

## Chapter 7

## System of Particles and Rotational Motion

## Solutions

## SECTION - A

## Objective Type Questions

## (Centre of Mass)

1. The linear mass density( $\lambda$ ) of a rod of length  $L$  kept along  $x$ -axis varies as  $\lambda = \alpha + \beta x$ ; where  $\alpha$  and  $\beta$  are positive constants. The centre of mass of the rod is at

(1)  $\frac{(2\beta + 3\alpha)L}{2(2\beta + \alpha L)}$  (2)  $\frac{(3\alpha + 2\beta)L}{3(2\alpha + \beta L)}$  (3)  $\frac{(3\beta + 2\alpha)L}{3(2\beta + \alpha L)}$  (4)  $\frac{(3\beta + 2\alpha)L}{3\beta + 2\alpha}$

**Sol.** Answer (2)

$$\lambda = \alpha + \beta x$$

$$dm = (\alpha + \beta x)dx$$

$$x_{cm} = \frac{\int_0^L x(\alpha + \beta x) dx}{\int_0^L (\alpha + \beta x) dx} = \frac{\alpha \int_0^L x dx + \beta \int_0^L x^2 dx}{\alpha \int_0^L dx + \beta \int_0^L x dx}$$

$$x_{cm} = \frac{\frac{\alpha L^2}{2} + \frac{\beta L^3}{3}}{\alpha L + \frac{\beta L^2}{2}}$$

2. A man of mass 60 kg is standing on a boat of mass 140 kg, which is at rest in still water. The man is initially at 20 m from the shore. He starts walking on the boat for 4 s with constant speed 1.5 m/s towards the shore. The final distance of the man from the shore is

(1) 15.8 m (2) 4.2 m (3) 12.6 m (4) 14.1 m

**Sol.** Answer (1)

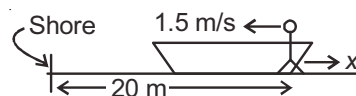
$$\begin{aligned} \text{Distance travelled by the man on boat in 4 second} &= (1.5) \times 4 \\ &= 6.0 \text{ m} \end{aligned}$$

$$140x = 60(6 - x)$$

$$140x = 360 - 60x$$

$$x = 1.8 \text{ m}$$

$$\text{So final distance of the man from the shore will be } 20 - (6 - 1.8) = 15.8 \text{ m}$$



3. A bomb of mass  $m$  is projected from the ground with speed  $v$  at angle  $\theta$  with the horizontal. At the maximum height from the ground it explodes into two fragments of equal mass. If one fragment comes to rest immediately after explosion, then the horizontal range of centre of mass is

(1)  $\frac{v^2 \sin^2 \theta}{g}$       (2)  $\frac{v^2 \sin \theta}{g}$       (3)  $\frac{v^2 \sin \theta}{2g}$       (4)  $\frac{v^2 \sin 2\theta}{g}$

**Sol.** Answer (4)

Path of the centre of mass will not change due to internal forces

$$R_{cm} = \frac{v^2 \sin 2\theta}{g}$$

4. Two blocks of masses 5 kg and 2 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 7 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(1) 30 m/s      (2) 20 m/s      (3) 10 m/s      (4) 5 m/s

**Sol.** Answer (4)

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{5(7) + 2(0)}{7} = 5 \text{ m/s}$$

### (Torque and Angular Momentum)

5. A particle starts from the point (0, 8) metre and moves with uniform velocity of  $\vec{v} = 3\hat{i}$  m/s. What is the angular momentum of the particle after 5 s about origin (mass of particle is 1 kg)?

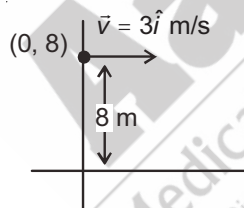
(1)  $-12\hat{k}$  kg m<sup>2</sup>/s      (2)  $-24\hat{k}$  kg m<sup>2</sup>/s      (3)  $-32\hat{k}$  kg m<sup>2</sup>/s      (4)  $-36\hat{k}$  kg m<sup>2</sup>/s

**Sol.** Answer (2)

$$L = mvr_{\perp}$$

$$= (1)(3)(8)$$

$$= 24(-\hat{k}) \text{ kgm}^2/\text{s}$$



6. A ball of mass 1 kg is projected with a velocity of  $20\sqrt{2}$  m/s from the origin of an xy co-ordinate axis system at an angle  $45^\circ$  with x-axis (horizontal). The angular momentum [In SI units] of the ball about the point of projection after 2 s of projection is [take  $g = 10 \text{ m/s}^2$ ] (y-axis is taken as vertical)

(1)  $-400 \hat{k}$       (2)  $200 \hat{i}$       (3)  $300 \hat{j}$       (4)  $-350 \hat{j}$

**Sol.** Answer (1)

$$\text{Time of flight } T = \frac{2u \sin \theta}{g} = \frac{2(20\sqrt{2}) \frac{1}{\sqrt{2}}}{10} = 4 \text{ second}$$

$\Rightarrow$  After 2 second particle will be at maximum height of the projectile

$$L = mvr_{\perp}$$

$$r_{\perp} = H_{\max} = \frac{u^2 \sin^2 \theta}{2g} = 20 \text{ m}$$

$$\text{So } L = (1) (20) (20) = 400 \text{ } (-\hat{k})$$

**(Moment of Inertia)**

7. The moment of inertia of a uniform semicircular wire of mass  $m$  and radius  $r$ , about an axis passing through its centre of mass and perpendicular to its plane is

(1)  $\frac{mr^2}{2}$

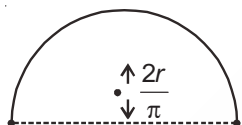
(2)  $mr^2$

(3)  $mr^2 \left( 1 - \frac{4}{\pi^2} \right)$

(4)  $mr^2 \left( 1 + \frac{4}{\pi^2} \right)$

**Sol.** Answer (3)

$$mr^2 = I_{cm} + m \left( \frac{2r}{\pi} \right)^2$$



$$I_{cm} = mr^2 \left[ 1 - \frac{4}{\pi^2} \right]$$

8. Moment of inertia of a uniform circular disc about its diameter is  $I$ . Its moment of inertia about an axis parallel to its plane and passing through a point on its rim will be

(1)  $3I$

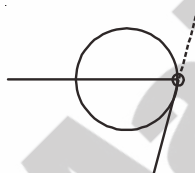
(2)  $4I$

(3)  $5I$

(4)  $6I$

**Sol.** Answer (3)

$$I = \frac{1}{4} mr^2$$



$$I' = \frac{1}{4} mr^2 + mr^2$$

$$I' = \frac{5}{4} mr^2$$

$$I' = 5I$$

9. Two discs of same mass and same thickness have densities as  $17 \text{ g/cm}^3$  and  $51 \text{ g/cm}^3$ . The ratio of their moment of inertia about their central axes is

(1)  $\frac{1}{3}$

(2)  $\frac{2}{3}$

(3)  $\frac{3}{1}$

(4)  $\frac{3}{2}$

**Sol.** Answer (3)

$$I = \frac{1}{2} V \rho r^2$$

$$I = \frac{1}{2} \pi r^2 t \rho r^2$$

$$I = \frac{\pi r^4 t \rho}{2}$$

$$\pi r_1^2 t \rho_1 = \pi r_2^2 t \rho_2$$

$$r_1^4 \rho_1^2 = r_2^4 \rho_2^2$$

$$\frac{r_1^4}{r_2^4} = \frac{\rho_2^2}{\rho_1^2}$$

$$\text{So, } \frac{I_1}{I_2} = \frac{\pi r_1^4 t \rho_1}{\pi r_2^4 t \rho_2} = \frac{\rho_2^2}{\rho_1^2} \cdot \frac{\rho_1}{\rho_2} = \frac{\rho_2}{\rho_1} = 3$$

10. A thin wire of length  $l$  and mass  $m$  is bent in the form of a semicircle. The moment of inertia about an axis perpendicular to its plane and passing through the end of the wire is

(1)  $\frac{ml^2}{2}$

(2)  $2mR^2$

(3)  $\frac{ml^2}{\pi^2}$

(4)  $\frac{2ml^2}{\pi^2}$

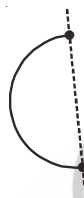
**Sol.** Answer (4)

$$\ell = \pi r$$

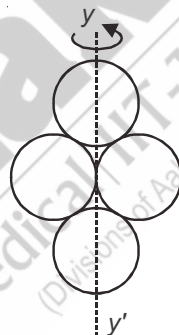
$$r = \frac{l}{\pi}$$

$$I = 2m \left( \frac{l}{\pi} \right)^2$$

$$= \frac{2ml^2}{\pi^2}$$



11. Four rings each of mass  $M$  and radius  $R$  are arranged as shown in the figure. The moment of inertia of the system about the axis  $yy'$  is



(1)  $2MR^2$

(2)  $3MR^2$

(3)  $4MR^2$

(4)  $5MR^2$

**Sol.** Answer (3)

$$\text{For upper and lower rings, } I_1 = \frac{MR^2}{2}$$

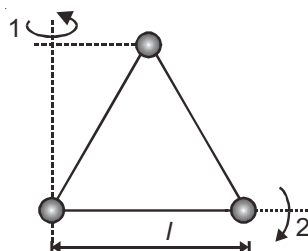
For middle rings, using parallel axis theorem

$$I_2 = \frac{MR^2}{2} + MR^2$$

$$= \frac{3}{2}MR^2$$

$$I = 2I_1 + 2I_2 = MR^2 + 3MR^2 = 4MR^2$$

12. Three particles each of mass  $m$  are placed at the corners of equilateral triangle of side  $l$



Which of the following is /are correct?

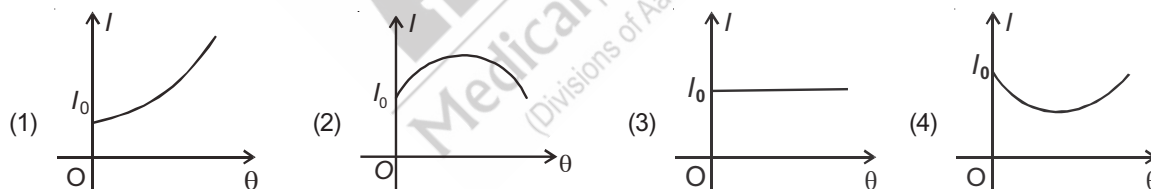
- (1) Moment of inertia about axis '1' is  $\frac{5}{4}ml^2$
- (2) Moment of inertia about axis '2' is  $\frac{3}{4}ml^2$
- (3) Moment of inertia about an axis passing through one corner and perpendicular to the plane is  $2ml^2$
- (4) All of these

**Sol.** Answer (4)

$$I_1 = ml^2 + m\left(\frac{l}{2}\right)^2 \quad I_2 = m\left(\frac{l\sqrt{3}}{2}\right)^2$$

$$I_1 = \frac{5ml^2}{4}, \quad I_2 = \frac{3ml^2}{4}, \quad I_3 = ml^2 + ml^2 = 2ml^2$$

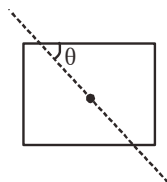
13. A square plate has a moment of inertia  $I_0$  about an axis lying in its plane, passing through its centre and making an angle  $\theta$  with one of the sides. Which graph represents the variation of  $I$  with  $\theta$ ?



