segment speed estimation

concept

more accurate to estimate for each segment, by breaking into components:
travel time = (stop time) + (acceleration time) + (deceleration time) + (time spent averaging a practical top speed)
where practical top speed = $\frac{nominal\ top\ speed}{terrain\ difficulty\ coefficient}$

part 1: acceleration

given

vmax = practical top speed (m/s)
m = mass of train (kg)
P = traction power (W)
F = max tractive effort (N)

find

t_acc = time to accelerate to vmax
d_acc = distance to accelerate to vmax
vavg_acc = average speed over d_acc

intermediates

v_1 = max v where F can be applied
P = Fv_1
v_1 = P/F
(**equation 1.1.1**)

t_1 = time when v_1 reached
F=ma
a=v_1/t_1
F=m*(v_1/t_1)
t_1 = m*(v_1/F)
t_1 = m*(P/F^2)
(**equation 1.1.2**)

t' = t - t_1

derive

ref:
https://physics.stackexchange.com/questions/350153/formula-to-determine-acceleration-based-on-constant-energy-input
https://kb.osu.edu/bitstream/handle/1811/2458/V30N04_218.pdf

assuming constant power:

$$P = \frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}m\frac{dv^2}{dt} = mv\frac{dv}{dt}$$

$$\int \frac{P}{m}dt = \int vdv$$

$$\frac{Pt}{m} = \frac{1}{2}v^2 + C$$

with initial velocity v_0: ($C = -½ v_0$)
$v = \sqrt{(v_0 + 2Pt/m)}$

for t ≤ t_1: accel limited by F

v = at
v = Ft/m
(**equation 1.2.1**)

t = mv/F
(**equation 1.2.2**)

d = ∫(0,t) (v(t)) dt
d = [0,t] ((F/2m)(t^2))
d = (F/2m)t^2
(**equation 1.2.3**)

for t > t_1: accel limited by P

let t' = t - t_1
v = √(v_1 + 2P(t-t_1)/m)
(**equation 1.2.4**)
v = √(v_1+ 2Pt'/m)

d = ∫(0,t') (v(t')) dt'
(https://www.integral-calculator.com/)

$$d = \frac{m\cdot\left(\left(\frac{2Pt'+am}{m}\right)^{3/2} - v_1^{3/2}\right)}{3P}$$

$$d = \frac{m\cdot\left(\left(\frac{2P(t-t_1)+am}{m}\right)^{3/2} - v_1^{3/2}\right)}{3P}$$

(**equation 1.2.5**)

v^2 = v_1 + 2P(t-t_1)/m
m(v^2 - v_1)= 2P(t-t_1)

$t = \frac{m(v^2 - v_1)}{2P} + t_1$

(**equation 1.2.6**)

eg.

m = 515 t = 467200 kg
P = 3900 hp = 2908229 w
F = 65000 lbf = 289134.4 n

v_1 = P/F = 10.058 m/s = 22.499 mph
t_1 = m*(P/F^2) = 16.253 s

let v = 30 mph = 13.411 m/s

t_acc = $\frac{m(v^2 - v_1)}{2P} + t_1$ = 28.892 s (to reach 30 mph)

+air resistance at high speeds

the above works well for finding t and d required to accelerate up to a low vmax
but for high vmax, t and d much larger due to negative power exerted by aerodynamic drag

$P_d = (½)\rho v^3 A C_d$
ρ = density of fluid (assume constant)
A = frontal area (assume constant)
C_d = coefficient of drag (assume constant)

the constants of drag

ref:
https://en.wikipedia.org/wiki/Density_of_air#:~:text=The%20density%20of%20air%20or,atmospheric%20pressure%2C%20temperature%20and%20humidity

ρ = 1.2041 kg/m^3 (dry air @ 20 °C and 101.325 kPa)
A = cross-section height * width (largest between locomotive and carriages)
C_d: start with 1; can only be obtained by simulation, or tuning t/v/d functions to match real data

abstract by calculating a single constant:

$D = (½)\rho A C_d$

(**equation 1.3.1**)

$P_d = Dv^3$

originally, P only accounted for traction power

now, accounting for P_d

$P_{tot} = P_{trac} - P_d$

$$\frac{P_{tot}t}{m} = \frac{1}{2}v^2 + C$$

$$\frac{(P_{trac} - P_d)t}{m} = \frac{1}{2}v^2 + C$$

$$\frac{(P_{trac} - Dv^3)t}{m} = \frac{1}{2}v^2 + C$$

$$t = \frac{m(\frac{1}{2}v^2 + C)}{P_{trac} - Dv^3}$$

for t ≤ t_1: accel limited by F

assume same v, t, d as previous— force of drag is insignificant for small v

for t > t_1: accel limited by P

then assume t' = t - t_1, $v_0 = v_1$

~~(from earlier C = -½ v_0)~~

solving for C

$$t' = \frac{m(\frac{1}{2}v^2 + C)}{P_{trac} - Dv^3}$$

at t = t_1, v = v_1

at t' = 0, v = v_1

$$0 = \frac{m(\frac{1}{2}{v_1}^2 + C)}{P_{trac} - D{v_1}^3}$$

$$C = \frac{1}{2}{v_1}^2$$

now plug C back in:

$$t' = \frac{m(\frac{1}{2}v^2 - \frac{1}{2}{v_1}^2)}{P_{trac} - Dv^3}$$

$$t = \frac{m(v^2 - {v_1}^2)}{2(P_{trac} - Dv^3)} + t_1$$

(**equation 1.3.2**)

note this is same as before but with $P_{tot} = P_{trac} - P_d$

solving for v by hand is impossible

provide: m, P_{trac}, D

calculate: t, t_1, v_1

then plug in and use polynomial solver:

$$\frac{m(v^2 - v_1^2)}{2(P_{trac} - Dv^3)} + t_1 - t = 0$$

$$mv^2 - mv_1^2 + 2P_{trac}t_1 - 2P_{trac}t - 2Dv^3t_1 + 2Dv^3t = 0$$

$$(2Dt - 2Dt_1)v^3 + mv^2 - mv_1^2 + 2P_{trac}t_1 - 2P_{trac}t = 0$$

$$2D(t - t_1)v^3 + mv^2 - mv_1^2 + 2P_{trac}(t_1 - t) = 0$$

$$av^3 + bv^2 + c = 0$$

where coefficients

a = $2D(t - t_1)$

b = m

c = $-mv_1^2 + 2P_{trac}(t_1 - t)$

(**equation 1.3.3**)

use the above to create a function in python, v(t), and use python to estimate:

d(t) = $\int_0^t v(t)dt$

(**equation 1.3.4**)

part 2: deceleration

concept

ref: http://www.railway-technical.com/books-papers--articles/high-speed-railway-capacity.pdf
“A more realistic approach is by Hunyadi (2011), who proposes a series of braking rates that vary with speed“

given

a_1 = deceleration rate (m/s^2) from v_1 (m/s) down to v_0 (m/s)
a_2 = deceleration rate from v_2 down to v_1
…
a_n = deceleration rate from v_n down to v_{n-1}

v_0 = 0

v_{max} = practical top speed (m/s) = initial speed

find

t_brake = time to brake from vmax

d_brake = distance to brake from vmax

deceleration

$a(v) = \{$
0 for $v = 0$
a_1 for $0 < v \le v_1$
a_2 for $v_1 < v \le v_2$
…
a_n for $v_{n-1} < v \le v_n$

time

given $v_{n-1} < v_{max} \le v_n$:
t_n = time to reach v_{n-1}
$t_n = (v_{max} - v_{n-1})/a_n$
$t_{n-1} = (v_{n-1} - v_{n-2})/a_{n-1} + t_n$
$t_{n-2} = (v_{n-2} - v_{n-3})/a_{n-2} + t_{n-1}$
…
$t_1 = (v_1 - v_0)/a_1 + t_2$

