Using Maple in Calculus: Part 2

FINDING LIMITS BY MAKING A TABLE OF VALUES

Just as we did in class, we can "explore" a limit In Maple by looking at a table of values. After first defining the function using the `f:=x->` command, we then use the `f(x)` command to find the y-value corresponding to the chosen x-value. (All of this is detailed in Part 1.)

Example: Conjecture the value of $\lim_{x\to 2}\frac{x^2-4}{x-2}$ by making a table of values.

We first enter the function into Maple using the following:

```
f:=x->(x^2-4)/(x-2);
```

We next take values of x close to 2 and compute the corresponding y-values, and put the results in a table. To find the y-values, we simply enter:

```
f(1.9);
f(1.99);
f(1.999);
f(2.1);
f(2.01);
f(2.001);
```

Putting the results in two tables, one for approaching from the left and another from the right, we get the following:

x	y
1.9	3.9
1.99	3.99
1.999	3.999

x	y
2.1	4.1
2.01	4.01
2.001	4.001

From these tables, we are ready to conjecture that $\lim_{x\to 2}\frac{x^2-4}{x-2}=4$.

FINDING LIMITS BY GRAPHING

Just as we did in class, we can "explore" a limit in Maple by looking at the function's graph. After first defining the function using the **`f:=x->`** command, we then use the **`plot`** command to graph the function. (All of this is detailed in Part 1.)

Example: Conjecture the value of $\lim_{x \to 0} \frac{\sin x}{x}$ by looking at its graph.

We first enter the function into Maple using the following:

```
f:=x->sin(x)/x;
```

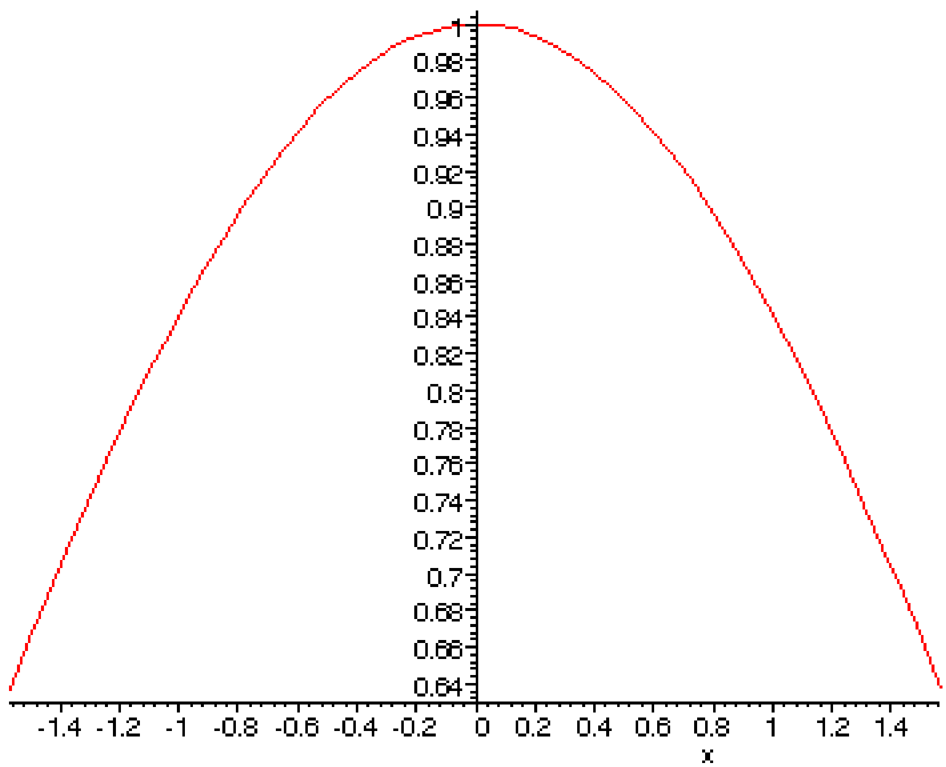
We next plot the graph with the following command:

```
plot(f(x),x=-Pi/2..Pi/2);
```

From the graph below, we can "see" that the limit appears to be 1, that is,

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

However, as we know, a graph only helps us *estimate* a value for a limit. It may or may not be the actual value.



