7MA102

09 01 2024

Curve Tracing

* Curve tracing - It is an anlytical method of drawing an approximate shape of a curve by the study of some of its important characterisitics. such as symmetry, intercepts, asymptotes, tangent, region of existent etc

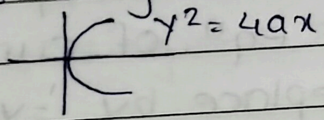
asymptotes - tangent at infinity

We study tracing of standard and other curve in
① cartesian form ② Polar form

* Tracing of curve in cartesian form

1 Symmetry

ⓐ The curve is symmetrical about x-axis is the eq^n of curve remains unchange when 'y' is replaced by '-y' i.e, if the eq^n contains only even powers of y.

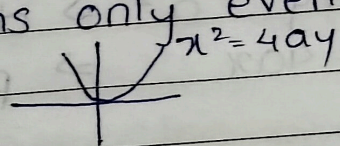


e.g. $y^2 = 4ax$

$(-y)^2 = 4ax$

$y^2 = 4ax$ symmetric about x axis.

ⓑ The curve is symmetrical about y-axis is the eq^n of curve remains unchanged when 'x' is replace by '-x' i.e, if the eq^n contains only even powers of x.



e.g. $x^2 = 4ay$

$(-x)^2 = 4ay$

$x^2 = 4ay$ symmetric about y axis.

The curve is symmetrical about both the axis if the powers of both x and y are eve

e.g. $x^2+y^2=r$ eq^n of circle

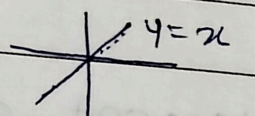
ⓓ The curve is symmetrical about the origin symmetric in opposite quadrant if the eq^n o curve remains unchange where 'x' is replace by 'y' is replace by '$-y$'

e.g.

$y=x$

$-y=-x$

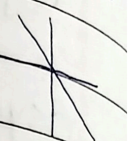
$y=x$



$y=-x$

$-y=x$

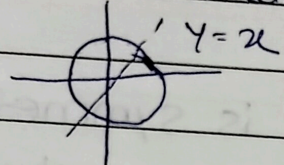
$y=-x$



ⓔ The curve is symmetrical about the line $y=x$ if the eq^n of curve remains unchanged when both x & y are interchanged.

e.g. $x^2+y^2=1$

$y^2+x^2=1$



ⓕ The curve is symmetrical about the line $y=-$ if the eq^n of curve remains unchanged whe 'x' is replace by '$-y$' & 'y' is replace by '$-x$'.

2 Origin

If the given curve passes through origin (0,0) then there will be no constant term in the eq^n

e.g. $x^2+y^2=a^2$ ⇒ does not pass through origin

$y^2=4ax$ ⇒ passes through origin

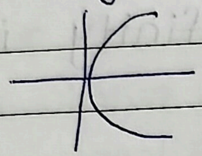
if the point (0,0) satisfies the eq^n of curve then the curve passes through origin otherwise not.

3 if origin lies on the curve then find the eq^n of tangent at origin

3 Tangent at origin

Tangents at origin are obtained by equating the lowest degree term of the eq^n to zero

e.g. $y^2 = 4ax$



$4ax$ is lowest degree term

$4ax = 0$

$x = 0$ i.e. tangent is y axis

4 Intersection with co-oridinate axis (or) intercepts

ⓐ x - intercept

the point where curve meet the x axis it is obtain by putting $y = 0$ in the given eq^n of curve & solving for x

ⓑ y - intercept

the point where curve meet the y axis it is obtain by putting $x = 0$ in the given eq^n of curve & solving for y

e.g. $x^2 + y^2 = a^2$

x - intercept $\Rightarrow x^2 = a^2 \Rightarrow x = \pm a$

y - intercept $\Rightarrow y^2 = a^2 \Rightarrow y = \pm a$

5 Point of intersection

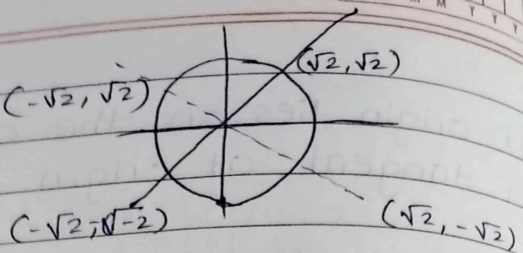
When the curve is symmetric about the line $y = x$ (or) $y = -x$ then the point of intersection are obtained by putting $y = x$ (or) $y = -x$ respectively. in the given eq^n of curve e.g. $x^2+y^2=4$

$2x^2=4$

$x^2=2$

$x=\sqrt{2}$

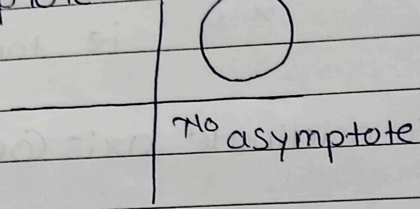
$y=\sqrt{2}$

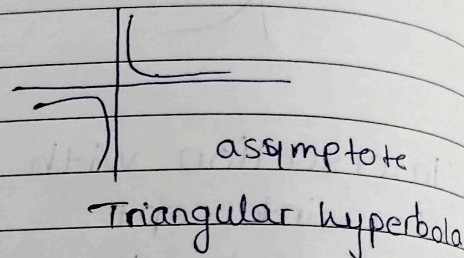


6 asymptote

tangent to the curve at infinity is called asymptote

e.g





There are two types of asymptote

① Parallel asymptote

② Oblique (or) inclined asymptote

ⓐ asymptote parallel to x axis

it is obtained by equating the coefficient of higher power of x to zero

ⓑ asymptote parallel to y axis

it is obtained by equating the coefficient of higher power of y to zero.

e.g $xy=k$ — triangular hyperbola

$\parallel^{el}$ to x axis

$y=0$

x axis itself is asymptote

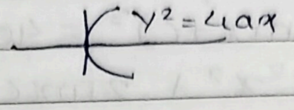
$\parallel^{el}$ to y axis

$x=0$

y axis itself is asymptote

In above two cases if the coefficient is const then there is no asymptote ||el to x axis & y axis.

eg. $y^2 = 4ax$



||el to axis => $4a = 0$ => not possible => No asymptote

||el to axis => $1 = 0$ => not possible => No asymptote

© oblique (or) inclined asymptote

It is obtain by using $y = mx + c$ in the given eqⁿ of curve simplify the eqⁿ and equate the coefficient of two highest power of x to zero.

