Level - I

Chapter 9

Mechanical Properties of Solids

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. Why are girders given I shape?

Sol. To reduce stress at top and bottom of girders.

2. The length of wire increases by 1 mm under 1 N. What will be increase in length under 2 N?

Sol. 2 mm

3. What is the SI unit of volumetric strain?

Sol. No unit

4. Which of the three Y, B or G is possible in all the three state of matter (solid, liquid and gas)?

Sol. Bulk modulus of elasticity (B) only.

5. Which type of strain is there, when a spiral spring is stretched by a force?

Sol. Shear strain

6. State Hooke's law.

Sol. Within elastic limit,

stress $\propto$ strain

$\Rightarrow$ stress = E × strain

7. What is the dimensional formula of Young's modulus?

Sol. $[ML^{-1}T^{-2}]$

8. D efine bulk modulus of elasticity.

Sol. The ratio of volume stress to volume strain is called bulk modulus.

9. What do you mean by Poisson's ratio?

Sol. The ratio of the lateral strain to the longitudinal strain is called the poisson's ratio.

10. What is the value of modulus of rigidity for a liquid?

Sol. Zero

Short Answer Type Questions :

11. Why is a spring made of steel, not of copper?

Sol. A spring will be better one, if a large restoring force is set up in it on being deformed, which in turn depends upon the elasticity of material of the spring. Since the Young's modulus of elasticity of steel is more than that of copper, hence steel is used.

12. An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support?

Sol. Breaking load = breaking stress × area, is free from the length of elastic wire.

13. Which is more elastic-rubber or steel? Explain.

Sol. Steel, because it sustains more deforming force.

14. Calculate the force required to increase length by 1% of a rod of area of cross-section 10^{-3} m^2. Modulus of elasticity is 1.2×10^{12} N m^{-2}.

Sol. $F = YA \cdot \frac{\Delta l}{l}$

$F = 1.2 \times 10^4$ N

15. Explain the terms (i) Young's modulus (ii) Bulk modulus.

Sol. (i) Young's modulus : The ratio of the longitudinal stress to the longitudinal strain is called Young's modulus.

(ii) Bulk modulus : The ratio of volume stress to volume strain is called bulk modulus.

16. A thick rope of density ρ and length L is hung from a rigid support. The Young's modulus of the material of rope is Y. What is the increase in length of the rope due to its own weight?

Sol. Let A be the area of cross-section of the rope. Weight of the rope, $F = AL\rho g$. It will be acting at the centre of gravity of the rope which lies at a distance $\frac{L}{2}$ from rigid support. Therefore,

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}} = \frac{AL\rho g/A}{\frac{\Delta L}{L/2}} \text{ or } \Delta L = \frac{L^2\rho g}{2Y}$$

17. Define elastic limit.

Sol. No body is perfectly elastic. However, a body behaves as a perfectly elastic body and recovers its original configuration completely when the deforming force does not exceed a particular limit. This limit is called the elastic limit.

18. When the pressure on a sphere is increased by 80 atm then its volume decreases by 0.01%. Find the Bulk modulus of elasticity of the material of sphere.

Sol. $P = 80 \times 1.013 \times 10^5$ N/m^2

$$\frac{\Delta V}{V} = \frac{0.01}{100}$$

$$\therefore \quad B = \frac{PV}{\Delta V} = \frac{80 \times 1.013 \times 10^5}{\frac{0.01}{100}} = 8.1 \times 10^{10} \text{ N/m}^2$$

19. A specimen of oil having an initial volume of 600 cm^3 is subjected to a pressure increase of 3.6×10^6 Pa and the volume is found to decrease by 0.45 cm^3. What is the Bulk modulus of the material?

Sol. $B = \frac{PV}{\Delta V} = \frac{3.6 \times 10^6 \times 600}{0.45} = 4.8 \times 10^9 \text{ N/m}^2$

20. Compare the mechanical properties of a steel cable, made by twisting many thin wires together, with the properties of a solid steel rod of the same diameter. What advantages does each have?

Sol. Refer theory.

21. The material in human bones and elephant bones is essentially same, but an elephant has much thicker legs. Explain why, in terms of breaking stress.

Sol. $\text{Stress} = \frac{\text{force}}{\text{area}}$

22. If a metal wire has its length doubled and its diameter tripled, then by what factor does its Young's modulus change?

Sol. Young's modulus of material is independent of stress and strain.

23. Calculate the longest length of steel wire that can hang vertically without breaking. Breaking stress for steel $= 7.982 \times 10^8$ N/m^2 and density for steel $d = 8.1 \times 10^3$ kg/m^3.

Sol. $\text{Breaking stress} = \frac{\text{force}}{\text{area}} = \frac{mg}{a}$

$$\Rightarrow \quad \text{Breaking stress} = \frac{\rho l a g}{a}$$

$$l = \frac{\text{Breaking stress}}{\rho g}$$

$$l = \frac{7.982 \times 10^8}{8.1 \times 10^3 \times 9.8} \text{ m}$$

$l = 1 \times 10^4$ m

24. A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

Sol. The copper and steel wires are under a tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A. We have, stress = strain × Young's modulus. Therefore,

$$\frac{W}{A} = Y_c \cdot \frac{\Delta L_c}{L_c} = Y_s \cdot \frac{\Delta L_s}{L_s}$$

Where the subscripts c and s refer to copper and stainless steel respectively, or,

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{Y_s}{Y_c}\right) \times \left(\frac{L_c}{L_s}\right)$$

Given, L_c = 2.2 m, L_s = 1.6 m

From table…… $Y_c = 1.1 \times 10^{11}$ Nm^{-2} and $Y_s = 2.0 \times 10^{11}$ Nm^{-2}

$$\frac{\Delta L_c}{\Delta L_s} = \left(\frac{2.0 \times 10^{11}}{1.1 \times 10^{11}}\right) \times \left(\frac{2.2}{1.6}\right) = 2.5$$

The total elongation is given to be

$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4}$ m

Solving the above equations,

$\Delta L_c = 5.0 \times 10^{-4}$ m and $\Delta L_s = 2.0 \times 10^{-4}$ m

Therefore, $w = \dfrac{(A \times Y_c \times \Delta L_c)}{L_c}$

$= \pi(1.5 \times 10^{-3})^2 \times [(5.0 \times 10^{-4} \times 1.1 \times 10^{11})/2.2]$

$= 1.8 \times 10^2$ N

25. A 2 m long wire is stretched by 0.5 cm. Calculate the elastic potential energy per unit volume if the Young's modulus of the material of the wire is $Y = 8 \times 10^{10}$ N/m^2.

Sol. $u = \dfrac{1}{2}(\text{stress}) \times \text{strain}$

$$= \frac{1}{2}\text{Young's modulus} \times (\text{strain})^2$$

$$= \frac{1}{2} \times 8 \times 10^{10} \times \left(\frac{0.5 \times 10^{-2}}{20}\right)^2$$

$$= 4 \times 10^{10} \times \frac{25 \times 10^{-4}}{400}$$

$= 2.5 \times 10^4$ J/m^3

Long Answer Type Questions :

26. A metal wire 3.50 m long and 0.70 mm in diameter has given the following test. A load weighing 20 N was originally hung from the wire to keep it straight. The position of the lower end of the wire was read on a scale as load was added.

Added Load (N)	Scale Reading (cm)
0	3.02
10	3.07
20	3.12
30	3.17
40	3.22
50	3.27
60	3.32
70	4.27

(i) Graph these values, plotting the increases in length horizontally and the added load vertically.

