Chapter 10

Mechanical Properties of Fluids

Solutions (Set-1)

SECTION - A

School/Board Exam. Type Questions

Very Short Answer Type Questions :

1. A block of wood is floting in a lake. What is apparent weight of the block?

Sol. The apperent weight of the floating block is equal to zero because weight of the block is balanced by force of buoyancy.

2. A boat carrying a large number of stones is floating in a water tank. What will happen to water level if the stones are unloaded into water?

Sol. Pascal's Law

The water level falls because volume of the water displaced by stones in water will be less than the volume of water displaced when stones are in the boat.

3. Why is pressure a scalar quantity?

Sol. The pressure acting on an area takes only the normal component of force acting on the area, and not the force vector.

4. What is the principle behind equation of continuity?

Sol. Conservation of mass.

5. What is the dimensional formula of Reynolds number?

Sol. $[M^0L^0T^0]$

6. What do you mean by critical Reynolds number?

Sol. The exact value of Reynolds number above which the flow of a liquid changes from streamline to turbulent is called critical Reynolds number.

7. What will happen when a body of density $2d$ is released at the surface of a liquid of density $\frac{d}{2}$?

Sol. The body sinks down in the liquid.

8. What is the shape of a liquid surface, in a container when the angle of contacts are respectively acute and obtuse?

Sol. Concave and convex.

9. How does the terminal velocity of a liquid drop falling down change when its radius increases by a factor 1.5?

Sol. $v_T \propto r^2$

$$\Rightarrow \quad \frac{v_T}{v_T'} = \left(\frac{r}{1.5r}\right)^2$$

$$\Rightarrow \quad v_T' = 2.25\, v_T$$

Thus, terminal velocity increases by a factor 2.25.

10. What does it imply if critical Reynolds number for two different fluids is almost same?

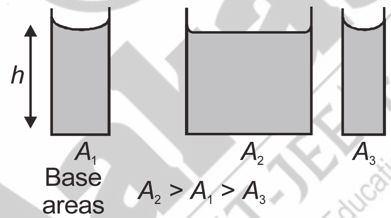
Sol. They flow through the pipes of same diameter.

11. In which type of capillary (material of capillary) will water descend and not rise?

Sol. A capillary made of paraffin wax.

Short Answer Type Questions :

12. A liquid is filled into three vessels upto same height as shown



For which vessel is the force exerted by the liquid at the bottom is maximum.

Sol. Since liquid is filled upto the same height

$\therefore\ P_1 = P_2 = P_3$

Respective forces exerted at the bottoms are

$F_1 = P_1A_1$ $\qquad$ $F_2 = P_2A_2$ $\qquad$ $F_3 = P_3A_3$

Since $A_2 > A_1 > A_3$

$\Rightarrow \quad F_2 > F_1 > F_3$

Thus liquid exerts maximum force at the bottom of the flask 2.

13. A 600 g of solid cube having an edge length of 10 cm floats in water. How much volume of the cube is inside water? (Given density of water = 10^3 kg m^{-3})

Sol. The upward buoyant force balances the weight of the cube. Let the volume of cube inside the water be V, then

$mg = V\rho g$

$(600 \text{ g})(10 \text{ ms}^{-2}) = V(\text{density of water})(10 \text{ ms}^{-2})$

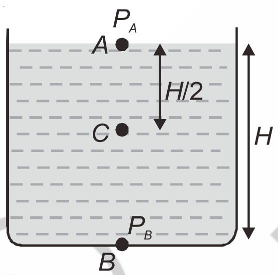
$\Rightarrow 0.6 \text{ kg} = V \times (10^3 \text{ kg m}^{-3})$

$$\Rightarrow V = \frac{0.6}{10^3}$$

$= 6 \times 10^{-4} \text{ m}^3$

$= 600 \text{ cm}^3$

14. In the shown figure, the pressure at points A and B are respectively P_A and P_B. Prove that pressure at point C is $\left(\frac{P_A + P_B}{2}\right)$.



Sol. At A, pressure = P_A

At B, $P_B = P_A + \rho gH$

At C, $P_C = P_A + \left(\rho g \frac{H}{2}\right)$

Now $P_A + P_B = P_A + P_A + \rho gH$

$$= 2\left(P_A + \rho g \frac{H}{2}\right)$$

$= P_A + P_B = 2(P_C)$

$$\Rightarrow \quad P_C = \frac{P_A + P_B}{2}$$

15. Water does not come out of the dropper unless its rubber bulb is pressed hard. Why?

Sol. Water is held inside the dropper against the atmospheric pressure. When the rubber bulb is pressed, pressure on water increases and pushes it out against the atmospheric pressure.

16. What happens if water is used in barometers instead of mercury?

Sol. (i) Density of water is 10^3 kg m^{-3} while that of mercury is 13.6×10^3 kg m^{-3}. Thus a barometer with water will require a tube of about 11 m to measure the atmospheric pressure. This is impractical.

(ii) Also water wets the barometer tube while mercury does not.

17. A barometer kept in an elevator accelerating downwards reads 76 cm of Hg. What is the air pressure inside the elevator?

Sol. Since the elevator is accelerating downwards

Net acceleration as experienced by the barometer tube = $g - a$

The height of mercury corresponding to pressure inside the elevator, $h = \dfrac{76 \times 13.6\,(g-a)}{13.6 \times g}$

18. Why don't two streamlines intersect?

Sol. The tangent at any point of a streamline gives the direction of flow of fluid particle at that point. If two streamlines intersect at a point, two tangents can be drawn at that point. Which means the oncoming fluid particle can go one way or the other. The flow will then no more be streamlined.

19. Why does the velocity decrease when water flow in a narrow pipe enters a broader pipe?

Sol. According to the equation of continuity,

Av = constant

where A = cross-sectional area of pipe

v = fluid velocity

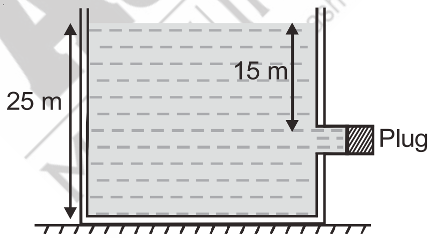
$\Rightarrow\ v \propto \dfrac{\text{constant}}{A}$

Therefore, as the cross-sectional area available to water increases, its velocity decreases.

20. What is the effect of temperature on viscosity and surface tension of a liquid?

Sol. Viscosity and surface tension both decrease with increase in temperature of a liquid. For gases viscosity increases with increase in temperature.

21. The water in a reservoir is 25 m deep. A horizontal pipe 5 cm in diameter passes through the reservoir 15 m below the water surface as shown. A plug secures the pipe opening. Find the force of friction between the plug and the pipe wall. [Take g = 10 m s^{-2}]



Sol. Since the plug is stationary, static friction acts between the plug and the pipe wall.

$\Rightarrow$ Static friction, f_s = Force of push by the water

= Gauge pressure × Area of cross-section of the plug

= $(\rho g h) \times (\pi r^2)$

Here $\rho = 10^3$ kg m^{-3}, g = 10 ms^{-2}, h = 15 m, r = 2.5 cm = 0.025 m

$\Rightarrow f_s = (10^3 \times 10 \times 15) \times 3.14 \times (0.025)^2$

$= 0.0294 \times 10^4$

= 294 N

22. An open tank contains water upto height H. Its side wall contains an orifice at depth h below the water surface. What is the maximum range of the water flowing out?

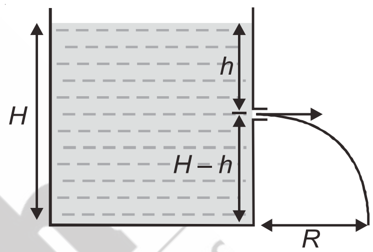
Sol. Velocity of efflux of water $v_{eff} = \sqrt{2gh}$, this velocity is in horizontal direction.

$\therefore$ Vertical velocity of water when it leaves the orifice is zero.

The time taken by water to fall through height $H - h$ is given by

$$t = \sqrt{\frac{2(H-h)}{g}}$$

The horizontal distance covered by water during this time



$R = v_{eff} \times t$

$$= \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$$

$$= 2\sqrt{H(H-h)}$$

This is maximum for $h = \frac{H}{2}$

$\Rightarrow \quad R = \frac{H}{2}$

23. What happens when a capillary tube of insufficient length is dipped in a liquid?

Sol. When a capillary tube of insufficient length is dipped in a liquid, the liquid rises to the top. The radius of curvature of the concave meniscus increases till the pressure on its concave side becomes equal to that on its other side. But the liquid does not overflow.

24. What is Reynold's number? Give its significance.

Sol. Reynold's number is a dimensionless parameter which decides the nature of flow of a liquid through a pipe.

It is given by $R_e = \frac{\rho v D}{\eta}$.

If R_e lies between 0 to 1000, the flow is laminar, for $1000 \le R_e \le 2000$, the flow is unsteady, for $R_e > 2000$, the flow is turbulent.

25. Write the limitations of Bernoulli's principle.

Sol. $\rightarrow$ Bernoulli's equation ideally applies to non-viscous fluids having streamlined flow.

$\rightarrow$ It is based on the assumption that there is no loss of energy due to friction.

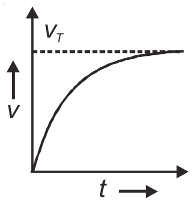
$\rightarrow$ It is applicable to incompressible fluids only.

26. What is terminal velocity? Draw a curve that correctly shows the variation of velocity v with time t for a small spherical ball falling vertically in a long column of viscous liquid.

Sol. The maximum constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity. For a ball of radius 'a' and density ρ falling through a fluid of viscosity η and density σ, the value of terminal velocity is given as

$$v_T = \frac{2a^2}{9\eta}(\rho - \sigma)g$$

Before attaining terminal velocity, the ball gets faster with time as shown in the following graph:



27. Two rain drops of the same radius fall through air with a steady velocity 5 cm/s. If the two drops coalesce, what is the terminal velocity of the new drop?

Sol. Terminal velocity $v_T = \frac{2a^2}{9\eta}(\rho - \sigma)g = 5\times10^{-2}\ \text{m s}^{-1}$ (initially)

When the two drops coalesce, the radius of the new drop becomes $R = \left(2^{\frac{1}{3}}\right)a$.