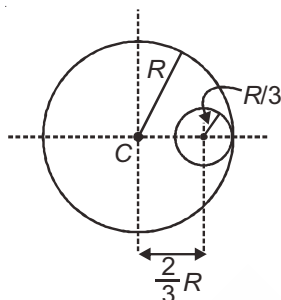
**Sol.** Answer (3)

Moment of inertia of a square lamina about any axis lying the plane of lamina and passing through the centre is

$$\frac{1}{12}ml^2.$$



14. From a uniform disc of radius  $R$  and mass  $9M$ , a small disc of radius  $\frac{R}{3}$  is removed as shown. What is the moment of inertia of remaining disc about an axis passing through the centre of disc and perpendicular to its plane?



- (1)  $\frac{32}{9}MR^2$  (2)  $10MR^2$  (3)  $\frac{40}{9}MR^2$  (4)  $4MR^2$

**Sol.** Answer (4)

$$I_1 = \frac{1}{2}(9M)(R^2) = \frac{9MR^2}{2}$$

Moment of inertia of removed disc about C

$$I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2 \quad (\text{mass of removed disc is } M)$$

$$= \frac{MR^2}{18} + \frac{4MR^2}{9} = \frac{9MR^2}{18} = \frac{MR^2}{2}$$

$$I = I_1 - I_2 = 4MR^2$$

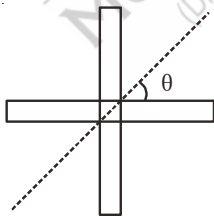
15. Two rods of equal lengths( $l$ ) and equal mass  $M$  are kept along  $x$  and  $y$  axis respectively such that their centre of mass lie at origin. The moment of inertia about an line  $y = x$ , is

- (1)  $\frac{ml^2}{3}$  (2)  $\frac{ml^2}{4}$  (3)  $\frac{ml^2}{12}$  (4)  $\frac{ml^2}{6}$

**Sol.** Answer (3)

$$I_{\text{Total}} = 2\left(\frac{ml^2}{12}\sin^2 45^\circ\right)$$

$$\frac{2ml^2}{12} \cdot \frac{1}{2} = \frac{ml^2}{12}$$



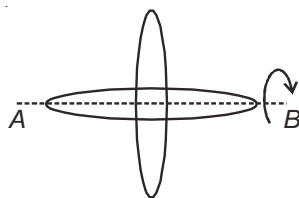
16. Two rings of same mass and radius  $R$  are placed with their planes perpendicular to each other and centres at a common point. The radius of gyration of the system about an axis passing through the centre and perpendicular to the plane of one ring is

- (1)  $2R$  (2)  $\frac{R}{\sqrt{2}}$  (3)  $\sqrt{\frac{3}{2}}R$  (4)  $\frac{\sqrt{3}R}{2}$

**Sol.** Answer (4)

$$I_{AB} = mr^2 + \frac{mr^2}{2}$$

$$= \frac{3mr^2}{2}$$



$$\frac{3mr^2}{2} = 2mk^2$$

$$k = \frac{\sqrt{3}}{2}R$$

17. A thin uniform wire of mass  $m$  and length  $l$  is bent into a circle. The moment of inertia of the wire about an axis passing through its one end and perpendicular to the plane of the circle is

(1)  $\frac{2mL^2}{\pi^2}$

(2)  $\frac{mL^2}{\pi^2}$

(3)  $\frac{mL^2}{2\pi^2}$

(4)  $\frac{mL^2}{3\pi^2}$

**Sol.** Answer (3)

$$2\pi r = L$$

$$r = \frac{L}{2\pi}, \quad I = 2mr^2 = 2m\left(\frac{L}{2\pi}\right)^2$$

18. The angular velocity of a body changes from  $\omega_1$  to  $\omega_2$  without applying a torque but by changing the moment of inertia about its axis of rotation. The ratio of its corresponding radii of gyration is

(1)  $\omega_1 : \omega_2$

(2)  $\sqrt{\omega_1} : \sqrt{\omega_2}$

(3)  $\omega_2 : \omega_1$

(4)  $\sqrt{\omega_2} : \sqrt{\omega_1}$

**Sol.** Answer (4)

Using angular momentum conservation,  $I_1\omega_1 = I_2\omega_2$

$$\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1}$$

$$\frac{mk_1^2}{mk_2^2} = \frac{\omega_2}{\omega_1}$$

$$\frac{k_1}{k_2} = \sqrt{\frac{\omega_2}{\omega_1}}$$

### (Theorems of Perpendicular Axes and Parallel Axes)

19. Four thin uniform rods each of length  $L$  and mass  $m$  are joined to form a square. The moment of inertia of square about an axis along its one diagonal is

(1)  $\frac{mL^2}{6}$

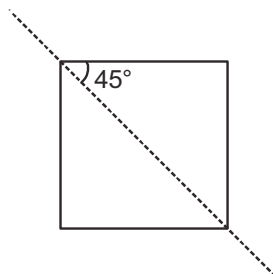
(2)  $\frac{2}{3}mL^2$

(3)  $\frac{3mL^2}{4}$

(4)  $\frac{4mL^2}{3}$

**Sol.** Answer (2)

$$\begin{aligned}
 I &= 4 \left( \frac{mL^2 \sin^2 45^\circ}{3} \right) \\
 &= \frac{4mL^2}{6} \\
 &= \frac{2}{3} mL^2
 \end{aligned}$$

**(Dynamics of Rotational Motion about a Fixed Axis)**

20. A hot solid sphere is rotating about a diameter at an angular velocity  $\omega_0$ . If it cools so that its radius reduces to  $\frac{1}{\eta}$  of its original value, its angular velocity becomes

- (1)  $\eta\omega_0$                       (2)  $\frac{\omega_0}{\eta}$                       (3)  $\frac{\omega_0}{\eta^2}$                       (4)  $\eta^2\omega_0$

**Sol.** Answer (4)

Angular momentum will be conserved

$$\frac{2}{5}mr^2\omega_0 = \frac{2}{5}m\left(\frac{r}{\eta}\right)^2\omega'$$

$$\omega' = \eta^2\omega_0$$

21. A thin rod of mass  $m$  and length  $l$  is suspended from one of its ends. It is set into oscillation about a horizontal axis. Its angular speed is  $\omega$  while passing through its mean position. How high will its centre of mass rise from its lowest position?

- (1)  $\frac{\omega^2 l^2}{2g}$                       (2)  $\frac{\omega^2 l^2}{3g}$                       (3)  $\frac{\omega^2 l^2}{g}$                       (4)  $\frac{\omega^2 l^2}{6g}$

**Sol.** Answer (4)

$$\frac{1}{2}ml^2\omega^2 = mgh \quad (\text{Energy conservation})$$

$$h = \frac{l^2\omega^2}{6g}$$

22. A solid body rotates about a fixed axis such that its angular velocity depends on  $\theta$  as  $\omega = k\theta^{-1}$  where  $k$  is a positive constants. At  $t = 0$ ,  $\theta = 0$ , then time dependence of  $\theta$  is given as

- (1)  $\theta = kt$                       (2)  $\theta = 2kt$                       (3)  $\theta = \sqrt{kt}$                       (4)  $\theta = \sqrt{2kt}$

**Sol.** Answer (4)

$$\omega = \frac{k}{\theta}$$

$$\frac{d\theta}{dt} = \frac{k}{\theta}$$

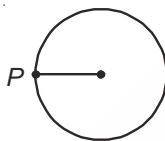


$$\int \theta d\theta = k \int dt$$

$$\frac{\theta^2}{2} = kt$$

$$\theta = \sqrt{2kt}$$

23. A uniform disc of mass  $m$  and radius  $R$  is pivoted at point  $P$  and is free to rotate in vertical plane. The centre  $C$  of disc is initially in horizontal position with  $P$  as shown in figure. If it is released from this position, then its angular acceleration when the line  $PC$  is inclined to the horizontal at an angle  $\theta$  is



(1)  $\frac{2g \cos \theta}{3R}$

(2)  $\frac{g \sin \theta}{2R}$

(3)  $\frac{2g \sin \theta}{R}$

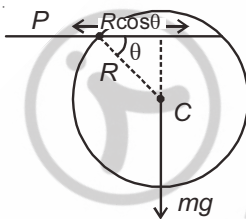
(4)  $\frac{2g \sin \theta}{3R}$

**Sol.** Answer (1)

$$\tau = I\alpha$$

$$mg(R \cos \theta) = \frac{3}{2}mr^2\alpha$$

$$\alpha = \frac{2g \cos \theta}{3r}$$



**(Angular Momentum in Case of Rotation about a Fixed Axis)**

24. A particle undergoes uniform circular motion. About which point in the plane of the circle, will the angular momentum of the particle remain conserved?

- (1) Centre of the circle
- (2) On the circumference of the circle
- (3) Inside the circle other than centre
- (4) Outside the circle

**Sol.** Answer (1)

External torque about centre will always be zero hence angular momentum of the particle will remain conserved.

