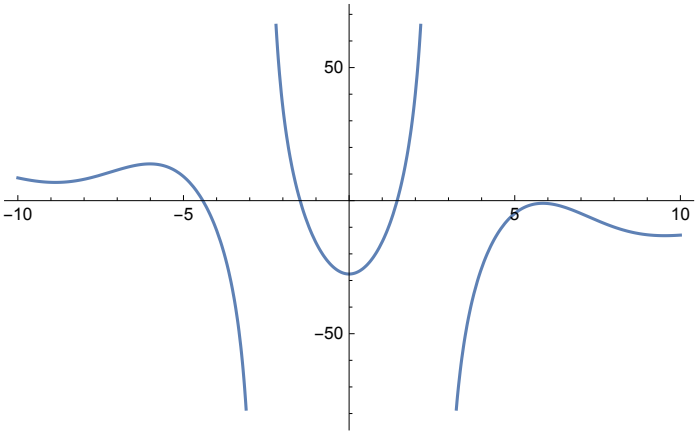
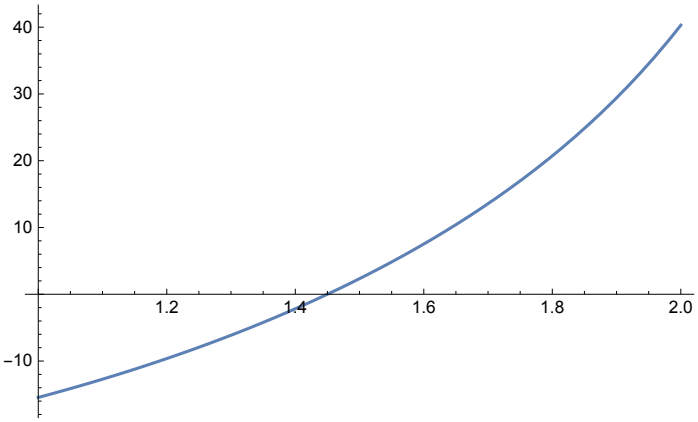
$$f[x\_] := \frac{-54\,(a+2)\,\text{Cos}[x] + x^3 + 23}{(b+2) - x^2}$$

```
a = 2; b = 5;
```

