Chapter 4

Motion in a Plane

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. Which of the following is a scalar quantity?

   Momentum, Acceleration, Work, Force.

Sol. Work is a scalar quantity, others are vectors.

2. Name two vector quantities.

Sol. Displacement and velocity.

3. Can three vectors of different magnitudes be combined to give a zero resultant?

Sol. If three vectors form the sides of a triangle when taken in the same order, the resultant vector is a null vector.

4. If $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, what is the angle between $\vec{A}$ and $\vec{B}$?

Sol. Let the angle between $\vec{A}$ and $\vec{B}$ be θ.

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$|\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$\Rightarrow$ $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$\Rightarrow$ $4AB\cos\theta = 0$

$\theta = 90°$

5. What is the average value of acceleration of an object in uniform circular motion in one complete revolution?

Sol. $\vec{\bar{a}} = \dfrac{\Delta\vec{v}}{\Delta t}$

In one complete rotation $\Delta\vec{v} = \vec{v} - \vec{v} = 0$. On reaching the same point, the object has same velocity.

$\therefore$ $\vec{\bar{a}} = 0$

6. Electric current has both magnitude and direction, so is it a vector?

Sol. No, electric current does not obey vector laws of addition.

7. Can the sum of two vectors be a scalar?

Sol. No, sum of two vectors is always a vector.

8. What is the magnitude of $\hat{i}+\hat{j}$?

Sol. The vector is $\hat{i}+\hat{j}$

$\therefore$ Its magnitude = $\sqrt{1^2+1^2}$

= $\sqrt{2}$

9. What is the direction of $\vec{A}+\vec{B}$ for parallel vectors $\vec{A}$ and $\vec{B}$?

Sol. It is along either $\vec{A}$ or $\vec{B}$, both have same direction.

10. When can the sum of two vectors be minimum and maximum?

Sol. Minimum, when two vectors are in opposite directions.

Maximum, when two vectors are parallel *i.e.*, in same direction.

Short Answer Type Questions :

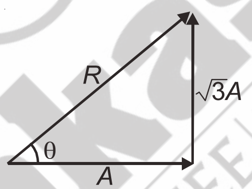
11. Two vectors having magnitudes *A* and $\sqrt{3}\,A$ are perpendicular to each other. What is the angle between their resultant and $\vec{A}$?

Sol. Let $\vec{R}$ be their resultant.

Then by law of cosines

$|\vec{R}|=\sqrt{A^2+(\sqrt{3}A)^2}$

$|\vec{R}|=2A$



Using law of sines, $\dfrac{|\vec{R}|}{\sin 90^\circ}=\dfrac{\sqrt{3}A}{\sin\theta}$

$\Rightarrow \quad \sin\theta=\dfrac{\sqrt{3}A}{2A}\times\sin 90^\circ$

$\sin\theta=\dfrac{\sqrt{3}}{2}$

$\theta = 60°$

12. Find the angle of the vector $\vec{A}=3\hat{i}+6\hat{j}-\hat{k}$ with *y*-axis.

Sol. Here $|\vec{A}|=\sqrt{3^3+6^2+(-1)^2}$

= $\sqrt{9+36+1}$

= $\sqrt{46}$

Let $\vec{A}$ make angle θ with y-axis, then

$|\vec{A}|\cos\theta=6$

$\Rightarrow \quad \cos\theta=\dfrac{6}{\sqrt{46}}=0.88465$

$\Rightarrow \quad \theta = \cos^{-1}(0.88465)$

$\theta = 27.79°$

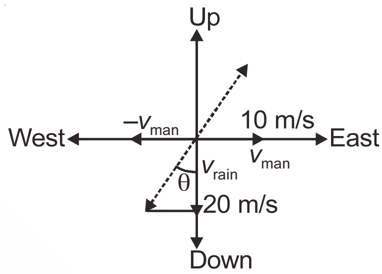
13. A boy sitting in a train moving with constant velocity, throws a ball vertically upwards. How does the ball appear to move to an observer, (i) inside the train, (ii) outside the train.

Sol. (i) To an observer sitting inside the train, the ball will appear to move straight vertically upwards and then downwards.

(ii) To an observer sitting outside the train, the ball will appear to move along the parabolic path.

14. Rain is falling vertically with speed 20 m/s. A man runs with speed of 10 m/s towards east. In which direction should he hold his umbrella?

Sol. Velocity of rain with respect to man



$\vec{v}_{rm} = \vec{v}_{rain} - \vec{v}_{man}$

$\Rightarrow \quad \tan\theta = \dfrac{10}{20}$

$= \dfrac{1}{2}$

Or θ = 26.56° inclined to vertical towards east direction.

15. Are the two vectors (2 kg) (4 m/s, towards east) and 2(4 m/s, towards east) same?

Sol. No, (2 kg) (4 m s^{-1}, towards east) shows the multiplication of a scalar quantity *i.e.*, mass of 2 kg with a velocity vector. So it is a momentum vector of magnitude 8 kg m/s directed towards east.

2 (4 m/s, towards east) is the velocity vector of magnitude 8 m/s. Hence, the two are not same.

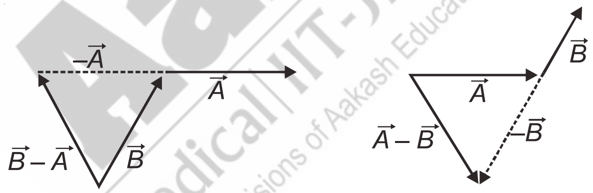
16. If $\vec{A}+\vec{B}=\vec{C}$ and $|\vec{C}|>|\vec{A}|$ and $|\vec{B}|$. Does that mean $|\vec{C}|>|\vec{A}|+|\vec{B}|$?

Sol. No, In above case, $\vec{A}, \vec{B}$ and $\vec{C}$ represent three sides of a triangle. And no side is ever greater than the sum of the other two.

Hence, $|\vec{C}|<|\vec{A}|+|\vec{B}|$

17. Show graphically that subtraction of two vectors is not commutative.

Sol. The two figures given show that $\vec{A}-\vec{B}$ and $\vec{B}-\vec{A}$ are two different vectors.



Hence, subtraction of vectors is not commutative.

18. A projectile is fired with kinetic energy 4 kJ. If its range is maximum, what is its K.E. at the highest point of its path?

Sol. Since the range is maximum, the angle of projection is 45°.

If velocity of projection = v

$\Rightarrow \quad 4\text{ kJ} = \dfrac{1}{2}mv^2$

Velocity at the highest point = v cos45° = $\dfrac{v}{\sqrt{2}}$

$\therefore$ K.E. at highest point = $\dfrac{1}{2}m\left(\dfrac{v}{\sqrt{2}}\right)^2$

$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad \frac{4\text{ kJ}}{2} = 2\text{ kJ}$

19. For a given velocity in projectile motion, name the quantities related to a projectile which have maximum values when the maximum height attained by the projectile is the largest.

Sol. (i) Vertical component of initial velocity

(ii) Angle of projection

(iii) Time of flight

20. Give a few examples of motion in two dimensions.

Sol. (i) Projectile motion

(ii) Uniform circular motion

(iii) Non-uniform circular motion : The object moving in a circle has different speeds at different points. For example, A stone tied with a thread whirled in a vertical plane.

21. Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.

Sol. $\therefore \quad H_{max} = R$

$$\Rightarrow \quad \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{v_0^2 \sin 2\theta_0}{g}$$

$$= \frac{\sin\theta_0}{2} = 2\cos\theta_0$$

$\Rightarrow \quad \tan\theta_0 = 4$ or $\theta_0 = 75.96°$

22. Why is centripetal acceleration called so? Is it a constant vector?

Sol. Centripetal acceleration always acts towards the centre of the circular path at every point. Hence, it is called centripetal acceleration which in Greek means “center seeking”. Since its direction changes continuously, it is not a constant vector.

23. What do you mean by resolution of a vector and components of a vector?

Sol. Splitting a vector into two or more component vectors is called resolution of the vector. A vector can be resolved into infinite number of components. However a vector can have only two rectangular components in a plane.

Components of vector : If vectors $\vec{A}, \vec{B}$ and $\vec{C}$, when added using vectors laws, result into vector $\vec{D}$, then they are said to be component vectors of $\vec{D}$ along directions $\vec{A}, \vec{B}$ and $\vec{C}$.

24. Find components of vector addition of $2\hat{i} + 4\hat{j} - \hat{k}$ and $3\hat{i} - 2\hat{j} + 2\hat{k}$.

Sol. Resultant $\quad \vec{R} = (2\hat{i} + 4\hat{j} - \hat{k}) + (3\hat{i} - 2\hat{j} + 2\hat{k})$

$$\vec{R} = (5\hat{i} + 2\hat{j} + \hat{k})$$

Hence, rectangular components of $\vec{R}$ are

$R_x = 5$ along x-axis

$R_y = 2$ along y-axis

$R_z = 1$ along z-axis

25. If the position of particle at time t is given by $\vec{r} = 2t^2\hat{i} + 6t\hat{j} + 8\hat{k}$. $\vec{r}$ has the unit m. Find the component of velocity along z-axis at time t = 4 s.

Sol. $\vec{r} = 2t^2\hat{i} + 6t\hat{j} + 8\hat{k}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + 6\hat{j} + 0\hat{k}$$

$$= 4t\hat{i} + 6\hat{j}$$

$\because$ v_z = 0, here. The particle does not have any velocity along z-axis.

26. An object is projected at an angle 30°. If its horizontal velocity is 50 km/h, what is its vertical velocity?

Sol. θ_0 = 30°

$v_0\cos\theta_0$ = 50 km/h

$v_0\sin\theta_0$ = ?

$$v_0 \sin\theta_0 = \frac{50}{\cos 30^\circ} \times \sin 30^\circ$$

$$= 50 \times \frac{1}{\sqrt{3}} \text{ km/h} = 28.86 \text{ km/h}$$

27. What is the relative velocity of a man swimming downstream with speed 12 km/h (in still water) with respect to a child running towards the river with speed 4 km/h in direction perpendicular to water flow. Speed of water flow = 4 km/h.

Sol. Velocity of man in still water = 12 km/h = $\vec{v}_m$

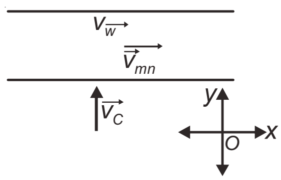
Velocity of water flow $\vec{v}_w = 4$ km/h

$\therefore$ Net velocity of man w.r.t. ground = $\vec{v}_{mn} = 16$ km/h $\hat{i}$

Velocity of child = $\vec{v}_C = 4$ km/h $\hat{j}$

$\therefore$ Velocity of man w.r.t. child

$$\vec{v}_{(mn)c} = 16\hat{i} - 4\hat{j}$$



$$|\vec{v}_{(mn)c}| = \sqrt{(16)^2 + (4)^2} = 4\sqrt{17} \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{-1}{4}\right)$$

= – 14.04° with x-axis (*i.e.*, direction of flow of river).

28. What do you mean by angular speed in uniform circular motion? How is it related to time period and centripetal acceleration?

Sol. **Angular speed (ω) :** The time rate of change of the angular displacement of an object having uniform circular motion is called angular speed (ω).

$$\omega = \frac{2\pi}{T} \text{ or } a_C = \omega^2 r$$

29. A cyclist going round a circular path with constant speed 10 km/h completes 42 revolutions in 30 minutes. Find its centripetal acceleration.

