Chapter 8

Gravitation

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. What is the value of G at the centre of the earth?

Sol. 6.67×10^{-11} N m^2/kg^2.

2. What is the apparent weight of an astronaut in a satellite revolving around the earth?

Sol. Zero.

3. What is the time period of geostationary satellite?

Sol. 24 hours

4. Write any two applications of Newton's law of gravitation.

Sol. (i) It helps to determine mass of the earth, the sun and the planets.

(ii) It helps to discover new stars and planets.

5. How does orbital velocity of a satellite vary with the mass of the satellite?

Sol. $V_0 = \sqrt{\dfrac{GM}{R+h}}$, where M = mass of planet.

It is independent of the mass of the satellite.

6. How far from the earth does the gravitational potential energy due to earth become zero?

Sol. At infinity.

7. What is the effect of rotation of the earth on the acceleration due to gravity?

Sol. It decreases due to rotation. This effect is maximum at equator and zero at poles.

8. Why does a rubber ball bounce higher on a hill than on ground?

Sol. At higher altitudes g decreases so the gravitational pull is less on hills.

9. Why is it not possible to place an artificial satellite in an orbit such that it is always visible over New Delhi?

Sol. Because a satellite remains always visible only if it is revolving around the earth in the equatorial plane with the period of revolution of 24 hours.

10. Why does a body weigh more at the poles than at the equator?

Sol. Because $g' = g - \omega^2 R\cos^2\phi$, and also earth is not of uniform radius

$\therefore \quad g_p > g_e$

Short Answer Type Questions :

11. Which is more fundamental, mass or weight of a body? Why?

Sol. Mass of a body is more fundamental because the value of weight changes from place to place as it depends on acceleration due to gravity but mass remains constant everywhere in the universe.

12. Why do the astronauts landing on the surface of moon tie heavy weights at their back before landing on moon?

Sol. The value of g is small on moon so the astronauts feel less weight hence to compensate for the loss in weight the astronauts tie heavy weights at their back.

13. Comment "Earth has atmosphere but moon has not"

Sol. Earth has atmosphere because the r.m.s velocity of air molecules is less than the escape speed from the surface of earth but at moon the r.m.s. velocity of air molecules is larger than the escape speed from the surface of moon. So moon does not have atmosphere.

14. To what factor does the time period of revolution of a planet be increased so that its distance from sun is increased 100 times?

Sol. $\dfrac{T_1^2}{T_2^2} = \dfrac{R_1^3}{R_2^3}$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{(100R_1)^3}$$

$$\therefore \quad T_2^2 = T_1^2(100)^3$$

$$\therefore \quad T_2 = T_1\ 1000$$

So time period of revolution of a planet increases to 1000 times.

15. Define orbital speed of satellite. What is its value for a satellite close to earth surface?

Sol. It is the speed of satellite with which it orbits around the earth.

Its value in the orbit close to earth is 7.92 km/s.

16. Two bodies of masses 10^{10} kg and 10^{20} kg are separated by a distance of 10^{15} m. What is the force of attraction between them?

Sol. $F = \dfrac{Gm_1m_2}{r^2}$

$$\therefore \quad F = \frac{6.67\times10^{-11}\times10^{10}\times10^{20}}{(10^{15})^2}$$

$$= 6.67 \times 10^{-11} \times \frac{10^{30}}{10^{30}}$$

$$\therefore \quad F = 6.67 \times 10^{-11}\ \text{N}$$

17. Two point mass having equal mass m are separated by a distance of 100 m. What is the value of m if the gravitational force of attraction between them is 66.7 N?

Sol. As $F = \dfrac{Gm_1m_2}{r^2}$

$\therefore \quad 66.7 = \dfrac{6.67 \times 10^{-11}\ m^2}{(100)^2}$

$\therefore \quad m^2 = \dfrac{100000}{10^{-11}}$

$\therefore \quad m^2 = 10^{16}$

$\therefore \quad m = 10^8$ kg

18. What will be the value of g at a height 0.75 times that of radius of the earth?

Sol. $g_h = g\left[\dfrac{R}{R+h}\right]^2$

$\therefore \quad g_h = g\left[\dfrac{R}{R+0.75R}\right]^2$

$= g\left[\dfrac{R}{1.75R}\right]^2$

$= g\left[\dfrac{4}{7}\right]^2$

$g_h = 9.8 \times \dfrac{16}{49} = 3.2$ m/s^2

19. Write any four properties of gravitational force.

Sol. (i) It is independent of the medium between the particles.

(ii) It is always attractive.

(iii) It is weakest force in nature.

(iv) It is conservative force.

20. The ratio of masses of two planets is 3 : 7 and the ratio of their radii is 9 : 7. What will be the ratio of the escape speed on both the planets?

Sol. $v_1 = \sqrt{\dfrac{2GM_1}{R_1}}$

$v_2 = \sqrt{\dfrac{2GM_2}{R_2}}$

$\therefore \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{M_1}{M_2} \times \dfrac{R_2}{R_1}}$

$= \sqrt{\dfrac{3}{7} \times \dfrac{7}{9}} = \sqrt{\dfrac{3}{9}}$

$= \sqrt{\dfrac{1}{3}}$

Hence, the ratio of their escape speed will be $1 : \sqrt{3}$.

21. A body weighs 100 N at the surface of earth. At what height will it weigh 36 N?

[Take R_e = 6400 km]

Sol. mg = 100 N, mg_h = 36 N

$$\therefore \quad \frac{g_h}{g} = \frac{36}{100}$$

$$g_h = g\left[\frac{R_e}{R_e + h}\right]^2$$

$$\therefore \quad \frac{g_h}{g} = \left[\frac{R_e}{R_e + h}\right]^2$$

$$\therefore \quad \frac{36}{100} = \frac{R_e^2}{(R_e + h)^2}$$

$$\frac{6}{10} = \frac{R_e}{R_e + h}$$

$$\therefore \quad 6R_e + 6h = 10R_e$$

$$\therefore \quad h = \frac{2R_e}{3}$$

$$= \frac{2 \times 6400}{3}$$

$$\therefore \quad h = 4266.6 \text{ km}$$

22. How deep a mine should be dug so that the weight of a body decreases by 75% as that on surface of the earth?

[Take R_e = 6400 km]

Sol. $$g_d = g\left[1 - \frac{d}{R_e}\right]$$

$$\therefore \quad 25\% \text{ of } g = g\left[1 - \frac{d}{R_e}\right]$$

$$\therefore \quad \frac{25}{100} = 1 - \frac{d}{R_e}$$

$$\therefore \quad \frac{d}{R_e} = 1 - \frac{25}{100}$$

$$d = \frac{75}{100} \times R_e$$

$$\therefore \quad d = \frac{75 \times 6400}{100}$$

$$\therefore \quad d = 4800 \text{ km}$$

23. What is the value of escape speed on a planet whose mass is four times that of earth and radii double that of earth?

Sol. $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$

$v_p = \sqrt{\dfrac{2GM_p}{R_p}}$

$\therefore \quad \dfrac{v_p}{v_e} = \sqrt{\dfrac{M_p}{M_e} \times \dfrac{R_e}{R_p}}$

$\therefore \quad v_p = 11.2\sqrt{\dfrac{4}{1} \times \dfrac{1}{2}}$

$= 11.2 \times 1.414$

$\simeq$ 15.8 km/s

24. What are the uses of geostationary satellites?

Sol. (i) They are used in communicating radio, T.V. and telephone signals across the world.

(ii) In weather forecasting.

(iii) In studying meteorites and cosmic radiations.

(iv) In studying upper region of atmosphere.

25. Deduce the relation between the orbital velocity of satellite close to earth's surface and escape velocity from earth surface.

Sol. $v_e = \sqrt{2gR_e}$

$\therefore \quad v_e = \sqrt{2}\sqrt{gR_e}$

As $\quad v_0 = \sqrt{gR_e}$

$\boxed{\therefore \; v_e = \sqrt{2}v_0}$

26. The radii of two planets are R and $2R$ respectively and their densities are ρ and $\dfrac{\rho}{2}$. What is the ratio of acceleration due to gravity at their surface?

