Chapter 10

Mechanical Properties of Fluids

Solutions

SECTION - A

Objective Type Questions

(Pressure)

1. The atmospheric pressure at a place is 10^5 Pa. If tribromomethane (specific gravity = 2.9) be employed as the barometric liquid, the barometric height is

   (1) 3.52 m (2) 1.52 m

   (3) 4.52 m (4) 2.52 m

Sol. Answer (1)

$1 \text{ atm} \simeq 10^5 \text{ Pa} = 76 \text{ cm of Hg}$

Density of Hg = 13.6

$\rho_{Hg} \times g \times h_{Hg} = \rho_{TBM} \times g \times h_{TBM}$

Substituting values

$13.6 \times 76 = 2.9 \times h$

$3.52 \text{ m} = h$

$\left\{\begin{array}{l} \rho_{Hg} = \text{density of Hg} \\ h_{Hg} = \text{height of Hg} \\ \rho_{TBM} = \text{density of tribromomethane} \\ h_{TBM} = \text{height of tribromomethane} \end{array}\right.$

2. A large vessel of height H, is filled with a liquid of density ρ, upto the brim . A small hole of radius r is made at the side vertical face, close to the base. The horizontal force is required to stop the gushing of liquid is

   (1) $(\rho g H)\pi r^2$ (2) $\rho g H$

   (3) $\rho g H \pi r$ (4) $\rho g \pi r^2$

Sol. Answer (1)

Pressure close to the base = $\rho g H$

Force required = pressure × area of hole = $\rho g H(\pi r^2)$

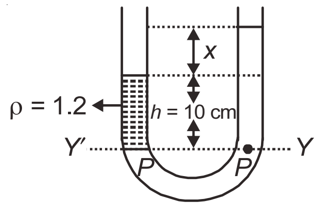
3. A vertical U-tube of uniform cross-section contains water in both the arms. A 10 cm glycerine column (R.D. = 1.2) is added to one of the limbs. The level difference between the two free surfaces in the two limbs will be

   (1) 4 cm (2) 2 cm (3) 6 cm (4) 8 cm

Sol. Answer (2)

Let the difference between 2 limbs be x

Pressure on the line YY' should be same below both limbs, so



$\rho_{glycerine} \times g \times h = \rho_{water} \times g \times (h + x)$

$\Rightarrow$ $1.2 \times 10 = 1 \times (h + x)$

$\Rightarrow$ $2 \text{ cm} = x$

4. The pressure at the bottom of a water tank is 4 P, where P is atmospheric pressure. If water is drawn out till the water level decreases by $\frac{3}{5}$th, then pressure at the bottom of the tank is

(1) $\frac{3P}{8}$ (2) $\frac{7P}{6}$ (3) $\frac{11P}{5}$ (4) $\frac{9P}{4}$

Sol. Answer (3)

Let height of water in tank be h

So, $4P - P = \rho_w gh$ ...(1)

$\because$ $\frac{3}{5}$ water taken out $\frac{2}{5}$th water is left to exert pressure

$$P' = P + \frac{2}{5}\rho_w gh$$

$\Rightarrow$ $P' = P + \frac{2}{5} \times 3P$ [From eq. (1)]

$\Rightarrow$ $P' = \frac{11P}{5}$

5. A air bubble rises from bottom of a lake to surface. If its radius increases by 200% and atmospheric pressure is equal to water coloumn of height H, then depth of lake is

(1) 21 H (2) 8 H (3) 9 H (4) 26 H

Sol. Answer (4)

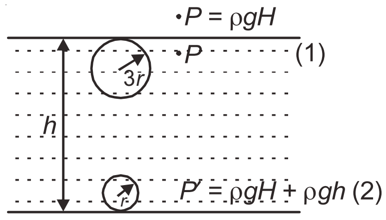
Let initial radius be = r

Final radius = r + 200% of r

= $3r$

Atmospheric pressure = ρgH

Let depth of the lake be h



So, pressure at the bottom of lake = $\rho gH + \rho gh$

Using $P_1V_1 = P_2V_2$

$$\rho gH \times \frac{4}{3}\pi(3r)^3 = (\rho gH + \rho gh) \times \frac{4}{3}\pi r^3$$

$$\rho gH \times \frac{4}{3}\pi \times 27r^3 = (\rho gH)\frac{4}{3}\pi r^3 + \rho gh \times \frac{4}{3}\pi r^3$$

Solving this equation we get

$26H = h$

6. A liquid mixture of volume V, has two liquids as its ingredients with densities α and β. If density of the mixture is σ, then mass of the first liquid in mixture is

(1) $\frac{\alpha V[\sigma\beta+1]}{\beta[\alpha+\sigma]}$ (2) $\frac{\alpha V[\sigma-\beta]}{[\sigma+\beta]}$ (3) $\frac{\alpha V(\beta-\sigma)}{\beta-\alpha}$ (4) $\frac{\alpha V[1-\sigma\alpha]}{\beta[\alpha-\sigma]}$

Sol. Answer (3)

Let mass of liquid with density $\alpha = M_1$

Let mass of liquid with density $\beta = M_2$

Total volume = V

Net density of mixture = σ

Now,

Total mass = $M_1 + M_2$

$\Rightarrow \quad V\sigma = M_1 + M_2 \qquad \left[\because \frac{\text{Total mass}}{V} = \sigma\right]$

$\Rightarrow \quad M_2 = V\sigma - M_1 \qquad ...(1)$

Now,

$$\sigma = \frac{\text{Total mass}}{\text{Total volume}} = \frac{(M_1+M_2)}{\left(\frac{M_1}{\alpha}\right)+\left(\frac{M_2}{\beta}\right)}$$

Substituting value of M_2 from equation (1)

$$\sigma = \frac{M_1+(V\sigma-M_1)}{\frac{M_1}{\alpha}+\frac{(V\sigma-M_1)}{\beta}}$$

Solving this we get

$$M_1 = \frac{\alpha V(\beta-\sigma)}{\beta-\alpha}$$

7. A barometer kept in an elevator reads 76 cm when the elevator is accelerating upwards. The most likely pressure inside the elevator (in cm of Hg) is

(1) 74 (2) 75 (3) 76 (4) 77

Sol. Answer (4)

8. A beaker containing a liquid of density ρ moves up with an acceleration 'a'. The pressure due to the liquid at a depth h below free surface of the liquid is

(1) $h\rho g$ (2) $h\rho\,(g-a)$

(3) $h\rho\,(g+a)$ (4) $2h\rho g\left(\frac{g+a}{g-a}\right)$

Sol. Answer (3)

Due to upward acceleration pseudo force will act downwards so value of acceleration due to gravity will increase by 'a'

$\therefore \quad g' = (g+a)$

$P = \rho g'h$

$\Rightarrow \quad P = \rho\,(g+a)h \qquad$ (Substitute g')

9. A barometer kept in an elevator reads 76 cm when it is at rest. If the elevator goes up with some acceleration, the reading will be

(1) 76 cm (2) > 76 cm (3) < 76 cm (4) Zero

Sol. Answer (3)

When elevator goes up with some acceleration upward, due to pseudo force acting downwards. Value of g increases to g'.

If g increases to g', *i.e.*, $g' = g + a$

$\because \quad P = \rho gh = \rho(g + a)h' = \text{constant}$

Then, $h' < h$

i.e., $h' < 76$ cm

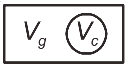
Archimedes' principle

10. A piece of gold weighs 10 g in air and 9 g in water. What is the volume of cavity?

(Density of gold = 19.3 g cm^{-3})

(1) 0.182 cc (2) 0.282 cc (3) 0.382 cc (4) 0.482 cc

Sol. Answer (4)



When dipped in water

$W_{app} = W_{air} - F_B$

$\Rightarrow$ 9 gm × g = 10 gm × g – F_B

$\Rightarrow$ 1 × g = F_B

Now (total volume displaced) × ρ_w × g = 1 × g

$(V_c + V_g) \times 1 = 1$

$$V_c = 1 - \frac{\text{Mass of gold in air}}{\rho_g} = 1 - \frac{10}{19.3} = 0.482 \text{ cc}$$

$\left\{\begin{array}{l}\text{Where,}\\ V_c = \text{volume of cavity}\\ V_g = \text{volume of gold}\\ W_{app} = 9 \text{ gm}\\ W_{air} = 10 \text{ gm}\\ F_B = \text{force of buoyancy}\\ \rho_w = \text{density of water} = 1\\ \rho_g = \text{density of gold} = 19.3\end{array}\right.$

11. A block of ice floats in an oil in a vessel when the ice melts, the level of oil will

(1) Go up (2) Go down

(3) Remain same (4) Go up or down depending on quantity of ice

Sol. Answer (2)

Since block of ice is displacing some oils to stay afloat when the ice block melts level of oil will go down.

12. An object suspended by a wire stretches it by 10 mm. When object is immersed in a liquid the elongation in wire reduces by $\frac{10}{3}$ mm. The ratio of relative densities of the object and liquid is

(1) 3 : 1 (2) 1 : 3 (3) 1 : 2 (4) 2 : 1

Sol. Answer (1)

$$\Delta L = \frac{FL}{AY}$$

$\left\{\begin{array}{l}\text{Let density of liquid} = \rho\\ \text{Let density of object} = \sigma\\ \text{Mass of object} = M\end{array}\right.$

$\Rightarrow$ Elongation $\propto$ force and force is due to weight

So elongation $\propto$ weight

$\Delta L_1 \propto$ weight ...(1) {When not submerged in liquid}

$\Delta L_2 \propto$ apparent weight ...(2) {When submerged in liquid}

Dividing (1) by (2)

$$\frac{10}{10-\frac{10}{3}} = \frac{Mg}{Mg-\frac{Mg\rho}{\sigma}}$$

$$\Rightarrow \quad \frac{1}{1-\frac{1}{3}} = \frac{1}{1-\frac{\rho}{\sigma}}$$

Solving this we get

$$\frac{\rho}{\sigma} = \frac{1}{3}$$

So relative densities of object (σ) and liquid (ρ) is 3 : 1

13. A spring balance reads 200 gf when carrying a lump of lead in air. If the lead is now immersed with half of its volume in brine solution, what will be the new reading of the spring balance? specific gravity of lead and brine are 11.4 and 1.1 respectively

(1) 190.4 gf (2) 180.4 gf (3) 210 gf (4) 170.4 gf

Sol. Answer (1)

$$W' = W - F_B$$

$$= v\sigma g - \frac{v}{2}\rho g$$

$$= v\sigma g\left(1-\frac{\rho}{2\sigma}\right)$$

$$\left\{\begin{array}{l} \text{Where,} \\ W' = \text{apparent weight} \\ W = \text{read weight = actual weight of body in vaccum} \\ \rho = \text{density of solution (1.1)} \\ \sigma = \text{density of material (11.4)} \end{array}\right.$$

$$W' = 200\left(1-\frac{1.1}{11.4\times 2}\right)$$

$= 190.35$ gf

14. A boat having some iron pieces is floating in a pond. If iron pieces are thrown in the liquid then level of liquid

(1) Increases (2) Decreases

(3) May increase or decrease (4) Neither increases nor decreases

Sol. Answer (2)

15. An ice cube contains a large air bubble. The cube is floating on the horizontal surface of water contained in a trough. What will happen to the water level, when the cube melts?