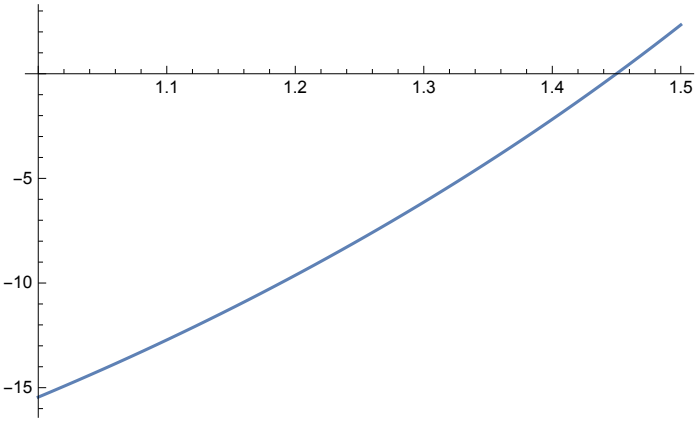
```
Plot[f[x], {x, -10, 10}]
```



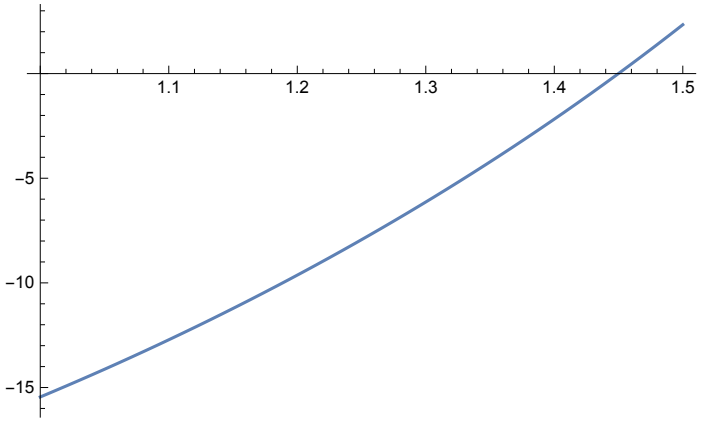

```
Plot[f[x], {x, 1, 2}]
```



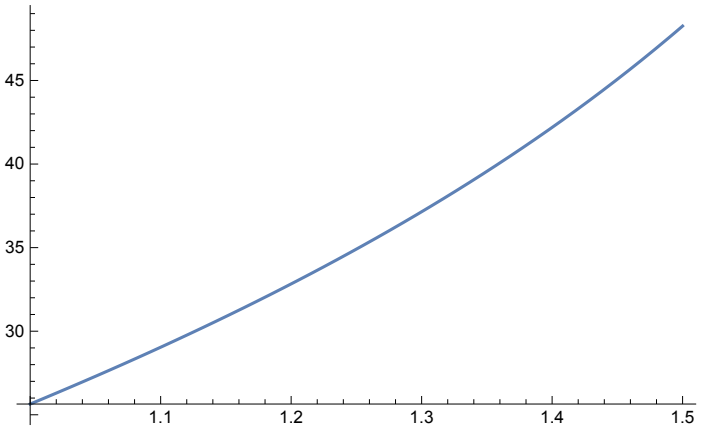

```
Plot[f[x], {x, 1, 1.5}]
```



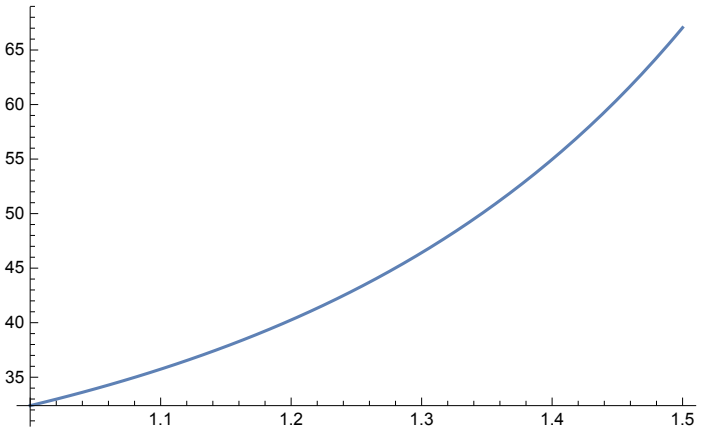
```
Plot[f[x], {x, 1, 1.5}]
```




```
Plot[f'[x], {x, 1, 1.5}]
(*първа производна*)
(* условието да е с посоянен знак за целия
 интервал е изпълнено priema samo otricatelni stoinosti *)
```




```
Plot[f''[x], {x, 1, 1.5}]
```




```
(*ф'' е по малко от 0 за целия и нтервал priema samo otricatelni stoinosti*)

f[1] * f[1.5]
```

```
-36.0925
```

```
(* f[1]*f[1.5]  e < 0 => има корен в интервала*)
(* всички условия на метода са изпълнени*)

(* избор за началмо приближение там където заедно
  със втората производна да има един и същ знак f(x)*f'' >0 *)

f[1.]
```

```
-15.4509
```

```
f[1.5]
(*ot tuk prawim izvoda f(1.5) e nasheto nachalno priblijeni,
zashtoto e otricatelna kakto i f'' e otricatelna  *)
```

```
2.33595
```

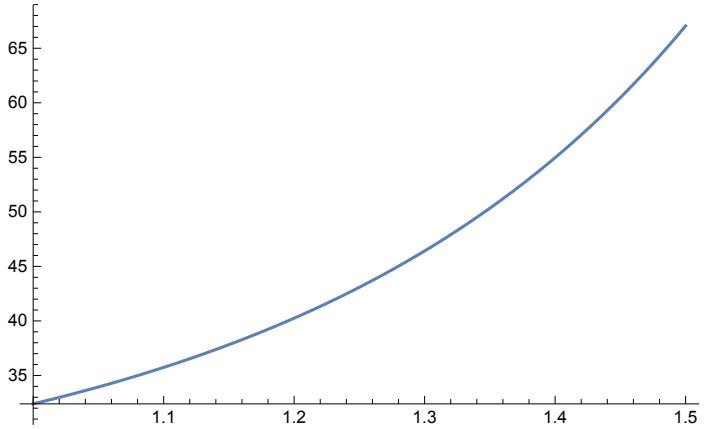
$$x1 = x - \frac{f[x]}{f'[x]} \text{(* formula za iteracii*)}$$

```
(*formula za greshkata*)
```