Sol. As $g = \dfrac{GM}{R^2}$

Also as $M = \dfrac{4}{3}\pi R^3\rho$

$\therefore \quad g = \dfrac{G4}{3}\dfrac{\pi R^3\rho}{R^2}$

$\therefore \quad g = \dfrac{4G\pi R\rho}{3}$

Now, $g_1 = \frac{4}{3}G\pi R\rho$

$$g_2 = \frac{4}{3}G\pi 2R \times \frac{\rho}{2}$$

$$\therefore \quad \frac{g_1}{g_2} = \frac{1}{1}$$

27. What will be the escape velocity from a point 3200 km above earth's surface? (Take R_e = 6400 km, g_e = 9.8 m/s^2)

Sol. $v_e = \sqrt{2g_h(R+h)}$

$$\therefore \quad v_e = \sqrt{\frac{2gR^2}{(R+h)^2} \times (R+h)}$$

$$\therefore \quad v_e = \sqrt{\frac{2gR^2}{(R+h)}}$$

$$\therefore \quad v_e = \sqrt{\frac{2 \times 9.8 \times (6.4 \times 10^6)^2}{6.4 \times 10^6 + 3.2 \times 10^6}}$$

$$= \sqrt{83.62 \times 10^6}$$

$= 9.1 \times 10^3$ m/s

= 9.1 km/s

28. Mention some necessary conditions for a geostationary satellite.

Sol. (i) It should revolve in an orbit concentric and coplanar with equatorial plane of the earth.

(ii) Its sense of rotation should be same as that of the earth.

(iii) It should have period of revolution of 24 hours and revolve at height of nearly 36,000 km.

29. Mention the conditions under which the weight of a body can become zero.

Sol. (i) When the body is kept at the centre of earth

(ii) At its free fall

(iii) When a body is orbiting around the earth

30. Prove that the weight at the centre of the earth is zero.

Sol. As weight = mg

but $g_d = g\left[1 - \frac{d}{R_e}\right]$

Now, $d = R_e$

$$\therefore \quad g_d = g\left[1 - \frac{R_e}{R_e}\right]$$

$\therefore \quad g_d = 0$

$\therefore$ Weight at centre $= m \times 0$

$= 0$

Long Answer Type Questions :

31. State and prove Kepler's 2nd law.

Sol. Law of areas : The line that joins any planet to the sun sweeps out equal areas in equal intervals of time.

It means that the area covered by the planet around the sun in given time intervals is constant *i.e.*, areal velocity is constant. This law comes from the observations that planets appear to move slower when they are farther from the sun than when they are nearer.

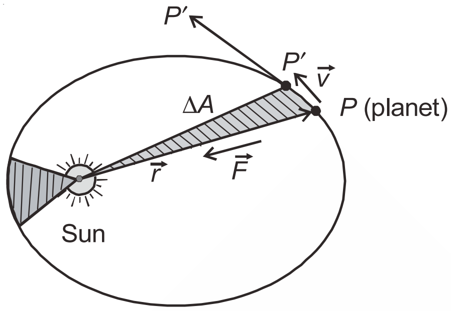


Fig : The plant P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt.

This law can be considered as a consequence of conservation of angular momentum which is valid for any force that is always directed towards or away from a fixed point *i.e.*, the central force. This area swept out by the planet of mass m in the time interval Δt is $\Delta\vec{A}$.

$\Delta\vec{A}$ is given by $\Delta\vec{A} = \frac{1}{2}(\vec{r}\times\vec{v}\Delta t)$

$$\therefore \quad \frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2}\vec{r}\times\vec{v}$$

$$= \frac{1}{2}\vec{r}\times\frac{\vec{p}}{m} \qquad \ldots\left(\vec{v} = \frac{\vec{p}}{m}\right)$$

$$\therefore \quad \frac{\Delta\vec{A}}{\Delta t} = \frac{\vec{L}}{2m}$$

Where $\vec{L} = \vec{r}\times\vec{p}$ = angular momentum, $\vec{p}$ = momentum and $\vec{r}$ is the position vector where sun is taken as the origin.

The torque acting on a planet due to the central force is clearly zero; *i.e.*, because $\vec{F}$ is anti parallel to $\vec{r}$.

Since, $\vec{\tau} = \vec{r}\times\vec{F} = rF\sin\theta$

$$\therefore \quad \vec{\tau} = 0$$

As $\vec{\tau} = \frac{d\vec{L}}{dt}$

$$\therefore \quad \frac{d\vec{L}}{dt} = 0$$

This implies that $\vec{L}$ is constant as the planet goes around.

Hence, $\frac{\Delta\vec{A}}{\Delta t}$ is a constant.

32. Draw and explain the setup made by Henry Cavendish to find the value of G.

Sol. The value of universal gravitational constant G was first determined experimentally by English scientist Henry Cavendish in 1798. The apparatus used by him is schematically shown in figure.

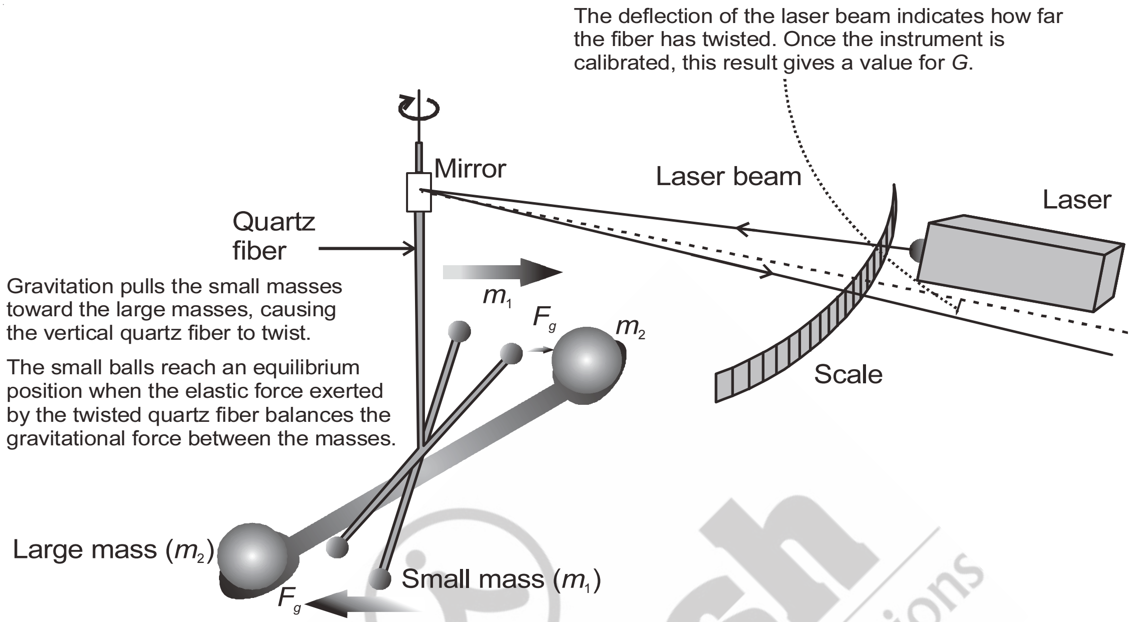


Fig : Experimental set up for finding the value of *G*

A light rod of length L is suspended from a rigid support by a thin quarty fibre. Two small identical lead spheres of mass m are attached at its ends to form a dumb-bell. Two large lead spheres of mass M are placed near the ends of the dumb-bell on opposite sides such that all the four spheres lie horizontally. The small spheres get attracted towards the large ones due to which torque arises and the suspended wire gets twisted till such time as the restoring torque of the wire equals the gravitational torque. The angle of rotation 'θ' is measured by the deflection of a light beam reflected from a mirror attached to the vertical suspension.

Now, gravitational torque = $F \times L$

$$= \frac{GMmL}{r^2}$$

and restoring torque = $\tau\theta$

where, τ is the restoring torque per unit angle of twist also known as torsion constant.

$$\therefore \quad \frac{GMmL}{r^2} = \tau\theta$$

or $G = \dfrac{\tau\theta r^2}{MmL}$

By knowing all the quantities, we can easily find the value of G. Since the time of Cavendish's experiment, the measurement of G has been refined and currently accepted value is

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2

33. Define gravitational potential. Write its expression. Also mention its relation with gravitational potential energy.

Sol. It is the potential energy associated with a unit mass at a point due to its position or we may say that the gravitational potential due to the gravitational force of the earth is defined as the potential energy of a particle of unit mass at that point.

$$V = \frac{\text{Work done}}{\text{Mass}}$$

$$\therefore \quad V = \frac{\frac{-GMm}{r}}{m}$$

$$\therefore \quad V = \frac{-GM}{r}$$

or simply we may also write that

Gravitational potential energy = Mass × gravitational potential

34. Define the time period of a satellite. Obtain its expression and hence show that it obeys Kepler's 3rd law.

Sol. It is the time taken by an earth satellite to complete one revolution around the earth.