Sol. Frequency $\nu = \dfrac{42}{30\times 60} = \dfrac{7}{300}\ \text{s}^{-1}$ ...(i)

$\therefore$ Time period $T = \dfrac{1}{\nu} = \dfrac{300}{7}$ s

Also $T = \dfrac{2\pi R}{v}$

$\Rightarrow R = \dfrac{vT}{2\pi}$

Given that v = 10 km h^{-1}

$\Rightarrow R = \dfrac{10\times\dfrac{5}{18}\times\dfrac{300}{7}}{2\times\dfrac{22}{7}}\ \text{m} = \dfrac{625}{33}\ \text{m}$ ...(ii)

$a_C = 4\pi^2\nu^2 R$

$= 4\times\dfrac{22}{7}\times\dfrac{22}{7}\times\dfrac{7}{300}\times\dfrac{7}{300}\times\dfrac{625}{33}$ [using equation (i) and (ii)]

$= \dfrac{11}{27}\ \text{m/s}^2$

$= 0.41\ \text{m s}^{-2}$

30. Find time of flight of an object projected at angle 30° with speed 60 m/s. [take g = 10 m/s^2]

Sol. $T = \dfrac{2v_0\sin\theta_0}{g} = \dfrac{2\times 60\times 1}{10\times 2} = 6$ s

Long Answer Type Questions :

31. State and prove the law of cosines.

Sol. Law of cosines : The resultant $\vec{R}$ of two vectors $\vec{P}$ and $\vec{Q}$ inclined at an angle θ is given by

$|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$.

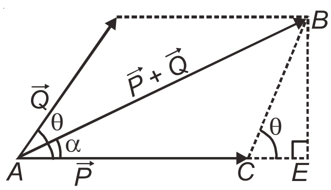
Let the two vectors $\vec{P}$ and $\vec{Q}$ are inclined at an angle θ with each other. Then, the resultant of their vector addition can be obtained by using parallelogram method as shown below.

Here, $\overrightarrow{AB} = \vec{P} + \vec{Q}$

From B, draw a line perpendicular to AC which meets it at E when extended.

Now in ΔABE

$AB^2 = AE^2 + BE^2$ [Pythagoras theorem]



$\Rightarrow AB^2 = (AC + CE)^2 + BE^2$ ...(i)

From ΔBCE, we can have

$CE = CB\cos\theta = Q\cos\theta$ [$CB = Q$, opposite sides of a parallelogram]

Also $BE = CB\sin\theta = Q\sin\theta$

Substituting these values in equation (i) above,

$AB^2 = (P + Q\cos\theta)^2 + Q^2\sin^2\theta$

$AB^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow \quad AB = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ ← Law of cosines

32. Define a unit vector.

Explain how can we express a vector using unit vectors along coordinate axes?

Find unit vector along the vector $6\hat{i} + 8\hat{j} + 10\hat{k}$.

Sol. A vector having unit magnitude is called a unit vector. The unit vector along the direction of vector $\vec{A}$ is denoted by $\hat{A}$. $\hat{A}$ is given by $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$.

It does not have any unit.

It is convenient to express a general vector in terms of its rectangular components using unit vectors. If the components of a vector $\vec{P}$ along the respective axes are P_x, P_y, P_z. Then

$$\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$$

$\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes.

Let $\vec{A} = 6\hat{i} + 8\hat{j} + 10\hat{k}$

$= |\vec{A}| = \sqrt{6^2 + 8^2 + 10^2}$

$= \sqrt{200}$

$= 10\sqrt{2}$

$\therefore$ Unit vector $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{6\hat{i}}{10\sqrt{2}} + \dfrac{8\hat{j}}{10\sqrt{2}} + \dfrac{1\hat{k}}{\sqrt{2}}$

$= \dfrac{3\hat{i}}{5\sqrt{2}} + \dfrac{4\hat{j}}{5\sqrt{2}} + \dfrac{1\hat{k}}{\sqrt{2}}$

33. Explain how can we resolve a vector into two components lying in its plane?

Sol. The process of splitting up of a vector into two or more vectors is known as the resolution of a vector.

Let us consider the components in a plane. Suppose we want to resolve a vector $\vec{P}$ along any two directions, say $\hat{l}$ and $\hat{m}$, which lie in a plane containing $\vec{P}$. Here $\hat{l}$ and $\hat{m}$ are the unit vectors along two directions.

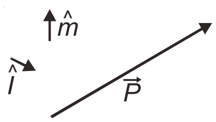


Figure (i)

Now if we draw a straight line parallel to $\hat{l}$ from the tail of $\vec{P}$ and another straight line parallel to $\hat{m}$ from the head of $\vec{P}$, and mark their intersection point as N (say), then we obtain an arrangement as shown in figure

(ii) below. $\overrightarrow{ON}$ is a vector parallel to $\hat{l}$, and $\overrightarrow{NS}$ is a vector parallel to $\hat{m}$.

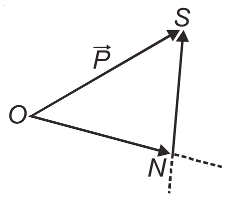


Figure (ii)

Then $\overrightarrow{ON} = \lambda\hat{l}$

And $\overrightarrow{NS} = \mu\hat{m}$

Where λ and μ are the real numbers denoting the magnitude of $\overrightarrow{ON}$ and $\overrightarrow{NS}$ respectively.

By applying triangle method of vector addition in figure (ii).

We get $\vec{P} = \overrightarrow{ON} + \overrightarrow{NS}$

$\Rightarrow \quad \vec{P} = \lambda\hat{l} + \mu\hat{m}$

Or we say λ and μ are the components of vector $\vec{P}$ along the directions $\hat{l}$ and $\widehat{m}$.

34. (i) Derive expression for the horizontal range of a projectile,

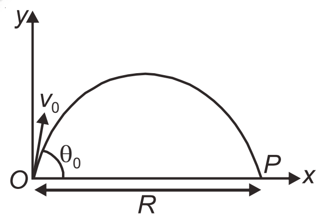
(ii) The maximum range of a projectile is $\frac{2}{\sqrt{3}}$ times the actual range for a given velocity of projection. What is the angle of projection for the actual range?

Sol. (i) The maximum horizontal distance travelled by the projectile during its flight is called the horizontal range of the projectile. This is the straight distance *OP* as shown in figure. It is denoted by *R*.

R can be calculated by using equation

$x = (v_0\cos\theta_0)t$

When $x = R$, t = Time of flight, $T_f = \frac{2v_0\sin\theta_0}{g}$



$\therefore \quad R = (v_0\cos\theta_0)\frac{(2v_0\sin\theta_0)}{g} = \frac{2v_0^2(\sin\theta_0)(\cos\theta_0)}{g}$

$R = \frac{v_0^2\sin 2\theta_0}{g}$ $\quad$ [$\because$ $2\sin\theta_0\cos\theta_0 = \sin(2\theta_0)$]

R is maximum when $2\theta_0 = 90°$ *i.e.*, $R_{max} = \frac{v_0^2}{g}$

(ii) $R_{max} = \frac{v_0^2}{g} = \frac{2}{\sqrt{3}}R$ $\quad$ [*R* = actual Range]

$\frac{v_0^2}{g} = \frac{2}{\sqrt{3}}\left(\frac{v_0^2\sin 2\theta_0}{g}\right)$

$\sin 2\theta_0 = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad 2\theta_0 = 60°$

$\Rightarrow \quad \theta_0 = 30°$

35. Can the resultant of three non parallel, coplanar forces of magnitudes 4 N, 8 N and 3 N acting on a particle be zero? Explain.

Sol. A vector having zero magnitude and having any arbitrary direction is called a null vector. It is denoted by $\vec{0}$.

Examples of null vector :

(i) Zero displacement

(ii) Zero change in velocity

(iii) Resultant of two equal and opposite forces acting at a point.

Properties of null vector $\vec{0}$.

$\vec{A}+\vec{0}=\vec{A}$

$\vec{A}-\vec{0}=\vec{A}$

$\vec{A}.0=\vec{0}$

$A.\vec{0}=\vec{0}$

Vector sum of three non-parallel and coplanar vectors will be zero only when the given vectors are represented by three sides of a triangle. From geometry, the sum of any two sides of a triangle must be greater, than the third side. Here in given problem (4 + 3) N < 8 N. The forces can not be represented by the side of a triangle. Hence their resultant can not be zero.

36. (i) Show that a projectile follows a parabolic path.

(ii) Find the horizontal distances travelled by a projectile projected at an angle 30° with horizontal with speed 15 m/s, at the time it is at height 2.5 m above the point of projection. [take g = 10 ms^{-2}]

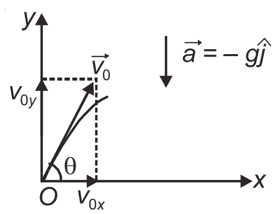
Sol. (i) Consider a stone projected with velocity v_0 at angle θ with x-axis.

$v_{0x} = v_0 \cos\theta$ [Horizontal speed at the time of release]

$v_{0y} = v_0 \sin\theta$ [Vertical speed at the time of release]

Along horizontal :

$\Rightarrow \quad \vec{v}_x = \vec{v}_{0x} = v_0 \cos\theta \hat{i}$



If we take the time at which the stone is projected as $t = 0$, then its horizontal displacement at any time t is

$x\hat{i} = (v_0 \cos\theta)t\hat{i}$

or $x = v_0 \cos\theta t$ ...(i)

Along vertical : The stone moves under a constant acceleration $\vec{a} = -g\hat{j}$. The velocity $\vec{v}_y$ of the stone at time t is given by

$v_y\hat{j} = v_{0y}\hat{j} + (-g)t\hat{j} \quad \Rightarrow \quad v_y = v_0 \sin\theta - gt$

Its vertical displacement y in this time is given by

$$y = v_0 \sin\theta t - \frac{1}{2}gt^2$$

Substituting $t = \dfrac{x}{v_0 \cos\theta}$ from equation (i)

$$y = v_0 \sin\theta\left(\frac{x}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{v_0^2\cos^2\theta}\right)$$

$\Rightarrow\quad y = (\tan\theta)\,x - \dfrac{1}{2}\dfrac{g}{(v_0^2\cos^2\theta)}x^2$ Equation of trajectory of the projectile.

(ii) $v_0 = 15$ m/s

$\theta = 30°$

$y = (v_0 \sin\theta)t - \dfrac{1}{2}gt^2 = 2.5$ m (vertical height)

$2.5 = (15\sin 30°)t - \dfrac{1}{2}\times 10t^2$

$2.5 = \dfrac{15}{2}t - 5t^2$

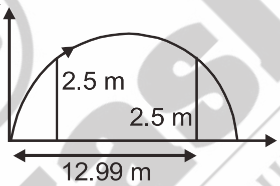
$\Rightarrow\quad 10t^2 - 15t + 5 = 0$

$\Rightarrow\quad t = 0.5$ s, 1 s

Horizontal distance travelled $x = v_0\cos\theta_0 t$

$= 15\cos 30° \times 0.5$ [Taking $t = 0.5$ s]

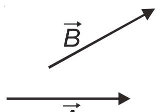
$= 15 \times \dfrac{\sqrt{3}}{2} \times 0.5 = 6.5$ m



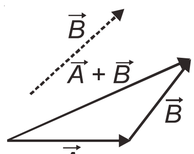
Similarly for $t = 1$ s, $x = 12.99$ m

37. Prove the commutative and associative properties of vector addition. How do we specify the position of an object using vectors?