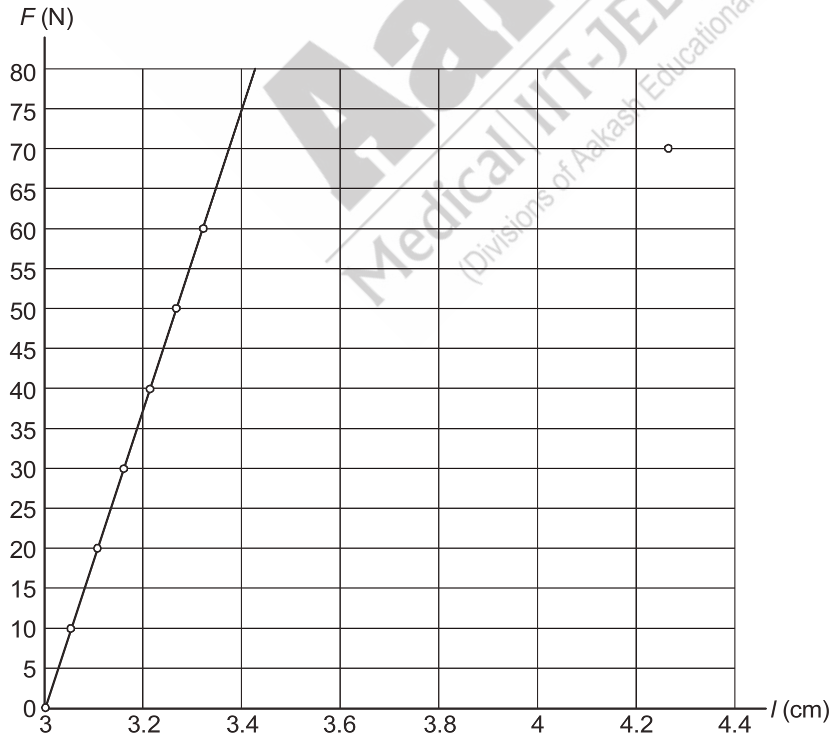
(ii) Calculate the value of Young's modulus.

(iii) The proportional limit occurred at a scale reading of 3.34 cm. What was the stress at this point?

Sol. $F_{\perp} = \left(\dfrac{YA}{l_0}\right)\Delta l$ so the slope of the graph in part (a) depends on Young's modulus.

$F_{\perp}$ is the total load, 20 N plus the added load.

(i) The graph is given in figure



(ii) The slope is $\dfrac{60\text{ N}}{(3.32-3.02)\times10^{-2}\text{ m}} = 2.0\times10^{4}\text{ N/m}$

$$Y = \left(\frac{l_0}{\pi r^2}\right)(2.0\times10^4\text{ N/m}) = \left(\frac{3.50\text{ m}}{\pi[0.35\times10^{-3}\text{ m}]^2}\right)(2.0\times10^4\text{ N/m}) = 1.8\times10^{11}\text{ Pa}$$

(iii) The stress is $F_{\perp}/A$. The total load at the proportional limit is 60 N + 20 N = 80 N

$$\text{stress} = \frac{80\text{ N}}{\pi(0.35\times10^{-3}\text{ m})^2} = 2.1\times10^{8}\text{ Pa}$$

The value of Y we calculated is close to the value for iron, nickel and steel.

27. A brass rod with a length of 1.40 m and a cross-sectional area of 2.00 cm^2 is fastened end to end to a nickel rod with length L and cross-sectional area 1.00 cm^2. The compound rod is subjected to equal and opposite pulls of magnitude 4.00×10^4 N at its ends.

(i) Find the length L of the nickel rod if the elongation of the two rods are equal.

(ii) What is the stress in each rod?

(iii) What is the strain in each rod?

Sol. Each piece of the composite rod is subjected to a tensile force of 4.00×10^4 N.

(i) $Y = \dfrac{F_{\perp}l_0}{A\Delta l}$ so $\Delta l = \dfrac{F_{\perp}l_0}{YA}$

$\Delta l_b = \Delta l_n$ gives that $\dfrac{F_{\perp}l_{0,b}}{Y_bA_b} = \dfrac{F_{\perp}l_{0,n}}{Y_nA_n}$ (b for brass and n for nickel); $l_{0,\,n} = L$

But the $F_{\perp}$ is same for both, so

$$l_{0,n} = \frac{Y_n}{Y_b}\frac{A_n}{A_b}l_{0,b}$$

$$L = \left(\frac{21\times10^{10}\text{ Pa}}{9.0\times10^{10}\text{ Pa}}\right)\left(\frac{1.00\text{ cm}^2}{2.00\text{ cm}^2}\right)(1.40\text{ m}) = 1.63\text{ m}$$

(ii) Stress $= \dfrac{F_{\perp}}{A} = \dfrac{T}{A}$

Brass : stress = $T/A = (4.00 \times 10^4\text{ N})/(2.00 \times 10^{-4}\text{ m}^2) = 2.00 \times 10^8$ Pa

Nickel : stress = $T/A = (4.00 \times 10^4\text{ N})/(1.00 \times 10^{-4}\text{ m}^2) = 4.00 \times 10^8$ Pa

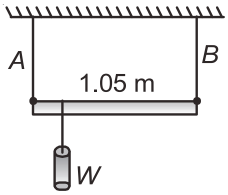
(iii) Y = stress/strain and strain = stress/Y

Brass : strain = $(2.00 \times 10^8\text{ Pa})/(9.0 \times 10^{10}\text{ Pa}) = 2.22 \times 10^{-3}$

Nickel : strain = $(4.00 \times 10^8\text{ Pa})/(21 \times 10^{10}\text{ Pa}) = 1.90 \times 10^{-3}$

Larger Y means less Δl and smaller A means greater Δl, so the two effects largely cancel and the lengths don't differ greatly. Equal Δl and nearly equal l means the strains are nearly the same. But equal tensions and A differing by a factor of 2 means the stresses differ by a factor of 2.

28. A 1.05 m long rod of negligible weight is supported at its ends by wires A and B of equal lengths as shown in figure. The cross-sectional area of A is 2.00 mm^2 and that of B is 4.00 mm^2. Young's modulus for wire A is 1.80×10^{11} Pa; and that for B is 1.20×10^{11} Pa. At what point along the rod should a weight w be suspended to produce



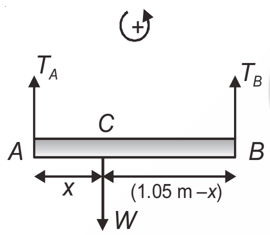
(i) Equal stress in A and B and

(ii) Equal strains in A and B?

Sol. (i) Stress = $F_{\perp}/A$, so equal stress implies T/A same for each wire.

$T_A/2.00\ mm^2 = T_B/4.00\ mm^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point C, a distance x to the right of wire A. The free-body diagram for the rod is given in the figure.



$$\sum \vec{\tau}_C = 0$$

$$+T_B(1.05\text{ m} - x) - T_A x = 0$$

But $T_B = 2.00T_A$ so $2.00T_A(1.05\text{ m} - x) - T_A x = 0$

$2.10\text{ m} - 2.00x = x$ and $x = 2.10\text{ m}/3.00 = 0.70$ m (measured from A).

(ii) Y = stress/strain gives that strain = stress/Y = $F_{\perp}/AY$.