Circumference of the orbit of satellite = $2\pi(R_e + h)$

$$v_0 = \text{orbital speed} = \frac{\text{Circumference}}{\text{Time period}}$$

$$\therefore \quad T = \frac{2\pi(R_e + h)}{V_0}$$

$$\therefore \quad T = \frac{2\pi(R_e + h)}{\sqrt{\frac{GM}{R_e + h}}} \qquad \ldots\left(v_0 = \sqrt{\frac{GM}{R_e + h}}\right)$$

$$\therefore \quad T = \frac{2\pi(R_e + h)^{3/2}}{\sqrt{GM_e}} \qquad \ldots(iv)$$

By squaring both sides we get

$$T^2 = \frac{4\pi^2}{GM_e}(R_e + h)^3$$

or $T^2 = k(R_e + h)^3$ …(v)

Where, $k = \frac{4\pi^2}{GM_e}$

$$\boxed{\therefore\ T^2 \propto R^3}$$

35. What do you mean by weightlessness? Explain it with any one practical illustration.

Sol. Weight of a body is the force with which it is attracted towards the centre of the earth.

According to Newton's second law $F = ma$

$\therefore$ $F = mg = W$ as $a = g$. SI unit of weight is newton.

Now, let us take a case of spring balance attached to a ceiling to weigh any object. If the spring balance is pulled from one side or say anything is attached to its free end then it would be stretched by certain amount. Suppose its top end suddenly breaks from the ceiling then in its free fall both ends of spring balance will move with identical acceleration g, hence the spring is not stretched and does not exert any upward force on the object which is moving down with acceleration g due to gravity. So, the reading becomes zero and we may say that the object in its free fall is weightless and this phenomenon is called weightlessness.

36. (i) State Kepler's 3rd law of planetary motion.

(ii) What is the time period in days of a planet at a distance 4 times that of earth from the sun.

Sol. (i) **Kepler's III law :** The square of the period of revolution of a planet around the sun is proportional to the cube of the semi-major axis of its elliptical path.

(ii) Here,

$T_1 = 365$ days , $R_1 = R$

$T_2 = ?$, $R_2 = 4R$

Now, $\dfrac{T_1^2}{T_2^2} = \dfrac{R_1^3}{R_2^3}$

$$\therefore \quad \frac{(365)^2}{T_2^2} = \frac{R^3}{(4R)^3}$$

$\therefore$ $T_2^2 = 64 \times 365 \times 365$

$\therefore$ $T_2 = 2920$ days

37. Write some uses of

(i) Geostationary satellites

(ii) Polar satellites

Sol. (i) Uses of geostationary satellites

(a) They are used in communicating radio, T.V. and telephone signals across the world.

(b) In weather forecasting.

(c) In studying meteorites and cosmic radiations.

(d) In studying upper region of atmosphere.

(ii) Uses of polar satellites

(a) For spying work for military purposes.

(b) For weather forecasting (even better than geostationary satellites).

38. What will be the minimum energy required to lift a body from earth's surface to an altitude five times the radius of earth?

Sol. Let the mass of satellite be m kg.

At earth's surfaces

$$U_i = -\frac{GMm}{R}$$

At altitude of $5R$

$$U_f = -\frac{GMm}{R+5R} = -\frac{GMm}{6R}$$

$\therefore$ Required energy $= U_f - U_i$

$$= \frac{GMm}{R}\left[1 - \frac{1}{6}\right]$$

$$= \frac{5}{6} \cdot \frac{GMm}{R}$$

$$= \frac{5}{6}\frac{gR^2m}{R}$$

$$= \frac{5}{6}mgR$$

39. A planet has its mass 4 times that of earth and radius thrice that of earth. What will be the weight of a block on its surface which weighs 100 N at earth's surface? [Take g = 10 ms^{-2}]

Sol. $g_p = \dfrac{GM_p}{R_p^2}$

$$g_e = \frac{GM_e}{R_e^2}$$

$$\therefore \quad \frac{g_p}{g_e} = \frac{M_p}{M_e}\frac{R_e^2}{R_p^2} = \frac{4}{1} \times \frac{1}{9}$$

$$\therefore \quad g_p = \frac{4}{9}g_e$$

Now, weight at earth = mg

$\therefore$ 100 = m × 10

$\therefore$ m = 10 kg

$\therefore$ Weight at a planet = mg_p

$$= 10 \times \frac{4}{9}$$

$$= \frac{40}{9} = 4.44 \text{ kg}$$

40. If the mass of earth is doubled by keeping its size unchanged, what will be the acceleration due to gravity at height 3200 km above earth's surface? [R_e = 6400 km and g = 9.8 m/s^2]

Sol. $g_2 = \dfrac{GM_2}{R^2}$

$g_1 = \dfrac{GM_1}{R^2}$

$\therefore \quad g_2 = 2g_1$

$\therefore \quad g_2 = 19.6$ m/s^2

Now,

$$g_h = g_2\left[\frac{R}{R+h}\right]^2$$

$$\therefore \quad g_h = 19.6\left[\frac{6400}{6400+3200}\right] = 19.6 \times \frac{6400}{9600}$$

$$\therefore \quad g_h = 19.6 \times \frac{64}{96}$$

$\therefore \quad g_h = 13.06$ m/s^2

41. (i) Earth is continuously pulling the moon towards its centre, still it does not fall to the earth. Why?

(ii) Why gravitational force is considered as the long range force?

Sol. (i) Gravitational force of attraction due to earth provides the necessary centripetal force, which keeps the moon in orbit around the earth. Moon will fall if its orbital velocity becomes zero.

(ii) As this force exists between the sun and planets, stars and our planet earth, different stars, even between each galaxies.

42. Calculate the work required in raising a body of mass m to a height h from the surface of the earth.

Sol. Let mass of earth be M and its radius be R.

As we know that, the acceleration due to gravity at earths surface is g.

Work done = change in P.E.

$$= -\frac{GMm}{R+h} - \left[-\frac{GMm}{R}\right]$$

$$= GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$= \frac{GMmh}{R(R+h)}$$

$$= \frac{gR^2mh}{R[R+h]} \qquad \ldots\left[\because GM = gR^2\right]$$

$$\therefore \quad \text{Work done} = \frac{mgh}{1+\dfrac{h}{R}}$$

43. A rocket is launched vertically from the surface of the earth with an initial velocity of 8 km/s. How far above the surface of the earth would it go? (Ignore the air resistance, take R_e = 6.4 × 10^6 m and g = 9.8 m/s^2)

Sol. Initial K.E. = gain in gravitational potential energy of rocket

$$\therefore \quad \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} - \left[-\frac{GMm}{R}\right]$$

$$\therefore \quad \frac{1}{2}v^2 = \frac{gR^2h}{R(R+h)} = \frac{gRh}{(R+h)}$$

$$\therefore \quad \frac{R+h}{h} = \frac{2gR}{v^2}$$

$$\therefore \quad \frac{R}{h} + 1 = \frac{2gR}{v^2}$$

$$\therefore \quad h = R\left[\frac{2gR}{v^2} - 1\right]^{-1}$$

$$\therefore \quad h = 6.4\times10^6\left[\frac{2\times9.8\times6.4\times10^6}{(8\times10^3)^2} - 1\right]^{-1}$$

$\therefore$ h = 6666.66 km

44. Mention some properties of gravitational force.

Sol. Properties of Gravitational Force

(i) It is independent of the medium between the particles.

(ii) It is always attractive in nature.

(iii) It is true from inter atomic distances to interplanetary distance.

(iv) It is weakest force in nature.

(v) It is conservative force, *i.e.*, work done by it doesn't depend on its path or work done in moving a particle round a closed path under the action of gravitational force is zero.

(vi) It is an action reaction pair *i.e.*, the force with which one body (say earth) attracts the second body (say apple) is equal to the force with which apple attracts the earth (in accordance with Newtons 2^{nd} law).

(vii) It is a two-body interaction *i.e.*, gravitational force between two particles is independent of the presence or absence of other particles.

(viii) It is a central force *i.e.*, acts along the line joining the two particles or centres of the interacting bodies.