Hence new terminal velocity

$$v_T' = \frac{2R^2}{9\eta}(\rho - \sigma)g$$

$$= \frac{2\left(2^{\frac{2}{3}}\right)a^2}{9\eta}(\rho - \sigma)g$$

$$= \left(2^{\frac{2}{3}}\right)v_T$$

$$= \left(2^{\frac{2}{3}}\right)\times 5\ \text{cm s}^{-1}$$

$= 6.3\ \text{cm s}^{-1}$

28. Give two examples of turbulent flow.

Sol. (i) When a fast-flowing stream encounters rocks, foamy whirlpool like regions are formed, which show turbulent flow.

(ii) The smoke rising up from a burning stack of wood is also an example of turbulence.

29. How do detergents enhance the cleansing action of water?

Sol. Molecules of detergents are hairpin shaped. Their one end is attracted to water and the other to the grease or oil. Thus, water grease interfaces are formed which decreases the surface tension of water. The greasy dirt is held suspended which otherwise is not made wet by water.

30. What is the excess pressure inside a liquid drop as compared to that in an air bubble of same radius?

Sol. A liquid drop has one free surface only. Hence, the excess pressure inside it is given by = $\frac{2S}{R}$

An air bubble has two free surfaces. Hence, for an air bubble of same radius, excess pressure inside it $= \frac{4S}{R}$.

Thus, excess pressure inside a liquid drop is half of that in an air bubble of same radius.

31. What is the mathematical expression for capillary rise?

Sol. Capillary rise, $h = \frac{2S\cos\theta}{r\rho g}$

S = surface tension

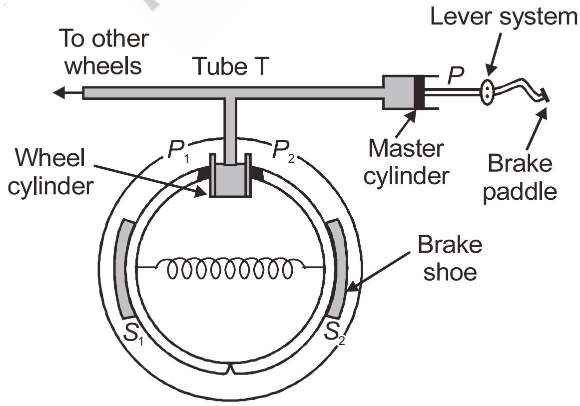
$\cos\theta$ = Angle of contact

Long Answer Type Questions :

32. Give the principle and explain the working of hydraulic brakes with a suitable diagram.

Sol. Hydraulic Brakes

Hydraulic brakes used in automobiles is based on Pascal's law. The wheels of the vehicle are connected to the brake pedal through an incompressible fluid called brake oil.



Hydraulic Brake

The piston directly connected to the brake paddle is called the master piston. When a little force is applied on the paddle with foot, the master piston moves inside. The pressure thus caused is transmitted undiminished to the other pistons which are directly connected to the wheels, (see figure). These pistons have smaller area of cross section. Applying Pascal's law, the braking forces produced at wheels is quite large as compared to the little force applied at the brake paddle.

One more advantage of using hydraulic brakes is that, an equal effect is produced at all the wheels of the vehicle. It is because pressure is transferred equally to all the pistons connected to the wheels. Thus, braking effort is equal on all wheels.

33. (i) Draw streamlines around a spinning ball.

(ii) Explain Magnus effect.

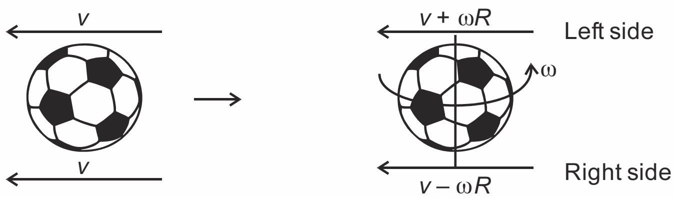
(iii) Derive equation of continuity.

Sol. (i) and (ii)

Dynamic Lift and Magnus Effect

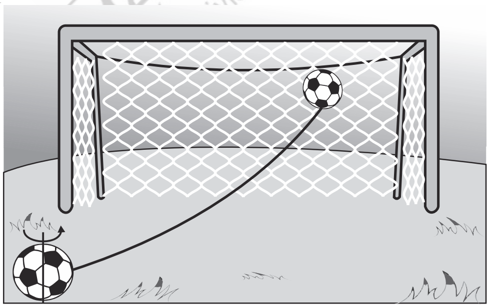
The deviation of a spinning ball from its parabolic path, while moving in air is called Magnus effect. The science behind it can be explained with the help of Bernoullis principle. Let us take an example of corner kick in football.

When a football is kicked without spinning, then relative to ball, air will move in backward direction, to fill the space left vacant by the football.



(a) Football without spinning (b) Football spinning about vertical axis

When football spins, it drags along a layer of air with itself so resultant velocity of air on left side becomes more than that of on the right side. From Bernoulli's equation, pressure on right side will be more and football will experience a force towards left side, which results in famous ***Bannana kick*** in football as shown in fig (c)



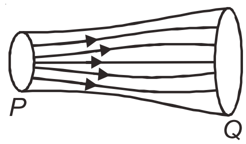
(c)

(iii) **Equation of continuity**

Consider a fluid in steady flow. The map of its flow for a particular section can be shown by the bundle of streamlines as shown below. Here area of cross-section is greater at Q than that at P. Hence, streamlines are closely spaced at P than that at Q. Let the area of cross-section and fluid velocity at P be A_P and v_P respectively. Let the corresponding quantities at Q be A_Q and v_Q.

Therefore, the volume of fluid moving in at P, in a small time interval Δt

$$= A_P v_P \Delta t$$



Similarly, the volume flowing out at Q, during the same interval Δt,

$$= A_Q v_Q \Delta t$$

By conservation of mass, for Δt

mass flowing in at P = mass flowing out at Q.

$\Rightarrow (A_P v_P \Delta t)\rho_P = (A_Q v_Q \Delta t)\rho_Q$ [ρ_P = fluid density at P, ρ_Q = fluid density at Q]

For flow of incompressible fluids

$\rho_P = \rho_Q$

$\Rightarrow \boxed{A_p v_P = A_Q v_Q}$

This expression is known as **equation of continuity**. It is a statement of **conservation of mass** in flow for incompressible fluids.

The equation of continuity can be stated as : For the streamline flow of an incompressible fluid through a pipe of varying cross-section, Av remains constant through out the flow.

$\therefore Av$ = constant

34. State and prove Bernoulli's principle.

Sol. Bernoulli's Principle

This principle is based on the law of conservation of energy for fluids in motion. It may be stated as follows:

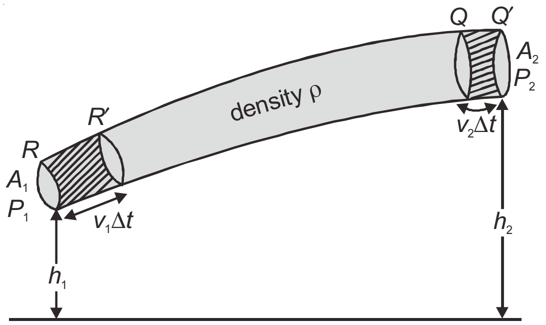
As we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{\rho v^2}{2}\right)$ and the potential energy per unit volume (ρgh) remains a constant.

Proof : According to the equation of continuity *i.e.*, Av = constant, fluid velocity changes with the change in cross-section. It means there must be some force acting on the fluid that causes this change in velocity. This resultant force arises due to the difference in pressure exerted by the fluid at different regions.

Consider the figure shown. It shows a portion of an incompressible fluid flowing steadily. Let the fluid initially be between the ends R and Q. Its velocity at end R is v_1 and that at Q is v_2. In an infinitesimal time interval Δt, fluid moves a distance $v_1\Delta t$ from R to R'. In the same time the fluid at the other end moves a distance $v_2\Delta t$ from Q to Q'.

Thus, the mass flowed in, in time Δt is

$\Delta m = (A_1 v_1 \Delta t)\rho = (A_2 v_2 \Delta t)\rho$



The pressure exerted by the oncoming fluid on this mass Δm at end R is P_1, and that at Q is P_2.

Thus, the force acting on the fluid inside the pipe at point R is $F_1 = P_1A_1$

This force moves the fluid by a distance $v_1\Delta t$.

Hence, the work done by the force is

$W_1 = (F_1)(v_1\Delta t)$

$W_1 = P_1A_1v_1\Delta t$

During the same interval Δt, the fluid inside the pipe pushes the fluid at point Q towards right by a distance $v_2\Delta t$.

Hence, the work done on the fluid inside the pipe

$W_2 = -P_2A_2v_2\Delta t$

So, the total work done on the fluid is

$W_1 + W_2 = P_1A_1v_1\Delta t - P_2A_2v_2\Delta t$

$= (P_1 - P_2)\Delta V$ ...(i) [By equation of continuity $A_1v_1\Delta t = A_2v_2\Delta t = \Delta V$]

This work done is related to the change in kinetic energy and change in its potential energy of the fluid by work energy theorem.

If the density of the fluid is ρ and $\Delta m = \rho A_1v_1\Delta t = \rho\Delta V$ is the mass passing through the pipe in time Δt, then change in its gravitational potential energy

$= \Delta mg(h_2 - h_1)$

$= (\rho\Delta V)g(h_2 - h_1)$ ...(ii)

Change in kinetic energy $= \frac{1}{2}\Delta m\left(v_2^2 - v_1^2\right)$

$= \frac{1}{2}(\rho\Delta V)\left(v_2^2 - v_1^2\right)$ ...(iii)

Using equations (i), (ii) and (iii); and applying work energy theorem,

$(P_1 - P_2)\Delta V = g(h_2 - h_1)(\rho\Delta V) + \frac{1}{2}(\rho\Delta V)(v_2{}^2 - v_1{}^2)$

$$\Rightarrow \quad P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

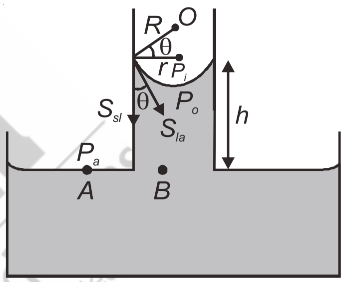
$$\Rightarrow \quad \boxed{P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2}$$

In general form $\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}}$

This result is known as Bernoulli's principle.

35. Derive an expression for the rise of liquid in a capillary tube of uniform diameter and sufficient length.

Sol. Let a capillary tube of radius r be dipped in a liquid of surface tension S and density ρ. If the angle of contact for this liquid and the capillary tube is acute, the liquid forms a concave meniscus as shown. The curved part can be considered to be a part of an air bubble inside the liquid. Thus, pressure at a point just above the liquid surface is greater than that just below it. It is given by $P_i - P_o = \frac{2S}{R}$



Here R is the radius of the concave curved part.