Sol. (i) Commutative Property : To obtain $\vec{A} + \vec{B}$, we shift $\vec{B}$ parallel to itself so that its tail coincides with the head of $\vec{A}$. The line segment joining the tail of $\vec{A}$ to the head of $\vec{A}$ represents $\vec{A} + \vec{B}$ and its direction is as shown in the figure (ii).

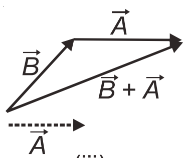


(i)



(ii)

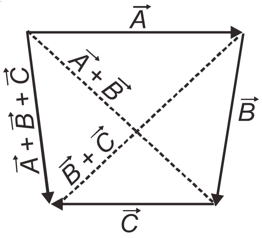
If instead of $\vec{B}$, we shift $\vec{A}$ parallel to itself such that the tail of $\vec{A}$ coincides with the head of $\vec{B}$, the vector obtained by joining the tail of $\vec{B}$ to the head of $\vec{B}$ gives $\vec{B} + \vec{A}$.



(iii)

If we compare $\vec{A} + \vec{B}$ and $\vec{B} + \vec{A}$ from figures (ii) and (iii), we find the two vectors are equal *i.e.,* $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

(ii) Associative Property : The figure shows that $\vec{A}+\vec{B}+\vec{C}=\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$



We can describe the position of an object using position vector. The position vector of a point *P* is the vector joining origin *O* to the point *P*. Thus it is the line *OP* having arrow at *P*.

38. The position of an object is described by the vector $\vec{r}=t^2\hat{i}+2t\hat{j}-t\hat{k}$ at any time *t*. Find its position, velocity and acceleration at time *t* = 6 s.

Sol. $\vec{r}=t^2\hat{i}+2t\hat{j}-t\hat{k}$

$\vec{v}=\frac{d\vec{r}}{dt}=2t\hat{i}+2\hat{j}-\hat{k}$

$\vec{a}=\frac{d\vec{v}}{dt}=2\hat{i}+0\hat{j}+0\hat{k}$

at t = 6 s,

(i) Position : $\vec{r}=6^2\hat{i}+12\hat{j}-6\hat{k} = 36\hat{i}+12\hat{j}-6\hat{k}$

(ii) Velocity : $\vec{v}=2\times 6\hat{i}+2\hat{j}-1\hat{k} = 12\hat{i}+2\hat{j}-\hat{k}$

$|\vec{v}|=\sqrt{12^2+2^2+(-1)^2} = \sqrt{149}$ m s^{-1}

(iii) Acceleration : $\vec{a}=2\hat{i}$

A constant acceleration of 2 m s^{-2} along positive *x*-axis.

39. If $\vec{A}=2\hat{i}-\hat{j}$ and $\vec{B}=\hat{i}-2\hat{j}$, find the scalar magnitude and directions of

(i) $\vec{A}$

(ii) $\vec{B}$ and

(iii) $\vec{A}+\vec{B}$

Sol. (i) $\vec{A}=2\hat{i}-\hat{j}$

Magnitude of $\vec{A}=\sqrt{(2)^2+(-1)^2} = \sqrt{5}$

Direction of $\vec{A}$ with *x*-axis.

$\theta=\tan^{-1}\left(\frac{-1}{2}\right)=-26.56°$

(ii) $\vec{B} = \hat{i} - 2\hat{j}$

Magnitude of $\vec{B} = \sqrt{(1)^2 + (-2)^2}$

$= \sqrt{5}$

Direction of $\vec{B}$ with x-axis

$$\phi = \tan^{-1}\left(\frac{-2}{1}\right) = -63.43°$$

(iii) $\vec{A} + \vec{B} = (2\hat{i} - \hat{j}) + (\hat{i} - 2\hat{j}) = (3\hat{i} - 3\hat{j})$

$\therefore$ Magnitude of $\vec{A} + \vec{B} = \sqrt{3^2 + (-3)^2} = 3\sqrt{2}$

Direction of $\vec{A} + \vec{B}$ with x-axis.

$$\alpha = \tan^{-1}\left(\frac{-3}{3}\right) = -45°$$

40. Show that a two-dimensional uniformly accelerated motion is a combination of two one-dimensional motions along perpendicular directions.

Sol. A body is said to be moving with uniform acceleration if its velocity vector suffers the same change in equal interval of time, however small. **For this case the average acceleration of the object is same as its instantaneous acceleration over a given time interval.**

The motion in a plane with uniform acceleration can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions. This can be shown as follows.

Let a constant acceleration $\vec{a}$ act on an object moving in a plane. This acceleration changes its velocity from $\overrightarrow{v_0}$ at time $t = 0$ to $\vec{v}$ at time $t = t$. Then

$$\vec{a} = \frac{\vec{v} - \overrightarrow{v_0}}{t - 0}$$

$$\Rightarrow \quad \vec{v} = \overrightarrow{v_0} + \vec{a}t \qquad \text{...(i)}$$

As we have studied in the last chapter, that for an object having constant acceleration, average velocity is given as

$$\bar{\vec{v}} = \frac{\overrightarrow{v_0} + \vec{v}}{2} \qquad \text{...(ii)}$$

From the definition of average velocity during the time interval $\Delta t = t - 0$, it can be expressed as $\bar{\vec{v}} = \dfrac{\vec{r} - \overrightarrow{r_0}}{t - 0}$.

Where $\vec{r}$ and $\vec{r}_0$ are the position vectors of the particle at time $t = t$ and $t = 0$ respectively.

$$\Rightarrow \quad \bar{\vec{v}}t = \vec{r} - \overrightarrow{r_0}$$

or $\vec{r} = \overrightarrow{r_0} + \bar{\vec{v}}t$

$$\vec{r} = \overrightarrow{r_0} + \left(\frac{\overrightarrow{v_0} + \vec{v}}{2}\right)t \qquad \text{[From equation (ii)]}$$

$$\Rightarrow \quad \vec{r} = \overrightarrow{r_0} + \left(\frac{\overrightarrow{v_0} + \overrightarrow{v_0} + \vec{a}t}{2}\right)t \qquad \text{[From equation (i)]}$$

$$\vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$$

Writing in component form

$$\Rightarrow \quad x\hat{i} + y\hat{j} = x_0\hat{i} + y_0\hat{j} + (v_{0_x}\hat{i} + v_{0_y}\hat{j})t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$$

Rearranging

$$\Rightarrow \quad x\hat{i} + y\hat{j} = \left(x_0 + v_{0x}t + \frac{1}{2}a_xt^2\right)\hat{i} + \left(y_0 + v_{0y}t + \frac{1}{2}a_yt^2\right)\hat{j}$$

Comparing two sides

$$\left.\begin{array}{ll} x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2 & \text{along x-axis} \\ y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2 & \text{along y-axis} \end{array}\right\} \quad ...(iii)$$

Thus we see that the motions in x and y-directions can be treated independently from each other. This result simplifies our study of motion in a plane.

A similar result can be obtained for the three-dimensional motion of an object. So for this case, we get a set of three equations as follows.

$$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$$

$$z = z_0 + v_{0z}t + \frac{1}{2}a_zt^2$$

41. Define the following,

(i) Relative velocity

(ii) Average velocity

(iii) Average acceleration

(iv) Centripetal acceleration

Sol. (i) **Relative velocity** : Let two objects A and B are moving in a plane with velocities $\vec{v}_A$ and $\vec{v}_B$ measured with respect to ground. The relative velocity of A with respect to B is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \quad ...(i)$$

Similarly relative velocity of B with respect to A is given by

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \quad ...(ii)$$

Comparing (i) and (ii)

$$\vec{v}_{AB} = -\vec{v}_{BA}$$

And $|\vec{v}_{AB}| = |\vec{v}_{BA}|$

(ii) **Average velocity** : Average velocity $\vec{\bar{v}}$ of an object is the ratio of its displacement and the corresponding time interval.

$$\therefore \quad \vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

Since $\vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t}$, the direction of average velocity is the same as that of $\Delta\vec{r}$.

The instantaneous velocity or simply the velocity is the limiting value of the average velocity when the time interval approaches zero.

(iii) **Average Acceleration :** The average acceleration $\vec{\bar{a}}$ is the ratio of the change in velocity and the corresponding time interval. If the velocity of an object changes from $\vec{v}$ to $\vec{v}_1$ in time interval Δt, then the acceleration of the object is given by the relation

$$\vec{\bar{a}} = \frac{\vec{v}' - \vec{v}}{\Delta t}$$

$$\Rightarrow \quad \vec{\bar{a}} = \frac{\Delta\vec{v}}{\Delta t} \quad \text{[where } \Delta\vec{v} = \vec{v}' - \vec{v}\text{]}$$

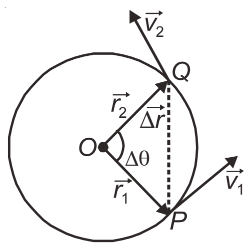
From the above relation, we can see that $\vec{\bar{a}}$ is along the direction of $\Delta\vec{v}$. The direction of $\Delta\vec{v}$ is different from that of $\vec{v}'$ and $\vec{v}$ as long as the object moves along a curve and not along a straight line. Or we say that for the motion along a curve, the direction of average acceleration is different from that of the velocity of the object. They may have any angle between 0° and 180° between them.

(iv) **Centripetal Acceleration :** Consider a particle moving on a circular path of radius *r* and centre *O*, with a uniform speed *v,* as shown in the figure below. Let the particle be at point *P* at time *t*, and at *Q* at time $t + \Delta t$. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors at *P* and *Q* directed along the tangents at *P* and *Q* respectively.

To find the change in velocity,

$$\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$$

This is the change in velocity during this time interval Δt.



By definition, the average acceleration is given by

$$\vec{\bar{a}} = \frac{\Delta\vec{v}}{\Delta t} \quad \textit{i.e.}, \ \vec{\bar{a}} \text{ is along } \Delta v.$$

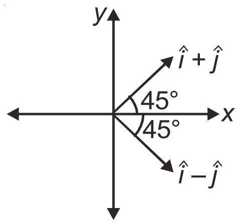
As $\Delta t \to 0$, the average acceleration becomes the instantaneous acceleration and $\Delta\theta$ also approaches zero.

Thus $\Delta\vec{v}$ and hence, $\vec{a}$ is perpendicular to velocity vector $\vec{v}_1$. But since $\vec{v}_1$ is directed along tangent at point *P*, so acceleration a_c acts along the radius towards the centre of the circle. That is why this acceleration is called **centripetal acceleration** which means 'centre seeking'. Numerically its value is $\frac{v^2}{r}$.

42. (i) Using graphical method, find the angle between $(\hat{i}+\hat{j})$ and $(\hat{i}-\hat{j})$.

(ii) Express the unit vector along above mentioned vectors.

