Type 5. Application to electrical circuit.

$I = \frac{dq}{dt}$

$q = \int i \cdot dt$

voltage drop across the resistance $= Ri$

— " — the Inductance $L = L\frac{di}{dt}$

— " — the capacitance $C = \frac{q}{c}$

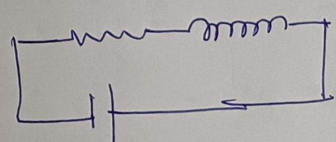
Kirchoff's law, the algebric sum of voltage around any closed ckt = Resultant EMF in ckt

The algebric sum of current flowing into any terminal is zero

Differential Equation

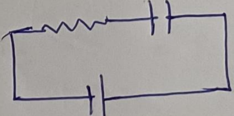
a) Consider a ckt containing resistance R and Inductance L in series with E then

By Kirchoff's law



$$Ri + L\frac{di}{dt} = E$$

b) Consider a ckt containing resistance R and capacitance C in series with voltage drop Emf E.



By Kirchof's law,
$$Ri + \frac{q}{C} = E$$

$$R \cdot \frac{dq}{dt} + \frac{q}{C} = E$$

c) $$\int e^{at} \sin bt \cdot dt = \frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt)$$

$$= \frac{e^{at}}{\sqrt{a^2+b^2}}\left(\frac{a}{\sqrt{a^2+b^2}}\sin bt - \frac{b}{\sqrt{a^2+b^2}}\cos bt\right)$$

$$= \frac{e^{at}}{\sqrt{a^2+b^2}} (\sin bt \cos\phi - \cos bt \sin\phi)$$

take $\cos\phi = \frac{a}{\sqrt{a^2+b^2}}$

$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}$

$$= \frac{e^{at}}{\sqrt{a^2+b^2}} \sin(bt-\phi)$$

$\therefore \tan\phi = \frac{b}{a}$

$\phi = \tan^{-1}\left(\frac{b}{a}\right)$

$$\int e^{at}\cos bt = \frac{e^{at}}{a^2+b^2}(a\cos bt + b\sin bt)$$

$$= \frac{e^{at}}{\sqrt{a^2+b^2}} \cos(bt-\phi)$$

$\phi = \tan^{-1}\left(\frac{b}{a}\right)$

Q: The equation of emf in terms of current i for electrical ckt having resistance r and condensor of capacity C in series is $E = Ri + \int \frac{i}{C} dt$ find current i at any time t when $E = E_0 \sin wt$

Given: $E = Ri + \int \frac{i}{c} dt$

$E_0 \sin wt = Ri + \int \frac{i}{c} dt$

diff w.r.t t.

$\frac{E_0 \cos wt \, w}{} = R\frac{di}{dt} + \frac{i}{c}$

dividing by R

$\frac{di}{dt} + \frac{1}{R}\frac{i}{C} = \frac{E_0 w \cdot \cos wt}{R}$

which is linear in i

$P = \frac{1}{RC}$, $Q = \frac{E_0 w \cos wt}{R}$

$I.f. = e^{\int \frac{1}{RC} \cdot dt} = e^{t/RC}$

→ Soln, $i . I.f. = \int Q . I.f. dt + C$

$i e^{t/RC} = \int \frac{E_0 w \cos wt \, e^{t/RC}}{R} dt + C$

$i e^{t/RC} = \frac{E_0 w}{R} \int e^{t/RC} \cos wt \cdot dt$

$i e^{t/RC} = \frac{E_0 w}{R} \frac{e^{t/RC}}{\sqrt{\frac{1}{R^2C^2} + w^2}} \cdot \cos(wt - \cancel{\theta}) + c$ $\theta = \tan^{-1}(wRC)$

Q. In ckt containing inductance L, resistance R, volltage $\mathcal{E}$ and diff-eq^n is

$$E = Ri + L\frac{di}{dt}$$

also given, $L = 640$ Hen. $R = 250\,\Omega$, $E = 500\,V$,

1) find time that lapces before it reaches 90% of it's maximum value. the current i being zero when $t=0$