FINDING LIMITS DIRECTLY IN MAPLE

Limits are found in Maple by using the `limit` command. After first defining the desired function (as detailed in Part 1), the `limit` command can then be used to evaluate that limit as x approaches the desired number. There are options for specifying that the limit is approaching from the left or from the right and for approaching $+\infty$ or $-\infty$, as shown in the examples that follow. The full syntax is `limit(f(x), x=a, [left,right])`.

Example: Find $\lim_{x\to 2} \frac{x^2-4}{x-2}$.

First define the function:

```
f:=x->(x^2-4)/(x-2);
```

Next use the `limit` command:

```
limit(f(x), x=2);
```

Maple returns a value of 4, that is,

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$$

FINDING ONE-SIDE LIMITS

By the addition of the word `left` or `right`, you can tell Maple to find a one-sided limit. First define the function as before, then issue the `limit` command to which you have added the word `left` or `right`.

Example: Find $\lim_{x\to 1^-} \frac{1}{1-x}$.

First define the function:

```
g:=x->1/(1-x);
```

Next use the `limit` command:

```
limit(g(x), x=1, left);
```

Maple returns a value of ∞, that is,

$$\lim_{x\to 1^-} \frac{1}{1-x} = +\infty$$

Example: Find $\lim\limits_{x \to 1^+} \dfrac{1}{1-x}$.

This is the same function as the previous example, so it has already been defined and is residing in Maple's memory as **`g(x)`**. Consequently, we only need issue the **`limit`** command with the word **`right`** included.

```
limit(g(x), x=1, right);
```

Maple returns a value of $-\infty$, that is,

$$\lim_{x \to 1^-} \frac{1}{1-x} = -\infty$$

NOTE: The previous two examples illustrate why we get ***undefined*** as the answer for $\lim\limits_{x \to 1} \dfrac{1}{1-x}$ (since the two one-sided limits are different).

Example: Find $\lim\limits_{x \to 2^+} \dfrac{4-x^2}{|x-2|}$.

First define the function, recalling that **`abs(x)`** is used for $|x|$ in Maple:

```
h:=x->(4-x^2)/abs(x-2);
```

Next find the limit:

```
limit(h(x),x=2,right);
```

Maple returns a value of –4, that is, . $\lim\limits_{x \to 2^+} \dfrac{4-x^2}{|x-2|} = -4$

LIMITS AT INFINITY

To find a limit as x increases or decreases without bound (approaches ∞ or $-\infty$), set the x-value equal to either `infinity` or `-infinity` in Maple.

Example: Find $\lim\limits_{x \to \infty} \dfrac{\sin x}{x}$.

First define the function:

```
f:=x->sin(x)/x;
```

Next find the limit:

```
limit(f(x),x=infinity);
```

Maple returns a value of 0, that is, $\lim\limits_{x \to \infty} \dfrac{\sin x}{x} = 0$.

Example: Find $\lim\limits_{x \to \infty} \dfrac{3x^2 - 5x + 2}{2x^2 + 7x - 1}$.

First define the function:

```
A:=x->(3*x^2-5*x+2)/(2*x^2+7*x-1);
```

Next find the limit:

```
limit(A(x),x=infinity);
```

Maple returns a value of $\frac{3}{2}$, that is, $\lim\limits_{x \to \infty} \dfrac{3x^2 - 5x + 2}{2x^2 + 7x - 1} = \dfrac{3}{2}$.

Example: Find $\lim\limits_{x\to-\infty}\dfrac{\sqrt{e^x}}{x}$.

First define the function, recalling that **`sqrt(x)`** is used for the $\sqrt{x}$ in Maple, and **`exp(x)`** is used for e^x:

```
g:=x->sqrt(exp(x))/x;
```