Sol. (i)



Angle that $\hat{i}+\hat{j}$ makes with *x*-axis

$$\tan\theta_1 = \frac{1}{1}$$

$$\theta_1 = 45°$$

Similarly, the angle that $\hat{i}-\hat{j}$ make with *x*-axis.

$$\tan\theta_2 = -\frac{1}{1}$$

$$\theta_2 = -45°$$

$\therefore$ Required angle = 90°

(ii) $|\hat{i}+\hat{j}| = \sqrt{1^2+1^2}$

$= \sqrt{2}$

Therefore unit vector along $\hat{i}+\hat{j}$, is $= \frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

$= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Similarly $|\hat{i}-\hat{j}| = \sqrt{2}$

$\therefore$ Unit vector along $\hat{i}-\hat{j}$, is $= \frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right) = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

43. The ceiling of a long hall is 30 m high. What is the maximum horizontal distance that a ball thrown with a speed 45 ms^{-1} can go without hitting the ceiling of the wall? [take g = 10 m/s^2]

Sol. v_0 = 4 5 ms^{-1}

Let θ_0 = angle for projection

Taking maximum height reached $H_{max} = \frac{v_0^2 \sin^2\theta_0}{2g}$

Taking maximum height = 30 m

We have $\frac{v_0^2 \sin^2\theta_0}{2g} = 30$

$$\Rightarrow \quad \sin^2\theta_0 = \frac{30\times 2\times 10}{v_0^2}$$

$$\Rightarrow \quad \sin^2\theta_0 = \frac{30\times 2\times 10}{45\times 45}$$

$$\Rightarrow \quad \sin^2\theta_0 = \frac{600}{45\times 45} = \frac{8}{27}$$

$$\Rightarrow \quad \sin\theta_0 = \frac{2\sqrt{2}}{3\sqrt{3}} \qquad ...(i)$$

$$\cos\theta_0 = \sqrt{1-\sin^2\theta_0}$$

$$= \sqrt{1-\frac{8}{27}} = \sqrt{\frac{15}{27}} = \frac{\sqrt{15}}{3\sqrt{3}} = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad \cos\theta_0 = \frac{\sqrt{5}}{3} \qquad ..(ii)$$

Horizontal range R for $\theta_0 = \sin^{-1}\left(\frac{2\sqrt{2}}{3\sqrt{3}}\right)$

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

$$= \frac{v_0^2}{g} 2\sin\theta_0\cos\theta_0$$

$$= \frac{v_0^2}{g}\times 2\times\frac{2\sqrt{2}}{3\sqrt{3}}\times\frac{\sqrt{5}}{3} \quad \text{[Using equation (i) and (ii)]}$$

$$= \frac{45\times 45}{10}\times\frac{2\times 2\sqrt{2}}{9\sqrt{3}}\times\sqrt{5}$$

= 164.3 m

44. A body of mass 10 kg revolves in a circle of diameter 0.8 m completing 420 revolutions in a minute. Calculate its

(i) Angular speed

(ii) Linear speed

(iii) Time period and

(iv) Centripetal acceleration

Sol. Given,

Diameter = 0.8 m

$\therefore$ Radius R = 0.4 m

$$\text{Frequency } (\nu) = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{420}{60}\text{ rev/s} = 7\text{ s}^{-1}$$

(i) Angular speed $\omega = 2\pi\nu \quad = 2\times\frac{22}{7}\times 7 = 44$ rad/sec

(ii) Linear speed $v = \dfrac{2\pi R}{T}$

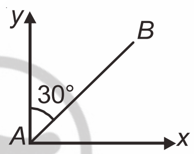
T = time period $= \dfrac{1}{\nu} = \dfrac{1}{7}$ s

$\therefore \quad v = 2\times\dfrac{22\times 0.4}{7\times 7} = \dfrac{44}{49}\times 0.4 = 0.3591$ m s^{-1}

(iii) Time period $T = \dfrac{1}{7} = 0.142$ s

(iv) Centripetal acceleration $a_C = \omega^2 R = (44)^2 \times 0.4 = 774.4$ m/s^2

45. A bird is flying with velocity $5\hat{i}+6\hat{j}$ w.r.t. wind. Wind blows along y-axis with velocity v. If bird is initially at A, and after sometime reaches B as shown. Find v, and also find the velocity of bird with respect to ground.



Sol. Given that velocity of bird with respect to wind is $\vec{v}_{bw} = 5\hat{i}+6\hat{j}$

Velocity of wind w.r.t. ground $\vec{v}_{wG} = v\hat{j}$

Now $\vec{v}_{bw} = \vec{v}_{bG} - \vec{v}_{wG}$ ...(i)

$5\hat{i}+6\hat{j} = \vec{v}_{bG} - \vec{v}_{wG}$

$\Rightarrow \quad \vec{v}_{bG} = 5\hat{i}+6\hat{j}+v\hat{j}$

$\vec{v}_{bG} = 5\hat{i}+(6+v)\hat{j}$

$\vec{v}_{bG}$ is shown by the vector $\overrightarrow{AB}$ in figure. Since velocity $\vec{v}_{bG}$ makes angle 30° with y-axis, it is inclined at 60° with x-axis. Hence

$\tan 60° = \dfrac{6+v}{5}$

$\Rightarrow \quad \sqrt{3} = \dfrac{6+v}{5}$

$\Rightarrow$ v = 2.66 m s^{-1}

$\therefore$ Velocity of bird with respect to ground

$v_{bG} = 5\hat{i}+6\hat{j}+2.66\hat{j}$

$= 5\hat{i}+8.66\hat{j}$

SECTION - B

Model Test Paper

1. Name two vector quantities.

Sol. Force, acceleration

2. Pick out the scalar quantity among the following : relative velocity, viscous drag, current, work, momentum.

Sol. Current, work

3. If an object having uniform circular motion undergoes an angular displacement $\Delta\theta$, what is the angle between its initial and final velocity vectors?

Sol. $\Delta\theta$

4. Give an example of null vector.

Sol. Zero displacement.

5. Add the vectors $2\hat{i}+3\hat{j}$, $\hat{i}-\hat{j}$ and $-3\hat{i}-2\hat{j}$.

Sol. Let $\vec{a}=2\hat{i}+3\hat{j}$

$\vec{b}=1\hat{i}-1\hat{j}$

$\vec{c}=-3\hat{i}-2\hat{j}$

$\vec{a}+\vec{b}+\vec{c}=(2+1-3)\hat{i}+(3-1-2)\hat{j}$

$= 0\hat{i}+0\hat{j}$

= Null vector

6. If a projectile is projected at 55°, at what other angle can it be projected to obtain the same range if projected with same speed?

Sol. 35°

7. When can $\vec{A}+\vec{B}$ be equal to $\vec{A}-\vec{B}$?

Sol. When $\vec{B}=\vec{O}$

$\vec{A}+\vec{B}=A$ and $\vec{A}-\vec{B}=\vec{A}$

$\Rightarrow \quad \vec{A}+\vec{B}=\vec{A}-\vec{B}$

When $\vec{B}$ is a null vector.

8. What is the time period of an object completing 40 revolutions in a minute?

Sol. $\nu=\dfrac{40}{60}\text{s}^{-1}=\dfrac{2}{3}\text{s}^{-1}$

Time period $T=\dfrac{1}{\nu}=\dfrac{3}{2}=1.5\,\text{sec}$

9. Keeping the angle of projection same, how does the horizontal range of a projectile vary, when its initial velocity is doubled?

Sol. Horizontal distance $R = \dfrac{v_0^2 \sin 2\theta_0}{g}$ ...(i)

Let $v_0' = 2v_0$

$\therefore \quad R' = \dfrac{(v_0')^2 \sin 2\theta_0}{g}$

$= \dfrac{4v_0^2 \sin 2\theta_0}{g}$

$R' = 4R$[From equation (i)]

10. Give two methods to specify a vector in a plane.

Sol. A vector $\vec{P}$ can be specified by two methods.

(i) By its magnitude $|\vec{P}|$ and its orientation θ with x-axis in x-y plane.

(ii) By its rectangular components along the corresponding co-ordinate axes *i.e.*, P_x and P_y.

$\Rightarrow \quad \vec{P} = P_x\hat{i} + P_y\hat{j}$

11. Find a unit vector parallel to the vector $3\hat{i} + 7\hat{j} + 4\hat{k}$.

Sol. Let $\vec{A} = 3\hat{i} + 7\hat{j} + 4\hat{k}$

$|\vec{A}| = \sqrt{3^2 + 7^2 + 4^2}$

$= \sqrt{9 + 49 + 16}$

$= \sqrt{74}$

$\therefore \quad \hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

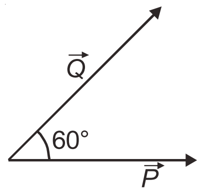
$\hat{A} = \dfrac{1}{\sqrt{74}}(3\hat{i} + 7\hat{j} + 4\hat{k})$

12. Find the resultant of two forces of magnitude 12 N and 7 N acting simultaneously on a body at angle 60° with each other.

Sol. $\vec{P} = 12\text{ N}$

$\vec{Q} = 7\text{ N}$

$\theta = 60°$



$\vec{R} = \vec{P} + \vec{Q}$

$R = \sqrt{P^2 + Q^2 + 2P\cos\theta}$

$R = \sqrt{12^2 + 7^2 + 2\times 12\times 7\times \frac{1}{2}}$

$= \sqrt{144+49+84}$

$= \sqrt{277}$

= 16.64 N

Let $\vec{R}$ makes $\angle\alpha$ with $\vec{P}$

Then $\frac{Q}{\sin\alpha} = \frac{R}{\sin 60°}$ [Law of sines]

$\Rightarrow \sin\alpha = \frac{Q\sin 60°}{R}$

$= \frac{7}{16.64}\times\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\sin\alpha$ = 0.364°

$\Rightarrow$ α = 21.36°

13. Show that the horizontal range of a projectile is maximum when projected at angle 45°, for given v_0.

Sol. $R = \frac{v_0^2}{g}\sin 2\theta_0$

For given value of v_0, R is maximum when $\sin 2\theta_0 = 1$

i.e., $2\theta_0 = 90°$

or $\theta_0 = 45°$

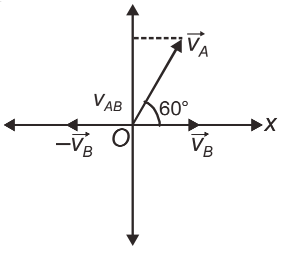
14. The angle between velocities $\vec{v}_A$ and $\vec{v}_B$ is 60°. Find $\vec{v}_{AB}$ if $|\vec{v}_A| = 16$ and $|\vec{v}_B| = 4$

Sol. Given $|\vec{v}_A| = 16$

$|\vec{v}_B| = 4$

Choose *x-y* plane such that

$\vec{v}_B$ lies along *x*-axis. (See figure)



Then $\vec{v}_B = 4\hat{i}$

$\vec{v}_A = (16\cos 60°)\hat{i} + (16\sin 60°)\hat{j}$

$= 8\hat{i} + 8\sqrt{3}\hat{j}$

By definition, relative velocity

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$= (8\hat{i} + 8\sqrt{3}\hat{j}) - 4\hat{i}$

$\vec{v}_{AB} = 4\hat{i} + 8\sqrt{3}\hat{j}$

Hence, $|\vec{v}_{AB}| = \sqrt{(4)^2 + (8\sqrt{3})^2}$

$= \sqrt{16 + 64 \times 3}$

= 14.42 m/s

Let θ be the angle of $\vec{v}_{AB}$ with direction of $\vec{v}_B$ *i.e.*, direction of *x*-axis.

Then $\theta = \tan^{-1}\left(\frac{8\sqrt{3}}{4}\right)$

$= \tan^{-1}(2\sqrt{3})$

= 73.89°

Hence, $\vec{v}_{AB}$ is 28 m/s at an angle 73.89° with $\vec{v}_B$.

15. How would a projectile motion be affected if air resistance becomes too large to be neglected?

Sol. If air resistance becomes too large, the path of the projectile will be different from its idealized trajectory. Maximum height attained and the horizontal range will be lesser than that attained in vacuum.

