Analysis of Variance

1. One-Way ANOVA
2. Two-Way ANOVA
3. Multivariate Analysis of Variance

Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Analysis of variance (ANOVA) is used

- to investigates the relationship between categorical independent variables and continuous dependent variables

```
InsectSprays[1:3,]
```

count	spray
10	A
7	A
20	A

Continuous Dependent Variable

Categorical Independent Variable

```
table(InsectSprays$spray)
```

```
 A  B  C  D  E  F 
12 12 12 12 12 12 
```

- to analyze the differences among group means and their associated procedures

One-way ANOVA is used to determine whether there are any statistically significant differences between the means of two or more independent groups.

Data Structure

Group	1	2	$\cdots$	k
	x_{11}	x_{21}	$\cdots$	x_{k1}
	x_{12}	x_{22}	$\cdots$	x_{k2}
	$\vdots$	$\vdots$		$\vdots$
	x_{1n_1}	x_{2n_2}	$\cdots$	x_{kn_k}
Group mean	μ_1	μ_2	$\cdots$	μ_k

Hypothesis

$$H_0: \quad \mu_1 = \mu_2 = \ldots = \mu_k$$
$$H_a: \quad \mu_1 \neq \mu_2 \neq \ldots \neq \mu_k$$

Sample Data : InsectSprays

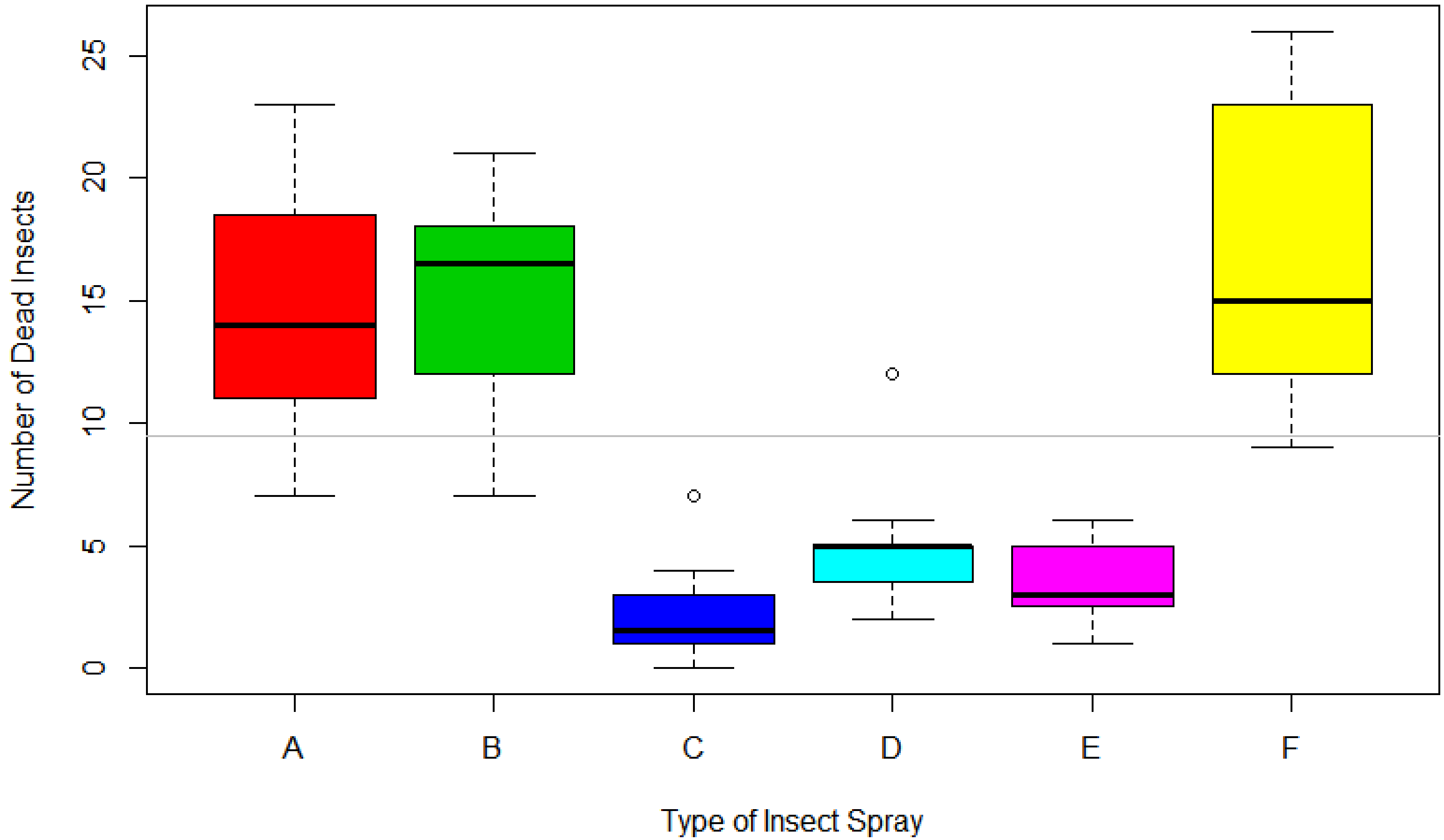
InsectSprays {datasets} Effectiveness of Insect Sprays
The counts of insects in agricultural experimental units treated with different insecticides.

```
> head(InsectSprays,4)
  count spray
1    10     A
2     7     A
3    20     A
4    14     A
> table(InsectSprays$spray)

 A  B  C  D  E  F
12 12 12 12 12 12
```