t_{stop} = total time to stop = t_1

speed

$v(t) = \{$
$v_{max} - a_n t$ for $0 < t < t_n$
$v_{n-1} - a_{n-1}t$ for $t_n < t < t_{n-1}$
$v_{n-2} - a_{n-2}t$ for $t_{n-1} < t < t_{n-2}$
…
$v_1 - a_1 t$ for $t_2 < t < t_1$
v_0 for $t > t_1$

dist

distance to stop = $d(t_1) = \int_0^{t_n} (v_{max} - a_n t)dt + \int_{t_n}^{t_{n-1}} (v_{n-1} - a_{n-1}t)dt + \ldots$

simpler case: only two rates

a_1 = deceleration rate (m/s^2) from v_1 (m/s) down to v_0 (m/s)

a_2 = deceleration rate for v above v_1
v_{max} = initial speed

time to stop = t_{stop}
= {
t_1 (time to stop from v_{max}) for …
t_2 (time to slow from v_{max} to v_1) + t_1 (time to stop from v_1) for …
= {
$v_{max}(1/a_1)$ for $v_{max} < v_1$
$(v_{max} - v_1)(1/a_2) + v_1(1/a_1)$ for $v_{max} > v_1$
(**equation 2.1.1**)
undefined for $v_{max} \leq 0$ "error: v_{max} must be positive"

t' = (time to slow from v_{max} to v_1) = t_2 - t' = $(v_{max} - v_1)(1/a_2)$

distance to stop = d_{stop}
= {
d_1 (distance to stop from v_1) for …
d_2 (distance to slow from v_{max} to v_1) + d_1 (distance to stop from v_1) for …
ref:
https://stackoverflow.com/questions/63085071/constant-acceleration-movement-with-minus-acceleration
(to get d from a: $d = v_0t + ½at^2$)

={
$0 + ½a_1t_1^2$ for …
$v_1t' + ½a_2t'^2 + ½a_1t_1^2$ for …
={
$½a_1(v_{max}/a_1)^2$ for …
$v_1((v_{max} - v_1)(1/a_2)) + ½a_2((v_{max} - v_1)(1/a_2))^2 + ½a_1(v_1/a_1)^2$ for …
={
$½(v_{max}^2/a_1)$ for $v_{max} < v_1$
$v_1(v_{max} - v_1)(1/a_2) + ½((v_{max} - v_1)^2(1/a_2)) + ½(v_1^2/a_1)$ for $v_{max} > v_1$
(**equation 2.1.2**)

speed at time t = v

if $v_{max} < v_1$:

$v = \max(0, v_{max} - a_1t)$
(**equation 2.1.3**)

if $v_{max} > v_1$:

$v = \max(0, \{$
$v_{max} - a_2 t$ for $t < t_1$
$v_1 - a_1(t - t_1)$ for $t > t_1$
$\}$

$= \{$
$v_{max} - a_2 t$ for $t < t_1$
$\max(0, (v_1 - a_1(t - t_1)))$ for $t > t_1$
$\}$
where $t_1 = \Delta v(1/a_1)$
$= (v_{max} - v_1)(1/a_1)$
(**equation 2.1.4**)

eg. for Amtrak Bi-Level passenger cars
ref (p.37):
https://web.archive.org/web/20160306083948/http://www.highspeed-rail.org/Documents/PRIIA_Bi-Level_Spec_305-001_Approved_rev%20C.1.pdf
“Braking rates: Full service: minimum of 1.35 miles per hour per second (mphps) (2.17 km/hr/s) deceleration from 125 mph (201 km/hr) down to 70 mph (113 km/hr), then increasing to not less than 2 miles per hour per second (mphps) (3 km/hr/s) average below 70 mph (113 km/hr)”