As it is clear from the figure

$$R = \frac{r}{\cos\theta}$$

$$\therefore \quad P_i - P_o = \frac{2S\cos\theta}{r} \quad \text{...(i)}$$

Since the region above the concave surface is open to the atmosphere, P_i = Atmospheric pressure P_a.

Also pressure at point A = atmospheric pressure P_a.

Since points A and B are at the same level

we have, $P_A = P_B = P_i$ ...(ii)

Using gauge pressure, pressure at point B is

$P_B = P_o + \rho g h$ ...(iii)

From equations (ii) and (iii), we have

$P_i = P_o + \rho g h$

or $P_i - P_o = \rho g h$

Substituting in equation (i) above

$$\rho g h = \frac{2S\cos\theta}{r}$$

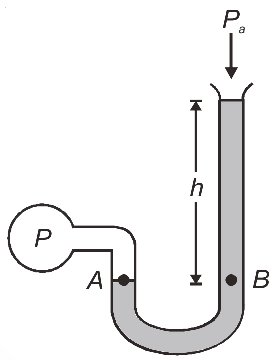
Thus, capillary rise $h = \frac{2S\cos\theta}{r\rho g}$.

If the liquid meniscus is convex, as in case of mercury, $\cos\theta$ is negative. Thus, h has a negative value. This tells that liquid will descend in this case.

36. What is an open tube manometer? Explain how does it work.

Sol. Open-Tube Manometer

An open-tube manometer is used for measuring pressure differences, or to measure the pressure of a gas enclosed in a vessel. It consists of a U-shaped tube open at both ends. A liquid of density ρ is filled in the tube. One end of the tube is left open to the atmosphere. The other end is connected with the system whose pressure is to be measured.



Due to the pressure exerted by the sample, and that by the atmosphere, the liquid in the U-tube rests at different levels in the two arms. Consider the two points A and B, as shown in the figure, lying at the same horizontal level. Then according to Pascal's law, pressures P_A and P_B at the respective points should be same. If h is the height of the liquid column above the point B, then

$P_A = P_B = P_a + \rho gh$

Here $P_A = P$ (pressure of the sample)

$\Rightarrow \boxed{P - P_a = \rho gh}$

What we normally measure is this gauge pressure *i.e.*, the difference between the sample pressure and the atmospheric pressure.

To measure small pressure differences, we use low density liquids like oil in the manometer. Whereas to measure large pressure differences, we use high density liquids like mercury.

37. Derive an expression for the velocity of efflux through an orifice in an enclosed container. Hence, derive Torricelli's law.

Sol. Speed of Efflux

The word 'efflux' means the outflow of the fluid. The expression for the velocity of efflux for a fluid, from a small hole of its container, can be found as follows.

The figure below shows a closed vessel filled with a liquid upto height l. Let the vessel contain a small hole (orifice) in its side at a level h below the top surface of liquid. Taking the liquid to be incompressible and its flow through the hole as streamline, we can apply the equation of continuity at points 1 and 2.

$A_2 v_2 = A_1 v_1$

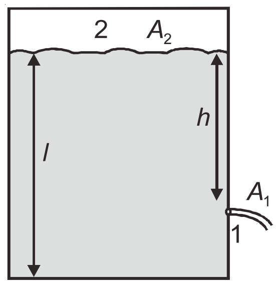
$\Rightarrow \boxed{v_2 = \dfrac{A_1 v_1}{A_2}}$ ...(i)

Applying Bernoulli's equation at the two points, we have

$P_2 + \frac{1}{2}\rho v_2^2 + \rho g l = P_1 + \frac{1}{2}\rho v_1^2 + \rho g(l - h)$ [Point 1 is above the ground by a level $l - h$]

If the cross-sectional area of the vessel

A_2 is much larger than that of hole *i.e.*, A_1



$v_2 \approx 0$

$$\Rightarrow \quad P_2 + \rho g l = P_1 + \frac{1}{2}\rho v_1^2 + \rho g(l - h)$$

$$\Rightarrow \quad \frac{1}{2}\rho v_1^2 = (P_2 - P_1) + \rho g h$$

$$v_1 = \sqrt{\frac{2}{\rho}\left[(P_2 - P_1) + \rho g h\right]}$$

Since the hole is open to the atmosphere, the pressure P_1 is same as the atmospheric pressure P_a.

$$\therefore \quad \boxed{v_1 = \sqrt{\frac{2}{\rho}(P_2 - P_a) + 2gh}}$$

This expression gives the velocity of efflux.

Torricelli's Law : In case the vessel containing the fluid is open *i.e.*, not covered, the pressure P_2 at the top of the liquid surface is same as the atmospheric pressure P_a.

Thus, the velocity of efflux becomes

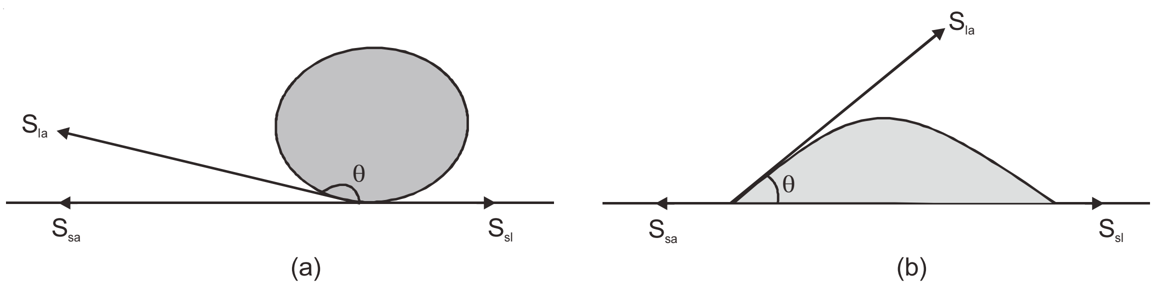
$$\boxed{v_{\text{eff}} = \sqrt{2gh}} \quad \text{...(ii)}$$

This is the same as the velocity acquired by a body after falling freely through a height h.

The expression (ii) is known as Torricelli's law.

38. What are the factors on which the angle of contact depends? What does the value of angle of contact signify?

Sol. Angle of contact is defined as the angle that the tangent to a liquid surface at the point of contact makes with the solid surface inside the liquid. The angle of contact depends on the nature of the solid and the liquid in contact. At the line of contact, the surface forces between the three media must be in equilibrium.



Different shapes of water drops with interfacial tensions
(a) on a lotus leaf (b) on a clean plastic plate.

(i) In the first case, the angle of contact θ is obtuse

Applying the condition of equilibrium for the three surface tensions at the point of contact, we get

$S_{sl} = S_{sa} + S_{la}\cos(\pi - \theta)$

$S_{sl} = S_{sa} - S_{la}\cos\theta$

$\Rightarrow S_{sl} > S_{sa}$

In this case liquid molecules are attracted strongly to themselves and weakly to those of solid. Hence, the liquid does not wet the solid surface.

For example, water on a greasy surface, and mercury on any surface. It is due to this that mercury splits into fine droplets when spread on our palm.

The surface of mercury when poured in a glass tube, bulges upward. We say it has a convex meniscus.

(ii) In the second case, the angle of contact is acute. Again applying the condition of equilibrium at the point of contact, we get $S_{sa} > S_{sl}$.

In such cases, liquid molecules are strongly attracted to those of solids and the liquid wets the solid.

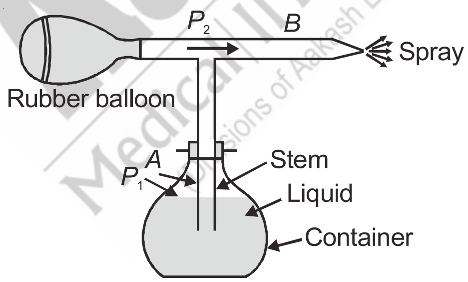
39. Explain three applications of Bernoulli's principle

Sol. (i) **Carburettor of automobiles**

There is a venturi channel provided in the carburettor, Air flows through it at large speed. It lowers the pressure at narrow neck. Petrol rises up in the chamber due to this and provides a correct mixture of air and fuel.

(ii) **Atomiser or Sprayer**

It is another application of Bernoulli's principle. When the balloon is pressed, the air rushes out of the horizontal tube *B* decreasing the pressure in it. It causes the liquid in the bottle to rise up in the vertical tube. When it collides with the high speed air in tube *B*, it breaks up into fine spray.



(iii) **Streamlined shape given to bodies.**

Aircrafts and submarines are given special shapes called streamlined shapes so as to increase the dynamic lift. This is one of the applications of Bernoulli's principle.

40. (i) A wooden block floats in water with one third of its volume submerged. In oil, the block floats with two third of its volume submerged. Find the density of (a) wood, (b) oil, if the density of water is 10^3 kg m^{-3}.

(ii) A golden ornament weighs 60 g in air and 55 g in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. Given specific gravity of gold = 20, specific gravity of copper = 10.

Sol. (i) Let the density of block is ρ_b and the density of oil be ρ_o

Let the volume of block = V

In water

Block floats with one-third of its volume submerged, thus

weight of block = weight of water displaced

$$\Rightarrow \quad V\rho_b g = \frac{V}{3}\rho_{\text{water}} g$$

$$\Rightarrow \quad \rho_b = \frac{\rho_{\text{water}}}{3} = \frac{10^3}{3} = 333.3 \text{ kg m}^{-3}$$

In oil

Blocks floats with two third of its volume submerged.

$\Rightarrow$ wt. of block = wt. of oil displaced

$$\Rightarrow \quad V\rho_b g = \frac{2}{3} V\rho_o g$$

$$\Rightarrow \quad \rho_b = \frac{2}{3}\rho_o$$

$$\Rightarrow \quad \rho_{\text{oil}} = \frac{3}{2}\rho_b = \frac{3}{2}\times 333.3 \approx 500 \text{ kg m}^{-3}$$

(ii) Let the amount of copper in the ornament = m

$\therefore$ The amount of gold in the ornament = $(60 - m)$

Density of gold, ρ_g = specific gravity of gold × 1 g cm^{-3}

= 20 g cm^{-3}

Similarly, Density of copper, ρ_c = 10 g cm^{-3}

Thus,

Volume of copper $V_1 = \dfrac{m}{10}$

Volume of gold $V_2 = \dfrac{60 - m}{20}$

When immersed in water

Decrease in weight = upthrust

$(60 - 55)g = (V_1 + V_2)\rho_w g$

$$5 = \frac{m}{10} + \frac{60 - m}{20} \qquad [\because \rho_w = 1]$$

$$5 = \frac{2m + 60 - m}{20}$$

$100 = m + 60$

$m = 40$

$m = 40$ g

Thus, amount of copper in the ornament is 40 g.

