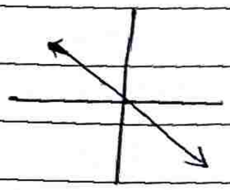
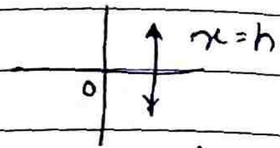
Curve Tracing

Type-1. cartesian coordinate.

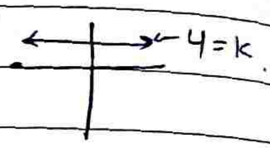
- $x=0 \rightarrow$ y-axis
- $y=0 \rightarrow$ x-axis
- $y=mx\ (m>0)$ = line passing through origine from 1st quadrant to 3rd quadrant.
- $y=mx\ (m<0)$ = line passing through origine from 2nd quadrant to 4th quadrant.



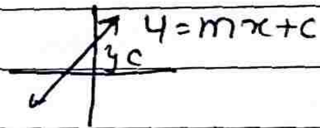
- $x=h$ = line parallel to y-axis



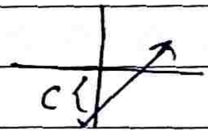
- $y=k$ = line parallel to x-axis



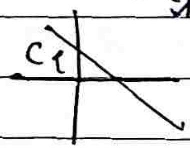
- $y=mx+c\ (m>0, c>0)$



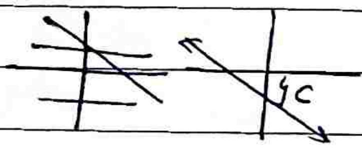
- $y=mx+c\ (m>0, c<0)$



- $y=mx+c\ (m<0, c>0)$



- $y=mx+c\ (m<0\ \ c>0)$



Rules for tracing a curve (cartesian co-ordinate).

* Summetry.

1] If Eqn contains even power of x then curve is symmetric about y-axis

2] If Eqn contains even power of y then curve is symmetric about x-axis.

3] If Eqn contains even power of x as well as y

the curve is symmetric about both axis

4] Replace $x/-x$ and $y/-y$ if the Eqn Remains same the curve is symmetric in opposite quadrant.

5] Replace x by y and y by x is the eqn remains same the curve is symmetric about in $y=x$.

* origin

1] If the Eqn has no any constant term then curve is passing through origin.

2] put $x=0$ $y=0$ if LHS = RHS of eqn then curve is passing through origin.

* Tangent to origin

(Find only if curve passing through origin)

1] It is obtain by lowest degree term of the eqn = 0

Eg., $x^2=4by$

$4by=0$

$y=0$

* point of intersection

1] To find the point of intersection of x axis put $y=0$ in the given eqn.

2] To find the point of intersection on y axis put $x=0$ in the given Eqn.

* Tangent (Find only if the point of intersection is other than origin)

1] differentiate given eqn w.r.t x and find dy/dx

2] put point of intersection other than origin in d

3] ① If $dy/dx = 0$ tangent $\parallel$ to x axis.

② If $dy/dx = \infty$ tangent $\perp$ ar to x-axis

③ dy/dx = constant → no tangent.

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* Asymplote

1] Asymplote is the tangent to the curve at ∞

2] coefficient of highest degree term of $x=0$

→ we get asymplote parallel to x-axis.

3] coefficient of highes degree term of $y=0$

→ we get asymplote ||el to y-axis

3] As $x \to \infty = y \to \infty$ then we may expect oblique asymptote let $y=mx+c$ with the oblique asymptote put $y=mx+c$ in the given eqn ie $f(x,y)=f(x,mx+c)$ Equate the coefficient of two sesccesive highest power of $x=0$. find $mx+c$ put the value of $mx+c$ in $y=mx+c$.

* extent of the curve.

Here we decide the region where the curve exist.

i] separate the term of x on one side and term of y another side.

ii] put any intermejet point betn two point of intersection.

iii] If $x(y)$ is finite the curve exist in that region

iv] If $x(y)$ is imagenacy then curve does not Exist.

* Tracing of the curve

Consider the different value of x for the region where the curve exist & put this value in the eqn of curve and find corresponding y and draw the curve.

