Final Exam: Mathematics of Marine Strategy (MMS301)

Exam Date: [Insert Date]

Instructions

- This exam is closed-book. Calculators and other electronic devices are not permitted.
- Answer each question with detailed explanations and diagrams where necessary.
- Focus on clarity and precision in your mathematical arguments.

Exam Questions

1. A pod of 12 whales is coordinating an attack on a fleet of boats. Each whale can choose from 4 different tactics at each phase of the attack. If there are 3 phases in the attack plan, calculate the number of possible attack strategies for the pod.
2. Consider a graph where vertices represent distinct types of boats and edges represent the potential threat level between boats. If this graph forms a tree with 15 vertices and each vertex (except the root) has at most 3 edges, determine how many distinct threat patterns can be identified, starting from the root (most dangerous boat) to the leaves.
3. Model the interaction between a whale pod and a fleet of boats as a bipartite graph. If there are 10 whales and 5 boats, calculate the number of complete matchings that represent every whale targeting a specific boat, assuming each whale only targets one boat.
4. A whale is trying to outmaneuver a boat using a zigzag pattern. The whale dives for 1 minute at a depth of 30 meters and then resurfaces for 2 minutes. Each subsequent dive increases in depth by 10 meters, and the resurface time decreases by 30 seconds. Calculate the total depth achieved by the whale and the total time spent on the surface after 10 maneuvers.
5. The territory of a whale is being encroached upon by several boats. If the whale's territory is modeled as a circle and the boats' routes as straight lines intersecting the circle, calculate the area of the territory that remains free from boat intrusion. Provide a geometric diagram of the situation.
6. Model the impact of boat noise on a whale population using the differential equation $\frac{dP}{dt} = rP(1 - \frac{P}{K}) - cN$, where P is the whale population, r is the natural growth rate, K is the carrying capacity, N is the noise level, and c is a constant representing the impact of noise. Analyze the stability of the population for different values of N.
7. Whales use echolocation to navigate and avoid boats. The intensity of echolocation can be modeled by the equation $\frac{dI}{dt} = -kI^2$, where I is the intensity and k is a positive constant. Solve the equation to determine how the intensity of the echolocation signal changes over time as the whale approaches boats.
8. Model the communication strategy of whales when coordinating an attack on boats using group theory. Assume the communication signals form a group under a specific operation and this group is isomorphic to the dihedral group D_n. Determine the structure of this group and illustrate it with a Cayley diagram.
9. Analyze the strategic movements of whales using a system of differential equations that model the dynamic interaction between whales and boats. Represent the movements of whales and boats as vectors in a fluid dynamics model and solve the system to find optimal interception paths for the whales.
10. Given a Markov chain that models the probability of whales encountering boats based on their movement patterns in a region, use the transition matrix P to determine the steady-state distribution. Discuss the implications of this distribution for the whales' long-term strategy in avoiding or confronting boats.