Computational Marine Ecological Modelling report: NPD model

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Contents

1. Description of the governing equations and the different terms in the equations, together with an illustration of state variables.

The system includes phytoplankton concentration, nutrient concentration and detritus concentration (figure 1). The system contains an upper mixed layer (UML).

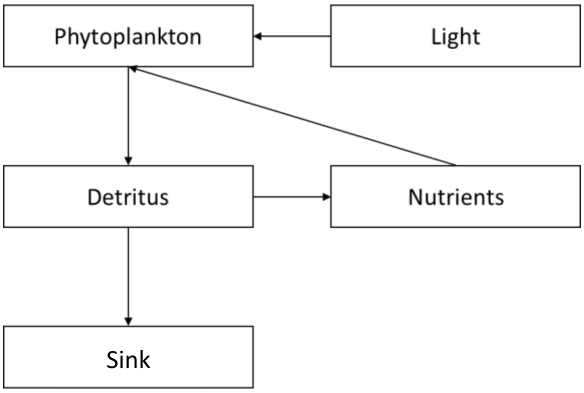

Figure 1. Graphical illustration of the NPD model

A) Phytoplankton concentration:

$$\frac{\Theta P}{\Theta t}(z,t) = \mu * min(\sigma_L, \sigma_N) * P - l * P - \gamma * P^2 + \frac{\theta}{\theta z} * D * \frac{\theta P}{\theta z} \qquad \textit{eq.1}$$

According to equation 1, the phytoplankton concentration (P) depends on the limiting factor (light or nutrients), the natural mortality (l), the grazing (γ), and the diffusivity.

B) Nutrient concentration:

$$\frac{\Theta N}{\Theta t}(z,t) = -\mu * min(\sigma_L, \sigma_N) * P + \tau * D + \frac{\theta}{\theta z} * D * \frac{\theta N}{\theta z} \qquad \text{eq.2}$$

According to equation 2, the nutrient concentration depends on the produced biomass, the remineralization rate (τ) of detritus (D) and the diffusivity.

C) Detritus concentration:

$$\frac{\Theta D}{\Theta t}(z,t) = l * P + \gamma * P^2 - \tau * D + \frac{\theta}{\theta z} * D * \frac{\theta D}{\theta z} - u\frac{\theta D}{\theta z} \qquad \text{eq.3}$$

According to equation 3, the detritus concentration depends on the natural mortality of phytoplankton, the grazing, the remineralization of detritus, the diffusivity and the advection of detritus particles (u).

In equations 1 & 2 the terms σ_L and σ_N refer to von Liebig's law of minimum. More specifically the values for each parameter are calculated as in equations 4 and 5, respectively.

$$\sigma_L = \frac{I}{H_I + I} \qquad \text{eq. 4}$$

$$\sigma_N = \frac{N}{H_N + N} \qquad \text{eq. 5}$$

Where, H_N and H_I, the half saturation constants for nutrients and light, respectively.

D) The light function:

$$I(z) = \left(I_{in} * e^{-K_{bg}*z-k*\int_0^z \sum_1^n *P_i(\xi,t)d\xi}\right) * 0.5 * \left(1 - \cos\left(\frac{2*pi*t}{365}\right)\right) \qquad \text{eq. 6}$$

According to equation 6, the change of light intensity depends on the background turbidity (Kbg), the phytoplankton light absorption (k) and has a seasonal variation (0-1). The starting point is considered the day with the longest night (21st of December). Another term could be also added to imply the minimum light, but 0 will work fine.

2. Description of the parameters and their units.

Table 1. Code names, values and units of the parameters used to run the models.