$$eps = 10^{-6}$$

$$epsk = \frac{M2}{2 * m1} (x1 - x)^2$$

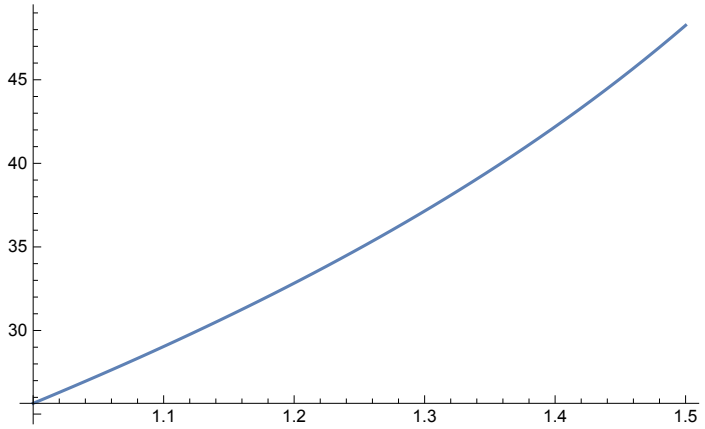
```
(* tyrsimm M2*)

Plot[Abs[f''[x]], {x, 1, 1.5}]
```




```
M2 = Abs[f''[1.5]]
```

```
67.0501
```

```
Plot[Abs[f'[x]], {x, 1, 1.5}]
```




```
m1 = Abs[f'[1.]]

x = 1.5; eps = 10^-6; M2 = Abs[f''[1.5]]; m1 = Abs[f'[1.]]; epsk = 1;

Print["k  = ", 0, " x1 = ", x, " f[x] = ", f[x], " f'[x] = ", f'[x]]

For[k = 1, epsk > eps, k++,
 x1 = x - f[x]/f'[x]; (*novo priblijenie*)
 epsk = M2/(2 * m1) (x1 - x)^2; (*greshka na novo priblijenie*)
 Print["k = ", k, " x1 = ", x1,
  " f[x] = ", f[x1], " f'[x] = ", f'[x1], " epsk = ", epsk];
 x = x1]
```

```
25.6427

k  = 0 x1 = 1.5 f[x] = 2.33595 f'[x] = 48.2562

k = 1 x1 = 1.45159 f[x] = 0.0759539 f'[x] = 45.1695 epsk = 0.00306357

k = 2 x1 = 1.44991 f[x] = 0.0000856689 f'[x] = 45.0677 epsk = 3.69671×10^-6

k = 3 x1 = 1.44991 f[x] = 1.09233×10^-10 f'[x] = 45.0676 epsk = 4.72413×10^-12
```

```
(*2.Zadacha ////////////////////////////////*)

A = {{40 + b - a, a, 0.3}, {a, -21, b}, {a + 1, 2 - b, 72}}

{{43, 2, 0.3}, {2, -21, 5}, {3, -3, 72}}

bzadacha2 = {{a}, {a - b}, {b + 1}}

{{2}, {-3}, {6}}

LinearSolve[A, bzadacha2]

{{0.0380967}, {0.167612}, {0.0887298}}

(*Iteratsionen procec metod na Qkobi*)
```

```
(*Forumuli
  za matrisa C
  Cij=-aij/aii,i≠j=1,2,...n
  Cii=0,i=1,2,...n
  za vektor d:
   di=bi/aii,i=1,2,...n
*)
```

```
n = Length[A]
```

```
3
```

```
(*inisializasiq matrisa C i wektor d*)
```

```
c = ( □ □ □
      □ □ □
      □ □ □ )
```

```
{{□, □, □}, {□, □, □}, {□, □, □}}
```

```
d = {0, 0, 0}
```

```
{0, 0, 0}
```

```
For[i = 1, i ≤ n, i++,
 c[[i]] = - A[[i]] / A[[i, i]];
 c[[i, i]] = 0;
 d[[i]] = bzadacha2[[i]] / A[[i, i]];

]
c // MatrixForm
d // MatrixForm
```