45. A satellite is launched into a circular orbit close to the earth's surface. What additional velocity is now to be imparted to the satellite in the orbit to overcome the gravitational pull? [Take R_e = 6400 km, g = 9.8 m/s^2]

Sol. Orbital velocity near the earth's surface ; $v_0 = \sqrt{gR_e}$

while, escape velocity = $\sqrt{2}v_0$

$\therefore$ Additional velocity required $= v_e - v_0$

$= (1.414 - 1)\sqrt{gR_e}$

$= 0.414\sqrt{9.8\times6400\times10^3}$

$= 3.278 \times 10^3$ m/s

= 3.278 km/s

SECTION - B

Model Test Paper

Very Short Answer Type Questions :

1. How far from the centre of the earth does the gravitational potential energy due to earth become zero?

Sol. At infinity

2. What is gravitational force?

Sol. A force of attraction between two bodies by the virtue of their masses.

3. How does orbital velocity of a satellite depend on the mass of the satellite?

Sol. $v_0 = \sqrt{\dfrac{GM}{R+h}}$

It is independent of mass of the satellite.

4. Suppose the size of the earth shrinks to half of its original size, what will be the gravitational force on a particle at its centre?

Sol. It will remain zero.

5. What is the time period of a geostationary satellite?

Sol. 24 hours

Short Answer Type Questions :

6. Show that weight of a body becomes zero at the centre of earth.

Sol. As weight = mg

At centre $d = R_e$

$$g_d = g\left[1 - \frac{d}{R_e}\right]$$

$\therefore \quad g_d = 0$

Hence, weight = 0

7. Why does moon have no atmosphere?

Sol. It is because the acceleration due to gravity on the surface of moon is very less. Due to which the value of escape speed on the surface of moon is small. The molecules of the atmospheric gases have thermal energies greater than the escape speed. That is why all the molecules of gases have escaped and there is no atmosphere.

8. How much will the gravitational force of attraction be increased between two objects when their masses are doubled and the distance between them is halved?

Sol. As, $F_0 = \dfrac{Gm_1m_2}{r^2}$

and $F = \dfrac{GM_1M_2}{R^2}$

$$\therefore \quad F = \frac{G2m_1 2m_2}{\left(\frac{r}{2}\right)^2}$$

$$\therefore \quad F = G4 \times 4 \times \frac{m_1 m_2}{r^2}$$

$$= 16\frac{Gm_1 m_2}{r^2}$$

$$\boxed{\therefore \; F = 16F_0}$$

9. Define escape speed. What is its value for earth?

Sol. The minimum speed with which the body has to be projected vertically upwards from the surface of earth so that it just crosses the gravitational force of attraction of earth.

Its value on earths surface is 11.2 km s^{-1}.

10. The distance of a planet from sun is 5.2 times that of the earth. Find the period of revolution of that planet around the sun.

So. We have, r_p = 5.2 r_e

T_p = ?, T_e = 1 year

$$\frac{T_p^2}{T_e^2} = \frac{r_1^3}{r_e^3}$$

$$\therefore \quad T_p = T_e\left(\frac{r_p}{r_e}\right)^{\frac{3}{2}}$$

$$= 1\left[\frac{5.2r_e}{r_e}\right]^{\frac{3}{2}}$$

T_p = 11.86 years

Short Answer Type Questions :

11. Where will a body weigh more, 10 km above the surface of earth or 10 km below the surface of the earth?

Sol. $g_h = g\left[1 - \frac{2h}{R}\right]$ and $g_d = g\left[1 - \frac{d}{R}\right]$

Here, h = d = 10 km

So, we can see clearly that $g_h < g_d$

As weight = Mass × Acceleration due to gravity

Hence, the body will weigh more at depth 10 km below the surface of earth.

12. Deduce the expression for gravitational potential energy.

Sol. Let us find the work done in lifting a particle from r_1 to r_2 along vertical path, then

$$W_{AB} = \int_{r_1}^{r_2} F dr$$

$$= \int_{r_1}^{r_2} \frac{GMm}{r^2} dr$$

$$= GMm \int_{r_1}^{r_2} \frac{dr}{r^2}$$

$$= GMm \left[\frac{-1}{r}\right]_{r_1}^{r_2}$$

$$\therefore \quad W_{AB} = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

This expression gives the value of gravitational potential energy.

13. Discuss the variation of acceleration due to gravity with altitude, depth and rotation of the earth.

Sol. (i) **Altitude :** With increase in altitude the value of g decreases.

(ii) **Depth :** With increase in depth the value of g decreases.

(iii) **Rotation of earth :** With rotation of earth the value of g decreases.

14. (i) State Newton's law of gravitation.

(ii) Show that Kepler's second law can be considered as a consequence of conservation of angular momentum.

Sol. (i) Every particle of matter in the universe attracts every other particle with a force that is directly proportional to the product of the masses of the particles and inversely proportional to the square of the distance between them.

(ii) Area swept out by the planet of mass m in the time interval Δt is $\Delta\vec{A}$

$\Delta\vec{A}$ is given by $\Delta\vec{A} = \frac{1}{2}(\vec{r} \times \vec{v}\Delta t)$

$$\therefore \quad \frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2}\vec{r} \times \vec{v} = \frac{1}{2}\vec{r} \times \frac{\vec{p}}{m}$$

$$\therefore \quad \frac{\Delta\vec{A}}{\Delta t} = \frac{\vec{L}}{2m}$$

As, $\vec{\tau} = \vec{r} \times \vec{F}$

$\therefore \quad \vec{\tau} = 0$

As $\vec{\tau} = \frac{d\vec{L}}{dt} = 0$

$\therefore$ L is constant.

15. A body weighs 45 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Sol. We have, $mg = 45$ N

and $h = \dfrac{R}{2}$

$$g_h = g\left[\frac{R}{R+h}\right]^2$$

$$g_h = g\left[\frac{R}{R+\frac{R}{2}}\right]^2$$

$$g_h = g\left[\frac{2}{3}\right]^2 = g\frac{4}{9}$$

$$\therefore \quad mg_h = \frac{4}{9}mg = \frac{4}{9}\times 45 = 20 \text{ N}$$

Hence, the gravitational force will be 20 N.

Long Answer Type Questions :

16. State Kepler's law of planetary motion. Deduce the universal law of gravitation using it.

Sol. Newton combined his laws of motion with Kepler's laws and deduced the law of gravitation. Path of planets around the sun is elliptical. For simplicity we can assume the orbit to be circular. Let us consider a planet of mass m moving with constant speed v in a circular orbit.

Its time period, $T = \dfrac{2\pi r}{v}$ …(i)

Where r is radius of circular path.

$T^2 = kr^3$ …(ii)

From Kepler's 3rd law, where k is a constant of proportionality.

From equation (i) and (ii)

$$\frac{4\pi^2 r^2}{v^2} = kr^3$$

or $v^2 = \dfrac{4\pi^2}{kr}$ …(iii)

We know that an object in a circular path is accelerated and its acceleration towards the centre is $\dfrac{v^2}{r}$.

$F = \dfrac{mv^2}{r}$ …[By Newtons 2nd law, $F = ma$]

$F = \dfrac{m.4\pi^2}{kr^2}$ using equation (iii)

$\Rightarrow \quad F \propto \frac{1}{r^2}$ and $F \propto m$

Force on the planet due to the sun = force on the sun due to the planet.

$\Rightarrow$ If the force is proportional to the mass of the planet, it should also be proportional to the mass of the sun.

$$\Rightarrow \quad F \propto \frac{Mm}{r^2}$$

or $\boxed{F = G\frac{Mm}{r^2}}$

17. Explain gravitational potential. Establish a relation for it.

Sol. It is the potential energy associated with a unit mass at a point due to its position or we may say that the gravitational potential due to the gravitational force of the earth is defined as the potential energy of a particle of unit mass at that point.

$$V = \frac{\text{Work done}}{\text{Mass}}$$

$$\therefore \quad V = \frac{\frac{-GMm}{r}}{m}$$

$$\therefore \quad V = \frac{-GM}{r}$$

or simply we may also write that

Gravitational potential energy = Mass × gravitational potential

❑ ❑ ❑

Solutions (Set-2)

Objective Type Questions

(Kepler's Laws)

1. According to Kepler, planets move in

 (1) Circular orbits around the sun

 (2) Elliptical orbits around the sun with sun at exact centre

 (3) Straight lines with constant velocity

 (4) Elliptical orbits around the sun with sun at one of its foci

Sol. Answer (4)

Kepler's first law,

Law of Orbits : All planets move in elliptical orbits, with the sun at one of the foci of the ellipse.

