Chapter 9

Level - II

Mechanical Properties of Solids

Solutions

SECTION - A

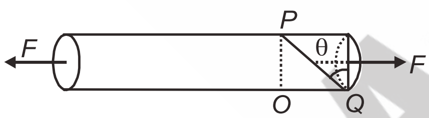
Objective Type Questions

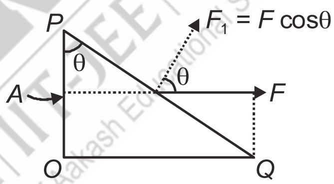
(Stress and Strain)

1. Two equal and opposite forces each of magnitude F is applied along a rod of transverse sectional area A. The normal stress to a section PQ inclined θ to transverse section is

(1) $\dfrac{F\sin\theta}{A}$ (2) $\dfrac{F}{A}\cos\theta$ (3) $\dfrac{F}{2A}\sin 2\theta$ (4) $\dfrac{F}{A}\cos^2\theta$

Sol. Answer (4)





$$\text{Stress} = \frac{F_{\text{normal}}}{\text{Area}} = \frac{F_1}{\text{Area}}$$

$$= \frac{F\cos\theta}{A/\cos\theta}$$

$$= \frac{F}{A}\cos^2\theta$$

Cross-sectional area of $PO = A$

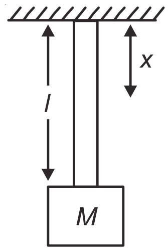
Cross-sectional area of $PQ = \dfrac{PO}{\cos\theta} = \dfrac{A}{\cos\theta}$

2. A vertical hanging bar of length l and mass m per unit length carries a load of mass M at lower end, its upper end is clamped to a rigid support. The tensile stress a distance x from support is ($A \to$ area of cross-section of bar)

(1) $\dfrac{Mg + mg(l - x)}{A}$ (2) $\dfrac{Mg}{A}$ (3) $\dfrac{Mg + mgl}{A}$ (4) $\dfrac{(M+m)gx}{Al}$

Sol. Answer (1)

$$\text{Tensile stress} = \frac{\text{Tension at point}}{\text{Area}}$$



Tension at distance x from top would be the amount of force acting due to all the weight below it

= Mass per unit length of rod × length of rod + Mg

= $m \times (l - x)\, g + Mg$

So Tensile stress $= \dfrac{m(l-x)g + Mg}{A}$

3. A wire of length 5 m is twisted through 30° at free end. If the radius of wire is 1 mm, the shearing strain in the wire is

(1) 30° (2) 0.36′

(3) 1° (4) 0.18°

Sol. Answer (2)

$$\theta = \frac{r}{L}\phi$$

$$\theta = \frac{1\times 10^{-3} \times 30^\circ}{5}$$

$$\theta = 6 \times 10^{-3}$$

$$= 0.36'$$

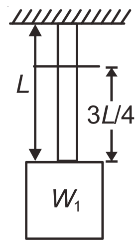
$\left\{\begin{array}{l}\text{Where}\\ \theta = \text{Angle of shear}\\ \phi = \text{Angle of twist}\\ r = \text{Radius of rod}\\ l = \text{length or rod}\end{array}\right.$

4. One end of uniform wire of length L and of weight W is attached rigidly to a point in roof and a weight W_1 is suspended from the lower end. If A is area of cross-section of the wire, the stress in the wire at a height $\frac{3L}{4}$ from its lower end is

(1) $\dfrac{W_1}{A}$ (2) $\dfrac{\left(W_1 + \dfrac{W}{4}\right)}{A}$ (3) $\dfrac{\left(W_1 + \dfrac{3W}{4}\right)}{A}$ (4) $\dfrac{W_1 + W}{A}$

Sol. Answer (3)

$$\text{Stress} = \frac{\text{Tension at point}}{\text{Area of cross-section}}$$



Tension = force due to weight hanging below the choosen point

That is $\left(\dfrac{3W}{4} + W_1\right)$

Stress $= \dfrac{3W/4 + W_1}{A}$

5. What is called the ratio of the breaking stress and the working stress?

(1) Elastic fatigue (2) Elastic after effect (3) Yield point (4) Factor of safety

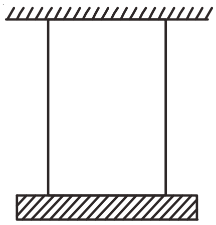
Sol. Answer (4)

$$\frac{\text{Breaking stress}}{\text{Working stress}} = n$$

n = Factor of safely

(Elastic Moduli)

6. Two wires of equal length and cross-sectional area are suspended as shown in figure. Their Young's modulii are Y_1 and Y_2 respectively. The equivalent Young's modulii will be

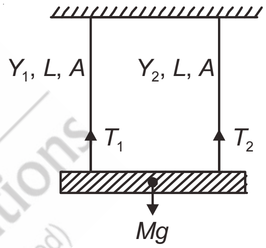


(1) $Y_1 + Y_2$ (2) $\frac{Y_1 Y_2}{Y_1 + Y_2}$ (3) $\frac{Y_1 + Y_2}{2}$ (4) $\sqrt{Y_1 Y_2}$

Sol. Answer (3)

Forces acting on both wires would be equal to T_1 and T_2 respectively by free body diagram

Let equivalent force constant of wire = K



$K_1 + K_2 = K$ $\left[\begin{array}{l}\text{Crossectional area will double}\\ \text{when both wires taken together}\end{array}\right]$

$$\frac{AY_1}{L} + \frac{AY_2}{L} = \frac{2AY}{L} \Rightarrow Y = \frac{Y_1 + Y_2}{2}$$

7. A uniform rod of length L has a mass per unit length λ and area of cross-section A. If the Young's modulus of the rod is Y. Then elongation in the rod due to its own weight is

(1) $\frac{2\lambda g L^2}{AY}$ (2) $\frac{\lambda g L^2}{AY}$ (3) $\frac{\lambda g L^2}{4AY}$ (4) $\frac{\lambda g L^2}{2AY}$

Sol. Answer (4)

Total mass can be assumed to be concentrated at center of mass at distance $\frac{L}{2}$ from top

$$\frac{M}{L} = \lambda$$

$$M = \lambda L$$

$$\Delta x = \frac{FL/2}{AY} = \frac{1}{2} \times \frac{\lambda L^2 g}{AY}$$

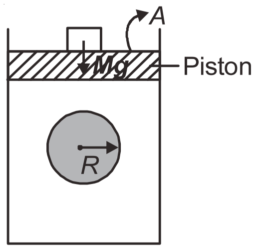
8. A solid sphere of radius R made of a material of bulk modulus B surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid. Find the fractional decrease in the radius of the sphere $\left(\frac{\Delta R}{R}\right)$ when a mass M is placed on the piston to compress the liquid

(1) $\frac{Mg}{AB}$ (2) $\frac{Mg}{4AB}$ (3) $\frac{Mg}{3AB}$ (4) $\frac{Mg}{2AB}$

Sol. Answer (3)

Pressure increased to weight M

$$P = \frac{\text{Force}}{\text{Area}} = \frac{Mg}{A} \quad ...(1)$$



And we know,

$$\frac{-\Delta V}{V} = \frac{P}{B}$$

$$\frac{-\Delta V}{V} = \frac{Mg}{AB} \quad \left[\because P = \frac{Mg}{A}\right] \quad ...(2)$$

$\left\{\begin{array}{l}\text{Where } -\dfrac{\Delta V}{V} = \text{Fractional decrease in volume} \\ P = \text{Pressure increased} \\ B = \text{Bulk modulus}\end{array}\right.$