Solving this we get values of m & c. if degree of curve is n then there are an oblique asymptote exist.

eg ① hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$b^2x^2 - a^2y^2 = a^2b^2$$

put $y = mx + c$

$$b^2x^2 - a^2(mx+c)^2 = a^2b^2$$

$$b^2x^2 - a^2(m^2x^2 + c^2 + 2mcx) - a^2b^2 = 0$$

$$(b^2 - a^2m^2)x^2 - a^2c^2 - 2mca^2x - a^2b^2 = 0$$

$$(b^2 - a^2m^2)x^2 - (2mca^2)x - (a^2c^2 + a^2 + b^2) = 0$$

$$b^2 - a^2m^2 = 0 \qquad\qquad -2mca^2 = 0$$

$$m^2 = b^2/a^2 \qquad\qquad c = 0$$

$$m = \pm b/a$$

$$y = \pm \frac{b}{a} x$$

② $y^2 = 4ax$

Consider $4a \neq 0$

but it is not possible

No asymptote parallel to x axis

similarly, $1 \neq 0$

No asymptote parallel to y axis

$\therefore y^2 = 4ax$

put $y = mx + c$

$(mx+c)^2 = 4ax$

$m^2x^2 + 2mxc + c^2 = 4ax$

$m^2 = 0 \quad , (2mc - 4a) = 0$

$\therefore c = 0$

7 * Region of existance 4
find out the region of the plane where no part
of the curve lies. prepare the table for certain
value of x and draw the curve.

8 * Tangent at the point of intersection
Find out dy/dx and check and value of dy/dx
at the point of intersection.

if $\left(\frac{dy}{dx}\right)_{(x_0, y_0)} = 0$ then tangent parallel to x-axis at (x_0, y_0)

if $\left(\frac{dy}{dx}\right)_{(x_0, y_0)} = N.D$ then tangent parallel to y-axis at (x_0, y_0)

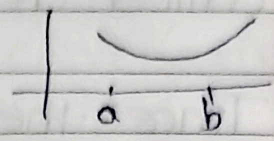
* Sign of first order Derivative

① In an closed interval (a,b) if $dy/dx > 0$ then the curve is increasing in $[a,b]$

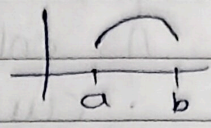
② If $dy/dx < 0$ then curve is decreasing in $[a,b]$

③ if at $x = x_0$ $dy/dx = 0$ then x_0, y_0 is a stationary point where maxima or minima can occur.

* Sign of 2nd order Derivative
In an close interval (a,b) if ① $\frac{d^2y}{dx^2} > 0$ then the curve is upwardly open

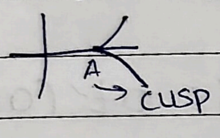


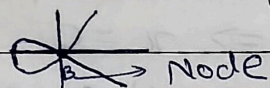
② if $\frac{d^2y}{dx^2} < 0$ then the curve is downwardly open



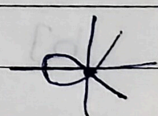
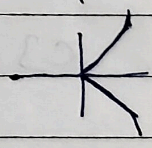
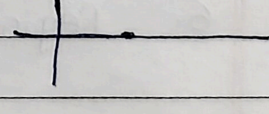
* Multiple point
A point through which 'r' branches of curve pass is called as multiple point of r^{th} order and has r tangent

r=2 ① Double point - A point through which 2 branches of curve pass is called Double point. At double point there are 2 tangent to the curve either distinct (or) coincident (or) imaginary.





Two tangent <
- realdistinct - Node
- coincident - cusp
- imaginary - conjucate (or) isolated point

	Tangent	value of $\frac{dy}{dx}$	
Node	Real & distinct	Real & distinct	
Cusp	Real & coincident	equal	
conjucate	imaginary	imaginary	

inflection - tangent curve ko intersect karta hai!

* Point of inflection
A point where the curve process the tangent is called as point of inflection

Q.1 Trace the curve $xy^2 = a^2(a-x)$

1. Symmetry :
As only even powers of y present
$\therefore$ curve is symmetry about x-axis

2. Origin : As constant term is present in the eq^n => curve doesn't pass through origin

3. Tangent at origin doesn't exist

4. Intersection with the co-ordinate Axis:
put $x=0$ => Not possible
put $y=0$ => $x=a$
$\therefore$ curve meet the x-axis at $(a,0)$

5. Asymptote :
a] Parallel to x axis
$$y^2 + a^2 = 0$$
Not Possible
b] parallel to y axis
$x = 0$ i.e. y-axis
c] No oblique asymptote exist

6. Region of existance
$$y^2 = \frac{a^2(a-x)}{x}$$
$$y = \pm a\sqrt{\frac{(a-x)}{x}}$$

15 01 2024

ⓐ For $x<0$

y is imaginary

curve does not exist in 2^{nd} & 3^{rd} quadrant

ⓑ $0 < x \le a \Rightarrow$ let $x = \frac{a}{2} \Rightarrow y = \pm a \Rightarrow$ Curve exist

ⓒ $x > a \Rightarrow$ let $x = 2a \Rightarrow y$ is imaginary

7. $a=1$ consider $\quad y = \pm\sqrt{\frac{1-x}{x}}$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
y	∞	± 3	± 1	± 1.53	± 1.22	± 1	± 0.81	± 0.65	± 0.5	± 0.33	0