Q $y^2(a+x) = x^2(3a-x)$

Given: $y^2(a+x) = x^2(3a-x)$ —— (I)

$ay^2 + xy^2 = 3ax^2 - x^3$ —— (II)

(1) Symmetry:

Here given Eqn contains Even power of y

So the curve is symmetric about x-axis.

(2) Origin:

put $x=0$, $y=0$ in (I)

$\therefore$ LHS $= 0$

RHS $= 0$

$\therefore$ Given Eqn of curve is passing through origin.

(3) Tangent through origin

lowest degree term $= 0$

$ay^2 - 3ax^2 = 0$

$y^2 = 3x^2$

$y = \pm\sqrt{3}x$

(4) point of intersection.

put $x=0$ in Eqn (I)

$ay^2 = 0$

$y = 0$

pt is $(0,0)$

put $y=0$ in Eqn (I)

$0 = (3a-x)x^2$

$x = 3a$ $\quad x = 0$

pt is $(3a, 0)$.

$\therefore$ point of intersection are $(0,0)$ $(3a,0)$

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⑤ Tangent

diff Eqn ⑪. with resp to x.

$$2ay.\frac{dy}{dx} + y^2 + 2xy.\frac{dy}{dx} = 6ax - 3x^2$$

$$(2ay + 2xy)\frac{dy}{dx} = 6ax - 3x^2 - y^2$$

$$\frac{dy}{dx} = \frac{6ax - 3x^2 - y^2}{2y(a+x)}$$

$$\left.\frac{dy}{dx}\right|_{(3a,0)} = \infty$$

ie, Tangent is parallel to y-axis at $(3a,0)$

⑥ Asymptote.

coefficient of Highest degree term of x is Equal to 0.

Here the highest degree term of x is $(-x^3)$ and its coefficient is -1.

$\therefore\ -1 = 0$ which is not possible.

$\therefore$ NO Asymptote parallel to x axis.

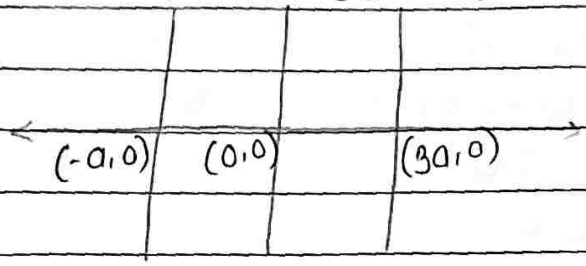
coefficient of Highest degree term of y is Equal to 0.

Here the highest degree term is y^2. and its coefficient is $a+x$.

$\therefore\ a + x = 0$

$\therefore\ x = -a$. is the asymptote parallel to y-axis.

(7) Extent of the curve



From Eqn (1), $y^2 = \frac{x^2(3a-x)}{a+x}$.

(a) For $x<-a$

put $x=-2a$

$y^2 = \frac{4a^2(5a)}{-a}$

$y \longrightarrow$ imaginary

for $x<-a$ curve does not exist.

(b) Betn the pts $(-a,0)$ and $(0,0)$.

put $x = -\frac{a}{2}$

$y^2 = \frac{\frac{a^2}{4}\left(3a+\frac{a}{2}\right)}{a/2}$

$= \frac{\frac{a^2}{4}\left(\frac{7a}{2}\right)}{\frac{a}{2}}$

$= \frac{7a^2}{4}$

y = Finite

∴ curve exist betn the point $(-a,0)$ and $(0,0)$

(c) Between the points (0,0) and (3a,0)

put $x = a$

$$y^2 = \frac{a^2(2a)}{2a}$$

$y^2 = a^2$

$y = \pm a$

$y \longrightarrow$ finite.