Equal strain thus implies

$$\frac{T_A}{(2.00\text{ mm}^2)(1.80\times10^{11}\text{ Pa})} = \frac{T_B}{(4.00\text{ mm}^2)(1.20\times10^{11}\text{ Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A$$

The $\sum \vec{\tau}_C = 0$ equation still gives $T_B(1.05\text{ m} - x) - T_A x = 0$

But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\text{ m} - x) - T_A x = 0$

$1.40\text{ m} = 2.33x$ and $x = 1.40\text{ m}/2.33 = 0.60$ m (measured from A)

Wire B has twice the cross-sectional area so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller Y for B partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

29. Explain modulus of elasticity and its various forms.

Sol. Refer theory.

30. Discuss experimental determination of Young's modulus of a metallic wire.

Sol. Refer theory.

SECTION - B

Model Test Paper

Very Short Answer Type Questions :

1. Which type of strain is there, when a spiral spring is stretched by a force?

Sol. Longitudinal strain

2. A wire 2 m in length suspended vertically stretches by 1 mm when mass of 20 kg is attached to the lower end. What is the elastic potential energy gained by the wire? (g = 10 m/s^2)

Sol. Elastic potential energy of a stretched wire,

$$U = \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume}$$

3. Define yield point.

Sol. Refer theory.

4. What do you mean by elastic limit?

Sol. Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely.

5. Define Bulk modulus.

Sol. Refer theory.

6. State Hooke's law.

Sol. From the experimental study made by Hooke in connection with the extension produced in the wire and load applied, he formulated a law known as Hooke's law.

7. What is the SI unit of shear strain?

Sol. Unitless.

8. Which one is more elastic : Water or air?

Sol. Bulk modulus for water is more than for air.

Short Answer Type Questions :

9. Define perfectly elastic body. Also write its two examples.

Sol. A body which regains its original configuration immediately after the removal of deforming force from it, is called perfectly elastic body. Quartz and phosphor bronze.

10. Explain the term elasticity.

Sol. The property of a material body by virtue of which it regains its original configuration on the removal of the deforming force is called elasticity.

11. The length of a wire increases by 4 mm, when a weight of 8 kg is hung. If the radius of same wire is doubled, what will be increase in length?

Sol. As we know that, $\Delta l \propto \frac{1}{r^2}$

$$\Rightarrow \quad \frac{\Delta l_1}{\Delta l_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \quad \Delta l_2 = \Delta l_1 \cdot \left(\frac{r_1}{r_2}\right)^2$$

$$\Delta l_2 = 4 \times \left(\frac{1}{2}\right)^2$$

Δl_2 = 1 mm

12. Define Poisson's ratio.

Sol. The ratio of the lateral strain to the longitudinal strain is called the poisson's ratio (ν).

$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

It is unitless and dimensionless quantity.

13. Explain ductile materials.

Sol. These are those material which show large plastic range beyond elastic limit. Examples of ductile materials are copper, silver, iron, etc.

14. What do you mean by modulus of rigidity?

Sol. The ratio of shear stress (*i.e.*, tangential stress) to the shear strain is called the shear modulus or modulus of rigidity. It is denoted by *G*.

$$G = \frac{\sigma_s}{\theta}$$

where, $\sigma_s \rightarrow$ Shear stress, $\theta \rightarrow$ shear strain

The units of *G* are the same as those of stress.

15. Why do spring balances show wrong readings after they have been used for a long time?

Sol. When spring balances have been used for long time, they develop elastic fatigue in them. The springs of such balances take time to recover their original configuration.

Short Answer Type Questions :

16. A 5 cm cube has its upper face displaced by 0.2 cm by a tangential force of 8 N. Calculate the shearing strain and shearing stress.

Sol. Given L = 5 cm = 5 × 10^{-2} m, ΔL = 0.2 cm = 0.2 × 10^{-2} m, F = 8 N

Shearing strain = $\frac{\Delta L}{L} = \frac{0.5}{5} = 0.04$

Shearing stress = $\frac{F}{L \times L} = \frac{8}{(5 \times 10^{-2})^2} = 3200 \text{ N/m}^2$

17. Define compressibility of a material.

Sol. Bulk modulus (B) of a material measures its tendency to recover its original volume, *i.e.*, it is a measure of incompressibility of the body.

The reciprocal of bulk modulus, *i.e.*, $\frac{1}{B}$ is called the compressibility of the body. *i.e.*,

Compressibility $= \frac{1}{B} = -\frac{\Delta V/V}{\Delta P}$

Thus, compressibility is defined as the fractional decreases in volume per unit increase in pressure.

18. What do you mean by stress? Explain its various types.

Sol. The internal restoring force developed per unit area of a deformed body is called stress.

The stress is of two types :

(i) Normal stress : When the elastic force developed is perpendicular (normal) to the surface, the stress is called the normal stress. Normal stress is of two types

(a) Tensile stress

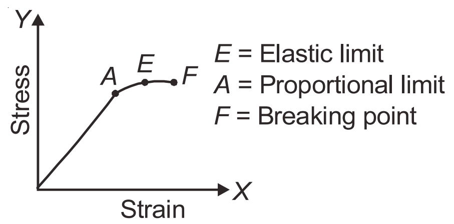
(b) Compressive stress

When there is an increase in length or volume, stress developed is tensile, while when there is a decrease in length or volume, stress is compressive.

(ii) Shearing stress : When the elastic force developed is parallel to the surface, the stress is called shearing or tangential stress.

19. Explain brittle materials. Also draw stress-strain curve for a brittle material.

Sol. These are those materials which show very small plastic range beyond elastic limit. For such materials, the breaking point lies close to the elastic limit. Such materials cannot be used in making the springs and sheets. Examples of brittle materials are cast iron, glass etc. The stress-strain curve for a brittle material is given below



20. Define elastic potential energy. Also write its various relation with stress, strain and volume.

Sol. Work has to be done by the applied force in deforming a body. This work done or energy spent is stored up in the body in the form of its potential energy. This energy is called elastic potential energy.

(i) Elastic potential energy = $\frac{1}{2}$ × stretching force × extension

(ii) Elastic potential energy per unit volume = $\frac{1}{2}$ × stress × strain

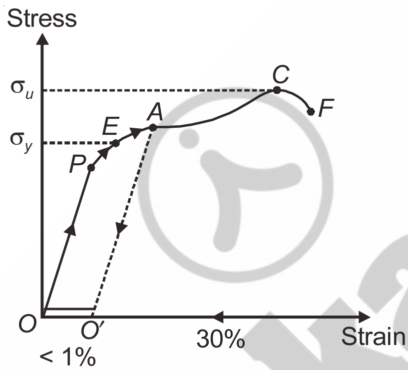
21. When we stretch a wire, we have to perform work, why?

Sol. On stretching the wire, its atoms are displaced from their respective positions and as such work is done against interatomic figure of attraction.

Long Answer Type Questions :

22. Explain stress-strain curve for a given material.

Sol. Typical stress-strain curve for a metal wire is shown in the figure. The stress is simple tensile stress and the strain is the percentage elongation.



P = Proportional limit
E = Elastic limit (or Yield point)
C = Tensile strength
F = Fracture point
OE = Elastic behaviour
EF = Plastic behaviour
OO′ = Permanent set

(i) During the first portion of the curve (*OP*) upto a strain of less than 1%, the stress and strain are proportional. The proportional relation between stress and strain in this region is called the Hooke's law.

(ii) From *P* to *E*, the stress and strain are not proportional to each other. But if the load is removed at any point between *O* and *E*, the curve will be retraced and the wire will be restored to its original length. In the region *OE*, the wire is said to be elastic or exhibits elastic behaviour. The point *E* is called the elastic limit or the yield point and the corresponding stress is called yield strength (σ_y).