41. How much energy is released when 8 identical water drops each of radius 0.5 cm combine to form one big drop? If the energy is converted to kinetic energy, find the velocity of the big drop.

[Surface tension of water = 0.07 N m^{-1}]

Sol. Given that radius of one small drop, r = 0.5 cm

= 0.005 m

8 such drops combine to form one big drop of radius R.

Then,

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

$\Rightarrow \quad R = 2r$

$= 2 \times 0.005$ m

$R = 0.01$ m

Now the energy released E, on combination of drops is given by

$E = S \times$ (decrease in area)

Where S is the surface tension of water, S = 0.07 N m^{-1}

$\Rightarrow \quad E = 0.07[(4\pi r^2)8 - 4\pi R^2]$

$= 0.07 \times 4\pi[8r^2 - R^2]$

$$= 0.07 \times 4 \times \frac{22}{7}\left[8(0.005)^2 - (0.01)^2\right]$$

$= 0.88 \times (0.02 - 0.01) \times 10^{-2}$

$= 8.8 \times 10^{-5}$ J

This energy gets converted to kinetic energy,

$$\Rightarrow 8.8 \times 10^{-5} = \frac{1}{2}mv^2$$

Where m = mass of the drop, v = velocity of the bigger drop

$$\Rightarrow \quad 8.8\times10^{-5} = \frac{1}{2}\times\rho V\times v^2$$

$$\Rightarrow \quad 8.8\times10^{-5} = \frac{1}{2}(10)^3\left(\frac{4}{3}\pi\times10^{-6}\right)\times v^2$$

$\Rightarrow$ $4.2 \times 10^{-2} = v^2$

$\Rightarrow$ $v = 0.205$ m/s

42. A plane flies horizontally at constant speed. Each of its two wings has an area of 30 m^2. If the speed of the air is 150 km/h below the wing and 245 km/h above it, what is the plane's mass?

[Given air density = 1 kg m^{-3}; g = 9.8 ms^{-2}]

Sol. Given v_{lower} = 150 km/h = $150\times\frac{5}{18} = \frac{125}{3}$ m s^{-1}

v_{upper} = 246 km/h = $246\times\frac{5}{18} = \frac{205}{3}$ m s^{-1}

Total area of the two wings = 2 × 30 m^2 = 60 m^2; ρ = 1 kg m^{-3}

Applying Bernoulli's equation,

$$P_{\text{lower}} + \frac{1}{2}\rho v_{\text{lower}}^2 = P_{\text{upper}} + \frac{1}{2}\rho v_{\text{upper}}^2$$

$$P_{\text{lower}} - P_{\text{upper}} = \frac{1}{2}\rho\left(v_{\text{upper}}^2 - v_{\text{lower}}^2\right)$$

$$= \frac{1}{2}\times\frac{1}{9}\left(205^2 - 125^2\right)$$

$$= \frac{1}{18}(42025 - 15625)$$

$= 1466.6$ N m^{-2}

Upward force on the plane = $(P_1 - P_2) \times A$

= 1466.6 × 60

= 88 × 10^3 N

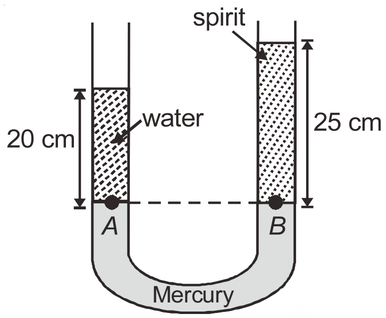
This force balances the weight of the plane.

Therefore $mg = 88 \times 10^3$

$$m = \frac{88\times10^3}{9.8}$$

Hence, the mass of the plane = 8979.5 kg

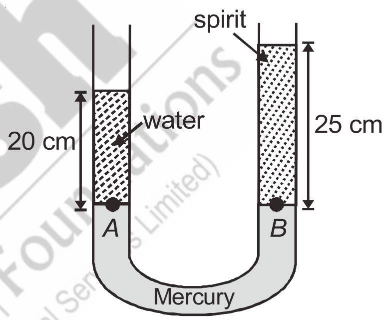
43. (i) A U-tube contains water and methylated spirit separated by mercury as shown below. What is the specific gravity of spirit?



(ii) In the above case if 10 cm of water and spirit each are further poured in the respective arms, find the difference of mercury levels in the two arms.

Sol. (i) The level of mercury is same in both the arms.

That means pressure at point A is the same as that at point B



$\Rightarrow$ $h_w \rho_w g = h_s \rho_s g$

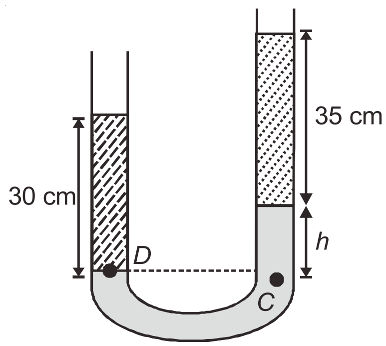
$\Rightarrow$ $h_w \rho_w = h_s \rho_s$

$\Rightarrow$ $\dfrac{\rho_s}{\rho_w} = \dfrac{h_w}{h_s}$

$\dfrac{\rho_s}{\rho_w} = \dfrac{20\text{ cm}}{25\text{ cm}}$

$\Rightarrow$ Specific gravity of spirit = 0.8.

(ii) When 10 cm of water and spirit each is poured in the respective arms, mercury level on the side of water goes down as water is denser than spirit.



Let the difference in the levels of mercury be h, now

equating the pressure at points C and D, we get

$h_w \rho_w g = h_s \rho_s g + h_m \rho_m g$

$\Rightarrow$ $h_w \rho_w = h_s \rho_s + h_m \rho_m$

$\Rightarrow$ $30 \times 1 = 35 \times 0.8 + h \times 13.6$

$\Rightarrow$ $h = 0.147$ cm

$\boxed{h = 1.47\text{ mm}}$

44. (i) Distinguish between streamline flow and turbulent flow.

(ii) Show that Bernoulli's equation is same as the equation due to Pascal's law in presence of gravity, if a liquid or a gas is at rest.

Sol. (i) **Streamline Flow**

1. In a streamline flow each following (oncoming) particle follows exactly the same path as that of its predecessor.
2. Flow is steady and different layers of liquid move parallel to each other.
3. Reynolds number is usually less than <1000 for a streamline flow.
4. Fluid velocity remains constant at any point of a streamline, but it may be different at different points of the same streamline.
5. It occurs at low speeds.
6. A streamline motion can be represented with help of a bundle of streamlines.

Turbulent Flow:

1. The haphazard and zig-zag flow of fluid particles is called turbulent flow.
2. It is accompanied by random irregular local circular currents called eddies.
3. It occurs at high flow speeds.
4. Mostly Reynolds number > 2000 for the flow to be turbulent.
5. It leads to dissipation of kinetic energy as heat.
6. Turbulent motion increases the rate of transfer of mass, momentum and energy.

(ii) According to Bernoulli's principle, $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$. If the fluid is at rest,

$\Rightarrow v_1 = v_2 = 0$

Then, $P_1 + \rho g h_1 = P_2 + \rho g h_2 \Rightarrow \boxed{P_1 - P_2 = \rho g(h_2 - h_1)}$

Which is a mathematical form of Pascal's law, *i.e.*, at $h_1 = h_2$, $P_1 - P_2 = 0$

45. Explain why

(i) Mercury does not wet glass

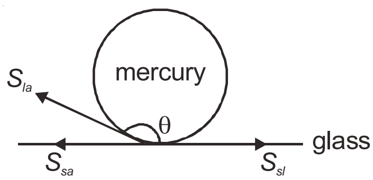
(ii) The hair of a paint brush cling together when taken out of water

(iii) Heavier drops flatten and are not spherical in shape.

Sol. (i) The angle of contact θ for mercury-glass interface is obtuse. The molecules of mercury are strongly attracted to themselves than that to glass molecules.

Hence mercury does not wet glass.

Applying the condition of equilibrium at the point of contact, we have



$S_{sa} + S_{la}\cos(\pi - \theta) = S_{sl}$

$\Rightarrow S_{sl} = S_{sa} - S_{la}\cos\theta$

Since θ is obtuse, $S_{sl} > S_{sa}$

(ii) When brush is taken out of water, thin water film is formed at the tips of the hair. It contracts due to surface tension and so the hair cling together.

(iii) In absence of any external force, a liquid surface acquires a spherical shape due to surface tension. It is because for a given volume, a sphere has the minimum surface area.

Therefore, small liquid drops are spherical in shape, large drops get flattened due to the action of gravity.

SECTION - B

Model Test Paper

Very Short Answer Type Questions :

1. What is the basis of Bernoulli's principle?

Sol. Conservation of energy.

2. What do you mean by an incompressible fluid?

Sol. If the density of fluids remains same with change in pressure, the fluid is said to be incompressible.

3. Express 1 atmosphere in N m^{-2} and bar.

Sol. 1 atm = 1.013×10^5 N m^{-2}

= 1.013 bar

4. For a fluid moving through a pipe, which of its layers moves fastest?

Sol. The layer flowing at the centre (along the axis) of the pipe moves fastest.

5. Why is the dam of water reservoir thick at the bottom?

Sol. If is so because the pressure of water in reservoir increases with depth.

6. How does pressure change at a point when fluid velocity decreases there?

Sol. Decrease in fluid velocity leads to increase in pressure at a point.

7. What is the angle of contact for a fluid in a tube, if it does not wet the tube?

Sol. Angle of contact is obtuse.

8. How does the size of a bubble change when it moves upto the surface from a point deep down the water?

Sol. As the bubble moves upto the surface, the absolute pressure at its outer surface decreases. It size grows bigger due to inner pressure pushing its walls outwards.

Short Answer Type Questions :

9. A piston of cross-section 400 cm^2 is fitted into the wall of a vessel containing fluid. The pressure at two points A and B in the fluid are 10 N m^{-2} and 13 N m^{-2} respectively. On pushing the piston with a force of 20 N, how much does the pressure difference between A and B become?