$\therefore$ The curve exist betn the pt (0,0) (3a,0)

(d) $x > 3a$

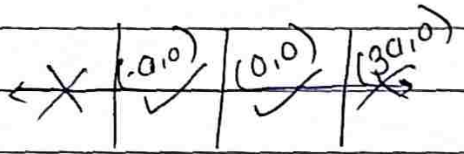
put $x = 4a$

$$y^2 = \frac{16a^2(-a)}{5a}$$

$y^2 = -ve$

$y \longrightarrow$ imaginary

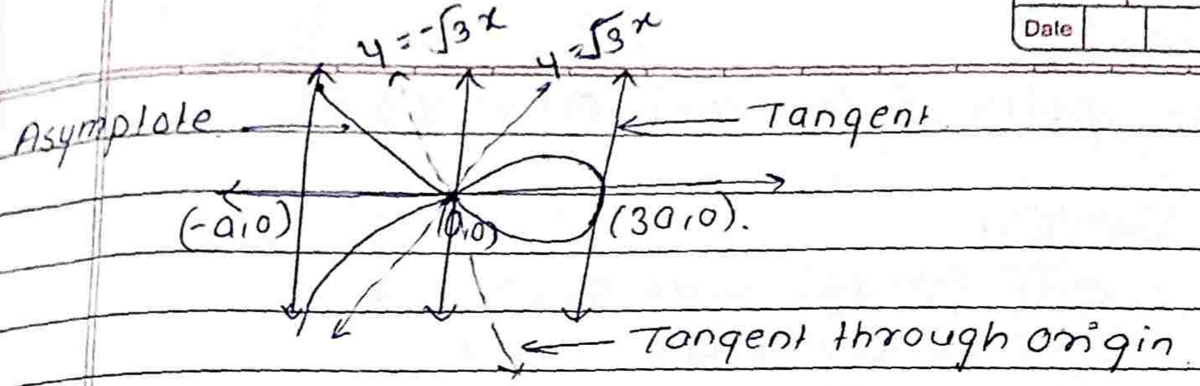
$\therefore$ curve does not Exist for $x > 3a$



(8) Tracing of the curve.

$$y^2 = \frac{x^2(3a-x)}{a+x} \quad \text{—— (iii)}$$

x	$-a$	$-a/2$	0	$a/2$	a	$2a$	$3a$
y	$\pm\infty$	$\pm 1.3a$	0	$\pm 0.6a$	$\pm a$	$\pm 1.1a$	0



Q Trace the curve

$xy^2 = a^2(a-x)$

$xy^2 = a^2(a-x)$ —— (I)

$xy^2 = a^3 - xa^2$ —— (II)

① Symmetry

Here given Eqn contains even power of y so the curve is symmetric about x-axis.

② origin

put $x=0$, $y=0$ in (II).

$LHS = 0$

$RHS = a^3$

$\therefore LHS \neq RHS$

$\therefore$ Given curve is not passing through origin.

③ Tangent through origin.

No tangent through origin

④ point of intersection

put $x=0$ in Eq ①

$0 = a^3$ which is not positive

put $y=0$

$0 = a^3(a-x)$

$x = a$

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$\therefore$ point of intersection is $(a, 0)$

⑤ Tangent

diff eqn ② with resp to x

$$x \cdot 2y \frac{dy}{dx} + y^2 = -a^2$$

$$2xy \frac{dy}{dx} = -a^2 - y^2$$

$$\frac{dy}{dx} = \frac{-a^2 - y^2}{2xy}$$

$$\left.\frac{dy}{dx}\right|_{(a,0)} = \infty$$

Tangent is parallel to y axis at $(a, 0)$

⑥ Asymptote.

Coefficient of Highest degree term of x is Equal to 0.

Here the highest degree term of x is x. and Coeffient is $y^2 + a^2$

$\therefore y^2 + a^2$

$y^2 = -a^2$

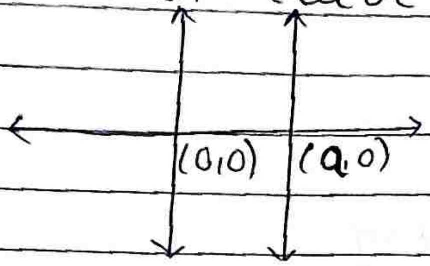
No Asymptote parallel to x-axis.

Coefficient of Highest degree term of y $\neq 0$.

Heighst degree term of y^2 and its coffis

$\therefore x = 0 \longrightarrow$ y-axis.

⑦ Extent of curve



From Eqn ②, $y^2 = \dfrac{a^3 - a^2 x}{x}$ —— (iii)

ⓐ For $x < 0$.

put $x = -a$.

$y^2 = \dfrac{2a^3}{-a}$

y = imaginary.