(1) It will remain unchanged (2) It will fall

(3) It will rise (4) First it will fall and then rise

Sol. Answer (1)

16. The reading of a spring balance when a block is suspended from it in air is 60 N. This reading is changed to 40 N when the block is submerged in water. The specific gravity of the block must be therefore

(1) 3 (2) 2 (3) 6 (4) 1.5

Sol. Answer (1)

17. The weight of a body in water is one third of its weight in air. The density of the body is

(1) 0.5 g/cm^3 (2) 1.5 g/cm^3

(3) 2.5 g/cm^3 (4) 3.5 g/cm^3

Sol. Answer (2)

18. A cubical block is floating in a liquid with one fourth of its volume immersed in the liquid. If whole of the system accelerates upward with acceleration $g/4$, the fraction of volume immersed in the liquid will be

(1) 1/4 (2) 1/2 (3) 3/4 (4) 2/3

Sol. Answer (1)

Upward acceleration just causes the acceleration due to gravity increases by some value, but since the term of 'g' gets cancelled out in the buoyancy equation.

Volume immersed × ρ_w × g = Total volume × ρ_{cube} × g

So, increasing it will not have any effect on the immersed volume.

19. A boat carrying a number of stones is floating in a water tank. If the stones are unloaded into water, the water level in the tank will

(1) Remain unchanged

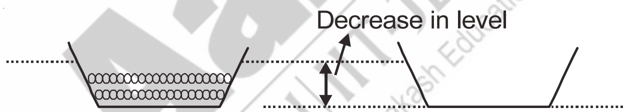
(2) Rise

(3) Fall

(4) Rise or fall depends on the number of stones unloaded

Sol. Answer (3)

Previously when stones are on the boat they are increasing the weight on the boat and to balance this weight boat needs to generate buoyancy force by displacing more water, but when stones are removed the boat starts displacing less amount of water hence the level of water in tank falls.



20. A block of ice is floating in a liquid of specific gravity 1.2 contained in a beaker. When the ice melts completely, the level of liquid in the vessel

(1) Increases (2) Decreases

(3) Remain unchanged (4) First increases then decreases

Sol. Answer (1)

Density of ice is less than water and density of liquid is more than water. So even when ice melts the level will rise.

If $\rho_{liquid} > \rho_{water}$ then level (liquid + water) will rise.

(Streamline Flow, Bernoulli's Principle and Viscosity)

21. Water flows in a stream line manner through a capillary tube of radius a. The pressure difference being P and the rate of flow is Q. If the radius is reduced to $\frac{a}{4}$ and the pressure is increased to $4P$, then the rate of flow becomes

(1) $4Q$ (2) $\frac{Q}{2}$ (3) Q (4) $\frac{Q}{64}$

Sol. Answer (4)

Rate of flow ∝ pressure difference × (radius)^4

$$Q \propto P \times a^4 \qquad \left\{\because \; Q = \frac{\pi P r^4}{8\eta L}\right\}$$

So, $\dfrac{Q_1}{Q_2} = \dfrac{P_1 a_1^4}{P_2 a_2^4}$

$$\frac{Q_1}{Q_2} = \frac{P \times a^4}{4P \times \left(\frac{a}{4}\right)^4} = \frac{64}{1}$$

$$\therefore \quad Q_2 = \frac{Q_1}{64} = \frac{Q}{64}$$

22. Three capillaries of length L, $\frac{L}{2}$ and $\frac{L}{3}$ are connected in series. Their radii are r, $\frac{r}{2}$ and $\frac{r}{3}$ respectively. Then if stream-line flow is to be maintained and the pressure across the first capillary is P, then

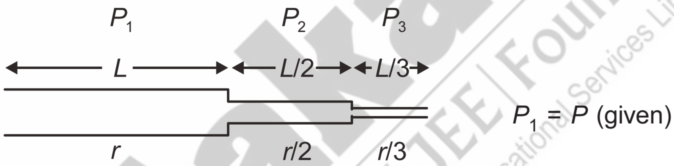
(1) The pressure difference across the ends of second capillary is $8P$

(2) The pressure difference across the third capillary is $43P$

(3) The pressure difference across the ends of second capillary is $16P$

(4) The pressure difference across the third capillary is $59P$

Sol. Answer (1)



$\because$ Rate of flow will be same across all pipes

So, pressure across the pipe $\propto \dfrac{\text{length}}{(\text{radius})^4}$ $\left\{\begin{array}{l}\text{rate of flow of liquid } (Q) \\ Q = \dfrac{\pi P r^4}{8\eta L}\end{array}\right.$

$$\frac{P_1}{P_2} = \frac{\left(L/r^4\right)}{\left(\dfrac{L/2}{(r/2)^4}\right)} = \frac{1}{8}$$

Then $P_2 = 8P_1$

23. Air streams horizontally past an air plane. The speed over the top surface is 60 m/s and that under the bottom surface is 45 m/s. The density of air is 1.293 kg/m^3, then the difference in pressure is

(1) 1018 N/m^2 (2) 516 N/m^2 (3) 1140 N/m^2 (4) 2250 N/m^2

Sol. Answer (1)

Applying Bernoullis equation

$$P_1 + \rho gh + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2} \times \rho\,[v_1^2 - v_2^2] = P_2 - P_1 = \Delta P$$

$$\Rightarrow \frac{1}{2} \times 1.293\,[(60)^2 - (45)^2] = \Delta P$$

$$\Rightarrow 1018 \text{ N/m}^2 \simeq \Delta P$$

24. Two water pipes P and Q having diameter 2×10^{-2} m and 4×10^{-2} m respectively are joined in series with the main supply line of water. The velocity of water flowing in pipe P is

(1) Four times that of Q (2) Two times that of Q

(3) $\frac{1}{2}$ times that of Q (4) $\frac{1}{4}$ times that of Q

Sol. Answer (1)

Rate of flow through both pipes will be same

i.e., $Q_1 = Q_2$

$$\frac{V_1}{t} = \frac{V_2}{t}$$

$$\frac{\pi r_1^2 l_1}{t} = \frac{\pi r_2^2 l_2}{t}$$

$$\left(\text{Where } \frac{l_1}{t} = V_P \text{ and } \frac{l_2}{t} = V_Q\right)$$

$$\Rightarrow \frac{\pi d_1^2}{4} V_P = \frac{\pi d_2^2}{4} \times V_Q$$

$$\Rightarrow V_P = \left(\frac{d_2}{d_1}\right)^2 V_Q$$

$$\Rightarrow V_P = \left(\frac{4 \times 10^{-2}}{2 \times 10^{-2}}\right)^2 V_Q$$

$$\Rightarrow V_P = 4V_Q$$

25. At what speed, the velocity head of water is equal to pressure head of 40 cm of mercury?

(1) 2.8 m/s (2) 10.32 m/s (3) 5.6 m/s (4) 8.4 m/s

Sol. Answer (2)

$$\frac{1}{2}\rho_{\text{water}} V^2 = \rho_{\text{mercury}}\, gh$$

$$V = \sqrt{2 \times \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} \times g \times h}$$

$$= \sqrt{2 \times 13.6 \times 9.8 \times \frac{40}{100}}$$

$\Rightarrow$ $V = 10.32$ m/s

26. If the terminal speed of a sphere of gold (density 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid.

(1) 0.2 m/s (2) 0.4 m/s (3) 0.1 m/s (4) 0.133 m/s

Sol. Answer (3)

$$V_{\text{terminal}} = \frac{2a^2}{9\eta}(\rho - \sigma)g$$

$$\left\{\begin{array}{l}\text{Where} \\ \rho = \text{density of material} \\ \sigma = \text{density of liquid}\end{array}\right.$$

$\Rightarrow$ $V_T \propto (\rho - \sigma)$

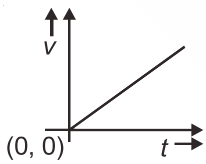
$$\Rightarrow \frac{V_{T_1}}{V_{T_2}} = \frac{\rho_{\text{gold}} - \sigma_{\text{liquid}}}{\rho_{\text{silver}} - \sigma_{\text{liquid}}}$$

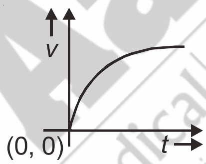
$$\Rightarrow \frac{0.2}{V} = \frac{19.5 - 1.5}{10.5 - 1.5}$$

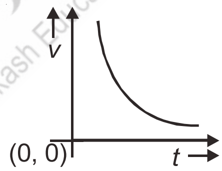
$\Rightarrow$ $V = 0.1$ m/s

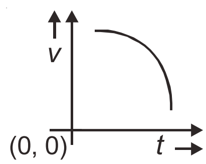
$$\left\{\begin{array}{l}\text{Given,} \\ V_{T_1} = 0.2 \text{ m/s} \\ V_{T_2} = V = \ ? \\ \rho_{\text{gold}} = 19.5 \text{ kg/m}^3 \\ \sigma_{\text{liquid}} = 1.5 \text{ kg/m}^3 \\ \rho_{\text{silver}} = 10.5 \text{ kg/m}^3\end{array}\right.$$

27. A spherical ball is dropped in a long column of viscous liquid. The speed v of the ball varies as function of time as

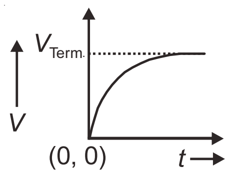
(1) 

(2) 

(3) 

(4) 

Sol. Answer (2)



Velocity does not increases after terminal speed is achieved.