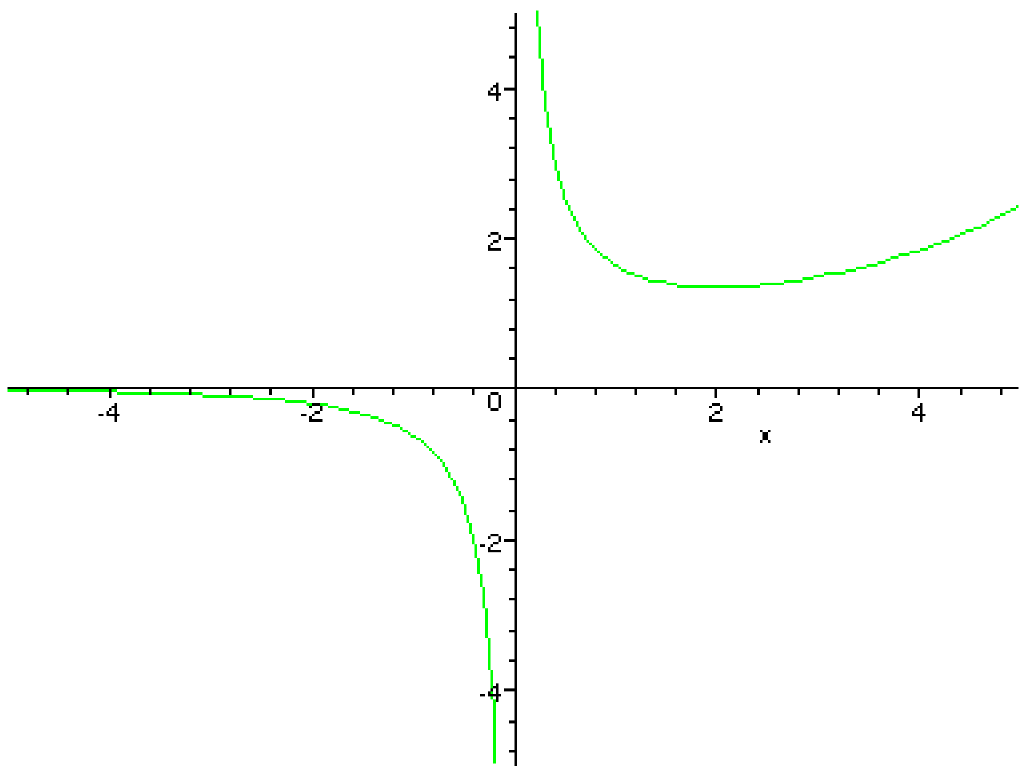
Next find the limit:

```
limit(g(x),x=-infinity);
```

Maple returns a value of 0, that is, $\lim\limits_{x\to-\infty}\dfrac{\sqrt{e^x}}{x}=0$.

Note that the graph appears to support this conclusion:

```
plot(g(x),x=-5..5,-5..5,discont=true);
```



PIECEWISE-DEFINED FUNCTIONS

You can use Maple to define piecewise-defined functions. Once defined, you can then graph them, evaluate them, and find limits involving them. The syntax for defining a piecewise-defined function is as follows:

```
f:=->piecewise(1st condition, 1st piece, 2nd condition,
     2nd piece, 3rd condition, 3rd piece, etc.);
```

where "condition" means an inequality involving *x*. Maple requires that you enter inequalities in a certain way:

$x < a$	**x<a**
$x \le a$	**x<=a**
$x > a$	**x>a**
$x \ge a$	**x>=a**

For so-called "double inequalities," separate the inequality into two parts and join them with a logical **and** (the same way as on a graphing calculator). Example: $-2 \le x < 3$ becomes:

```
-2<=x and x<3
```

Example; $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3x-5 & \text{if } x \ge 2 \end{cases}$ is entered into Maple as:

```
f:=x->piecewise(x<2,x^2,x>=2,3*x-5);
```

Once defined, you can evaluate $f(x)$ in the normal way, and Maple will automatically choose the "right" piece.