8. tangent at the point of intersection $(a,0)$

$$xy^2 = a^3 - a^2x$$

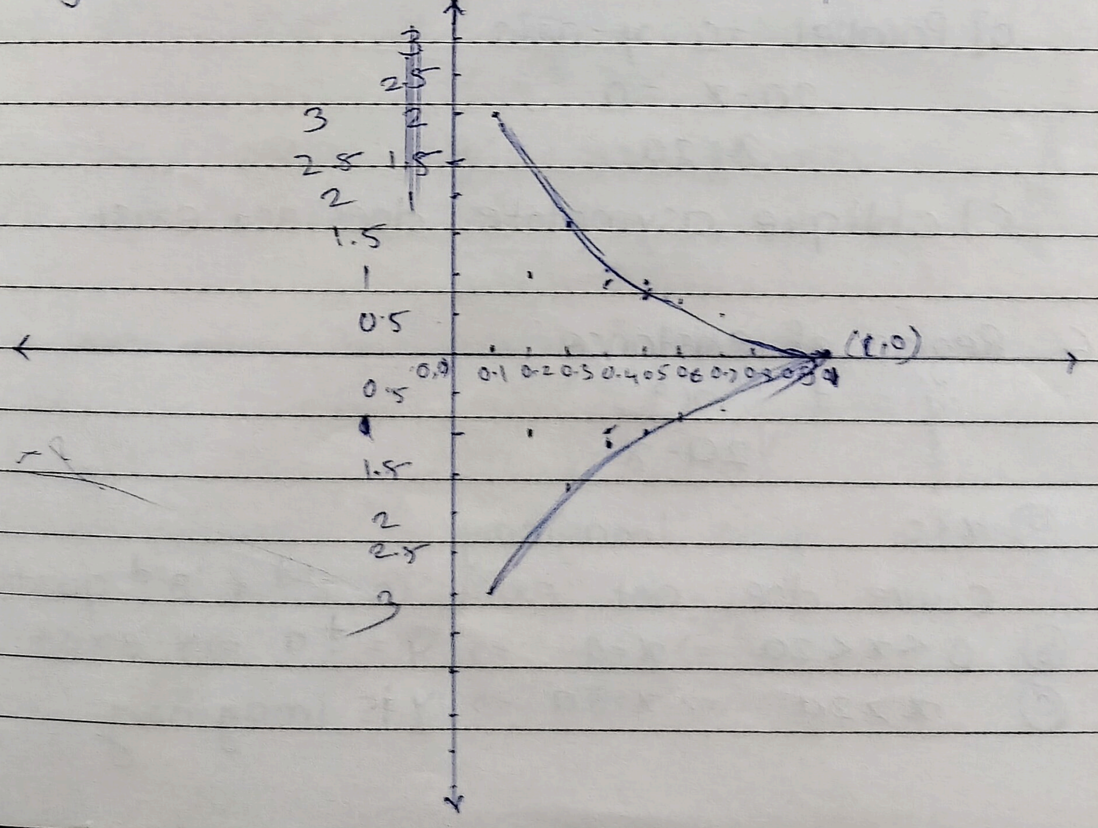
w.r.t x

$$2yx\frac{dy}{dx} + y^2 = -a^2$$

$$\frac{dy}{dx} = \frac{-(a^2+y^2)}{2xy}$$

$$\left.\frac{dy}{dx}\right|_{(a,0)} = \text{N.D } (\infty)$$

tangent at $(a,0)$ is parallel to y-axis



Q Trace the curve $y^2(2a-x) = x^3$

1. Symmetry:
   Curve symmetry about x axis

2. Origin: doesn't pass through origin

3. tangent at origin: $2ay^2 = 0$
   $y^2 = 0$ ie, $y=0$ is double tangent
   $\therefore$ x axis is a double tangent

4. Intersection with co-ordinate axis
   put $x=0$ $\quad 2ay^2 = 0 \quad y=0$
   put $y=0$ $\quad x^3 = 0 \quad x=0$
   curve meets the axes origin only.

5. Asymptote
   a] Parallel to x-axis
   $1 \neq 0$ Not possible
   Asymptote parallel to x-axis does not exist

   b] Parallel to y-axis
   $2a - x = 0$
   $x = 2a$

   c] oblique asymptote does not exist

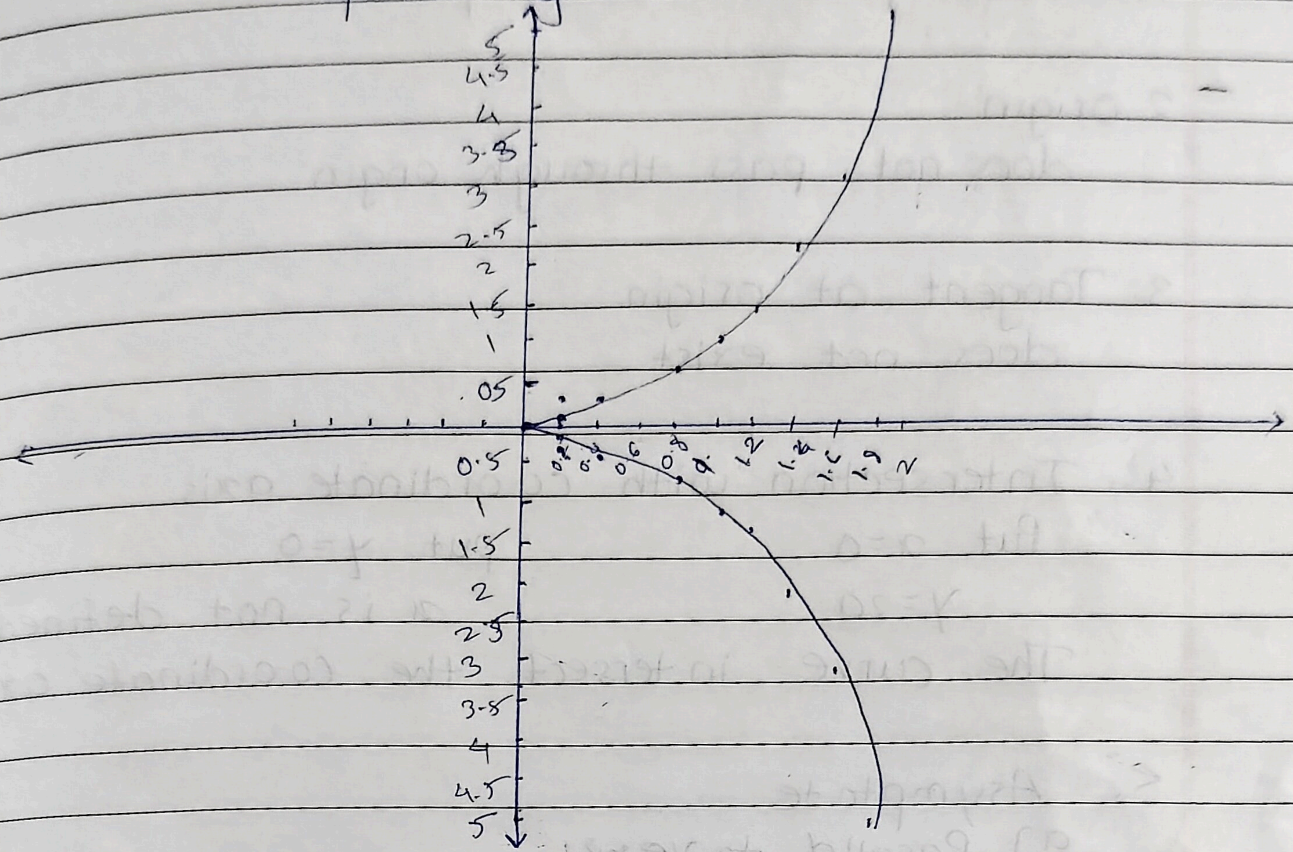
6. Region of existance
   $$y = \pm \sqrt{\frac{x^3}{2a-x}}$$