(Surface Tension)

28. If T is the surface tension of a fluid, then the energy needed to break a liquid drop of radius R into 64 equal drops is

(1) $6\pi R^2 T$ (2) $\pi R^2 T$ (3) $12\pi R^2 T$ (4) $8\pi R^2 T$

Sol. Answer (3)

Work done = surface tension × change in area

Since volume will remain equal

Let us assume radius of new drop = r each

$$\Rightarrow \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{R}{4} = r$$

$W = T \cdot \Delta A$

$= T[n \times 4\pi r^2 - 4\pi R^2]$

$$= T\left[64 \times 4\pi\left(\frac{R}{4}\right)^2 - 4\pi R^2\right] = 12\pi R^2 T$$

29. The excess pressure inside a spherical drop of water is four times that of another drop. Then their respective mass ratio is

(1) 1 : 16 (2) 1 : 64 (3) 1 : 4 (4) 1 : 8

Sol. Answer (2)

$$\Delta P = \frac{2T}{R}$$

$$\left\{\begin{array}{l}\text{Where,}\\ P_0 = \text{Excess pressure}\\ T = \text{Surface tension}\\ R = \text{Radius}\end{array}\right.$$

Pressure $\propto \frac{1}{\text{Radius}}$

$$\Rightarrow \frac{\Delta P_1}{\Delta P_2} = \frac{R_2}{R_1}$$

$$\Rightarrow \frac{4P}{P} = \frac{R_2}{R_1}$$

$\Rightarrow 4R_1 = R_2$

$$\Rightarrow \frac{R_1}{R_2} = \frac{1}{4}$$

$M = V \times \rho$

And $V \propto R^3 \Rightarrow M \propto \rho R^3$

ρ is same for both

$M \propto R^3$

$$\left\{\begin{array}{l}\text{Where,}\\ M = \text{Mass}\\ V = \text{Volume}\\ \rho = \text{Density}\end{array}\right.$$

So, $\frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

30. The work done in blowing a soap bubble of 10 cm radius is (surface tension of soap solution is 0.03 N/m).

(1) 37.68×10^{-4} J (2) 75.36×10^{-4} J (3) 126.82×10^{-4} J (4) 75.36×10^{-3} J

Sol. Answer (2)

Work done = surface tension × change in area × number of free surfaces = $S \times \Delta A \times 2$

$= 0.03 \times 4\pi \times (10 \times 10^{-2})^2 \times 2$

$= 75.36 \times 10^{-4}$ J

31. A glass capillary tube of inner diameter 0.28 mm is lowered vertically into water in a vessel. The pressure to be applied on the water in the capillary tube so that water level in the tube is same as that in the vessel is (surface tension of water = 0.07 N/m and atmospheric pressure = 10^5 N/m^2).

(1) 10^3 (2) 99×10^3 (3) 100×10^3 (4) 101×10^3

Sol. Answer (4)

Height of liquid in capillary $= \dfrac{2T}{r\rho g} = h$

Pressure we need to apply = $\rho gh + P_0$

Substitute value of h

$$P = \rho g \times \frac{2T}{r\rho g} + P_0 = \frac{2T}{r} + P_0 = \frac{4T}{d} + P_0$$

$$\Rightarrow P = \frac{4 \times 0.07}{(0.28 \times 10^{-3})} + P_0 = 1000 \text{ Nm}^{-2} + 10^5 \text{ Nm}^{-2}$$

$\Rightarrow P = (10^3 + 10^5)$ Nm^{-2} = 101×10^3 Nm^{-2}

$$\begin{cases} \text{Where,} \\ T = \text{Surface tension} \\ r = \text{Radius of capillary} \\ \rho = \text{Density of liquid} \\ P_0 = \text{Atmospheric pressure} \end{cases}$$

$$\begin{cases} \text{Given,} \\ T = 0.07 \text{ N/m} \\ d = 0.28 \text{ mm} \end{cases}$$

32. Water rises to a height of 10 cm in a capillary tube and mercury falls to a depth of 3.42 cm in the same capillary tube. If the density of mercury is 13.6 kg/m^3 and angle of contact is 135°, the ratio of surface tension for water and mercury is (angle of contact for water and glass is 0°).

(1) 1 : 0.5 (2) 1 : 3

(3) 1 : 6.5 (4) 1.5 : 1

Sol. Answer (3)

$$h = \frac{2T\cos\theta}{r\rho g}$$

For water,

$$10 \text{ cm} = \frac{2 \times T_w \times \cos 0^\circ}{r \times 1 \times g} \quad ...(1)$$

{T_w – Surface tension of water

For mercury,

$$-3.42 \text{ cm} = \frac{2 \times T_M \times \cos 135^\circ}{r \times 13.6 \times g} \quad ...(2)$$

{T_M – Surface tension of mercury

Dividing Eq^n (1) by (2)

$$\frac{10}{-3.42} = \frac{2 \times T_w \times 1 \times r \times 13.6 \times g}{r \times 1 \times g \times 2 \times T_M \times \frac{-1}{\sqrt{2}}}$$

$$\Rightarrow \frac{10}{3.42} = \sqrt{2} \times 13.6 \times \frac{T_w}{T_M}$$

$$\Rightarrow \frac{10}{3.42\times1.41\times13.6} = \frac{T_w}{T_M}$$

$$\Rightarrow \frac{1}{6.5} = \frac{T_w}{T_M}$$

33. A spherical drop of water has 1 mm radius. If the surface tension of water is 75 × 10^{-3} N/m, then difference of pressure between inside and outside of the drop is

(1) 35 N/m^2 (2) 70 N/m^2 (3) 140 N/m^2 (4) 150 N/m^2

Sol. Answer (4)

Excess pressure $= \frac{2T}{R}$ $\begin{cases} T = \text{surface tension} \\ R = \text{radius} \end{cases}$

$$= \frac{2\times75\times10^{-3}}{1\times10^{-3}}$$

= 150 N/m^2

34. A capillary tube is dipped in water and it is 20 cm outside water. The water rises upto 8 cm. If the entire arrangement is put in freely falling elevator the length of water column in the capillary tube will be

(1) 20 cm (2) 4 cm (3) 10 cm (4) 8 cm

Sol. Answer (1)

If entire arrangement is in free fall then the weight of water in capillary will be balanced by pseudo force which would be equal to the weight of water.

Hence, surface tension has no weight to balance so full capillary will be filled with water.

35. If the excess pressure inside a soap bubble is balanced by an oil column of height 2 mm, then the surface tension of soap solution will be (r = 1 cm, density of oil = 0.8 g/cm^3)

(1) 3.9 N/m (2) 3.9 × 10^{-2} N/m (3) 3.9 × 10^{-3} N/m (4) 3.9 × 10^{-1} N/m

Sol. Answer (2)

Pressure due to oil column = $\rho_{oil} \times g \times h_{oil} = \frac{0.08\times10^{-3}}{(10^{-2})^3}\times9.8\times2\times10^{-3} = 15.68$

Now, excess pressure = pressure due to oil column

$$\Rightarrow \frac{4T}{R} = 15.68$$

$$\Rightarrow \frac{4\times T}{1\times10^{-2}} = 15.68$$

$\Rightarrow$ T = 3.92 × 10^{-2} N/m

36. There is small hole in a hollow sphere. The water enters in it when it is taken to a depth of 40 cm under water. The surface tension of water is 0.07 N/m. The diameter of hole is

(1) 7 mm (2) 0.07 mm (3) 0.0007 mm (4) 0.7 m

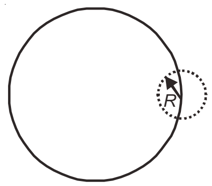
Sol. Answer (2)

Let take g = 10 m/s^2

For water to enter the sphere, pressure required is = ρgh

$$= 1\times 10\times \frac{40}{100}\times 1000 \quad (\rho = 1000\ \text{kg/m}^3)$$

$$= 4000\ \frac{\text{N}}{\text{m}^2} = \text{excess pressure}$$



Let the hole have radius = R

Excess pressure $= \frac{2T}{R}$ [One surface air, one surface water]

$\Rightarrow \quad 4000 = \frac{2\times 0.07}{R}$

$\Rightarrow \quad 2R = 0.07 \times 10^{-3}$ m

$\Rightarrow \quad d = 0.07$ mm

37. Two equal drops are falling through air with a steady velocity of 5 cm/second. If two drops coalesce, then new terminal velocity will be

(1) $5 \times (4)^{1/3}$ cm/s (2) $5\sqrt{2}$ cm/s (3) $\frac{5}{\sqrt{2}}$ cm/s (4) 5×2 cm/s

Sol. Answer (1)

$V_{\text{Terminal}} \propto r^2$

If initial radius = r, let new radius = R

Then $2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow (2)^{1/3} r = R$

$\Rightarrow V_T \propto R^2$

$\propto (2)^{2/3} r^2$ (For bigger drops)

$$\frac{V_{T\ \text{smaller drop}}}{V_{T\ \text{bigger drop}}} = \frac{r^2}{(2)^{2/3} r^2}$$

$\Rightarrow \frac{5}{x} = \frac{1}{(2)^{2/3}}$

$\Rightarrow 5 \times (2)^{2/3} = x$

$\Rightarrow 5 \times (4)^{1/3}$ cm/s $= x$

38. A small drop of water falls from rest through a large height h in air; the final velocity is

(1) Proportional to $\sqrt{h}$ (2) Proportional to h

(3) Inversely proportional to h (4) Almost independent of h

Sol. Answer (4)

Since drop is falling from a large height it achieves its terminal velocity and then there is no further increase in velocity so v is independent of 'h' if 'h' is very large.