25. When a planet moves around sun, then its

- (1) Angular velocity is constant
- (2) Areal velocity is constant
- (3) Linear velocity is constant
- (4) Linear momentum is conserved

**Sol.** Answer (2)

**(Rolling Motion)**

26. A force  $F$  is applied at the centre of a disc of mass  $M$ . The minimum value of coefficient of friction of the surface for rolling is

(1)  $\frac{F}{2Mg}$

(2)  $\frac{F}{3Mg}$

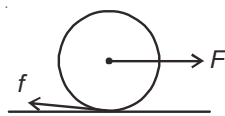
(3)  $\frac{2F}{5Mg}$

(4)  $\frac{2F}{7Mg}$

**Sol.** Answer (2)

$$F - f = Ma \quad \dots(1)$$

$$fr = \frac{1}{2}Mr^2 \frac{a}{r} \quad \dots(2)$$



Using (1) and (2),

$$F = \frac{3}{2}Ma \Rightarrow a = \frac{2F}{3M}$$

$$f = \frac{1}{2}M \cdot \frac{2F}{3M}$$

$$f = \frac{F}{3} = \mu Mg$$

$$\Rightarrow \mu = \frac{F}{3Mg}$$

27. When a rolling body enters onto a smooth horizontal surface, it will

- |                      |                                 |
|----------------------|---------------------------------|
| (1) Continue rolling | (2) Starts slipping             |
| (3) Come to rest     | (4) Slipping as well as rolling |

**Sol.** Answer (1)

Smooth surface won't be able to change  $w$  or  $v$  of the body. So to conserve its angular momentum it will continue to roll on the smooth surface.

28. A hollow sphere of mass  $m$  and radius  $R$  is rolling downward on a rough inclined plane of inclination  $\theta$ . If the coefficient of friction between the hollow sphere and incline is  $\mu$ , then

- |   |                                       |
|---|---------------------------------------|
| (1) Friction opposes its translation                    | (2) Friction supports rotation motion |
| (3) On decreasing $\theta$ , frictional force decreases | (4) All of these                      |

**Sol.** Answer (4)29. A heavy solid sphere is thrown on a horizontal rough surface with initial velocity  $u$  without rolling. What will be its speed, when it starts pure rolling motion?

- |                    |                    |                    |                    |
|--------------------|--------------------|--------------------|--------------------|
| (1) $\frac{3u}{5}$ | (2) $\frac{2u}{5}$ | (3) $\frac{5u}{7}$ | (4) $\frac{2u}{7}$ |
|--------------------|--------------------|--------------------|--------------------|

**Sol.** Answer (3)

Using angular momentum conservation,  $mur = mvr + \frac{2}{5}mr^2\left(\frac{v}{r}\right)$

$$u = 7\frac{v}{5}$$

$$v = \frac{5u}{7}$$

30. A cylinder rolls down two different inclined planes of the same height but of different inclinations

- (1) In both cases the speed and time of descent will be different
- (2) In both cases the speed and time of descent will be same
- (3) The speed will be different but time of descent will be same
- (4) The time of descent will be different but speed will be same

**Sol.** Answer (4)

31. A disc of mass 3 kg rolls down an inclined plane of height 5 m. The translational kinetic energy of the disc on reaching the bottom of the inclined plane is

- (1) 50 J
- (2) 100 J
- (3) 150 J
- (4) 175 J

**Sol.** Answer (2)

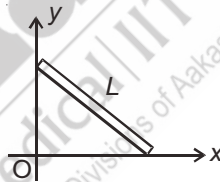
Using mechanical energy conservation,  $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$3(5)(10) = \frac{1}{2}mv^2 + \frac{1}{2}ml^2\left(\frac{v^2}{r^2}\right)$$

$$150 = \frac{3}{4}mv^2 \Rightarrow mv^2 = 200$$

$$\frac{1}{2}mv^2 = 100 \text{ J} = \text{K.E.}_{\text{Translation}}$$

32. A rod of length  $L$  leans against a smooth vertical wall while its other end is on a smooth floor. The end that leans against the wall moves uniformly vertically downward. Select the correct alternative



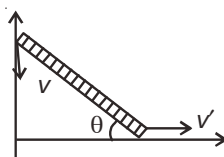
- (1) The speed of lower end increases at a constant rate
- (2) The speed of the lower end decreases but never becomes zero
- (3) The speed of the lower end gets smaller and smaller and vanishes when the upper end touches the ground
- (4) The speed of the lower end remain constant till upper end touches the ground

**Sol.** Answer (3)

Using constraint motion relation

$$v \cos \theta = v' \sin \theta$$

$$v' = v \tan \theta$$



As  $\theta$  keeps on decreasing,  $\tan \theta$  will also decrease and at last  $\theta$  will become zero and  $v' = 0$

33. What is the minimum coefficient of friction for a solid sphere to roll without slipping on an inclined plane of inclination  $\theta$ ?

(1)  $\frac{2}{7} \tan \theta$

(2)  $\frac{1}{3} g \tan \theta$

(3)  $\frac{1}{2} \tan \theta$

(4)  $\frac{2}{5} \tan \theta$

**Sol.** Answer (1)

$$\mu = \frac{I \tan \theta}{I + mr^2} = \frac{\frac{2}{5} mr^2 \tan \theta}{\frac{2}{5} mr^2 + mr^2} = \frac{2 \tan \theta}{7}$$

## SECTION - B

### Previous Years Questions

1. A disc of radius 2 m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s. How much work is needed to stop it?

[NEET-2019]

(1) 3 J

(2) 30 kJ

(3) 2 J

(4) 1 J

**Sol.** Answer (1)

Work required = change in kinetic energy

Final KE = 0

$$\text{Initial KE} = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2 = \frac{3}{4} mv^2$$

$$= \frac{3}{4} \times 100 \times (20 \times 10^{-2})^2 = 3 \text{ J}$$

$$|\Delta \text{KE}| = 3 \text{ J}$$

2. A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop after  $2\pi$  revolutions is

[NEET-2019]

(1)  $2 \times 10^{-6} \text{ N m}$

(2)  $2 \times 10^{-3} \text{ N m}$

(3)  $12 \times 10^{-4} \text{ N m}$

(4)  $2 \times 10^6 \text{ N m}$

**Sol.** Answer (1)

Work energy theorem.

$$W = \frac{1}{2} I (\omega_f^2 - \omega_i^2) \theta = 2\pi \text{ revolution}$$

$$= 2\pi \times 2\pi = 4\pi^2 \text{ rad}$$

$$W_i = 3 \times \frac{2\pi}{60} \text{ rad/s} \Rightarrow -\tau \theta = \frac{1}{2} \times \frac{1}{2} mr^2 (0^2 - \omega_i^2)$$

$$\Rightarrow -\tau = \frac{\frac{1}{2} \times \frac{1}{2} \times 2 \times (4 \times 10^{-2}) \left( -3 \times \frac{2\pi}{60} \right)^2}{4\pi^2}$$

$$\Rightarrow \tau = 2 \times 10^{-6} \text{ N m}$$

3. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere?

[NEET-2018]

- (1) Angular velocity (2) Moment of inertia  
(3) Angular momentum (4) Rotational kinetic energy

**Sol.** Answer (3)

$$\tau_{\text{ext}} = 0$$

$$\text{So, } \frac{dL}{dt} = 0 \text{ i.e., } L = \text{constant}$$

So angular momentum remains constant.

4. A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy ( $K_t$ ) as well as rotational kinetic energy ( $K_r$ ) simultaneously. The ratio  $K_t : (K_t + K_r)$  for the sphere is [NEET-2018]

- (1) 7 : 10 (2) 5 : 7 (3) 2 : 5 (4) 10 : 7

**Sol.** Answer (2)

$$K_t = \frac{1}{2}mv^2$$

$$K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2$$

$$\text{So, } \frac{K_t}{K_t + K_r} = \frac{5}{7}$$

5. Three objects, A : (a solid sphere), B : (a thin circular disk) and C : (a circular ring), each have the same mass  $M$  and radius  $R$ . They all spin with the same angular speed  $\omega$  about their own symmetry axes. The amounts of work ( $W$ ) required to bring them to rest, would satisfy the relation [NEET-2018]

- (1)  $W_C > W_B > W_A$  (2)  $W_A > W_B > W_C$   
(3)  $W_A > W_C > W_B$  (4)  $W_B > W_A > W_C$

**Sol.** Answer (1)

Work done required to bring them rest,  $\Delta W = \Delta KE$

$$\Delta W = \frac{1}{2}I\omega^2$$

$\Delta W \propto I$  for same  $\omega$

$$\begin{aligned} W_A : W_B : W_C &= \frac{2}{5}MR^2 : \frac{1}{2}MR^2 : MR^2 \\ &= \frac{2}{5} : \frac{1}{2} : 1 = 4 : 5 : 10 \end{aligned}$$

$$\Rightarrow W_C > W_B > W_A$$

6. The moment of the force,  $\vec{F} = 4\hat{i} + 5\hat{j} - 6\hat{k}$  at  $(2, 0, -3)$ , about the point  $(2, -2, -2)$ , is given by [NEET-2018]

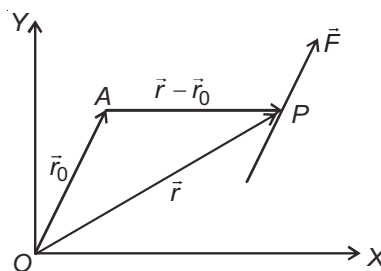
- (1)  $-8\hat{i} - 4\hat{j} - 7\hat{k}$  (2)  $-4\hat{i} - \hat{j} - 8\hat{k}$  (3)  $-7\hat{i} - 4\hat{j} - 8\hat{k}$  (4)  $-7\hat{i} - 8\hat{j} - 4\hat{k}$

**Sol.** Answer (3)

$$\vec{\tau} = (\vec{r} - \vec{r}_0) \times \vec{F} \quad \dots(i)$$

$$\vec{r} - \vec{r}_0 = (2\hat{i} + 0\hat{j} - 3\hat{k}) - (2\hat{i} - 2\hat{j} - 2\hat{k}) = 0\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & -1 \\ 4 & 5 & -6 \end{vmatrix} = -7\hat{i} - 4\hat{j} - 8\hat{k}$$



7. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? [NEET-2017]

- (1) 25 m/s<sup>2</sup>                      (2) 0.25 rad/s<sup>2</sup>                      (3) 25 rad/s<sup>2</sup>                      (4) 5 m/s<sup>2</sup>

**Sol.** Answer (3)

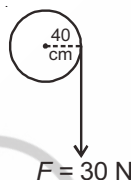
$$\tau = I \alpha$$

$$F \times R = MR^2 \alpha$$

$$30 \times 0.4 = 3 \times (0.4)^2 \alpha$$

$$12 = 3 \times 0.16 \alpha$$

$$400 = 16 \alpha \Rightarrow \alpha = 25 \text{ rad/s}^2$$



8. Which of the following statements are correct?

- (a) Centre of mass of a body always coincides with the centre of gravity of the body.  
 (b) Centre of mass of a body is the point at which the total gravitational torque on the body is zero  
 (c) A couple on a body produce both translational and rotational motion in a body.  
 (d) Mechanical advantage greater than one means that small effort can be used to lift a large load.