   (a) $x < 0$ $\quad y$ is imaginary
   Curve does not exist in 2^{nd} & 3^{rd} quadrant
   (b) $0 < x < 2a \Rightarrow x = a \Rightarrow y = \pm a \Rightarrow y$ exist
   (c) $x > 2a \Rightarrow x = 3a \Rightarrow y$ is imaginary

x	0	0.2	0.4	0.8	1	1.2	1.4	1.6	1.8	2
y	0	± 0.067	± 0.2	± 0.65	1	± 1.47	± 2.13	± 3.2	± 5.4	∞

7. Tangent at point of intersection $(2a, 0)$

$y^2 2a - y^2 x$



Q Trace the curve $y = \frac{8a^3}{x^2 + 4a^2}$ $a > 0$

Q Trace the curve $xy^2 = 4a^2(2a - x)$

Q Trace the curve $y^2 = ax^3$, $a > 0$

Q Trace the curve $y = \frac{8a^3}{x^2+4a^2}$ $a>0$

1. Symmetry

The curve is symmetric about y-axis because the even power of x is present

2. Origin

does not pass through origin

3. Tangent at origin

does not exist

4. Intersection with co-ordinate axis

Put $x=0$ | put $y=0$
$y=2a$ | x is not defined

The curve intersect the co-ordinate axis $(0,2a)$

5. Asymptote

a] Parallel to x axis

b] Parallel to y axis

c] oblique asymptote does not exist

6. Region of existance

$a>0$ the y take only +ve value

So curve is in the I^{st} & II^{nd} quadrant

Q Trace $xy^2 = 4a^2(2a-x)$

1. Symmetry

Curve symmetry about x axis

2. Origin

doesn't pass through origin

3. Tangent at origin doesn't exist

4. intersection with co-ordinate axis

put $x=0$ $\quad 8a^3 = 0$ $\quad$ Not possible

put $y=0$ $\quad 8a^3 = 8a^2 x$ $\quad \Rightarrow 2a = x$

curve meet at x-axis at $(2a,0)$

5. Asymptote

a] Parallel to x-axis

~~$y^2 = 0$~~ $y^2 + 4a^2 = 0$ $\quad$ Not possible

b] Parallel to y axis

$x = 0$ i.e, y-axis

c] oblique asymptote does not exist.

6. Region of existance

$$y^2 = \frac{4a^2(2a-x)}{x}$$

$$y = \pm \sqrt{\frac{4a^2(2a-x)}{x}}$$

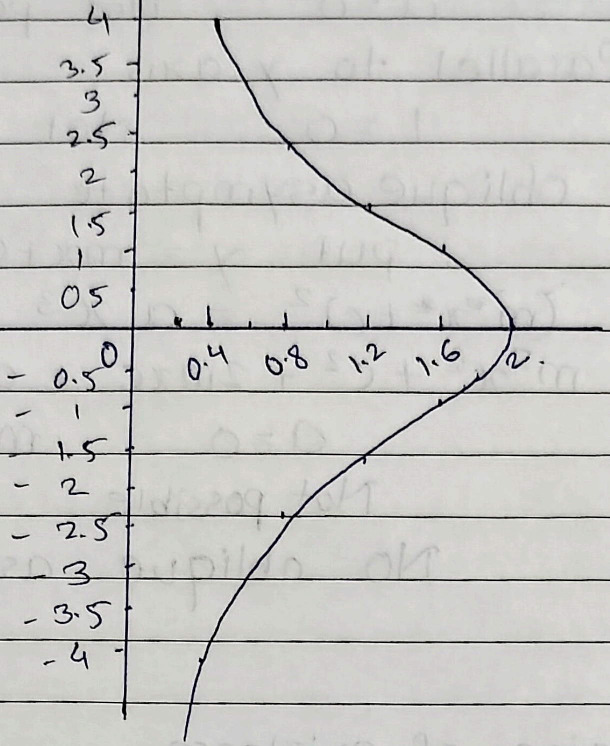
ⓐ for $x < 0$

y is imaginary $\quad$ curve does not exist in 2nd & 3rd quadrant

ⓑ $0 < x < 2a \Rightarrow x = a \Rightarrow y = \pm 2a \Rightarrow y$ exist ✓

ⓒ $x > 2a \Rightarrow x = 3a \Rightarrow y$ is imaginary

x	0	0.4	0.8	1.2	1.6	2
y	∞	4	2.45	1.63	1	0



Q $y^2 = ax^3$ Trace the curve

1. Symmetry
   Curve symmetry about x axis

2. origin
   Pass through origin

3. Tangent at origin
   $y^2=0$ i.e, $y=0$ is double tangent
   x axis is double tangent.

4. Intersection with co-ordinate axis
   put $x=0$ $y^2=0$ $y=0$
   put $y=0$ $ax^3=0$ $x=0$
   Curve meet at origin only.