16. Find the magnitude of change in momentum of an oblique projectile during its whole journey if velocity and angle of projection are v_0 and θ_0 respectively.

Sol. Given that

Velocity of projection = v_0

Angle of projection = θ_0

In vector form, velocity at the time of projection

$\vec{v}_0 = v_0 \cos\theta_0 \hat{i} + v_0 \sin\theta_0 \hat{j}$

Velocity at the time of landing

$\vec{v}' = v_0 \cos\theta \hat{i} - v_0 \sin\theta \hat{j}$ [Projectile hits back with the same velocity, in downward direction]

$\therefore$ Change in momentum

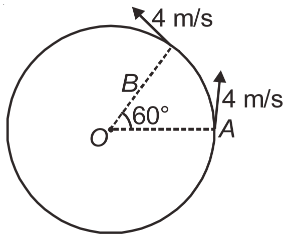
$\Delta p = m(\vec{v}' - \vec{v}_0)$

$= m\left[(v_0 \cos\theta_0 \hat{i} - v_0 \sin\theta_0 \hat{j}) - (v_0 \cos\theta_0 \hat{i} + v_0 \sin\theta_0 \hat{j})\right]$

$\Delta p = -2mv_0 \sin\theta_0 \hat{j}$

So change in momentum is $2mv_0\sin\theta_0$ in downward direction.

17. Find the magnitude of change in velocity for the uniform circular motion shown below.



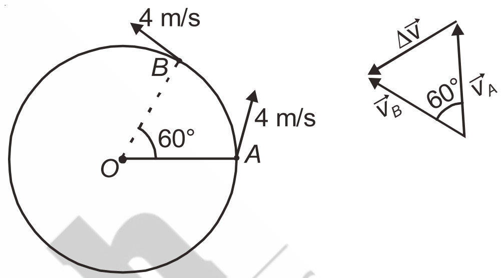
Sol. Velocities at points A and B are

$\vec{v}_A = 4$ m/s normal to OA

$\vec{v}_B = 4$ m/s normal to OB

Since angle between OA and OB is 60°.

Hence, angle between $\vec{v}_A$ and $\vec{v}_B$ is also 60°.



$\therefore$ Change in velocity $\Delta\vec{v} = \vec{v}_B - \vec{v}_A$

$$|\Delta\vec{v}| = \sqrt{v_A^2 + v_B^2 - 2v_A.v_B\cos 60^\circ}$$

$$= \sqrt{16 + 16 - 2\times 16\times\frac{1}{2}}$$

$\Rightarrow$ $\Delta\vec{v} = 4$ m/s Along the perpendicular bisector of the angle AOB.

18. Define the following terms

(i) Uniform circular motion

(ii) Angular displacement

(iii) Frequency of rotation

Sol. (i) **Uniform circular motion :** An object is said to have uniform circular motion if it travels in a circular path with constant speed.

(ii) **Angular displacement :** Angle that an object in circular motion subtends at the centre of its circular path, while going from one point to another, is called its angular displacement, in a given time.

(iii) **Frequency of rotation :** Number of rotations completed in unit time is called the frequency of rotation.

19. Give one example of each of the following

(i) A vector without unit

(ii) A vector not having any specific direction

(iii) A vector which is not constant

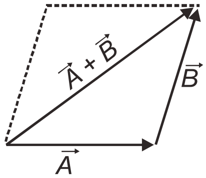
Sol. (i) Unit vector does not have any unit or dimension.

(ii) A null vector does not have a specific direction as its magnitude is zero.

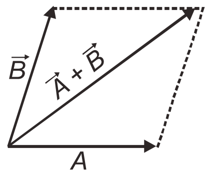
(iii) Centripetal acceleration is not a constant vector. Its direction changes continuously.

20. Show that parallelogram method and triangle method of vector addition are equivalent.

Sol. Let $\vec{A}$ and $\vec{B}$ be two vectors. Their addition by triangle method and parallelogram method is shown below graphically.



Triangle method



Parallelogram method

It is clear that the two methods bear the same result.

21. Find the velocity of an object after 3 s if its position at t = 0 is given as $\vec{r} = 3t\hat{i} + 2t^2\hat{j}$

Sol. Given $\vec{r} = 3t\hat{i} + 2t^2\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = 3\hat{i} + 4t\hat{j}$$

At t = 3, $\vec{v} = 3\hat{i} + 12\hat{j}$

$$|\vec{v}| = \sqrt{(3)^2 + (12)^2}$$

$$= \sqrt{153}$$

Angle made with x-axis $\theta = \tan^{-1}(4)$

22. State and prove the law of sines.

Sol. Let $\vec{A}$ and $\vec{B}$ be two vectors, inclined at angle θ with each other.

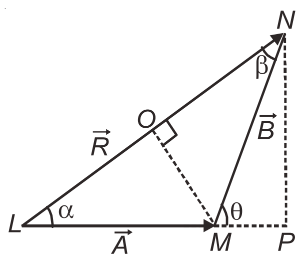
Let their resultant $\vec{R} = \vec{A} + \vec{B}$, make angle α with $\vec{A}$ and angle β with $\vec{B}$ as shown in the figure.

Draw $OM \perp LN$. Draw $NP \perp LM$ which meets LM at point P when extended.

In ΔPNM

$$\sin\theta = \frac{PN}{MN} = \frac{PN}{|\vec{B}|} \quad \text{...(i)}$$

In ΔPNL, $\sin\alpha = \dfrac{PN}{LN} = \dfrac{PN}{|\vec{R}|}$ ...(ii)



From (i) and (ii)

$$|\vec{B}|\sin\theta = |\vec{R}|\sin\alpha$$

$$\Rightarrow \frac{|\vec{B}|}{\sin\alpha} = \frac{|\vec{R}|}{\sin\theta} \quad \text{...(iii)}$$

In ΔMOL, $\sin\alpha = \dfrac{OM}{LM} = \dfrac{OM}{|\vec{A}|}$ ...(iv)

In ΔMON, $\sin\beta = \dfrac{OM}{MN} = \dfrac{OM}{|\vec{B}|}$ ...(v)

From (iv) and (v),

$$|\vec{A}|\sin\alpha = |\vec{B}|\sin\beta$$

$$\Rightarrow \quad \frac{|\vec{A}|}{\sin\beta} = \frac{|\vec{B}|}{\sin\alpha} \qquad \text{...(vi)}$$

From, equation (iii) and (vi) we have,

$$\frac{|\vec{A}|}{\sin\beta} = \frac{|\vec{B}|}{\sin\alpha} = \frac{|\vec{R}|}{\sin\theta} \qquad \leftarrow \text{Law of sines}$$

23. What do you mean by relative velocity? A boat goes in a river with speed 15 km/h (in still water) while a man crosses it with speed 8 km/h (in still water). The speed of water flow is 4 km/h and the boat goes downstream. How fast and in which direction does the man swim according to a man in boat?

Sol. **Relative velocity** : The velocity of an object *A* as observed by another object *B* (moving or at rest) is called relative velocity of *A* with respect to *B*.

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

Where $\vec{v}_A$ and $\vec{v}_B$ are respective velocities with respect to a common reference point.

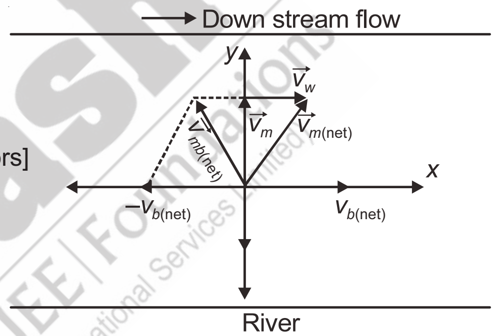
Let the direction of river flow be as shown in the figure.

Given that

Speed of boat in still water, v_b = 15 km/h

Speed of water flow v_w = 4 km/h

As the boat goes downstream, net velocity of boat



$$v_{b(\text{net})} = \vec{v}_b + \vec{v}_w$$

$$= 19\hat{i} \text{ km/h} \qquad \text{[Parallel vectors]}$$

Velocity of man in still water = $\vec{v}_m = 8\hat{j}$ km/h [see figure]

Net speed of man in river, $\vec{v}_{m(\text{net})} = \vec{v}_m + \vec{v}_w$

$$= 8\hat{j} + 4\hat{i}$$

Required velocity *i.e.,* relative velocity of man w.r.t. boat

$$\vec{v}_{mb(\text{net})} = \vec{v}_{m(\text{net})} - \vec{v}_{b(\text{net})}$$

$$= (8\hat{j} + 4\hat{i}) - 19\hat{i}$$

$$= 8\hat{j} - 15\hat{i} = -15\hat{i} + 8\hat{j}$$

$$|\vec{v}_{mb(\text{net})}| = \sqrt{(-15)^2 + 8^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289}$$

$$= 17 \text{ km/h}$$

Direction $\theta = \tan^{-1}\left(\frac{8}{-15}\right)$

$$= -28.07°$$

The man appears to swim at speed 17 km/h making angle 28.07° with the upstream.

❑ ❑ ❑

Solutions (Set-2)

Objective Type Questions

(Scalars and Vectors, Multiplication of Vectors by Real Numbers, Addition and Subtraction of Vectors - Graphical Methods, Resolution of Vectors, Vector Addition - Analytical Method)

1. Which of the following is a vector?

(1) Current (2) Time (3) Acceleration (4) Volume

Sol. Answer (3)

Acceleration is a vector quantity.

2. The change in a vector may occur due to

(1) Rotation of frame of reference (2) Translation of frame of reference

(3) Rotation of vector (4) Both (1) & (3)

Sol. Answer (3)

Change in a vector may occur due to rotation of vector and not due to rotation of frame of reference.

3. Which one of the following pair cannot be the rectangular components of force vector of 10 N?

(1) 6 N & 8 N (2) 7 N & $\sqrt{51}$ N (3) $6\sqrt{2}$ N & $2\sqrt{7}$ N (4) 9 N & 1 N

Sol. Answer (4)

The vector magnitude = $\sqrt{A_{x^2} + A_{y^2}}$

Vector magnitude = 10

But (4) option gives the magnitude

$\Rightarrow \sqrt{9^2 + 1^2} = \sqrt{82} \neq 10$ [by trial method check options]

4. The resultant of two vectors at an angle 150° is 10 units and is perpendicular to one vector. The magnitude of the smaller vector is

(1) 10 units (2) $10\sqrt{3}$ units (3) $10\sqrt{2}$ units (4) $5\sqrt{3}$ units

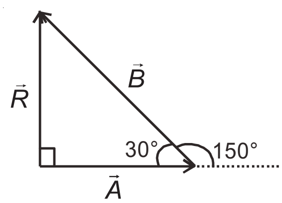
Sol. Answer (2)

$\Rightarrow R^2 + A^2 = B^2$ .....(1)

$R = 10$

Also $\tan 30° = \dfrac{\text{Perpendicular}}{\text{Base}}$

$\dfrac{1}{\sqrt{3}} = \dfrac{R}{A}$



From equation (1) $A = 10\sqrt{3}$

$(10)^2 + (10\sqrt{3})^2 = B^2$

$B = 20$

5. Two vectors, each of magnitude A have a resultant of same magnitude A. The angle between the two vectors is

(1) 30° (2) 60° (3) 120° (4) 150°

Sol. Answer (3)

$|\vec{A}| = |\vec{B}| = |\vec{R}|$

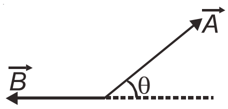
$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

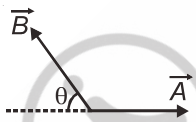
$A^2 = A^2 + A^2 + 2A^2\cos\theta$

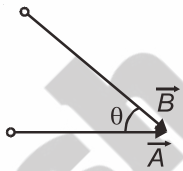
$-A^2 = 2A^2\cos\theta$

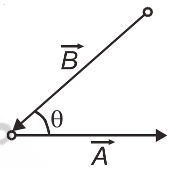
$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

6. Let θ be the angle between vectors $\vec{A}$ and $\vec{B}$. Which of the following figures correctly represents the angle θ?