[NEET-2017]

- (1) (b) and (d)                      (2) (a) and (b)                      (3) (b) and (c)                      (4) (c) and (d)

**Sol.** Answer (1)

Centre of mass may or may not coincide with centre of gravity.

9. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities  $\omega_1$  and  $\omega_2$ . They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is [NEET-2017]

- (1)  $\frac{1}{2}I(\omega_1 + \omega_2)^2$                       (2)  $\frac{1}{4}I(\omega_1 - \omega_2)^2$                       (3)  $I(\omega_1 - \omega_2)^2$                       (4)  $\frac{1}{8}(\omega_1 - \omega_2)^2$

**Sol.** Answer (2)

$$\Delta KE = \frac{1}{2} \frac{I_1 I_2}{I_1 + I_2} (\omega_1 - \omega_2)^2$$

$$= \frac{1}{2} \frac{I^2}{(2I)} (\omega_1 - \omega_2)^2$$

$$= \frac{1}{4} I (\omega_1 - \omega_2)^2$$

10. Two rotating bodies  $A$  and  $B$  of masses  $m$  and  $2m$  with moments of inertia  $I_A$  and  $I_B$  ( $I_B > I_A$ ) have equal kinetic energy of rotation. If  $L_A$  and  $L_B$  be their angular momenta respectively, then [NEET (Phase-2) 2016]

- (1)  $L_A = \frac{L_B}{2}$  (2)  $L_A = 2L_B$  (3)  $L_B > L_A$  (4)  $L_A > L_B$

**Sol.** Answer (3)

$$E = \frac{L^2}{2I}$$

$$\Rightarrow E_A = E_B$$

$$\Rightarrow \frac{L_A^2}{2I_A} = \frac{L_B^2}{2I_B}$$

$$I_B > I_A$$

$$\Rightarrow L_B > L_A$$

11. A solid sphere of mass  $m$  and radius  $R$  is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation ( $E_{\text{sphere}} / E_{\text{cylinder}}$ ) will be [NEET (Phase-2) 2016]

- (1) 2 : 3 (2) 1 : 5 (3) 1 : 4 (4) 3 : 1

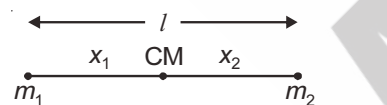
**Sol.** Answer (2)

$$\frac{E_{\text{sphere}}}{E_{\text{cylinder}}} = \frac{\frac{1}{2} \left( \frac{2}{5} m R^2 \right) \omega^2}{\frac{1}{2} \left( \frac{1}{2} m R^2 \right) (2\omega)^2} = \frac{1}{5}$$

12. A light rod of length  $l$  has two masses  $m_1$  and  $m_2$  attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is [NEET (Phase-2) 2016]

- (1)  $\frac{m_1 m_2}{m_1 + m_2} l^2$  (2)  $\frac{m_1 + m_2}{m_1 m_2} l^2$  (3)  $(m_1 + m_2) l^2$  (4)  $\sqrt{m_1 m_2} l^2$

**Sol.** Answer (1)



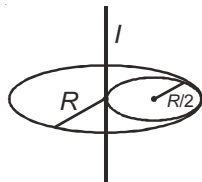
$$x_1 = \frac{m_2 l}{m_1 + m_2} \quad \text{and} \quad x_2 = \frac{m_1 l}{m_1 + m_2}$$

$$I = m_1 x_1^2 + m_2 x_2^2 = \frac{m_1 m_2}{m_1 + m_2} l^2$$

13. From a disc of radius  $R$  and mass  $M$ , a circular hole of diameter  $R$ , whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre? [NEET-2016]

- (1)  $\frac{9MR^2}{32}$  (2)  $\frac{15MR^2}{32}$  (3)  $\frac{13MR^2}{32}$  (4)  $\frac{11MR^2}{32}$

**Sol.** Answer (3)



$$\begin{aligned}
 I &= I_{\text{remain}} + I_{(R/2)} \Rightarrow I_{\text{remain}} = I - I_{(R/2)} \\
 &= \frac{MR^2}{2} - \left[ \frac{\frac{M}{4} (R/2)^2}{2} + \frac{M}{4} \left( \frac{R}{2} \right)^2 \right] = \frac{MR^2}{2} - \left[ \frac{MR^2}{32} + \frac{MR^2}{16} \right] \\
 &= \frac{MR^2}{2} - \left[ \frac{MR^2 + 2MR^2}{32} \right] \\
 &= \frac{MR^2}{2} - \frac{3MR^2}{32} = \frac{16MR^2 - 3MR^2}{32} = \frac{13MR^2}{32}
 \end{aligned}$$

14. A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first? **[NEET-2016]**

- (1) Depends on their masses (2) Disk  
(3) Sphere (4) Both reach at the same time

**Sol.** Answer (3)

$$a_{\text{sphere}} > a_{\text{disc}}$$

$$\text{Acceleration } (a) = \frac{g \sin \theta}{1 + K^2/r^2}, \text{ independent of mass and radius.}$$

15. An automobile moves on a road with a speed of  $54 \text{ km h}^{-1}$ . The radius of its wheels is  $0.45 \text{ m}$  and the moment of inertia of the wheel about its axis of rotation is  $3 \text{ kg m}^2$ . If the vehicle is brought to rest in  $15 \text{ s}$ , the magnitude of average torque transmitted by its brakes to the wheel is **[Re-AIPMT-2015]**

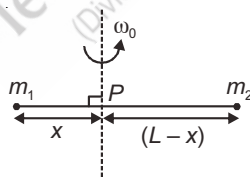
- (1)  $2.86 \text{ kg m}^2\text{s}^{-2}$  (2)  $6.66 \text{ kg m}^2\text{s}^{-2}$  (3)  $8.58 \text{ kg m}^2\text{s}^{-2}$  (4)  $10.86 \text{ kg m}^2\text{s}^{-2}$

**Sol.** Answer (2)

$$\tau = I\alpha = I \frac{a}{r} = I \frac{V}{tr} = \frac{3 \times 54 \times \frac{5}{18}}{15 \times 0.45}$$

$$\tau = 6.66 \text{ kg m}^2\text{s}^{-2}$$

16. Point masses  $m_1$  and  $m_2$  are placed at the opposite ends of a rigid rod of length  $L$ , and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point  $P$  on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity  $\omega_0$  is minimum, is given by **[Re-AIPMT-2015]**



- (1)  $x = \frac{m_2 L}{m_1 + m_2}$  (2)  $x = \frac{m_1 L}{m_1 + m_2}$  (3)  $x = \frac{m_1}{m_2} L$  (4)  $x = \frac{m_2}{m_1} L$

**Sol.** Answer (1)

Minimum work  $\Rightarrow$  Minimum rotational kinetic energy

$\Rightarrow$  Maximum angular momentum  $\Rightarrow$  Minimum moment of inertia

So, its rotation should be about CM

$$x = \frac{m_2 L}{m_1 + m_2}$$



17. A force  $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$  is acting at a point  $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$ . The value of  $\alpha$  for which angular momentum about origin is conserved is [Re-AIPMT-2015]

(1) 1 (2) -1 (3) 2 (4) Zero

**Sol.** Answer (2)

Angular momentum conserved  $\Rightarrow \tau = 0$

$$\Rightarrow \vec{r} \times \vec{F} = 0$$

$$\Rightarrow \frac{F_x}{x} = \frac{F_y}{y} = \frac{F_z}{z}$$

$$\Rightarrow \frac{\alpha}{2} = \frac{3}{-6} = \frac{6}{-12}$$

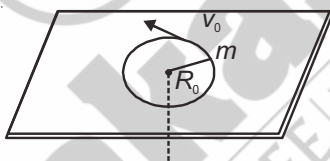
$$\Rightarrow \alpha = -1$$

18. A rod of weight  $W$  is supported by two parallel knife edges  $A$  and  $B$  and is in equilibrium in a horizontal position. The knives are at a distance  $d$  from each other. The centre of mass of the rod is at distance  $x$  from  $A$ . The normal reaction on  $A$  is [AIPMT-2015]

(1)  $\frac{W(d-x)}{d}$  (2)  $\frac{Wx}{d}$  (3)  $\frac{Wd}{x}$  (4)  $\frac{W(d-x)}{x}$

**Sol.** Answer (1)

19. A mass  $m$  moves in a circle on a smooth horizontal plane with velocity  $v_0$  at a radius  $R_0$ . The mass is attached to a string which passes through a smooth hole in the plane as shown.



The tension in the string is increased gradually and finally  $m$  moves in a circle of radius  $\frac{R_0}{2}$ . The final value of the kinetic energy is [AIPMT-2015]

(1)  $\frac{1}{2}mv_0^2$  (2)  $mv_0^2$  (3)  $\frac{1}{4}mv_0^2$  (4)  $2mv_0^2$

**Sol.** Answer (4)

20. Three identical spherical shells, each of mass  $m$  and radius  $r$  are placed as shown in figure. Consider an axis  $XX'$  which is touching to two shells and passing through diameter to third shell. Moment of inertia of the system consisting of these three spherical shells about  $XX'$  axis is [AIPMT-2015]



(1)  $4mr^2$  (2)  $\frac{11}{5}mr^2$  (3)  $3mr^2$  (4)  $\frac{16}{5}mr^2$

**Sol.** Answer (1)

21. Two spherical bodies of mass  $M$  and  $5M$  and radii  $R$  and  $2R$  are released in free space with initial separation between their centres equal to  $12R$ . If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is [AIPMT-2015]

(1)  $1.5R$  (2)  $2.5R$  (3)  $4.5R$  (4)  $7.5R$

**Sol.** Answer (4)

22. A solid cylinder of mass  $50\text{ kg}$  and radius  $0.5\text{ m}$  is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of  $2\text{ rev/s}^2$  is [AIPMT-2014]

(1)  $25\text{ N}$  (2)  $50\text{ N}$  (3)  $78.5\text{ N}$  (4)  $157\text{ N}$

**Sol.** Answer (4)

Use  $\tau = I\alpha$

$$\text{T.R.} = \frac{mR^2}{2} \cdot \alpha$$

23. The ratio of the accelerations for a solid sphere (mass  $m$  and radius  $R$ ) rolling down an incline of angle  $\theta$  without slipping and slipping down the incline without rolling is [AIPMT-2014]

(1)  $5 : 7$  (2)  $2 : 3$  (3)  $2 : 5$  (4)  $7 : 5$

**Sol.** Answer (1)

$$\frac{a_1}{a_2} = \frac{\frac{g \sin \theta}{\left(\frac{2}{5} + 1\right)}}{g \sin \theta} = 5 : 7$$