Sol. Initially

Pressure at point A = 10 N m^{-2}

Pressure at point B = 13 N m^{-2}

Pressure difference between A and B = 3 N m^{-2}

When the piston is pushed in with a force 20 N,

$$\text{Pressure exerted} = \frac{20\text{N}}{\text{Cross Section of piston}}$$

$$= \frac{20 \text{ N}}{400 \times 10^{-4} \text{ m}^2}$$

$$= 500 \text{ N m}^{-2}$$

According to Pascal's law, this increase in pressure is transmitted undiminished to all parts of the fluid.

Hence, Now pressure at point A = 10 + 500

= 510 N m^{-2}

Pressure at point B = 13 + 500

= 513 N m^{-2}

Pressure difference between A and B = 513 – 510

= 3 N m^{-2}

So the pressure difference still remains 3 N m^{-2}

10. Name three factors on which, the viscous force acting on a spherical body moving through a fluid depends.

Sol. These factors are :

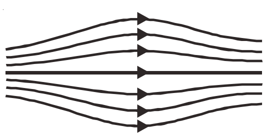
(i) Nature of the fluid *i.e.*, viscosity of the fluid

(ii) Radius of the spherical body

(iii) Velocity of the body

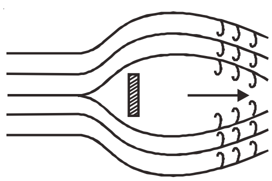
11. Show diagrammatically, both streamline and turbulent flows.

Sol. (i) Streamline flow



Streamlines do not intersect. They are crowded more at the region of greater flow speed.

(ii) Turbulent flow



Flow is haphazard.

12. For the water flowing through a horizontal pipe, the pressure at two points are 3.5×10^5 N m^{-2} and 2.0×10^5 N m^{-2}. What is the difference of kinetic energy per unit volume at these points?

Sol. According to Bernoulli's principle for a horizontal flow,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$3.5 \times 10^5 + \frac{1}{2}\rho v_1^2 = 2.0 \times 10^5 \text{ Nm}^{-2} + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = (3.5 - 2.0) \times 10^5 \text{ Nm}^{-2} = 1.5 \times 10^5 \text{ Nm}^{-2}$$

This is the difference in kinetic energy per unit volume.

13. Define the following:

(i) Angle of contact

(ii) Viscosity

Sol. (i) **Angle of contact :** The angle between the tangent to the liquid surface at the point of contact, and the solid surface inside the liquid is called the angle of contact. If it is acute, the liquid wets the surface; If it is obtuse, the liquid does not wet the solid surface.

(ii) **Viscosity :** The property of a fluid due to which it offers resistance to the motion of a body moving through it, or the resistance that one layer of fluid offers to the relative motion of the other layer is called viscosity. A thicker fluid is more viscous than a thinner one.

14. What is Reynolds number? What is its significance?

Sol. Reynold's number R_e is a dimensionless parameter which is mathematically given by the relation

$$R_e = \frac{\rho v d}{\eta}$$

where ρ = density of the fluid

v = fluid velocity

η = viscosity of the fluid

and d = diameter of the pipe through which the fluid flows.

Significance : The value of Reynold's number tells the nature of fluid flow.

The flow is laminar or steady for $R_e < 1000$

The flow is unsteady for $1000 < R_e < 2000$

$R_e > 2000$ means turbulent flow.

15. A liquid drop of radius R breaks up into eight tiny drops. If the surface tension of the liquid is S, what is the change in surface energy?

Sol. For bigger drop

Radius = R

Surface energy, $E_1 = 4\pi R^2 \times S$ ...(i)

Let the radius of a tiny drop = r

then volume of big drop = 8 × volume of a tiny drop

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = 8\times\frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R = 2r$$

$$\Rightarrow \quad \boxed{r = \frac{R}{2}}$$

Surface energy of one tiny drop = $4\pi r^2 \times S$

$$= 4\pi\left(\frac{R}{2}\right)^2 \times S$$

Surface energy for eight such drops $= 8\times 4\pi\left(\frac{R}{2}\right)^2 S$

$$= 8\pi R^2 S \quad \text{...(ii)}$$

Comparing equations (i) and (ii) the surface energy becomes double.

Short Answer Type Questions :

16. Give reasons for the following:

(i) A bucket of water weighs lesser in water than in air.

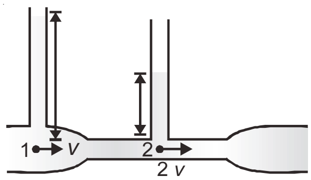
(ii) A spinning ball gets deviated from its path.

Sol. (i) When inside water, the bucket is acted upon by buoyant force of water. This force acts upwards and makes the bucket appear lighter.

According to Archimedes' principle, the buoyant force is equal to the weight of water displaced by the bucket.

(ii) For a spinning ball, the relative velocity of streamlines of air moving past becomes smaller at one end and greater at the other. This change in relative velocity gives rise to pressure difference at the two ends. The ball deviates towards the region of low pressure. This is called magnus effect.

17. In the shown Venturimeter, fluid velocities at two points are indicated. What is the difference in the levels of liquid in tube at the two points? [Given density of the liquid = ρ]



Sol. Let the levels in the two arms be H and h

By Bernoulli's equation

$$\frac{1}{2}\rho v^2 + \rho gH = \frac{1}{2}\rho(2v)^2 + \rho gh$$

$$\Rightarrow \rho g(H-h) = \frac{1}{2}\rho\left(4v^2 - v^2\right)$$

$$\rho g(H-h) = \frac{3}{2}\rho v^2$$

$$\boxed{H-h = \frac{3\rho v^2}{2g}}$$

18. Find the excess pressure inside an air bubble of radius 0.8 cm. Surface tension of the soap solution is 5.6×10^{-2} N m^{-1}.

Sol. For an air bubble,

excess pressure inside it = $\frac{4S}{R}$

$$= \frac{4 \times 5.6 \times 10^{-2}}{0.8 \times 10^{-2}}$$

= 28 N m^{-2}

19. A liquid of density 1.7×10^3 kg m^{-3} and viscosity 0.85 N s m^{-2} flows through a pipe of radius 2 cm with speed 25 cm s^{-1}. Find the nature of flow.

Sol. Nature of flow is determined by Reynold's number

$$R_e = \frac{\rho v D}{\eta}$$

$\rho = 1.7 \times 10^3$ kg m^{-3}

v = 25 cm s^{-1} = 25×10^{-2} m s^{-1}

$D = 4 \text{ cm} = 4 \times 10^{-2}$ m

$\eta = 0.85 \text{ Ns m}^{-2}$

$$\Rightarrow R_e = \frac{1.7\times10^3\times25\times10^{-2}\times4\times10^{-2}}{0.85}$$

$$= 20$$

$\because R_e < 1000$, Flow is laminar.

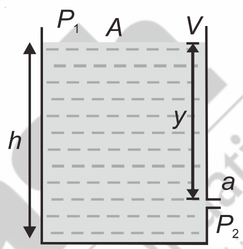
20. Explain Torricelli's law of efflux.

Sol. Let a tank contains a liquid of density ρ upto a height h. It has a small hole on its side at a depth y from the liquid surface. Let the area of cross-section at the top and at the hole be A and a respectively.

From equation of continuity

$AV = av_1$

From Bernoulli's principle



$$P_1+\frac{1}{2}\rho V^2+\rho gy = P_2+\frac{1}{2}\rho v_1^2+0$$

If the tank is open, $P_1 = P_2 = P_a$

$$\Rightarrow \quad \frac{1}{2}\rho V^2+\rho gy = \frac{1}{2}\rho v_1^2$$

$\Rightarrow$ If $A >> a$, then [equation of continuity]

$V \approx 0$

$$\Rightarrow \quad \frac{1}{2}\rho v_1^2 = \rho gy$$

$\boxed{v_1 = \sqrt{2gy}}$ Torricelli's law

The velocity of efflux of a liquid is equal to the velocity which a body acquires in falling freely from the level of free liquid surface to the level of orifice.

21. What is the cause of surface tension?

Sol. The excess energy possessed by the molecules at the surface of a liquid makes the liquid surface behave like an elastic stretched membrane. This gives rise to surface tension.

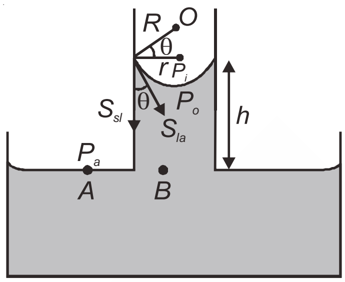
Long Answer Type Questions :

22. (i) Derive an expression for the rise of liquid in capillary tube of uniform diameter and sufficient length.

(ii) Water rises to a height of 25 mm in a capillary. If another capillary of same material and one third radius of previous value is dipped in water, to what height will water rise in new capillary?

Sol. (i) A tube of very fine bore is called a capillary. 'Capilla' is a Latin word which means hair. Thus, capillary is a very thin tube. When such a tube, open at both ends, is dipped in a beaker containing water, water rises in it against gravity.

Let us find an expression for the height *h*, upto which a liquid rises in a capillary tube. Let a capillary tube of radius *r* be dipped in a liquid of surface tension *S* and density ρ. If the angle of contact for this liquid and the capillary tube is acute, the liquid forms a concave meniscus as shown.



The curved part can be considered to be a part of an air bubble inside the liquid. Thus, pressure at a point just above the liquid surface is greater than that just below it.

It is given by $P_i - P_o = \dfrac{2S}{R}$

Here *R* is the radius of the concave curved part

having its centre at *O* (see figure).

As it is clear from the figure

$$R = \frac{r}{\cos\theta}$$

$$\therefore \quad P_i - P_o = \frac{2S\cos\theta}{r} \qquad \text{...(i)}$$

Since region above the concave surface is open to the atmosphere, P_i = Atmospheric pressure P_a.

Also, pressure at point *A* = Atmospheric pressure, P_a.

Since points *A* and *B* are at the same levels.

We have, $P_A = P_B = P_i$ ...(ii)

Using gauge pressure, Absolute pressure at point *B* is

$P_B = P_o + \rho gh$ ...(iii)

From equations (ii) and (iii), we have

$P_i = P_o + \rho gh$

or $P_i - P_o = \rho gh$

Substituting in equation (i)

$$\rho g h = \frac{2S\cos\theta}{r}.$$

Thus, capillary rise $h = \dfrac{2S\cos\theta}{r\rho g}$

If the liquid meniscus is convex, as in case of mercury, θ is obtuse and $\cos\theta$ is negative. Thus h has a negative value. This tells that liquid descends in this case.

The expression for capillary rise shows that it is

- Inversely proportional to the radius of the tube and the density of liquid.
- Directly proportional to the surface tension of the liquid.