$$\begin{pmatrix} 0 & -\frac{2}{43} & -0.0069767441860465115` \\ \frac{2}{21} & 0 & \frac{5}{21} \\ -\frac{1}{24} & \frac{1}{24} & 0 \end{pmatrix}$$ (*glavniq diagonal zadiljitelno trabwa da e 0*)

$$\begin{pmatrix} \frac{2}{43} \\ \frac{1}{7} \\ \frac{1}{12} \end{pmatrix}$$

```
(* a)*)

43 x1 + 2 x2 + 0.3 x3 = 2
2 x1 + -21 x2 + 5 x3 = -3
3 x1 - 3 x2 + 72 x3 = 6
(*za i-tiq red prehvirlqm vsichko,koeto e bez xi ot dqsnata s obraten znak*)
43 x1 = -2 x2 - 0.3 x3 + 2
-21 x2 = -2 x1 - 5 x3 - 3
72 x3 = -3 x1 + 3 x2 + 6

(*za i-tiq red delim na koefisienta pred xi*)
x1^(k+1) = -2/43 x2^(k) - 0.3/43 x3^(k) + 2/43
x2^(k+1) = -2/-21 x1^(k) -5/-21 x3^(k) -3/-21
x3^(k+1) = -3/72 x1^(k) + 3/72 x2^(k) + 6/72


(*k=0,1,2..*)

(* b)proverka na shodimost *)

Norm[c]

0.256999

(*po-malka ot 1 izvoda e shte bide shodqsht *)



(* v) izbor na nachalno priblijenie*)

(*formula
  x^(k+1)=C.x^(k)+d
*)

x0 = {3, 6, 68}(*nqma znachenie*)
Print["k = ", 0, "x^(k) = ", x0]
For[k = 1, k ≤ 5, k++,
 x2 = c.x0 + d;
 x0 = x2;
 Print["k = ", k, "x^(k) = ", x0]
]

{3, 6, 68}

k = 0x^(k) = {3, 6, 68}

k = 1x^(k) = {{-0.706977}, {16.619}, {0.208333}}

k = 2x^(k) = {{-0.727921}, {0.125129}, {0.805251}}

k = 3x^(k) = {{0.0350736}, {0.265258}, {0.118877}}

k = 4x^(k) = {{0.0333447}, {0.174502}, {0.0929243}}

k = 5x^(k) = {{0.037747}, {0.168158}, {0.0892149}}
```

```
(*za sravnenie gore LinearSolve
 {{0.0380966973576075`},{0.16761153905085596`},{0.08872978507055201`}}
*)



(* d) otsenka na greshka*)

(*formula
  e^(k)=||x*-x^(k)||≤||C||^(k) (||x^(0)||+||d||/(1-||C||))
*)

x0 = {3, 6, 68}(*nqma znachenie*)
nx0 = Norm[x0];
eps1 = Norm[c]^0 (nx0 + Norm[d]/(1 - Norm[c]));
Print["k = ", 0, "x^(k) = ", x0, "e^(k) = ", eps1]
For[k = 1, k ≤ 5, k++,
 x2 = c.x0 + d;
 x0 = x2;
 eps1 = Norm[c]^k (nx0 + Norm[d]/(1 - Norm[c]));
 Print["k = ", k, "x^(k) = ", x0, "e^(k) = ", eps1]
]
{3, 6, 68}

k = 0x^(k) = {3, 6, 68}e^(k) = 68.5613

k = 1x^(k) = {{-0.706977}, {16.619}, {0.208333}}e^(k) = 17.6202

k = 2x^(k) = {{-0.727921}, {0.125129}, {0.805251}}e^(k) = 4.52836

k = 3x^(k) = {{0.0350736}, {0.265258}, {0.118877}}e^(k) = 1.16378

k = 4x^(k) = {{0.0333447}, {0.174502}, {0.0929243}}e^(k) = 0.299091

k = 5x^(k) = {{0.037747}, {0.168158}, {0.0892149}}e^(k) = 0.0768661

(*Zadacha 3 ////////////////////////////////////*)

(* a) sistavqne na tablitsata*)
(*
x_1=10+b+i(0.3);i=0,10
*)

xt = Table[10 + b + i * (0.3), {i, 0, 10}]
{15., 15.3, 15.6, 15.9, 16.2, 16.5, 16.8, 17.1, 17.4, 17.7, 18.}

(*ot 1 zadacha*)

f[x_] := (-54 (a + 2) Cos[x] + x^3 + 23) / ((b + 2) - x^2)

f[xt](*prilagane xt v f*)
{-16.3399, -16.746, -17.0679, -17.3089, -17.4793,
 -17.5941, -17.6724, -17.7345, -17.8013, -17.8913, -18.0201}

x_i    15.      15.3   □   17.4      17.7      18.
y_i  -16.3399  -16.746  □  -17.8013  -17.8913  -18.0201
```

```
Length[xt]
```

```
11
```

```
(* b) izbirane podhodqshti tochki L2*)
```

```
k = 10 + b + 0.25 * a + 0.02
```

```
15.52
```

```
(*izbirame podhodqshto cislo ot gornata tablitsa*)
```

```
      x0       x1       x2
x_i   15.      15.3     15.6
y_i  -16.3399  -16.746  -17.0679
```

```
(* v) Postroqvane na interpotsionen polinom*)
```

```
(**formula
   L2(x)=y0(((x-x1)(x-x2))/((x0-x1)(x0-x2)))+
   y1(((x-x0)(x-x2))/((x1-x0)(x1-x2)))+y2(((x-x0)(x-x1))/((x2-x0)(x2-x1)))
)
```

```
L2[x_] := -16.3399 * (((x - 15.3) (x - 15.6)) / ((15. - 15.3) (15. - 15.6))) +
  -16.746 * (((x - 15.) (x - 15.6)) / ((15.3 - 15.) (15.3 - 15.6))) +
  -17.0679 * (((x - 15.) (x - 15.3)) / ((15.6 - 15.) (15.6 - 15.3)))
```

```
L2[x]
```