2) show that the current will apporach to 2 Amp, whe $T \to \infty$

$\Rightarrow$ $$L\frac{di}{dt} + Ri = E$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$ which is linear in i

$$P = \frac{R}{L}, \quad Q = \frac{E}{L}$$

$$I.F. = e^{\int \frac{R}{L} \cdot dt}$$

$$I.F = e^{\frac{Rt}{L}}$$

Sol^n. $$i\, I.F. = \int Q \cdot I.F\, dt + C$$

$$i e^{\frac{Rt}{L}} = \int \frac{E}{L} \cdot e^{Rt/L}\, dt + C$$

$$i e^{\frac{Rt}{L}} = \frac{E}{\cancel{L}} \times \frac{\cancel{L}}{R}\, e^{Rt/L} + C$$

$$i e^{Rt/L} = \frac{E}{R} e^{Rt/L} + C$$

$$\therefore i = \frac{E}{R}\frac{\cancel{e^{Rt/L}}}{\cancel{e^{Rt/L}}} + \frac{C}{e^{Rt/L}} \quad \text{--- (1)}$$

$\therefore$ given that at $t=0$, $i=0$.

$$\therefore 0 = \frac{E}{R}\cancel{e^{0}} + C.$$

$$C = \frac{-E}{R}$$

from (1) $$i = \frac{E}{R} - \frac{E}{R}e^{-Rt/L} \quad \text{--- (2)}$$

maximum value of i be i_{max} which is obtain at $t \to \infty$

$i_{max} = \frac{E}{R}$

we have to find time t when current reaches to 90% of it max value

$\therefore i = \frac{90}{100} \times i_{max} = \frac{9}{10}\frac{E}{R}$

Eqⁿ (2) becomes.

$$\frac{9}{10}\frac{E}{R} = \frac{E}{R} - \frac{E}{R} e^{-Rt/L}$$

$$e^{-Rt/L} = \frac{1}{10}$$

$$-Rt/L = \log 10^{-1}$$

$$\frac{Rt}{L} = \log_e 10$$

$$t = \frac{L}{R} \log_e(10)$$

$$t = 5.89$$

$T \to \infty \quad i = i_{max}$

$$i = \frac{E}{R} = \frac{500}{250} = 2A.$$

-Eqⁿ is

-it

3 At 200 Ω resistance is connected in series with $C = 0.001$ F and Emf $E = 400 \times e^{-3t}$. if $Q = 0$ at $t = 0$ find time t when max charge is on the capacitor

→ Here, the ckt contains Resistance R and cap C.
By kirchoff's law, voltage law across R $= Ri$
across C $= \frac{q}{C}$ ~~= 400e^{-3t}~~

$$\therefore \quad Ri + \frac{q}{C} = 400e^{-3t}$$

put $R = 200$, $C = 1 \times 10^{-3}$.

$$200\frac{dq}{dt} + \frac{q}{RC} = \frac{400e^{-3t}}{R}.$$

$$\frac{dq}{dt} + 5q = 2e^{-3t}$$

$P = 5 \qquad Q = 2e^{-3t}.$

$$I.F. = e^{\int 5 \cdot dt} = e^{5t}.$$

Soln. $\quad q\, I.F. = \int 2e^{-3t} e^{5t} \cdot dt + K.$

$$qe^{5t} = e^{2t} + K$$

$$q = e^{-3t} + Ke^{-5t}.$$

when $t = 0$ $q = 0$

$$0 = 1 + K$$

$$K = -1$$

$$\therefore \boxed{q = e^{-3t} - e^{-5t}} \quad \text{—} (1).$$

for max charge $\frac{dq}{dt} = 0$

$$\therefore \frac{dq}{dt} = -3e^{-3t} + 5e^{-5t} = 0.$$

$$\frac{3}{5} = e^{-2t} \qquad t = \frac{1}{2} \log\left(\frac{5}{3}\right)$$