(iii) If the wire is loaded further, the strain increases rapidly. But when the load is removed at some point beyond *E*, say at *A*, the wire does not regain its original length but traverses the dashed line *AO′*. The length at zero stress is now greater than the original length and the material is said to have a permanent set or permanent extension *OO′* (which is the strain corresponding to zero stress) is a measure of the permanent set.

(iv) Further increase in load beyond *A* produces a large increase in strain until a point *F* is reached at which the wire fractures. The point *F* is called the fracture point or the breaking point.

(v) From *E* to *F*, the wire is said to undergo plastic flow or plastic deformation. There is a marked change in the internal structure of the wire which is brought about by the slipping of crystal planes.

(vi) If large plastic deformation takes place between the elastic limit and the fracture point, metal is said to be ductile.

(vii) If fracture occurs soon after the elastic limit, the metal is said to be brittle.

(viii) The stress corresponding to the highest point *C* in the stress-strain curve (after which the strain increases even if the wire is unloaded, *i.e.*, wire literally flows) is called the tensile strength or ultimate strength (σ_u) of the metal.

23. Write important application of elasticity.

Sol. Application of elasticity

(i) Bridge and buildings : A bridge is so designed that apart from its own load, it should be able to withstand (a) the load of the traffic flowing over it and (b) the force of wind. For this we use beams and columns. Similarly in the design of buildings, beams and columns are used.

(ii) If A is the cross-sectional area of crane wire required to lift a load Mg, then

Stress on the rope $= \frac{Mg}{A}$

To prevent the permanent deforming of the wire, its yield strength (σ_y) should be greater than the stress acting on it, *i.e.*, $\sigma_y \geq \frac{Mg}{A}$ or $A \geq \frac{Mg}{\sigma_y}$

(iii) Hollow cylinders are preferred to solid cylinders for transmitting torque. A hollow rod is a better shaft than solid one of the same mass.

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Solutions (Set-2)

Objective Type Questions

(Elastic Behaviour of Solids)

1. Select the correct alternative(s)

   (1) Elastic forces are not always conservative

   (2) Elastic forces are always conservative

   (3) Elastic forces are conservative only when Hooke's law is obeyed

   (4) Elastic forces are not conservative

Sol. Answer (1)

Since at every value of force material is not able to gain its shape. Therefore elastic forces are not always conservative.

2. Which of the following affects the elasticity of a substance?

   (1) Change in temperature (2) Impurity in substance

   (3) Hammering (4) All of these

Sol. Answer (4)

Elasticity is hampered by change in temperature as it changes the structure of grains of the material. Impurity also changes elasticity.

By hammering also grain shape gets changes and effects elasticity.

3. Select the wrong definition

   (1) Deforming Force – force that changes configuration of body

   (2) Elasticity – property of regaining original configuration

   (3) Plastic body – which can be easily melted

   (4) Elastic limit – beyond which material begins to flow

Sol. Answer (3)

Plastic body is defined as a body which cannot regain its shape and size after deforming force is removed.

(Stress and Strain)

4. The shear strain is possible in

   (1) Solids (2) Liquids

   (3) Gases (4) All of these

Sol. Answer (1)

Shear strain is possible in solids only, as only solids have a definite surface.

5. The ratio of radii of two wires of same material is 2 : 1. If these wires are stretched by equal force, the ratio of stresses produced in them is

   (1) 2 : 1 (2) 1 : 2

   (3) 1 : 4 (4) 4 : 1

Sol. Answer (3)

We know,

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\left\{\begin{array}{l}\sigma = \text{Stress}\\ F = \text{Force}\\ A = \text{Area}\\ r = \text{radius}\end{array}\right\}$$

So, Stress × Area = Force

$\sigma \times A = F$

∵ (Since) Force applied on the wires is equal we can relate two conditions as

$$\left\{\begin{array}{l}\text{Where}\\ \sigma_1 - \text{Stress in } 1^{st} \text{ wire}\\ A_1 - \text{Area of } 1^{st} \text{ wire}\\ r_1 - \text{Radius of } 1^{st} \text{ wire}\\ \sigma_2 - \text{Stress in } 2^{nd} \text{ wire}\\ A_2 - \text{Area of } 2^{nd} \text{ wire}\\ r_2 - \text{Radius of } 2^{nd} \text{ wire}\end{array}\right\}$$

$\sigma_1 A_1 = \sigma_2 A_2$

$$\frac{\sigma_1}{\sigma_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{r^2}{(2r)^2} = \frac{r^2}{4r^2} = \frac{1}{4}$$

6. A steel wire of diameter 2 mm has a breaking strength of 4×10^5 N. What is the breaking force of similar steel wire of diameter 1.5 mm?

(1) 2.3×10^5 N (2) 2.6×10^5 N (3) 3×10^5 N (4) 1.5×10^5 N

Sol. Answer (1)

We know,

Breaking force = Breaking stress × area

As breaking stress is dependent on material.

So we can use

$$\frac{F_1}{F_2} = \frac{d_1^2}{d_2^2}$$

$$\left\{\begin{array}{l}F_1 = 4\times10^5 \text{ N}\\ d_1 = 2 \text{ mm}\\ F_2 = ?\\ d_2 = 1.5 \text{ mm}\end{array}\right\}$$

Substituting values

$$\frac{4\times10^5}{F_2} = \frac{(2)^2}{(1.5)^2}$$

$F_2 = 2.3 \times 10^5$ N

7. A steel wire is 1 m long and 1 mm^2 in area of cross-section. If it takes 200 N to stretch this wire by 1 mm, how much force will be required to stretch a wire of the same material as well as diameter from its normal length of 10 m to a length of 1002 cm?

(1) 1000 N

(2) 200 N

(3) 400 N

(4) 2000 N

Sol. Answer (3)

$$\frac{FL}{AY} = \Delta x$$

Since A, Y remain constant in given case

We can say

$FL \propto \Delta x$

or $\dfrac{F_1L_1}{F_2L_2} = \dfrac{\Delta x_1}{\Delta x_2}$

$$\begin{cases} F_1 = 200 \text{ N} \\ \Delta x_1 = 1 \text{ mm} \\ \Delta x_2 = 10.02 \text{ m} - 10 \text{ m} = 0.02 \text{ m} = 20 \text{ mm} \\ L_1 = 1 \text{ m} \\ L_2 = 10 \text{ m} \end{cases}$$

Substitute the values

$$\frac{200\times 1}{F_2 \times 10} = \frac{1 \text{ mm}}{20 \text{ mm}}$$

F_2 = 400 N

8. What is the percentage increase in length of a wire of diameter 2.5 mm, stretched by a force of 100 kg wt? Young's modulus of elasticity of wire = 12.5 × 10^{11} dyne/cm^2

(1) 0.16% (2) 0.32%

(3) 0.08% (4) 0.12%

Sol. Answer (1)

$Y = \dfrac{FL}{A\Delta L}$ $\Rightarrow$ Percentage increase $\dfrac{\Delta L}{L}\times 100 = \dfrac{F}{AY}\times 100$

Diameter = 2.5 mm $d = \dfrac{2.5}{1000}$ m

Area $= \dfrac{\pi d^2}{4} = \dfrac{\pi}{4}\left(\dfrac{2.5}{1000}\right)^2$ m^2 Y = 12.5 × 10^{11} dyne/cm^2 $\left\{\dfrac{1 \text{ dyne}}{\text{cm}^2} = \dfrac{0.1 \text{ N}}{\text{m}^2}\right\}$