24. A small object of uniform density rolls up a curved surface with an initial velocity  $v$ . It reaches upto a maximum height of  $\frac{3v^2}{4g}$  with respect to the initial position. The object is: [NEET-2013]

(1) Solid sphere (2) Hollow sphere (3) Disc (4) Ring

**Sol.** Answer (3)

$$U_i + K_i = U_f + K_f \Rightarrow 0 + \left( \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 \right) = mg\left(\frac{3v^2}{4g}\right) + 0$$

Solving this, we get  $I = \frac{Mr^2}{2} \Rightarrow$  body is disc

25. A rod  $PQ$  of mass  $M$  and length  $L$  is hinged at end  $P$ . The rod is kept horizontal by a massless string tied to point  $Q$  as shown in figure. When string is cut, the initial angular acceleration of the rod is: [NEET-2013]

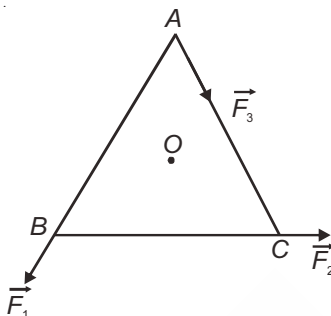


(1)  $\frac{g}{L}$  (2)  $\frac{2g}{L}$  (3)  $\frac{2g}{3L}$  (4)  $\frac{3g}{2L}$

**Sol.** Answer (4)

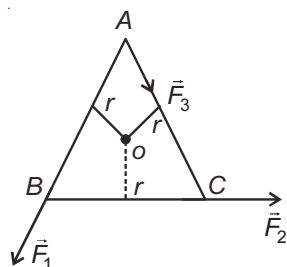
26.  $ABC$  is an equilateral triangle with  $O$  as its centre  $\vec{F}_1, \vec{F}_2$  and  $\vec{F}_3$  represent three forces acting along the sides  $AB, BC$  and  $AC$  respectively. If the total torque about  $O$  is zero then the magnitude of  $\vec{F}_3$  is

[AIPMT (Prelims)-2012]



- (1)  $\frac{F_1 + F_2}{2}$       (2)  $2(F_1 + F_2)$       (3)  $F_1 + F_2$       (4)  $F_1 - F_2$

Sol. Answer (3)



$$F_1 r + F_2 r = F_3 r \Rightarrow F_3 = F_1 + F_2$$

27. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along

[AIPMT (Prelims)-2012]

- (1) The radius  
(2) The tangent to the orbit  
(3) A line perpendicular to the plane of rotation  
(4) The line making an angle of  $45^\circ$  to the plane of rotation

Sol. Answer (3)

28. Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3.0 m and weighs 100 kg. The 55 kg man walks up to the 65 kg man and sits with him. If the boat is in still water the center of mass of the system shifts by

[AIPMT (Prelims)-2012]

- (1) Zero      (2) 0.75 m      (3) 3.0 m      (4) 2.3 m

Sol. Answer (1)

Net external force on the man and boat is zero and centre of mass is initially at rest. So centre of mass will not move.

29. A circular platform is mounted on a frictionless vertical axle. Its radius  $R = 2$  m and its moment of inertia about the axle is  $200 \text{ kg m}^2$ . It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of  $1 \text{ ms}^{-1}$  relative to the ground. Time taken by the man to complete one revolution is

[AIPMT (Mains)-2012]

- (1)  $\pi \text{ s}$       (2)  $\frac{3\pi}{2} \text{ s}$       (3)  $2\pi \text{ s}$       (4)  $\frac{\pi}{2} \text{ s}$

**Sol. Answer (3)**

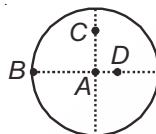
$$0 = (50)(1)(2) - 200\omega$$

$$\omega = \frac{1}{2} \text{ rad/s}$$

$$v_{rel} = 1 + 2\left(\frac{1}{2}\right) = 2$$

$$T = \frac{(2\pi)(2)}{2} = 2\pi \text{ s}$$

30. The moment of inertia of uniform circular disc is maximum about an axis perpendicular to the disc and passing through **[AIPMT (Mains)-2012]**



- (1) B (2) C (3) D (4) A

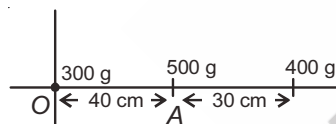
**Sol. Answer (1)**

$$I_{new} = I_{cm} + md^2 \text{ (parallel axis theorem)}$$

$I_{cm}$  is same for all points but  $d$  is maximum for B

31. Three masses are placed on the x-axis: 300 g at origin, 500 g at  $x = 40$  cm and 400 g at  $x = 70$  cm. The distance of the centre of mass from the origin is **[AIPMT (Mains)-2012]**

- (1) 40 cm (2) 45 cm (3) 50 cm (4) 30 cm

**Sol. Answer (1)**

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{300(0) + 500(40) + 400(70)}{1200} = 40 \text{ cm}$$

32. A mass  $m$  moving horizontally (along the x-axis) with velocity  $v$  collides and sticks to a mass of  $3m$  moving vertically upward (along the y-axis) with velocity  $2v$ . The final velocity of the combination is **[AIPMT (Mains)-2012]**

- (1)  $\frac{2}{3}v\hat{i} + \frac{1}{3}v\hat{j}$  (2)  $\frac{3}{2}v\hat{i} + \frac{1}{4}v\hat{j}$  (3)  $\frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}$  (4)  $\frac{1}{3}v\hat{i} + \frac{2}{3}v\hat{j}$

**Sol. Answer (3)**

33. The moment of inertia of a thin uniform rod of mass  $M$  and length  $L$  about an axis passing through its midpoint and perpendicular to its length is  $I_0$ . Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is **[AIPMT (Prelims)-2011]**

- (1)  $I_0 + ML^2$  (2)  $I_0 + \frac{ML^2}{2}$  (3)  $I_0 + \frac{ML^2}{4}$  (4)  $I_0 + 2ML^2$

**Sol. Answer (3)**

$$I_0 = \frac{ML^2}{12}$$

$$I = I_0 + M\left(\frac{L}{2}\right)^2 = I_0 + \frac{ML^2}{4}$$

34. The instantaneous angular position of a point on a rotating wheel is given by the equation,  $\theta(t) = 2t^3 - 6t^2$ . The torque on the wheel becomes zero at **[AIPMT (Prelims)-2011]**

- (1)  $t = 2$  s (2)  $t = 1$  s  
(3)  $t = 0.5$  s (4)  $t = 0.25$  s

**Sol. Answer (2)**

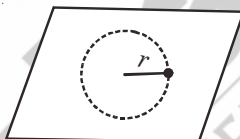
$$\theta = 2t^3 - 6t^2$$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 12t$$

$$\alpha = \frac{d\omega}{dt} = 12t - 12$$

$$\alpha = 0 \Rightarrow 12t - 12 = 0 \Rightarrow t = 1 \text{ s}$$

35. A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will **[AIPMT (Mains)-2011]**



- (1) Increase by a factor of 4 (2) Decrease by a factor of 2  
(3) Remain constant (4) Increase by a factor of 2

**Sol. Answer (1)**

Use angular momentum conservation,  $m_1 v_1 r_1 = m_2 v_2 r_2$

36. A circular disk of moment of inertia  $I_t$  is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed  $\omega_i$ . Another disk of moment of inertia  $I_b$  is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed  $\omega_f$ . The energy lost by the initially rotating disc to friction is: **[AIPMT (Prelims)-2010]**

- (1)  $\frac{1}{2} \frac{I_b^2}{(I_t + I_b)} \omega_i^2$  (2)  $\frac{1}{2} \frac{I_t^2}{(I_t + I_b)} \omega_i^2$  (3)  $\frac{I_b - I_t}{(I_t + I_b)} \omega_i^2$  (4)  $\frac{1}{2} \frac{I_b \cdot I_t}{(I_t + I_b)} \omega_i^2$

**Sol. Answer (4)**

37. Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are  $v$  and  $2v$  at any instant, then the speed of centre of mass of the system will be: **[AIPMT (Prelims)-2010]**

- (1)  $2v$  (2) Zero (3)  $1.5v$  (4)  $v$

**Sol. Answer (2)**

$$\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{cm} = 0$$

$$\Rightarrow (v_{cm})_i = (v_{cm})_f = 0$$

38. A ball moving with velocity 2 m/s collides head on with another stationary ball of double the mass. If the coefficient of restitution is 0.5 then their velocities (in m/s) after collision will be **[AIPMT (Prelims)-2010]**

- (1) 0, 2 (2) 0, 1  
(3) 1, 1 (4) 1, 0.5

**Sol. Answer (2)**

39. A man of 50 kg mass is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0.5 kg mass downwards with a speed 2 m/s. When the stone reaches the floor, the distance of the man above the floor will be **[AIPMT (Prelims)-2010]**

- (1) 20 m (2) 9.9 m (3) 10.1 m (4) 10 m

**Sol. Answer (3)**

40. From a circular disc of radius  $R$  and mass  $9M$ , a small disc of mass  $M$  and radius  $\frac{R}{3}$  is removed concentrically. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is **[AIPMT (Mains)-2010]**

- (1)  $\frac{40}{9} MR^2$  (2)  $MR^2$  (3)  $4MR^2$  (4)  $\frac{4}{9} MR^2$

**Sol. Answer (1)**

$$I_1 = \frac{1}{2}(9M)R^2 = \frac{9MR^2}{2}$$

$$I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{MR^2}{18}$$

$$I = I_1 - I_2 = \frac{9MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$$

41. A solid cylinder and a hollow cylinder, both of the same mass and same external diameter are released from the same height at the same time on a inclined plane. Both roll down without slipping. Which one will reach the bottom first? **[AIPMT (Mains)-2010]**

- (1) Both together only when angle of inclination of plane is  $45^\circ$   
(2) Both together  
(3) Hollow cylinder  
(4) Solid cylinder

**Sol. Answer (4)**

$$a = r\alpha = \frac{r\tau}{I} \quad (\because \tau = I\alpha)$$

$$\text{So } a = \frac{mgr^2 \sin \theta}{I + mr^2}$$

If  $I$  is less,  $a$  is more,  $t$  is less

42. (a) Centre of gravity (C.G.) of a body is the point at which the weight of the body acts  
 (b) Centre of mass coincides with the centre of gravity if the earth is assumed to have infinitely large radius  
 (c) To evaluate the gravitational field intensity due to any body at an external point, the entire mass of the body can be considered to be concentrated at its C.G.  
 (d) The radius of gyration of any body rotating about an axis is the length of the perpendicular dropped from the C.G. of the body to the axis

Which one of the following pairs of statements is correct ?