$y^2 = ax^3$

5. Asymptote

a] Parallel to x axis

$a = 0$ Not possible

b] Parallel to y axis

$1 = 0$ Not possible

c] oblique asymptote

put $y = mx + c$

$(mx+c)^2 = ax^3$

$m^2x^2 + c^2 + 2mxc = ax^3$

$a = 0$ $m^2 = 0$

Not possible $m = 0$

No oblique asymptote

6. Region of existance

$y = \pm\sqrt{ax^3}$

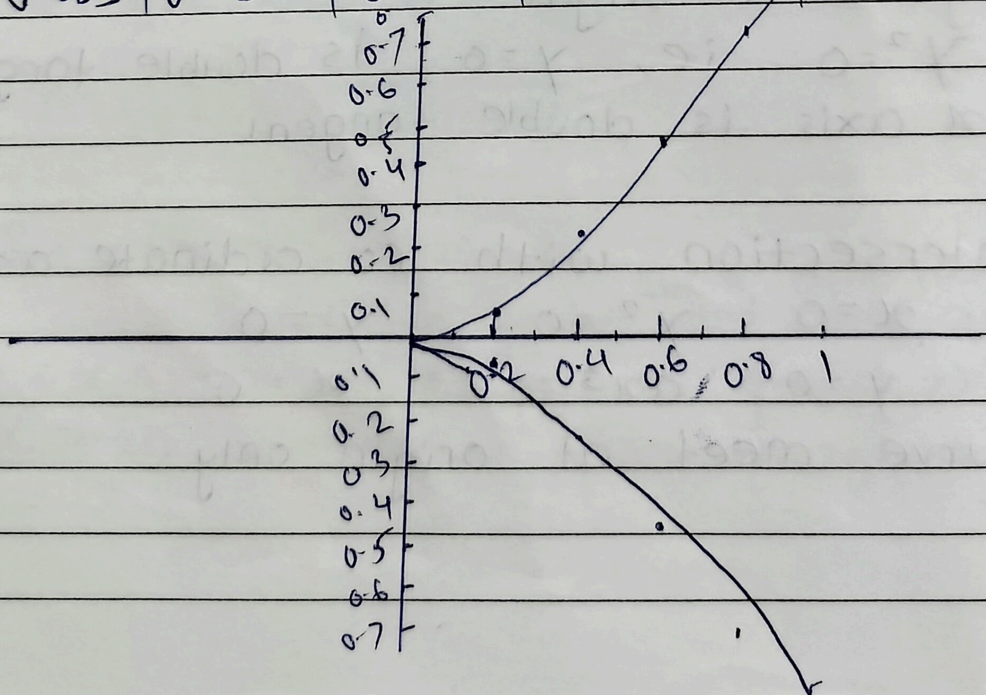
(a) $x < 0$ imaginary

(b) $x > 0$, 1st & 3rd quandrant exist

$a > x > 0$

$a = 1$

x	0.2	0.4	0.6	0.8	1	0
y	0.089	0.25	0.46	0.71	1	0



Maths Tutorial No: 1

① Find the oblique asymptotes of $x^3+y^3=3axy$

put $y = mx+c$

$x^3 + (mx+c)^3 = 3ax(mx+c)$

$x^3 + m^3x^3 + c^3 + 3m^2x^2c + 3mxc^2 = 3amx^2 + 3axc$

$x^3(1+m^3) + x^2(3m^2c - 3ma) + x(3mc^2 + 3ac) + c^3 = 0$

③ Trace the curve $y^2(a+x) = x^2(b-x)$

$y^2a + y^2x = x^2b - x^3$

1. Symmetry

As only even powers of y present

$\therefore$ Curve is symmetric about x-axis

2. Origin

There is no constant term in equation

$\therefore$ curve doesn't pass through origin

3. Tanget at origin doesnot exist

4. Intersection with co-ordinate axis

put $x=0$ $\quad y^2a = 0$ $\quad$ ~~Not possible~~ $\quad y=0$

put $y=0$ $\quad x^2b - x^3$

5. Asymptote

a) Parallel to x axis

$-1 = 0$ Not possible

b) Parallel to y axis

$a+x=0$ Not possible

c) oblique asymptote

$(mx+c)^2(a+x) = x^2(b-x)$

$(m^2x^2 + c^2 + 2mxc)(a+x) = x^2b - x^3$

$m^2x^2a + m^2x^3 + c^2a + xc^2 + 2mxca + 2mx^2c$

$x^2(m^2a + 2mc) + x^3 m^2 + x(c^2 + 2mca) + c^2a$

Friday 19 01 2023

Polar form:

1 Symmetrical:

- If all replacing θ by $-\theta$ the eq^n of curve remains unchange then the curve is symmetrical about initial line.
- If all replacing r by $-r$ the eq^n of curve remain unchange then the curve is symmetrical about the pole ie, if eq^n contain only even power of r then the curve is symmetric about the pole in this case, pole is called as centre of curve
- curve is symmetric about line $\pi/2$ if the eq^n remains unchange when θ is replace by $\pi - \theta$
- Curve is symmetric about the line $\theta = \pi/2$, if the eq^n of curve remains unchange when θ is replace by $\pi/2 - \theta$
- Curve is symmetric about the line $\theta = 3\pi/4 = 135^0$ if eq^n of curve remains unchange when θ is replace by $3\pi/2 - \theta$

2 Pole:

Value of θ for which $r=0$ are poles of the curve

e.g. $r = a(1+\cos\theta)$ $\quad a>0$

put $r=0$

$a(1+\cos\theta)=0$

$\cos\theta = -1$

$\theta = \pi, 3\pi, 5\pi, \ldots$

These are the poles for the curve.