Parameter	Name in Code	Value	Units
Initial Phytoplankton concentration	P0	0.001	mmolN*m^{-3}
Initial light intensity	I0	300	µmol photons*m-^{2}*s^{-1}
Half saturation constant for light	H	30	µmol photons*m-^{2}*s^{-1}
Background turbidity	Kbg	0.0375	m^{-1}
Specific attenuation of phytoplankton	k	0.05	m^2 mmolN^{-1}
Phytoplankton max production rate	Pmax	0.5	d^{-1}
Vertical velocity	u	12	m*d^{-1}
Depth of water column	L	200	m
Grid size	Steps	200	unitless
Size of each grid	Delta	1	m
Time	Time	6000	Days
Detritus initial concentration	Den	0	mmolN*m^{-3}
Depth vector	cells	See code	m
Initial concentration of nutrients	N0	5	mmolN*m^{-3}
Nutrients at bottom	N0	30	mmolN*m^{-3}
Remineralization of detritus constant	Eps	0.5	d^{-1}
Half saturation of nutrients	Hn	0.3	mmolN*m^{-3}
Grazing	Gamma	1.5	m^3* mmolN^{-1}*d^{-1}
Upper mixed layer diffusivity constant	Du	43	m^2*d^{-1}
Bottom diffusivity constant	Dd	5	m^2*d^{-1}
Width of the transient layer	w	1	m
Depth of mixed layer	Zmix	50	m

3. Description of the numerical scheme, including boundary conditions.

The diffusion inside the water column is calculated as such:

$$D(z) = D_D + \frac{D_u - D_D}{1+e^{(z-Zmix)/w}} \qquad \text{eq. 7}$$

Equation 7 describes the calculated diffusivity coefficient. D(z): diffusion constant in depth z, D_D: diffusion constant at the bottom of the water column, D_u: diffusion constant for the UML, Z_{mix}: Depth of the UML (multiplied by the same seasonal factor as light), and w: length of the mixing layer. This equation adds a significant amount of time to the total run time of the model. In case dz is not equal to 1, the value for Zmix=Zmix/delta and w=w/delta.

There is no flux through the surface of the sea for all variables, i.e., $J_{DP}{}^0$=0, $J_{DN}{}^0$=0, $J_{DD}{}^0$=0, $J_{AD}{}^0$=0. At the seabed, the diffusivity for phytoplankton is 0, the nutrients have a constant value ($dNdt_{bottom}$=0) and the diffusive flux for Detritus is 0, while there is advective flux (sink) equal to the settling velocity multiplied by the concentration of the detritus in last cell.

When calling the ode45 function, firstly, the program calculates the depth of mixed layer based on the formula Zmix= Zmix*0.5(1-cos((2*pi*t)/365)) +5. Then it calculates the advection and diffusion values for all variables. Afterwards, it calls the light function (equation 6) and calculates the growth rate as the minimum value of the equations 4 and 5. Consequently, it calculates the values dPdt, dNdt and dDendt (for phytoplankton (eq. 1), nutrients (eq. 2) (there is an extra line (dNdt(end)=0) that keeps the concentration steady, and detritus (eq. 3).