[AIPMT (Mains)-2010]

- (1) (d) and (a)                      (2) (a) and (b)                      (3) (b) and (c)                      (4) (c) and (d)

**Sol.** Answer (2)

43. A thin circular ring of mass  $M$  and radius  $r$  is rotating about its axis with constant angular velocity  $\omega$ . Two objects each of mass  $m$  are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity given by

[AIPMT (Mains)-2010]

- (1)  $\frac{(M+2m)\omega}{2m}$                       (2)  $\frac{2M\omega}{M+2m}$   
 (3)  $\frac{(M+2m)\omega}{M}$                       (4)  $\frac{M\omega}{M+2m}$

**Sol.** Answer (4)

44. A thin circular ring of mass  $M$  and radius  $R$  is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity  $\omega$ . If two objects each of mass  $m$  be attached gently to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity:

[AIPMT (Prelims)-2009]

- (1)  $\frac{\omega M}{M+2m}$                       (2)  $\frac{\omega(M+2m)}{M}$                       (3)  $\frac{\omega M}{M+m}$                       (4)  $\frac{\omega(M-2m)}{M+2m}$

**Sol.** Answer (1)

Using angular momentum conservation,  $(Mr^2\omega) = (M+2m)r^2\omega'$

$$\omega' = \frac{M}{M+2m}\omega$$

45. An explosion blows a rock into three parts. Two parts go off at right angles to each other. These two are, 1 kg first part moving with a velocity of  $12 \text{ ms}^{-1}$  and 2 kg second part moving with a velocity of  $8 \text{ ms}^{-1}$ . If the third part flies off with a velocity of  $4 \text{ ms}^{-1}$ , its mass would be

[AIPMT (Prelims)-2009]

- (1) 7 kg                      (2) 17 kg                      (3) 3 kg                      (4) 5 kg

**Sol.** Answer (4)

46. If  $\vec{F}$  is the force acting on a particle having position vector  $\vec{r}$  and  $\vec{\tau}$  be the torque of this force about the origin, then:

[AIPMT (Prelims)-2009]

- (1)  $\vec{r} \cdot \vec{\tau} > 0$  and  $\vec{F} \cdot \vec{\tau} < 0$                       (2)  $\vec{r} \cdot \vec{\tau} = 0$  and  $\vec{F} \cdot \vec{\tau} = 0$   
 (3)  $\vec{r} \cdot \vec{\tau} = 0$  and  $\vec{F} \cdot \vec{\tau} \neq 0$                       (4)  $\vec{r} \cdot \vec{\tau} \neq 0$  and  $\vec{F} \cdot \vec{\tau} = 0$

**Sol.** Answer (2)

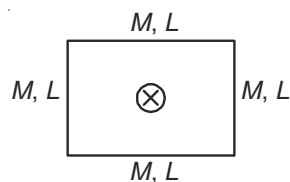
$\vec{\tau}$  will be perpendicular to  $\vec{F}$  and  $\vec{r}$  as  $\vec{\tau} = \vec{r} \times \vec{F}$



47. Four identical thin rods each of mass  $M$  and length  $\ell$ , form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is: [AIPMT (Prelims)-2009]

(1)  $\frac{2}{3} M\ell^2$  (2)  $\frac{13}{3} M\ell^2$  (3)  $\frac{1}{3} M\ell^2$  (4)  $\frac{4}{3} M\ell^2$

Sol. Answer (4)



$$I = I_{cm} + Md^2$$

$$I_{Total} = 4I = 4 \left[ \frac{ML^2}{12} + M \left( \frac{L}{2} \right)^2 \right] = \frac{4ML^2}{3}$$

48. Two bodies of mass 1 kg and 3 kg have position vectors  $\hat{i} + 2\hat{j} + \hat{k}$  and  $-3\hat{i} - 2\hat{j} + \hat{k}$ , respectively. The centre of mass of this system has a position vector: [AIPMT (Prelims)-2009]

(1)  $-2\hat{i} - \hat{j} + \hat{k}$  (2)  $2\hat{i} - \hat{j} - 2\hat{k}$  (3)  $-\hat{i} + \hat{j} + \hat{k}$  (4)  $-2\hat{i} + 2\hat{k}$

Sol. Answer (1)

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{(1\hat{i} + 2\hat{j} + \hat{k}) + (3(-3\hat{i} - 2\hat{j} + \hat{k}))}{1 + 3} = \frac{-8\hat{i} - 4\hat{j} + 4\hat{k}}{4} \Rightarrow \vec{r}_{cm} = -2\hat{i} - \hat{j} + \hat{k}$$

49. The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is [AIPMT (Prelims)-2008]

(1)  $\sqrt{2} : \sqrt{3}$  (2)  $\sqrt{3} : \sqrt{2}$  (3)  $1 : \sqrt{2}$  (4)  $\sqrt{2} : 1$

Sol. Answer (3)

$$\frac{M_1 K_1^2}{M_2 K_2^2} = \frac{M_1 r^2}{M_2 r^2}$$

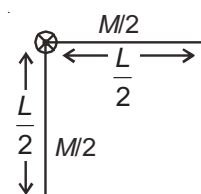
$$\text{Given } M_1 = M_2$$

$$\frac{K_1}{K_2} = \frac{1}{\sqrt{2}}$$

50. A thin rod of length  $L$  and mass  $M$  is bent at its midpoint into two halves so that the angle between them is  $90^\circ$ . The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is [AIPMT (Prelims)-2008]

(1)  $\frac{\sqrt{2}ML^2}{24}$  (2)  $\frac{ML^2}{24}$  (3)  $\frac{ML^2}{12}$  (4)  $\frac{ML^2}{6}$

Sol. Answer (3)



$$I = 2I = 2 \left[ \frac{M}{2} \left( \frac{L}{2} \right)^2 \right] = \frac{ML^2}{12}$$

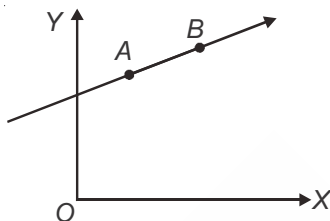


51. A wheel has angular acceleration of  $3 \text{ rad/sec}^2$  and an initial angular speed of  $2 \text{ rad/sec}$ . In a time of 2 sec it has rotated through an angle (in radian) of [AIPMT (Prelims)-2007]

(1) 4 (2) 6 (3) 10 (4) 12

**Sol.** Answer (3)

52. A particle of mass  $m$  moves in the  $XY$  plane with a velocity  $V$  along the straight line  $AB$ . If the angular momentum of the particle with respect to origin  $O$  is  $L_A$  when it is at  $A$  and  $L_B$  when it is at  $B$ , then [AIPMT (Prelims)-2007]

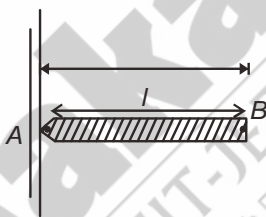


- (1)  $L_A < L_B$   
 (2)  $L_A > L_B$   
 (3)  $L_A = L_B$   
 (4) The relationship between  $L_A$  and  $L_B$  depends upon the slope of the line  $AB$

**Sol.** Answer (3)

Perpendicular distance from  $O$  of line  $AB$  will be constant. Hence angular momentum will be constant.

53. A uniform rod  $AB$  of length  $l$ , and mass  $m$  is free to rotate about point  $A$ . The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about  $A$  is  $\frac{ml^2}{3}$ , the initial angular acceleration of the rod will be [AIPMT (Prelims)-2007]



- (1)  $\frac{3g}{2l}$  (2)  $\frac{2g}{3l}$  (3)  $mg \frac{1}{2}$  (4)  $\frac{3}{2} gl$

**Sol.** Answer (1)

54. The moment of inertia of a uniform circular disc of radius  $R$  and mass  $M$  about an axis touching the disc at its diameter and normal to the disc is : [AIPMT (Prelims)-2006]

- (1)  $MR^2$  (2)  $\frac{2}{5} MR^2$  (3)  $\frac{3}{2} MR^2$  (4)  $\frac{1}{2} MR^2$

**Sol.** Answer (3)

55. A uniform rod of length  $l$  and mass  $m$  is free to rotate in a vertical plane about  $A$ . The rod initially in horizontal position is released. The initial angular acceleration of the rod is (Moment of inertia of rod about  $A$  is  $\frac{ml^2}{3}$ ) [AIPMT (Prelims)-2006]



- (1)  $\frac{3g}{2l}$  (2)  $\frac{2l}{3g}$  (3)  $\frac{3g}{2l^2}$  (4)  $mg \frac{l}{2}$

**Sol. Answer (1)**

$$\tau = I\alpha$$

$$\frac{mgl}{2} = \frac{ml^2}{3}\alpha$$

$$\alpha = \frac{3g}{2l}$$

56. A drum of radius  $R$  and mass  $M$ , rolls down without slipping along an inclined plane of angle  $\theta$ . The frictional force **[AIPMT (Prelims)-2005]**

- (1) Converts translational energy to rotational energy      (2) Dissipates energy as heat  
(3) Decreases the rotational motion      (4) Decreases the rotational and translational motion

**Sol. Answer (1)**

57. Two bodies have their moments of inertia  $I$  and  $2I$  respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio **[AIPMT (Prelims)-2005]**

- (1) 1 : 2      (2)  $\sqrt{2} : 1$       (3) 2 : 1      (4) 1 :  $\sqrt{2}$

**Sol. Answer (4)**

$$\frac{L_1}{L_2} = \frac{\sqrt{2IK}}{\sqrt{2(2I)K}} = \frac{1}{\sqrt{2}}$$

58. The moment of inertia of a uniform circular disc of radius  $R$  and mass  $M$  about an axis passing from the edge of the disc and normal to the disc is **[AIPMT (Prelims)-2005]**

- (1)  $\frac{1}{2}MR^2$       (2)  $MR^2$   
(3)  $\frac{7}{2}MR^2$       (4)  $\frac{3}{2}MR^2$

**Sol. Answer (4)**

$$I = I_{\text{cm}} + MR^2$$

$$= \frac{MR^2}{2} + MR^2$$

$$= \frac{3}{2}MR^2$$



59. A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity  $4 \text{ ms}^{-1}$ . It collides with a horizontal spring of force constant  $200 \text{ Nm}^{-1}$ . The maximum compression produced in the spring will be **[AIPMT (Prelims)-2012]**

- (1) 0.7 m      (2) 0.2 m  
(3) 0.5 m      (4) 0.6 m

**Sol. Answer (4)**

Use energy conservation

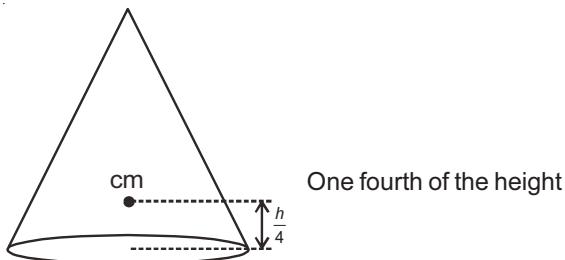
$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2 + 0$$

60 The centre of mass of a solid cone along the line from the center of the base to the vertex is at

- (1) One-fourth of the height (2) One-third of the height  
(3) One-fifth of the height (4) None of these

**Sol.** Answer (1)



61. The centre of mass of a system of particles does not depend on

- (1) Position of the particles  
(2) Relative distances between the particles  
(3) Masses of the particles  
(4) Forces acting on the particles

**Sol.** Answer (4)

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

So  $x_{cm}$  or  $y_{cm}$  does not depend upon force acting on the particles.

