Using Maple in Calculus: Part 3

FINDING DERIVATIVES

Finding derivatives in Maple is very easy. As before, the best route to take is to first define the function and then issue the following derivative command:

```
diff(f(x),x);
```

where **f(x)** is the function you previously defined, and **x** is the independent variable in the function.

Example: Find the derivative of $f(x) = 3x^2 - 5x + 2$.

We first enter the function into Maple using the following:

```
f:=x->3*x^2-5*x+2;
```

(Remember that we have to use ***** for multiplication and **^** for exponents.)

We next issue the derivative command:

```
diff(f(x),x);
```

and we get:

```
6x - 5
```

(which is exactly what we had expected).

SIMPLIFYING ANSWERS

For more complicated functions, Maple does not automatically give a “pretty” answer.

Example: Find the derivative of $f(x) = \dfrac{2x-1}{3x+2}$.

We first enter the function into Maple using the following:

```
f:=x->(2*x-1)/(3*x+2);
```

(Remember that we have to use parentheses around both the numerator and denominator.)

We next issue the derivative command:

```
diff(f(x),x);
```

and we get:

$$2\frac{1}{3x+2}-3\frac{2x-1}{(3x+2)^2}$$

which I think you will agree is not very “pretty”!

We can force Maple to simplify this answer by issuing the **`simplify`** command. However, we have a slight problem. The simplify command expects us to tell us the name of what we want to simplify. Since we didn’t give a name to the derivative (we just told Maple to find the derivative), we don’t have any name to refer to.

Fortunately, since we have the derivative just sitting there on the last line, there is a way to tell Maple to simplify the expression on that last line. Anytime (not just for simplifying) you want Maple to perform some command on the last line, simply use a percent sign (`%`) to refer to the expression on that last line.

In the current example, all we have to do is enter the following:

```
simplify(%);
```

and we will get a “prettier” answer:

$$7\frac{1}{(3x+2)^2}$$

which isn’t perfect (we’d rather have $\frac{7}{(3x+2)^2}$), but it is a definite improvement.

HIGHER-ORDER DERIVATIVES

To find a higher-order derivative, such as the second derivative, we only need to add another **`x`** to the command.

Find the second derivative $f''(x)$ for $f(x)=\frac{1}{x^2}$.

As always, we first define the function in Maple:

```
f:=x->1/x^2;
```

We next issue the derivative command, but add an extra **`x`**:

```
diff(f(x),x,x);
```

and we get:

$$6\frac{1}{x^4} \text{ or } \frac{6}{x^4}$$

To find the third derivative, just add another **x**, and to find the fourth derivative, another **x**, and so on. Obviously, this could get tiresome for the 10th derivative, for example. Fortunately, there is another way to get a higher-order derivative, especially when the order is larger than 3. The inclusion of a dollar sign (**$**) and the number signifying the order does the job. The following example demonstrates the command.

Example: Find the eighth derivative $f^{(8)}(x)$ for $f(x) = \sqrt{x}$.

First define the function:

```
f:=x->sqrt(x);
```

Next issue the derivative command, modifying it to indicate we want the eighth derivative:

```
diff(f(x),x$8);
```

We get:

$$-\frac{135135}{256}\frac{1}{x^{(15/2)}}$$

which know can (and should) be rewritten as:

$$\frac{-135135}{256\sqrt{x^{15}}} \quad \text{or} \quad \frac{-135135}{256x^7\sqrt{x}}$$

TANGENT LINES

Now that we know how to find the derivative, we can use it to find and graph a tangent line to the curve at any point. Remember that one interpretation of the derivative is that it is a *formula* for the slope of the tangent line for any value of *x*.

Example: Find the slope of the tangent line to the graph of $f(x) = 3x^2 - 5x + 1$ when $x = 2$.

First define the function:

```
f:=x->3*x^2-5*x+1;
```

Since we want to use the derivative later on, this time we are going to "name" it so that we can refer back to at any later time. Since we are finding the first derivative, we are going to call it **f1** (however, you can use any name you like):

```
f1:=diff(f(x),x);
```

and we get:

```
6x - 5
```

Now anytime we want to do anything with the derivative, we can simply refer to it as **f1**.

What we want to do with it now is evaluate it for $x = 2$. We do this by issuing the **subs** command:

```
subs(x=2,f1);
```

which gives us an answer of **7**. Recall that this number is the slope of the tangent line when $x = 2$.

Now that we have the slope of the tangent line, we can write the equation of the tangent line using the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point on the graph that the tangent line runs through.

Our only problem is that we don't know what y_1 is. But y_1 is simply the value of the function for $x = 2$, and we get that value by issuing the **subs** command on the *original* function (not the derivative):

```
subs(x=2,f(x));
```

We get a value of **3**.

Now we have all the pieces: the point $(2,3)$ and the slope $m = 7$. Putting them into the point-slope form, we finally get:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= 7(x - 2) \\ y - 3 &= 7x - 14 \\ y &= 7x - 11 \end{aligned}$$

The last thing we might want to do in this example is graph the original function and the line we just found to verify that it indeed does look like a tangent line. We do this by first entering the equation of the tangent line into Maple as a function (we used the letter *t* for tangent, but you may use any name you like):

```
t:=x->7*x-11;
```

Finally, we issue the **`plot`** command to graph both the original function $f(x)$ and the tangent line $t(x)$:

```
plot([f(x),t(x)],x=-1..3,y=-2..5,scaling=constrained);
```

(Because the point in question is $(2,3)$, the values for *x* were chosen to be a little to the left and right of $x = 2$ and the values for *y* to be a little above and below $y = 3$. The added option **`scaling=constrained`** forces Maple to use the same scale for the *x* and *y* axes.)