$\Rightarrow$ F = 100 × 10 = 1000 N

$$\Rightarrow \frac{1000\times 100}{\dfrac{3.14\times(2.5)^2}{4\times(1000)^2}\times 12.5\times 10^{11}\times 0.1} = \frac{\Delta L}{L}\times 100$$

= 0.16%

9. The Poisson's ratio of a material is 0.5. If a force is applied to a wire of this material, there is a decrease in the cross-sectional area by 4%. The percentage increase in the length is

(1) 1% (2) 2% (3) 2.5% (4) 4%

Sol. Answer (4)

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \nu$$

$$-\left(\frac{\Delta r / r}{\Delta l / l}\right) = 0.5$$

Substitute $\Delta r/r$ = (–2/100)

$$\frac{\Delta l}{l} = \frac{4}{100}$$

$\because$ $A \propto r^2$

So $\dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r}$

$$-\frac{4}{100} = 2\times\frac{\Delta r}{r}$$

$$-\frac{2}{100} = \frac{\Delta r}{r}$$

$\therefore$ % increase $= \dfrac{\Delta l}{l}\times 100 = 4\%$

10. A uniform cubical block is subjected to volumetric compression, which decreases its each side by 2%. The Bulk strain produced in it is

(1) 0.03 (2) 0.02

(3) 0.06 (4) 0.12

Sol. Answer (3)

Volume = (side)^3

$V = (a)^3$

So $\frac{\Delta V}{V} = \frac{3\Delta a}{a}$ $\left\{\text{given } \frac{\Delta a}{a} = -2\%\right\}$

$\therefore$ $\frac{\Delta V}{V} = 3 \times -2$ Side decreases so we used (–)ve sign

= –6%

So bulk strain produced is 0.06

11. The Poisson's ratio cannot have a value of

(1) 0.7 (2) 0.2

(3) 0.1 (4) 0.5

Sol. Answer (1)

Poisson's ratios value can't be practically more than 1/2 so only value above 1/2 is 0.7

12. A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 3 × 10^{-3}, what will be percentage increase in volume?

(1) 2% (2) 3% (3) 5% (4) 0%

Sol. Answer (4)

$$\frac{\Delta V}{V} = (1 - 2\nu)\frac{\Delta L}{L}$$

As ν = 0.5; $\frac{\Delta V}{V} = 0$

(Hooke's Law)

13. Hooke's law is applicable for

(1) Elastic materials only (2) Plastic materials only

(3) Elastomers only (4) All of these

Sol. Answer (1)

Hooke's law is applicable only for elastic materials as only they follow the stress-strain proportionality.

14. When a load of 10 kg is suspended on a metallic wire, its length increase by 2 mm. The force constant of the wire is

(1) 3×10^4 N/m (2) 2.5×10^3 N/m (3) 5×10^4 N/m (4) 7.5×10^3 N/m

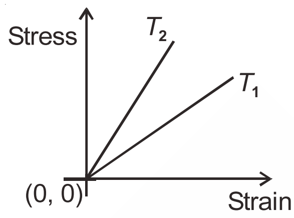
Sol. Answer (3)

Force constant $(K) = \dfrac{\text{Force}}{\text{Elongation}} = \dfrac{F}{\Delta x}$ $\quad \begin{cases} F = 10 \text{ kg} = 100 \text{ N} \\ \Delta x = 2 \text{ mm} = 0.002 \text{ m} \end{cases}$

Substituting values

$$K = \frac{100}{0.002} = 5 \times 10^4 \text{ N/m}$$

15. Figure shows graph between stress and strain for a uniform wire at two different temperatures. Then



(1) $T_1 > T_2$ (2) $T_2 > T_1$

(3) $T_1 = T_2$ (4) None of these

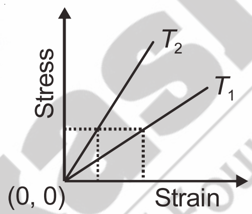
Sol. Answer (1)

From the graph we can see young's modulus is less for T_1 as compared to T_2

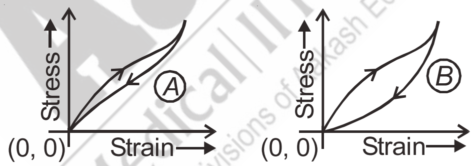
(Y = slope of stress-strain curve)

As T increases, Y decreases

So $T_1 > T_2$



16. Two different types of rubber are found to have the stress-strain curves as shown. Then



(1) A is suitable for shock absorber

(2) B is suitable for shock absorber

(3) B is suitable for car tyres

(4) None of these

Sol. Answer (2)

One with higher hysterysis loss suitable for shock absorber because high hysterysis loss will lead to dampen shocks in a easy manner.

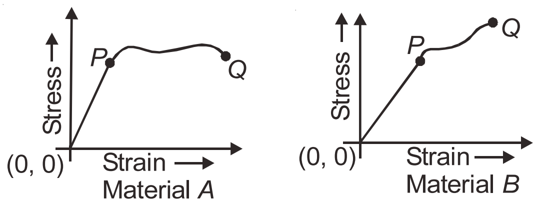
One with lower hysterysis loss suitable for tyres because it will have lesser energy dissipated into heat.

Area between loop gives amount of hysterysis loss. More area more loss, less area less loss.

Therefore, B is suitable for shock absorber and A for tyres.

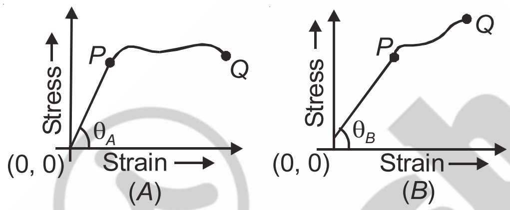
(Stress-Strain Curve)

17. The stress strain graphs for two materials *A* and *B* are shown in figure. The graphs are drawn to the same scale. Select the correct statement



(1) Material *A* has greater Young's Modulus
(2) Material *A* is ductile
(3) Material *B* is brittle
(4) All of these

Sol. Answer (4)



Slope of stress strain curve (tan θ) gives the value of young's modulus for given material

$\Rightarrow$ $\tan\theta = Y$

And from the graph we can clearly see

$\tan\theta_A > \tan\theta_B$

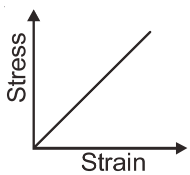
So material *A* has greater young's modulus

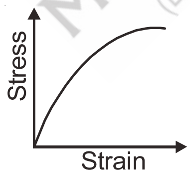
P to *Q* distance in material *A* is greater than *P* to *Q* distance in material *B*

Which implies more deformation is possible in *A* as compared to *B*

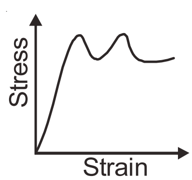
Hence we can say *A* is ductile, *B* is brittle.

18. Which of the following is the graph showing stress-strain variation for elastomers?

(1) 

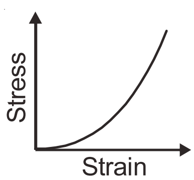
(2) 

(3) 

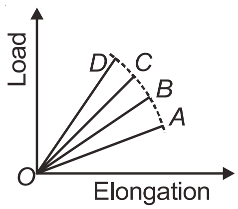
(4) 

Sol. Answer (3)

In elastomers stress varies exponentially with strain e.g. Rubber



19. The load versus elongation graph for four wires of same length and the same material is shown in figure. The thinnest wire is represented by line



(1) OC (2) OD

(3) OA (4) OB

Sol. Answer (3)

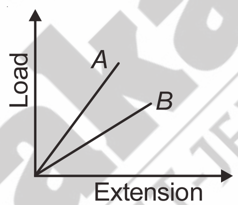
For the same load wire with maximum elongation has minimum cross-section area

As $\dfrac{FL}{AY} = \Delta x$

F, L, Y are fixed so $\dfrac{1}{A} \propto \Delta x$

$\Rightarrow$ OA is the thinnest.

