30/30

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Midterm Exam #1: Astronomy 160b, spring 2007, February 8, 2007

Below is a summary of a fictional proposal to identify terrestrial planets around the nearest sunlike star α Centauri. The test consists of five problems (worth a total of 30 points) related to this proposal. The test will last 50 minutes — plan your time accordingly! Please put your name on *both* pages of the test, and note that we're using both side of the test paper. Please do all problems on the test paper — if you need more space, continue on a separate piece of paper labelled with your name and the problem number. Use a different piece of paper for each problem you need to continue. **The test is open book, but electronic devices such as calculators are not allowed.**

1 year $= 3 \times 10^7$ seconds

1 A.U. $= 1.5 \times 10^{11}$m

$1M_\odot = 2 \times 10^{30}$ kg

$1M_J = 10^{-3} M_\odot$

$1M_E = 3 \times 10^{-6} M_\odot$

$P_J \approx 11$ years

$a_J \approx 5$ A.U.

$c = 3 \times 10^8$ m/s

$G = 7 \times 10^{-11}$ in mks units

1 parsec $= 3 \times 10^{16}$m

1 radian $= 2 \times 10^5$ arcseconds

$a^3 = P^2 GM/(4\pi^2)$

$\alpha = D_2/D_1$

$V = 2\pi a/P$

$V_* M_* = V_p M_p$

$\Delta\lambda/\lambda = V_R/c$

$\rho = M/(4\pi R^3/3)$

A Proposal to Identify Terrestrial Planets Around α Centauri

Proposal Summary

We propose to develop a new spectrograph capable of extremely high precision radial velocity measurements. Our design is such that we require extremely bright target stars. The only solar type star sufficiently bright is α Centauri, which is located only one parsec from the Earth. Current planetary formation theories suggest that the companion star orbiting around α Cen is likely to prevent the formation of gas giant planets, and indeed the most sensitive astrometric measurements available rule out the presence of Jupiter-mass planets with orbital periods of one year around α Cen. However, there is no reason to think that terrestrial (rocky) planets could not form relatively close to the star. In the first three years of operation ("stage one" of the project) we expect to be able to identify Earth-mass planets within 0.5 A.U. of α Cen. In a further three years ("stage two"), by incorporating expected technological improvements, we anticipate extending our range to include Earth-mass planets within 2 A.U. of α Cen. This would represent the first Doppler search sensitive to planets with Earth-like masses and Earth-like orbits around a Sun-like star.

Our proposed new instrument will also monitor the brightness of α Cen, allowing us to identify planetary transits if they occur. Of course, it would require considerable luck for transits to occur, even if planets exist near α Cen. However the potential scientific return of being able to determine the density of the planet is so great that we believe it is worth the extra effort to carry out the relevant measurements, even if there is a relatively low probability of success.

1 (5 points). The proposers claim that by the end of both stages of the project they could detect Earth-like planets orbiting at a distance of 2 AU from α Cen. Calculate the orbital period of a planet in a circular orbit at a distance of precisely 2 AU. Give your answer to the nearest year.

2

$a^3 = P^2 M$ → solar masses

AU ↗ ↳ yr

$M = 1 M_\odot$
$a = 2$ AU

$2^3 = 8 = P^2 \times 1$

$P = \sqrt{8}$

3

$\boxed{P = 3 \text{ yrs}}$

2 (6 points). The proposers say that it would require luck to be able to observe a transit. What lucky situation would have to exist for a transit to be observable? To what extent is this same kind of luck required for their proposed radial velocity measurements?

3

To observe a transit, the system must be viewed "edge-on" - otherwise the planet won't obscure the star. Since star systems are aligned randomly, it would be lucky if this particular system turned out to be edge-on.

3

Radial velocity measurements also favor edge-on systems, but less drastically. The size of the radial velocity change is greatest with edge-on systems, and gradually diminishes as the orbit becomes less edge-on. But unlike transits, and edge-on system is not required, just helpful.

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3 (7 points). Suppose α Cen has a planet of Earth-mass, and that a transit of the planet can be observed. If the planet's density is less than that of Earth, one would expect the transit to be deeper (that is, the fraction of light obscured by the planet is greater) than a transit of Earth across the Sun as observed by an alien astronomer. Explain why. If the transit is four times deeper, calculate the radius of the planet. (Hint: the radius of the Earth is 7×10^6 meters).