3. Tangent at the pole:
If $\theta_1, \theta_2, \ldots$ are poles of the given curve then eq^n of tangent at pole is given by $\theta = \theta_1, \theta = \theta_2, \ldots$

4. Point of intersection:
POI of the curve with initial line $\theta = 0$ and with line $\theta = \pi/2$ are obtained by using $\theta = 0$ and $\theta = \pi/2$ respectively in the given eq^n of curve

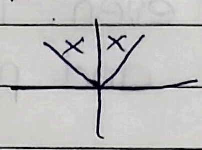
5. Region of existance:
Find out the min^m and max^m value of r, let a & b are min^m & max^m values of r then the curve lies in the anulus region b/w the two circle of radii a & b
If r min^m = 0 then the curve lies within the circle of radius b.
Curve does not exist for values of θ for which r is imaginary

e.g. $r^2 = a^2 \cos 2\theta$

for $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$

let $\theta = \frac{\pi}{2}$

$r^2 = -a^2$



6) for certain values of θ find r for eq^n involving periodic fxn generally θ varies from 0 to 2π

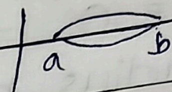
7. Angle b/w radius vector and tangent at any point (r, θ)
It is obtain from $\tan\phi = r \cdot \frac{d\theta}{dr}$

where ϕ is angle b/w radius vector & tangent

- If $\phi = 0$ then tangent ~~to~~ is final to initial line at (r,θ)
- If $\phi = \pi/2$ then tangent is $\perp^{ar}$ to initial line at (r,θ)

Remark:

1. If $\frac{dr}{d\theta} > 0$ then r increases
2. If $\frac{dr}{d\theta} < 0$ then r decreases
3. Loop- If the curve meets initial line at point a & b and if it is symmetric about the initial line then the loop of the curve exist b/w point a & b.



3. Curve of the type $r = a.\sin n\theta$ & $r = a.\cos n\theta$ are called roses. they consist of either n and $2n$ similar loop according as n is odd or n is even. Divide each quadrant into n equal parts and plot r ~~for~~ θ

1 Trace the q curve $r = a.\cos 3\theta$

It is a rose curve

As $n = 3 \Rightarrow$ odd curve has 3 similar loop.

1. Symmetry

As eq^n remains unchange when θ is replace by $-\theta$ curve is symmetric about initial line.

2. Pole

put $r = 0$, $a > 0$

$a.\cos 3\theta = 0$

$3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{------}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

$\theta = 30, 90, 150, 210, 270, 330$

3. Tangent at the pole

eq^n of tangents at

$\theta = \frac{\pi}{6}$, $\theta = \frac{\pi}{2}$, $\theta = \frac{5\pi}{6}$

$\theta = \frac{7\pi}{6}$, $\theta = \frac{3\pi}{2}$, $\theta = \frac{11\pi}{6}$

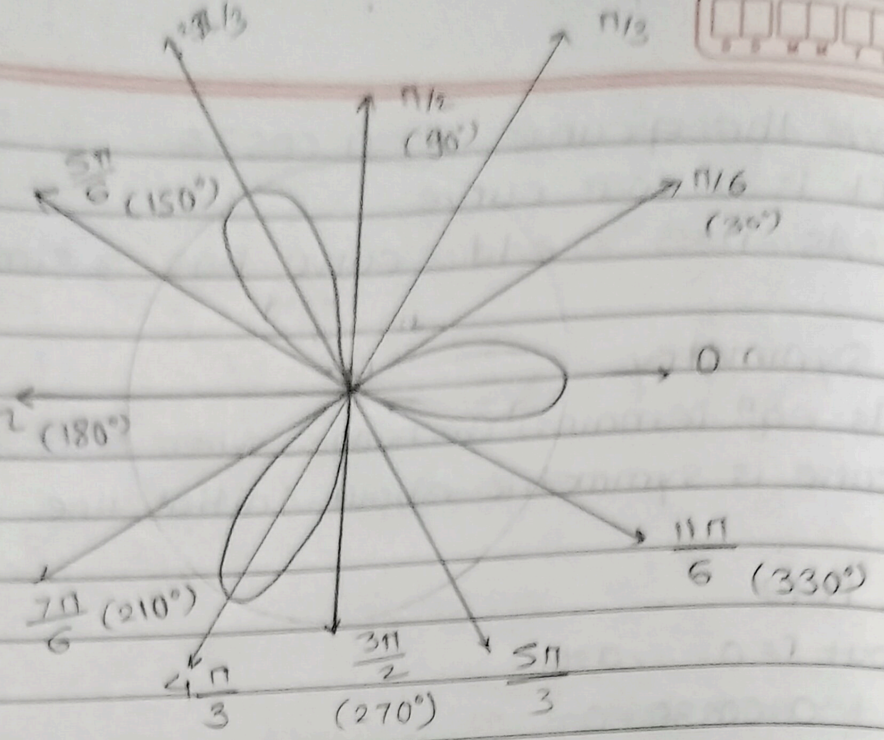
4. Region of existance

$r = a.\cos 3\theta$

for any θ , $-1 \leq \cos\theta \leq 1$

$\therefore r_{min} = 0$ $r_{max} = a$

$\therefore$ Curve lies within the circle of radius a



θ	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
r	a	$-a$	a	$-a$	a	$-a$

Angle b/w radius vector & tangent

$$\tan\phi = \frac{r.d\theta}{dr} \qquad \frac{dr}{d\theta} = -3a\sin 3\theta$$

$$\tan\phi = \frac{r}{dr/d\theta}$$

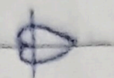
$$\tan\phi = \frac{a\cos 3\theta}{-3a\sin 3\theta} = -\frac{1}{3}\cot 3\theta$$

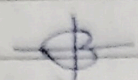
$$\tan\phi = -\frac{1}{3}\cot 3\theta$$

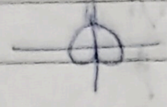
at $\theta = 0$ $\quad \tan\phi = \infty \quad \phi = \pi/2$

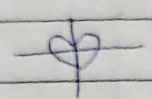
Thus tangent at $\theta = 0$ is $\perp^{ar}$ to the initial line i.e, radius vector.