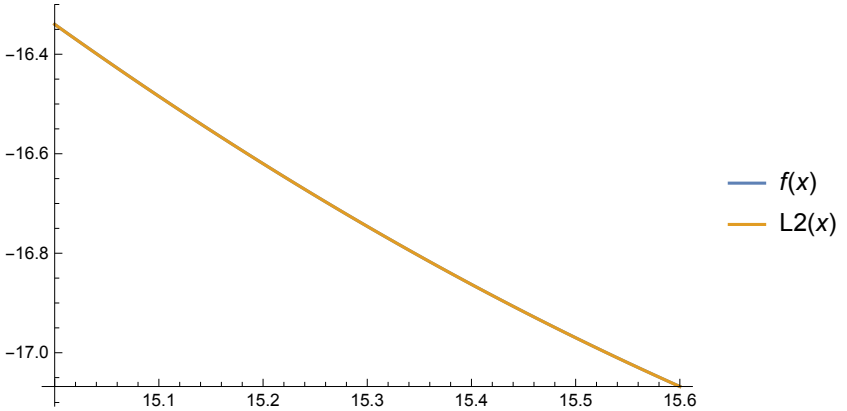
$$-90.7772\,(-15.6+x)\,(-15.3+x)+186.067\,(-15.6+x)\,(-15.+x)-94.8217\,(-15.3+x)\,(-15.+x)$$

```
Expand[L2[x]]
```

$$111.32-15.5273\,x+0.467778\,x^2$$

```
(* g)Proverka dali postroeniq polinom udevlotvorqvane*)
```

```
Plot[{f[x], L2[x]}, {x, 15., 15.6}, PlotLegends → "Expressions"]
```




```
(*proverka dali sivpada*)
```

```
      x0       x1       x2
x_i   15.      15.3     15.6
y_i  -16.3399  -16.746  -17.0679
```

```
L2[15.]
L2[15.3]
L2[15.6]
```

```
-16.3399
```

```
-16.746
```

```
-17.0679
```

```
(* d) namirane na priblijenata stoynost na funksiata f(k)*)
```

```
k = 10 + b + 0.25 * a + 0.02
```

```
15.52
```

```
L2[15.52]
```

```
-16.9903
```