20. In the given figure, if the dimensions of the two wires are same but materials are different, then Young's modulus is



(1) More for A than B

(2) More for B than A

(3) Equal for A and B

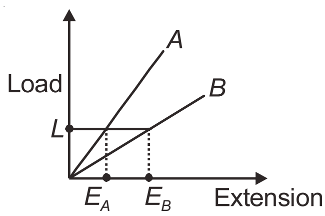
(4) None of these

Sol. Answer (1)

At same value of load

A has less elongation than B ...(1)

$\dfrac{FL}{AY} = \Delta L$ $\left\{\begin{array}{l} \because\ L, A \text{ are same} \\ F - \text{Load is also taken same} \end{array}\right\}$



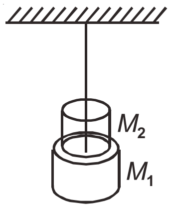
So $\dfrac{1}{Y} \propto \Delta L$ ...(2)

Using conditions (1) and (2)

We can say

$Y_A > Y_B$ {Young's modulus of A greater than B}

21. The length of wire, when M_1 is hung from it, is l_1 and is l_2 with both M_1 and M_2 hanging. The natural length of wire is



(1) $\frac{M_1}{M_2}(l_1 - l_2) + l_1$ (2) $\frac{M_2 l_1 - M_1 l_2}{M_1 + M_2}$ (3) $\frac{l_1 + l_2}{2}$ (4) $\sqrt{l_1 l_2}$

Sol. Answer (1)

Let the natural length of wire be = l

When only M_1 hanging

Using $\Delta l = \frac{FL}{AY}$

$$(l_1 - l) = \frac{M_1 g \cdot l}{AY} \quad ...(1)$$

When both M_1, M_2 hanging

$$(l_2 - l) = \frac{(M_1 + M_2)\, g \cdot l}{AY} \quad ...(2)$$

Dividing (1) by (2)

$$\frac{l_1 - l}{l_2 - l} = \frac{M_1}{M_1 + M_2}$$

Solving this we get

$$l = \frac{M_1}{M_2}(l_1 - l_2) + l_1$$

22. The substances having very short plastic region are

(1) Ductile (2) Brittle

(3) Malleable (4) All of these

Sol. Answer (2)

Substances with short plastic region are brittle because less amount of permanent deformation could be done in them.

(Elastic Moduli)

23. Due to addition of impurities, the modulus of elasticity

(1) Decreases (2) Increases

(3) Remains constant (4) May increase or decrease

Sol. Answer (4)

It depends on the elastic property of impurities if they themselves more elastic, elasticity will increase. If they are less elastic, elasticity will decrease.

24. A load of 2 kg produces an extension of 1 mm in a wire of 3 m in length and 1 mm in diameter. The Young's modulus of wire will be

(1) 3.25×10^{10} N m^{-2}
(2) 7.48×10^{12} N m^{-2}
(3) 7.48×10^{10} N m^{-2}
(4) 7.48×10^{-10} N m^{-2}

Sol. Answer (3)

We know

$$Y = \frac{F \times L}{A \times \Delta L}$$

Substituting values

$$Y = \frac{20 \times 3}{\pi \times \frac{1}{4} \times 10^{-6} \times 1 \times 10^{-3}}$$

$= 7.48 \times 10^{10}$ Nm^{-2}

25. Young's modulus depends upon

(1) Stress applied on material
(2) Strain produced in material
(3) Temperature of material
(4) All of these

Sol. Answer (3)

Young's modulus is a material property and it also depends on temperature of material.

26. The value of Young's modulus for a perfectly rigid body is

(1) 1
(2) Less than 1
(3) Zero
(4) Infinite

Sol. Answer (4)

For perfectly rigid body the condition is that there should not be any elognation ($\Delta L = 0$) for any value of force

So from the formulae we know $\frac{FL}{A \cdot \Delta L} = Y$

If we put $\Delta L = 0$

We get Y as ∞

27. A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 100 atm. The Bulk modulus of its material is

(1) 1.01×10^{11} N m^{-2}
(2) 1.01×10^{12} N m^{-2}
(3) 1.01×10^{10} N m^{-2}
(4) 1.0×10^{13} N m^{-2}

Sol. Answer (1)

We know $\frac{\Delta V}{V} = -\frac{P}{B}$

Substituting values

$$\frac{-\frac{0.01}{100} \times V}{V} = \frac{-100}{B} \times 1.01 \times 10^5$$ {1 atm = 1.01×10^5 Pa or N m^{-2}}

$B = 1.01 \times 10^{11}$ N m^{-2}

28. A metallic rod of length l and cross-sectional area A is made of a material of Young's modulus Y. If the rod is elongated by an amount y, then the work done is proportional to

(1) y (2) $\frac{1}{y}$ (3) y^2 (4) $\frac{1}{y^2}$

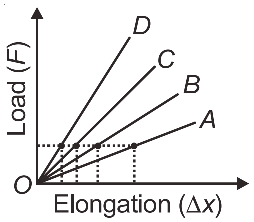
Sol. Answer (3)

Work done = energy stored

$$W = \frac{1}{2} \times \frac{\Delta L}{L} \times A \times Y \times \Delta L$$

$$W = \frac{1}{2}\frac{AY}{L} \times \Delta L^2$$

$$W \propto \Delta L^2$$



29. If the Bulk modulus of lead is 8.0 × 10^9 N/m^2 and the initial density of the lead is 11.4 g/cc, then under the pressure of 2.0 × 10^8 N/m^2, the density of the lead is

(1) 11.3 g/cc (2) 11.5 g/cc (3) 11.6 g/cc (4) 11.7 g/cc

Sol. Answer (4)

We know,

$$\rho_2 = \frac{\rho_1 B}{(B - P)}$$

Where
$P = 2 \times 10^8$ N/m^2
$B = 8 \times 10^9$ N/m^2
$\rho_1 = 11.4$ g/cc
$\rho_2 = ?$

Substituting the values

We get

$$\rho_2 = 11.7 \text{ g/cc}$$

30. When the temperature of a gas is constant at 20°C and pressure is changed from P_1 = 1.01 × 10^5 Pa to P_2 = 1.165 × 10^5 Pa, then the volume changes by 10%. The Bulk modulus of the gas is

(1) 1.55 × 10^5 Pa (2) 1.01 × 10^5 Pa (3) 1.4 × 10^5 Pa (4) 0.115 × 10^5 Pa

Sol. Answer (1)

$$\frac{\Delta V}{V} = \frac{-\Delta P}{B}$$

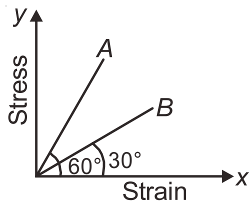
Substituting the values

$$\frac{-10}{100} = \frac{-(1.165 \times 10^6 - 1.01 \times 10^6)}{B}$$

$$\frac{1}{10} = \frac{.155 \times 10^5}{B}$$

B = 1.55 × 10^5 Pa

31. The stress versus strain graph for wires of two materials A and B are as shown in the figure. If Y_A and Y_B are the Young's moduli of the materials, then