spray	A	B	C	D	E	F
	10	11	0	3	3	11
	7	17	1	5	5	9
	20	21	7	12	3	15
	14	11	2	6	5	22
	14	16	3	4	3	15
	12	14	1	3	6	16
	10	17	2	5	1	13
	23	17	1	5	1	10
	17	19	3	5	3	26
	20	21	0	5	2	26
	14	7	1	2	6	24
	13	13	4	4	4	13
mean	14.5	15.3	2.1	4.9	3.5	16.7

Step 1. Examine the mean differences

```
boxplot(count~spray, data=InsectSprays,
        xlab='Type of Insect Spray',
        ylab='Number of Dead Insects', col=2:7)
abline(h=mean(InsectSprays$count),col='gray')
```



Step 2. Compute ANOVA table for a fitted model

aov(formula, data, ...)

Fit an analysis of variance model by a call to lm for each stratum.

formula: y(continuous variable) ~ X (categorical variable with two or more groups)

```
#Conducting One-Way ANOVA
aov.out <- aov(count~spray,data=InsectSprays)
summary(aov.out)
```

```
            Df Sum Sq Mean Sq F value Pr(>F)    
spray        5   2669   533.8    34.7 <2e-16 ***
Residuals   66   1015    15.4
```

#mean square
=sample variance
=sum of squares/Df

#SST = sum of the squares of the deviations of all observations

SS_{Total} = Total Sums of Squares $= \sum_{j=1}^{J} \sum_{i=1}^{n_j} (y_{ij} - \bar{\bar{y}})^2$ $\quad \bar{\bar{y}}$ = *grand mean*

SS_A = Explanatory Variable A's Sums of Squares $= \sum_{j=1}^{J} \sum_{i=1}^{n_j} (\bar{y}_j - \bar{\bar{y}})^2$

SS_E = Error (Residual) Sums of Squares $= \sum_{j=1}^{J} \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2$

$$SS_{Total} = SS_A + SS_E$$

https://tinyurl.com/y6eag2dk https://tinyurl.com/y6a8l7gd

anova(object, ...)

Compute analysis of variance tables for one or more fitted model objects.

```
> an3 <- anova(lm(count ~ spray))
> an3
Analysis of Variance Table

Response: count
          Df Sum Sq Mean Sq F value    Pr(>F)
spray      5 2668.8  533.77  34.702 < 2.2e-16 ***
Residuals 66 1015.2   15.38
```

Anova Analysis Result

	Sum Sq	Df	Mean Sq	F	p-value
spray	2668.8	5	533.8	34.702	0.000
Residuals	1015.2	66	15.38		
Sum	3684.0	71			

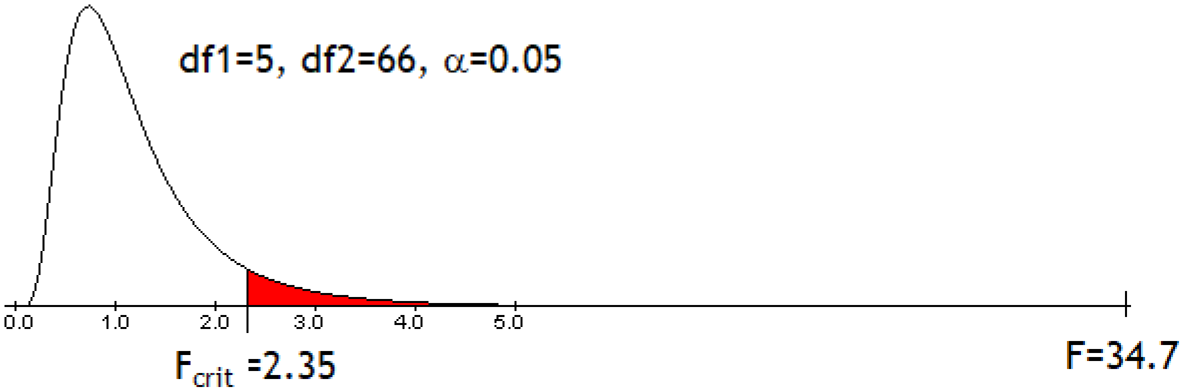
The difference between lm and aov is mainly in the <u>form of the output.</u> If you have multiple error terms then you must use aov because lm does not support the Error term.