Volume of a sphere is $V = \frac{4}{3}\pi R^3$ $\left\{\begin{array}{l}\text{Where} \\ r \text{ is radius}\end{array}\right\}$

$$\Rightarrow \frac{\Delta V}{V} = \frac{3\Delta R}{R}$$

Using (2)

$$-\frac{Mg}{3AB} = \frac{\Delta R}{R} \quad \text{[(–)ive sign indicates decrease]}$$

$\therefore$ Fractional decrease in radius is $\frac{Mg}{3AB}$

9. A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find Bulk modulus of the material of sphere

(1) 9.8×10^6 N/m^2 (2) 1.2×10^{10} N/m^2 (3) 9.8×10^{10} N/m^2 (4) 9.8×10^{11} N/m^2

Sol. Answer (3)

Pressure at bottom of sea = $\rho_w gh$

$\rho_w = 1000$ kg/m^3 = 1 g/cc, $g = 9.8$ m/s^2, $h = 1000$ m

$P = 10^3 \times 9.8 \times 1000$ N/m^2

Now $\frac{-\Delta V}{V} = \frac{P}{B}$ $\left\{\frac{-\Delta V}{V} = \frac{0.01}{100}\right.$ (given)

$$\frac{0.01}{100} = \frac{10^3 \times 9.8 \times 1000}{B}$$

$B = 9.8 \times 10^{10}$ N/m^2

10. A solid cube of copper of edge 10 cm subjected to a hydraulic pressure of 7×10^6 Pa. If Bulk modulus of copper is 140 GPa, then contraction in its volume will be

(1) 5×10^{-8} m^3 (2) 4×10^{-8} m^3

(3) 2×10^{-8} m^3 (4) 10^8 m^3

Sol. Answer (1)

Initial volume $V = (\text{side})^3 = (10 \times 10^{-2})^3 = 10^{-3}$ m^3

$P = 7 \times 10^6$ Pa

$B = 140 \times 10^9$ Pa

We know

$$\frac{-\Delta V}{V} = \frac{P}{B} \qquad \{-\Delta V = \text{Contraction in volume}\}$$

$$\frac{-\Delta V}{10^{-3}} = \frac{7\times10^6}{140\times10^9}$$

$-\Delta V = 5 \times 10^{-8}\ m^3$

11. Three bars having length l, $2l$ and $3l$ and area of cross-section A, $2A$ and $3A$ are joined rigidly end to end. Compound rod is subjected to a stretching force F. The increase in length of rod is (Young's modulus of material is Y and bars are massless)

(1) $\frac{13Fl}{2AY}$ (2) $\frac{Fl}{AY}$ (3) $\frac{9Fl}{AY}$ (4) $\frac{3Fl}{AY}$

Sol. Answer (4)

If extension of rod = x

$x = x_1 + x_2 + x_3$ Where x_1, x_2, x_3 are individual extensions in rod 1, 2, 3

$$x_1 = \frac{Fl}{AY},\ x_2 = \frac{2Fl}{2AY},\ x_3 = \frac{3Fl}{3AY}$$

So $x = \frac{3Fl}{AY}$

12. An ideal gas has adiabatic exponent γ. It contracts according to the law $PV = \alpha$, where α is a positive constant. For this process, the Bulk modulus of the gas is

(1) P (2) $\frac{P}{\alpha}$ (3) αP (4) $(1-\alpha)P$

Sol. Answer (1)

From $PV = \alpha$

$P\Delta V + V\Delta P = 0$

$$\Rightarrow P = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

$\Rightarrow P = B$

So $B = P$

13. Two wire A and B are stretched by same force. If, for A and B, $Y_A : Y_B = 1 : 2$, $r_A : r_B = 3 : 1$ and $l_A : l_B = 4 : 1$, then ratio of their extension $\left(\frac{\Delta l_A}{\Delta l_B}\right)$ will be

(1) 10 : 13 (2) 11 : 7 (3) 8 : 9 (4) 6 : 5

Sol. Answer (3)

$$\Delta x = \frac{FL}{AY}$$

For wire A

$$\Delta L_A = \frac{F\cdot L_A}{\pi r_A^2 \cdot Y_A} \quad ...(1)$$

For wire B

$$\Delta L_B = \frac{F\cdot L_B}{\pi r_B^2 \cdot Y_B} \quad ...(2)$$

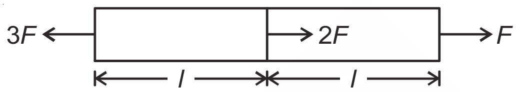
Divide (1) by (2)

$$\frac{\Delta L_A}{\Delta L_B} = \frac{F \cdot L_A}{\pi r_A^2 \cdot Y_A} \times \frac{\pi r_B^2 \cdot Y_B}{F \times L_B} = \frac{L_A}{L_B} \times \left(\frac{r_B}{r_A}\right)^2 \times \frac{Y_B}{Y_A}$$

Substituting the value of ratio's

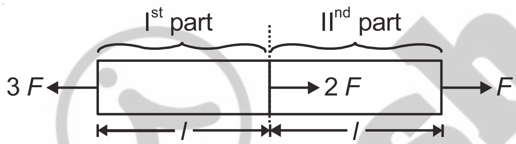
$$\frac{\Delta L_A}{\Delta L_B} = \frac{4}{1} \times \left(\frac{1}{3}\right)^2 \times \frac{2}{1} = \frac{8}{9}$$

14. A bar is subjected to axial forces as shown. If E is the modulus of elasticity of the bar and A is its cross-section area. Its elongation will be



(1) $\frac{Fl}{AE}$ (2) $\frac{2Fl}{AE}$ (3) $\frac{3Fl}{AE}$ (4) $\frac{4Fl}{AE}$

Sol. Answer (4)



Elongation in Ist part,

$$\Delta x_1 = \frac{3Fl}{AE}$$

Elongation in IInd part,

$$\Delta x_2 = \frac{Fl}{AE}$$

So, Net elongation, $\Delta x = \Delta x_1 + \Delta x_2 = \frac{3Fl}{AE} + \frac{FL}{AE} = \frac{4Fl}{AE}$

15. A metal ring of initial radius r and cross-sectional area A is fitted onto a wooden disc of radius $R > r$. If Young's modulus of metal is Y then tension in the ring is

(1) $\frac{AYR}{r}$ (2) $\frac{AY(R-r)}{r}$ (3) $\frac{Y}{A}\left(\frac{R-r}{r}\right)$ (4) $\frac{Yr}{AR}$

Sol. Answer (2)

r – radius of metal ring

R – radius of wooden disc

Given, $R > r$

So $2\pi R > 2\pi r$

To get the metal ring fitted on wooden disc the circumfrence should be increased by $(2\pi R - 2\pi r)$ of metal ring

$\therefore \quad \Delta L = 2\pi(R - r)$

F = tension developed in ring

$$\therefore \quad 2\pi(R-r) = \frac{T(2\pi r)}{AY} \qquad \left(\Delta L = \frac{FL}{AY}\right)$$

$$\frac{AY(R-r)}{r} = T$$

16. Two wires A and B of same length and of same material have radii r_1 and r_2 respectively. Their one end is fixed with a rigid support and at other end equal twisting couple is applied. Then ratio of the angle of twist at the end of A and the angle of twist at the end of B will be