39. Two soap bubbles having radii 3 cm and 4 cm in vacuum, coalesce under isothermal conditions. The radius of the new bubble is

(1) 1 cm (2) 5 cm (3) 7 cm (4) 3.5 cm

Sol. Answer (2)

Energy initial = Energy final

$\Rightarrow$ $8\pi(3)^2S + 8\pi(4)^2S = 8\pi(r)^2S$ {Surface tension remains constant throughout process

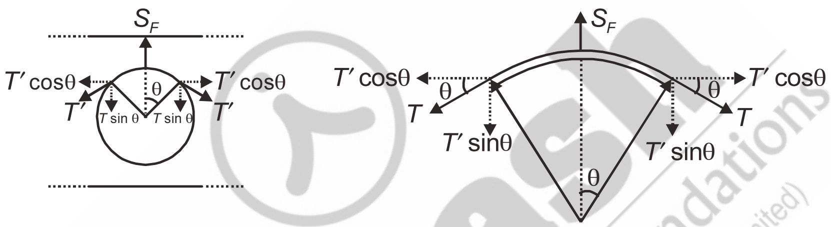
$\Rightarrow$ $(3)^2 + (4)^2 = (r)^2$

$\Rightarrow$ $5 \text{ cm} = r$

40. A massless inextensible string in the form of a loop is placed on a horizontal film of soap solution of surface tension T. If film is pierced inside the loop and it convert into a circular loop of diameter d, then the tension produced in string is

(1) Td (2) πTd (3) πd^2T (4) $\dfrac{\pi d^2T}{4}$

Sol. Answer (1)



By force balancing in vertical direction

$S_F = 2T'\sin\theta$

$S_F = 2T'\theta$ $\left\{\begin{array}{l}\because\ \theta \text{ is small} \\ \sin\theta \simeq \theta\end{array}\right.$

$S \times 2r \times 2\theta = 2 \times T' \times \theta$

$S \times 2r = \text{Tension}$ $\left\{\begin{array}{l}r - \text{radius} \\ d - \text{diameter}\end{array}\right.$

$S \times d = \text{Tension}$

$\because$ $S = T$

So, Tension = Td

$\left\{\begin{array}{l}\text{Where,} \\ S_F = \text{Force due to surface tension} \\ T' = \text{Tension in string} \\ \theta = \text{Small angle} \\ S \text{ or } T = \text{Surface tension}\end{array}\right.$

(Miscellaneous)

41. A vessel contain a liquid has a constant acceleration 19.6 m/s^2 in horizontal direction. The free surface of water get sloped with horizontal at angle

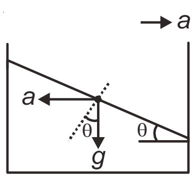
(1) $\tan^{-1}\left[\dfrac{1}{2}\right]$ (2) $\sin^{-1}\left[\dfrac{1}{\sqrt{3}}\right]$ (3) $\tan^{-1}\left[\sqrt{2}\right]$ (4) $\sin^{-1}\left[\dfrac{2}{\sqrt{5}}\right]$

Sol. Answer (4)

$$\tan\theta = \frac{a}{g} = \frac{19.6}{9.8} = 2$$

$\tan\theta = 2$

$\Rightarrow$ $\sin\theta = \dfrac{2}{\sqrt{5}}$ $\Rightarrow$ $\theta = \sin^{-1}\left[\dfrac{2}{\sqrt{5}}\right]$

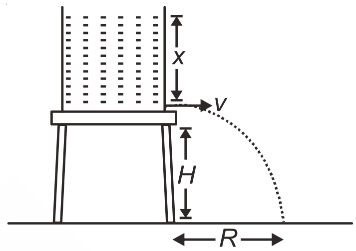


42. A cylinder containing water, stands on a table of height H. A small hole is punched in the side of cylinder at its base. The stream of water strikes the ground at a horizontal distance R from the table. Then the depth of water in the cylinder is

(1) H (2) R (3) $\sqrt{RH}$ (4) $R^2/4H$

Sol. Answer (4)

Let depth of water in cylinder be x



So velocity (v) of efflux $= \sqrt{2gx}$

Time taken (t) by water to travel vertical distance of H

$$= \sqrt{\frac{2H}{g}}$$

$\Rightarrow$ Range = $v \times t$

$$R = \sqrt{2gx} \times \sqrt{\frac{2H}{g}}$$

Solving this we get

$$\frac{R^2}{4H} = x$$

43. A large open tank has two holes in its wall. One is a square of side a at a depth x from the top and the other is a circular hole of radius r at depth $4x$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then r is equal to

(1) $2\pi a$ (2) a (3) $\frac{a}{\sqrt{2\pi}}$ (4) $\frac{a}{\pi}$

Sol. Answer (3)

Since quantities of water flowing out of both holes is same

$\Rightarrow$ Area of hole × velocity of efflux = constant

So, $A_1 \times V_1 = A_2 \times V_2$

Substituting values.

$$a^2 \times \sqrt{2gx} = \pi r^2 \times \sqrt{8gx}$$

$\Rightarrow$ $a^2 = 2\pi r^2$

$$\Rightarrow \frac{a}{\sqrt{2\pi}} = r$$

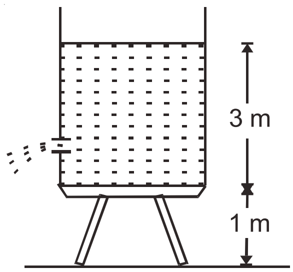
A_1 = Area of square hole

V_1 = Velocity of efflux from square hole $= \sqrt{2gx}$

A_2 = Area of circular hole

V_2 = Velocity of efflux from circular hole $= \sqrt{2g(4x)}$

44. Water is filled in a tank upto 3 m height. The base of the tank is at height 1 m above the ground. What should be the height of a hole made in it, so that water can be sprayed upto maximum horizontal distance on ground?



(1) 3 m from ground (2) 1.5 m from ground

(3) 1.5 m from base of tank (4) 2 m from ground

Sol. Answer (4)

Let height of hole from the base of container be h

Velocity of efflux $= \sqrt{2g(3-h)}$

$R = 2[(h + 1)(3 - h)]^{1/2}$ [R = Range proved in Q. 47]

$\Rightarrow$ $R = 2(-h^2 + 2h + 3)^{1/2}$

$$\frac{dR}{dh} = -2h + 2$$

If $\frac{dR}{dh} = 0,$ then range would be more for corresponding height

So, $0 = -2h + 2$ $\Rightarrow$ $h = 1$

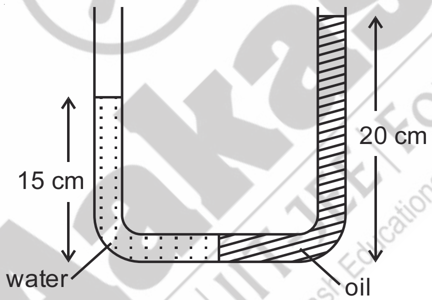
$\therefore$ Height from the ground = 1 + 1 = 2 m

SECTION - B

Previous Years Questions

1. In a u-tube as shown in the fig. water and oil are in the left side and right side of the tube respectively. The heights from the bottom for water and oil columns are 15 cm and 20 cm respectively. The density of the oil is

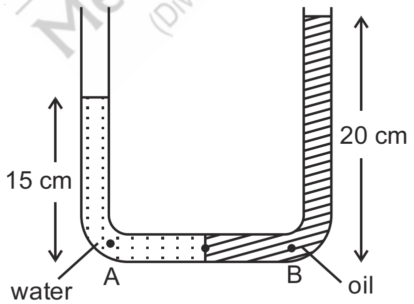
[take ρ_{water} = 1000 kg/m^3] **[NEET- 2019 (Odisha)]**



(1) 1333 kg/m^3 (2) 1200 kg/m^3 (3) 750 kg/m^3 (4) 1000 kg/m^3

Sol. Answer (3)

In equilibrium



Pressure at A = Pressure at B.

$P_a + 0.15 \times 10^3 \times g = P_a + 0.20 \times d_0 g$

$$d_0 = \frac{0.15 \times 10^3}{0.20}$$

$= 0.75 \times 10^3 = 750$ kg/m^3

2. Two small spherical metal balls, having equal masses, are made from materials of densities ρ_1 and ρ_2 ($\rho_1 = 8\rho_2$) and have radii of 1 mm and 2 mm, respectively, they are made to fall vertically (from rest) in a viscous medium whose coefficient of viscosity equals η and whose density is $0.1\rho_2$. The ratio of their terminal velocities would be,

[NEET- 2019 (Odisha)]

(1) $\frac{79}{36}$ (2) $\frac{79}{72}$ (3) $\frac{19}{36}$ (4) $\frac{39}{72}$

Sol. Answer (1)

As $V_T = \frac{2a^2}{9\eta}(\rho - \sigma)\,g$

$$V_{T_1} = \frac{2\times(1)^2}{9\eta}(\rho_1 - 0.1\rho_2)\,g$$

$$V_{T_1} = \frac{2\times1}{9\eta}(8\rho_2 - 0.1\rho_2)\,g \qquad \text{...(i)}$$

$$V_{T_2} = \frac{2\times(2)^2}{9\eta}(\rho_2 - 0.1\rho_2)\,g \qquad \text{...(ii)}$$

$$\therefore\ \frac{V_{T_1}}{V_{T_2}} = \frac{7.9}{4(0.9)} = \frac{79}{36}$$

3. A soap bubble, having radius of 1 mm, is blown from a detergent solution having a surface tension of 2.5×10^{-2} N/m. The pressure inside the bubble equals at a point Z_0 below the free surface of water in a container. Taking g = 10 m/s^2, density of water = 10^3 kg/m^3, the value of Z_0 is : **[NEET- 2019]**

(1) 100 cm (2) 10 cm (3) 1 cm (4) 0.5 cm

Sol. Answer (3)

Excess pressure = $\frac{4T}{R}$, Gauge pressure = $\rho g Z_0$

$$P_0 + \frac{4T}{R} = P_0 + \rho g Z_0$$

$$Z_0 = \frac{4T}{R\times\rho g}$$

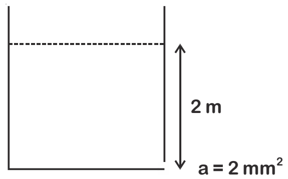
$$Z_0 = \frac{4\times2.5\times10^{-2}}{10^{-3}\times1000\times10}\ \text{m}$$

