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USE OF IT & MAPLE IN TEACHING PRE-CALCULUS

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Abstract: Use of technology in the classroom now a days, is becoming more popular for almost all levels of education, particularly at university. With Mathematics, IT is a great help to make this essential subject for all disciplines more live, realistic, and accessible, and with the aid of Computer Software and tools, modules of Mathematics becoming easier to understand and acceptable/live.
At Zayed University in UAE one of the Learning Out Comes of this University is to assure graduates will be critically aware of the implication of IT on individual and society, so be able to use IT in communication and solve problems in an ethical way .
With this principle in mind, MS Power Point and MAPLE software have been used for teaching Pre-calculus, along with teacher Web Site on the University Intranet, which had a great impact on learning results, as my pedagogical approach included a comprehensive overview of concepts and presenting solutions to examples from multiple perspectives of Algebraic, Graphics, and Numeric.
Students have shown enhanced capability to learn and apply mathematical methods in solving real life problems and also utilizing such methods in critical thinking and reasoning to make decision.
For example, in evaluating and graphing different kinds of functions by MAPLE (Mathematics Software). I usually ask my students to use this software in solving their exercises, and when difficulties experienced by the students in doing so, interactive action takes place by working with the students on their Laptops in the class to achieve better understanding of the subject, and overall comprehension.

INTRODUCTION

It is quite common within the community of students, and society in general, that mathematics is a complicated and difficult subject to grasp. This barrier of fear from mathematics have been the subject of discussion world wide, as well as many conferences and seminars related to education.
With this in mind I have, like many others teaching mathematics, always thought of ways and means of making mathematics simple, interesting, lively and attractive. Thus innovative tools were utilized so as to achieve the goals of making life easy with mathematics, such as the use of simple slide rule, in the old days, to utilization of technology now a days.
At university level the above mentioned psychological barrier is even stronger when you talk to students of art and political sciences, particularly those doing majors in business, finance and information systems. The use of technology helped in minimizing the fear, and encouraged students to play with the tools of mathematics to achieve results in problem solving. In this paper I am trying to demonstrate an example of using a software (namely MAPLE), with the aid of lap-top in the class room as an interactive tool, to help students in gaining confidence of handling mathematical problems. For instance in the field of graphing, which is useful for real life problems, the effect and results measured through talking to students at Zayed University proved to be positive and encouraging.

PRE-CALCULUS

Pre-Calculus introduce and perform mathematical operations on different mathematical functions representing these models to produce meaningful results. In Pre-calculus different kinds of functions can be recognized, some important basic characteristics of functions are illustrated, also solve , graph and principles related to above functions are understood. Functions like Linear, Quadratic, Polynomials, Rational, Exponential, Logarithmic and Trigonometric Functions.

In this paper I am tackling Exponential functions and their graphs, how to recognize and evaluate exponential function with base a (Standard Exponential Function). How to graph this kind of function and its transformation in the same window. Also introduce the inverse of exponential function which is a Logarithmic function, taking in consideration the characteristics of this function.

Using IT tools for productivity is essential to a professional standard. It is more challenging and interesting to show students available ways of using Technology in gaining their knowledge , such as Computer Software to help them in solving problems and exploring further new knowledge. So I have , like others in Zayed University utilized Maple.

MAPLE

"MAPLE is a Symbolic Computation System or Computer Algebra System. Both phrases refer to MAPLE ability to manipulate information in a symbolic or algebraic manner. Conventional mathematical programs require numerical values for all variables. By contrast MAPLE maintains and manipulates the underlying symbols and expressions"[(1)], so we can use this software to estimate and obtain exact analytical solutions to many mathematical problems.

Like every other Software, such as Mathematica, MATLAB, etc, MAPLE has the capability of tackling mathematical problems, namely:

1. Check and validate answers to problems obtained by using algebraic methods.
2. Approximate solutions to equations involving functions.
3. Perform complicated mathematical procedures, such as those found in many real life applications.
4. Evaluate and simplification of expressions, sorting algebraic assumptions, etc.
5. Finding exact solutions, including simple solve, numerical solve (fsolve). Expanding polynomials as sums, collecting the coefficients of like degrees, root finding and factorization.
6. Graph functions (graphing in two dimensions 2D , three dimensions 3D, animation and manipulating graphical objects).
7. Handling calculus examples, including derivative, integral, differential equations and partial differential equations.

With Pre-Calculus, in particular, this Software can be used to utilize most of the aforementioned capabilities.

Functions are the heart of calculus in general, It is a rule that assigns to each number x in the domain, a unique number f(x). MAPLE recognizes functions, and has a special notation for defining them, which is a bit different than the usual notation that we use in calculus.

For example to define a function in MAPLE we use **f:=x->rule**. Where f is the name of the function, and x is the independent variable, and **rule** is the rule that defines how the function works. MAPLE also recognize exponential, natural exponential and logarithmic functions by their standard names exp and lin respectively.