(1) $\frac{r_1^2}{r_2^2}$ (2) $\frac{r_2^2}{r_1^2}$ (3) $\frac{r_2^4}{r_1^4}$ $\frac{r_1^4}{r_2^4}$

Sol. Answer (3)

$$r_1^4\,\phi_A = r_2^4\,\phi_B$$

$$\frac{\phi_A}{\phi_B} = \frac{(r_B)^4}{(r_A)^4} = \left(\frac{r_2}{r_1}\right)^4$$

17. For a given material, the Young's modulus is 2.4 times its modulus of rigidity. Its Poisson's ratio is

(1) 0.2 (2) 0.4 (3) 1.2 (4) 2.4

Sol. Answer (1)

$Y = 2G\,[1 + \nu]$

$\Rightarrow \quad 2.4G = 2G\,[1 + \nu]$

$\Rightarrow \quad 0.2 = \nu$

$\left\{\begin{array}{l}\text{Where}\\ Y = \text{Young's modulus}\\ G = \text{Poisson's ratio}\\ \nu = \text{Modulus of rigidity}\end{array}\right.$

18. The ratio of adiabatic to isothermal elasticity of a diatomic gas is

(1) 1.67 (2) 1.4 (3) 1.33 (4) 1.27

Sol. Answer (2)

$K_{\text{isothermal}}$ = Pressure of gas (P)

$K_{\text{adiabatic}} = \gamma \times$ pressure of gas $(\gamma \cdot P)$

$$\text{Ratio} = \frac{\gamma P}{P} = \gamma$$

$$\gamma \text{ of diatomic gas} = \frac{7}{5} = 1.4$$

19. For an elastic material

(1) $Y > G$ (2) $Y < G$

(3) $YG = 1$ (4) $Y = G$

Sol. Answer (1)

Y = Young's modulus

G = modulus of rigidity

We have a formulae

$Y = 2G[1 + \nu]$

Since $0 < \nu \le 0.5$

Using maximum value of ν

$Y = 3G$

$\Rightarrow$ $Y > G$

(Applications of Elastic Behaviour of Materials)

20. When a small mass m is suspended at lower end of an elastic wire having upper end fixed with ceiling. There is loss in gravitational potential energy, let it be x, due to extension of wire, mark correct option

(1) The lost energy can be recovered

(2) The lost energy is irrecoverable

(3) Only $\frac{x}{2}$ amount of energy is recoverable

(4) Only $\frac{x}{3}$ amount of energy is recoverable

Sol. Answer (3)

ΔU (loss in gravitational potential energy) = $mg \times \Delta l$

$\Delta U = x$ (given)

So $x = mg \times \Delta l$ $\left\{\begin{array}{l}\text{Where} \\ m = \text{mass suspended} \\ \Delta l = \text{elongation in wire}\end{array}\right.$

Elastic potential energy gained $= \frac{1}{2} \times \text{Force} \times \text{Elongation}$

$$= \frac{1}{2} \times Mg \times \Delta l$$

$$= \frac{1}{2} Mg \times \Delta l \qquad [\because\ Mg\Delta l = x]$$

$$= \frac{1}{2} x$$

So only $\frac{x}{2}$ amount of energy is recoverable which is stored as elastic potential energy in wire.

21. A mild steel wire of length $2l$ meter cross-sectional area A m^2 is fixed horizontally between two pillars. A small mass m kg is suspended from the mid point of the wire. If extension in wire are within elastic limit. Then depression at the mid point of wire will be

(1) $\left(\frac{Mg}{YA}\right)^{1/3}$ (2) $\left(\frac{Mg}{lA}\right)^{1/3}$ (3) $\left(\frac{Mgl^3}{YA}\right)^{1/3}$ (4) $\frac{Mg}{2YA}$

Sol. Answer (3)

Let $OC = x$ [depression]

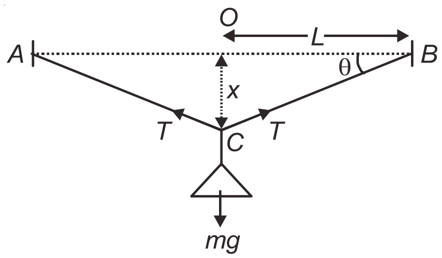
and θ be small angle

$\because$ x is small

ΔL (Extension in OB part of wire) = $BC - OB$

$BC = (L^2 + x^2)^{1/2}$ and $OB = L$

$\Delta L = \{(L^2 + x^2)^{1/2} - L\}$



We know $F = \dfrac{YA\,\Delta L}{L}$

So $F = \dfrac{YA}{L}\left\{(L^2 + x^2)^{1/2} - L\right\}$

$$= YA\left\{\left(1 + \frac{x^2}{L^2}\right)^{1/2} - 1\right\}$$

$$= YA\left\{1 + \frac{x^2}{2L^2} - 1\right\}$$

$\left\{\begin{array}{l}\text{Using binomial expansion} \\ \dfrac{x^2}{L^2} << 1\end{array}\right.$

$$F = \frac{YAx^2}{2L^2}$$

So $\left(1 + \dfrac{x^2}{L^2}\right)^{1/2} \simeq 1 + \dfrac{x^2}{2L^2}$

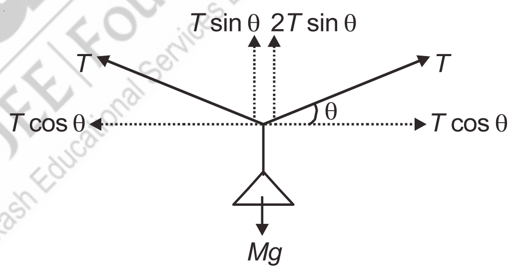
Tension in each part of wire will be equal to 'F'

By vertical equilibrium

$Mg = 2T\sin\theta$

$= 2T\,\theta$ $\quad$ {If θ is small. So $\sin\theta \simeq \theta$}

$\because$ $T = F$



$$Mg = 2 \times \frac{YAx^2}{2L^2} \times \theta$$

$\left\{\begin{array}{l}\text{in } \Delta\, OBC \\ \dfrac{x}{L} = \tan\theta\end{array}\right.$

$$Mg = 2 \times \frac{YAx^2}{2L^2} \times \frac{x}{L}$$

$\because$ θ is small, $\tan\theta \simeq \sin\theta \simeq \theta$

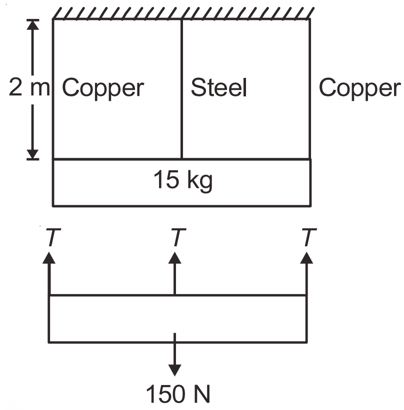
We get,

$$\left(\frac{MgL^3}{YA}\right)^{1/3} = x$$

22. A rigid bar of mass 15 kg is supported symmetrically by three wire each of 2 m long. These at each end are of copper and middle one is of steel. Young's modulus of elasticity for copper and steel are 110×10^9 N/m^2 and 190×10^9 N/m^2 respectively. If each wire is to have same tension, ratio of their diameters will be

(1) $\sqrt{\dfrac{11}{19}}$ (2) $\sqrt{\dfrac{19}{11}}$ (3) $\sqrt{\dfrac{30}{11}}$ (4) $\sqrt{\dfrac{11}{30}}$