3

If a planet has Earth mass but low density, it must have a larger radius than Earth, since $\rho = \frac{\text{mass}}{\text{volume}}$ volume $= \frac{4}{3}\pi R^3$ — low ρ, same $M \to$ larger R. The size of a transit is determined by the size of the planet, so a lower $\rho \to$ bigger $R \to$ bigger transit.

1 (for eqn)

$\left(\frac{R_E}{R_*}\right)^2$ = Earth transit $\left(\frac{R_p}{R_*}\right)^2$ = planet transit

$$4 = \frac{\text{planet transit}}{\text{Earth transit}} = \left(\frac{R_p}{\cancel{R_*}}\right)^2 \left(\frac{\cancel{R_*}}{R_E}\right)^2 = (R_p/R_E)^2$$

$\frac{R_p}{R_E} = 2$ $\boxed{R_p = 2 \times 7 \times 10^6 = 1.5 \times 10^7 m}$ 3 for result

It's also possible to do it the hard way, by calculating the depth of Earth transit (or quote it) multiply by 4, and calculate radius. That's fine, just more work. (2 pts for intermediate result)

4 (4 points). Given the information presented in the proposal, and in the lectures, circle the best answer for each of the following questions.

Current astrometric measurements of α Cen allow the presence of

a) Jupiter-mass planets with $a = 0.5$A.U.
b) Jupiter-mass planets with $a = 2$A.U.
c) planets with masses $= 0.2$ that of Jupiter and orbits with $a > 10$A.U.
d) both (b) and (c).

The "migration" theory that has been proposed to explain hot Jupiters provides a straightforward explanation for

a) hot Jupiters *and* terrestrial rocky planets in the same planetary system.
b) planetary systems containing a hot Jupiter and a second large gas/ice planet further away from the star (a "warm" Jupiter).
c) Jupiter-mass rocky planets in the outer Kuiper belt region of planetary systems.
d) hot Jupiters in double star systems like α Cen.

5 (8 points). In the "second stage" of their project, the proposers claim that improved sensitivity will enable them to detect Earth-mass planets in orbits with semi-major axes up to 2 AU. This additional sensitivity could also be used to reduce the minimum mass of planets they can detect in a smaller orbit. Explain why improved measurements of radial velocity can lead to detections of planets with either larger orbits or less mass (or some combination of these). Calculate the mass of the least massive detectable planet in a circular orbit with $a = 0.5$ AU at the conclusion of the second stage of the project.

3

In the radial velocity method we observe V_*, and

$$V_* = \frac{M_p}{M_*} V_p$$

Note: M_* is always the same.

If we improve sensitivity, the V_* can be smaller. Thus either V_p or M_p can be smaller. V_p gets smaller as the size of the orbit increases. So more sensitivity to V_* can ~~~~ generate more sensitivity EITHER to longer orbits OR to lower planet masses.

$$V^2 = \frac{GM}{a}$$

$$V_{*\,min} = \frac{M_E}{M_\odot} V_p \;\; \text{at } 2\,AU$$

involving this equation for calculation 1 pt

$$= \frac{M_E}{M_\odot} \sqrt{\frac{GM_*}{2\,AU}}$$

$$V_{*\,min} = \frac{M_{planet}}{M_\odot} V_p \;\; \text{at } 0.5\,AU$$

"$V^2 = \frac{GM}{a}$ — a is 4 times shorter so V is twice as big" — that's fine - you don't have to do it formally as here

$$\text{so} \quad \frac{M_{planet}}{\cancel{M_\odot}} \sqrt{\frac{\cancel{GM_*}}{\tfrac{1}{2}\,AU}}^{=\sqrt{2}} = \frac{M_E}{\cancel{M_\odot}} \sqrt{\frac{\cancel{GM_*}}{2\,AU}}$$

$$\frac{M_{planet}}{M_{EARTH}} = \sqrt{\tfrac{1}{2}} \Big/ \sqrt{2} = \frac{1}{\sqrt{2}}\,\frac{1}{\sqrt{2}} = \frac{1}{2}$$

4 pts getting to answer

this answer is sufficient

$$M_{planet} = \tfrac{1}{2} M_{EARTH} = \tfrac{1}{2} \times 2\times10^{30} \times 3\times10^{-6} = 3\times10^{24}\,kg$$

Again, there's a harder way to do it - actually calculate the limit $V_{*\,min}$, and then using it with $a = \tfrac{1}{2}$ AU. Again, that's fine if one gets the right answer. (2 pts for intermediate result)