https://tinyurl.com/y4ox47ax

Step 3. Conclude with the F test result

qf(p, df1, df2, ncp, lower.tail=TRUE)
Quantile function for the F distribution with df1 and df2 degrees of freedom

```
>
> df1 = 5; df2 = 66
> a = 0.05
> Fc = qf(1-a, df1, df2)  #critical F
> Fc
[1] 2.353809
```



Interpretation:

(1) $p < 0.05$
(2) $F = 34.7$ (Large F-value indicates that the model is significant.)
(3) We reject the null hypothesis.
(4) We conclude that count of insects varies with respect to spray type.

model.tables(x, ...)

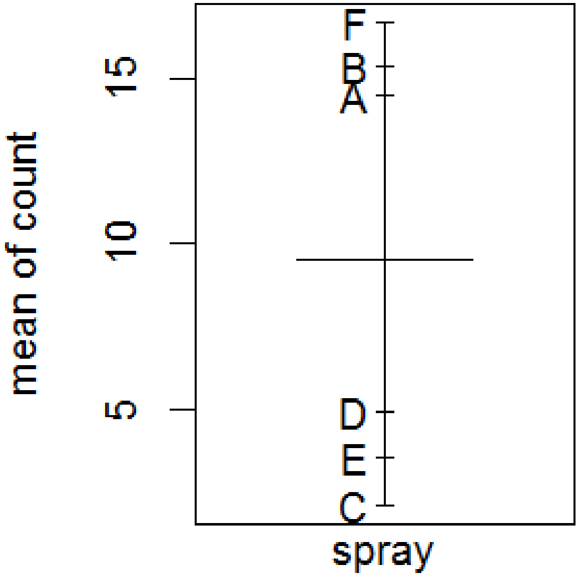
Computes summary tables for model fits, especially complex aov fits.

```
> print(model.tables(aov.out,"means"),digits=3)
Tables of means
Grand mean
9.5
spray
    A     B     C     D     E     F
14.50 15.33  2.08  4.92  3.50 16.67
```

plot.design(x, fun, data=NULL, ...)

Plot univariate effects of one or more factors, typically for a designed experiment as analyzed by aov().