Sol. Answer (2)



Tension is same (given)

From free body diagram

$3T = 150$ N

$T = 50$ N

Since the bar has to be supported symmetrically

Therefore extension in each wire will be same

We know $\Delta x = \dfrac{FL}{AY}$

Compare 1 copper wire with another steel wire

$$\frac{FL}{A_C Y_C} = \frac{FL}{A_S Y_S}$$

Where,
- A_C – Area of copper wire
- Y_C – Young's modulus copper
- A_S – Area of steel wire
- Y_S – Young's modulus steel

$$\Rightarrow \frac{A_S}{A_C} = \frac{Y_C}{Y_S}$$

Substitutuing value of Y_C and Y_s

$$\frac{d_S^2}{d_C^2} = \frac{110\times10^9}{190\times10^9}$$

$$\frac{d_S}{d_C} = \sqrt{\frac{11}{19}}$$

- d_S – diameter of steel wire
- d_C – diameter of copper wire

$$\frac{d_C}{d_S} = \sqrt{\frac{19}{11}}$$

23. The strain energy stored in a body of volume V due to shear strain ϕ is (shear modulus is G)

(1) $\dfrac{\phi^2 V}{2G}$ (2) $\dfrac{\phi V^2}{2G}$ (3) $\dfrac{\phi^2 V}{G}$ (4) $\dfrac{1}{2}G\phi^2 V$

Sol. Answer (4)

$$\text{Shear modulus} = \frac{\text{Shear stress}}{\text{Shear stress}}$$

$$G = \frac{\text{Shear stress}}{\phi}$$

$G\phi$ = Shear stress

Strain energy per unit volume = $\dfrac{1}{2}$ × shear stress × shear strain

$$\Rightarrow \frac{\text{Strain energy}}{\text{Volume}} = \frac{1}{2}\times G\phi\times\phi$$

Strain energy = $\dfrac{1}{2}G\phi^2 V$

24. A metal wire having Poisson's ratio 1/4 and Young's modulus 8×10^{10} N/m^2 is stretched by a force, which produces a lateral strain of 0.02% in it. The elastic potential energy stored per unit volume in wire is [in J/m^3]

(1) 2.56×10^4 (2) 1.78×10^6

(3) 3.72×10^2 (4) 2.18×10^5

Sol. Answer (1)

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \text{Poisson's ratio}$$

$$\frac{0.02/100}{\Delta l / l} = \frac{1}{4}$$

$$\frac{\Delta l}{l} = \frac{0.08}{100}$$

$$\begin{cases} Y = \text{(Young's modulus)} \\ \quad = 8 \times 10^{10} \quad \text{(given)} \\ \text{Poission's ratio} = \frac{1}{4} \quad \text{(given)} \\ \text{Lateral strain} = 0.02\% \quad \text{(given)} \end{cases}$$

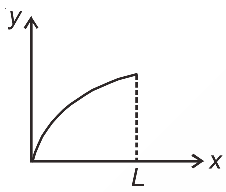
ΔU (Elastic potential energy per unit volume = $\frac{1}{2} \times Y \times (\text{Longitudinal strain})^2$

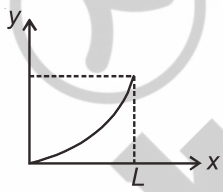
Substituting values

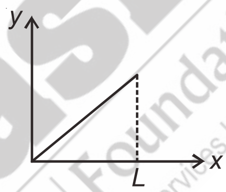
$$\Delta U = \frac{1}{2} \times 8 \times 10^{10} \times \left(\frac{0.08}{100}\right)^2$$

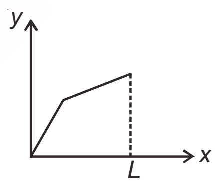
$$\Delta U = 2.56 \times 10^4 \text{ J/m}^3$$

25. Which of the following curve represents the correctly distribution of elongation (y) along heavy rod under its own weight $L \rightarrow$ length of rod, $x \rightarrow$ distance of point from lower end?

(1) 

(2) 

(3) 

(4) 

Sol. Answer (2)

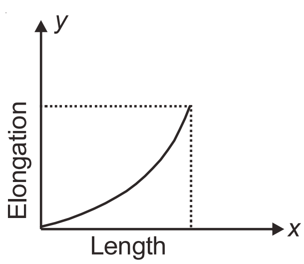
For elongation of rod under its own weight

We know $\Delta x = \frac{\rho g x^2}{2Y}$

We can clearly see that elongation $\propto (x^2)$

So graph of Δx *vs* x should be a upward parabola.

$$\begin{cases} \text{Where,} \\ \Delta x = \text{Elongation} \\ \rho = \text{Density of rod} \\ Y = \text{Young's modulus} \\ L = \text{Length} \\ g = \text{Acceleration due to gravity} \\ x = \text{Distance of point from lower end} \end{cases}$$



26. A wire can sustain a weight of 15 kg. If it cut into four equal parts, then each part can sustain a weight

(1) 5 kg
(2) 45 kg
(3) 15 kg
(4) 30 kg

Sol. Answer (3)

$$\text{Stress} = \frac{F}{A}$$

So $\text{Stress} \propto \frac{1}{A}$

Since, we are not reducing the crossectional area of the wire. Therefore each part can still sustain same force i.e, 15 kg weight.

27. The normal density of gold is ρ and its modulus is B. The increase in density of piece of gold when pressure P is applied uniformly from all sides

(1) $\frac{\rho P}{2B}$ (2) $\frac{\rho B}{2P}$ (3) $\frac{\rho P}{B-P}$ (4) $\frac{\rho B}{B-P}$

Sol. Answer (3)

We know

$$\frac{\Delta V}{V} = \frac{P}{B} \quad ...(1) \qquad \begin{cases} \Delta V - \text{Change in volume} \\ P - \text{Pressure applied} \\ B - \text{Bulk modulus} \end{cases}$$

And $\rho = \frac{M}{V}$ ...(2) $\begin{cases} \rho = \text{Density} \\ M = \text{Mass} \\ V = \text{Volume} \end{cases}$

From (2)

$$\Delta\rho = \frac{M}{V - \Delta V} - \frac{M}{V}$$

$$\Delta\rho = \frac{M}{V} \times \frac{\Delta V}{V - \Delta V}$$

$$\Delta\rho = \rho \times \frac{1}{\frac{V}{\Delta V} - 1} \quad \text{[From eq. (2)]}$$

$$\Delta\rho = \rho \times \frac{1}{\frac{B}{P} - 1} \quad \text{[From eq. (1)]}$$

$$\Delta\rho = \frac{\rho P}{B - P}$$

28. A uniform wire of length L and radius r is twisted by an angle α. If modulus of rigidity of the wire is G, then the elastic potential energy stored in wire, is

(1) $\frac{\pi G r^4 \alpha}{2L^2}$ (2) $\frac{\pi G r^4 \alpha^2}{4L}$ (3) $\frac{\pi G r^4 \alpha}{4L^2}$ (4) $\frac{\pi G r^4 \alpha^2}{2L}$

Sol. Answer (2)

$$U = \text{Work done} = \frac{1}{2} C\phi^2, \quad \text{where } C = \frac{\pi G r^4}{2L} \text{ (torsional constant)}$$

$$= \frac{\pi G r^4 \phi^2}{4L} \quad (\phi = \text{angle of twist} = \alpha,\ G = \text{Modulus of rigidity})$$

Substituting values, $U = \frac{\pi \eta r^4 \alpha^2}{4L}$

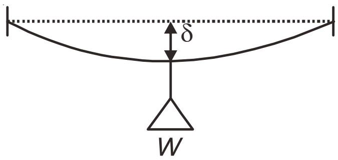
29. If δ is the depression produced in a beam of length L, breadth b and thickness d, when a load is placed at the mid point, then

(1) $\delta \propto L^3$ (2) $\delta \propto \frac{1}{b^3}$ (3) $\delta \propto \frac{1}{d}$ (4) All of these

Sol. Answer (1)

Since we know

$$\delta = \frac{WL^3}{4Y\,bd^3}$$



So we can say $\delta \propto L^3$

SECTION - B

Previous Years Questions

1. The stress-strain curves are drawn for two different materials X and Y. It is observed that the ultimate strength point and the fracture point are close to each other for material X but are far apart for material Y.