Hence, a liquid rises or descends more in a narrower tube than a wider one.

(ii) Rise of liquid $h = \dfrac{2S\cos\theta}{r\rho g}$

For this case hr = constant

$\Rightarrow$ $h_1r_1 = h_2r_2$

$$(25\text{ mm})r = h_2\left(\frac{r}{3}\right)$$

$\Rightarrow$ $\boxed{h_2 = 75\text{ mm}}$

23. (i) State Stoke's law. Derive an expression for terminal velocity using it.

(ii) What are the limitations of Bernoulli's principle?

Sol. (i) Stoke's law states that the viscous drag F on a sphere of radius 'a' moving with velocity v through a fluid of viscosity η is given as

$F = -6\pi\eta av$.

Thus, the force is directly proportional to the velocity of the sphere but acts in the direction opposite to that of velocity.

Derivation of Terminal velocity :

When the sphere falls freely in a fluid, its speed increases due to acceleration of gravity. As a result viscous drag of the fluid increases in the opposite direction. At some stage the viscous drag and the upward buoyant force become sufficient enough to balance the weight of the sphere in the fluid. The sphere then comes down with a constant velocity called terminal velocity v_T.

Thus at v_T

Viscous drag + upward buoyant force = Weight of the sphere

$6\pi\eta a v_T + V\sigma g = V\rho g$ [ρ = density of sphere, σ = density of fluid, V = volume of sphere]

$\Rightarrow \quad 6\pi\eta a v_T = V(\rho - \sigma)g$

But $V = \frac{4}{3}\pi a^3$

$\Rightarrow 6\pi\eta a v_T = \frac{4}{3}\pi a^3 (\rho - \sigma)g$

$\Rightarrow \boxed{v_T = \frac{2a^2}{9\eta}(\rho - \sigma)g}$

(ii) **Limitations of Bernoulli's Principle**

Bernoulli's equation is based on the principle of conservation of energy it is valid for

(a) Non-viscous fluids so that no energy is dissipated due to viscous drag

(b) Streamline flow

(c) Incompressible fluids

(d) Irrotational flow

❑ ❑ ❑

Solutions (Set-2)

Objective Type Questions

(Pressure)

1. The term 'fluid' is used for

(1) Liquids only

(2) Gases only

(3) A mixture of liquid and gas only

(4) Both liquids and gases

Sol. Answer (4)

Substances that can flow is called fluid. Thus both liquids and gases are fluids.

2. Select wrong statement about pressure

(1) Pressure is a scalar quantity

(2) Pressure is always compressive in nature

(3) Pressure at a point is same in all directions

(4) None of these

Sol. Answer (4)

Pressure is scalar as it is not added vectorially. Pressure is compressive in nature, it is same in all the directions at a point.

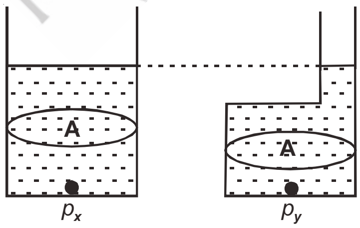
3. Gauge pressure

(1) May be positive (2) May be negative (3) May be zero (4) All of these

Sol. Answer (4)

Gauge pressure depends on the reference chosen, it can be positive, negative or zero.

4. Figure shows two containers *P* and *Q* with same base area *A* and each filled upto same height with same liquid. Select the correct alternative



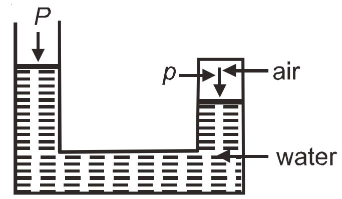
(1) $p_x = p_y$ (2) $p_x > p_y$ (3) $p_y > p_x$ (4) Cannot say

Sol. Answer (1)

The level of water above both points is same so by hydrostatic paradox.

$p_x = p_y$

5. The pressure of confined air is p. If the atmospheric pressure is P, then



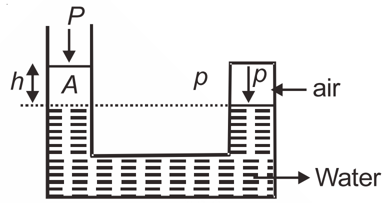
(1) P is equal to p

(2) P is less than p

(3) P is greater than p

(4) P may be less or greater than p depending on the mass of the confined air

Sol. Answer (2)



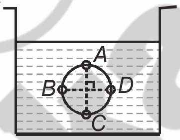
Pressure at point $A = P$ (Hydrostatic paradox)

and $P + \rho_w gh$ = pressure at $A = p$

$\Rightarrow \quad P = p - \rho_w gh$

So, $P < p$

6. Figure shows a container filled with a liquid of density ρ. Four points A, B, C and D lie on the diametrically opposite points of a circle as shown. Points A and C lie on vertical line and points B and D lie on horizontal line. The incorrect statement is (p_A, p_B, p_C, p_D are absolute pressure at the respective points)

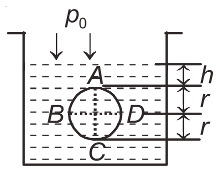


(1) $p_D = p_B$ (2) $p_A < p_B = p_D < p_C$ (3) $p_D = p_B = \dfrac{p_C - p_A}{2}$ (4) $p_D = p_B = \dfrac{p_C + p_A}{2}$

Sol. Answer (3)

Points at same height have same pressure, points with height difference say 'h' will have difference of ρgh.

Let radius of circle is r



$p_A = p_0 + h\rho g$

$p_B = p_D = p_0 + (h + r)\rho g$

$p_C = p_0 + (h + 2r)\rho g$

Then,

$p_C \approx p_A = [p_0 + (h + 2r)\rho g] + [p_0 + h\rho g]$

$= p_0 + h\rho g + 2r\rho g + p_0 + h\rho g$

$= 2[p_0 + (h + r)\rho g]$

$$\frac{p_C + p_A}{2} = p_0 + (h + r)\rho g$$

$$\frac{p_C + p_A}{2} = p_B = p_D$$

i.e., option (1), (2) and (4) gives correct statement but incorrect statement is (3)

7. The volume of an air bubble is doubled as it rises from the bottom of lake to its surface. The atmospheric pressure is 75 cm of mercury. The ratio of density of mercury to that of lake water is $\frac{40}{3}$. The depth of the lake in metre is

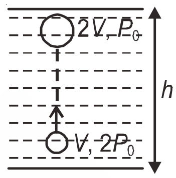
(1) 10 (2) 15 (3) 20 (4) 25

Sol. Answer (1)

$2P_0 = P_0 + \rho gh$

$\Rightarrow P_0 = \rho gh$

$\Rightarrow P_0 = 75$ cm mercury [Atmospheric pressure]



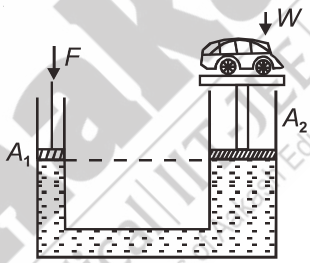
$\Rightarrow \rho_{mercury} \times g \times \frac{75}{100} = \rho_{water} \times g \times h$

$\Rightarrow \frac{\rho_m}{\rho_w} \times \frac{75}{100} = h$

$\Rightarrow \frac{40}{3} \times \frac{75}{100} = h \quad \left[\because \frac{\rho_m}{\rho_w} = \frac{40}{3} \text{ (given)}\right]$

$\Rightarrow h = 10$ m

8. In a hydraulic jack as shown, mass of the car $W = 800$ kg, $A_1 = 10$ cm^2, $A_2 = 10$ m^2. The minimum force F required to lift the car is



(1) 1 N (2) 0.8 N (3) 8 N (4) 16 N

Sol. Answer (2)

Pressure in a liquid is divided equally so we can say pressure at both the pistons should be same

$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\left\{\begin{array}{l}\text{Where,}\\ F_1 = F\\ A_1 = 10 \text{ cm}^2\\ A_2 = 10 \text{ m}^2 = 10\times 10^4 \text{ cm}^2\\ F_2 = 8000 \text{ N}\end{array}\right.$

Substituting values,

$\frac{F}{10} = \frac{8000}{10\times 10^4}$

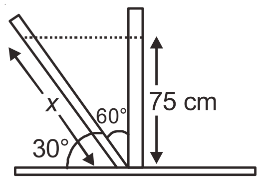
$\Rightarrow F = 0.8$ N $\left\{\begin{array}{l}\text{Take}\\ g = 10 \text{ m/s}^2\end{array}\right\}$

9. A barometer tube reads 75 cm of Hg. If tube is gradually inclined at an angle of 30° with horizontal, keeping the open end in the mercury container, then find the length of mercury column in the barometer tube

(1) 86.7 cm (2) 150 cm (3) 75 cm (4) 92.5 cm

Sol. Answer (2)

$\because$ We are not changing the atmospheric pressure, so height of Hg from the surface should not change.



$$\frac{75}{x} = \cos 60°$$

$\Rightarrow$ x = 150 cm

(Archimedes' principle)

10. A wooden cube just floats inside water with a 200 gm mass placed on it. When the mass is removed, the cube floats with its top surface 2 cm above the water level. What is the side of the cube ?

(1) 6 cm (2) 8 cm (3) 10 cm (4) 12 cm

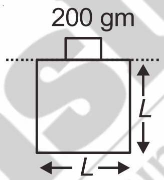
Sol. Answer (3)

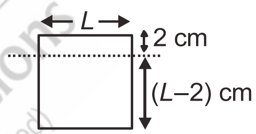
Mass × g = Volume of part of cube × ρ × g

$\Rightarrow$ 200 × g = L^2 (2 × ρ_w × g)

$\Rightarrow$ 100 = L^2 $\{\because \ \rho_w = 1\}$

$\Rightarrow$ 10 cm = L





From the two figures we can see that the 200 gm block is provided with required buoyant force but a part of cube which is afloat in 2nd figure.