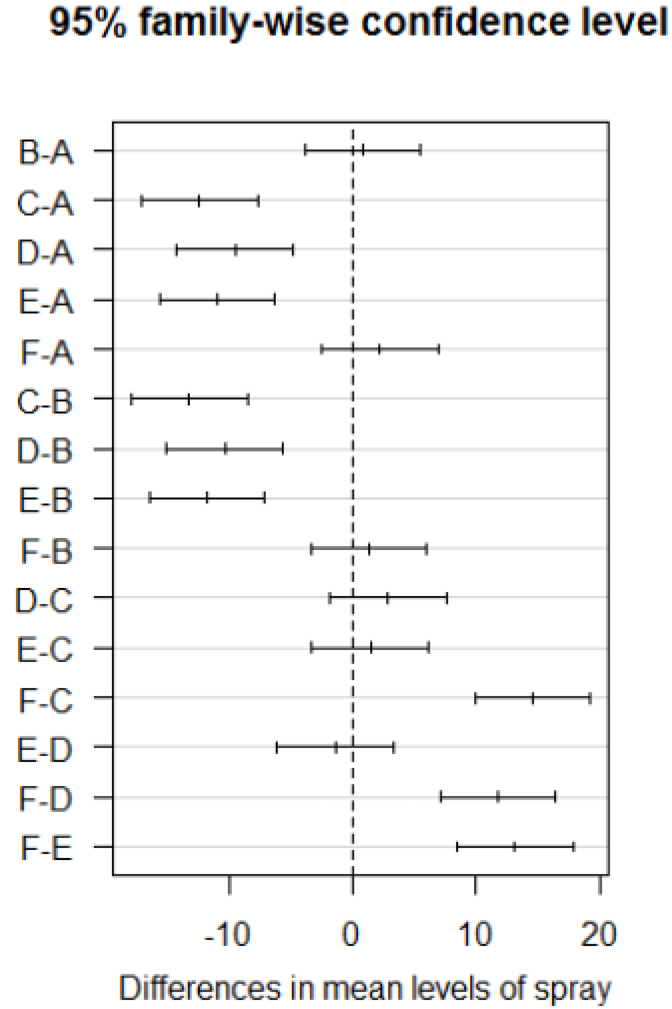
```
plot.design(InsectSprays)
```



Step 4. Perform Tukey HSD (Honestly Significant Difference) Post Hoc Test

```
> tkh <- TukeyHSD(aov.out, conf.level=0.95)
> tkh
  Tukey multiple comparisons of means
    95% family-wise confidence level

$spray
           diff        lwr       upr     p adj
B-A   0.8333333  -3.866075  5.532742 0.9951810
C-A -12.4166667 -17.116075 -7.717258 0.0000000
D-A  -9.5833333 -14.282742 -4.883925 0.0000014
E-A -11.0000000 -15.699409 -6.300591 0.0000000
F-A   2.1666667  -2.532742  6.866075 0.7542147
C-B -13.2500000 -17.949409 -8.550591 0.0000000
D-B -10.4166667 -15.116075 -5.717258 0.0000002
E-B -11.8333333 -16.532742 -7.133925 0.0000000
F-B   1.3333333  -3.366075  6.032742 0.9603075
D-C   2.8333333  -1.866075  7.532742 0.4920707
E-C   1.4166667  -3.282742  6.116075 0.9488669
F-C  14.5833333   9.883925 19.282742 0.0000000
E-D  -1.4166667  -6.116075  3.282742 0.9488669
F-D  11.7500000   7.050591 16.449409 0.0000000
F-E  13.1666667   8.467258 17.866075 0.0000000
```

```
plot(tkh, las=1)
```



This output indicates that the differences B-A, F-A, F-B, D-C, E-C, and E-D are not significant. Meanwhile, C-A, D-A, E-A, C-B, D-B, E-B, F-C, F-D and F-E are significant.

The **two-way analysis of variance (ANOVA)** examines the influence of two different categorical independent variables (called factors) on one continuous dependent variable.

Understand if there is an interaction between the two variables. Two-Way ANOVA is referred to a Factorial ANOVA.

Source of variation	**DF**	**Sum of squares SS**	**Mean square MS**	**F**	**P-value**
Factor A	DFA = I -1	SSA	SSA/DFA	MSA/MSE	**for F_A**
Factor B	DFB = J - I	SSB	SSB/DFB	MSB/MSE	**for F_B**
Interaction	DFAB = $(I-1)(J-1)$	SSAB	SSAB/DFAB	MSAB/MSE	**for F_{AB}**
Error	DFE = N - IJ	SSE	SSE/DFE		
Total	DFT = N – 1 =DFA+DFB+DFAB+DFE	SST =SSA+SSB+SSAB+SSE	SST/DFT		

Examples:

One-way ANOVA : Do mice weigh more in early or late mating season?

Two-way ANOVA : Are mice heavier in early or late mating season and does that depend on the gender of the mice?

Hypothesis for two factors (*A, B*) and a dependent variable (*D*)

Interaction Effect:

Two variables interact if a combination of variables affects results that would not be anticipated from their main effects.

H0: A and B interaction will have no significant effect on D

Main effect :

A: A will have no significant effect on D (H0)

B: B will have no significant effect on D (H0)

Sample Data : schooldays

data("**schooldays**") {HSAUR3}

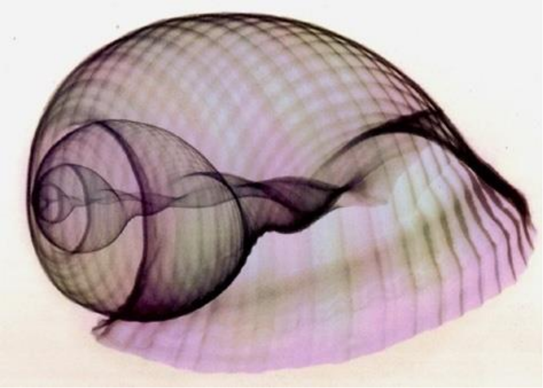
Data from a sociological study, the number of days absent from school is the response variable.

race : race of the child, a factor with levels aboriginal and non-aboriginal.

school : the school type, a factor with levels F0 (primary), F1 (first), F2 (second) and F3 (third form).

absent : number of days absent from school.

```
> library(HSAUR3)
> head(schooldays,2)
        race gender school learner absent
1 aboriginal   male     F0    slow      2
2 aboriginal   male     F0    slow     11
```