(1) $Y_B = 2Y_A$ (2) $Y_A = 3Y_B$ (3) $Y_B = 3Y_A$ (4) $Y_A = Y_B$

Sol. Answer (2)

$Y = \tan\theta$

$Y_A = \tan 60°$, $Y_B = \tan 30°$

$= \sqrt{3}$ $= 1/\sqrt{3}$

$\Rightarrow Y_A = 3Y_B$

32. When a rubber ball is taken to the bottom of a sea of depth 1400 m, its volume decreases by 2%. The Bulk modulus of rubber ball is [density of water is 1 g cc and g = 10 m/s^2]

(1) 7×10^8 N/m^2 (2) 6×10^8 N/m^2

(3) 14×10^8 N/m^2 (4) 9×10^8 N/m^2

Sol. Answer (1)

Pressure at the bottom of sea = $\rho_w gh$ = 1000 kg/m^3 × 10 m/s^2 × 1400 m = 14000000 N/m^2

Also we know

$$\frac{\Delta V}{V} = -\frac{P}{B} \qquad \left\{\frac{\Delta V}{V} = \frac{-2}{100}\right\}$$

$$\Rightarrow \frac{-2}{100} = \frac{-14000000}{B} \Rightarrow B = 7 \times 10^8 \text{ N/m}^2$$

33. A spherical ball contracts in volume by 0.02%, when subjected to a normal uniform pressure of 50 atmosphere. The Bulk modulus of its material is

(1) 1×10^{11} N/m^2 (2) 2×10^{10} N/m^2

(3) 2.5×10^{10} N/m^2 (4) 1×10^{13} N/m^2

Sol. Answer (3)

$$\frac{\Delta V}{V} = -\frac{P}{B} \qquad \left\{\frac{\Delta V}{V} = \frac{-0.02}{100}\right\}$$

P = 50 atm = 50 × 1.01 × 10^5 Pa or N/m^2

So $B = 50 \times 1.01 \times 10^5 \times \dfrac{100}{0.02} = 2.5 \times 10^{10}$ N/m^2

34. Correct pair is

(1) Change in shape – Longitudinal strain

(2) Change in volume – Shear strain

(3) Change in length – Bulk strain

(4) Reciprocal of Bulk modulus – Compressibility

Sol. Answer (4)

B = bulk modulus and $\frac{1}{B}$ is defined as compressibility

(Applications of Elastic Behaviour of Materials)

35. The breaking stress of aluminium is 7.5×10^7 N m^{-2}. The greatest length of aluminium wire that can hang vertically without breaking is (Density of aluminium is 2.7×10^3 kg m^{-3})

(1) 283×10^3 m (2) 28.3×10^3 m

(3) 2.83×10^3 m (4) 0.283×10^3 m

Sol. Answer (3)

Breaking stress = $\rho \times g \times L_{max}$

Substitute values from the question

Breaking stress = 7.5×10^7 Nm^{-2}

$\rho = 2.7 \times 10^3$ kg m^{-3}

$g = 9.8$ m/s

$7.5 \times 10^7 = 2.7 \times 10^3 \times 9.8 \times L_{max}$

$$\frac{7.5 \times 10^7}{2.7 \times 10^3 \times 9.8} = L_{max}$$

2.83×10^3 m $= L_{max}$

36. A wire 2 m in length suspended vertically stretches by 10 mm when mass of 10 kg is attached to the lower end. The elastic potential energy gain by the wire is (take g = 10 m/s^2)

(1) 0.5 J (2) 5 J

(3) 50 J (4) 500 J

Sol. Answer (1)

Change in potential energy , $\Delta U = \frac{1}{2} \cdot F \cdot \Delta L$

Substituting values

$$\Delta U = \frac{1}{2} \times 100 \times \frac{10}{1000}$$

$\Delta U = 0.5$ J

37. A wire of length L and cross-sectional area A is made of material of Young's modulus Y. The work done in stretching the wire by an amount x is

(1) $\frac{YAx^2}{L}$ (2) $\frac{YAx^2}{2L}$ (3) $\frac{2YAx^2}{L}$ (4) $\frac{4YAx^2}{L}$

Sol. Answer (2)

$\because \quad W = \frac{1}{2}Fx$

and $Y = \frac{FL}{Ax}$

$\therefore \quad F = \frac{YAx}{L}$

$$W = \frac{1}{2}\left(\frac{YAx}{L}\right)x$$

$$W = \frac{1}{2}\frac{YAx^2}{L}$$

38. Two exactly similar wires of steel and copper are stretched by equal forces. If the total elongation is 2 cm, then how much is the elongation in steel and copper wire respectively? Given, Y_{steel} = 20 × 10^{11} dyne/cm^2, Y_{copper} = 12 × 10^{11} dyne/cm^2.

(1) 1.25 cm; 0.75 cm (2) 0.75 cm; 1.25 cm (3) 1.15 cm; 0.85 cm (4) 0.85 cm; 1.15 cm

Sol. Answer (2)

Let us say that elongation in copper = x

Than elongation in steel = 2 – x

We know

$$\frac{FL}{AY} = \Delta x$$

$\because \quad F, A, L$ are same only material is different

We can say

$$\frac{1}{Y} \propto \Delta x$$

$$\frac{Y_2}{Y_1} = \frac{\Delta x_1}{\Delta x_2}$$

$$\left\{\begin{array}{l}\text{Where}\\ Y_2 = Y_{steel}\\ Y_1 = Y_{copper}\\ \Delta x_1 = \text{elongation in copper} = x\\ \Delta x_2 = \text{elongation in steel} = 2 - x\end{array}\right.$$

Substituting values

$$\frac{20\times10^{11}}{12\times10^{11}} = \frac{x}{2-x}$$

$\Rightarrow \quad x$ = 1.25 cm

So Δx_{copper} = 1.25 cm, Δx_{steel} = 0.75 cm

39. A steel rod has a radius 10 mm and a length of 1.0 m. A force stretches it along its length and produces a strain of 0.32%. Young's modulus of the steel is 2.0×10^{11} N m^{-2}. What is the magnitude of the force stretching the rod?

(1) 100.5 kN (2) 201 kN (3) 78 kN (4) 150 kN

Sol. Answer (2)

Strain = 0.32%

$\Rightarrow \frac{\Delta L}{L} \times 100 = 0.32$

$\Rightarrow \frac{\Delta L}{L} = \frac{0.32}{100}$

$A = \pi r^2 = 3.14 \times \left(\frac{10}{1000}\right)^2$

$Y = 2 \times 10^{11}$ Nm^2

We know

$\frac{FL}{AY} = \Delta L$

$F = \left(\frac{\Delta L}{L}\right) \times A \times Y$

Substituting values

$F = \frac{0.32}{100} \times 3.14 \times \left(\frac{10}{1000}\right)^2 \times 2 \times 10^{11}$

F = 201 kN

40. The proportional limit of steel is 8×10^8 N/m^2 and its Young's modulus is 2×10^{11} N/m^2. The maximum elongation, a one metre long steel wire can be given without exceeding the proportional limit is

(1) 2 mm (2) 4 mm (3) 1 mm (4) 8 mm

Sol. Answer (2)

At proportional limit

Stress $\propto$ strain

Stress = $Y \times$ strain {Y = Young's Modulus}

Stress = $Y \times \frac{\Delta L}{L}$

$$\begin{cases} \text{Stress} = 8 \times 10^8 \text{ N/m}^2 \\ Y = 2 \times 10^{11} \text{ N/m}^2 \\ L = 1 \text{ m} \end{cases}$$