Curve does not exist for $x < 0$.

ⓑ Betn the pt $(0,0)$ and $(a,0)$

put $x = a/2$

$$y^2 = \dfrac{a^3 - \dfrac{a^3}{2}}{\dfrac{a}{2}}$$

$$= \dfrac{a^3/2}{a/2}$$

y is finite.

$\therefore$ Curve Exist betn the point $(0,0)$ and $(a,0)$

$y^2 = \frac{a^3 - a^2x}{x}$

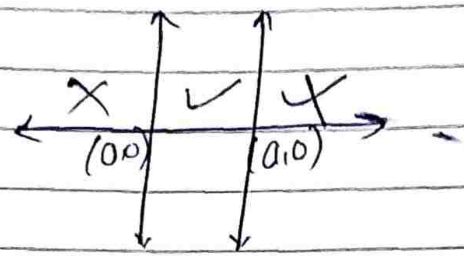
ⓒ for $x > a$

put $x = 2a$

$y^2 = \frac{-a^3}{2a}$

y = imaginary

$\therefore$ curve does not Exist for $x > a$.



⑧ Tracing of the curve

x	0	$a/4$	$a/2$	$3a/4$	a
y	∞	$\pm 1.7a$	$\pm a$	$\pm 0.5a$	

Q Trace the curve.

$$y(x^2+4a^2) = 8a^3 \quad \text{--- (1)}$$

$$yx^2 + 4a^2y - 8a^3 = 0 \quad \text{--- (2)}$$

(1) Symmetry

Here given eqn contains even power of x. So the curve is symmetry about y-axis

(2) Origin.

put $x=0$ in Eqn (1)

$LHS = 0$

$RHS = 8a^3$

$LHS \neq RHS$

$\therefore$ Given curve is not passing thr origin.

(3) Tangent through origin.

No

(4) point of intersection.

put $x=0$ in Eqn (1)

$4a^2y - 8a^3 = 0$

$y = 2a$.

put $y=0$

$8a^3 = 0$

which is not possible.

pt of intersecn $(0, 2a)$

(5) Tangent

diff Eqn (2) w.r.t x.

$$y(2x) + x^2\frac{dy}{dx} + 4a^2\frac{dy}{dx} = 0.$$

$$2xy + (x^2+4a^2)\frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+4a^2}$$

$$\left.\frac{dy}{dx}\right|_{(0,2a)} = 0$$

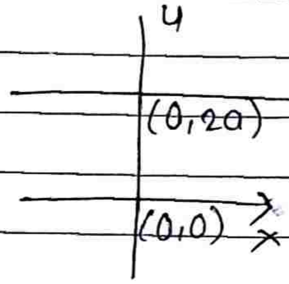
Tangent is parallel to $(0,2a)$

⑥ Asymptote

coefficient of Highest degree term of x is x^2 and coefficient is 'y'.

x-axis is asymptote.

⑦ Extent of the curve



From ① $y = \frac{8a^3}{x^2+4a^2}$ —— (B) ~~$x2=$~~

$$x^2+4a^2 = \frac{8a^3}{y}$$

(a) ~~for $y>$~~ $2a$

$$x^2 = \frac{8a^3}{y} - 4a^2.$$

ⓐ For $y > 2a$

put $y = 4a$

$$x^2 = \frac{8a^3}{4a} - 4a^2 \quad \text{———} \; (3)$$

x is imaginary.

ⓑ put $y = a$.

$$x^2 = \frac{8a^3}{a} - 4a^2$$

$x = 4a^2$.

$x = \pm 2a$ finite.

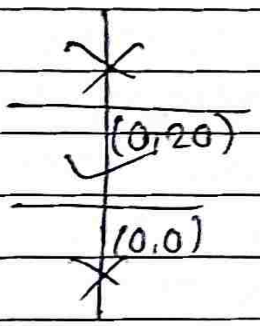
curve exist at pts $(0,0)$ $(0,2a)$

ⓒ For $y < 0$.