11. A block of steel of size 5 × 5 × 5 cm^3 is weighed in water. If relative density of steel is 7, its apparent weight is

(1) 6 × 5 × 5 × 5 g wt (2) 4 × 4 × 4 × 7 g wt

(3) 5 × 5 × 5 × 7 g wt (4) 4 × 4 × 4 × 6 g wt

Sol. Answer (1)

Apparent weight = $\rho_s V_b g - \rho_w V_b g$

$= V_b g(\rho_s - \rho_w)$

$= 5 \times 5 \times 5 \times g \times (7 - 1)$ $\left[\begin{array}{l}\rho_w = 1\\ \rho_s = 7 \ \text{(given)}\end{array}\right.$

$= 6 \times 5 \times 5 \times 5 \times g$ wt

$\left\{\begin{array}{l}\text{Where,}\\ \rho_s - \text{density of steel}\\ \rho_w - \text{density of water}\\ V_b - \text{volume of block}\\ \quad (\text{side} \times \text{side} \times \text{side})\end{array}\right.$

12. A block of wood floats in water with $\frac{4}{5}$th of its volume submerged, but it just floats in another liquid. The density of liquid is (in kg/m^3)

(1) 750 (2) 800 (3) 1000 (4) 1250

Sol. Answer (2)

$$\frac{4}{5} V_b \times \rho_w \times g = V_b \times \rho_b \times g$$

$\left\{\begin{array}{l}\text{Where,}\\ V_b = \text{volume of block}\\ \rho_w = \text{density of water} = 1000 \text{ kg/m}^3\\ \rho_b = \text{density of block}\end{array}\right.$

$$\Rightarrow \quad \frac{\rho_w}{\rho_b} = \frac{5}{4}$$

$$\rho_b = \frac{4}{5} \times 1000 = 800 \text{ kg/m}^3$$

And when block is put in liquid of density ρ_l it just floats

So, $V_b \times \rho_b \times g = V_b \times \rho_l \times g$

$\Rightarrow \quad \rho_b = \rho_l$

So, $\rho_l = 800$ kg/m^3

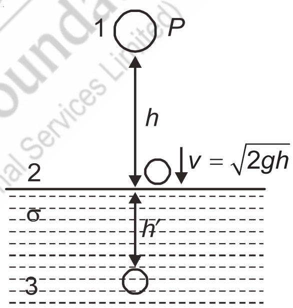
13. A body of density ρ is dropped from rest from a height h into a lake of density σ, where $\sigma > \rho$. Neglecting all dissipative forces, the maximum depth to which the body sinks before returning to float on surface

(1) $\dfrac{h}{\sigma - \rho}$ (2) $\dfrac{h\rho}{\sigma}$

(3) $\dfrac{h\rho}{\sigma - \rho}$ (4) $\dfrac{h\sigma}{\sigma - \rho}$

Sol. Answer (3)

Between point 2 and 3



$$P_0 + \rho g h' + \frac{1}{2}\rho v^2 = \sigma g h' + P_0 \qquad [P_0 = \text{atmospheric pressure}]$$

$$\Rightarrow \quad \rho g h' + \rho g h = \sigma g h' \qquad \left[\because\ v = \sqrt{2gh}\right]$$

$$\text{So, } h' = \frac{h\rho}{\sigma - \rho}$$

14. Two liquids having densities d_1 and d_2 are mixed in such a way that both have same mass. The density of the mixture is

(1) $\dfrac{d_1 + d_2}{2}$ (2) $\dfrac{d_1 + d_2}{d_1 d_2}$

(3) $\dfrac{d_1 d_2}{d_1 + d_2}$ (4) $\dfrac{2d_1 d_2}{d_1 + d_2}$

Sol. Answer (4)

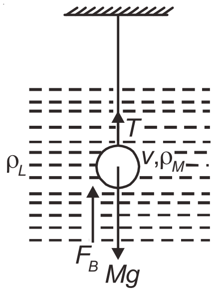
Let each have mass = M and densities d_1 and d_2

$$d_{\text{mix}} = \frac{M_{\text{mix}}}{V_{\text{mix}}} = \frac{M + M}{\left(\frac{M}{d_1}\right) + \left(\frac{M}{d_2}\right)} = \frac{2d_1 d_2}{d_1 + d_2}$$

15. A metallic sphere weighing 3 kg in air is held by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of metallic is 10. The tension in the string is

(1) 18.7 N (2) 42.5 N (3) 32.7 N (4) 27.6 N

Sol. Answer (4)



Where,
F_B – Force of Buoyancy
ρ_L – Density of liquid
ρ_M – Density of metal
M – Mass of sphere
T – Tension in string

$T = Mg$ – Buoyant Force

$\Rightarrow T = \rho_M Vg - \rho_L Vg \qquad \left(\rho_M = \frac{M}{V}\right)$

$= (\rho_M - \rho_L)\, Vg$

$= (10 - 0.8) \times \frac{3}{10} \times 10$

$\Rightarrow T = 9.2 \times 3 = 27.6$ N

16. A rectangular block is 10 cm × 10 cm × 15 cm in size is floating in water with 10 cm side vertical. If it floats with 15 cm side vertical, then the level of water will

(1) Rise (2) Fall

(3) Remain same (4) Change according to density of block

Sol. Answer (3)

Mass of block remains same, volume displaced of water will also remain same so level of water will not change.

17. Two cubical blocks identical in dimensions float in water in such a way that 1st block floats with half part immersed in water and second block floats with 3/4 of its volume inside the water. The ratio of densities of blocks is

(1) 2 : 3 (2) 3 : 4 (3) 1 : 3 (4) 1 : 4

Sol. Answer (1)

$\frac{3}{4} V \times \rho_2 \times g = \frac{1}{2} V \times \rho_1 \times g$

$\Rightarrow \frac{\rho_2}{\rho_1} = \frac{2}{3}$

Where,
V = Volume of block
ρ_1 = density of liquid 1
ρ_2 = density of liquid 2

(Streamline Flow, Bernoulli's Principle)

18. Which of the following is not the property of an ideal fluid?

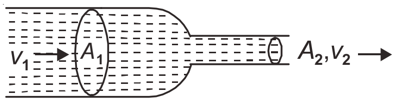
(1) Fluid flow is irrotational (2) Fluid flow is streamline

(3) Fluid is incompressible (4) Fluid is viscous

Sol. Answer (4)

An ideal fluid is not viscous.

19. A liquid flows in the tube from left to right as shown in figure. A_1 and A_2 are the cross-sections of the portions of the tube as shown. The ratio of speed $\frac{v_1}{v_2}$ will be



(1) $\frac{A_1}{A_2}$ (2) $\frac{A_2}{A_1}$ (3) $\sqrt{\frac{A_2}{A_1}}$ (4) $\sqrt{\frac{A_1}{A_2}}$

Sol. Answer (2)

By equation of continuity

$v_1A_1 = v_2A_2$

So, $\frac{v_1}{v_2} = \frac{A_2}{A_1}$

20. Water (ρ = 1000 kg/m^3) and kerosene (σ = 800kg/m^3) are filled in two identical cylindrical vessels. Both vessels have small holes at their bottom. The speed of the water and kerosene coming out of their holes are v_1 and v_2 respectively. Select the correct alternative

(1) $v_1 = v_2$ (2) $v_1 = 0.8\, v_2$ (3) $0.8\, v_1 = v_2$ (4) $v_1 = \sqrt{0.8}\, v_2$

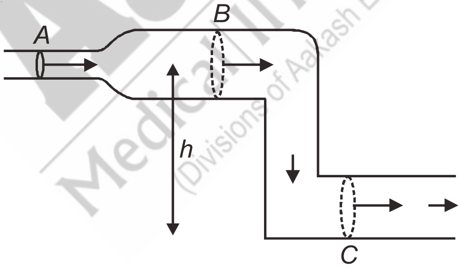
Sol. Answer (1)

Velocity of efflux for small holes $= \sqrt{2gh}$

Which clearly is independent of 'ρ' (density)

So, $v_1 = v_2$

21. Water is flowing through a channel (lying in a vertical plane) as shown in the figure. Three sections A, B and C are shown. Sections B and C have equal area of cross section. If P_A, P_B and P_C are the pressures at A, B and C respectively then



(1) $P_A > P_B = P_C$ (2) $P_A < P_B < P_C$

(3) $P_A < P_B = P_C$ (4) $P_A > P_B > P_C$

Sol. Answer (2)

Solution by using Bernoulli's principle and equation of continuity

Comparing points A and B

$A_Av_A = A_Bv_B$ {equation of continuity}

$\because$ $A_A < A_B$

$v_A > v_B$

$$P_A + \frac{1}{2}\rho V_A^2 + \rho gh = P_B + \frac{1}{2}\rho V_B^2 + \rho gh$$ {Bernoulli's equation}

$\because$ $v_A > v_B$

$\Rightarrow$ $\frac{1}{2}\rho V_A^2 > \frac{1}{2}\rho V_B^2$

$\therefore$ $P_A < P_B$ ...(1)

Now comparing *C* and *B*

$A_B = A_C \Rightarrow v_B = v_C$ [equation of continuity]

$$P_B + \frac{1}{2}\rho V^2 + \rho g h_B = P_C + \frac{1}{2}\rho V^2 + \rho g h_C$$

$\Rightarrow$ $P_B + \rho g h_B = P_C + \rho g h_C$

$\because$ $h_B > h_C$ then ...(2)

$P_B < P_C$

Using (1) and (2)

We can say, $P_A < P_B < P_C$

(Viscosity)

22. A flat plate of area 0.1 m^2 is placed on a flat surface and is separated from it by a film of oil 10^{-5} m thick whose coefficient of viscosity is 1.5 N sm^{-2}. The force required to cause the plate to slide on the surface at constant speed of 1 mm s^{-1} is

(1) 10 N (2) 15 N (3) 20 N (4) 25 N

Sol. Answer (2)

$$F = \eta A \frac{v}{l}$$

Substituting values,

$$= 1.5 \times 0.1 \times \frac{1\times 10^{-3}}{10^{-5}}$$

= 15 N

23. A ball of density σ and radius *r* is dropped on the surface of a liquid of density ρ from certain height. If speed of ball does not change even on entering in liquid and viscosity of liquid is η, then the height from which ball dropped is

(1) $2g\left[\frac{(\sigma-\rho)r}{9\eta}\right]^2$ (2) $\frac{2g(\sigma-\rho)^2 r^2}{9\eta}$ (3) $\frac{2(\sigma-\rho)gr^2}{9\eta}$ (4) $2g\left[\frac{(\sigma-\rho)r^2}{9\eta}\right]^2$

Sol. Answer (4)

The ball has already reached the magnitude of velocity which is equal to its terminal velocity in fluid.