(1) 

(2) 

(3) 

(4) 

Sol. Answer (3)

To find angle between vectors, they will be joined either head to head or tail to tail.

7. $\vec{A}$ is a vector of magnitude 2.7 units due east. What is the magnitude and direction of vector $4\vec{A}$?

(1) 4 units due east (2) 4 units due west (3) 2.7 units due east (4) 10.8 units due east

Sol. Answer (4)

$\vec{A} = 2.7\,\hat{i}$

Vector $4\vec{A}$

$\Rightarrow 4(2.7\hat{i}) = 10.8\hat{i}$ or 10.8 units due east.

8. Two forces of magnitude 8 N and 15 N respectively act at a point. If the resultant force is 17 N, the angle between the forces has to be

(1) 60° (2) 45° (3) 90° (4) 30°

Sol. Answer (3)

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$A = 8,\ B = 15,\ R = 17$

$17^2 = 8^2 + 15^2 + 2 \times 8 \times 15 \times \cos\theta$

$289 = 64 + 225 + 240\cos\theta$

$\Rightarrow 289 = 289 + 24\cos\theta$

$24\cos\theta = 0$

$\cos\theta = 0 \Rightarrow \theta = 90°$

9. Two forces of 10 N and 6 N act upon a body. The direction of the forces are unknown. The resultant force on the body may be

(1) 15 N (2) 3 N

(3) 17 N (4) 2 N

Sol. Answer (1)

The resultant of two vectors always lie between ($A + B$) & ($A - B$).

So the resultant of 10 N & 6 N should lie between 16 N & 4 N.

So answer is 15 N.

10. The vector $\overrightarrow{OA}$ where O is origin is given by $\overrightarrow{OA} = 2\hat{i} + 2\hat{j}$. Now it is rotated by 45° anticlockwise about O. What will be the new vector?

(1) $2\sqrt{2}\ \hat{j}$ (2) $2\ \hat{j}$

(3) $2\ \hat{i}$ (4) $2\sqrt{2}\ \hat{i}$

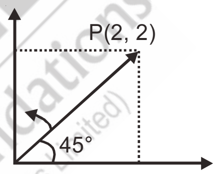
Sol. Answer (1)

$\overrightarrow{OA} = 2\hat{i} + 2\hat{j}$

$|\overrightarrow{OA}| = \sqrt{4+4} \Rightarrow 2\sqrt{2}$

On rotating by an angle of 45º anticlockwise it will lie along y-axis.

So $\vec{A} = 2\sqrt{2}\ \hat{j}$



11. If the sum of two unit vectors is also a unit vector, then magnitude of their difference and angle between the two given unit vectors is

(1) $\sqrt{3}, 60°$ (2) $\sqrt{3}, 120°$

(3) $\sqrt{2}, 60°$ (4) $\sqrt{2}, 120°$

Sol. Answer (2)

$|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$|\vec{A}| = |\vec{B}| = |\vec{R}| = 1$

$1 = 1 + 1 + 2 \times 1 \times 1 \times \cos\theta$

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$

$|\vec{R}| = |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 120°}$

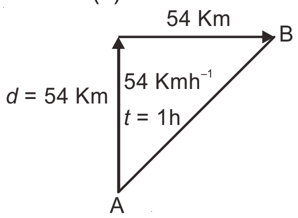
$= \sqrt{1^2 + 1^2 - 2\times1\times1\times\left(-\frac{1}{2}\right)} = \sqrt{3} = |\vec{A} - \vec{B}|$

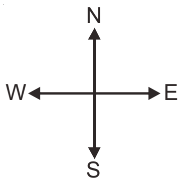
(Motion in a Plane, Motion in a Plane with Constant Acceleration, Relative Velocity in Two Dimensions)

12. A car moves towards north at a speed of 54 km/h for 1 h. Then it moves eastward with same speed for same duration. The average speed and velocity of car for complete journey is

(1) 54 km/h, 0 (2) 15 m/s, $\frac{15}{\sqrt{2}}$ m/s (3) 0, 0 (4) 0, $\frac{54}{\sqrt{2}}$ km/h

Sol. Answer (2)





$\text{Displacement} = \frac{54\sqrt{2}}{\text{Km}}$

Distance = 2 × 54 = 108 Km

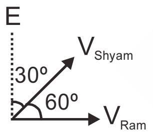
$\text{Average speed} = \frac{108}{2} = 54\,\text{Kmh}^{-1} \times \frac{5}{18} = 15\,\text{ms}^{-1}$

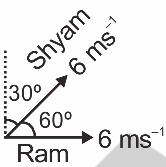
$\text{Average velocity} = \frac{\text{disp.}}{\text{time}} = \frac{54\sqrt{2}}{2} \Rightarrow 27\sqrt{2} \times \frac{5}{18} \Rightarrow \frac{15}{\sqrt{2}}\,\text{m/s}$

13. Ram moves in east direction at a speed of 6 m/s and Shyam moves 30° east of north at a speed of 6 m/s. The magnitude of their relative velocity is

(1) 3 m/s (2) 6 m/s (3) $6\sqrt{3}$ m/s (4) $6\sqrt{2}$ m/s

Sol. Answer (2)

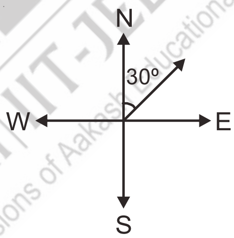




$|\vec{V}_{RS}| = \sqrt{V_R^2 + V_S^2 - 2V_RV_S\cos\theta}$

$= \sqrt{6^2 + 6^2 - 2\times 6^2 \times \frac{1}{2}}$

= 6 ms^{-1}



14. A train is running at a constant speed of 90 km/h on a straight track. A person standing at the top of a boggey moves in the direction of motion of the train such that he covers 1 meters on the train each second. The speed of the person with respect to ground is

(1) 25 m/s (2) 91 km/h (3) 26 km/h (4) 26 m/s

Sol. Answer (4)

$V_T = 90\,\text{Kmh}^{-1} = 90 \times \frac{5}{18} = 25\,\text{ms}^{-1}$

$V_m = ?$

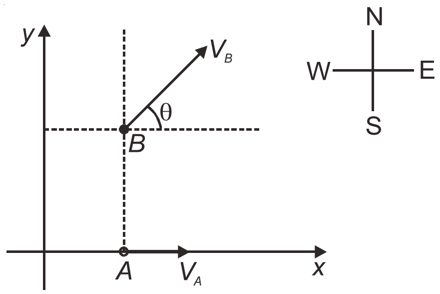
d = speed × time

$d_{net} = V_{net} \times t$

$1 = (V_m - 25) \times 1$

$V_m = 26\,\text{ms}^{-1}$

15 . Figure shows two ships moving in *x*-*y* plane with velocities V_A and V_B. The ships move such that B always remains north of *A*. The ratio $\frac{V_A}{V_B}$ is equal to



(1) cosθ (2) sinθ (3) secθ (4) cosecθ

Sol. Answer (1)

If ship *B* is always north of ship *A* then, their horizontal component should be equal, so,

$V_A = V_B \cos\theta$

$\Rightarrow \frac{V_A}{V_B} = \cos\theta$

16. A person, reaches a point directly opposite on the other bank of a flowing river, while swimming at a speed of 5 m/s at an angle of 120° with the flow. The speed of the flow must be

(1) 2.5 m/s (2) 3 m/s (3) 4 m/s (4) 1.5 m/s

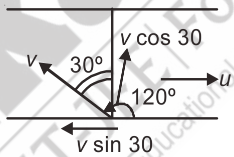
Sol. Answer (1)

For drift to be zero

$u = v \sin 30°$

$= 5 \times \frac{1}{2}$

$= 2.5 \text{ ms}^{-1}$



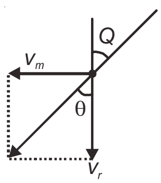
17. A car with a vertical windshield moves in a rain storm at a speed of 40 km/hr. The rain drops fall vertically with constant speed of 20 m/s. The angle at which rain drops strike the windshield is

(1) $\tan^{-1}\frac{5}{9}$ (2) $\tan^{-1}\frac{9}{5}$ (3) $\tan^{-1}\frac{3}{2}$ (4) $\tan^{-1}\frac{2}{3}$

Sol. Answer (1)

$$\tan\theta = \frac{v_m}{v_r} = \frac{\frac{20\times 5}{9}}{20}$$

$$\theta = \tan^{-1}\left(\frac{5}{9}\right)$$



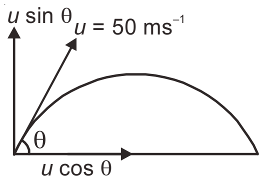
(Projectile Motion)

18. A body of mass 1 kg is projected from ground at an angle 30º with horizontal on a level ground at a speed 50 m/s. The magnitude of change in momentum of the body during its flight is (g = 10 m/s^2)

(1) 50 kg ms^{-1} (2) 100 kg ms^{-1} (3) 25 kg ms^{-1} (4) Zero

Sol. Answer (1)

$\Rightarrow$ The change in momentum = $-2mu\sin\theta\hat{j}$



$|\Delta\vec{p}| = 2mu\sin\theta$

$= 2 \times 1 \times 50 \times \sin 30°$

$|\Delta\vec{p}| = 50$ Kg ms^{-1}

19. Two projectiles are projected at angles $\left(\frac{\pi}{4}+\theta\right)$ and $\left(\frac{\pi}{4}-\theta\right)$ with the horizontal, where $\theta < \frac{\pi}{4}$, with same speed. The ratio of horizontal ranges described by them is

(1) $\tan\theta : 1$ (2) $1 : \tan^2\theta$ (3) $1 : 1$ (4) $1 : \sqrt{3}$

Sol. Answer (3)

The horizontal range is same when the angles of projection are complimentary to each other.

20. A shell is fired vertically upwards with a velocity v_1 from a trolley moving horizontally with velocity v_2. A person on the ground observes the motion of the shell as a parabola, whose horizontal range is

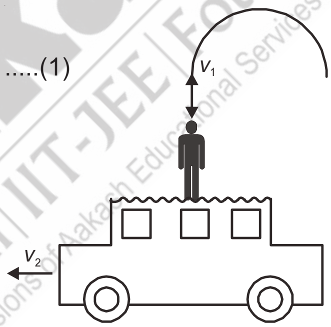
(1) $\frac{2v_1^2v_2}{g}$ (2) $\frac{2v_1^2}{g}$ (3) $\frac{2v_2^2}{g}$ (4) $\frac{2v_1v_2}{g}$

Sol. Answer (4)

There is no acceleration in the horizontal direction.

