Using Maple in Calculus: Part 5

THE RECTANGLE METHOD FOR FINDING AREA

Maple has built into it a routine for finding the area under a curve by using the rectangle method. As discussed in class, we can use either the left end, right end, or middle of each rectangle for the height of each small rectangle in our sum. In order to use this routine, we must first "load" it with the following command:

```
with(student);
```

The command for finding the sum of the areas of the small rectangles has four parts:

(1) the function, defined as $f(x)$ in what follows;
(2) the left x-value, denoted by a;
(3) the right x-value, denoted by b; and
(4) the number of small rectangles, denoted by n.

With these four parameters, the command for finding the sum of the small rectangles (using the right ends as the heights) is given by:

```
evalf(rightsum(f(x),x=a..b,n));
```

Example: Use the rectangle method to approximate the area under the curve $f(x) = x^2$ between $x = 0$ and $x = 1$, using 4 small rectangles.

First we define $f(x)$ in the usual way:

```
f:=x->x^2;
```

Then we issue the new command:

```
evalf(rightsum(f(x),x=0..1,4));
```

and we get the answer

.4687500000

Since we have the function defined, we can easily change the number of small rectangles to 1000 or 1,000,000, or more, and Maple will dutifully calculate the sum of the areas for us:

```
evalf(rightsum(f(x),x=0..1,1000));
```

(ans: .3338335000)

```
evalf(rightsum(f(x),x=0..1,1000000));
```

(ans: .3333338333)

From this, it appears as if the answer is approaching 1/3 as the number of small rectangles gets larger and larger.

To use the left side of the small rectangles as the heights, we use:

```
evalf(leftsum(f(x),x=a..b,n));
```

and for the middle of the small rectangles as the heights, we use:

```
evalf(middlesum(f(x),x=a..b,n));
```

ANTIDERIVATIVES (INDEFINITE INTEGRALS)

Maple can compute antiderivatives, although it will omit the arbitrary constant. The command is:

```
int(f(x),x);
```

Example: To find the antiderivative of $f(x) = x^2$, we first define $f(x)$ in the usual way:

```
f:=x->x^2;
```

Then we issue the **int** command:

```
int(f(x),x);
```

to get

$$\frac{1}{3}x^3$$

We can also find the antiderivative without first defining $f(x)$.

Example: To find the antiderivative of $f(x) = \cos x$, we simply issue the command:

```
int(cos(x),x);
```

to get

$$\sin x.$$

DEFINITE INTEGRALS

Maple can also compute definite integrals. We need only add the two limits of integration. The command is:

```
int(f(x),x=a..b);
```

Again, we can either define $f(x)$ first or enter it directly into the command.

Example: Find the value of the definite integral of $f(x) = x^2 - x - 2$ from $x = -1$ to $x = 2$.

Since $f(x) = x^2 - x - 2$, $a = -1$, and $b = 2$, the command is simply:

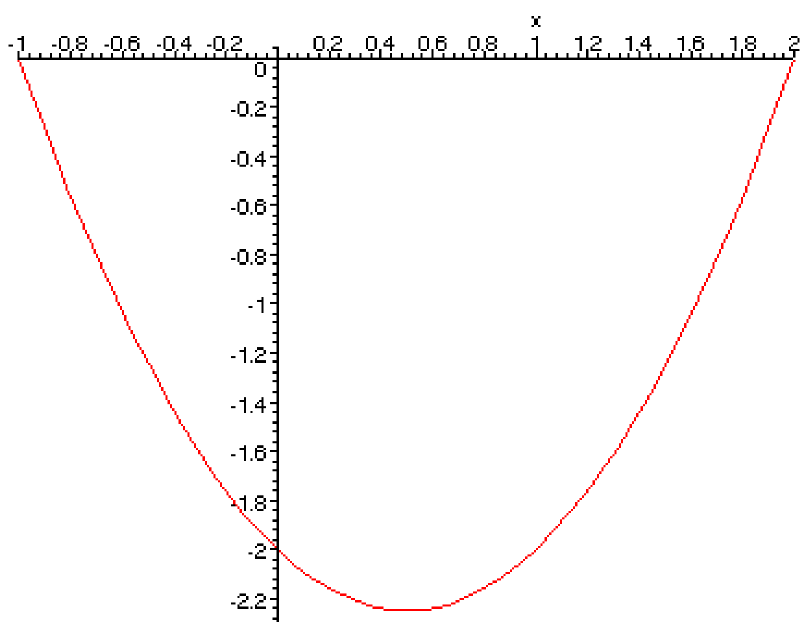
```
int(x^2-x-2,x=-1..2);
```

which yields an answer of $\frac{-9}{2}$.

To "explore" why the answer is negative, we can graph the function using the command:

```
plot(x^2-x-2,x=-1..2);
```

The result is the following:



Since the graph is entirely below the x-axis, we shouldn't be surprised by the negative answer.

In some situations, we may want a decimal answer (i.e., "floating point") instead of an exact answer (i.e., fractions, radicals, etc.). In this case, we simply add the **`evalf`** command in front of the **`int`** command:

```
evalf(int(f(x),x=a..b));
```

Example: Find the decimal value of $\int_{\pi/6}^{\pi/4} \sin(3x)\, dx$.

Maple command: **`evalf(int(sin(3*x),x=Pi/6..Pi/4));`**

Answer: **`.2357022604`**

Note: the exact answer is $\frac{1}{6}\sqrt{2}$ (found by not including the **`evalf`** command).