$v = \sqrt{2gh}$ {Velocity of body fallen form height $h = \sqrt{2gh}$ }

$$v_{\text{Terminal}} = \frac{2r^2}{9\eta}(\sigma-\rho)g$$

Equating both

$$\sqrt{2gh} = \frac{2r^2}{9\eta}(\sigma - \rho)g$$

$$\Rightarrow \quad h = 2g\left[\frac{(\sigma-\rho)r^2}{9\eta}\right]^2$$

24. Viscous drag force depends on

(1) Size of body (2) Velocity with which it moves

(3) Viscosity of fluid (4) All of these

Sol. Answer (4)

$$F = \eta A \frac{v}{d}$$

$$\begin{cases} \text{Where,} \\ F = \text{Drag Force} \\ \eta = \text{Viscosity of fluids} \\ A = \text{Area} \propto \text{size of body} \\ v = \text{Velocity} \end{cases}$$

25. The terminal velocity of a small sized spherical body of radius r falling vertically in a viscous liquid is given by the proportionality

(1) $v \propto \frac{1}{r^2}$ (2) $v \propto r^2$ (3) $v \propto \frac{1}{r}$ (4) $v \propto r$

Sol. Answer (2)

$$v_T = \frac{2r^2}{9\eta}[\sigma - \rho]g$$

So, $v_T \propto r^2$

(Surface Tension)

26. On putting a capillary tube in a pot filled with water, the level of water rises upto a height of 4 cm in the tube. If a tube of half the diameter is used instead, the water will rise to a height of nearly

(1) 2 cm (2) 4 cm (3) 8 cm (4) 11 cm

Sol. Answer (3)

For capillary tube

$$h = \frac{2T}{r\rho g}$$

We can say

$$h \propto \frac{1}{r} \text{ or } h \propto \frac{1}{d}$$

So, $\frac{h_1}{h_2} = \frac{d_2}{d_1}$

$$\Rightarrow \frac{4}{x} = \frac{d}{2d}$$

$\Rightarrow$ x = 8 cm

27. Soap helps in cleaning clothes, because

(1) It attracts the dirt particles

(2) It decreases the surface tension of water

(3) It increases the cohesive force between water molecules

(4) It increases the angle of contact

Sol. Answer (2)

Soap helps cleaning clothes because, it decreases the surface tension of water thus water molecules penetrate easily into dirt and oil.

28. On increasing temperature of a liquid, its surface tension generally

(1) Increases (2) Decreases

(3) Remains constant (4) First increases and then decreases

Sol. Answer (2)

On increasing the temperature energy increases hence surface tension decreases. Because surface tension is nothing but some extra energy required by surface molecules to stay at the place.

29. The raincoats are made water proof by coating it with a material, which

(1) Absorb water (2) Increase surface tension of water

(3) Increase the angle of contact (4) Decreases the density of water

Sol. Answer (3)

Raincoats are coated with material which increase the angle of contact, so water does not penetrates inside the layer.

30. An iron needle slowly placed on the surface of water floats because

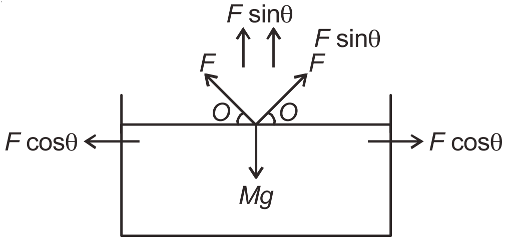
(1) It displaces water more than its weight

(2) The density of material of needle is less than that of water

(3) Of surface tension

(4) Of its shape

Sol. Answer (3)



Needle floats due to surface tension of water which balances the weight of needle.

In equilibrium $2F\sin\theta = mg$

31. Two water droplets merge with each other to form a larger droplet. In this process

(1) Energy is liberated (2) Energy is absorbed

(3) Energy is neither liberated nor absorbed (4) Some mass is converted into energy

Sol. Answer (1)

Work is done when we break a drop into 'n' drops equal to $4\pi R^2\sigma(n^{1/3} - 1)$

So energy will be liberated if we merge back those drops.

32. The radius of a soap bubble is r. The surface tension of soap solution is 'S'. Keeping temperature constant, the radius of the soap bubble is doubled. The energy necessary for this will be

(1) $24\ \pi r^2 S$ (2) $8\ \pi r^2 S$ (3) $16\ \pi r^2 S$ (4) $12\ \pi r^2 S$

Sol. Answer (1)

Work done in making a soap bubble of radius $r = 4\ \pi r^2 S \times 2\ = 8\ \pi r^2 S$ $\left\{\begin{matrix}\text{Multiply by 2 due}\\ \text{to two free surface}\end{matrix}\right\}$

$\therefore$ Energy of bubble $= 8\ \pi r^2 S = E_r$

Work done in making a $2r$ radius soap bubble $= 4\ \pi(2r)^2\ S \times 2 = 32\ \pi r^2 S$

$\therefore$ Energy of bubble $= 32\ \pi r^2 S = E_{2r}$

So energy required to expand a bubble from r to $2r$ will be equal to $E_{2r} - E_r$

Substituting values

We get,

$32\ \pi r^2 S - 8\ \pi r^2 S = 24\ \pi r^2 S$

33. The surface tension of a liquid is 5 N/m. If a film is held on a ring of area 0.02 m^2, its surface energy is about

(1) 5×10^{-2} J (2) 2.5×10^{-2} J (3) 2×10^{-1} J (4) 3×10^{-1} J

Sol. Answer (3)

Surface energy = surface tension × area of film × number of free surface

$= 5 \times 0.02 \times 2$

$= 2 \times 10^{-1}$ J

34. The excess pressure in a soap bubble is double that in other one. The ratio of their volume is

(1) 1 : 2 (2) 1 : 8 (3) 1 : 4 (4) 1 : 1

Sol. Answer (2)

Excess pressure in soap bubble $= \dfrac{4S}{R}$

Let for first bubble,

$$P = \frac{4S}{R} \qquad \left\{\begin{matrix}S - \text{Surface tension}\\ R - \text{Radius}\end{matrix}\right.$$

For second bubble,

$$2P = \frac{4S}{x} \qquad \begin{cases} S - \text{Surface tension} \\ x - \text{Radius} \end{cases}$$

Substitute value of P

$$2 \times \frac{4S}{R} = \frac{4S}{x}$$

$$\Rightarrow \quad x = \frac{R}{2}$$

Ratio of Radii $= \frac{R/2}{R} = \frac{1}{2}$

So, Ratio of volume = (Ratio of Radii)^3

$$= \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

35. The work done to break a spherical drop of radius R in n drops of equal size is proportional to

(1) $\frac{1}{n^{2/3}} - 1$ (2) $\frac{1}{n^{1/3}} - 1$ (3) $n^{1/3} - 1$ (4) $n^{4/3} - 1$

Sol. Answer (3)

$\because$ Volume = constant

i.e., $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

Radius of each new droplet $= \frac{R}{n^{1/3}}$

Work done to break into 'n' drops $= S\,[n \times 4\pi r^2 - 4\pi R^3]$

$= S \times 4\pi R^2\,[n^{1/3} - 1]$

36. The kerosene oil rises up in the wick of a lamp

(1) Due to high surface tension of oil (2) Because the wick attract s the oil

(3) Because wick decreases the surface tension of oil (4) Due to capillaries formed in the wick

Sol. Answer (4)

Capillary action is responsible.

Wick has a lot of capillaries which help the oil rise.

37. Ploughing help to retain water by soil

(1) By creating capillaries (2) By breaking capillaries

(3) By turning the soil upside down (4) None of these

Sol. Answer (2)

By breaking capillaries as they do not allow water to seep inside.

38. A capillary tube of radius r is immersed in a liquid and mass of liquid, which rises up in it is M. If the radius of tube is doubled, then the mass of liquid which will rise in capillary tube will be

(1) $2M$ (2) M (3) $M/2$ (4) $M/4$

Sol. Answer (1)

$M = \rho v$

$\Rightarrow$ Mass $\propto$ volume of tube

Mass $\propto$ height × Area

$$\begin{cases} \text{Area} \propto r^2 \\ \text{Height} \propto \dfrac{1}{r} \\ \text{Where } r = \text{radius of cube} \end{cases} \qquad \left(h = \dfrac{2S}{rgh}\right)$$

$\Rightarrow$ Mass $\propto \dfrac{1}{r} \times r^2$

Mass $\propto r$

$\therefore$ If radius doubles, mass of liquid that rises up also doubles.

(Miscellaneous)

39. A tank is filled with water to a height H. A hole is made in one of the walls at a depth D below the water surface. The distance x from the foot of the wall at which the stream of water coming out of the tank strikes the ground is given by

(1) $x = 2\,[D\,(H - D)]^{1/2}$ (2) $x = 2\,(gD)^{1/2}$

(3) $x = 2\,[D\,(H + D)]^{1/2}$ (4) None of these

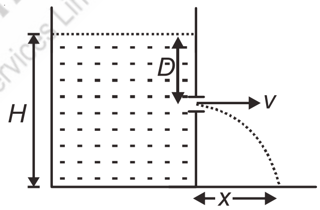
Sol. Answer (1)

Velocity of efflux $= \sqrt{2gD} = v$

Say time taken by water to travel the vertical distance of $(H - D)$ = 't'

Using $s = ut + \dfrac{1}{2}at^2$

$$\begin{cases} \text{Where,} \\ s = H - D \\ u = 0 \\ a = g \end{cases}$$



We get,

$$'t' = \sqrt{\frac{2(H-D)}{g}}$$

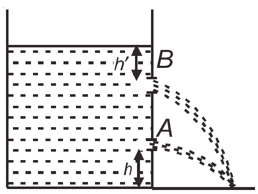
Now, $x = v \times t$

Substituting the values

$$x = \sqrt{2gD} \times \sqrt{\frac{2(H-D)}{g}}$$

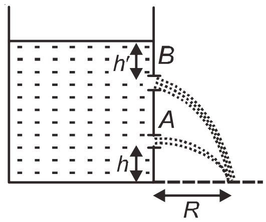
$\Rightarrow$ $x = 2[D\,(H - D)]^{1/2}$

40. A tank is filled with water and two holes A and B are made in it. For getting same range, ratio of h'/h is



(1) 2 (2) $\dfrac{1}{2}$ (3) $\dfrac{1}{3}$ (4) 1

Sol. Answer (4)



For hole 'A'

Velocity of efflux $= \sqrt{2g(x+h')}$

$R = 2[(x + h')h]^{1/2}$ ...(1)

For hole 'B'

Velocity of efflux $= \sqrt{2gh'}$

$R = 2[h'(x + h)]^{1/2}$ ...(2)

Equating (1) and (2)

We get

$2[(x + h')h]^{1/2} = 2[h'(x + h)]^{1/2}$

$\Rightarrow$ $(x + h')h = h'(x + h)$

$\Rightarrow$ $h = h'$

$\Rightarrow$ $\dfrac{h'}{h} = 1$

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