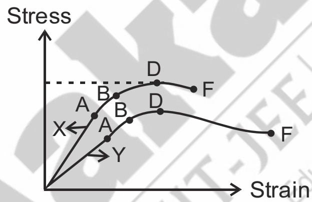
We can say that materials X and Y are likely to be (respectively), **[NEET-2019]**

(1) Plastic and ductile (2) Ductile and brittle (3) Brittle and ductile (4) Brittle and plastic

Sol. Answer (3)

As given that fracture point and ultimate strength point is close for material X, hence X is brittle in nature and both points are far apart for material Y hence it is ductile.

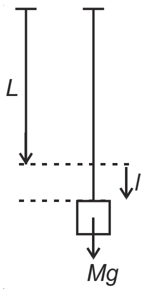
i.e X is brittle and Y is ductile in nature.



2. When a block of mass M is suspended by a long wire of length L, the length of the wire becomes $(L + l)$. The elastic potential energy stored in the extended wire is : **[NEET-2019]**

(1) Mgl (2) MgL (3) $\frac{1}{2}Mgl$ (4) $\frac{1}{2}MgL$

Sol. Answer (3)



$U = \frac{1}{2}$ (work done by gravity)

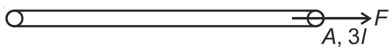
$U = \frac{1}{2}Mgl$

3. Two wires are made of the same material and have the same volume. The first wire has cross-sectional area A and the second wire has cross-sectional area 3*A*. If the length of the first wire is increased by Δl on applying a force *F*, how much force is needed to stretch the socond wire by the same amount? **[NEET-2018]**

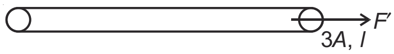
(1) 9*F*   (2) 6*F*   (3) *F*   (4) 4*F*

Sol. Answer (1)

Wire 1 :



Wire 2 :



For wire 1, $\Delta l = \left(\dfrac{F}{AY}\right)3l$ …(i)

For wire 2, $\dfrac{F'}{3A} = Y\dfrac{\Delta l}{l}$

$\Rightarrow \quad \Delta l = \left(\dfrac{F'}{3AY}\right)l$ …(ii)

From equation (i) & (ii),

$$\Delta l = \left(\frac{F}{AY}\right)3l = \left(\frac{F'}{3AY}\right)l \Rightarrow \boxed{F' = 9F}$$

4. The bulk modulus of a spherical object is *B*. If it is subjected to uniform pressure *p*, the fractional decrease in radius is **[NEET-2017]**

(1) $\dfrac{p}{B}$ (2) $\dfrac{B}{3p}$

(3) $\dfrac{3p}{B}$ (4) $\dfrac{p}{3B}$

Sol. Answer (4)

$$B = \frac{p}{\left(\dfrac{\Delta V}{V}\right)}$$

$$\frac{\Delta V}{V} = \frac{p}{B}$$

$$3\frac{\Delta r}{r} = \frac{p}{B}$$

$$\frac{\Delta r}{r} = \frac{p}{3B}$$

5. The Young's modulus of steel is twice that of brass. Two wires of same length and of same area of cross section, one of steel and another of brass are suspended from the same roof. If we want the lower ends of the wires to be at the same level, then the weights added to the steel and brass wires must be in the ratio of **[Re-AIPMT-2015]**

(1) 1 : 1   (2) 1 : 2   (3) 2 : 1   (4) 4 : 1

Sol. Answer (3)

6. The approximate depth of an ocean is 2700 m. The compressibility of water is 45.4×10^{-11} Pa^{-1} and density of water is 10^3 kg/m^3. What fractional compression of water will be obtained at the bottom of the ocean?

[AIPMT-2015]

(1) 1.4×10^{-2} (2) 0.8×10^{-2} (3) 1.0×10^{-2} (4) 1.2×10^{-2}

Sol. Answer (4)

$$B = \frac{P}{\Delta V / V} \Rightarrow \frac{\Delta V}{V} = \frac{P}{B}$$

$$\frac{\Delta V}{V} = \frac{hdg}{B}$$

$$= \frac{2700 \times 10^3 \times 10}{\frac{1}{45.4 \times 10^{-11}}}$$

$$= 1.2 \times 10^{-2}$$

7. Copper of fixed volume V is drawn into wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. Which of the following graphs is a straight line? **[AIPMT-2014]**

(1) Δl versus $\frac{1}{l}$ (2) Δl versus l^2

(3) Δl versus $\frac{1}{l^2}$ (4) Δl versus l

Sol. Answer (2)

$$V = A \cdot L$$

$$Y = \frac{FL}{A\Delta L} \Rightarrow \Delta L = \frac{FL}{\frac{V}{L}Y}$$

$$\Delta L = \frac{FL^2}{VY}$$

$$\Rightarrow \Delta L \propto L^2$$

Thus, ΔL versus L^2 is straight line.

8. The following four wires of length L and radius r are made of the same material. Which of these will have the largest extension, when the same tension is applied? **[NEET-2013]**

(1) L = 400 cm, r = 0.8 mm (2) L = 300 cm, r = 0.6 mm

(3) L = 200 cm, r = 0.4 mm (4) L = 100 cm, r = 0.2 mm

Sol. Answer (4)

We know, $\Delta x = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y}$

$$\Rightarrow \Delta x \propto \frac{L}{r^2}$$

Δx directly proportional to L

And Δx inversely proportional to r^2

For option (1), $\frac{L}{r^2} = \frac{400 \times 10}{(0.8)^2} = 6250$

For option (2), $\frac{L}{r^2} = \frac{300 \times 10}{(0.6)^2} = 8333.33$

For option (3), $\frac{L}{r^2} = \frac{200 \times 10}{(0.4)^2} = 12,500$

For option (4), $\frac{L}{r^2} = \frac{100 \times 10}{(0.2)^2} = 25,000$

For option (4) we are getting maximum value of $\frac{L}{r^2}$

$\Rightarrow$ Δx also maximum for L = 100 cm and r = 0.2 mm

9. A rope 1 cm in diameter breaks, if the tension in it exceeds 500 N. The maximum tension that may be given to similar rope of diameter 3 cm is

(1) 500 N (2) 3000 N (3) 4500 N (4) 2000 N

Sol. Answer (3)

Tension $\propto$ (radius)^2

$$\left[\Delta x = \frac{FL}{\pi r^2 Y}\right]$$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^2$$

So $T \propto r^2$

Substituting values

$$\left.\begin{array}{l} T_1 = 500\text{ N} \\ r_1 = 1\text{ cm} \\ r_2 = 3\text{ cm} \end{array}\right\} \text{given}$$