Some Applications From My Lessons

The most effective way to use MAPLE or any other software in the classroom, is through presentation on a screen that students can see properly. In the mean time explain and solve on the white board. As an example some of my class work, which is a bout The Exponential Functions, are shown hereunder.

To introduce any new topic and likewise exponential functions, I regularly start with a Power Point Presentation through which Definition and Characteristics of this function explained , also

showing the graphs of these functions and comparing them, how transformations and reflections on the axis is changing the expression of that exponential functions. What we mean by the inverse of a function, and what is the inverse of exponential function , how we can see the relationship between them in one graph.
(Example of this Power Point Presentation is illustrated below).

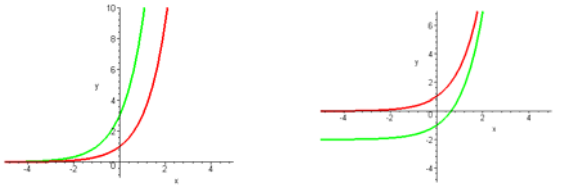
Exponential Functions

Exponential Functions and Their Graphs
Definition
The Exponential Function *f* with base a is denoted by
$f(x) = a^x$ where $a > 0$, $a \neq 1$ and x is any Real No.
If $a > 1$ the function is **Exponential Growth Function**.
If $0 < a < 1$ the function is **Exponential**

Graphs of Exponential Functions
$f(x) = 2^x$ **$F(x) = 2^{-x}$**
$g(x) = 4^x$ **$G(x) = 4^{-x}$**

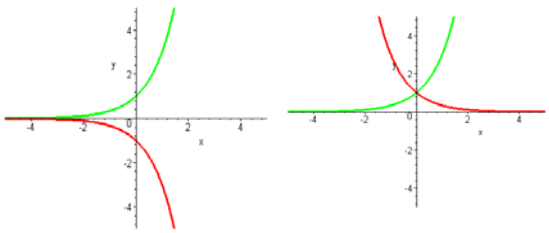
Comparing the functions:
$F(x) = 2^{-x} = f(-x)$ ***and***
$G(x) = 4^{-x} = g(-x)$
The graph of F is a reflection of the graph *f* (*in y-axis*).
The Graph of exponential functions have: one y-intercept and one horizontal Asymptote (x-axis).

Transformations of Graph of Exponential Functions:
$g(x) = 3^{x+1} = f(x+1)$ $h(x) = 3^x - 2 = f(x) - 2$

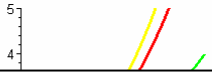


Exponential Functions and its Reflections

$k(x) = -3^x = -f(x)$ $j(x) = 3^{-x} = f(-x)$



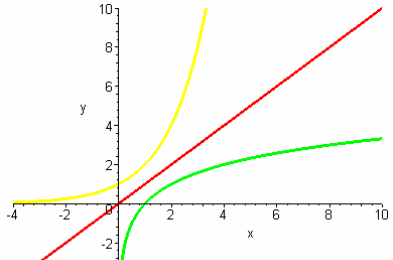
The Natural Base e
$e \approx 2.71828$ **this is called the Natural Base**
The function $f(x) = e^x$ is the Natural Exponential Function



The basic characteristics of exponential functions are:

Graph of $y = a^x$, $a > 1$	Graph of $y = a^{-x}$, $a > 1$
•Domain: ($-\infty, \infty$)	- Domain:($-\infty, \infty$)
•Range: ($0, \infty$)	- Range: ($0, \infty$)
•Intercept: (0, 1)	- Intercept: (0, 1)
•Increasing	- Decreasing
•x-axis is a horizontal horizontal asymptote	- x-axis is a horizontal asymptote
•Continuous	-Continuous

Exponential and Its Inverse Functions In One window



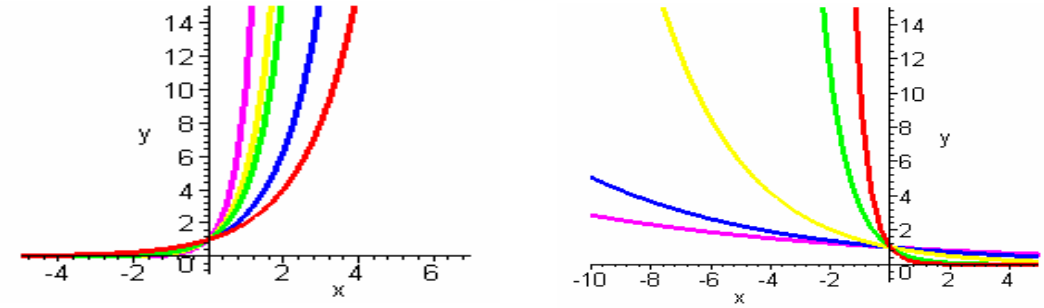
Following this explanation, its helpful to demonstrate the capabilities of MAPLE in this area, by graphing some exponential and transformed exponential functions.
How to solve these functions visualize their characteristics and also how to find its inverse . In the following pages a MAPLE work sheet showing us what we mentioned above.