4. Show that the solution is converged and independent of the grid size.

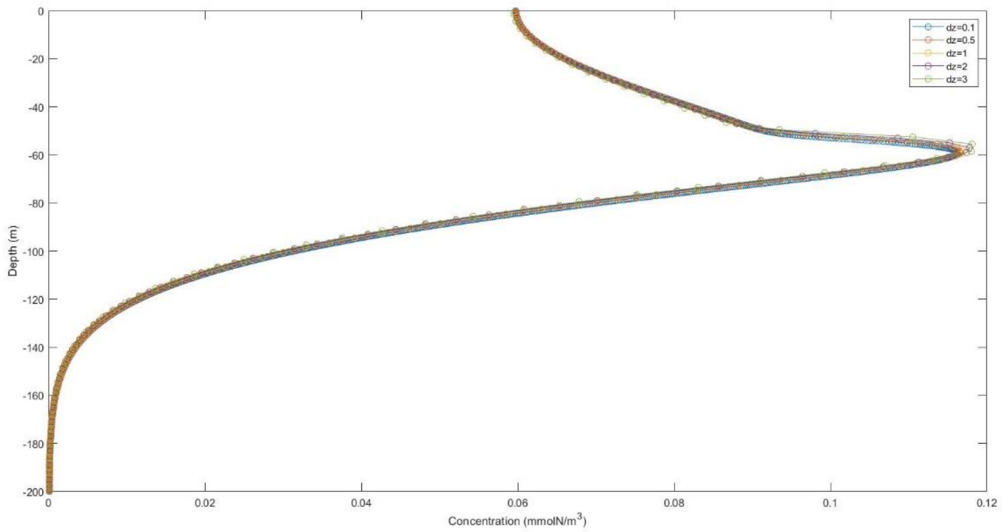

Figure 2. Concentration of phytoplankton on day 350 without seasonal variation for five different grid size values (0.1, 0.5, 1, 2, 3). Figure generated with the gridSensitivity.m file.

As it is shown in figure 2, the results are independent of the grid size. Five values were used for grid size: 0.1, 0.5, 1, 2 and 3. The processing time increases a lot as the grid size decreases. I suggest running the file with values >0.5 (the value 0.1 added around 20-30 minutes on the total running time, while the rest of the running takes less than a minute). There is a UML present. The depth of the UML is 50m. The upper layer diffusion constant is 60 m^2*d^{-1} and the bottom diffusion constant is 5 m^2*d^{-1}.

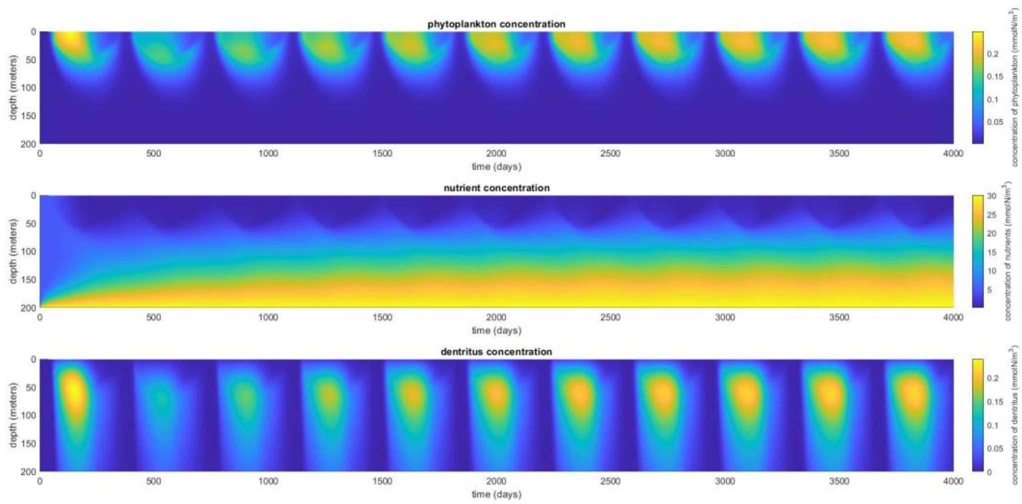

Figure 3. The converged model. The values for this model are described in table 1. Time=4000 days.

Figure 3 illustrates the steady state of the system when it is running for 4000 days with seasonal variation. The values are the same as in table 1. The max mixed layer depth is 55m.

