HW1 20120888 조민기

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a) $p \rightarrow (q \rightarrow r) = \neg p \wedge \neg q \wedge r$

p	q	r	¬p	¬q	$p \rightarrow (q \rightarrow r)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

b) $(p \rightarrow q) \rightarrow r = \neg(\neg p \wedge q) \wedge r = (p \vee \neg q) \wedge r$

p	q	r	¬q	$(p \rightarrow q) \rightarrow r$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	T	T
F	F	F	T	F

c) $(\mathrm{p} \wedge \mathrm{q}) \rightarrow r = (\neg p \vee \neg q) \wedge r$

p	q	r	$\neg\mathrm{p}$	$\neg\mathrm{q}$	$(\mathrm{p} \rightarrow \mathrm{q}) \rightarrow \mathrm{r}$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

d) $\forall\mathrm{x}\left(\mathrm{x} \rightarrow \left(\mathrm{p} \rightarrow (\mathrm{q} \rightarrow \mathrm{r})\right)\right) = \forall x(\neg x \wedge \neg p \wedge \neg q \wedge r)$

p	q	r	$\neg\mathrm{p}$	$\neg\mathrm{q}$	$\forall\mathrm{x}\left(\mathrm{x} \rightarrow \left(\mathrm{p} \rightarrow (\mathrm{q} \rightarrow \mathrm{r})\right)\right)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

e) $\left(\exists x(x)\right) \rightarrow \left(p \rightarrow (q \rightarrow r)\right) = True \rightarrow \left(p \rightarrow (q \rightarrow r)\right) = \left(p \rightarrow (q \rightarrow r)\right) \quad \because \left(\exists x(x)\right) = Ture$

p	q	r	$\neg p$	$\neg q$	$p \rightarrow (q \rightarrow r)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

f) $p \rightarrow \left(q \rightarrow \left(r \rightarrow \forall x(x)\right)\right) = p \rightarrow \left(q \rightarrow (r \rightarrow \text{False})\right) = \neg p \wedge \neg q \wedge (\neg r \wedge False)$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$p \rightarrow \left(q \rightarrow \left(r \rightarrow \forall x(x)\right)\right)$
T	T	T	F	F	F	F
T	T	F	F	F	T	T
T	F	T	F	T	F	T
T	F	F	F	T	T	T
F	T	T	T	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	T	T	T	T

g) $p \to \big(q \to \forall x(r \to x)\big) = \neg p \land \neg q \land \forall x(r \to x)$

p	q	r	$\neg p$	$\neg q$	$\forall x(r \to x)$	$p \to \Big(q \to \big(r \to \forall x(x)\big)\Big)$
T	T	T	F	F	F	F
T	T	F	F	F	T	T
T	F	T	F	T	F	T
T	F	F	F	T	T	T
F	T	T	T	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	T
F	F	F	T	T	T	T

h) $p \to \forall x\big(q \to (r \to x)\big) = \neg p \land \forall x\big(q \to (r \to x)\big)$

p	q	r	$\neg p$	$\forall x\big(q \to (r \to x)\big)$	$p \to \forall x\big(q \to (r \to x)\big)$
T	T	T	F	F	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

i)

a) When S_{13} is a true, $S_1 \sim S_{12}$ need to be a true but that's not possible. And for n>1, when the $S_{13} \sim S_{n+1}$is false, S_n cannot be a true because all of $S_1 \sim S_n$ cannot be a true. Only S_1 can be a true. Or, every statements are false.

b) S_{13} is true because true statement can't be more than 13 which is total number of all statements. Since S_{13} is true, S_1 cannot be true because there is already one true statement. And $S_{12}$is true because there are only 12 statements except S_1 which cannot be a true. We can keep doing this until S_7, so $S_{13} \sim S_7$ is true and other statements are false.

c) When n is the biggest number that S_n is true, even if the every sentences beneath $S_n$are true, S_n cannot be true. Which means all sentences are false.

d) Two cases are available. Only S_1can be true and other statements are false, or every statements are false.

$$\neg S_2 \vee \neg S_3$$

e) Over half of statements from the top is true and rest are false.

$$\neg S_1 \vee S_2 \vee S_3$$

f) Every statements are false.

$$\neg S_1 \vee \neg S_2 \vee \neg S_3$$