Z_0 = 1 cm

4. A small hole of area of cross-section 2 mm^2 is present near the bottom of a fully filled open tank of height 2 m. Taking g = 10 m/s^2, the rate of flow of water through the open hole would be nearly **[NEET- 2019]**

(1) 12.6×10^{-6} m^3/s (2) 8.9×10^{-6} m^3/s (3) 2.23×10^{-6} m^3/s (4) 6.4×10^{-6} m^3/s

Sol. Answer (1)



Rate of flow liquid

$Q = au = a\sqrt{2gh}$

$= 2\times10^{-6}\ m^2 \times \sqrt{2\times10\times2}\ m/s$

= $2 \times 2 \times 3.14 \times 10^{-6}$ m^3/s

= 12.56×10^{-6} m^3/s

= 12.6×10^{-6} m^3/s

5. A small sphere of radius r falls from rest in a viscous liquid. As a result, heat is produced due to viscous force. The rate of production of heat when the sphere attains its terminal velocity, is proportional to

[NEET- 2018]

(1) r^3 (2) r^2 (3) r^4 (4) r^5

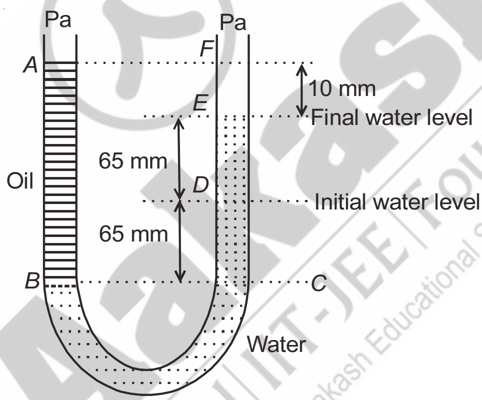
Sol. Answer (4)

Power = $6\pi\eta r V_T \cdot V_T = 6\pi\eta r V_T^2$

$V_T \propto r^2 \Rightarrow$ Power $\propto r^5$

6. A U tube with both ends open to the atmosphere, is partially filled with water. Oil, which is immiscible with water, is poured into one side until it stands at a distance of 10 mm above the water level on the other side. Meanwhile the water rises by 65 mm from its original level (see diagram). The density of the oil is

[NEET-2017]



(1) 650 kg m^{-3} (2) 425 kg m^{-3} (3) 800 kg m^{-3} (4) 928 kg m^{-3}

Sol. Answer (4)

$h_{oil}\ \rho_{oil}\ g = h_{water}\ \rho_{water}\ g$

$140 \times \rho_{oil} = 130 \times \rho_{water}$

$\rho_{oil} = \frac{13}{14} \times 1000\ \text{kg/m}^3$

$\rho_{oil} = 928$ kg m^{-3}

7. A rectangular film of liquid is extended from (4 cm × 2 cm) to (5 cm × 4 cm). If the work done is 3×10^{-4} J, the value of the surface tension of the liquid is **[NEET (Phase-2) - 2016]**

(1) 0.250 Nm^{-1} (2) 0.125 Nm^{-1} (3) 0.2 Nm^{-1} (4) 8.0 Nm^{-1}

Sol. Answer (2)

$W = 2(A_f - A_i)T$

$\Rightarrow T = \frac{W}{(A_f - A_i)\times 2}$

$$= \frac{3\times10^{-4}\ J}{2[5\times4\times10^{-4} - 4\times2\times10^{-4}]} = 0.125\ Nm^{-1}$$

8. Three liquids of densities ρ_1, ρ_2 and ρ_3 (with $\rho_1 > \rho_2 > \rho_3$), having the same value of surface tension T, rise to the same height in three identical capillaries. The angles of contact θ_1, θ_2 and θ_3 obey

[NEET (Phase-2) - 2016]

(1) $\frac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 \geq 0$ (2) $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

(3) $\frac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$ (4) $\pi > \theta_1 > \theta_2 > \theta_3 > \frac{\pi}{2}$

Sol. Answer (2)

$$h = \frac{2T\cos\theta}{r\rho g}$$

$\Rightarrow$ $r \propto \cos\theta$ (as T, h and r are constants)

$\rho \uparrow \ \Rightarrow \ \theta \downarrow$

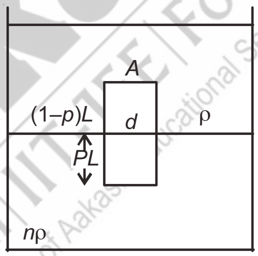
$\theta_1 < \theta_2 < \theta_3$

Its rise so $0 \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

9. Two non-mixing liquids of densities ρ and $n\rho$ ($n > 1$) are put in a container. The height of each liquid is h. A solid cylinder of length L and density d is put in this container. The cylinder floats with its axis vertical and length pL ($p < 1$) in the denser liquid. The density d is equal to **[NEET-2016]**

(1) $\{1 + (n - 1)p\}\rho$ (2) $\{1 + (n + 1)p\}\rho$ (3) $\{2 + (n + 1)p\}\rho$ (4) $\{2 + (n - 1)p\}\rho$

Sol. Answer (1)



Weight of cylinder = $Th_1 + Th_2$

$ALdg = (1 - P)\ LA\rho g + (PLA)\ n\rho g$

$\Rightarrow$ $d = (1 - P)\ \rho + Pn\rho$

$\Rightarrow$ $= \rho - P\rho + nP\rho$

$= \rho + (n - 1)P\rho$

$= \rho\ [1 + (n - 1)P]$

10. The cylindrical tube of a spray pump has radius R, one end of which has n fine holes, each of radius r. If the speed of the liquid in the tube is V, the speed of the ejection of the liquid through the holes is

[Re-AIPMT-2015]

(1) $\frac{V^2R}{nr}$ (2) $\frac{VR^2}{n^2r^2}$ (3) $\frac{VR^2}{nr^2}$ (4) $\frac{VR^2}{n^3r^2}$

Sol. Answer (3)

11. Water rises to a height h in capillary tube. If the length of capillary tube above the surface of water is made less than h, then **[Re-AIPMT-2015]**

(1) Water does not rise at all

(2) Water rises upto the tip of capillary tube and then starts overflowing like a fountain

(3) Water rises upto the top of capillary tube and stays there without overflowing

(4) Water rises upto a point a little below the top and stays there

Sol. Answer (3)

12. A wind with speed 40 m/s blows parallel to the roof of a house. The area of the roof is 250 m^2. Assuming that the pressure inside the house is atmospheric pressure, the force exerted by the wind on the roof and the direction of the force will be (P_{air} = 1.2 kg/m^3) **[AIPMT-2015]**

(1) 2.4×10^5 N, downwards (2) 4.8×10^5 N, downwards

(3) 4.8×10^5 N, upwards (4) 2.4×10^5 N, upwards

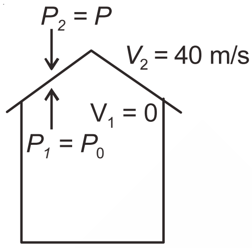
Sol. Answer (4)

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = P_1 - P_2$$

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{F}{A}$$

$$\frac{1}{2}\times 1.2\times\left(40^2 - 0^2\right) = \frac{F}{250}$$



$F = 2.4 \times 10^5$ N, upward (because $P_1 > P_2$)

13. A certain number of spherical drops of a liquid of radius r coalesce to form a single drop of radius R and volume V. If T is the surface tension of the liquid, then **[AIPMT-2014]**

(1) Energy = $4VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released (2) Energy = $3VT\left(\frac{1}{r} + \frac{1}{R}\right)$ is absorbed

(3) Energy = $3VT\left(\frac{1}{r} - \frac{1}{R}\right)$ is released (4) Energy is neither released nor absorbed

Sol. Answer (3)

Let n drops of radius r coaslece to form a big drop of radius R

$$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 = V$$

$n \times 4\pi r^3 = 4\pi R^3 = 3V$ ...(1)

Energy = $T.\Delta A$

$= T\,[n \times 4\pi r^2 - 4\pi R^2]$

$$= T\left[\frac{n\times 4\pi r^3}{r} - \frac{4\pi R^3}{R}\right]$$

$$= T\left[\frac{3V}{r} - \frac{3V}{R}\right]$$

$$= 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$$

14. The wettability of a surface by a liquid depends primarily on : **[NEET-2013]**

(1) Surface tension

(2) Density

(3) Angle of contact between the surface and the liquid

(4) Viscosity

Sol. Answer (3)

15. The neck and bottom of a bottle are 3 cm and 15 cm in radius respectively. If the cork is pressed with a force 12 N in the neck of the bottle, then force exerted on the bottom of the bottle is

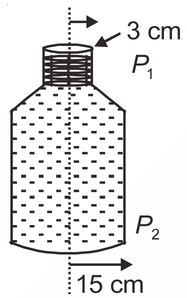
(1) 30 N

(2) 150 N

(3) 300 N

(4) 600 N

Sol. Answer (3)



Pressure applied on 1 point in a liquid spreads equally

So let P_1 be pressure at neck, P_2 be pressure at bottom

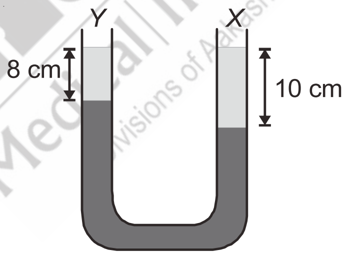
$$P_1 = P_2$$

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \left[\because P = \frac{F}{A}\right]$$

$$\Rightarrow \frac{12}{\pi \times 9} = \frac{F_2}{\pi \times 225}$$

$$\Rightarrow 300 \text{ N} = F_2$$

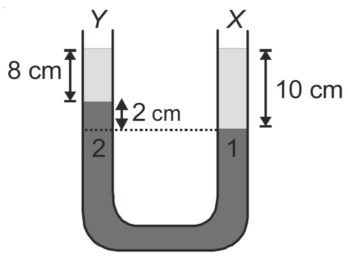
16. A liquid X of density 3.36 g cm^{-3} is poured in a U-tube, which contains Hg. Another liquid Y is poured in left arm with height 8 cm, upper levels of X and Y are same what is density of Y?