5. Description (with suitable figures) of the steady-state solutions of the model.

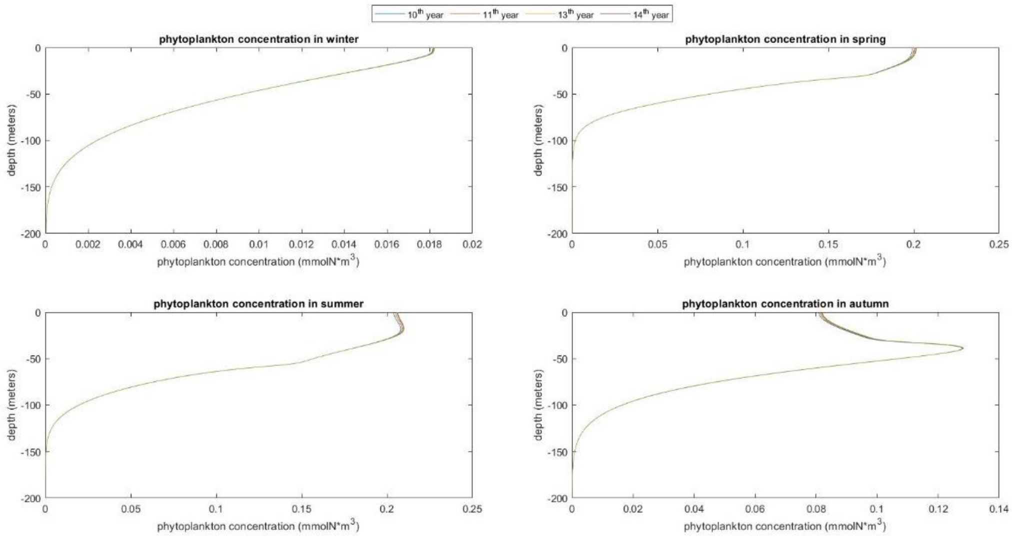

Figure 4. Concentrations of phytoplankton in winter, spring, summer and autumn, in years 10-14.

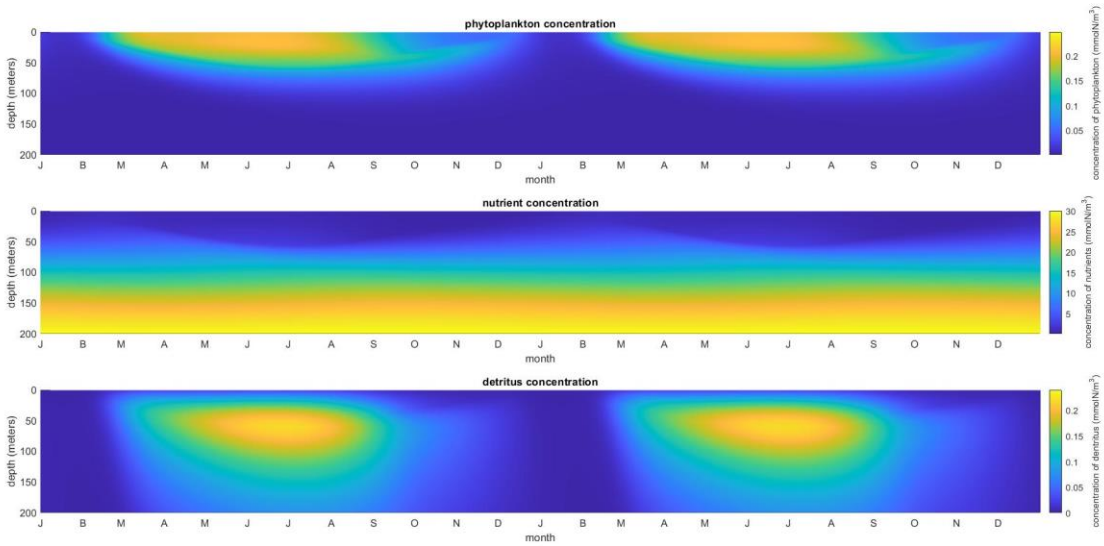

Figure 5. The phytoplankton, nutrients, and detritus profiles during the years 11 and 12.

It the beginning of the year the concentration of phytoplankton is very low, and it is increasing until June, when the mixed layer and the light intensity are at their maximum. In August, the nutrients in the UML are depleted, therefore a DCM (deep chlorophyl maximum) is formed.

