

**Astronomy 160b - Spring 2007**  
Problem Set #3 — due Feb. 22 in class

As noted in class, there's lots of useful information on the black hole website  
<http://www.cmi.yale.edu/bh>

I (8 points). A given object will form a black hole if its radius is less than its Schwarzschild radius. This leads to a very peculiar feature of black holes — the more massive they are, the less dense the material that forms them needs to be. Remember that density is equal to mass divided by volume, or in symbols  $\rho = M/((4/3)\pi R^3)$ .

- a) What density would a human being need to be crushed down to in order to become a black hole?
- b) What density would the Earth need to be crushed down to in order to become a black hole?
- c) Suppose you had a huge spherical cosmic ocean of water. How big and how massive would the ocean have to be to form a black hole (without additional compression)?
- d) Derive a general expression relating the mass of a black hole to the density required for a black hole of that mass to form.

II (8 points). *Note that you will need information from Tuesday's lecture to answer this question.* There is a relativistic expression for the addition of velocities (that is, for the total observed velocity  $v_{tot}$  of something that moves at velocity  $v_1$  with respect to another object that itself moves at velocity  $v_2$  with respect to the observer). This expression is

$$v_{tot} = (v_1 + v_2)/(1 + v_1 v_2/c^2).$$

Show that in the limit where both  $v_1$  and  $v_2$  approach zero, the Newtonian result is recovered. Show that if either  $v_1$  or  $v_2$  is equal to  $c$ , that  $v_{tot}$  is also  $c$ . Explain why this latter result shows that the speed of light is the same for all observers. Apply the approximation  $(1 + \epsilon)^n \approx 1 + n\epsilon$  to the denominator of the above expression to determine the post-Newtonian correction to the Newtonian result.

III (4 points). We've posted a short excerpt from Kip Thorne's book about black holes, in which he describes his interactions with Carl Sagan about wormholes (see under "resources" on the classes server). Thorne suggests that the questions physicists try to answer are of three kinds: questions about what occurs in the world naturally; questions about what can be accomplished using plausible technologies; and "Sagan questions", which involve infinitely advanced technologies. Pose a question about black holes of each of these kinds (you don't have to answer them!). The questions should *not* be the ones Thorne uses as examples. Discuss the extent to which any plausible question in physical science falls into one of these three categories.