightarrow v' = \frac{-4v}{234}$

46. An object of mass 80 kg moving with velocity 2 ms^{-1} hit by collides with another object of mass 20 kg moving with velocity 4 ms^{-1}. Find the loss of energy assuming a perfectly inelastic collision

(1) 12 J (2) 24 J (3) 30 J (4) 32 J

Sol. Answer (4)

Loss in kinetic energy

$$\Delta k = \frac{m_1 m_2}{2(m_1 + m_2)}(u_1 - u_2)^2$$

$\Delta k = 32$ J

47. A ball of mass m moving with velocity v collides head-on with the second ball of mass m at rest. If the coefficient of restitution is e and velocity of first ball after collision is v_1 and velocity of second ball after collision is v_2 then

(1) $v_1 = \frac{(1-e)u}{2}, v_2 = \frac{(1+e)u}{2}$ (2) $v_1 = \frac{(1+e)u}{2}, v_2 = \frac{(1-e)u}{2}$

(3) $v_1 = \frac{u}{2}, v_2 = -\frac{u}{2}$ (4) $v_1 = (1 + e)u$, $v_2 = (1 - e)u$

Sol. Answer (1)

$e = \dfrac{\text{Velocity of separation}}{\text{Velocity of approach}}$

So, $V_1 = \dfrac{(1-e)u}{2}, v_2 = \dfrac{(1+e)}{2}u$

48. Particle *A* makes a perfectly elastic collision with another particle *B* at rest. They fly apart in opposite direction with equal speeds. If their masses are m_A & m_B respectively, then

(1) $2m_A = m_B$ (2) $3m_A = m_B$ (3) $4m_A = m_B$ (4) $\sqrt{3}m_A = m_B$

Sol. Answer (2)

From conservation of momentum and mechanical energy conservation

$3m_A = m_B$

49. A shell of mass *m* moving with a velocity *v* breakes up suddenly into two pieces. The part having mass $\dfrac{m}{3}$ remains stationary. The velocity of the other part will be

(1) v (2) $2\,v$ (3) $\dfrac{2}{3}v$ (4) $\dfrac{3}{2}v$

Sol. Answer (4)

Momentum will be conserved, $P_i = P_f$

$$mv = \frac{m}{3}(0) + \frac{2m}{3}(v^1) \Rightarrow v^1 = \frac{3}{2}v$$

50. A particle of mass *m* moving towards west with speed *v* collides with another particle of mass *m* moving towards south. If two particles stick to each other, the speed of the new particle of mass 2 *m* will be

(1) $v\sqrt{2}$ (2) $\dfrac{v}{\sqrt{2}}$ (3) $\dfrac{v}{2}$ (4) v

Sol. Answer (2)

Using conservation of momentum, $P_i = P_f$

$$mv(-\hat{i}) + mv(-\hat{j}) = 2m\,\vec{v}^1$$

$$|v^1| = \frac{v}{\sqrt{2}}$$

51. A body of mass 10 kg moving with speed of 3 ms^{-1} collides with another stationary body of mass 5 kg. As a result, the two bodies stick together. The KE of composite mass will be

(1) 30 J (2) 60 J (3) 90 J (4) 120 J

Sol. Answer (1)

Using momentum conservation

$10(3) = 15\,V$

$V = 2$ m/s

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2}(15)(2)^2 = 30 \text{ J}$$

52. Select the false statement

(1) In elastic collision, KE is not conserved during the collision

(2) The coefficient of restitution for a collision between two steel balls lies between 0 and 1

(3) The momentum of a ball colliding elastically with the floor is conserved

(4) In an oblique elastic collision between two identical bodies with one of them at rest initially, the final velocities are perpendicular

Sol. Answer (3)

Momentum will be conserved.

53. A bullet of mass m moving with a velocity u strikes a block of mass M at rest and gets embedded in the block. The loss of kinetic energy in the impact is

(1) $\frac{1}{2}mMu^2$ (2) $\frac{1}{2}(m+M)u^2$ (3) $\frac{mMu^2}{2(m+M)}$ (4) $\left(\frac{m+M}{2mM}\right)u^2$

Sol. Answer (3)

$$\Delta k = \frac{1}{2}\frac{m_1M_2}{(m_1+M_2)}(u_1-u_2)^2 = \frac{mMx^2}{2(m+M)}$$

54. A bullet of mass m moving with velocity v strikes a suspended wooden block of mass M. If the block rises to height h, the initial velocity of the bullet will be

(1) $\sqrt{2gh}$ (2) $\frac{M+m}{m}\sqrt{2gh}$ (3) $\frac{m}{M+m}\sqrt{2gh}$ (4) $\frac{M+m}{M}\sqrt{2gh}$

Sol. Answer (2)

$P_i = P_f$

$mv + 0 = mv^1 + Mv^1$

$mv = (m + M)v^1$

$$v^1 = \frac{mv}{m+M} = \sqrt{2gh}$$

$$v = \frac{(m+M)\sqrt{2gh}}{m}$$

55. A ball is allowed to fall from a height of 10 m. If there is 40% loss of energy due to impact, then after one impact ball will go up by

(1) 10 m (2) 8 m (3) 4 m (4) 6 m

Sol. Answer (4)

When ball just reaches the ground

k_1 = mg (10)

40 % of energy is lost after impact. Using mechanical energy conservation

$U_i + k_i = U_f + k_f$

$0 + (0.6)\, k_1 = mgh + 0$

(0.6) mg $(10) = mgh$

$h = 6$ m

56. A bullet weighing 10 g and moving with a velocity 300 m/s strikes a 5 kg block of ice and drop dead. The ice block is kept on smooth surface. The speed of the block after the collision is

(1) 6 cm/s (2) 60 cm/s (3) 6 m/s (4) 0.6 cm/s

Sol. Answer (2)

Using conservation of momentum

$P_i = P_f$

$$\frac{10}{1000}.(300) = \left(5 + \frac{10}{1000}\right)v$$

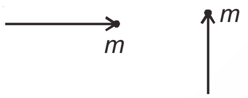
$$5 + \frac{10}{1000} \approx 5$$

So, v = 0.6 m/s Or 60 cm/s

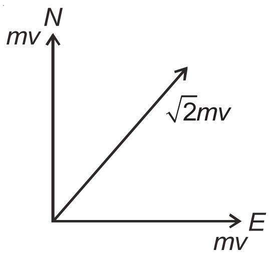
57. A particle of mass m moving eastward with a speed v collides with another particle of the same mass moving northwards with same speed v. The two particles coalesce on collision. The new particle of mass 2m will move with velocity.

(1) $\frac{v}{2}$ North-East (2) $\frac{v}{\sqrt{2}}$ South-West (3) $\frac{v}{2}$ North-West (4) $\frac{v}{\sqrt{2}}$ North-East

Sol. Answer (4)



Using momentum conservation



$$2mv^1 = \sqrt{2}mv$$

$$v^1 = \frac{v}{\sqrt{2}} \text{ North-East}$$

58. Two perfectly elastic particles A and B of equal masses travelling along the line joining them with velocity 15 m/s and 10 m/s respectively, collide. Their velocities after the elastic collision will be (in m/s), respectively

(1) 0, 25 (2) 3, 20 (3) 10, 15 (4) 20, 5

Sol. Answer (3)

Velocities will interchange as mass is same and collision is elastic

u_1 = 10 m/s , u_2 = 15 m/s

v_1 = 15 m/s , v_2 = 10 m/s

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