Q Trace the curve $r = a(1+\cos\theta)$ → Cardioid

$r = a(1+\cos\theta)$ 

$r = a(1-\cos\theta)$ 

$r = a(1+\sin\theta)$ 

$r = a(1-\sin\theta)$ 

1. Symmetry

$r = a(1+\cos(-\theta)) = a(1+\cos\theta)$

symmetry about initial line

2. Pole

put $r=0$ $\qquad a>0$

$a(1+\cos\theta) = 0$

$\cos\theta = -1$

$\theta = \pi, 3\pi, 5\pi, \ldots$

3. Tangent at pole

$\theta = \pi, \ \theta = 3\pi, \ \theta = 5\pi, \ \ldots$

All tangents are coincides.

4. Region of existance

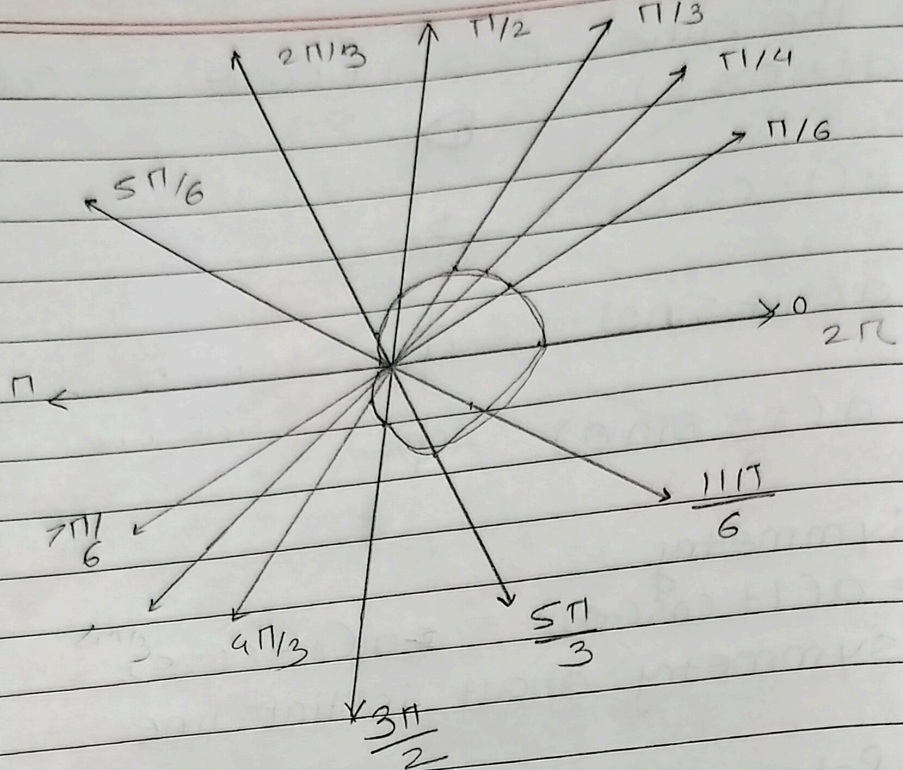
$r_{max} = 2a$ $\qquad \because -1 \le \cos\theta \le 1$

$r_{min} = 0$

Curve lies within the cirzle with radius 2a.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
r	$2a$	$1.86a$	$1.70a$	$1.5a$	a	$0.5a$	$0.13a$	0

θ	$7\pi/6$	$4\pi/3$	$3\pi/2$	$5\pi/3$	$11\pi/6$	2π
r	$0.13a$	$0.5a$	a	$1.5a$	$1.86a$	$2a$