(1) 0.8 gcc^{-1} (2) 1.2 gcc^{-1} (3) 1.4 gcc^{-1} (4) 1.6 gcc^{-1}

Sol. Answer (1)

Pressure at 1 and 2 will be same



$$\rho_X gH_X = \rho_Y gH_Y + \rho_{Hg} g \times 2$$

$$\Rightarrow 3.36 \times 10 = \rho_Y \times 8 + 13.6 \times 2$$

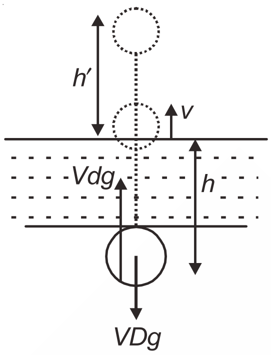
Solving this we get

$$\rho_Y = 0.8 \text{ g cc}^{-1}$$

17. A wooden ball of density D is immersed in water of density d to a depth h below the surface of water and then released. Upto what height will the ball jump out of water?

(1) $\frac{d}{D}h$ (2) $\left(\frac{d}{D}-1\right)h$ (3) h (4) Zero

Sol. Answer (2)



Force acting on ball at depth 'h' (*i.e.* apparent weight)

$F = Vg\,[d - D]$

Acceleration $(a) = \frac{Vg[d-D]}{VD}$ [Mass = volume × density]

Velocity $= \sqrt{2ah} = v$ [Using $v^2 - u^2 = 2as$]

h' (height above water) $= \frac{v^2}{2g} = \frac{2\times Vg[d-D]h}{2\times gVD} = \left[\frac{d}{D}-1\right]h$

18. A piece of solid weighs 120 g in air, 80 g in water and 60 g in a liquid. The relative density of the solid and that of the liquid are respectively

(1) 3, 2 (2) $2, \frac{3}{4}$ (3) $\frac{3}{2}, 2$ (4) $3, \frac{3}{2}$

Sol. Answer (4)

$w' = w\left[1-\frac{\rho}{\sigma}\right]$ $\left\{\begin{array}{l}\text{where,}\\ \rho = \text{density of liquid}\\ \sigma = \text{density of body}\end{array}\right.$

Inside water

$\Rightarrow\ 80 = 120\left[1-\frac{\rho_{water}}{\rho_{solid}}\right]$

$\Rightarrow\ \rho_{solid} = 3$ [$\because\ \rho_{water} = 1$]

Inside liquid

$60 = 120\left[1-\frac{\rho_{liquid}}{\rho_{solid}}\right]$

Using $\rho_{solid} = 3$

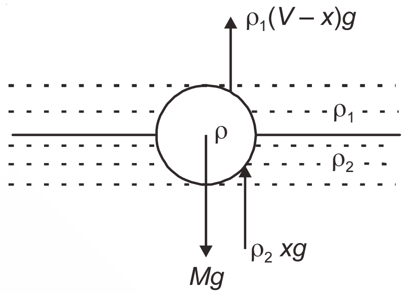
We get $\rho_{liquid} = \frac{3}{2}$

19. A solid sphere of volume V and density ρ floats at the interface of two immiscible liquids of densities ρ_1 and ρ_2 respectively. If $\rho_1 < \rho < \rho_2$, then the ratio of volume of the parts of the sphere in upper and lower liquids is

(1) $\dfrac{\rho_2 - \rho}{\rho - \rho_1}$ (2) $\dfrac{\rho + \rho_1}{\rho + \rho_2}$ (3) $\dfrac{\rho + \rho_2}{\rho + \rho_1}$ (4) $\dfrac{\sqrt{\rho_1 \rho_2}}{\rho}$

Sol. Answer (1)

$\rho_1 < \rho < \rho_2$ (given)



Let volume of sphere in lower liquid = x

Force of buoyancy by lower liquid = $\rho_2 xg$

Force of buoyancy by upper liquid = $\rho_1(V - x)g$

Force of gravity on sphere = Mg = $V\rho g$

Balancing all the forces for vertical equilibrium

We get

$V\rho g = \rho_1(V - x)\, g + \rho_2 xg$

Solving this we get

$x = \dfrac{V(\rho - \rho_1)}{(\rho_2 - \rho_1)}$

So $\dfrac{V - x}{x} = \dfrac{\rho_2 - \rho}{\rho - \rho_1}$

$\left\{\begin{array}{l}\text{Where,}\\ V - x = \text{volume of sphere in upper liquid}\\ x = \text{volume of sphere in lower liquid}\end{array}\right.$

20. Ice pieces are floating in a beaker A containing water and also in a beaker B containing miscible liquid of specific gravity 1.2. When ice melts, the level of

(1) Water increases in A (2) Water decreases in A

(3) Liquid in B decreases (4) Liquid in B increases

Sol. Answer (4)

For beaker 'A'

Ice is floating in water

$\rho_{ice}\, v_{ice}\, g = \rho_{water}\, v_{water\ displaced}\, g$

$\because\ \rho_{ice} \simeq \rho_{water}$

So we can say

$v_{ice} \simeq v_{water\ displaced}$

So after the ice melts the level of water will not change.

For beaker 'B'

Ice is floating in liquid with density 1.2

clearly $\rho_{liquid} > \rho_{ice}$

So from above analogy

$v_{ice} > v_{liquid\ displaced}$

So when ice melts the level in beaker 'B' increases.

21. A vessel contains oil (density 0.8 g cm^{-3}) over mercury (density 13.6 g cm^{-3}). A homogenous sphere floats with half volume immersed in mercury and the other half in oil. The density of the material of the sphere in g cm^{-3} is

(1) 12.8 (2) 7.2 (3) 6.4 (4) 3.3

Sol. Answer (2)

Let density of sphere be ρ

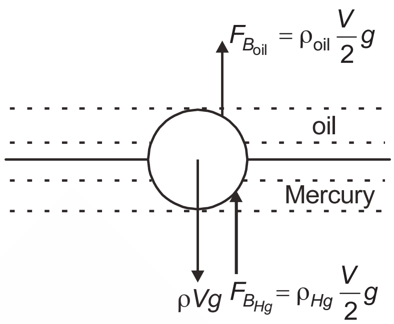
And volume be v

Balancing forces for vertical equilibrium

$$\rho Vg = \frac{\rho_{Hg} Vg}{2} + \frac{\rho_{\text{oil}} Vg}{2}$$

$$\Rightarrow \quad \rho = \frac{13.6}{2} + \frac{0.8}{2}$$

$\Rightarrow$ ρ = 7.2 g cm^{-3}



22. Two solid pieces, one of steel and the other of aluminium when immersed completely in water have equal weights. When the solid pieces are weighed in air

(1) The weight of aluminium is half the weight of steel

(2) Steel peice will weigh more

(3) They have the same weight

(4) Aluminium piece will weigh more

Sol. Answer (4)

Apparent weight = weight in air – F_{Buoyancy}

$\because$ Apparent weight of steel and aluminium is same

So weight of aluminium – F_B on Aluminium = weight of steel – F_B on steel ...(1)

$\rho_{\text{Al}} V_{\text{Al}}\, g - \rho_{\text{water}} V_{\text{Al}}\, g = \rho_{\text{steel}} V_{\text{steel}}\, g - \rho_{\text{water}} V_{\text{steel}}\, g$

$\rho_{\text{steel}} > \rho_{\text{Al}}$ and $\rho_{\text{water}} = 1$

So $(\rho_{\text{Al}} - 1) V_{\text{Al}} = (\rho_{\text{steel}} - 1) V_{\text{steel}}$

$\because$ $\rho_{\text{steel}} > \rho_{\text{Al}}$

$(\rho_{\text{steel}} - 1) > (\rho_{\text{Al}} - 1)$

So $V_{\text{Al}} > V_{\text{steel}}$

Also $\rho_{\text{water}} V_{\text{Al}}\, g > \rho_{\text{water}} V_{\text{steel}}\, g$

$\Rightarrow$ Force of buoyancy on Aluminium > Force of buoyancy on steel.

Using this condition in equation (1)

We get,

weight of Aluminium – weight of steel > 0

$\Rightarrow$ weight of Aluminium > weight of steel

23. A piece of wood is floating in water. When the temperature of water rises, the apparent weight of the wood will

(1) Increase (2) Decrease

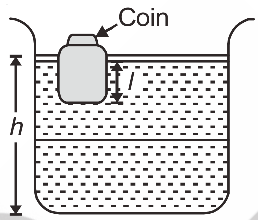
(3) May increase or decrease (4) Remain same

Sol. Answer (3)

When temperature of water is raised from 0 to 4°C, its density increases and after 4°C density decreases and apparent weight $\propto F_{\text{Buoyancy}} \propto$ density of water

So apparent weight may increase or decrease.

24. A wooden block, with a coin placed on its top, floats in water as shown in the figure. The distances h and l are shown there. After some time, the coin falls into the water, then



(1) Both l and h increase

(2) Both l and h decrease

(3) l decreases and h increases

(4) l increases and h decreases

Sol. Answer (2)

When coin falls into water block has to displace lesser volume to stay afloat.

Implies that block will go up and water will go down.

Hence both l and h will decrease.

25. An iceberg is floating in water. The density of ice in the iceberg is 917 kg m^{-3} and the density of water is 1024 kg m^{-3}. What percentage fraction of the iceberg would be visible?