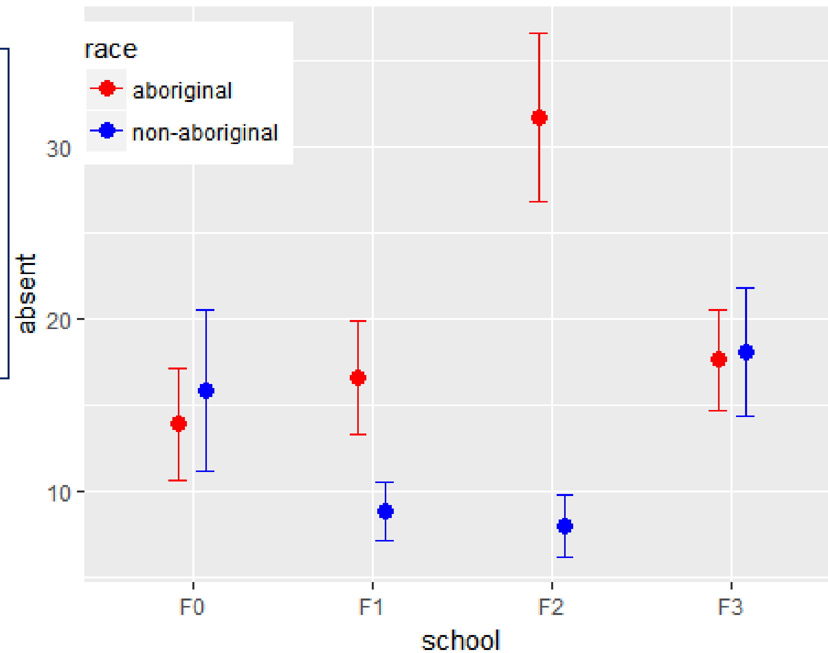


Means and Summary Statistics by Group

```
> library(Rmisc)
> sum = summarySE(schooldays,measurevar="absent",
+                 groupvars=c("race","school"))
> sum
            race school  N    absent        sd       se        ci
1     aboriginal     F0 13 13.846154 11.859217 3.289155  7.166453
2     aboriginal     F1 20 16.550000 14.752252 3.298704  6.904267
3     aboriginal     F2 20 31.650000 21.947965 4.907714 10.271964
4     aboriginal     F3 21 17.571429 13.253571 2.892166  6.032953
5 non-aboriginal     F0 14 15.785714 17.493641 4.675372 10.100528
6 non-aboriginal     F1 28  8.821429  9.059693 1.712121  3.512982
7 non-aboriginal     F2 18  7.944444  7.741958 1.824797  3.849985
8 non-aboriginal     F3 20 18.050000 16.693759 3.732838  7.812920
```

Standard Error Plot Using Summary Statistics

```
library(ggplot2)
pd = position_dodge(0.3)
ggplot(sum, aes(x=school,y=absent,color=race)) +
 geom_errorbar(aes(ymin=absent-se,ymax=absent+se),
              width=0.2,size=0.7,position=pd) +
 geom_point(shape=16, size=3, position=pd) +
  scale_color_manual(values=c("red","blue")) +
  theme(legend.position=c(0.13,0.85))
```



Conducting Two-Way ANOVA

```
> aov2 <- aov(absent ~ race*school, data=schooldays)
> summary(aov2)
             Df Sum Sq Mean Sq F value   Pr(>F)    
race          1   2646  2645.7  12.461 0.000556 ***
school        3   1326   441.8   2.081 0.105245    
race:school   3   3738  1246.1   5.869 0.000822 ***
Residuals   146  30997   212.3                     
```

#race*school = race + school + race:school

Main effect 1:

P-value for race is 0.001. We <u>reject</u> the null hypothesis that the means of absent evaluated according to the race are equal.

Main effect 2:

P-value for school is 0.105. We <u>retain</u> the null hypothesis that the means of absent evaluated according to the school are equal.

Interaction effect:

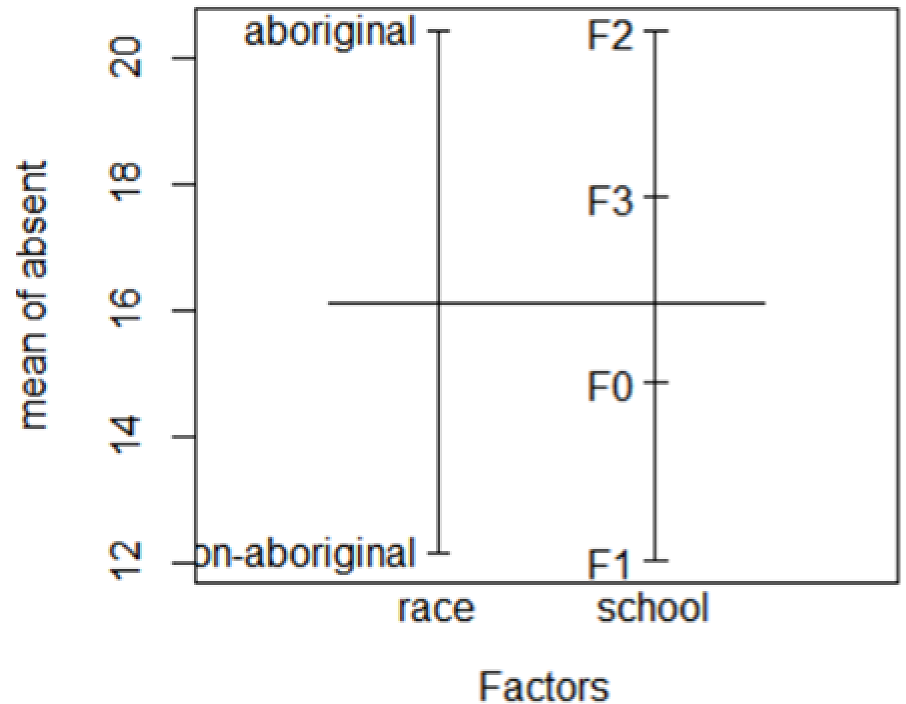
P-value for race:school is 0.001.The interaction between race and school is statistically significant and we <u>reject</u> the null hypothesis.

Summary Tables for ANOVA Model Fits

```
> print(model.tables(aov2,"means"),digits=4)
Tables of means
Grand mean

16.13636

 race
     aboriginal non-aboriginal
          20.45          12.15
rep       74.00          80.00   <= n

 school
       F0    F1    F2    F3
    14.84 12.57 20.04 17.54
rep 27.00 48.00 38.00 41.00

 race:school
               school
race            F0    F1    F2    F3
  aboriginal    13.85 16.55 31.65 17.57
  rep           13.00 20.00 20.00 21.00
  non-aboriginal 15.79  8.82  7.94 18.05
  rep           14.00 28.00 18.00 20.00
```

Univariate Effects of race and school Factors

```
plot.design(absent ~ race+school,
            data=schooldays)
```



Interaction Plot

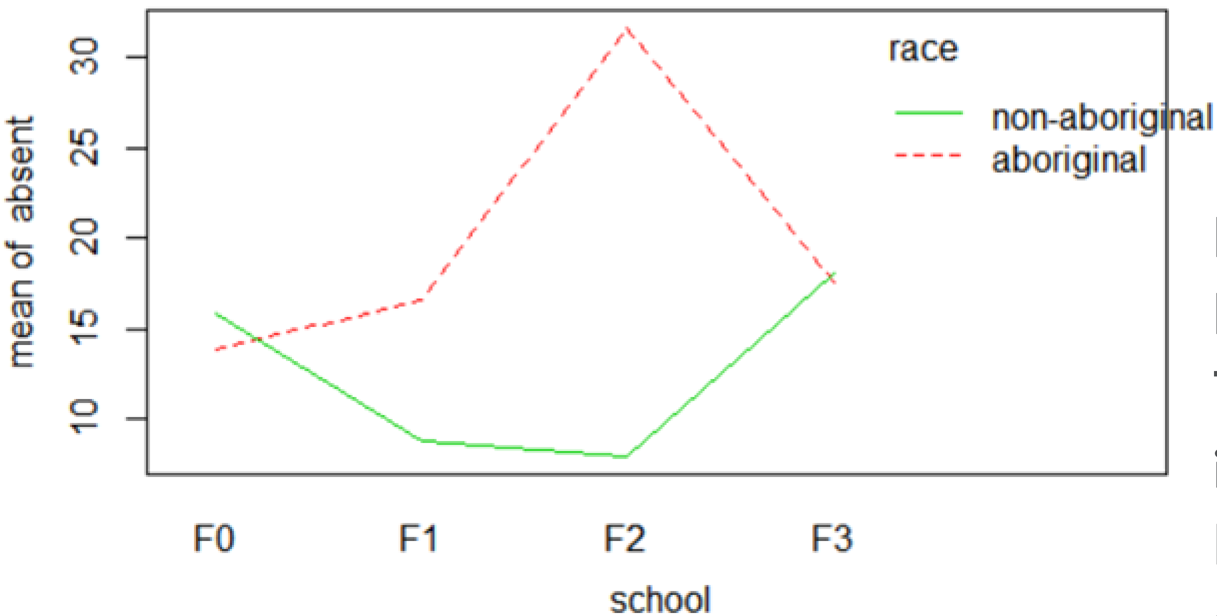
An interaction plot is used to characterize the relationship between variables.

interaction.plot(x.factor, trace.factor, response, fun=mean, ...) {stats}

Two-way Interaction Plot

Plots the mean (or other summary) of the response for two-way combinations of factors, thereby illustrating possible interactions.

```
with(schooldays, interaction.plot(x.factor=school,
     trace.factor=race, response=absent, col=2:3))
```



In this interaction plot, the lines are not parallel. This interaction effect indicates the relationship between school and race.