62. Consider a system of two particles having masses  $m_1$  and  $m_2$ . If the particle of mass  $m_1$  is pushed towards  $m_2$  through a distance  $d$ , by what distance should be particle of mass  $m_2$  be moved so as to keep the centre of mass of the system of particles at the original position?

- (1)  $\frac{m_1}{m_1 + m_2} d$  (2)  $\frac{m_1}{m_2} d$  (3)  $d$  (4)  $\frac{m_2}{m_1} d$

**Sol.** Answer (2)

$$m_1 d = m_2 x$$

$$x = \frac{m_1 d}{m_2}$$

63. Three identical metal balls, each of the radius  $r$  are placed touching each other on a horizontal surface such that an equilateral triangle is formed when centres of three balls are joined. The centre of the mass of the system is located at

- (1) Line joining centres of any two balls (2) Centre of one of the balls  
(3) Horizontal surface (4) Point of intersection of the medians

**Sol.** Answer (4)



Centre of mass will lie on the centroid of this triangle i.e., point of intersection of the medians.

64. A rod of length 3 m has its mass per unit length directly proportional to distance  $x$  from one of its ends then its centre of gravity from that end will be at

- (1) 1.5 m (2) 2 m (3) 2.5 m (4) 3.0 m

**Sol.** Answer (2)

$$dm = kx dx \quad \{ \text{as } \lambda = kx \}$$

$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int_0^3 x kx dx}{\int_0^3 kx dx} = \frac{\left[ \frac{kx^3}{3} \right]_0^3}{\left[ \frac{kx^2}{2} \right]_0^3} = \frac{2}{3}(3) = 2 \text{ m}$$

65. The ratio of radii of gyration of a circular ring and a circular disc, of the same mass and radius, about an axis passing through their centres and perpendicular to their planes are

- (1)  $\sqrt{2} : 1$  (2)  $1 : \sqrt{2}$  (3)  $3 : 2$  (4)  $2 : 1$

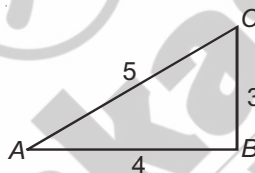
**Sol.** Answer (1)

$$\frac{M_1 K_1^2}{M_2 K_2^2} = \frac{I_1}{I_2} = \frac{M_1 r^2}{\left( \frac{M_1 r^2}{2} \right)}$$

$$M_1 = M_2 \quad (\text{given})$$

$$\text{So } \frac{K_1}{K_2} = \sqrt{2}$$

66. The ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure.  $I_{AB}$ ,  $I_{BC}$  and  $I_{CA}$  are the moments of inertia of the plate about AB, BC and CA respectively. Which one of the following relations is correct?



- (1)  $I_{AB} + I_{BC} = I_{CA}$  (2)  $I_{CA}$  is maximum (3)  $I_{AB} > I_{BC}$  (4)  $I_{BC} > I_{AB}$

**Sol.** Answer (4)

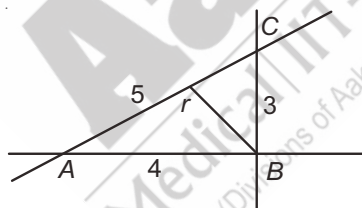
$$I_{AB} = m(3)^2$$

$$I_{BC} = m(4)^2$$

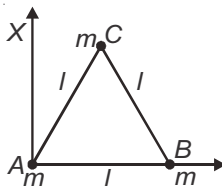
$$I_{CA} = mr^2$$

$$r < 4$$

$$\Rightarrow I_{BC} > I_{AB}$$



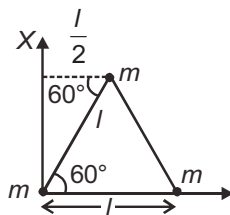
67. Three particles, each of mass  $m$  gram, are situated at the vertices of an equilateral triangle ABC of side  $l$  cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in  $\text{gcm}^2$  units will be



- (1)  $\frac{3}{4} ml^2$  (2)  $2ml^2$  (3)  $\frac{5}{4} ml^2$  (4)  $\frac{3}{4} ml^2$

**Sol. Answer (3)**

$$\begin{aligned}
 I &= I_1 + I_2 + I_3 \\
 &= 0 + m\left(\frac{l}{2}\right)^2 + ml^2 \\
 &= \frac{5ml^2}{4}
 \end{aligned}$$



68. A circular disc is to be made by using iron and aluminium so that it acquires maximum moment of inertia about geometrical axis. It is possible with

- (1) Aluminium at interior and iron surround to it
- (2) Iron at interior and aluminium surround to it
- (3) Using iron and aluminium layers in alternate order
- (4) Sheet of iron is used at both external surface and aluminium sheet as interna layers

**Sol. Answer (1)**

As density of iron is higher than Aluminium. So iron should be farther from the rotational axis.

69. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is

- (1) 2 : 3
- (2) 2 : 1
- (3)  $\sqrt{5} : \sqrt{6}$
- (4) 1 :  $\sqrt{2}$

**Sol. Answer (3)**

For disc, using parallel axis theorem first and then using perpendicular axis theorem

$$I_{disc} = \frac{5}{4}Mr^2$$

$$I_{ring} = \frac{3}{2}Mr^2$$

$$\frac{I_{disc}}{I_{ring}} = \frac{5(2)}{4(3)} = \frac{5}{6} = \frac{K_1^2}{K_2^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \sqrt{\frac{5}{6}}$$

70. The reduced mass of two particles having masses  $m$  and  $2m$  is

- (1)  $2m$
- (2)  $3m$
- (3)  $\frac{2m}{3}$
- (4)  $\frac{m}{2}$

**Sol. Answer (3)**

$$\text{Reduced mass } \mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{m(2m)}{m + 2m} = \frac{2m}{3}$$

71. What is the torque of the force  $\vec{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$  N acting at the point  $\vec{r} = 3\hat{i} + 2\hat{j} + 3\hat{k}$  m about origin?

- (1)  $-6\hat{i} + 6\hat{j} - 12\hat{k}$  (2)  $-17\hat{i} + 6\hat{j} + 13\hat{k}$  (3)  $6\hat{i} - 6\hat{j} + 12\hat{k}$  (4)  $17\hat{i} - 6\hat{j} - 13\hat{k}$

**Sol.** Answer (4)

$$\tau = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix}$$

$$\hat{i}(17) - \hat{j}(6) + \hat{k}(-13)$$

$$17\hat{i} - 6\hat{j} - 13\hat{k}$$

72. A couple produces

- (1) Linear and rotational motion (2) No motion  
(3) Purely linear motion (4) Purely rotational motion

**Sol.** Answer (4)

73. The angular speed of a fly-wheel making 120 revolutions/minute is

- (1)  $4\pi$  rad/s (2)  $4\pi^2$  rad/s (3)  $\pi$  rad/s (4)  $2\pi$  rad/s

**Sol.** Answer (1)

$$120 \text{ rev/min} = \frac{2\pi(120)}{60} = 4\pi \text{ rad/s}$$

74. Two discs are rotating about their axes, normal to the discs and passing through the centres of the discs. Disc  $D_1$  has 2 kg mass and 0.2 m radius and initial angular velocity of  $50 \text{ rad s}^{-1}$ . Disc  $D_2$  has 4kg mass, 0.1 m radius and initial angular velocity of  $200 \text{ rad s}^{-1}$ . The two discs are brought in contact face to face, with their axes of rotation coincident. The final angular velocity (in  $\text{rad.s}^{-1}$ ) of the system is

- (1) 40 (2) 60 (3) 100 (4) 120

**Sol.** Answer (3)

Using angular momentum conservation

$$I_1 \omega_1 + I_2 \omega_2 = (I_1 + I_2)\omega$$

$$\frac{1}{2}(2)(0.2)^2(50) + \frac{1}{2}4(0.1)^2(200)$$

$$= \left[ \frac{1}{2}(2)(0.2)^2 + \frac{1}{2}(4)(0.1)^2 \right] \omega$$

$$6 = \frac{6}{100} \omega$$

$$\omega = 100 \text{ rad/s}$$

75. A wheel having moment of inertia  $2 \text{ kgm}^2$  about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

- (1)  $\frac{2\pi}{15} \text{ Nm}$  (2)  $\frac{\pi}{12} \text{ Nm}$  (3)  $\frac{\pi}{15} \text{ Nm}$  (4)  $\frac{\pi}{18} \text{ Nm}$

**Sol. Answer (3)**

$$\alpha = \frac{0 - \frac{60 \times 2\pi}{60}}{60} = \frac{-2\pi}{60} \text{ rad/s}^2$$

$$\tau = I\alpha = (2) \left( \frac{\pi}{30} \right) = \frac{\pi}{15} \text{ Nm}$$

76. What is the value of linear velocity, if  $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$  and  $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$ ?