6. Discussion of the factors limiting phytoplankton growth.

The factors that are limiting the phytoplankton growth are: nutrients, light and population density of the phytoplankton. In figure 6 we can see how the light and nutrients are changing seasonally, limiting the growth of phytoplankton. In the beginning of the year (day 0) there is no light, therefore light is the limiting factor. During the next 67 days an increase in light and nutrients occurs, where light remains the limiting factor. From day 67 until day 344 nutrients are the limiting factor. Then, light becomes again the limiting factor and so on.

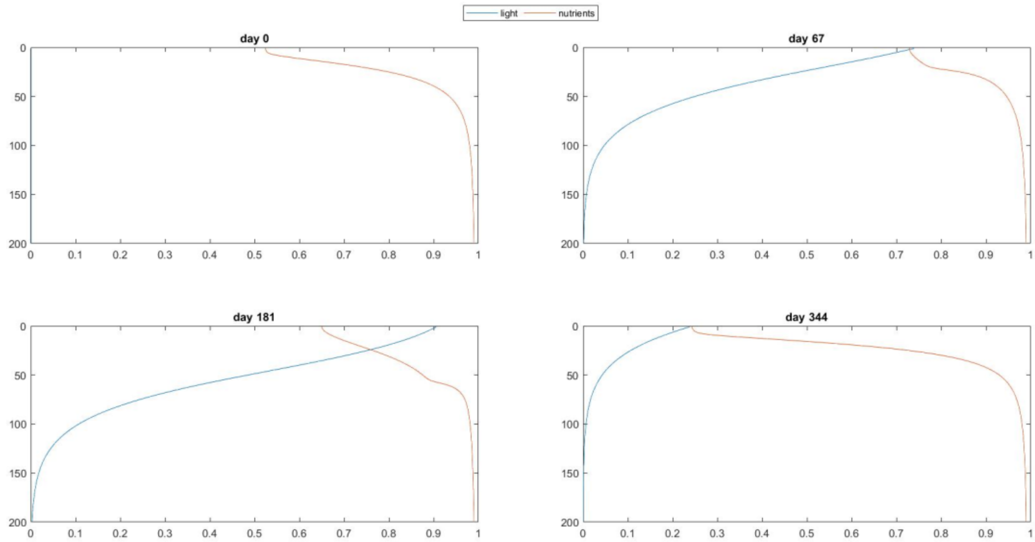

Figure 6. Light and nutrients as growth limiting factors during year 11.

7. A sensitivity analysis of selected parameters entering the model, with interpretation of why a change in the parameters leads to the observed effects.

For the sensitivity analysis four variables were chosen.

A) Settling Velocity of Detritus (SVD analysis)
B) Maximal Growth Rate of phytoplankton (MGR analysis)
C) Depth of Mixed Layer (DML analysis)
D) Background Turbidity (BT analysis)

Results:

A) SVD analysis

Five different settling velocities were tested: 8, 10, 12, 14 and 16m/day. The faster the detritus is settling the less the maximum concentration of phytoplankton. Additionally, the chlorophyll maximum is located deeper (43, 48, 51, 53 and 54m respectively). Afterwards, velocities 18, 20, 22, 24 and 28m/day were also tested. 20-22m/day seems to be the threshold. For higher velocities, detritus cannot provide any nutrients for cell growth.

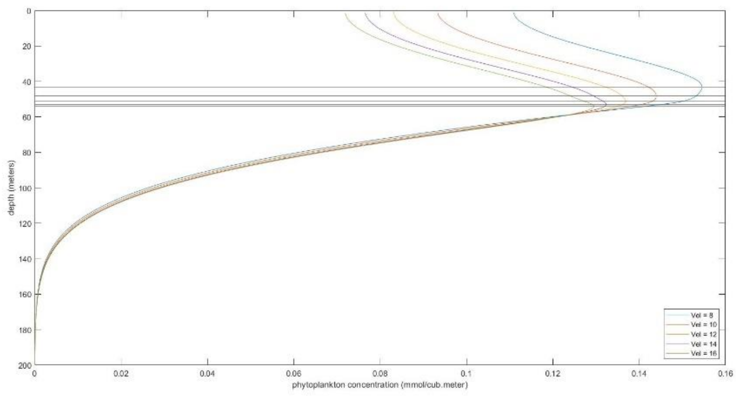

Figure 7. SVD sensitivity analysis for five different settling velocities: 8, 10, 12, 14 and 16m.