$a_2 = 1.35$ mph/s $= 0.60$ m/s^2
$a_1 = 2$ mph/s $= 0.89$ m/s^2
$v_2 = 125$ mph $= 55.88$ m/s
$v_1 = 70$ mph $= 31.29$ m/s
$v_0 = 0$ m/s
(**constants 2.1.5**)

part 3: stop-to-stop estimation

concept

given two consecutive stops S_A and S_B, find the arrival-to-arrival travel time from S_A to S_B (or vice versa)

given

v_{max} = practical top track speed (m/s)
calculate from earlier: t_{acc}, d_{acc}; t_{brake}, d_{brake}

t_{dwell} = dwell time at S_A or S_B (assume same for either) (t)
d_{total} = track distance between S_A or S_B (m)
$d_{total} = d_{acc} + d_{vmax} + d_{brake}$

find

t_{total} = total arrival-to-arrival travel time from S_A to S_B (assume equal for vice versa)
v_{avg} = average speed from S_A to S_B

intermediates

d_{vmax} = distance traveled at v_{max}
$d_{vmax} = d_{total} - d_{acc} - d_{brake}$
(**equation 3.1.1**)

t_{vmax} = time traveled at v_{max}
$t_{vmax} = d_{vmax} / v_{max}$
(**equation 3.1.2**)

tying it all together

$t_{total} = t_{dwell} + t_{acc} + t_{vmax} + t_{brake}$
(**equation 3.2.1**)
$v_{avg} = d_{total} / t_{total}$
(**equation 3.2.2**)

catch: constrained by d_{total}

$d_{total} = d_{acc} + d_{vmax} + d_{brake}$
d_{acc} and d_{brake} are w.r.t. v_{max}
property: d_{vmax} must be non-negative
property that follows: $d_{acc} + d_{brake} \leq d_{total}$
(**property 3.3.1**)
practically, this means can only reach a speed of v_{max} if the distance required to accelerate to v_{max}, plus the distance required to brake from v_{max} to a stop, is less than the total distance between the two stops

before, v_{max} was defined by other factors: track conditions, curves, grades etc
but now, must also consider d_{tot} constraint on d_{acc} and d_{brake}, and thus v_{max}

adjustment algorithm

if v_{max} provided is too high, it must be lowered to satisfy: $d_{acc} + d_{brake} \leq d_{total}$
due to the complexity of d_{acc} and d_{brake} w.r.t. v_{max}, it may not be feasible to solve for optimal v_{max}
instead, insert a **brute force** adjustment of v_{max}:

while not($d_{acc} + d_{brake} \leq d_{total}$):

lower v_{max} by a constant factor (eg. 0.5%)
recalc d_{acc}, d_{brake}

break after certain number of iterations, eg. 100, in case of bad params (eg. unit conversion errors)
(**algorithm 3.3.2**)

(old, ignore) segment speed estimation

concept

within a segment, the closer the (highest) speed limit is to the service max speed, the lower average speed is expected relative to that speed limit; estimating for dwell/ accel/ decel times, lesser speed limits, curves, grades
eg: on a segment of regional service (max 100), if the highest speed limit is 100: estimated average speed = 50
but if the highest speed limit is 60, estimated average speed = 42

given

p = segment length (pixels), measured

v = highest speed limit within segment
m = service max speed
c = loss coefficient ([0, 1], higher = slower)
high speed: c = 0.375 (~25% faster due to significant speed up efforts: higher + more consistent limits within segment, grade/terrain separation)
otherwise: c = 0.5 (real world examples: service max speeds vs. average speeds)

find

L = segment length (mi)
f = efficiency: average speed as a factor of v

μ = segment avg spd (mph) = f * v
t = segment time = L / μ

line stats

T = E2E time

s = num trainsets
F = frequency = round_up(T * 2 / s)