$s_x = u_xT + \frac{1}{2}a_0 \times T^2$

$R = u_xT$ .....(1)



$s_y = u_yT + \frac{1}{2}g_yT^2$

$O = v_1T - \frac{1}{2}gT^2$

$\Rightarrow v_1T = \frac{1}{2}gT^2$

$T = \frac{2v_1}{g}$

We know,

(R) range = (Horizontal velocity u_x) × time of fight (T)

i.e., $R = u_x \times T$

$R = v_2 \times \frac{2v_1}{g} \Rightarrow \frac{2v_1v_2}{g}$

21. The position coordinates of a projectile projected from ground on a certain planet (with no atmosphere) are given by $y = (4t - 2t^2)$m and $x = (3t)$ metre, where t is in second and point of projection is taken as origin. The angle of projection of projectile with vertical is

(1) 30° (2) 37° (3) 45° (4) 60°

Sol. Answer (2)

$y = 4t - 2t^2$

$x = 3t$

$v = v_{x\hat{i}} + v_{y\hat{j}}$

$v_x = \dfrac{dx}{dt},\ v_y = \dfrac{dy}{dt}$

$v_x = 3,\ v_y = 4 - 4t$

for $t = 0$, $v_y = 4$

$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{4}{3}$

$\theta = 53°$ with horizontal

With vertical

$\theta = 37°$

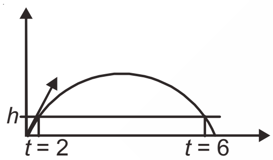
22. A particle is projected from ground with speed 80 m/s at an angle 30° with horizontal from ground. The magnitude of average velocity of particle in time interval t = 2 s to t = 6 s is [Take g = 10 m/s^2]

(1) $40\sqrt{2}$ m/s (2) 40 m/s (3) Zero (4) $40\sqrt{3}$ m/s

Sol. Answer (4)

Average velocity of the projectile when it is at the same vertical height is : $u\cos\theta$.

$\Rightarrow 80 \times \cos 30° \Rightarrow 40\sqrt{3}$ m/s.

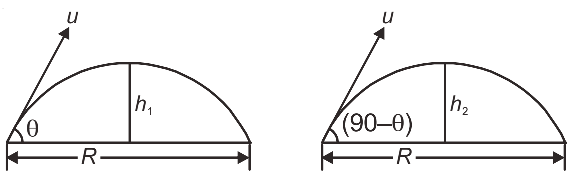


23. A stone projected from ground with certain speed at an angle θ with horizontal attains maximum height h_1. When it is projected with same speed at an angle θ with vertical attains height h_2. The horizontal range of projectile is

(1) $\dfrac{h_1 + h_2}{2}$ (2) $2h_1h_2$ (3) $4\sqrt{h_1h_2}$ (4) $h_1 + h_2$

Sol. Answer (3)

When the angles are complimentary the range is same,



$h_1 = \dfrac{u^2\sin^2\theta}{2g}$,

$h_2 = \dfrac{u^2\sin^2(90-\theta)}{2g}$

$h_1 = \dfrac{u^2\sin^2\theta}{2g}$

$h_2 = \dfrac{u^2\cos^2\theta}{2g}$

$h_1h_2 = \dfrac{u^4\sin^2\theta\cos^2\theta}{4g^2} \Rightarrow \left(\dfrac{2u\sin\theta\cos\theta}{g}\right)^2 \times \dfrac{1}{4g} \times \dfrac{1}{4}$

$$h_1h_2 = R^2\frac{1}{16} \Rightarrow R^2 = 16\,h_1h_2$$

$$R = 4(\sqrt{h_1h_2})$$

24. Two objects are thrown up at angles of 45° and 60° respectively, with the horizontal. If both objects attain same vertical height, then the ratio of magnitude of velocities with which these are projected is

(1) $\sqrt{\frac{5}{3}}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\sqrt{\frac{3}{2}}$

Sol. Answer (4)

$h_1 = h_2$

$$\frac{u_1^2\sin^2 45^\circ}{2g} = \frac{u_2^2\sin^2 60^\circ}{2g}$$

$$\frac{u_1^2}{u_2^2} = \frac{\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{3}{2}$$

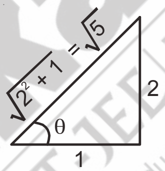
$$\frac{u_1}{u_2} = \sqrt{\frac{3}{2}}$$

25. For an object projected from ground with speed u horizontal range is two times the maximum height attained by it. The horizontal range of object is

(1) $\frac{2u^2}{3g}$ (2) $\frac{3u^2}{4g}$ (3) $\frac{3u^2}{2g}$ (4) $\frac{4u^2}{5g}$

Sol. Answer (4)

$R = 24$ also, $\frac{H}{R} = \frac{1}{4}\tan\theta$



$$\frac{H}{R} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{4}\tan\theta$$

$$\tan\theta = 2 = \frac{P}{B}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$R = \frac{2u^2}{g}.\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{5}}$$

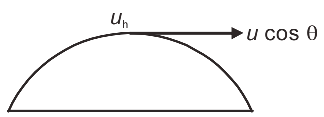
$$R = \frac{4u^2}{5g}$$

26. The velocity at the maximum height of a projectile is $\frac{\sqrt{3}}{2}$ times its initial velocity of projection (u). Its range on the horizontal plane is

(1) $\frac{\sqrt{3}u^2}{2g}$ (2) $\frac{3u^2}{2g}$ (3) $\frac{3u^2}{g}$ (4) $\frac{u^2}{2g}$

Sol. Answer (1)

$$u_h = u\cos\theta$$



$$\frac{\sqrt{3}}{2}u = u\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$\theta = 30°$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{u^2 \sin 60°}{g} \Rightarrow \frac{\sqrt{3}u^2}{2g} = R$$

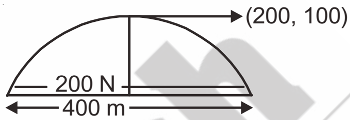
27. A projectile is thrown into space so as to have a maximum possible horizontal range of 400 metres. Taking the point of projection as the origin, the co-ordinates of the point where the velocity of the projectile is minimum are

(1) (400, 100) (2) (200, 100) (3) (400, 200) (4) (200, 200)

Sol. Answer (2)

R_{max} = 400 m

The velocity is minimum at the highest point



$$\Rightarrow H \rightarrow \frac{R}{2}$$

$R = 4H$

$400 = 4 \times H$

$H = 100$ m

28. If the time of flight of a bullet over a horizontal range R is T, then the angle of projection with horizontal is

(1) $\tan^{-1}\left(\frac{gT^2}{2R}\right)$ (2) $\tan^{-1}\left(\frac{2R^2}{gT}\right)$ (3) $\tan^{-1}\left(\frac{2R}{g^2T}\right)$ (4) $\tan^{-1}\left(\frac{2R}{gT}\right)$

Sol. Answer (1)

$$T = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{gT}{2\sin\theta}$$

$$R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

$$R = \frac{2u\sin\theta}{g} \times u\cos\theta$$

$R = T \times u \cos\theta$

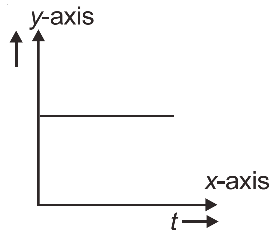
$$R = T \times \frac{gT\cos\theta}{2\sin\theta}$$

$$R = \frac{gT^2}{2}\frac{1}{\tan\theta}$$

$$\tan\theta = \frac{gT^2}{2R}$$

$$\theta = \tan^{-1}\left(\frac{gT^2}{2R}\right)$$

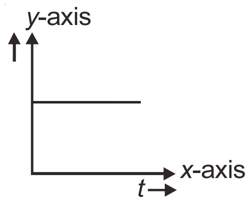
29. In the graph shown in figure, which quantity associated with projectile motion is plotted along y-axis?



(1) Kinetic energy (2) Momentum (3) Horizontal velocity (4) None of these

Sol. Answer (3)

It is the horizontal component of velocity that remains constant throughout the motion as there is no acceleration in that direction $a_x = 0$, u_x = constant



30. The equation of a projectile is $y = ax - bx^2$. Its horizontal range is

(1) $\frac{a}{b}$ (2) $\frac{b}{a}$ (3) $a + b$ (4) $b - a$

Sol. Answer (1)

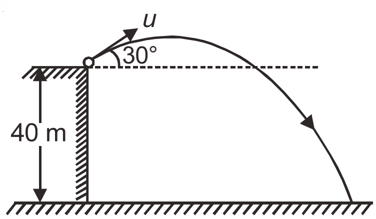
$y = ax - bx^2$

When the body lands then $y = 0$, $x = R$, $0 = aR - bR^2$

$aR = bR^2$

$R = \frac{a}{b}$

31. Figure shows a projectile thrown with speed u = 20 m/s at an angle 30° with horizontal from the top of a building 40 m high. Then the horizontal range of projectile is



(1) $20\sqrt{3}$ m (2) $40\sqrt{3}$ m

(3) 40 m (4) 20 m

Sol. Answer (2)

$S_y = u_y T + \frac{1}{2} g_y T^2$

$-40 = 4\sin 30T - \frac{1}{2} g T^2$

$-40 = 20 \times \frac{1}{2} T - 5T^2$

$-8 = 2T - T^2$

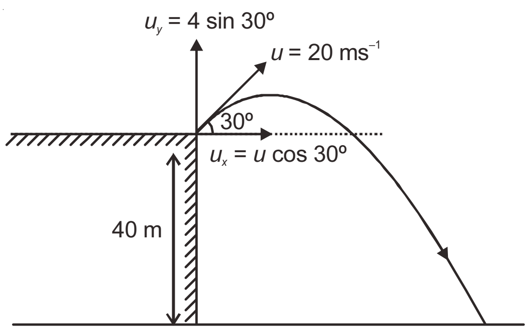
$T^2 - 2T - 8 = 0$

$T^2 - 4T + 2T - 8 = 0$

$T = -2, 4$

$R = u\cos\theta T = 20 \times \frac{\sqrt{3}}{2} \times 4$

$R = 40\sqrt{3}$ m



32. When a particle is projected at some angle to the horizontal, it has a range R and time of flight t_1. If the same particle is projected with the same speed at some other angle to have the same range, its time of flight is t_2, then

(1) $t_1 + t_2 = \dfrac{2R}{g}$ (2) $t_1 - t_2 = \dfrac{R}{g}$ (3) $t_1 t_2 = \dfrac{2R}{g}$ (4) $t_1 t_2 = \dfrac{R}{g}$

Sol. Answer (3)

The angles has to be complimentary *i.e.,* if $\theta_1 \to \theta,\ \theta_2 \to (90-\theta)$

$$t_1 = \frac{2u\sin\theta}{g},\ t_2 = \frac{2u\sin(90-\theta)}{g}$$

$$t_2 = \frac{2u\cos\theta}{g}$$

$$t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

$$t_1 t_2 = \frac{2R}{g}$$

33. A projectile is thrown with velocity v at an angle θ with horizontal. When the projectile is at a height equal to half of the maximum height, the vertical component of the velocity of projectile is

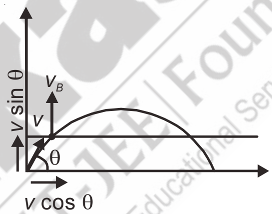
(1) $v \sin\theta \times 3$ (2) $\dfrac{v\sin\theta}{3}$ (3) $\dfrac{v\sin\theta}{\sqrt{2}}$ (4) $\dfrac{v\sin\theta}{\sqrt{3}}$

Sol. Answer (3)

$$v_B^2 = v^2\sin^2\theta - \frac{2g}{2}\left(\frac{u^2\sin^2\theta}{2g}\right)$$

$$v_B^2 = \frac{v^2\sin^2\theta}{2}$$

$$v_B = \frac{v\sin\theta}{\sqrt{2}}$$



34. Two paper screens A and B are separated by distance 100 m. A bullet penetrates A and B, at points P and Q respectively, where Q is 10 cm below P. If bullet is travelling horizontally at the time of hitting A, the velocity of bullet at A is nearly

(1) 100 m/s (2) 200 m/s (3) 600 m/s (4) 700 m/s

Sol. Answer (4)

10 cm $\Rightarrow$ 10 × 10^{-2} m $\Rightarrow$ 10^{-1} $\Rightarrow$ 0.1 m

It is a case of horizontal projectile.