Q Trace the curve $r = a.\sin 2\theta$

It is a rose curve

as $n = 2 \Rightarrow 4$ loops

1. Symmetry

Replace θ by $\pi/2 - \theta$

$r = a \sin 2(\pi/2 - \theta)$

$= a \sin(\pi - 2\theta) = a \sin 2\theta$ ∴ eqⁿ remains same

∴ Curve is symmetric about $\theta = \pi/4$

Replace θ by $3\pi/2 - \theta$

$r = a \sin 2\left(\frac{3\pi}{2} - \theta\right) = a \sin(3\pi - 2\theta)$

$= a \sin 2\theta$

Curve is symmetric $\theta = 3\pi/4$

2. Pole

put $r=0$, $a\sin 2\theta = 0$

$2\theta = 0, \pi, 2\pi$

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ ---

These are the pole of the curve

3. Tangent at pole

They are represented by 4 eqⁿ

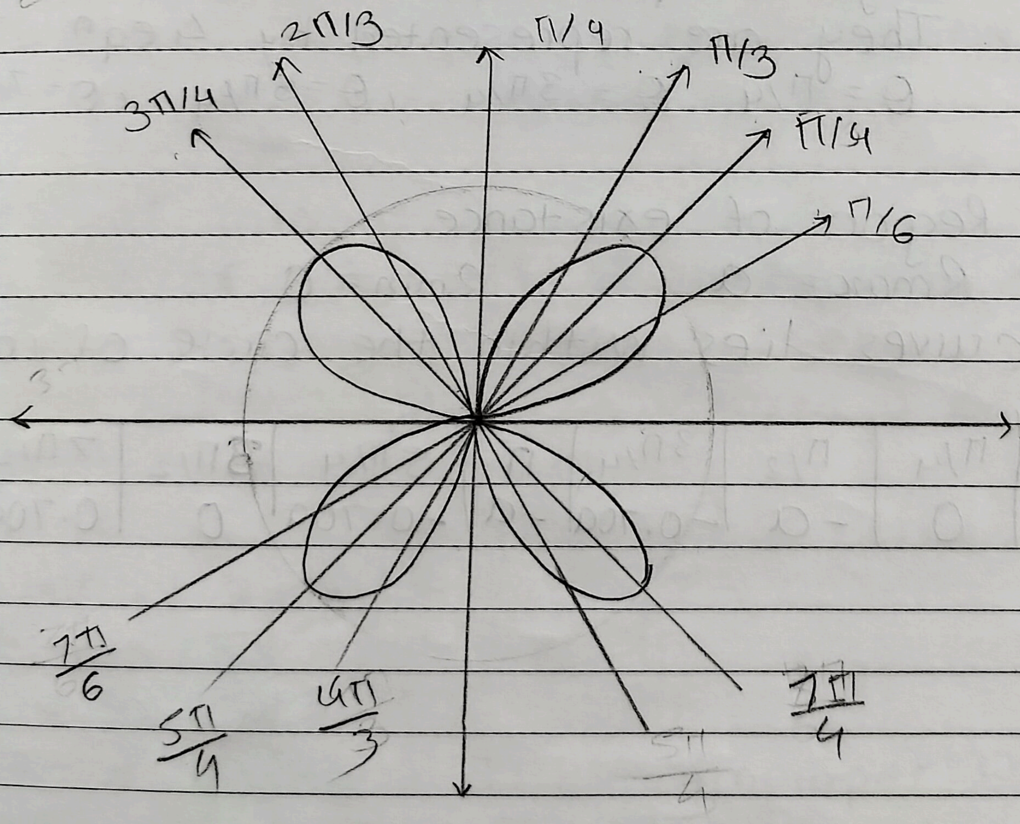
$\theta = 0$, $\theta = \pi/2$, $\theta = 3\pi/2$, $\theta = \pi$

4. Region of existance

$r_{max} = a$ $r_{min} = 0$

Curve lies within the circle of radius a

	45	135	225	315	60	30	120	210	330	
θ	$\pi/4$	$3\pi/4$	$5\pi/4$	$7\pi/4$	$\pi/3$	$\pi/6$	$2\pi/3$	$7\pi/6$	$11\pi/6$	2π
r	a	$-a$	a	$-a$	0.860	$0.86a$	$-0.86a$	$0.86a$	-0.86	0



Q Trace the curve $r = a\cos 2\theta$

rose curve
$n=2$
loop

1. Symmetry

Curve is symmetrical about initial line

$r = a\cos 2(-\theta) = a\cos 2\theta$

$r = a\cos 2(\pi - \theta)$

$= a\cos(2\pi - 2\theta)$

Curve is symmetrical about $\pi/2$

2. Pole

put $r = 0$

$a\cos 2\theta = 0$

$\cos 2\theta = 0$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

3. Tangent at pole

They are represented by 4 eq^n

$\theta = \pi/4 \quad \theta = 3\pi/4, \theta = 5\pi/4, \theta = 7\pi/4$

4. Region of existance

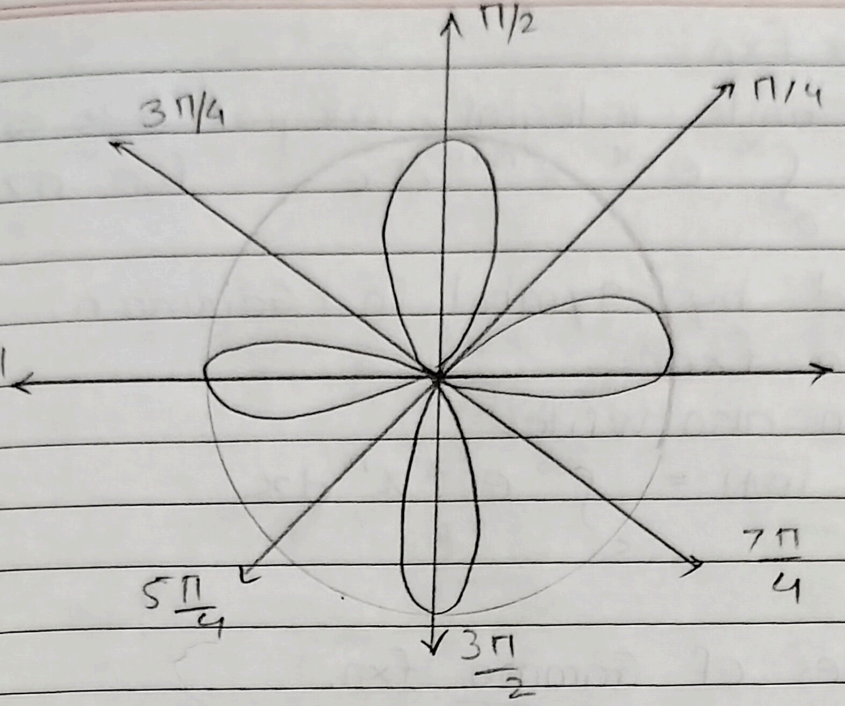
$R_{max} = a \qquad R_{min} = 0$

curves lies within the circle of radius a

θ	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
r	0	$-a$	$-0.70a$	$-a$	$-0.70a$	0	$0.70a$	a

Tangent ki value ko consider Nhi karte

$r = a(1+\sin\theta)$



Module 2 :

* Beta Gamma Function

Intro : Beta & Gamma fxn are define in terms of certain improper integral which can not be evaluated by usual method. Gamma & Beta Fxn are related to each other. This fxn occurs frequently in statistics, physics & engineering etc

Improper integral

If one or both units of integration of a definite intergal are infinite or the integrand is unbounded on the interval, then the integral is called impropar or singular integral

eg. $I_1 = \int_2^{\infty} x e^{-x}\, dx$

$I_2 = \int_0^{10} \frac{e^{-x}}{x}\, dx$

The 1^{st} integral I_1 is improper because one unit is infinite, in 2^{nd} integral, integrand tends to infinity when x tends to zero.