(1) 5% (2) 10% (3) 12% (4) 8%

Sol. Answer (2)

ρ_{ice} × volume of ice × g = ρ_{water} × volume of ice inside water × g

917 × volume of ice = 1024 × volume of ice inside water

Let volume of ice = V

$$\therefore \quad \text{\% Volume visible} = \frac{V - \text{volume inside water}}{V} \times 100$$

$$= \left(\frac{V - \dfrac{917V}{1024}}{V}\right) \times 100$$

$$= \frac{\left(\dfrac{1024 - 917}{1024}\right)V}{V} \times 100$$

= 10%

26. A piece of wax weighs 18.03 g in air. A piece of metal is found to weigh 17.03 g in water. It is tied to the wax and both together weigh 15.23 g in water. Then, the specific gravity of wax is

(1) $\dfrac{18.03}{17.03}$ (2) $\dfrac{17.03}{18.03}$ (3) $\dfrac{18.03}{19.83}$ (4) $\dfrac{15.03}{17.03}$

Sol. Answer (3)

Weight of wax in air = 18.03 g

Apparent weight of metal in water = 17.03 g

Apparent weight = weight in air – $\rho_{water}\ V_{metal}\ g$

So weight of metal in air = apparent weight + $V_{metal}\ g$ $[\because\ \rho_{water} = 1]$

= 17.03 + $V_{metal} \times g$

When wax and metal are tied together

Total weight in air = 18.03 + 17.03 + $V_{metal} \times g$

And apparent weight in water = 15.23 = weight in air – $\rho_{water}\ V_{wax}\ g - \rho_{water}\ V_{metal}\ g$

$\Rightarrow$ 15.23 = 18.03 + 17.03 + $V_{metal}\ g - V_{wax}\ g - V_{metal}\ g$

$\Rightarrow$ $V_{wax}\ g = 20.1$

$\Rightarrow \dfrac{\text{Mass of wax}}{\text{density}} \times g = 19.83$ $\left[\because\ \rho = \dfrac{M}{V}\right]$

$\Rightarrow \dfrac{18.03}{g \times \rho} \times g = 19.83$ $\left[\text{Mass} = \dfrac{\text{weight}}{g}\right]$

So specific gravity of wax $= \dfrac{18.03}{19.83}$

27. Eight equal drops of water are falling through air with a steady velocity of 10 cm^{-1}. If the drops combine to form a single drop big in size, then the terminal velocity of this big drop is

(1) 80 cms^{-1} (2) 30 cms^{-1} (3) 10 cms^{-1} (4) 40 cms^{-1}

Sol. Answer (4)

Let radius of smaller drops be r, and bigger be R

When 8 such drops combine to form a bigger drop the total volume of water remains same

So, $8 \times \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$\therefore$ $2r = R$

And we know,

$V_{\text{Terminal}} \propto r^2$

$\therefore \dfrac{V_{T\,\text{smaller}}}{V_{T\,\text{bigger}}} = \dfrac{r^2}{R^2} = \dfrac{r^2}{4r^2}$

$\Rightarrow \dfrac{10}{V_{T\,\text{bigger}}} = \dfrac{1}{4}$ $[\because\ V_{T\,\text{smaller}} = 10\text{ cms}^{-1}\text{ (given)}]$

$\Rightarrow V_{T\,\text{bigger}} = 40\text{ cms}^{-1}$

28. Streamline flow is more likely for liquid with

(1) High density and low viscosity
(2) Low density and high viscosity
(3) High density and high viscosity
(4) Low density and low viscosity

Sol. Answer (2)

Streamline flow is more likely for liquid with low density and high viscosity.

29. An air bubble of radius 10^{-2} m is rising up at a steady rate of 2×10^{-3} ms^{-1} through a liquid of density 1.5×10^3 kg m^{-3}, the coefficient of viscosity neglecting the density of air, will be (g = 10 ms^{-2})

(1) 23.2 units
(2) 83.5 units
(3) 334 units
(4) 167 units

Sol. Answer (4)

$$V_T = \frac{2a^2}{9\eta} g(\rho - \sigma)$$

Substituting values

$$2\times10^{-3} = \frac{2\times10^{-4}\times10\times1.5\times10^3}{9\eta}$$

$\Rightarrow$ $\eta \sim 167$ units

$$\begin{cases} \text{Where given is} \\ V_T = 2\times10^{-3}\ \text{ms}^{-1} \\ a = 10^{-2}\ \text{m} \\ \rho = 1.5\times10^3\ \text{kg m}^{-3} \\ \sigma \sim 0 \\ g = 10\ \text{ms}^{-2} \end{cases}$$

30. The flow of liquid is laminar or streamline is determined by

(1) Rate of flow of liquid
(2) Density of fluid
(3) Radius of the tube
(4) Coefficient of viscosity of liquid

Sol. Answer (1)

It is decided by rate of flow of liquid

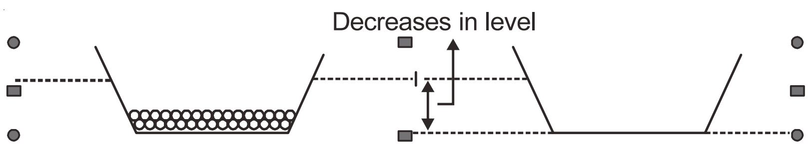
Given by Reynolds number $= \frac{\rho v d}{n}$

31. A boat carrying a number of large stones is floating in a water tank. What would happen to the water level, if a few stones are unloaded into water?

(1) Rises
(2) Falls
(3) Remains unchanged
(4) Rises till half the number of stones are unloaded and then begins to fall

Sol. Answer (2)

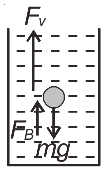
Previously when stones are on the boat they are increasing the weight on the boat and to balance this weight boat needs to generate buoyancy force by displacing more water, but when stones are removed the boat starts displacing less amount of water hence the level of water in tank falls.



32. The velocity of a small ball of mass M and density d_1 when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is d_2, the viscous force acting on the ball is

(1) $Mg\left(1-\frac{d_2}{d_1}\right)$ (2) $Mg\frac{d_1}{d_2}$ (3) $mg(d_1 - d_2)$ (4) mgd_1d_2

Sol. Answer (1)



$$F_v = mg - F_v = vd_1g - vd_2g = vd_1g\left(1-\frac{d_2}{d_1}\right) = mg\left(1-\frac{d_2}{d_1}\right)$$

33. There are two holes one each along the opposite sides of a wide rectangular tank. The cross-section of each hole is 0.01 m^2 and the vertical distance between the holes is one metre. The tank is filled with water. The net force on the tank in newton when the water flows out of the holes is (density of water = 1000 kgm^{-3})

(1) 100 (2) 200 (3) 300 (4) 400

Sol. Answer (2)

Net force = $F_2 - F_1$

$= \rho A v_2^2 - \rho A v_1^2$

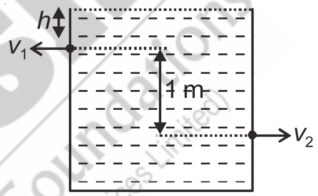
$$F = ma = v\rho\times\left(\frac{v}{t}\right) = Ah\rho\left(\frac{v}{t}\right) = A\rho v^2$$

$= 2\rho g\,(h + 1)\,A - 2\rho ghA \quad \left[v = \sqrt{2gx}\right]$

$= 2\rho gA$

$= 1000 \times 10 \times 0.01 \times 2$

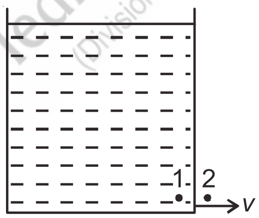
$= 200$ N



34. A hole is made at the bottom of the tank filled with water (density 1000 kg/m^3). If the total pressure at the bottom of the tank is 3 atm (1 atm = 10^5 N/m^2), then the velocity of efflux is

(1) $\sqrt{200}$ m/s (2) $\sqrt{400}$ m/s (3) $\sqrt{500}$ m/s (4) $\sqrt{800}$ m/s

Sol. Answer (2)



Apply Bernoulli's theorem

$$\underbrace{P+\rho gH}_{\text{Total pressure}} + \frac{1}{2}\rho v^2 = \text{constant} \qquad \begin{cases}\text{Given} \\ \text{Total pressure} = 3 \text{ atm}\end{cases}$$

At point 1, 3 atm + 0 = constant ...(i)

At point 2, 1 atm + $\frac{1}{2}\rho v^2$ = constant ...(ii)

Equate (i) and (ii)

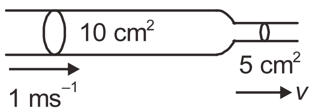
$$3 = 1 + \frac{1}{2}\rho v^2 \qquad [\text{Use } \rho = 1000 \text{ and } 1 \text{ atm} = 10^5 \text{ N/m}^2]$$

We get, $v = \sqrt{400}$ m/s

35. A horizontal pipe line carries water in stremline flow. At a point where the cross-sectional area is 10 cm^2 the water velocity is 1 ms^{-1} and pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is

(1) 200 Pa (2) 400 Pa (3) 500 Pa (4) 800 Pa

Sol. Answer (3)



P = 2000 Pa

$A_1V_1 = A_2V_2$ (equation of continuity)

$10 \times 1 = 5 \times v$

So, $v = 2$ ms^{-1}

Apply Bernoulli theorem at both the points,

$$2000 + \frac{1}{2} \times 1000 \times 1^2 = P + \frac{1}{2} \times 1000 \times 4 \Rightarrow P = 500 \text{ Pa}$$

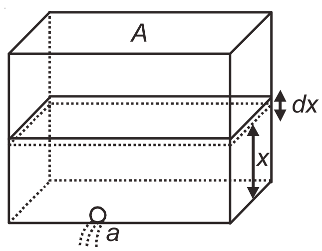
36. A rectangular vessel when full of water, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when half filled with water?

(1) 9 min (2) 7 min (3) 5 min (4) 3 min

Sol. Answer (2)

Let time taken by height 'x' to get reduced by $dx = dt$

$$\therefore \quad dt = \frac{\text{volume}}{\text{efflux speed}} = \frac{A \times dx}{\sqrt{2gx}} \qquad \{A \text{ is area of cross-section}\}$$



$$\int_0^T dt = \int_0^h \frac{A}{\sqrt{2g}} \frac{dx}{\sqrt{x}}$$

$$\Rightarrow \quad T = \frac{A}{a}\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad T \propto \sqrt{h}$$

So we can use

$$\frac{T_1}{T_2} = \frac{\sqrt{h_1}}{\sqrt{h_2}}$$

$$\frac{10 \text{ min}}{t \text{ min}} = \frac{\sqrt{h}}{\sqrt{h/2}}$$

$$\Rightarrow \quad t = \frac{10}{\sqrt{2}} \sim 7 \text{ min}$$

37. A metal plate of area 10^3 cm^2 rests on a layer of oil 6 mm thick. A tangential force 10^{-2} N is applied on it to move it with a constant velocity of 6 cms^{-1}. The coefficient of viscosity of the liquid is

(1) 0.1 poise (2) 0.5 poise

(3) 0.7 poise (4) 0.9 poise

Sol. Answer (1)

$$F = \eta A \frac{v}{d}$$

$$\Rightarrow \quad 10^{-2} = \eta \times (10^3 \times 10^{-4}) \times \frac{6\times 10^{-2}}{6\times 10^{-3}} = 0.01 \text{ poiseuille}$$

$$= 0.1 \text{ poise}$$

Where,
F = Force
η = Coefficient of viscosity
A = Area
v = Velocity
d = Thickness of layer

38. With an increase in temperature, surface tension of liquid (except molten copper and cadmium)

(1) Increases (2) Remain same

(3) Decreases (4) First decreases then increases

Sol. Answer (3)

When we increase the temperature, we are providing energy to the molecules. This increase in potential energy causes the surface energy to drop and become less negative hence decreasing surface tension because surface tension is nothing but surface energy per unit area.