Let $T_2 = x$

$$\frac{500}{x} = \frac{1^2}{3^2}$$

x = 4500 N

$\Rightarrow$ T_2 = 4500 N

10. A wire of length L and radius r fixed at one end and a force F applied to the other end produces an extension l. The extension produced in another wire of the same material of length $2L$ and radius $2r$ by a force $2F$, is

(1) l (2) $2l$ (3) $4l$ (4) $\frac{l}{2}$

Sol. Answer (1)

$$L = \frac{FL}{\pi r^2 Y} \quad ...(1) \qquad \left[\Delta x = \frac{FL}{AY}\right]$$

Now, new parameters

$F = 2F$

$L = 2L$

$r = 2r$

Substituting new parameters in eq. (1)

$$L' = \frac{2F \times 2L}{\pi(2r)^2 Y}$$

$$= \frac{FL}{\pi r^2 Y} \qquad \left\{\because \frac{FL}{\pi r^2 Y} = l \text{ from equation 1}\right\}$$

$L' = L$

11. The increase in pressure required to decrease the 200 L volume of a liquid by 0.008% in kPa is (Bulk modulus of the liquid = 2100 MPa is)

(1) 8.4 (2) 84 (3) 92.4 (4) 168

Sol. Answer (4)

V = 200 L

ΔV = –0.008% of 200 L (Decrease in volume so we use (–)ive sign)

$$= \frac{0.008}{100} \times 200 = -0.016 \text{ L}$$

B = 2100 MPa = 21 × 10^8 Pa

We know

$$\frac{-\Delta V}{V} = \frac{\Delta P}{B}$$

$$\frac{0.016}{200} = \frac{\Delta P}{21 \times 10^8}$$

168 × 10^3 Pa = ΔP

168 kPa = ΔP

12. Which of the following relations is true?

(1) $Y = 2G(1 - 2\nu)$ (2) $Y = 2G(1 + 2\nu)$

(3) $Y = 2G(1 - \nu)$ (4) $(1 + \nu)2G = Y$

Sol. Answer (4)

$Y = 2G(1 + \nu)$

$$\left\{\begin{array}{l}\text{Where,}\\ Y = \text{Young's modulus}\\ G = \text{Shear modulus}\\ \nu = \text{Poisson's ratio}\end{array}\right.$$

13. A 5 m long aluminium wire ($Y = 7 \times 10^{10}$ N m^{-2}) of diameter 3 mm supports a 40 kg mass. In order to have the same elongation in the copper wire ($Y = 12 \times 10^{10}$ N m^{-2}) of the same length under the same weight, the diameter should now be (in mm)

(1) 1.75 (2) 1.5 (3) 2.3 (4) 5.0

Sol. Answer (3)

For aluminium wire

$$\Delta x_1 = \frac{FL}{AY} = \frac{4FL}{\pi d^2 Y}$$

For copper wire

$$\Delta x_2 = \frac{4FL}{\pi d^2 Y}$$

Since $\Delta x_1 = \Delta x_2$ [given condition]

$$\frac{4\times 400\times 5}{\pi\times(3)^2\times 7\times 10^{10}} = \frac{4\times 400\times 5}{\pi d^2\times 12\times 10^{10}}$$

Solving this we get

$$d = \frac{\sqrt{21}}{2} \simeq 2.3 \text{ mm}$$

14. Two wires of same material and radius have their lengths in ratio 1 : 2. If these wires are stretched by the same force, the strain produced in the two wires will be in the ratio

(1) 2 : 1 (2) 1 : 1 (3) 1 : 2 (4) 1 : 4

Sol. Answer (2)

$$\text{Strain} = \frac{\Delta l}{l}$$

We know

$$\Delta l = \frac{FL}{AY}$$

$$\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi r^2 Y}$$

For wire 1

$$S_1 = \text{strain} = \frac{\Delta l_1}{L} = \frac{F}{\pi r^2 Y} \quad ...(1)$$

For wire 2

$$S_2 = \text{strain} = \frac{\Delta l_2}{2L} = \frac{F}{\pi r^2 Y} \quad ...(2)$$

Therefore,

$$\text{Ratio of strains } = \frac{S_1}{S_2} = \frac{F\times \pi r^2 Y}{\pi r^2 Y\times F} = \frac{1}{1}$$

1 : 1

15. A steel wire of cross-sectional area 3×10^{-6} m^2 can withstand a maximum strain of 10^{-3}. Young's modulus of steel is 2×10^{11} N m^{-2}. The maximum mass the wire can hold is (take g = 10 m s^{-2})

(1) 40 kg (2) 60 kg (3) 80 kg (4) 100 kg

Sol. Answer (2)

$$\text{Strain} = \frac{\Delta l}{l} = \frac{F}{AY}$$

$$\left\{\begin{array}{l}\text{Given,}\\ \dfrac{\Delta l}{l} = 10^{-3}\\ A = 3\times 10^{-6}\text{ m}^2\\ Y = 2\times 10^{11}\text{ Nm}^{-2}\end{array}\right.$$

Substituting values

$$10^{-3} = \frac{F}{3\times 10^{-6}\times 2\times 10^{11}}$$

600 N = F

$$\text{Therefore maximum mass } = \frac{F}{g} = \frac{600}{10} = 60 \text{ kg}$$

16. The hollow shaft is ...... than a solid shaft of same mass, material and length.

(1) Less stiff (2) More stiff (3) Equally stiff (4) None of these

Sol. Answer (2)

Let C' = restoring couple per unit twist for hollow cylinder

So $C' = \dfrac{\pi G(r_2^4 - r_1^4)}{2L}$ $\left\{\begin{array}{l}\text{Where,}\\ r_2 \text{ and } r_1 \text{ are outer and inner radii}\end{array}\right\}$

And

C = restoring couple per unit twist for solid cylinder

$$C = \frac{\pi G r^4}{2L}$$

$\Rightarrow \dfrac{C'}{C} = \dfrac{r_2^4 - r_1^4}{r^4} = \dfrac{(r_2^2 - r_1^2)(r_2^2 + r_1^2)}{r^4}$ $\left\{\begin{array}{l}\text{If mass of both cylinder same than}\\ \pi r^2 L\rho = \pi(r_2^2 - r_1^2)L\rho\\ \text{or } r^2 = r_2^2 - r_1^2\end{array}\right.$

$\therefore \dfrac{C'}{C} = \dfrac{r_2^2 + r_1^2}{r_2^2 - r_1^2} > 1$ Hence hollow cylinder more stronger than solid one.

17. The Bulk modulus for an incompressible liquid is

(1) Zero (2) Unity

(3) Infinity (4) Between 0 and 1

Sol. Answer (3)

We know

$$\frac{-\Delta V}{V} = \frac{P}{B}$$

$$B = \frac{-P \cdot V}{\Delta V}$$

For incompressible liquid

ΔV (Change in volume) = 0

For every value of pressure applied

Put $\Delta V = 0$

$\Rightarrow B = \infty$ (Infinity)

18. A copper rod length L and radius r is suspended from the ceilling by one of its ends. What will be elongation of the rod due to its own weight when ρ and Y are the density and Young's modulus of the copper respectively?