$y = -a$

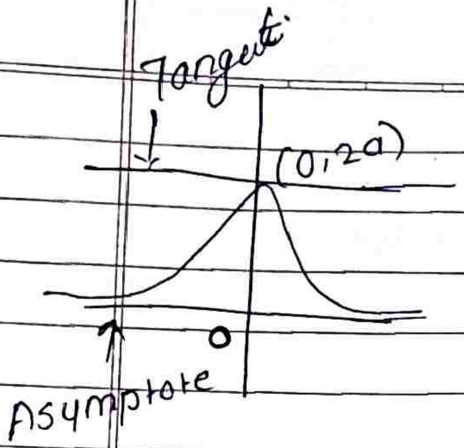
$$x^2 = \frac{8a^3}{-a} - 4a^2.$$

$x \to$ imaginary.



⑧ Tracing of the curve.

~~at~~ y	0	$a/2$	a	$3a/2$	$2a$
x	$\pm\infty$	$\pm 3.4a$	$\pm 2a$	$\pm 1.1a$	0



Q. $a^2y^2 = x^2(x-a)(2a-x)$

$2ax^3 - x^4 - 2a^2x^2 + ax^3 = a^2y^2$

$3ax^3 - 2a^2x^2 - x^4 - a^2y^2 = 0$

Given eqn contain even power of y.

① Symmetry

about x-axis.

② Origin.

put $x=0$ $y=0$

LHS $=0$

RHS $=0$

$\therefore$ LHS $=$ RHS

$\therefore$ curve is passing through origin.

③ Tangent thr origin.

$\therefore$ lowest degree term $=0$

$a^2y^2 + 2a^2x^2 = 0$

$y^2 = -2x^2$.

$\therefore$ Tangent thr origin is not possible.

④ pt of intersection

put $x=0$ in eqn ①

$y=0$

$\therefore$ pt is $(0,0)$

put $y=0$ in ①

$$x^2(x-a)(2a-x)=0$$

$$x=0,\quad x=a,\quad x=2a$$

pts are $(0,0)$ $(a,0)$ $(2a,0)$

diff: eqn ② w.r.t x.

$$2a^2\frac{dy}{dx}=9ax^2-4x^3-4a^2x.$$

$$\frac{dy}{dx}=\frac{9ax^2-4x^3-4a^2x}{2a^2y}$$

$$\left.\frac{dy}{dx}\right|_{(a,0)}=\infty.$$

$$\left.\frac{dy}{dx}\right|_{(2a,0)}=\infty.$$

∴ Tangent is parallel to y-axis at $(a,0)$ $(2a,0)$

⑥ Asymptote.

coefficient of Higghest degree term of $x=0$

Highest degree x^4. and coefficient $=1$.

∴ $1=0$.

∴ no asymptote

coefficient of higgest degree term of $y=0$.

Highest degree y^2 coefficient $=a^2$.

$a^2=0$.

∴ No asymptote.

of $x\to\infty$, $y\to\infty$ so we may expect oblique asymptote

let $y=mx+c$ be the eqn of oblique asymptot put $y = mx + c$ in Eqn ②

$$a^2(mx+c)^2 = 3ax^3 - x^4 - 2a^2x^2.$$

$$a^2(m^2x^2 + 2mxc + c^2) = 3ax^3 - x^4 - 2a^2x^2$$

$$x^4 - 3ax^3 + (a^2m^2 + 2a^2)x^2 + 2a^2mcx + a^2c^2$$

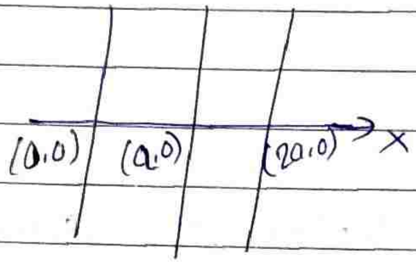
Equate the coefficient of two successive highest power of x.

$\therefore 1 = 0.$ not possibe

$-3a = 0$ — " —

No oblique asymptote.

⑦ Extent of the curve.



$$y^2 = \frac{x^2(x-a)(2a-x)}{a^2}$$

~~for~~

① For $x<0$

put $x = -a$

$$y^2 = \frac{a^2(-2a)a}{a^2} \quad \text{③}$$

y = imaginary

$\therefore$ curve does not exist for $x<0$.