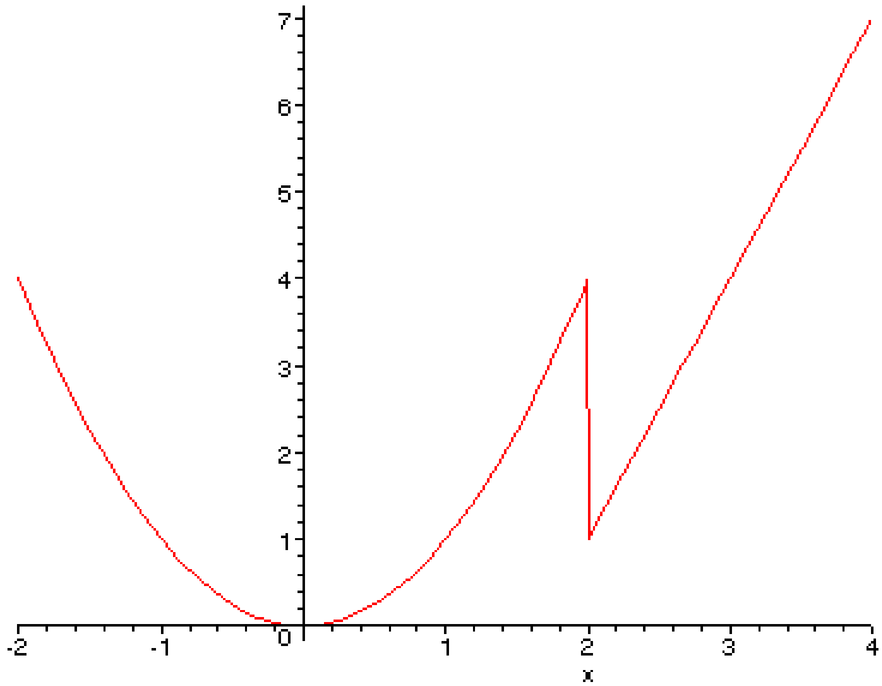
f(1); yields a value of 1, which is 1^2

f(2); yields a value of 1, which is $3(2)-5$

f(3); yields a value of 4, which is $3(3)-5$

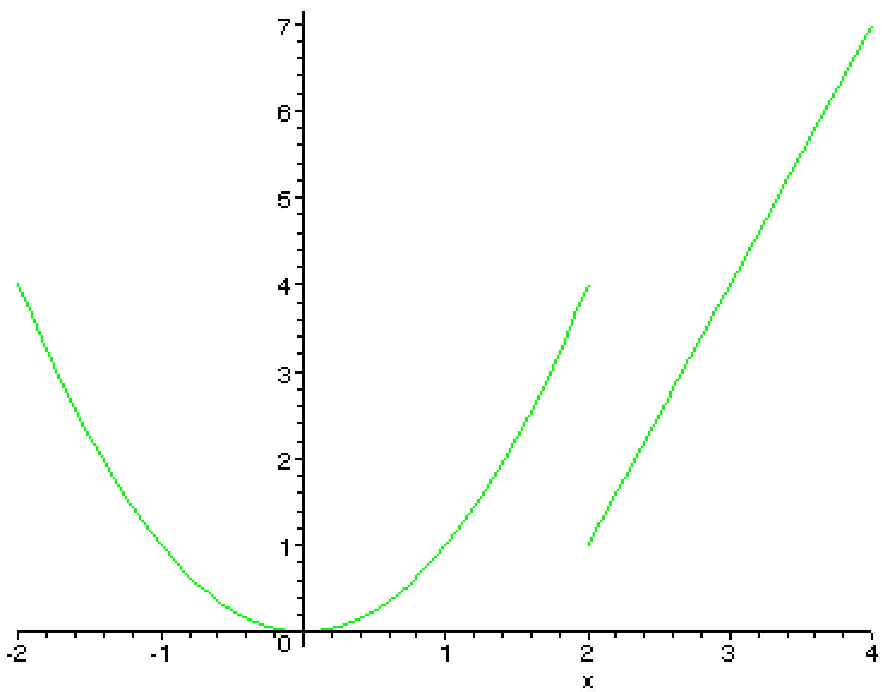
You can also graph $f(x)$ in the normal way:

```
plot(f(x),x=-2..4);
```



Note that the discontinuity at 2 is "connected" by a vertical line, which we know is not correct. We remove this line in the usual way, by adding **`discont=true`** to the **`plot`** command:

```
plot(f(x),x=-2..4,discont=true);
```



To find a limit, we proceed in the normal way. Since the “action” is at 2, we will find the limit as x approaches 2.

Find $\lim_{x \to 2} f(x)$

Since we have already defined $f(x)$, we simply enter

```
limit(f(x),x=2);
```

Not surprisingly, Maple gives us an answer of *undefined* (since the left and right limits are not the same as seen on the graph).

If we specify left or right, we will get actual values.

```
limit(f(x),x=2,left);
```

which gives an answer of 4 as the graph indicates.

```
limit(f(x),x=2,right);
```

which gives an answer of 1 as the graph again indicates.

Example: $g(x) = \begin{cases} 2x & \text{if } x < -1 \\ x^3 & \text{if } -1 \le x < 2 \\ x+3 & \text{if } x \ge 2 \end{cases}$ is entered into Maple this way:

```
g:=x->piecewise(x<-1,2*x,-1<=x and x<2,x^3,x>=2,x+3);
```

Find $\lim_{x \to -1^-} g(x)$. In Maple, we enter

```
limit(g(x),x=-1,left);
```

and we get $\lim_{x \to -1^-} g(x) = -2$.