- (1)  $4\hat{i} - 13\hat{j} + 6\hat{k}$       (2)  $-18\hat{i} - 13\hat{j} + 2\hat{k}$       (3)  $6\hat{i} + 2\hat{j} + 3\hat{k}$       (4)  $6\hat{i} - 2\hat{j} + 8\hat{k}$

**Sol. Answer (2)**

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$$\hat{i}(-18) - \hat{j}(13) + \hat{k}(2)$$

$$= -18\hat{i} - 13\hat{j} + 2\hat{k}$$

77. If  $|\vec{A} \times \vec{B}| = \sqrt{3} \vec{A} \cdot \vec{B}$  then the value of  $|\vec{A} + \vec{B}|$  is

- (1)  $(A^2 + B^2 + AB)^{1/2}$       (2)  $\left( A^2 + B^2 + \frac{AB}{\sqrt{3}} \right)^{1/2}$       (3)  $A + B$       (4)  $(A^2 + B^2 \sqrt{3} + AB)^{1/2}$

**Sol. Answer (1)**

$$|\vec{A}| |\vec{B}| \sin \theta = \sqrt{3} |\vec{A}| |\vec{B}| \cos \theta$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 60^\circ$$

$$|\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A} \cdot \vec{B} = (A^2 + B^2 + AB)^{1/2}$$

78. If the angle between the vectors  $\vec{A}$  and  $\vec{B}$  is  $\theta$ , the value of the product  $(\vec{B} \times \vec{A}) \cdot \vec{A}$  is equal to

- (1)  $BA^2 \sin \theta$       (2)  $BA^2 \cos \theta$       (3)  $BA^2 \sin \theta \cos \theta$       (4) Zero

**Sol. Answer (4)**

$(\vec{B} \times \vec{A})$  and  $\vec{A}$  will be perpendicular to each other so cross product will be zero

79. A round disc of moment of inertia  $I_1$  about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia  $I_2$  rotating with an angular velocity  $\omega$  about the same axis. The final angular velocity of the combination of discs is

- (1)  $\frac{I_2 \omega}{I_1 + I_2}$       (2)  $\omega$   
 (3)  $\frac{I_1 \omega}{I_1 + I_2}$       (4)  $\frac{(I_1 + I_2) \omega}{I_1}$

**Sol. Answer (1)**Using angular momentum conservation,  $I_1(0) + I_2(\omega) = (I_1 + I_2)\omega'$ 

$$\omega' = \frac{I_2\omega}{I_1 + I_2}$$

80. A disc is rotating with angular speed  $\omega$ . If a child sits on it, what is conserved?

- (1) Linear momentum      (2) Angular momentum      (3) Kinetic energy      (4) Potential energy

**Sol. Answer (2)**81. A solid cylinder is rolling without slipping on a plane having inclination  $\theta$  and the coefficient of static friction  $\mu_s$ . The relation between  $\theta$  and  $\mu_s$  is

- (1)  $\tan \theta > 3\mu_s$       (2)  $\tan \theta \leq 3\mu_s$       (3)  $\tan \theta < 3\mu_s^2$       (4) None of these

**Sol. Answer (2)**

$$\mu_s \geq \frac{\frac{1}{2}mr^2 \tan \theta}{\frac{1}{2}mr^2 + mr^2}$$

$$\mu_s \geq \frac{\tan \theta}{3}$$

$$3\mu \geq \tan \theta$$

82. A solid spherical ball rolls on a table. Ratio of its rotational kinetic energy to total kinetic energy is

- (1)  $\frac{1}{2}$       (2)  $\frac{1}{6}$       (3)  $\frac{7}{10}$       (4)  $\frac{2}{7}$

**Sol. Answer (4)**

83. A hollow cylinder and a solid cylinder are rolling without slipping down an inclined plane, then which of these reaches earlier?

- (1) Solid cylinder      (2) Hollow cylinder  
(3) Both simultaneously      (4) Can't say anything

**Sol. Answer (1)**Body of smaller  $\frac{K^2}{R^2}$  will take less time. Solid cylinder has smaller  $\frac{K^2}{R^2}$ 84. A disc is rolling such that the velocity of its centre of mass is  $v_{cm}$ . Which one will be correct?

- (1) The velocity of highest point is  $2v_{cm}$  and point of contact is zero  
(2) The velocity of highest point is  $v_{cm}$  and point of contact is  $v_{cm}$   
(3) The velocity of highest point is  $2v_{cm}$  and point of contact is  $v_{cm}$   
(4) The velocity of highest point is  $2v_{cm}$  and point of contact is  $2v_{cm}$

**Sol. Answer (1)**



85. A solid sphere of radius  $R$  is placed on a smooth horizontal surface. A horizontal force  $F$  is applied at height  $h$  from the lowest point. For the maximum acceleration of centre of mass, which is correct?

- (1)  $h = R$
- (2)  $h = 2R$
- (3)  $h = 0$
- (4) Centre of mass has same acceleration in each case

**Sol.** Answer (4)

Acceleration of CM is independent of point of application of force.

86. A point  $P$  is the contact point of a wheel on ground which rolls on ground without slipping. The value of displacement of the point  $P$  when wheel completes half of rotation (If radius of wheel is 1 m)

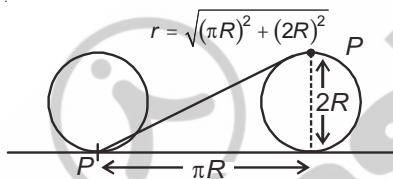
- (1) 2 m
- (2)  $\sqrt{\pi^2 + 4}$  m
- (3)  $\pi$  m
- (4)  $\sqrt{\pi^2 + 2}$  m

**Sol.** Answer (2)

Use pythagoras theorem

$$r = R\sqrt{\pi^2 + 4}$$

$$= \sqrt{\pi^2 + 4} \text{ m}$$



87. A solid cylinder of mass  $M$  and radius  $R$  rolls without slipping down an inclined plane of length  $L$  and height  $h$ . What is the speed of its centre of mass when the cylinder reaches its bottom?

- (1)  $\sqrt{2gh}$
- (2)  $\sqrt{\frac{3}{4}gh}$
- (3)  $\sqrt{\frac{4}{3}gh}$
- (4)  $\sqrt{4gh}$

**Sol.** Answer (3)

Using mechanical energy conservation

$$Mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2}$$

$$Mgh = \frac{3}{4}Mv^2$$

$$v = \sqrt{\frac{4gh}{3}}$$

88. A drum of radius  $R$  and mass  $M$ , rolls down without slipping along an inclined plane of angle  $\theta$ . The frictional force

- (1) Dissipates energy as heat
- (2) Decreases the rotational motion
- (3) Decreases the rotational and translational motion
- (4) Converts translational energy to rotational energy

**Sol.** Answer (4)

89. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is  $K$ . If radius of the ball be  $R$ , then the fraction of total energy associated with its rotational energy will be

(1)  $\frac{K^2 + R^2}{R^2}$       (2)  $\frac{K^2}{R^2}$       (3)  $\frac{K^2}{K^2 + R^2}$       (4)  $\frac{R^2}{K^2 + R^2}$

**Sol.** Answer (3)

$$K_{Rot} = \frac{1}{2} MK^2 \frac{v^2}{R^2}$$

$$K_{Total} = \frac{1}{2} MK^2 \frac{v^2}{R^2} + \frac{1}{2} Mv^2$$

$$\frac{K_{Rot}}{K_{Total}} = \frac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}} = \frac{K^2}{K^2 + R^2}$$

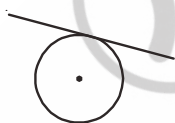
90. The moment of inertia of a disc of mass  $M$  and radius  $R$  about an axis, which is tangential to the circumference of the disc and parallel to its diameter is

(1)  $\frac{5}{4} MR^2$       (2)  $\frac{2}{3} MR^2$       (3)  $\frac{3}{2} MR^2$       (4)  $\frac{1}{2} MR^2$

**Sol.** Answer (1)

$$I = \frac{MR^2}{4} + MR^2$$

$$= \frac{5MR^2}{4}$$



## SECTION - C

### Assertion - Reason Type Questions

- A : Centre of mass of a system may or may not lie inside the system.  
 R : The position of centre of mass depends on distribution of mass within the system.

**Sol.** Answer (1)
- A : The position of centre of mass relative to body is independent of the choice of coordinate system.  
 R : Centre of mass does not shift its position in the absence of an external force.

**Sol.** Answer (3)
- A : A bomb at rest explodes. The centre of mass of fragments moves along parabolic path.  
 R : Under the effect of gravity only the path followed by centre of mass is always parabolic.

**Sol.** Answer (4)
- A : If an object is taken to the centre of earth, then its centre of gravity cannot be defined.  
 R : At the centre of earth acceleration due to gravity is zero.

**Sol.** Answer (1)
- A : It is very difficult to open or close a door if force is applied near the hinge.  
 R : The moment of applied force is minimum near the hinge.

**Sol.** Answer (1)

6. A : The moment of force is maximum for a point if force applied on it and its position vector w.r.t. the point of rotation are perpendicular.

R : The magnitude of torque is independent of the direction of application of force.

**Sol.** Answer (3)

7. A : If angular momentum of an object is constant about a point, then net torque on it about that point is zero.

R : Torque is equal to the rate of change of angular momentum.

**Sol.** Answer (1)

8. A : Two rings of equal mass and radius made of different materials, will have same moment of inertia.

R : Moment of inertia depends on mass as well as distribution of mass in the object.

**Sol.** Answer (1)

9. A : In pure rolling motion all the points of a rigid body have same linear velocity.

R : Rolling motion is not possible on smooth surface.

**Sol.** Answer (4)

10. A : For an object in rolling motion rotational kinetic energy is always equal to translational kinetic energy.

R : For an object in rolling motion magnitude of linear speed and angular speed are equal.

**Sol.** Answer (4)

11. A : The work done by friction force on an object during pure rolling motion is zero.

R : In pure rolling motion, there is relative motion at the point of contact.

**Sol.** Answer (3)

12. A : When a rigid body rotates about any fixed axis, then all the particles of it move in circles of different radii but with same angular velocity.

R : In rigid body relative position of particles are fixed.

**Sol.** Answer (1)

13. A : A rigid body can't be in a pure rolling on a rough inclined plane without giving any external force.

R : Since there is no torque providing force acting on the body in the above case, the body can't come in a rolling condition.

**Sol.** Answer (4)

14. A : When a ring moves in pure rolling condition on ground, it has 50% translational and 50% rotational energy.

$$R : \frac{KE_{\text{trans}}}{KE_{\text{rot}}} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}I\omega^2} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}(MR^2)\frac{V^2}{R^2}} = 1:1$$

**Sol.** Answer (1)

15. A : For a body to be in equilibrium the net torque acting on the body about any point and net force should be zero.

R : For net torque to be zero, net force should also be zero.

**Sol.** Answer (3)