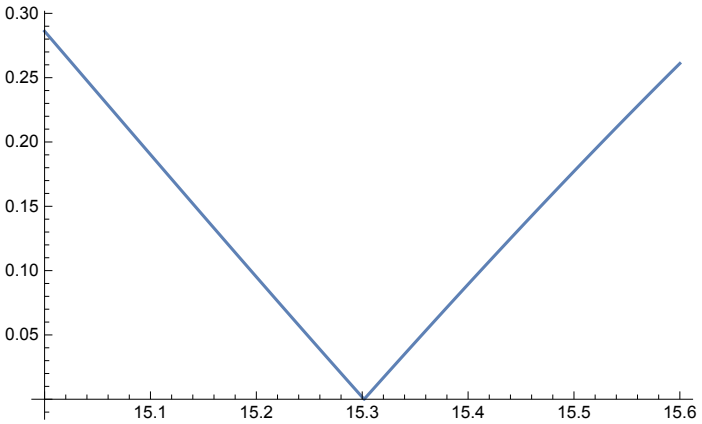
```
Print["Priblijena stoynost na funksiata = ",
 L2[15.52], "Tochnata stoynost na funksiata = ", f[15.52],
 "Istinskata greshka na priblijenieto e ", Abs[L2[15.52] - f[15.52]]]
```

```
Priblijena stoynost na funksiata = -16.9903Tochnata stoynost na funksiata =
 -16.9902Istinskata greshka na priblijenieto e 0.0000958898
```

```
(* e) Teoretichna otsenka na greshkata*)
(*formula
  |Rn(x)| = |f(x) - Ln(x)| ≤ Mn+1/(n+1)! (x-x0)...(x-xn)
    R2= za polinom ot vtora stepen
*)
```

$$R_2(x) \le \frac{M_3}{3!} \left| (x - x_0)\,(x - x_1)\,(x - x_3) \right|, \text{ kideto } M_3 = \max \left| f'''(x) \right|$$

```
Plot[Abs[f'''[x]], {x, 15., 15.6}]
```




```
M3 = Abs[f'''[15.6]]
```

```
0.261035
```

$$R2[x\_] := \frac{M3}{3!}\,\text{Abs}\left[(x - 15.)\,(x - 15.3)\,(x - 15.6)\right]$$

```
k = 10 + b + 0.25 * a + 0.02
```

```
15.52
```

```
R2[15.52]
```

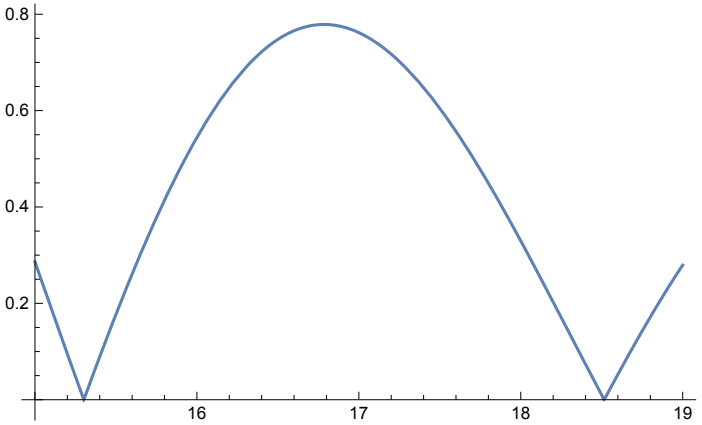
```
0.000398166
```

```
(*za sravnenie*)
```

```
"Istinskata greshka na priblijenieto e "0.0000958898
```

```
(*v sluchay na ekstrapolatsia*)
```

```
Plot[Abs[f'''[x]], {x, 15., 19}]
```




```
M3 = 0.8
```