② Betn the pt $(0,0)$ and $(a,0)$.

$$x = \frac{a}{2}$$

$$y^2 = \frac{\frac{a^2}{4}\left(\frac{-a}{2}\right)\left(\frac{3a}{2}\right)}{a^2}$$

y imaginary

curve does not Exist betn (0,0)(2a,0)

③ Betn the pt (a,0) (2a,0).

$$y^2 = \frac{\frac{9a^2}{4}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}{a^2}$$

y = finite.

∴ Curve Exist Betn the pt (0,0)(2a,0).

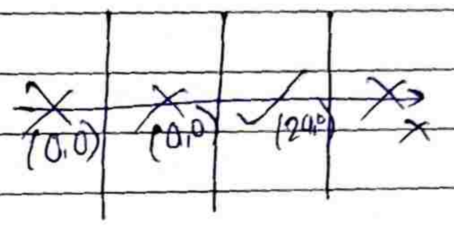
④ For $x > 2a$

$x = 3a$

$$y^2 = \frac{9a^2(2a)(-a)}{a^2}$$

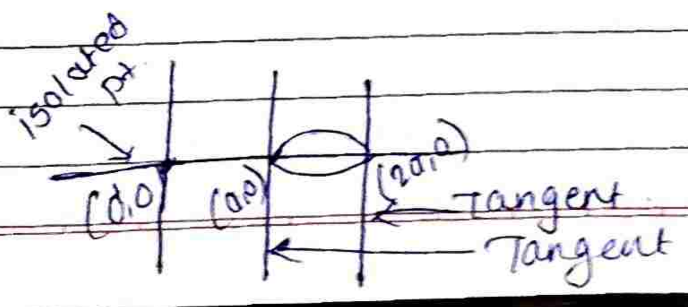
y imaginary

∴ curve does not Exist in $x > 2a$



⑧ Tracing the curve

x	a	$\frac{5a}{4}$	$\frac{3a}{2}$	$\frac{7a}{4}$	$2a$
y	0	$\pm 0.5a$	$\pm 0.7a$	$\pm 0.8a$	0



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Q. Trace the curve $x^2(2a-y)=y^3$ —— (1)

$2ax^2 - x^2y = y^3$ —— (11)

(1) Symmetry

Eqn contain Even power of x.

$\therefore$ curve is symmetry about y-axis

(2) Origin. put $x=0$ $y=0$

$LHS = 0$ ~~LHS = RHS~~

$RHS = 0$

$\therefore$ Curve is passing through origin

(3) Tangent thr origin.

lowest degree term $=0$

$2a=0$

$\therefore$ y axis is the tangent thr origin.

(4) pt of intersection

put $x=0$

$y=0$

pt is $(0,0)$

put $y=0$

$x=0$

pt is $(0,0)$

pts ~~a~~ is $(0,0)$.

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(5) Asymptote.

coefficient of Highest degree term of $x = 0$.

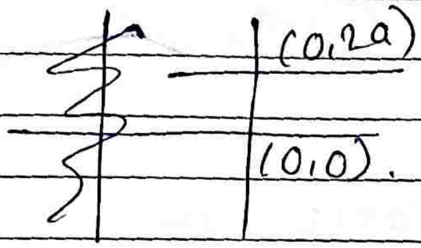
~~Highest degree term~~ $\cancel{x^2}$

$$(2a - y) = 0$$

$$y = 2a$$

is asymptote $\parallel$ to a-xis.

(7) Extent of the curve



$$x^2 = \frac{y^3}{2a - y}.$$

(1) for $y > 2a$

$y = 3a$.

$$x^2 = \frac{9a^2}{-a}.$$

x imaginary

curve does not exist.

(2) betn pts $(0,0)$ and $(0,2a)$

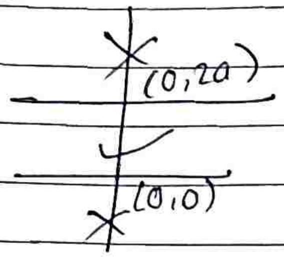
put $y = a$

$$x^2 = \frac{a^3}{a}$$

x finite.

curve exist betn $(0,0)$ and $(0,2a)$

For $y > 0$ put $y = 3a$.



⑧ Tracing of curve.

y	0	$a/2$	0	$3a/2$	$2a$
x					