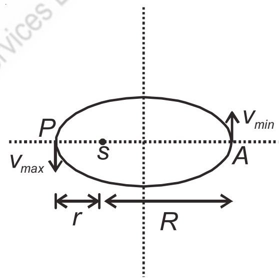
2. The minimum and maximum distances of a planet revolving around sun are r and R. If the minimum speed of planet on its trajectory is v_0, its maximum speed will be

 (1) $\frac{v_0R}{r}$ (2) $\frac{v_0r}{R}$ (3) $\frac{v_0R^2}{r^2}$ (4) $\frac{v_0r^2}{R^2}$

Sol. Answer (1)

According to Kepler's second law.

Low of Areas : The line that joins any planet to the sun sweeps out equal areas in equal intervals of time. Thus planets appear to move slower when they are farther from sun than when they are nearer.



Now, for planets moving around the sun in an elliptical orbit, Angular momentum is conserved.

$\Rightarrow \quad |\vec{L_P}| = |\vec{L_A}|$

$mv_{max}r = mv_0R$

$v_{max} = \frac{v_0R}{r}$

3. A planet of mass m moves around the sun of mass M in an elliptical orbit. The maximum and minimum distances of the planet from the sun are r_1 and r_2 respectively. The time period of the planet is proportional to

 (1) $r_1^{3/2}$ (2) $r_2^{3/2}$ (3) $(r_1 + r_2)^{3/2}$ (4) $(r_1 - r_2)^{3/2}$

Sol. Answer (3)

Law of periods : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

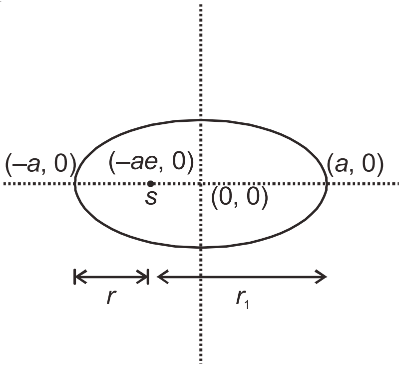
$T^2 \propto a^3$

where,

T = Time period of revolution of a planet.

a = Semi-major axis of the elliptical orbit traced by the planet.

From figure, $r_1 + r_2 = 2a$



$$\Rightarrow \quad T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

$$\Rightarrow \quad T \propto (r_1 + r_2)^{3/2}$$

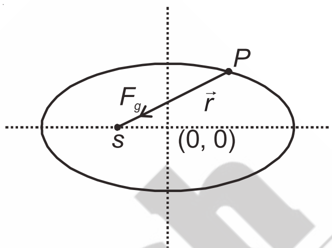
4. The torque on a planet about the centre of sun is

(1) Zero (2) Negative

(3) Positive (4) Depend on mass of planet

Sol. Answer (1)

Force of gravity is acting on the planet,



Torque of force of gravity = $\vec{r} \times \overrightarrow{F_g} = rF_g \sin\theta$

Since θ = 180°, τ = 0

5. During motion of a planet from perihelion to aphelion the work done by gravitational force of sun on it is

(1) Zero

(2) Negative

(3) Positive

(4) May be positive or negative

Sol. Answer (2)

According to Kepler's Law of areas, $v_A < v_P$

v_A = speed of planet at aphelion

v_P = speed of planet at perihelion

Now, work done by gravitational force of sun $= \Delta\text{K.E} = \frac{1}{2}m\left(v_A^2 - v_P^2\right)$

$\Rightarrow$ $W_{\text{gravitation force}}$ is negative.

6. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius $4R$ is

(1) $4T$ (2) $\frac{T}{4}$ (3) $8T$ (4) $\frac{T}{8}$

Sol. Answer (3)

Using Kepler's third law,

$$T^2 \propto R^3$$

$$\Rightarrow \quad \frac{T_2}{T} = \left(\frac{4R}{R}\right)^{3/2}$$

$$\Rightarrow \quad T_2 = T \times 2^3$$

$$= 8T$$

(Universal Law of Gravitation)

7. Gravitation is the phenomenon of interaction between

   (1) Point masses only (2) Any arbitrary shaped masses

   (3) Planets only (4) None of these

Sol. Answer (2)

Gravitation is the phenomenon of interaction between any arbitrary shaped bodies.

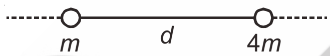
8. Force of gravitation between two masses is found to be F in vacuum. If both the masses are dipped in water at same distance then, new force will be

   (1) $> F$ (2) $< F$ (3) F (4) Cannot say

Sol. Answer (3)

Force of gravitation is independent of the medium. Force is F when masses are in vacuum. When masses are dipped in water force will be same.

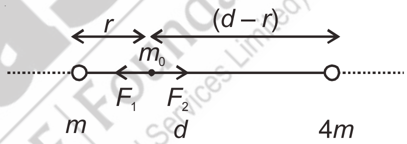
9. Two point masses m and $4m$ are separated by a distance d on a line. A third point mass m_0 is to be placed at a point on the line such that the net gravitational force on it is zero.



The distance of that point from the m mass is

   (1) $\frac{d}{2}$ (2) $\frac{d}{4}$ (3) $\frac{d}{3}$ (4) $\frac{d}{5}$

Sol. Answer (3)



Force of gravitation on m_0 due to $m = \frac{Gmm_0}{r^2} = F_1$

Force of gravitation on m_0 due to $4m = \frac{G4mm_0}{(d-r)^2} = F_2$

Net force = 0

$\Rightarrow F_1 = F_2$

$$\frac{Gmm_0}{r^2} = \frac{4Gmm_0}{(d-r)^2}$$

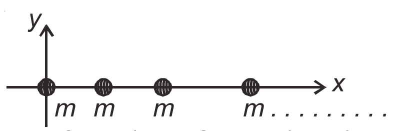
$\Rightarrow (d - r)^2 = (2r)^2$

$\Rightarrow d - r = 2r$

$\Rightarrow d = 3r$

Thus, $r = \frac{d}{3}$

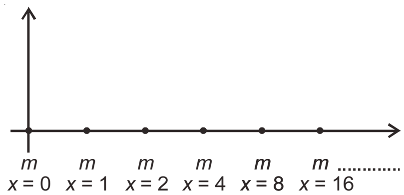
10. A large number of identical point masses m are placed along x-axis, at x = 0, 1, 2, 4, ... The magnitude of gravitational force on mass at origin (x = 0), will be



   (1) Gm^2 (2) $\frac{4}{3}Gm^2$ (3) $\frac{2}{3}Gm^2$ (4) $\frac{5}{4}Gm^2$

Sol. Answer (2)

Let, F_1, F_2 , F_4, F_8 ...... be the forces of gravitation due masses 'm' at x = 1, 2, 4, 8 … respectively.



$$\Rightarrow \quad F_1 = \frac{Gm^2}{1^2}$$

$$F_2 = \frac{Gm^2}{2^2}$$

$$F_4 = \frac{Gm^2}{4^2}$$

$$F_8 = \frac{Gm^2}{8^2}$$

$$F_1 + F_2 + F_4 + F_8 \ldots = \quad Gm^2 \underbrace{\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \ldots\right)}_{\text{infinite G.P. with common ratio } = \frac{1}{4}}$$

For an infinite G.P, sum = $\left(\dfrac{a}{1-r}\right)$

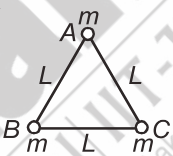
a is the first term

r is the common ratio

$$\Rightarrow \quad \text{Sum} = \frac{1}{1-\frac{1}{4}} = \left(\frac{4}{3}\right)$$

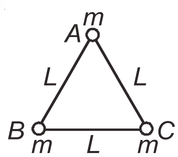
$$\Rightarrow \quad F_1 + F_2 + F_4 + F_8 \ldots\ldots = \frac{4}{3} Gm^2$$

11. Three particles A, B and C each of mass m are lying at the corners of an equilateral triangle of side L. If the particle A is released keeping the particles B and C fixed, the magnitude of instantaneous acceleration of A is



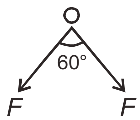
(1) $\sqrt{3}\dfrac{Gm^2}{L^2}$ (2) $\sqrt{2}\dfrac{Gm^2}{L^2}$ (3) $\sqrt{2}\dfrac{Gm}{L^2}$ (4) $\sqrt{3}\dfrac{Gm}{L^2}$

Sol. Answer (4)



At this moment,

Forces acting on particle at A can be shown,



where, $F = \dfrac{Gm^2}{L^2}$

$\Rightarrow$ Net force will be resultant of both,

$$F_{\text{resultant}} = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ} = \sqrt{3}\, F$$

$$\Rightarrow \quad F_{\text{resultant}} = \frac{\sqrt{3}\, Gm^2}{L^2}$$

$$a = \frac{F}{m} = \frac{\sqrt{3}\, Gm}{L^2}$$

(The Gravitational Constant, Acceleration Due to Gravity of the Earth)

12. The gravitational constant depends upon

(1) Size of the bodies (2) Gravitational mass

(3) Distance between the bodies (4) None of these

Sol. Answer (4)

Gravitational constant '*G*' is independent of size of bodies, gravitational mass and distance between the bodies.