Substituting values

$\frac{8 \times 10^8 \times 1}{2 \times 10^{11}} = \Delta L$

4 mm = ΔL

41. In a series combination of copper and steel wires of same length and same diameter, a force is applied at one of their ends while the other end is kept fixed. The combined length is increased by 2 cm. The wires will have

(1) Same stress and same strain

(2) Different stress and different strain

(3) Different stress and same strain

(4) Same stress and different strain

Sol. Answer (4)

Stress = $\frac{F}{A}$

Force is same, A is same

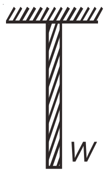
So same stress

Strain = $\frac{\Delta L}{L}$

L is same, but due to different young's modulus

ΔL would be different so strain is different

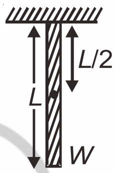
42. A rod of uniform cross-sectional area A and length L has a weight W. It is suspended vertically from a fixed support. If Young's modulus for rod is Y, then elongation produced in rod is



(1) $\frac{WL}{YA}$ (2) $\frac{WL}{2YA}$ (3) $\frac{WL}{4YA}$ (4) $\frac{3WL}{4YA}$

Sol. Answer (2)

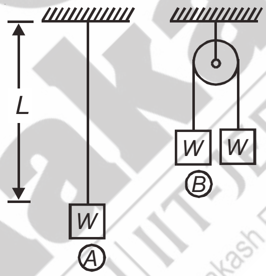
Center of mass is at $\frac{L}{2}$ distance from top so it can be assumed for easy calculation that W weight is hanged to a $\frac{L}{2}$ length string



Now use $\frac{FL}{AY} \cdot \Delta L$

$$\Delta L = \frac{W \times L}{2AY}$$

43. If in case A, elongation in wire of length L is l, then for same wire elongation in case B will be



(1) $4l$ (2) $2l$ (3) l (4) $l/2$

Sol. Answer (3)

Since tension in both cases is same and all other parametrs (Y, A, L) are also same

$\Rightarrow$ Elongation will be same in both cases.

44. Two wires A and B of same material have radii in the ratio 2 : 1 and lengths in the ratio 4 : 1. The ratio of the normal forces required to produce the same change in the lengths of these two wires is

(1) 1 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Sol. Answer (1)

From $\frac{FL}{AY} = \Delta x$ $\{\because \Delta x,\ Y \text{ same}\}$

We using $F \propto \frac{L}{A} \propto \frac{L}{r^2}$

So $\frac{F_1}{F_2} = \frac{L_1}{r_1^2} \times \frac{r_2^2}{L_2} = \left(\frac{L_1}{L_2}\right) \times \left(\frac{r_2}{r_1}\right)^2$

Substitute the ratio's

We get $\frac{F_1}{F_2} = \frac{1}{1}$ or $F_1 : F_2 :: 1 : 1$

45. Energy stored per unit volume in a stretched wire having Young's modulus Y and stress 'σ' is

(1) $\frac{Y\sigma}{2}$ (2) $\frac{\sigma^2 Y}{2}$ (3) $\frac{\sigma^2}{2Y}$ (4) $\frac{\sigma}{2Y}$

Sol. Answer (3)

$$\Delta U = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y}$$

$$= \frac{1}{2} \times \frac{\sigma^2}{Y}$$

46. A wire suspended vertically from one end is stretched by attaching a weight 200 N to the lower end. The weight stretches the wire by 1 mm. The elastic potential energy gained by the wire is

(1) 0.1 J (2) 0.2 J (3) 0.4 J (4) 10 J

Sol. Answer (1)

Elastic potential energy = $\frac{1}{2}$ × force × elongation

$$= \frac{1}{2} \times 200 \times \frac{1}{1000} = 0.1 \text{ J}$$

47. Work done by restoring force in a wire within elastic limit is –10 J. Maximum amount of heat produced in the wire is

(1) 10 J (2) 20 J (3) 5 J (4) 15 J

Sol. Answer (1)

Work done by external constant force = heat produced + potential energy

20 J = ΔH + 10 J

$\Rightarrow$ ΔH = 10 J

48. The work done per unit volume to stretch the length of area of cross-section 2 mm^2 by 2% will be

[$Y = 8 \times 10^{10}$ N/m^2]

(1) 40 MJ/m^3 (2) 16 MJ/m^3 (3) 64 MJ/m^3 (4) 32 MJ/m^3

Sol. Answer (2)

Work done per unit volume in stretching

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} Y \times (\text{Strain})^2$$

$$= \frac{1}{2} \times 8 \times 10^{10} \times \left(\frac{2}{100}\right)^2$$

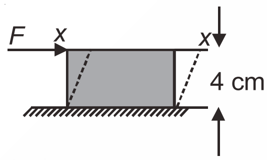
= 16 MJ/m^3

Work done per unit volume

Substitute values $= 8 \times 10^{10} \cdot \left(\frac{2}{100}\right)^2 \times \frac{1}{2}$

= 16 MJ/m^3

49. A steel plate of face area 1 cm^2 and thickness 4 cm is fixed rigidly at the lower surface. A tangential force F = 10 kN is applied on the upper surface as shown in the figure. The lateral displacement x of upper surface w.r.t. the lower surface is (Modulus of rigidity for steel is 8×10^{11} N/m^2)



(1) 5×10^{-5} m

(2) 5×10^{-6} m

(3) 2.5×10^{-3} m

(4) 2.5×10^{-4} m

Sol. Answer (2)

Modulus of rigidity (G) $= \frac{\text{Force} \times \text{Length}}{\text{Area} \times \text{Lateral displacement}} = \frac{FL}{A \times \Delta x}$

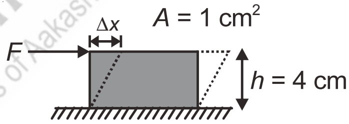
F = 10 kN = 10×10^3 N

L = 4 cm = 0.04 m

A = 1 cm^2 = 1×10^{-4} m^2

G = 8×10^{11} N/m^2

Substituting values



$$8 \times 10^{11} = \frac{10 \times 10^3 \times 0.04}{1 \times 10^{-4} \times \Delta x}$$

$$\Delta x = \frac{10 \times 10^3 \times 0.04}{1 \times 10^{-4} \times 8 \times 10^{11}} = 5 \times 10^{-6} \text{ m}$$

50. When a uniform metallic wire is stretched the lateral strain produced in it is β. If ν and Y are the Poisson's ratio and Young's modulus for wire, then elastic potential energy density of wire is

(1) $\frac{Y\beta^2}{2}$

(2) $\frac{Y\beta^2}{2\nu^2}$

(3) $\frac{Y\nu\beta^2}{2}$

(4) $\frac{Y\nu^2}{2\beta}$

Sol. Answer (2)

β = Strain (lateral)

ν = Poisson's ratio

Y = Young's modulus

Elastic potential energy density = $\frac{1}{2} \times Y \times (\text{strain longitudinal})^2$ ...(1)

Also $\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \text{Poisson's ratio}$

$$\frac{\beta}{\text{Longitudinal strain}} = \nu$$

$\Rightarrow$ Longitudinal strain $= \frac{\beta}{\nu}$

Substituting the value in equation (1)

$$E.P.E = \frac{1}{2} Y \times \left(\frac{\beta}{\nu}\right)^2 = \frac{Y\beta^2}{2\nu^2}$$

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