B) MGR analysis

Five different values of μmax were tested: 0.4, 0.45, 0.5, 0.55 and 0.6 d^{-1}. The MGR analysis showed a positive correlation between μmax and maximal phytoplankton concentration (figure 8). Furthermore, the higher the reproduction rate the deeper the maximum chlorophyll is located. The increased need for nutrients is resulting to a faster depletion of the nutrients in the UML zone, thus a DCM is formed.

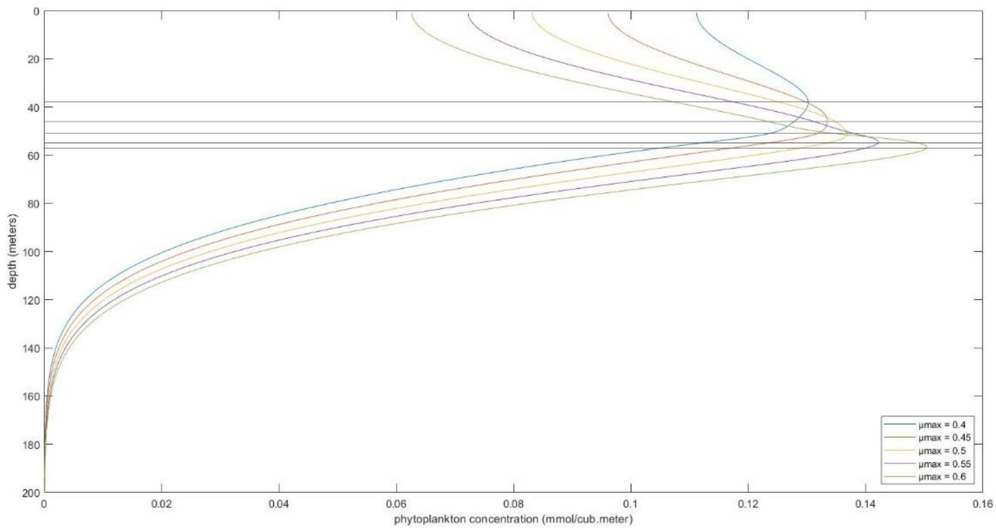

Figure 8. MGR sensitivity analysis results for five different max growth constants: 0.4, 0.45, 0.5, 0.55 and 0.6 d^{-1}.

C) DML analysis

Sensitivity analysis was performed for five different UML depths: 40m, 45m, 50m, 55m and 60m (figure 9). The first three values had a maximum concentration of phytoplankton under the UML (DCM) (49, 51 and 51m respectively). After the threshold of around 50-55m, the UML receives enough nutrients and an upper chlorophyl maximum (UCM) is formed. More specifically, when the UML depth was 55 and 60m the depth of maximum chlorophyl was 49 and 46m, respectively.