13. Two planets have same density but different radii. The acceleration due to gravity would be

(1) Same on both planets (2) Greater on the smaller planet

(3) Greater on the larger planet (4) Dependent on the distance of planet from the sun

Sol. Answer (3)

Acceleration due to gravity at the surface of a planet, $g = \frac{GM}{R^2}$, where *M* is the mass of planet, *R* is the radius of the planet,

Also, $M = pV$

$$\Rightarrow \quad g = \frac{G}{R^2} \times \left(\frac{4}{3}\pi GR^3\rho\right)$$

Thus, $g = \frac{4}{3}\pi GR\rho$

Thus $g \propto$ Radius of the planet,

Thus, acceleration due to gravity would be greater on the larger planet.

14. If the radius of earth shrinks by 1.5% (mass remaining same), then the value of gravitational acceleration changes by

(1) 2% (2) –2% (3) 3% (4) –3%

Sol. Answer (3)

$$g = \frac{GM}{R^2}$$

$$g' = \frac{GM}{(0.985\ R)^2}$$

$$g' = (1.0306)\frac{GM}{R^2}$$

$$\Rightarrow \quad g' = 1.0306\ g$$

$\Rightarrow$ Acceleration changes by

$$\frac{\Delta g}{g} \times 100 = +3\%$$

Alternate method:

$$g' = \frac{GM}{(R+\Delta R)^2}$$

$$g' = GM(R+\Delta R)^{-2}$$

$$g' = \frac{GM}{R^2}\left(1+\frac{\Delta R}{R}\right)^{-2}$$

for $\frac{\Delta R}{R} \ll 1$, we can use binomial and approximately,

$$g' = \frac{GM}{R^2}\left(1-\frac{2\Delta R}{R}\right)$$

$$\Rightarrow \quad g' = g - g\frac{2\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{-2\Delta R}{R} = -2\times\left(\frac{-1.5}{100}\right) = \frac{+3}{100} = 3\% \qquad [g' - g = \Delta g]$$

15. If density of a planet is double that of the earth and the radius 1.5 times that of the earth, the acceleration due to gravity on the surface of the planet is

(1) $\frac{3}{4}$ times that on the surface of the earth (2) 3 times that on the surface of the earth

(3) $\frac{4}{3}$ times that on the surface of the earth (4) 6 times that on the surface of the earth

Sol. Answer (2)

Acceleration due to gravity on the surface of a planet is given by, $g = \frac{GM}{R^2}$

$M \rightarrow$ Mass of the planet

$R \rightarrow$ Radius of the planet

Also, $M = \frac{4}{3}\pi R^3 \times \rho$

$\Rightarrow \quad g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\rho G \pi R$

$\rho \rightarrow$ Density of the planet.

$\Rightarrow$ Acceleration due to gravity α ρR

$\Rightarrow \quad \frac{g_{\text{planet}}}{g_{\text{earth}}} = \frac{2\rho_e \times 1.5\, R_e}{\rho_e \times R_e} = 3$

$\Rightarrow$ Acceleration due to gravity on the surface of planet is 3 times that on the surface of earth.

16. The gravitational force on a body of mass 1.5 kg situated at a point is 45 N. The gravitational field intensity at that point is

(1) 30 N/kg (2) 67.5 N/kg (3) 46.5 N/kg (4) 43.5 N/kg

Sol. Answer (1)

Gravitation force = mg

g = gravitation field intensity.

$\Rightarrow$ 45 = 1.5 × g

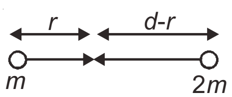
$\Rightarrow \quad g = \frac{45}{1.5} = 30$ N/kg

17. Two point masses having mass m and $2m$ are placed at distance d. The point on the line joining point masses, where gravitational field intensity is zero will be at distance

(1) $\frac{2d}{\sqrt{3}+1}$ from point mass "$2m$" (2) $\frac{2d}{\sqrt{3}-1}$ from point mass "$2m$"

(3) $\frac{d}{1+\sqrt{2}}$ from point mass "m" (4) $\frac{d}{1-\sqrt{2}}$ from point mass "m"

Sol. Answer (3)



Gravitational field intensity will be zero,

$$\Rightarrow \frac{Gm}{r^2} = \frac{2Gm}{(d-r)^2}$$

$$\Rightarrow \frac{1}{r} = \frac{\sqrt{2}}{d-r}$$

$$\Rightarrow d - r = \sqrt{2}r$$

$$\Rightarrow r\left(1+\sqrt{2}\right) = d$$

$$\Rightarrow r = \frac{d}{\left(1+\sqrt{2}\right)}$$

(Acceleration Due to Gravity above the Surface of Earth, Acceleration Due to Gravity below the Surface of Earth)

18. At what height above the surface of earth the value of "*g*" decreases by 2%? [radius of the earth is 6400 km]

(1) 32 km (2) 64 km (3) 128 km (4) 1600 km

Sol. Answer (2)

Acceleration due to gravity above the surface of earth at a height *h* is given $g' = g\left(1 - \frac{2h}{R_e}\right)$

here, g' = 0.98 g

$$\Rightarrow 0.98 = 1 - \frac{2h}{R_e}$$

$$\Rightarrow \frac{2h}{R_e} = 0.02$$

h = 0.01 R_e

= 0.01 × 6400 km

= 64 km

19. If *R* is the radius of earth and *g* is the acceleration due to gravity on the earth's surface. Then mean density of earth is

(1) $\frac{4\pi G}{3gR}$ (2) $\frac{3\pi R}{4gG}$ (3) $\frac{3g}{4\pi RG}$ (4) $\frac{\pi Rg}{12G}$

Sol. Answer (3)

Acceleration due to gravity at earth's surface is given by,

$$g = \frac{GM}{R^2}$$

$$M = \frac{4}{3}\pi R^3 \rho$$

$M \rightarrow$ Mass of earth

$\rho \rightarrow$ Density of earth

$R \rightarrow$ Radius of earth

$$\Rightarrow \quad g = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$$

$$\Rightarrow \quad \rho = \frac{3g}{4\pi GR}$$

20. The value of g at the surface of earth is 9.8 m/s^2. Then the value of 'g' at a place 480 km above the surface of the earth will be nearly (radius of the earth is 6400 km)

(1) 9.8 m/s^2 (2) 7.2 m/s^2 (3) 8.5 m/s^2 (4) 4.2 m/s^2

Sol. Answer (3)

$$g_h = g\left[\frac{R}{R+h}\right]^2$$

$$\Rightarrow \quad g_h = 9.8\left[\frac{6400}{6400+480}\right] = 8.48 \text{ m/s}^2$$

21. If the change in the value of 'g' at a height 'h' above the surface of the earth is same as at a depth x below it, then (x and h being much smaller than the radius of the earth)

(1) $x = h$ (2) $x = 2h$ (3) $x = \frac{h}{2}$ (4) $x = h^2$

Sol. Answer (2)

$$g_h = g\left(1 - \frac{2h}{R_e}\right)$$

$$g_x = g\left(1 - \frac{x}{R_e}\right)$$

According to the question,

$$g_h - g = g_x - g$$

$$\Rightarrow \quad g\left(-\frac{2h}{R_e}\right) = g\left(\frac{-x}{R_e}\right)$$

$$\Rightarrow \quad x = 2h$$

22. The acceleration due to gravity on a planet is 1.96 m/s^2. If it is safe to jump from a height of 3 m on the earth, the corresponding height on the planet will be

(1) 3 m (2) 6 m (3) 9 m (4) 15 m

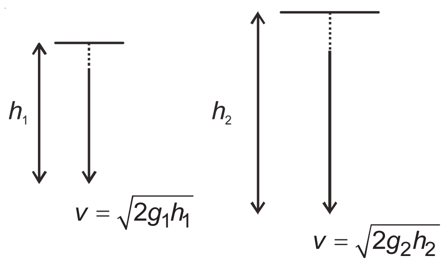
Sol. Answer (4)

