Using Maple in Calculus: Part 6

VOLUMES OF REVOLUTION

Maple has built into it a routine for finding and displaying volumes of revolution (around either the *x*- or *y*-axis). To use this routine, we must first issue the "with" command as follows:

```
with(Student[Calculus1]);
```

Maple gives us three choices for the "output":

(1) the actual volume of the 3D solid;
(2) the integral necessary for finding the volume; and
(3) a "plot" of the 3D solid.

The basic command has three parameters: (1) the function $f(x)$, (2) the left end of the curve $x = a$, and (3) the right end of the curve $x = b$. The default axis of revolution is the *x*-axis. The basic command then is given by:

```
VolumeOfRevolution(f(x), x = a..b);
```

Example: Find the volume generated by revolving the curve $f(x) = x^2$ between $x = 2$ and $x = 5$ around the *x*-axis.

The command for doing this is simply:

```
VolumeOfRevolution(x^2, x = 2..5);
```

and we get $\frac{3093}{5}\pi$

To see the integral that is used to compute this volume, we need to add an "option" to the command, namely, "output = integral":

```
VolumeOfRevolution(x^2, x = 2..5, output=integral);
```

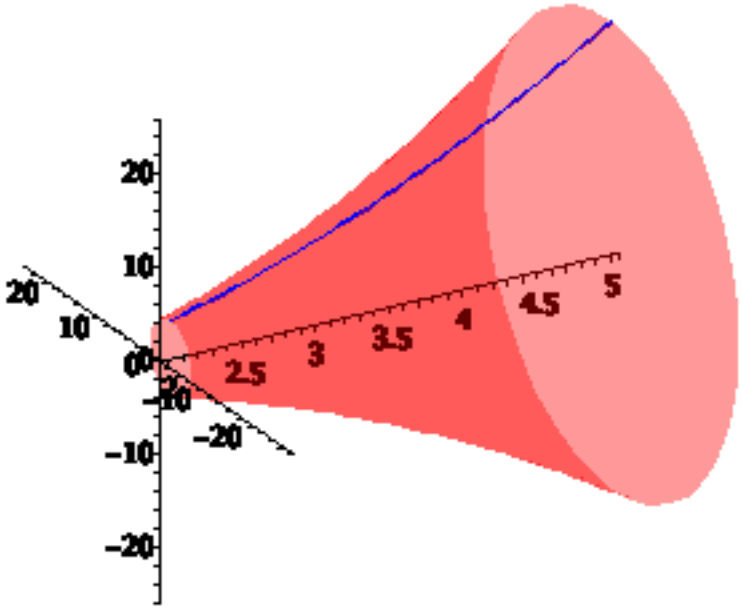
which gives us the answer $\int_2^5 \pi x^4 dx$.

Our third option is to have Maple give us a three-dimensional "plot" of the solid. This time the option is given by "output=plot":

```
VolumeOfRevolution(x^2, x = 2..5, output=plot);
```

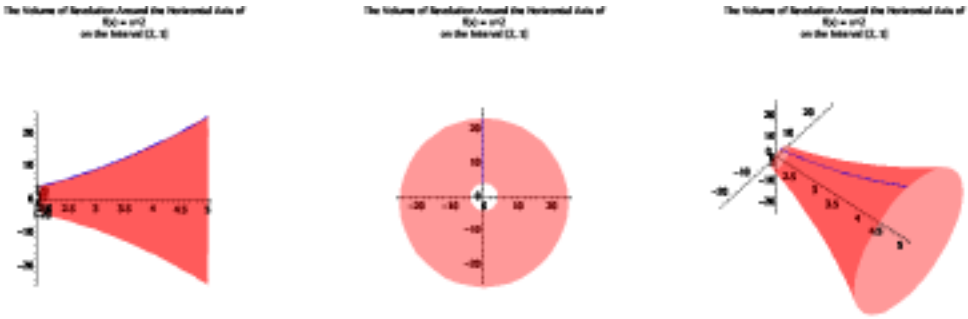
The result is shown on the following page.

The Volume of Revolution Around the Horizontal Axis of
f(x) = x^2
on the Interval [2, 5]



The blue curve represents the piece of the curve that we rotated. The red shape represents the resulting solid.

This plot is "moveable." By clicking and dragging with the mouse, we are able to rotate this graph to see it from any angle.



To revolve the curve around the *y*-axis instead, we need to add "axis=vertical" to the VolumeOfRevolution command.

Example: Find the volume generated by revolving the curve $f(x) = \sin x$ between $x = 0$ and $x = \pi$ around the *y*-axis.

```
VolumeOfRevolution(sin(x), x = 0..Pi, axis=vertical);
```

which gives us the somewhat surprising answer of 2π.

As before, we can add the "output=integral" option to get the necessary integral or the "output=plot" option to get the graph.

VOLUMES OF REVOLUTION BETWEEN TWO CURVES

To find the volume generated by revolving an area between two curves, we simply have to add the second function to the list of parameters.

Example: Find the integral necessary to find the volume generated by revolving the area between the curves $f(x) = x^2$ and $g(x) = x^3$ between $x = 0$ and $x = 1$ around the *x*-axis.

```
VolumeOfRevolution(x^2, x^3, x = 0..1, output=integral);
```

Answer: $\frac{2}{35}\pi$

NOTE 1: Even though these two curves intersect at $x = 0$ and $x = 1$, thereby definitively defining the area to be revolved, Maple still requires that we insert the `x = 0..1` part of the command.

NOTE 2: Maple assumes that the first function is the "top" function and the second one is the "bottom" function. If entered in the wrong order, the result will be negative.

MODIFYING THE GRAPH

We can modify the way the graph looks by adding some "plot3d/options."

Labeling the Axes

To add labels to the axes, add the `labels=[x,y,z]` option to the command:

```
VolumeOfRevolution(x^3, x=0..1, output=plot, labels=[x,y,z]);
```

Changing the Thickness of the Curve

To change the thickness of the curve, add the `thickness=n` option to the command, where `n` is an integer between 1 and 10. The default setting is 1.

```
VolumeOfRevolution(x^2, x=0..1, output=plot,thickness=3);
```

Removing the Axes

To eliminate the axes from the graph, add the **axes=none** option to the command.

```
VolumeOfRevolution(cos(x), x=0..Pi/2,output=plot,axes=none);
```

Putting a Box around the Graph

Sometimes it may be helpful to put a box around the graph, especially to help visualize the three-dimensional nature of the solid as you move and rotate it with the mouse. To do so, add the **axes=boxed** option to the command.

```
VolumeOfRevolution(ln(x), x=1..2, output=plot, axes=boxed);
```