

Name of the Student: \_\_\_\_\_

**There are 3 questions in the exam. You have 30 minutes to finish the exam.**

**Question 1 (20%)**

State true or false:

- (a) \_\_\_ For any sets  $A$  and  $B$ , if  $A \subseteq B$ , then  $A \subset B$ .
- (b) \_\_\_ For any sets  $A$ ,  $B$  and  $C$ , if  $A \subseteq B$  and  $B \subset C$ , then  $A \subseteq C$ .
- (c) \_\_\_ For any sets  $A$ ,  $B$  and  $C$ , if  $A \subseteq B$  and  $C \subseteq B$ , then  $A \cap C \neq \emptyset$ .
- (d) \_\_\_  $\{x \in \mathbb{N} \mid x < 2 \text{ and } x > 2\} = \{x \in \mathbb{Z} \mid x^2 + 5 = 0\}$ ,  
where  $\mathbb{N}$  is the set of natural numbers (=zero and all the positive whole numbers), and  $\mathbb{Z}$  is the set of integers (=natural numbers and all the negative whole numbers).

**Question 2 (30%)**

Remember that for a set of sets  $A$ ,

$$\bigcup A = \{a \mid a \in B \text{ for some } B \in A\}$$

$$\bigcap A = \{a \mid a \in B \text{ for each } B \in A\}$$

State whether true or false:

- (a) \_\_\_ Given any set  $K$ ,  $\bigcup \{X \mid X \subseteq K\} \subseteq K$
- (b) \_\_\_ Given any set  $K$ ,  $\bigcap \{X \mid X \subseteq K \text{ and } |X| < 3\} = \emptyset$

**Question 3 (50%)**

Given a set  $A$  and a relation  $R \subseteq A \times A$ . We say that  $R$  is *connected* if and only if for every *distinct*  $x$  and  $y$  in  $A$ ,  $(x, y)$  or  $(y, x)$  or both are in  $R$ .

Given  $A = \{1, 2, 3, 4\}$ , what is the smallest number of elements a connected relation  $R$  on  $A$  can have. Give an example for such a relation.