

Name of the Student: _____

There are 3 questions in the exam. You have 30 minutes to finish the exam.

Question 1 (30%)

State true or false:

- (a) ___ $A \subseteq B$ if and only if there exists no a such that $a \in A$ and $a \notin B$.
- (b) ___ Let $\mathcal{P}(A)$ be the set of all subsets of A ; $\bigcup \mathcal{P}(A) \in \mathcal{P}(A)$.
- (c) ___ There are as many b 's in *trick* as in *crick*.
- (d) ___ $\emptyset \subset \{\emptyset\}$
- (e) ___ For any sets A, B, C , if $A \subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.
- (f) ___ If $a, b \in A$, then $\{\{a\}, \{a, b\}\} \in \mathcal{P}(\mathcal{P}(A))$.

Question 2 (40%)

Remember the definitions of reflexivity, symmetry and transitivity:

A relation $R \subseteq A \times A$ is **reflexive** if and only if for each $x \in A$, $(x, x) \in R$.

A relation $R \subseteq A \times A$ is **symmetric** if and only if for each $(x, y) \in R$, (y, x) is also in R .

A relation $R \subseteq A \times A$ is **transitive** if and only if whenever (x, y) and (y, z) are in R , then (x, z) is also in R .

Given two relations R_1 and R_2 in $A \times A$ for some set A , state true or false:

- (a) ___ if both R_1 and R_2 are reflexive, then $R_1 \cap R_2$ is also reflexive.
- (b) ___ if both R_1 and R_2 are symmetric, then $R_1 \cup R_2$ is also symmetric.
- (c) ___ if both R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is also symmetric.
- (d) ___ if both R_1 and R_2 are transitive, then $R_1 \cap R_2$ is also transitive.
- (e) ___ if both R_1 and R_2 are transitive, then $R_1 \cup R_2$ is also transitive.

Question 3 (30%)

Let $A = \{1, 2, 3, 4\}$; give a relation R in $A \times A$, that is:

- (a) reflexive, symmetric, but not transitive;
- (b) reflexive, transitive, but not symmetric;
- (c) transitive, symmetric, but not reflexive.