It is safer to jump from a height of 3 m on earth,

$\Rightarrow$ Corresponding velocity attained = $\sqrt{2g_1h_1}$

It will be safer to jump from a height on other planet

If the velocity attained is same = $\sqrt{2g_2h_2}$



$$\Rightarrow \quad \sqrt{2g_1h_1} = \sqrt{2g_2h_2}$$

$$9.8 \times 3 = 1.96 \times h_2$$

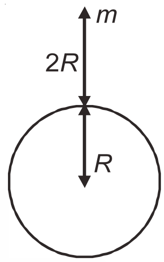
$$\Rightarrow \quad h_2 = 5 \times 3 = 15 \text{ m}$$

(Gravitational Potential Energy)

23. An object is taken to height 2*R* above the surface of earth, the increase in potential energy is [*R* is radius of earth]

(1) $\frac{mgR}{2}$ (2) $\frac{mgR}{3}$ (3) $\frac{2mgR}{3}$ (4) $2\ mgR$

Sol. Answer (3)



Potential energy at surface $= -\frac{GMm}{R}$

Potential energy at height, $2R = -\frac{GMm}{3R}$

Change in potential energy $= -\frac{GMm}{3R} + \frac{GMm}{R}$

$$= \frac{GMm}{R}\left(\frac{-1+3}{3}\right)$$

$$= \frac{2}{3}\frac{GMm}{R}$$

$$= \frac{2}{3}\left(\frac{GM}{R^2}\right)mR$$

$$= \frac{2}{3}mgR$$

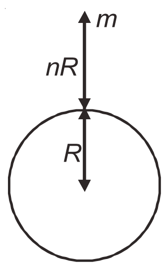
24. The change in potential energy when a body of mass *m* is raised to height *nR* from the earth's surface is (*R* is radius of earth)

(1) $mgR\left(\frac{n}{n-1}\right)$ (2) $nmgR$

(3) $mgR\left(\frac{n}{n+1}\right)$ (4) $mgR\left(\frac{n^2}{n^2+1}\right)$

Sol. Answer (3)

Potential energy at the surface $= -\frac{GMm}{R}$



Potential energy at height, $nR = -\frac{GMm}{(n+1)R}$

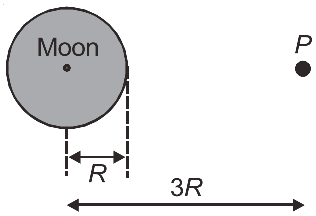
Change in potential energy $= -\frac{GMm}{(n+1)R} + \frac{GMm}{R}$

$$= \frac{GMm}{R}\left(\frac{-1+n+1}{n+1}\right)$$

$$= \left(\frac{n}{n+1}\right)\left(\frac{GM}{R^2}\right)m \times R$$

$$= mgR\left(\frac{n}{n+1}\right)$$

25. A stationary object is released from a point P at a distance $3R$ from the centre of the moon which has radius R and mass M. Which of the following gives the speed of the object on hitting the moon?



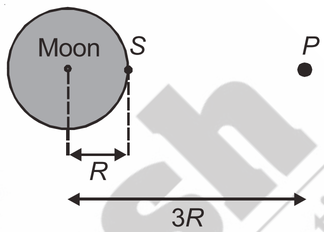
(1) $\left(\frac{2GM}{3R}\right)^{1/2}$ (2) $\left(\frac{4GM}{3R}\right)^{1/2}$ (3) $\left(\frac{GM}{3R}\right)^{1/2}$ (4) $\left(\frac{GM}{R}\right)^{1/2}$

Sol. Answer (2)

Conserving mechanical energy between points P and S,

$$-\frac{GMm}{3R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

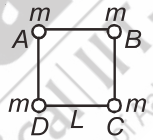
$$\Rightarrow \frac{1}{2}mv^2 = -\frac{GMm}{3R} + \frac{GMm}{R} = \frac{GMm}{R}\left(\frac{-1}{3}+1\right)$$



$$\frac{1}{2}\cancel{m}v^2 = \frac{2GM\cancel{m}}{3R}$$

$$\Rightarrow v = \sqrt{\frac{4GM}{3R}}$$

26. Four particles A, B, C and D each of mass m are kept at the corners of a square of side L. Now the particle D is taken to infinity by an external agent keeping the other particles fixed at their respective positions. The work done by the gravitational force acting on the particle D during its movement is



(1) $2\frac{Gm^2}{L}$ (2) $-2\frac{Gm^2}{L}$ (3) $\frac{Gm^2}{L}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$ (4) $-\frac{Gm^2}{L}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$

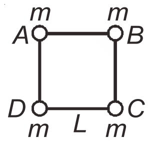
Sol. Answer (4)

Work done by the gravitational force acting on the particle D during its movement

$= -\Delta U$

$= -(U_{final} - U_{initial})$

$= U_{initial} - U_{final}$



Now, when the particle is at infinity, $U = 0$

$\Rightarrow U_{final} = 0$

$\Rightarrow$ Work done $= U_{initial}$

$$U_{initial} = -\frac{Gm^2}{L} - \frac{Gm^2}{L} - \frac{Gm^2}{\sqrt{2}L} = -\frac{Gm^2}{L}\left(2+\frac{1}{\sqrt{2}}\right) = -\frac{Gm^2}{L}\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)$$

27. If an object is projected vertically upwards with speed, half the escape speed of earth, then the maximum height attained by it is [R is radius of earth]

(1) R (2) $\frac{R}{2}$ (3) $2R$ (4) $\frac{R}{3}$

Sol. Answer (4)

$$V_e = \sqrt{\frac{2GM}{R}}$$

$M \rightarrow$ mass of earth

$R \rightarrow$ Radius of earth

Now, conserving potential energy at the surface of earth and highest point,

$$-\frac{GMm}{R} + \frac{1}{2}m\left(\frac{1}{2}\sqrt{\frac{2GM}{R}}\right)^2 = -\frac{GMm}{r}$$

$$-\frac{GMm}{R} + \frac{GMm}{4R} = -\frac{GMm}{r}$$

$$-\frac{3GMm}{4R} = -\frac{GMm}{r}$$

$$\Rightarrow \quad r = \frac{4R}{3}$$

$$\Rightarrow \quad R + h = \frac{4R}{3}$$

$$\Rightarrow \quad h = \left(\frac{R}{3}\right)$$

28. If a satellite of mass 400 kg revolves around the earth in an orbit with speed 200 m/s then its potential energy is

(1) –1.2 MJ (2) –8.0 MJ

(3) –16 MJ (4) –2.4 MJ

Sol. Answer (3)

For a satellite,

$$\text{P.E} = -\frac{GMm}{r}$$

m = mass of satellite

r = radius of orbit

$$\text{K.E} = \frac{1}{2}mv^2 = \frac{GMm}{2r} = -\frac{\text{P.E}}{2}$$

$\Rightarrow$ P.E = – mv^2

= – 400 × 4 × 10^4

= – 16 MJ

29. An artificial satellite revolves around a planet for which gravitational force(F) varies with distance r from its centre as $F \propto r^2$. If v_0 its orbital speed, then

(1) $v_0 \propto r^{-1/2}$ (2) $v_0 \propto r^{3/2}$ (3) $v_0 \propto r^{-3/2}$ (4) $v_0 \propto r$

Sol. Answer (2)

Gravitational force (F) provides the necessary centripetal force to keep the satellite in orbit,

$$\Rightarrow \frac{mv_0^2}{r} \propto F$$

$$\frac{mv_0^2}{r} \propto r^2$$

$v_0 \rightarrow$ Orbital speed

$r \rightarrow$ Radius of orbit

$$\Rightarrow v_0 \propto r^{3/2}$$

30. If the gravitational potential on the surface of earth is V_0, then potential at a point at height half of the radius of earth is

(1) $\frac{V_0}{2}$ (2) $\frac{2}{3}V_0$ (3) $\frac{V_0}{3}$ (4) $\frac{3V_0}{2}$

Sol. Answer (2)

Gravitational potential on the surface,

$$V_0 = -\frac{GM_e}{R_e}$$

Gravitational potential at height h,

$$V_n = -\frac{GM_e}{\left(R_e + \frac{R_e}{2}\right)}$$

$$= -\frac{2}{3}\frac{GM_e}{R_e}$$

$$= \frac{2}{3}V_0$$

(Escape Speed)