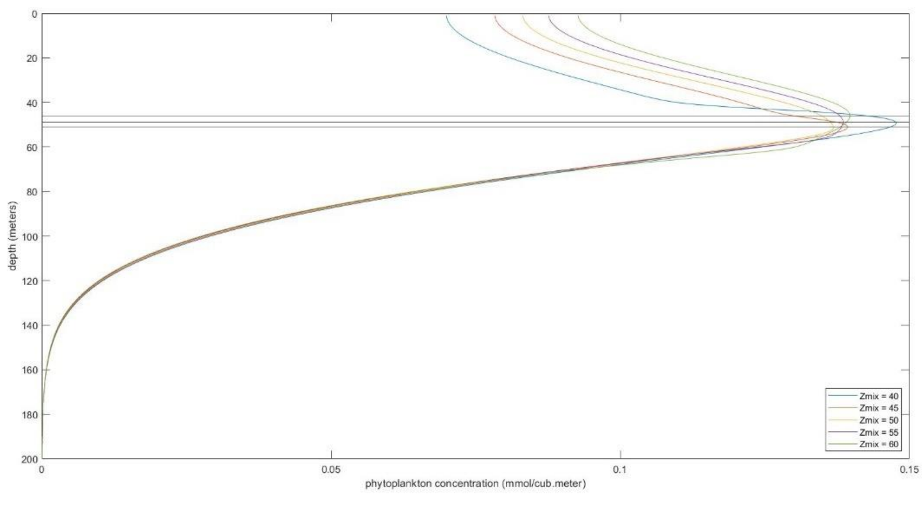

Figure 9. Depth of maximum phytoplankton concentration for five different depths of UML.

D) BT analysis

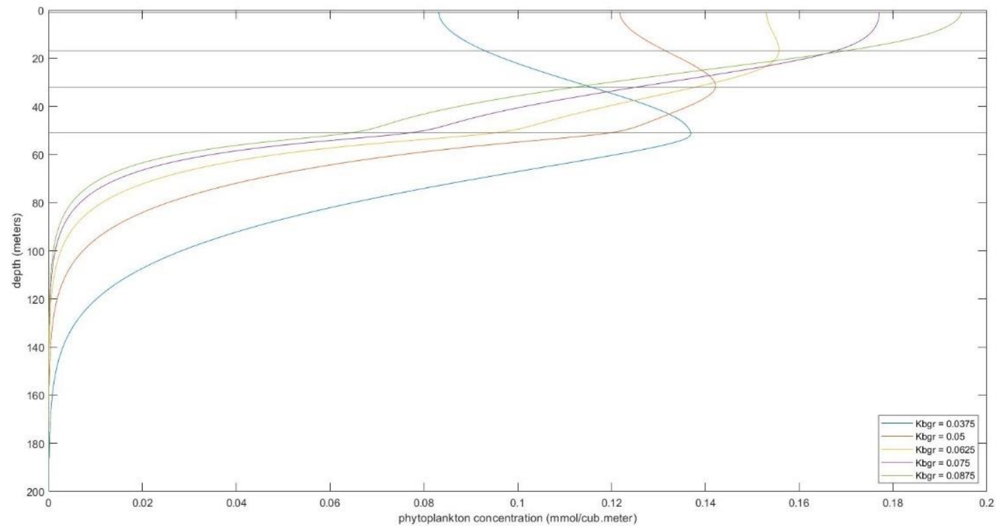

Figure 10. Sensitivity analysis for background turbidity.

Sensitivity analysis for background turbidity was executed for the following values: 0.0375, 0.05, 0.0625, 0.0750 and 0.0875 m^{-1} (figure 10). The results showed that with increased background turbidity comes shallower chlorophyll maximum. This outcome stems from the limitation of light that penetrates the water and the cells can be multiplied only near the surface. There is a higher phytoplankton concentration as well. This indicates that more nutrients can reach the upper layer, therefore the phytoplankton cells have more building blocks available there.

8. A description of the seasonal succession of plankton.

The seasonal succession of plankton depends on the environmental parameters. The absence of light, during winter results in a sharp decrease for a short period of time. During this 'dark' period, nutrients can reach the surface layers. When light is abundant again, planktonic organisms can use these nutrients and expand their populations. The increasing of the UML also brings more nutrients in UML; therefore, the population can maintain its size. After 0.5 year, UML starts shrinking and nutrients cannot reach the UML. This results in the creation of a DCM, which follows the maximum depth of the UML until the light becomes insufficient for reproduction. Then, the population sharply decreases its size, and the annual circle begins again.

[The code is available on GitHub]