Exponential Functions and Their Graphs

1) **Use Maple to graph different exponential functions in one window,**
if a > 0 and a is not = 1

```
plot([2^x,4^x,5^x,2.5^x,10^x],x=-5..7,y=-1..15);
```

if 0< a < 1 and a is not = 1



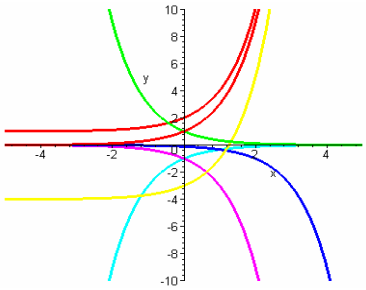

```
plot([0.1^x,0.3^x,0.7^x,0.85^x,0.9^x],x=-10..5,y=-1..15);
```

2) Graph of the standard function and its transformations

a) Graph the standard function f(x) = 3^x and the transformations below in the same window,

3^{-x}, 3^x-4, -3^{x-2}, -3^x, -3^{-x}, 3^x+1

```
>plot([3^x,3^(-x),3^x-4,-3^(x-2),-3^x,-3^(-x),3^x+1],x=-
 5..5,y=-                                          10..10);
```




```
fsolve(3^x-4,x);
```

1.261859507

```
> fsolve(-3^(-x),x);
```

b) Using the above graph identify the following features of the functions:
Asymptotes, Intercepts, Increasing and decreasing intervals

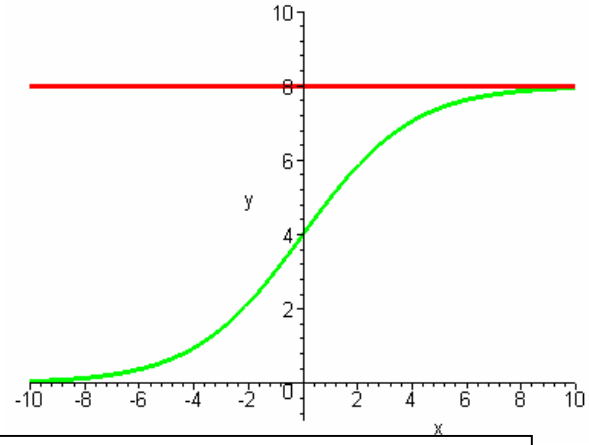
3) Graph the function and find any asymptotes numerically by Constructing a table of values for the function $f(x)=\frac{8}{1+e^{-0.5x}}$

```
f:=x->(8/(1+exp(-0.5*x)));
```

$$f := x \to 8\,\frac{1}{1+e^{(-.5\,x)}}$$

➢ `plot({f(x),8},x=-10..10,y=-.10);`

	A	B	C
1	x	$8\,\frac{1}{1+e^{(-.5\,x)}}$	
2	-30	$.2447217816\ 10^{-5}$	
3	-20	.0003631829497	
4	-10	.05354280739	
5	0	4.000000000	
6	10	7.946457193	
7	20	7.999636817	
8	30	7.999997552	
9	40	7.999999984	
10	50	8.000000000	
11	55	8.000000000	



Asymptotes are at y = 0 and y = 8

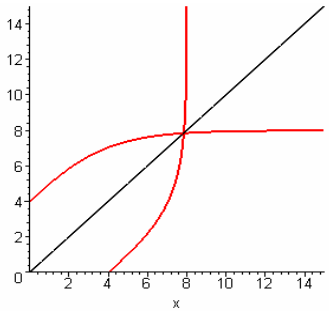
To find the Inverse of above function, the following commands can be used:

> `with(plots):with(plottools):` Warning, the name arrow has been redefined

> `f1:=x->(8/(1+exp(-0.5*x)));`

$$f1 := x \to 8\,\frac{1}{1+e^{(-.5\,x)}}$$

```
> l1:=plot(x,x=0..15,colour=black,linestyle=3,thickness=2):>
p1:=plot(f1(x),x=0..15,y=0..15,thickness=2):
p2:=reflect(p1,[[0,0],[1,1]],thickness=2):
display([l1,p1,p2]);
```



EFFECTS/CONCLUSION Some use Graphing Calculator, others Mathematical Software such as MAPLE, as valuable tools to visualize mathematical principles, verifying solution to equation, exploring mathematical ideas, and developing mathematical modules.

1) One way to view the importance of mathematical software is through graphing: Graphs of functions in general and exponential functions in particular is important in developing students mathematical imagination, literacy and capabilities in this area.

   The other way is through solving exponential function finding a solution and relating the graphs with this solution , where the curve cutting the x-axis .

It is noticed that students are more motivated to this subject, and work more actively and creatively.

3) Comprehend and apply mathematical software as a tool for thinking, through: first

exploring graphs related to exponential functions as in above examples, this leads to discover more through consulting with peers, or performing trial and error test, then , they can analyze, interpret the result of problems related to exponential functions.

REFERENCES 1- MAPLE V, Learning Guide, by K.M. Heal, M.L. Hansen, K.M. Rickard., 2- Pre-Calculus, Functions and Graphs 3rd Edition, by Larson, Hostetler, Edwards