Conducting Two-Way ANOVA

Finding the minimal adequate model:

In the previous slide, factor “school” alone was not significant. So here we remove this factor to arrive at a minimal adequate model.

Step	Procedure	Explanation
1	Fit the maximal model	Fit all the factors, interactions and covariates of interest. Note the residual deviance. If you are using Poisson or binomial errors, check for overdispersion and rescale if necessary
2	Begin model simplification	Inspect the parameter estimates using **summary**. Remove the least significant terms first, using **update -**, starting with the highest order interactions
3	If the deletion causes an insignificant increase in deviance	Leave that term out of the model Inspect the parameter values again Remove the least significant term remaining
4	If the deletion causes a significant increase in deviance	Put the term back in the model using **update +** . These are the statistically significant terms as assessed by deletion from the maximal model
5	Keep removing terms from the model	Repeat steps 3 or 4 until the model contains nothing but significant terms This is the minimal adequate model If none of the parameters is significant, then the minimal adequate model is the null model

https://tinyurl.com/y23zm38p https://tinyurl.com/y54krxxp

3. Multivariate Analysis of Variance

The multivariate analysis of variance (MANOVA) is a type of multivariate analysis used to analyze data that involves more than one dependent variable at a time.

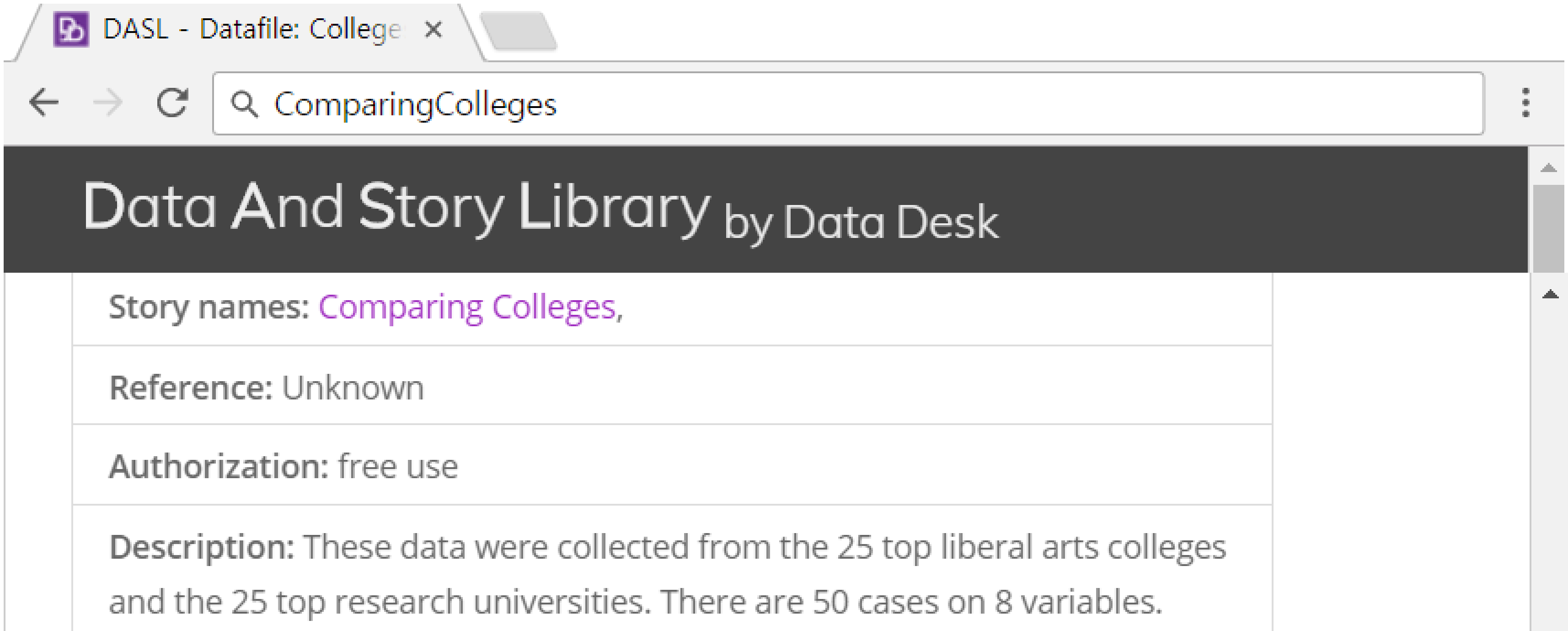
MANOVA allows us to test hypotheses regarding the effect of one or more independent variables on two or more dependent variables.