$$R2[x\_] := \frac{M3}{3!}\,Abs\left[(x-15.)\,(x-15.3)\,(x-15.6)\right]$$

```
R2[0.8]
```

```
132.576
```

```
(* j) Postroqvame polinom na lineyna regresia*)
```

```
xt
```

```
{15., 15.3, 15.6, 15.9, 16.2, 16.5, 16.8, 17.1, 17.4, 17.7, 18.}
```

```
yt = f[xt]
```

```
{-16.3399, -16.746, -17.0679, -17.3089, -17.4793,
 -17.5941, -17.6724, -17.7345, -17.8013, -17.8913, -18.0201}
```

```
xt^2
```

```
{225., 234.09, 243.36, 252.81, 262.44, 272.25, 282.24, 292.41, 302.76, 313.29, 324.}
```

```
xt * yt
```

```
{-245.098, -256.214, -266.259, -275.212, -283.164,
 -290.303, -296.896, -303.261, -309.742, -316.675, -324.362}
```

```
Np = Length[xt]
```

```
11
```

$$\sum_{i=1}^{Np} xt[[i]]$$

```
181.5
```

$$\sum_{i=1}^{Np} yt[[i]]$$

$$-191.656$$

$$\sum_{i=1}^{Np} xt[[i]]^2$$

$$3004.65$$

$$\sum_{i=1}^{Np} xt[[i]]\, yt[[i]]$$

$$-3167.19$$

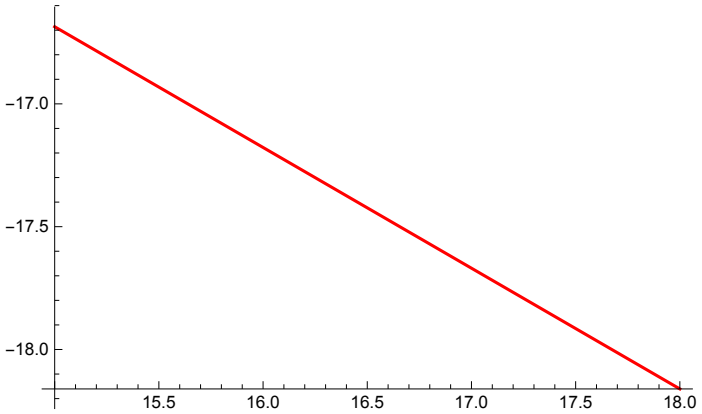
$$11\, a_0 + 181.5\, a_1 = -191.656$$

$$181.5\, a_0 + 3004.65 = -3167.19$$

```
Azadacha3 = ( 11     181.5   );
            ( 181.5  3004.65 )
bzadacha3 = {-191.656, -3167.19};

LinearSolve[Azadacha3, bzadacha3]
```

$$\{-9.31327, -0.491515\}$$

```
P1[x_] := -0.491515 x + -9.31327

grPl = Plot[P1[x], {x, xt[[1]], xt[[Np]]}, PlotStyle → Red]
```



$$greshka = \sqrt{\sum_{i=1}^{Np} \left(yt[[i]] - P1[xt[[i]]]\right)^2}$$

$$0.536198$$

```
(*lineyna regresia ot vtora stepen*)

xt^3
```

$$\{3375., 3581.58, 3796.42, 4019.68, 4251.53,\ 4492.13, 4741.63, 5000.21, 5268.02, 5545.23, 5832.\}$$

```
xt^4
```

$$\{50\,625., 54\,798.1, 59\,224.1, 63\,912.9, 68\,874.8,\ 74\,120.1, 79\,659.4, 85\,503.6, 91\,663.6, 98\,150.6, 104\,976.\}$$

```
xt^2 * yt
```

```
{-3676.47, -3920.07, -4153.63, -4375.87, -4587.26,
 -4790., -4987.85, -5185.76, -5389.51, -5605.15, -5838.51}
```

```
Sum[xt[[i]]^3, {i, 1, Np}]
```

```
49 903.4
```

```
Sum[xt[[i]]^4, {i, 1, Np}]
```

```
831 508.
```

```
Sum[xt[[i]]^2 yt[[i]], {i, 1, Np}]
```