39. Determine the energy stored in the surface of a soap bubble of radius 2.1 cm if its tension is 4.5×10^{-2} Nm^{-1}.

(1) 8 mJ (2) 2.46 mJ (3) 4.93×10^{-4} J (4) None of these

Sol. Answer (3)

Energy = surface tension × surface area × number of free surfaces

$= (4.5 \times 10^{-2}) \times 4\pi \times (2.1 \times 10^{-2}) \times 2 = 4.98 \times 10^{-4}$ J

40. A mercury drop of radius 1.0 cm is sprayed into 10^6 droplets of equal sizes. The energy expended in this process is (surface tension of mercury is equal to 32×10^{-2} Nm^{-1})

(1) 3.98×10^{-4} J (2) 8.46×10^{-4} J (3) 3.98×10^{-2} J (4) 8.46×10^{-2} J

Sol. Answer (3)

Energy expended = surface tension × increase in area (Formulae)

So, volume initially = volume of 10^6 drops

$$\Rightarrow \quad \frac{4}{3}\pi\left(\frac{1}{100}\right)^3 = 10^6 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \left[\left(\frac{1}{100}\right)^3 \times \frac{1}{10^6}\right]^{1/3} = r \qquad \text{[Let radius of small drops = } r\text{]}$$

$\Rightarrow \quad 10^{-4}$ m = r

So increase in surface area $= 4\pi\left[\left(\frac{1}{10000}\right)^2 \times 10^6 - \left(\frac{1}{100}\right)^2\right] = 4\pi\left[\frac{1}{100} - \frac{1}{10000}\right]$

$$\Rightarrow \quad \Delta A = \frac{4\pi \times 0.99}{100}$$

Using this value in formulae

Energy $= 32\times10^{-2}\times\dfrac{4\pi\times0.99}{100}$ [$\because$ Surface tension = 32 × 10^{-2} (given)]

= 3.98 × 10^{-2} J

41. When a glass capillary tube of radius 0.015 cm is dipped in water, the water rises to a height of 15 cm within it. Assuming contact angle between water and glass to be 0°, the surface tension of water is [ρ_{water} = 1000 kg m^{-3}, g = 9.81 ms^{-2}]

(1) 0.11 Nm^{-1} (2) 0.7 Nm^{-1} (3) 0.072 Nm^{-1} (4) None of these

Sol. Answer (1)

$$h = \frac{2\,S\cos\theta}{\rho rg}$$

Substituting values

$$\frac{15}{100} = \frac{2\times S\times1\times100}{1000\times0.015\times9.81}$$

$\therefore$ S = 0.11 Nm^{-1}

$$\left\{\begin{array}{l}\text{Where,}\\ S = \text{surface tension} = ?\\ h = \text{height of water in capillary} = 15\text{ cm}\\ r = \text{radius of capillary} = 0.015\text{ cm}\\ \theta = \text{angle of contact} = 0^\circ\\ g = 9.8\text{ ms}^{-2}\end{array}\right.$$

42. A liquid does not wet the sides of a solid, if the angle of contact is

(1) Obtuse (2) 90° (3) Acute (4) Zero

Sol. Answer (1)

Solid will not get wet if the liquid has high surface tension (example mercury) and liquids with high surface tension have obtuse angle of contact.

43. Two drops of equal radius coalesce to form a bigger drop. What is ratio of surface energy of bigger drop to a smaller one?

(1) $2^{1/2}$: 1 (2) 1 : 1 (3) $2^{2/3}$: 1 (4) None of these

Sol. Answer (3)

Surface energy = surface tension × surface area

Let the radius of smaller drops be r

And that of bigger drop be R

Then ratio of surface energies = ratio of surface area [$\because$ Surface tension is same for both]

= $4\pi R^2 : 4\pi r^2$

= $R^2 : r^2$ ...(1)

$\because$ 2 smaller drops are forming 1 big drop so $2\times\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

So, $2^{1/3}r = R$ ...(2)

$\Rightarrow$ Using 1 and 2 we can say that ratio of surface energies = $2^{2/3}r^2 : r^2 = 2^{2/3} : 1$

44. The excess pressure inside a spherical drop of water is frour times that of another drop. Then their respective mass ratio is

(1) 1 : 16 (2) 8 : 1 (3) 1 : 4 (4) 1 : 64

Sol. Answer (4)

Excess pressure $= \dfrac{2T}{r}$ {Where, r = radius of drop

$\Delta P \propto \dfrac{1}{r}$

$\dfrac{\Delta P_1}{\Delta P_2} = \dfrac{r_2}{r_1} = \dfrac{4}{1}$ $\left[\because \dfrac{\Delta P_1}{\Delta P_2} = \dfrac{1}{4}\right]$

$V \propto r^3$

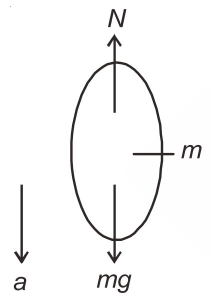
$\dfrac{V_1}{V_2} = \left(\dfrac{r_1}{r_2}\right)^3 = \left(\dfrac{1}{4}\right)^3 = \dfrac{1}{64}$

$\therefore$ $M = V \times \rho$ then $\dfrac{M_1}{M_2} = \dfrac{V_1}{V_2} = \dfrac{1}{64}$

45. A balloon with mass m is descending down with an acceleration a (where $a < g$). How much mass should be removed from it so that it starts moving up with an acceleration a?

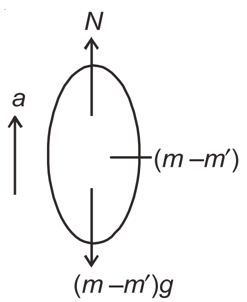
(1) $\dfrac{2ma}{g+a}$ (2) $\dfrac{2ma}{g-a}$ (3) $\dfrac{ma}{g+a}$ (4) $\dfrac{ma}{g-a}$

Sol. Answer (1)



$mg - N = ma$ ..... (1)

Let m mass remove from ballon



$N - (m - m')g = (m - m')a$ ..... (2)

After addition of equation (1) & (2), then

$\boxed{m' = \dfrac{2ma}{g+a}}$

SECTION - C

Assertion - Reason Type Questions

1. A : Hydraulic lift is based on Pascal's Law.

   R : Hydrostatic pressure is a scalar quantity

Sol. Answer (2)

A : is true

R : is true

But reason is not the correct explanation. Correct explanation is, change in pressure is transferred, undiminished from one point to the other.

2. A : The apparent weight of a body floating on the surface of a liquid is zero.

   R : The net force on a body floating on the surface of a liquid is zero.

Sol. Answer (1)

A : is true

R : is true

And reason is also the correct explanation.

3. A : It is better to wash cloths in hot water than cold water.

   R : On increasing temperature surface tension of water decreases.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

4. A : The impurities added to water may increase or decrease surface tension.

   R : The change in surface tension depends on the nature of impurities.

Sol. Answer (1)

A is true, R is true and correct explanation.

5. A : If air blows over the roof of a house, the force on the roof is upwards.

   R : When air blows over the roof, the pressure over it from out side decreases.

Sol. Answer (1)

A is true, R is true and correct explanation.

6. A : When rain drops fall through air some distance, they attain a constant velocity.

   R : The viscous drag of air just balances the weight of rain drops.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

7. A : Bernoulli's theorem holds good only for non-viscous and incompressible liquid.

   R : Bernoulli's theorem is based on the conservation of energy.

Sol. Answer (2)

A : is true

R : is true

Correct explanation of 'A' is Bernoulli's equation does not take into account the elastic energy of the fluids.

8. A : At high altitudes (mountains), it is very difficult to stop bleeding from a cut in the body.

R : At high altitude the atmospheric pressure is less than the blood pressure inside the body.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

9. A : When liquid drops merge into each other to form a large drop, energy is released.

R : When liquid drops merge to form a large drop surface tension decreases.

Sol. Answer (3)

A : is true

R : is false, because when large drop is formed, surface area gets reduced. Hence surface energy gets reduced due to reduction in surface area not the surface tension.

10. A : Excess pressure inside a soap bubble is $\frac{4T}{r}$ (symbols have their usual meanings).

R : The pressure difference across a curved surface of radius of curvature r is $\frac{2T}{r}$. There are two surfaces in a soap bubble.

Sol. Answer (1)

A : is true

R : is true and correct explanation.

11. A : Buoyant force is always vertically upward.

R : Buoyant force is always opposite to the direction of acceleration due to gravity.

Sol. Answer (4)

A : is wrong, Buoyant force is not always vertically upwards

R : is wrong, Buoyant force is always opposite to the direction of effective acceleration.

12. A : Equation of continuity is $A_1v_1\rho_1 = A_2v_2\rho_2$ (symbols have their usual meanings).

R : Equation of continuity is valid only for incompressible liquids.

Sol. Answer (3)

A : is true

R : is false,

13. A : Atomizer is based on the principle of Bernoulli's theorem.

R : Bernoulli's theorem is based on the conservation of energy.

Sol. Answer (2)

A : is true

R : is true, but not the correct explanation

Correct explanation is, decrease in pressure forces the water to move up the tube and get sprayed.

14. A : The spiders and insects can run on the surface of water.

R : Buoyant force balances the weight of insects.

Sol. Answer (3)

A : true

R : is false, surface tension balances the weight of insect.

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