(1) $\dfrac{\rho^2 g L^2}{2Y}$ (2) $\dfrac{\rho g L^2}{2Y}$ (3) $\dfrac{\rho^2 g^2 L^2}{2Y}$ (4) $\dfrac{\rho g L}{2Y}$

Sol. Answer (2)

Let W be the total weight acting downwards

Let the centre of mass

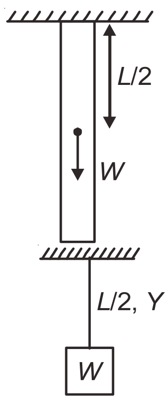
which is at the distance of $\frac{L}{2}$ for the top

So it can be assumed that a mass W is hung by a massless

wire of length $\frac{L}{2}$, Young's modulus Y,

ρ = density of wire, $W = Mg = \rho \times \pi r^2 L \times g$

Using $\Delta L = \frac{FL}{AY} = \frac{\rho \pi r^2 Lg}{\pi r^2} \times \frac{L}{2} \times \frac{1}{Y} = \frac{\rho g L^2}{2Y}$



19. Which of the following substances has the highest elasticity?

(1) Steel (2) Copper

(3) Rubber (4) Sponge

Sol. Answer (1)

Substance which requires more force for per unit elongation have more elasticity

OR

Less stretchable means more elastic

So, steel is least stretchable

$\Rightarrow$ Most elastic.

20. When a wire of length 10 m is subjected to a force of 100 N along its length, the lateral strain produced is 0.01×10^{-3} m. The Poisson's ratio was found to be 0.4. If the area of cross-section of wire is 0.025 m^2, its Young's modulus is

(1) 1.6×10^8 N m^{-2} (2) 2.5×10^{10} N m^{-2} (3) 1.25×10^{11} N m^{-2} (4) 16×10^9 N m^{-2}

Sol. Answer (1)

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\frac{\Delta l}{\underbrace{l}_{\text{Longitudinal strain}}} = \frac{F}{AY}$$

So $\nu = \dfrac{\text{Lateral strain}}{F/AY}$

Therefore

$$Y = \frac{\nu \times F}{\text{Lateral strain} \times A}$$

$$\left\{\begin{array}{l} \text{Given,} \\ F = 100 \text{ N} \\ \text{Lateral strain} = 0.01 \times 10^{-3} \text{ m} \\ \nu = 0.4 \\ A = 0.025 \text{ m}^2 \end{array}\right.$$

Substituting values

$$Y = \frac{0.4 \times 100}{0.01 \times 10^{-3} \times 0.025} = 1.6 \times 10^8 \text{ N m}^{-2}$$

21. Two wires of length l, radius r and length $2l$, radius $2r$ respectively having same Young's modulus are hung with a weight mg. Net elongation is

(1) $\frac{3\,mgl}{\pi r^2 Y}$ (2) $\frac{2\,mgl}{3\pi r^2 Y}$ (3) $\frac{3\,mgl}{2\pi r^2 Y}$ (4) $\frac{3\,mgl}{4\pi r^2 Y}$

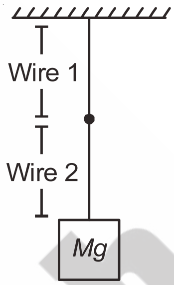
Sol. Answer (3)

Tension in both wires will be same

Let elongation in wire 1 be = Δl_1

$$\Delta l_1 = \frac{mgl}{\pi r^2 Y} \qquad \left[\Delta x = \frac{FL}{AY}\right]$$

Let elongation in wire 2 be = Δl^2



$$\Delta l_2 = \frac{mg \times 2l}{\pi(2r^2)Y}$$

Net elongation = $\Delta l_1 + \Delta l_2$

$$= \frac{mgl}{\pi r^2 Y} + \frac{mgl}{2\pi r^2 Y}$$

$$= \frac{3mgl}{2\pi r^2 Y}$$

22. A cube of side 40 mm has its upper face displaced by 0.1 mm by a tangential force of 8 kN. The shearing modulus of cube is

(1) 2×10^9 N m^{-2} (2) 4×10^9 N m^{-2} (3) 8×10^9 N m^{-2} (4) 16×10^9 N m^{-2}

Sol. Answer (1)

Shear modulus $= \frac{F \cdot h}{A \cdot x}$

Substituting values

$$= \frac{8000 \times 40 \times 10^{-3}}{1600 \times 10^{-6} \times 0.1 \times 10^{-3}}$$

$= 2 \times 10^9$ Nm^{-2}

23. A rod of length l and radius r is joined to a rod of length $\frac{l}{2}$ and radius $\frac{r}{2}$ of same material. The free end of small rod is fixed to a rigid base and the free end of larger rod is given a twist of $\theta°$, the twist angle at the joint will be

(1) $\frac{\theta}{4}$ (2) $\frac{\theta}{2}$ (3) $\frac{5\theta}{6}$ (4) $\frac{8\theta}{9}$

Sol. Answer (4)

Torque will be same

$\Rightarrow$ ϕ × Restoring couple per unit twist will be same for both the rods

$$\frac{\pi G r^4}{2L} \times \phi = \tau$$

We can use

$$\frac{\pi G r_1^4}{2L_1}\phi_1 = \frac{\pi G r_2^4}{2L_2}\phi_2$$

$$\frac{r_1^4}{L_1}\times\phi_1 = \frac{r_2^4}{L_2}\phi_2$$

Substituting values

$$\frac{r^4}{L}\times\phi_1 = \frac{(r/2)^4}{l/2}\times\phi_2$$

$$\phi_1 = \frac{\phi_2}{8}$$

Also $\phi_1 + \phi_2 = \theta$ (given)

$$\frac{\phi_2}{8} + \phi_2 = \theta$$

$$\phi_2 = \frac{8\theta}{9}$$

24. The Young's modulus of the material of a wire is 2×10^{10} N m^{-2}. If the elongation strain is 1%, then the energy stored in the wire per unit volume (in Jm^{-3}) is

(1) 10^6 (2) 10^8 (3) 2×10^6 (4) 2×10^8

Sol. Answer (1)

$$u = \frac{1}{2}Y(\text{strain})^2$$

Substituting values

$$u = \frac{1}{2}\times 2\times 10^{10}\times\left(\frac{1}{100}\right)^2$$

$u = 10^6$

25. A wire of natural length l, Young's modulus Y and area of cross-section A is extended by x. Then the energy stored in the wires is given by

(1) $\frac{1}{2}\frac{YA}{l}x^2$ (2) $\frac{1}{3}\frac{YA}{l}x^2$ (3) $\frac{1}{2}\frac{Yl}{A}x^2$ (4) $\frac{1}{2}\frac{YA}{l^2}x^2$

Sol. Answer (1)

Energy density per unit volume $= \frac{1}{2}\times(\text{strain})^2\times Y$

Volume = length × area of cross-section

$\therefore$ Energy (total) $= \frac{1}{2}\times(\text{strain})^2\times Y\times L\times A$

$$= \frac{1}{2}\frac{x^2}{L^2}YLA$$

$$E = \frac{1}{2}\frac{YA}{L}x^2$$

26. When a force is applied on a wire of uniform cross-sectional area 3×10^{-6} m^2 and length 4 m, the increase in length is 1 mm. Energy stored in it will be ($Y = 2 \times 10^{11}$ N/m^2)

(1) 6250 J (2) 0.177 J (3) 0.075 J (4) 0.150 J

Sol. Answer (3)

Energy stored $= \frac{1}{2} \times$ work done

$$= \frac{1}{2} \times F \times \Delta x$$

$$= \frac{1}{2} \times \frac{YA}{L} \Delta x \cdot \Delta x$$

Substituting values

$$E = \frac{1}{2} \times \frac{2 \times 10^{11} \times 3 \times 10^{-6} \times 1 \times 10^{-3} \times 10^{-3}}{4}$$

