Sensors

Computational topology - group project

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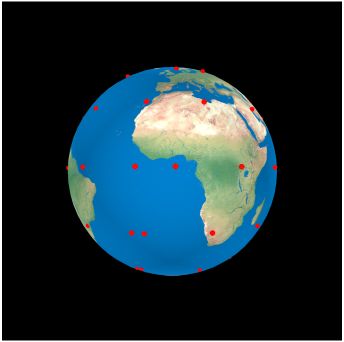
Problem description

Number of sensors on the sphere of radius 1 (Earth):



- each sensor gathers data from the surrounding area in the shape of a sphere of radius R,
- each sensor can communicate with other sensors which are at most r away.

Goals

1. The sensor network is connected.
2. The sensor network covers the whole sphere.
3. Values of r and R are as small as possible.
4. There are no obsolete sensors.
5. Find optimal distribution of 50 sensors on the sphere.

Vietoris-Rips complex $\longrightarrow r$

Connected sensor network $\longrightarrow$ Such r so that Vietoris-Rips complex $VR_r(S)$ is connected.

- sensors: S $(S_i = (r_i, \phi_i, \theta_i))$,
- sensor connections $\{S_i, S_j\} \subset S; d(S_i, S_j) \leq 2r$,
- $F \subset S$ is a simplex in $VR_r(S)$, if diam $F \leq 2r$.

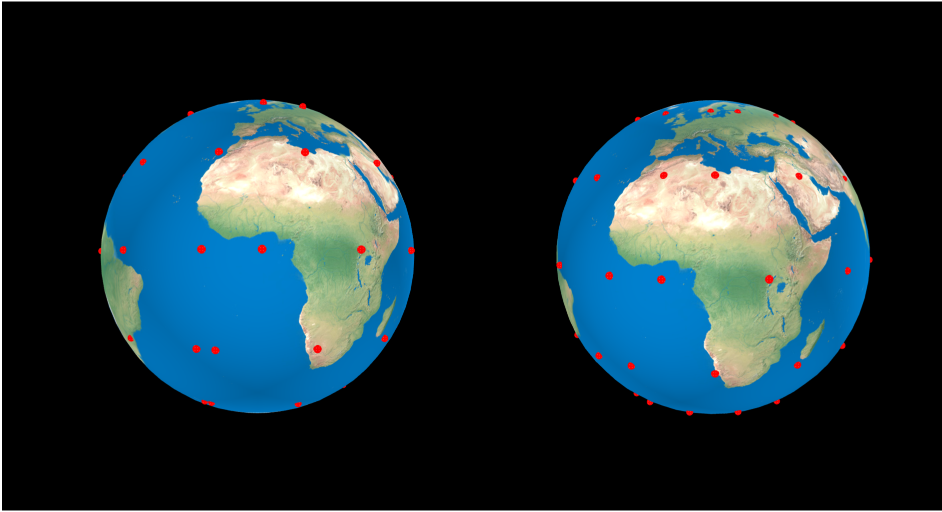
Čech complex $\longrightarrow R$

The sensor network covers the whole sphere $\longrightarrow$ Such R so that Euler characteristic of Čech complex should be that of a sphere.

- sensors: S $(S_i = (r_i, \phi_i, \theta_i))$,
- $B_R(x)$ closed ball with radius R around x,
- $\check{C}_R = \{\sigma \subset S, \cap_{x\in\sigma} B_R(x) \neq \emptyset\}$.

In practice, instead of calculating Euler characteristic we checked first two Betti numbers.

Two different initial distributions of sensors on Earth

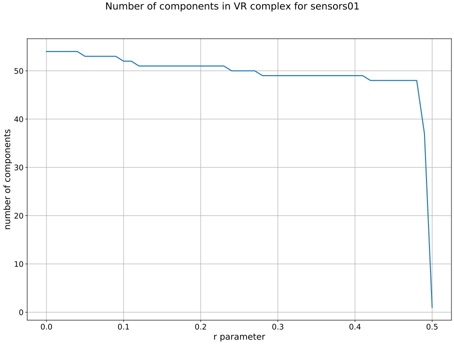


(a) Sensors 1.