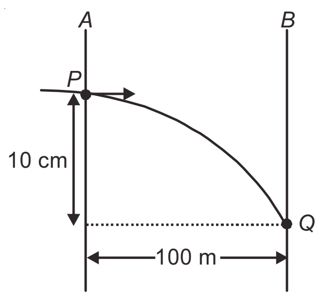
So, $a_x = 0$, $u_x = 4$, $u_y = 0$, $a_y = -g$

R = 100m, $T = \sqrt{\dfrac{2H}{g}} \Rightarrow$ Time of flight

$R = u_x T$

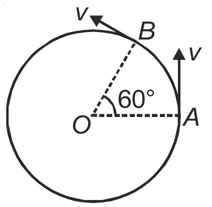
$$100 = u\sqrt{\frac{2\times 0.1}{100}} \Rightarrow \frac{u\sqrt{2}}{10} = 100$$

$$u = \frac{1000}{\sqrt{2}} \approx 707\ \text{ms}^{-1}$$



(Uniform Circular Motion)

35. A particle is moving in a circle of radius *r* having centre at *O*, with a constant speed *v*. The magnitude of change in velocity in moving from *A* to *B* is



(1) $2v$ (2) 0 (3) $\sqrt{3}\,v$ (4) v

Sol. Answer (4)

$$[\Delta\vec{v}] = 2v\sin\frac{\theta}{2} = 2\times v\times\sin\left(\frac{60^\circ}{2}\right) = 2\times v\times\frac{1}{2} \Rightarrow v = |\Delta\vec{v}|$$

36. A car is going round a circle of radius R_1 with constant speed. Another car is going round a circle of radius R_2 with constant speed. If both of them take same time to complete the circles, the ratio of their angular speeds and linear speeds will be

(1) $\sqrt{\frac{R_1}{R_2}}, \frac{R_1}{R_2}$ (2) 1, 1 (3) $1, \frac{R_1}{R_2}$ (4) $\frac{R_1}{R_2}, 1$

Sol. Answer (3)

The angular speed is given by

$$\omega = \frac{2\pi}{T}$$

$$\omega \propto \frac{1}{T} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

if $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

So, ratio $\Rightarrow$ 1 : 1

and linear speed $v = R\omega$

$$v \propto R$$

$$\frac{v_1}{v_2} = \frac{R_1}{R_2}$$

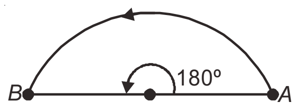
37. A body revolves with constant speed *v* in a circular path of radius *r*. The magnitude of its average acceleration during motion between two points in diametrically opposite direction is

(1) Zero (2) $\frac{v^2}{r}$ (3) $\frac{2v^2}{\pi r}$ (4) $\frac{v^2}{2r}$

Sol. Answer (3)

$$a_{avg} = \frac{2v\sin\left(\frac{\theta}{2}\right)}{\left(\frac{r\theta}{v}\right)}$$

$$a_{avg} = \frac{2v^2\sin\left(\frac{\theta}{2}\right)}{r\theta}$$



Here, $\theta = \pi$ rad

$$a_{avg} = \frac{2v^2 \sin\left(\frac{\pi}{2}\right)}{r \times \pi}$$

$$a_{avg} = \frac{2v^2}{\pi r}$$

38. An object of mass m moves with constant speed in a circular path of radius R under the action of a force of constant magnitude F. The kinetic energy of object is

(1) $\frac{1}{2}FR$ (2) FR (3) $2FR$ (4) $\frac{1}{4}FR$

Sol. Answer (1)

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{F}{a} \times v^2 = \frac{1}{2}\frac{F \times v^2}{\left(\frac{v^2}{R}\right)} = \frac{1}{2}FR$$

39. The angular speed of earth around its own axis is

(1) $\frac{\pi}{43200}$ rad/s (2) $\frac{\pi}{3600}$ rad/s (3) $\frac{\pi}{86400}$ rad/s (4) $\frac{\pi}{1800}$ rad/s

Sol. Answer (1)

Angular speed = $\frac{2\pi}{T}$

$T \rightarrow$ Time period of earth = 24 h

$$\omega = \frac{2\pi}{24 \times 60 \times 60} = \frac{\pi}{43200} \text{ rad s}^{-1}$$

40. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle is (in m/s^2)

(1) π^2 (2) $8\pi^2$ (3) $4\pi^2$ (4) $2\pi^2$

Sol. Answer (3)

$a = r\omega^2$

$$a = \frac{25}{100}(2 \times 2\pi)^2$$

$a = 4\pi^2$ m/s^2

41. A particle is revolving in a circular path of radius 25 m with constant angular speed 12 rev/min. Then the angular acceleration of particle is

(1) $2\pi^2$ rad/s^2 (2) $4\pi^2$ rad/s^2 (3) π^2 rad/s^2 (4) Zero

Sol. Answer (4)

Angular acceleration is the rate of change of angular speed or angular velocity if $\vec{\omega}$ remains constant then $\alpha = 0$

42. Two particles are moving in circular paths of radii r_1 and r_2 with same angular speeds. Then the ratio of their centripetal acceleration is

(1) 1 : 1
(2) $r_1 : r_2$
(3) $r_2 : r_1$
(4) $r_2^2 : r_1^2$

Sol. Answer (2)

Centripetal acceleration is given by

$$a = \frac{v^2}{r} = r\omega^2$$

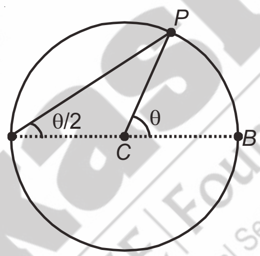
For same 'ω'

$$a_c \propto r \Rightarrow \frac{a_1}{a_2} = \frac{r_1}{r_2}$$

43. A particle P is moving in a circle of radius r with uniform speed v. C is the centre of the circle and AB is diameter. The angular velocity of P about A and C is in the ratio

(1) 4 : 1
(2) 2 : 1
(3) 1 : 2
(4) 1 : 1

Sol. Answer (3)



$$\omega_{P/C} = \frac{d\theta}{dt}$$

$$\omega_{P/A} = \frac{1}{2}\frac{d\theta}{dt}$$

$$\omega_{P/A} = \frac{1}{2}\omega_{P/C}$$

$$\frac{\omega_{P/A}}{\omega_{P/C}} = \frac{1}{2} = 1:2$$

44. A car is moving at a speed of 40 m/s on a circular track of radius 400 m. This speed is increasing at the rate of 3 m/s^2. The acceleration of car is

(1) 4 m/s^2
(2) 7 m/s^2
(3) 5 m/s^2
(4) 3 m/s^2

Sol. Answer (3)

v = 40 ms^{-1}

r = 400 m

a_T = 3 ms^{-2}

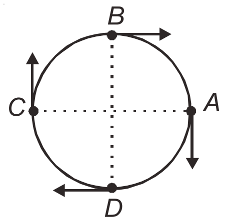
$$a_c = \frac{v^2}{r} = \frac{40 \times 40}{400} = 4\text{ ms}^{-2}$$

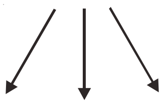
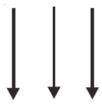
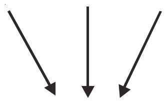
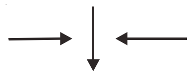
$$a = \sqrt{a_C^2 + a_T^2}$$

$$a = \sqrt{4^2 + 3^2} = 5\text{ ms}^{-2}$$

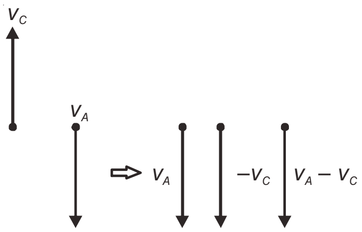
a = 5 ms^{-2}

45. Four particles A, B, C and D are moving with constant speed v each. At the instant shown relative velocity of A with respect to B, C and D are in directions

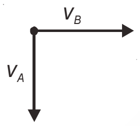


(1)  (2)  (3)  (4) 

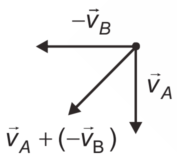
Sol. Answer (1)

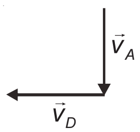
(i) 

$\vec{v}_{AC} = \vec{v}_A - \vec{v}_C \Rightarrow$  $\vec{v}_A - \vec{v}_C$

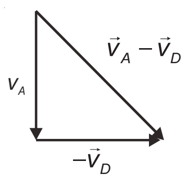
(ii) 

$\Rightarrow \vec{v}_{AB} = \vec{v}_A - \vec{v}_B \Rightarrow \vec{v}_A + (-\vec{v}_A)$



(iii) 

$\Rightarrow \vec{v}_{AD} = \vec{v}_A - \vec{v}_D = \vec{v}_A + (-\vec{v}_D)$

$\Rightarrow$ 

46. The ratio of angular speeds of minute hand and hour hand of a watch is

(1) 6 : 1 (2) 12 : 1 (3) 60 : 1 (4) 1 : 60

Sol. Answer (2)

ω_{mh} = Angular speed of minute hand

ω_{hh} = Angular speed of hour hand

$$\omega_{mh} = \frac{2\pi}{60\,m} = \frac{2\pi}{60\times60}\text{rad s}^{-1}$$

$$\omega_{hh} = \frac{2\pi}{12\text{ h}} = \frac{2\pi}{12\times60\times60}\text{rad s}^{-1}$$

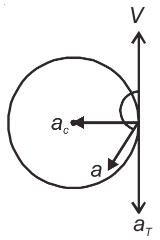
$$\frac{\omega_{mh}}{\omega_{hh}} = \frac{\frac{2\pi}{60\times60}}{\frac{2\pi}{12\times60\times60}} = \frac{1}{1}\times\frac{12}{1}$$

$\omega_{mh} : \omega_{hh} \Rightarrow 12 : 1$

47. If θ is angle between the velocity and acceleration of a particle moving on a circular path with decreasing speed, then

(1) $\theta = 90°$ (2) $0° < \theta < 90°$ (3) $90° < \theta < 180°$ (4) $0° \le \theta \le 180°$

Sol. Answer (3)



θ between v & Q is

$90° < \theta < 180°$

48. If speed of an object revolving in a circular path is doubled and angular speed is reduced to half of original value, then centripetal acceleration will become/remain

(1) Same (2) Double (3) Half (4) Quadruple

Sol. Answer (1)

$a_c = r\omega^2 = (r\omega)(\omega)$

$a_c = v\omega$

$a_c = (2v)\left(\frac{\omega}{2}\right) = v\omega = a_c$

❑ ❑ ❑