One-Way MANOVA

The one-way MANOVA is presented using the data set that contains one independent variable and two or more dependent variables.

Example : ComparingColleges

http://dasl.datadesk.com/datafiles



The ComparingColleges dataset contains information on six continuous dependent variables along with a single discrete variable School Type. Twenty-five of each type of school were surveyed.

1. School: Contains the name of each school
2. School_Type: Coded 'LibArts' for liberal arts and 'Univ' for university
3. SAT: Median combined Math and Verbal SAT score of students
4. Acceptance: % of applicants accepted
5. StudentP: Money spent per student in dollars
6. Top10P: % of students in the top 10% of their h.s. graduating class
7. PhDP: % of faculty at the institution that have PhD degrees
8. GradP: % of students at institution who eventually graduate

```
> cdt <- read.csv("ComparingColleges.csv",header=T)
> attach(cdt); dim(cdt)
[1] 50  8
> head(cdt)
       School School_Type  SAT Acceptance StudentP Top10P PhDP GradP
1     Amherst    Lib Arts 1315         22    26636     85   81    93
2 Swarthmore    Lib Arts 1310         24    27487     78   93    88
3   Williams    Lib Arts 1336         28    23772     86   90    93
4    Bowdoin    Lib Arts 1300         24    25703     78   95    90
5  Wellesley    Lib Arts 1250         49    27879     76   91    86
6     Pomona    Lib Arts 1320         33    26668     79   98    80
> table(cdt$School_Type)

Lib Arts     Univ 
      25       25 
```

▪ Conducting one-Way MANOVA

manova(formula, data, ...) {stats}

A class for the multivariate analysis of variance.

We will use MANOVA to determine if the **school types** differ across all six dependent variables (SAT, Acceptance, StudentP, PhdP, GradP, Top10P) simultaneously.

```
> Y <- cbind(SAT,Acceptance,StudentP,PhDP,GradP,Top10P)
> table(School_Type)
School_Type
Lib Arts      Univ
      25        25
> fit <- manova(Y ~ School_Type)
> summary(fit)
            Df  Pillai approx F num Df den Df    Pr(>F)
School_Type  1 0.54081   8.4405      6     43 4.507e-06 ***
Residuals   48
```

We reject the null hypothesis at α=0.05.

MANOVA of the data with School_Type as the factor reveals that there is a significant difference between research universities and liberal arts colleges at the 5% level.

The **summary.aov()** function will yield the ANOVA univariate statistics for each of the dependent variables.

```
> summary.aov(fit)
 Response SAT :
            Df Sum Sq Mean Sq F value Pr(>F)
School_Type  1   2679  2679.1  0.6852 0.4119
Residuals   48 187685  3910.1

 Response Acceptance :
            Df Sum Sq Mean Sq F value Pr(>F)
School_Type  1  369.9  369.92  2.1187  0.152
Residuals   48 8380.8  174.60

 Response StudentP :
            Df     Sum Sq    Mean Sq F value    Pr(>F)
School_Type  1 3605397494 3605397494  22.146 2.177e-05
Residuals   48 7814347896  162798914

 Response PhDP :
            Df  Sum Sq Mean Sq F value  Pr(>F)
School_Type  1  269.12 269.120  4.2034 0.04583 *
Residuals   48 3073.20  64.025

 Response GradP :
            Df  Sum Sq Mean Sq F value Pr(>F)
School_Type  1   20.48  20.480  0.3539 0.5547
Residuals   48 2778.00  57.875

 Response Top10P :
            Df Sum Sq Mean Sq F value    Pr(>F)
School_Type  1 2592.0 2592.00  19.567 5.559e-05
Residuals   48 6358.3  132.47
```

The differences lie in the 3 dependent variables.

- Which school type has the higher mean for the three significant variables?

```
> tapply(StudentP,School_Type,mean)
Lib Arts     Univ
21755.56 38738.84
> tapply(PhDP,School_Type,mean)
Lib Arts     Univ
   88.24    92.88
> tapply(Top10P,School_Type,mean)
Lib Arts     Univ
   67.24    81.64
```

```
par(mai=c(0.8,0.8,0.2,0.2),mfrow=c(1,3))
plot.design(StudentP~School_Type)
plot.design(PhDP ~ School_Type)
plot.design(Top10P ~ School_Type)
```

The liberal arts colleges have more students per $, which means that the universities spend more per student.

Universities have more PhD's and Top 10% students than liberal arts colleges.