31. The total mechanical energy of an object of mass m projected from surface of earth with escape speed is

(1) Zero (2) Infinite (3) $-\frac{GMm}{2R}$ (4) $-\frac{GMm}{3R}$

Sol. Answer (1)

Total mechanical energy = K.E + P.E

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \text{Total mechanical energy} = \frac{1}{2}m \times \frac{2GM}{R} - \frac{GMm}{R} = 0$$

32. The escape velocity of a body from earth is about 11.2 km/s. Assuming the mass and radius of the earth to be about 81 and 4 times the mass and radius of the moon, the escape velocity in km/s from the surface of the moon will be

(1) 0.54 (2) 2.48 (3) 11 (4) 49.5

Sol. Answer (2)

$$v_{\text{escape}} = \sqrt{\frac{GM}{R}}$$

$$\frac{v_{\text{escape Earth}}}{v_{\text{escape moon}}} = \sqrt{\frac{M_e}{R_e} \times \frac{R_m}{M_m}} = \sqrt{\frac{81}{4}} = \left(\frac{9}{2}\right)$$

$$\Rightarrow \quad v_{\text{moon}} = \frac{2}{9} \times 11.2 = 2.48 \text{ km/s}$$

33. If M is mass of a planet and R is its radius then in order to become black hole [c is speed of light]

(1) $\sqrt{\frac{GM}{R}} \le c$ (2) $\sqrt{\frac{GM}{2R}} \ge c$ (3) $\sqrt{\frac{2GM}{R}} \ge c$ (4) $\sqrt{\frac{2GM}{R}} \le c$

Sol. Answer (3)

A planet can become a black hole if its mass and radius are such that it has an immense force of gravity on its surface. The force of attractum has to be so large that even light cannot escape from its surface.

Speed of light = c

$$v_e = \sqrt{\frac{2GM}{R}}$$

If $v_e \ge c$

$\Rightarrow$ Even light can't escape from the surface of such planet making it appear black.

34. The atmosphere on a planet is possible only if [where v_{rms} is root mean square speed of gas molecules on planet and v_e is escape speed on its surface]

(1) $v_{\text{rms}} = v_e$ (2) $v_{\text{rms}} > v_e$ (3) $v_{\text{rms}} \le v_e$ (4) $v_{\text{rms}} < v_e$

Sol. Answer (4)

The atmosphere on a planet is possible only if $v_{\text{rms}} < v_e$

If $v_{\text{rms}} \ge v_{\text{escape}}$ the gas molecules will leave the surroundings of the planet, i.e., will be free from gravitational pull of the planet.

35. When speed of a satellite is increased by x percentage, it will escape from its orbit, where the value of x is

(1) 11.2% (2) 41.4% (3) 27.5% (4) 34.4%

Sol. Answer (2)

For a satellite near Earth's surface,

$$v_0 = \sqrt{\frac{GM_e}{R_e}}, \quad v_e = \sqrt{\frac{2GM_e}{R_e}}$$

$$v_e = \sqrt{2}\, v_0$$

$$\Rightarrow \quad \text{\% increase, } x = \left(\frac{\sqrt{2}-1}{1}\right) \times 100 = 41.4\%$$

(Earth Satellite, Energy of an Orbiting Satellite)

36. In an orbit if the time of revolution of a satellite is T, then PE is proportional to

(1) $T^{1/3}$ (2) T^3 (3) $T^{-2/3}$ (4) $T^{-4/3}$

Sol. Answer (3)

According to Kepler's third law,

$T^2 \propto r^3$

r = radius of orbit

For a satellite of mass m orbiting in an orbit of radius r around planet of mass M,

$$\text{Potential energy (PE)} = \frac{-GMm}{r}$$

$$\Rightarrow \quad \text{PE} \propto \frac{GMm}{T^{2/3}}$$

$$\Rightarrow \quad \text{PE} \propto T^{-2/3}$$

37. A small satellite is revolving near earth's surface. Its orbital velocity will be nearly

(1) 8 km/s (2) 11.2 km/s (3) 4 km/s (4) 6 km/s

Sol. Answer (1)

For a satellite revolving near earth's surface, $v_0 = \sqrt{\frac{GM_e}{R_e}} = \sqrt{gR_e}$

Taking g = 9.81 m/s^2 and R_e = 6400 km

$$v_0 = \sqrt{\frac{9.8}{1000} \times 6400} = 7.92 \text{ km/s} \simeq 8 \text{ km/s}$$

38. If potential energy of a satellite is –2MJ, then the binding energy of satellite is

(1) 1 MJ (2) 2 MJ (3) 8 MJ (4) 4 MJ

Sol. Answer (1)

For a satellite of mass m revolving around a planet of mass in a circular orbit of radius r,

$$\text{P.E} = -\frac{GMm}{r}$$

$$\text{K.E} = \frac{1}{2}m\frac{GM}{r} = \frac{GMm}{2r}$$

$$\text{T.E} = -\frac{GMm}{2r}$$

$$\text{Binding energy} = |\text{T.E}| = \frac{GMm}{2r}$$

$$= \frac{|\text{P.E}|}{2} = 1\,\text{MJ}$$

Alternate method,

$$\text{Binding energy} = -\text{T.E}$$

$$= -\frac{\text{P.E}}{2}$$

$$= 1 \text{ MJ}$$

(Geostationary and Polar Satellites)

39. The time period of polar satellites is about

(1) 24 hr (2) 100 min

(3) 84.6 min (4) 6 hr

Sol. Answer (2)

Time period of polar satellites is about 100 minutes polar satellites are low altitude satellites. ($h \simeq$ 500 – 800 km)

40. The mean radius of earth is R, and its angular speed on its axis is ω. What will be the radius of orbit of a geostationary satellite?

(1) $\left(\frac{Rg}{\omega^2}\right)^{1/3}$ (2) $\left(\frac{R^2g}{\omega^2}\right)^{1/3}$ (3) $\left(\frac{R^2g}{\omega}\right)^{1/3}$ (4) $\left(\frac{R^2\omega^2}{g}\right)^{1/3}$

Sol. Answer (2)

Time period of rotation of earth $=\frac{2\pi}{\omega}$

(Duration of one day)

Geostationary satellite has same time period, $T=\frac{2\pi}{\omega}$. Let r be the radius of orbit of satellite

$\Rightarrow$ Time period of satellite $=\frac{2\pi r^{3/2}}{\sqrt{GM_e}}$

Also, $g=\frac{GM_e}{R_e^2}$

$$\Rightarrow \quad T=\frac{2\pi\, r^{3/2}}{\sqrt{g}(R_e)}=\frac{2\pi\, r^{3/2}}{R_e\sqrt{g}}=\frac{2\pi}{\omega}$$

$$\Rightarrow \quad r^{3/2}=\frac{R_e}{\omega}\sqrt{g}$$

$$\Rightarrow \quad r=\left(\frac{R_e^2}{\omega^2}g\right)^{1/3}$$

41. A satellite of the earth is revolving in a circular orbit with a uniform speed v. If the gravitational force suddenly disappears, the satellite will

(1) Continue to move with velocity v along the original orbit

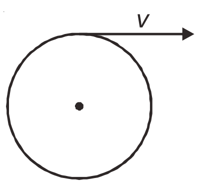
(2) Move with a velocity v, tangentially to the original orbit

(3) Fall down with increasing velocity

(4) Ultimately come to rest somewhere on the original orbit

Sol. Answer (2)

For a satellite revolving in a circular orbit, gravitational force provides the necessary centripetal force. If the gravitational force suddenly disappears, the satellite will move with a velocity v, tangentally to the original orbit.



42. The relay satellite transmits the television signals continuously from one part of the world to another because its

(1) Period is greater than the period of rotation of the earth

(2) Period is less than the period of rotation of the earth

(3) Period has no relation with the period of the earth about its axis

(4) Period is equal to the period of rotation of the earth about its axis

Sol. Answer (4)

A relay satellite transmits the television signals continuously from one part of the world to another because its period is equal to the period of rotation of the earth about its axis.

43. If height of a satellite from the surface of earth is increased, then its

(1) Potential energy will increase

(2) Kinetic energy will decrease

(3) Total energy will increase

(4) All of these

Sol. Answer (4)

For a satellite orbiting at height h from earth,

$$P.E = -\frac{GM_e m_s}{(R_e + h)}$$

$$K.E = \frac{GM_e m_s}{2(R_e + h)}$$

$$T.E = -\frac{GM_e m_s}{2(R_e + h)}$$

If h is increased, P.E increases (becomes less negative)

K.E decreases

T.E increases (becomes less negative)

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