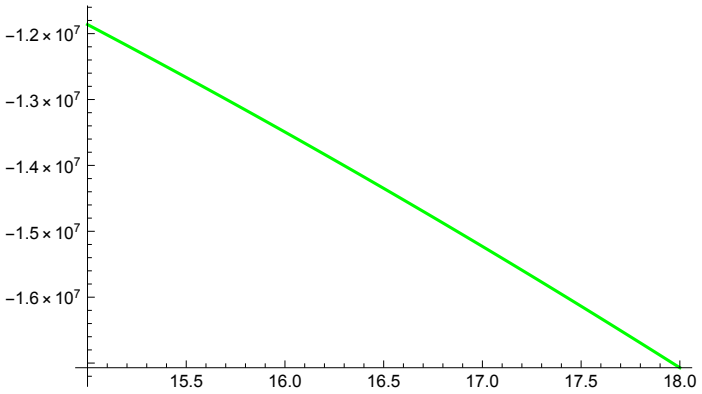
```
-52 510.1
```

```
Azadacha3 .1 = {{11, 181.5, 3004.65}, {181.5, 3004.65, 49 903.4}, {3004.65, 49 903.4, 831 508.}};
bzadacha3 .1 = {-191.656, -3167.19, -52 510.1};
LinearSolve[Azadacha3 .1, bzadacha3 .1]
```

```
{-9.31327, -0.491515}
```

```
P2[x_] := -52 510.1 x^2 - 3167.19 x + -191.656
```

```
grP2 = Plot[P2[x], {x, xt[[1]], xt[[Np]]}, PlotStyle → Green]
```




```
greshkazad3 = Sqrt[Sum[(yt[[i]] - P2[xt[[i]]])^2, {i, 1, Np}]]
```

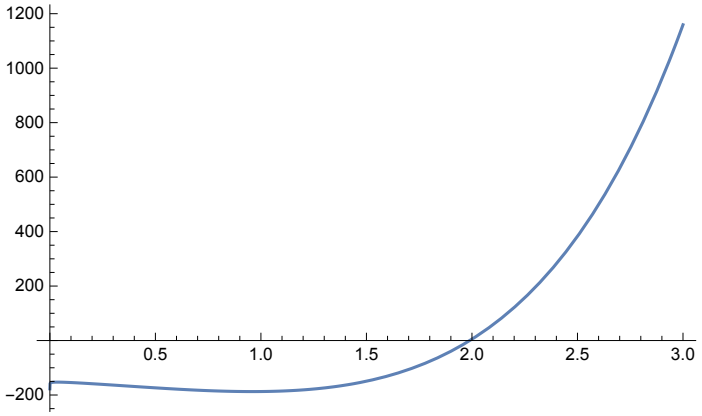
```
4.80563 × 10^7
```

```
(*4.Zadacha*)

(*
    y'=y-1n(x^2+1)+(2x)/(x^2+1)
    y(a)=b
    x ∈ |a,a+2|
*)

Clear[x, y, f]
```

```
DSolve[{y'[x] == y[x] - 1 n (x^2 + 1) + (2 x) / x^2 + 1, y[2] == 5}, y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, -\frac{1}{e^2}\left(e^2 - 6\, e^x - 3\, e^2\, n + 11\, e^x\, n - 2\, e^2\, n\, x - e^2\, n\, x^2 + 2\, e^{2+x}\, \text{ExpIntegralEi}[-2] - 2\, e^{2+x}\, \text{ExpIntegralEi}[-x]\right)\right]\right\}\right\}$$

```
yex[x_] := -1/e^2 (e^2 - 6 e^x - 3 e^2 n + 11 e^x n - 2 e^2 n x -
     e^2 n x^2 + 2 e^(2+x) ExpIntegralEi[-2] - 2 e^(2+x) ExpIntegralEi[-x])
Plot[
 yex[
  x],
 {x,
  0,
  3}]
```




```
a = 2; b = 4;

h = 0.02;
n = b - a / h;
y0 = 5;

f[x_, y_] := y - 1 n (x^2 + 1) + (2 x) / (x^2 + 1)

f[2, 0]
```

480.8

```
% // N
```

480.8

```
yex[2]
```

5.