E = 0.075 J

27. If in a wire of Young's modulus Y, longitudinal strain X is produced then the potential energy stored in its unit volume will be

(1) 0.5 YX^2 (2) 0.5 Y^2X

(3) 2 YX^2 (4) YX^2

Sol. Answer (1)

Potential energy per unit volume $= \frac{1}{2} \times$ (strain)^2 × Young's modulus

Substituting data from question

We get, $U = \frac{1}{2} X^2 Y$

28. A material has Poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(1) 0.6 (2) 0.4

(3) 0.2 (4) Zero

Sol. Answer (4)

Poisson's ratio = 0.50

$$\frac{-\text{Lateral strain}}{\text{Longitudinal strain}} = 0.50$$

$$\Rightarrow \quad \frac{-\Delta r / r}{\Delta L / L} = \frac{1}{2}$$

$$\frac{-2\Delta r}{r} = \frac{\Delta L}{L} \qquad ...(1)$$

Volume = area × length

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta A}{A} + \frac{\Delta L}{L}$$

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta L}{L} \qquad \begin{bmatrix} A \propto r^2 \\ \frac{\Delta A}{A} = \frac{2\Delta r}{r} \end{bmatrix}$$

Using equation (1)

We get

$$\frac{\Delta V}{V} = 0$$

So $\frac{\Delta V}{V} \times 100 = 0\%$

29. There is no change in the volume of a wire due to the change in its length on stretching. The Poisson's ratio of the material of the wire is

(1) $+\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $+\frac{1}{4}$ (4) $-\frac{1}{4}$

Sol. Answer (1)

$V = A \times L$

$V = \pi r^2 L$

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta L}{L}$$

$$\frac{\Delta V}{V} = 0 \qquad \text{[given]}$$

$$\frac{-2\Delta r}{r} = \frac{\Delta L}{L} \qquad \text{...(1)}$$

Poisson's ratio $= \frac{-\Delta r}{r} \Big/ \frac{\Delta L}{L}$ ...(2)

Using equation (1) in (2)

$$\nu = \frac{-\Delta r}{r} \Big/ 2\left(\frac{-\Delta r}{r}\right) \Rightarrow \nu = \frac{1}{2}$$

30. If Young's modulus of elasticity Y for a material is one and half times its rigidity coefficient G, the Poisson's ratio ν will be

(1) $+\frac{2}{3}$ (2) $-\frac{1}{4}$ (3) $+\frac{1}{4}$ (4) $-\frac{2}{3}$

Sol. Answer (2)

$Y = \frac{3}{2}G$ [given]

$\nu = ?$

And we know, $Y = 2G(1 + \nu)$

$$\frac{3G}{2} = 2G(1+\nu)$$

Solving we get, $\nu = -\frac{1}{4}$

SECTION - C

Assertion - Reason Type Questions

1. A : Hooke's law is obeyed only for small values of strain.

   R : The deformation beyond elastic limit is called plasticity.

Sol. Answer (2)

Statement (A) is true

Statement (R) is also true

But (R) is not the correct explaination of (A)

Because correct reason is Hooke's law is obeyed in elastic limit only.

2. A : Strain is a dimensionless quantity.

   R : Strain is internal force per unit area of a body.

Sol. Answer (3)

(A) Is true

$$\text{Strain} = \frac{\Delta L}{L}$$

(R) Is false because strain is change in dimension by original dimension.

3. A : Diamond is more elastic than rubber.

   R : When same deforming force is applied diamond deforms less than rubber.

Sol. Answer (1)

(A) Is true because modulus of elasticity is more for diamond so less deformation in diamond than rubber when same deforming force applied

(R) Is true and correct explanation.

4. A : Bulk modulus for a perfectly plastic body is zero.

   R : For perfect plastic material, there is no restoring force.

Sol. Answer (1)

(A) Is true because a perfectly plastic body cannot regain its shape even when the deforming forces are removed because restoring forces are absent

(R) Is true and correct explanation for (A)

5. A : The railway bridges are declared unfit after their use for a long period.

   R : Due to repeated strain the elasticity of material decreases.

Sol. Answer (1)

(A) Is true because after a long use the material weakens and shows dangerous deformation when load is applied because its elasticity has decreased gradually over the time.

(R) Is true and correct explanation for (A)

6. A : Spring balances show wrong readings after they have been used for a long time.

R : Spring in spring balance temporary losses elasticity due to repeated alternating deforming force.

Sol. Answer (1)

(A) Is true because after a long use elasticity decreases and small temporary deformation remains these which in turn tend to be the reason of wrong readings.

(R) Is true and correct explanation for (A)

7. A : Modulus of elasticity is independent of dimensions of the body.

R : Modulus of elasticity depends on the material of the body.

Sol. Answer (2)

(A) True because modulus of elasticity is a material property

(R) True

But (R) is not the correct explanation because no where it reasons why modulus of elasticity is independent of dimensions of the body.

8. A : Adiabatic elasticity of a gas is greater than isothermal elasticity.

R : $\dfrac{E_{\text{adiabatic}}}{E_{\text{isothermal}}} = \gamma$.

Sol. Answer (1)

(A) True

Because $\dfrac{E_{\text{adiabatic}}}{E_{\text{isothermal}}} = \gamma$ and γ always greater than 1

So $E_{\text{adiabatic}}$ is always greater than $E_{\text{isothermal}}$

(R) True and also correct explanation.

9. A : When a beam is bent only tensile strain is produced.

R : The depression produced in a rectangular beam is directly proportional to its width.

Sol. Answer (4)

(A) Is false because strain is there so stress will also be present

$\because$ Stress $\propto$ strain

(R) False depression $\propto \dfrac{1}{\text{width}}$

10. A : To minimise the depression in a beam, it is designed as 'I' shape girder.

R : The 'I' shape girders have large load bearing surface, which decreases the stress.

Sol. Answer (1)

(A) Is true because having more surface area means less force per unit area i.e. less stress

(R) Is true and correct explanation of (A)

11. A : Iron is more elastic than copper.

R : Under a given deforming force, Iron is deformed less than copper.

Sol. Answer (1)

(A) Is true because less deformation under a similar deforming force means more elasticity

(R) Is true and correct explanation of (A)

12. A : Lateral strain is directly proportional to the longitudinal strain within the elastic limit.

R : Poisson's ratio for a given material at a constant temperature is constant.

Sol. Answer (1)

(A) Is true because,

$$\frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \nu$$

$\Rightarrow$ Lateral strain $\propto$ longitudinal strain

As ν is constant

(R) Is true and correct explanation of (A)

13. A : Equal amount of work is done when two identical springs of steel and copper are equally stretched.

R : Both springs have same spring constant.

Sol. Answer (4)

(A) Is wrong because amount of work done is not same because the spring constants are different.

(R) Is wrong.

14. A : Increase in temperature of a substance decreases the modulus of elasticity.

R : With increase in temperature, interatomic separation increases..

Sol. Answer (1)

(A) Is true because when we increase the temperature the average distance between the molecules tend to increase hence decreasing the modulus of elasticity.

(R) Is true and correct explanation.

15. A : It is the breaking stress and not the breaking strength which depends on the material.

R : Breaking strength = $\frac{\text{Breaking stress}}{\text{Area}}$.

Sol. Answer (4)

(A) Is wrong both depend on the material because breaking strength is maximum stress a body can take

(R) Is wrong

Breaking strength = breaking stress × area

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