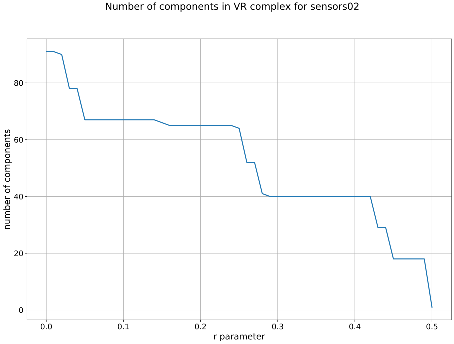
(b) Sensors 2.

Vietoris-Rips $\longrightarrow$ 1 component

Sensor network must be connected.

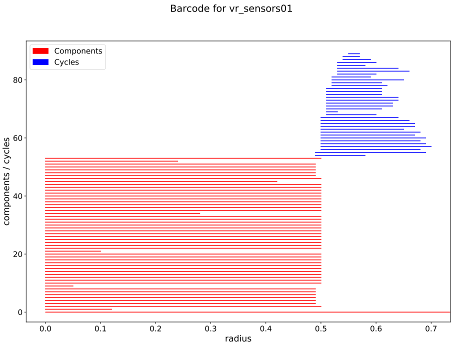


(a) $r = 0.5$

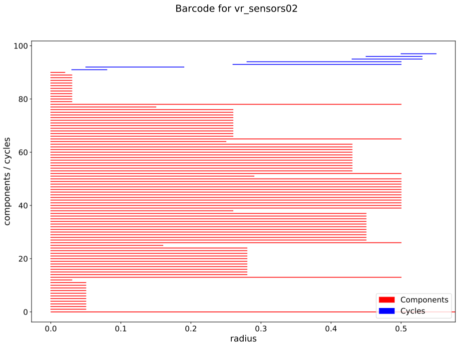


(b) $r = 0.5$

Barcode for Vietoris-Rips



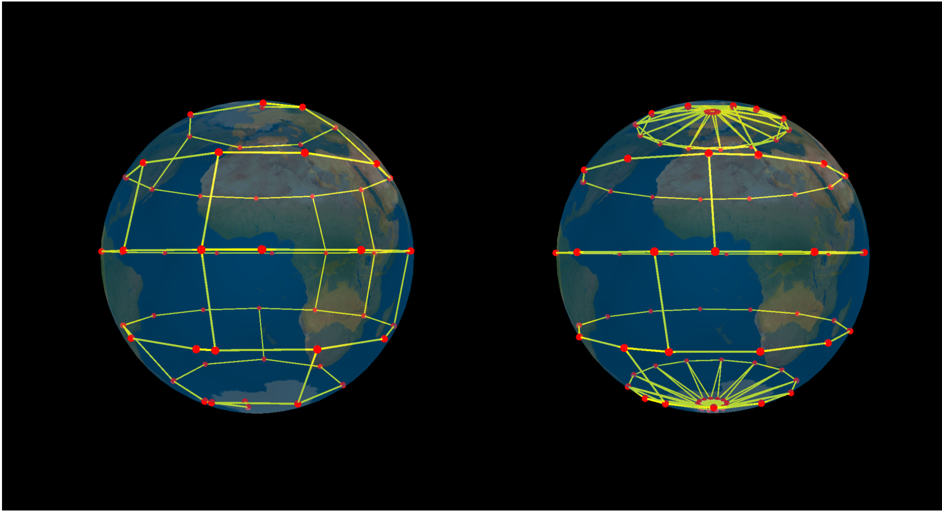
(a) Sensors 1.



(b) Sensors 2.

Connections in sensor network

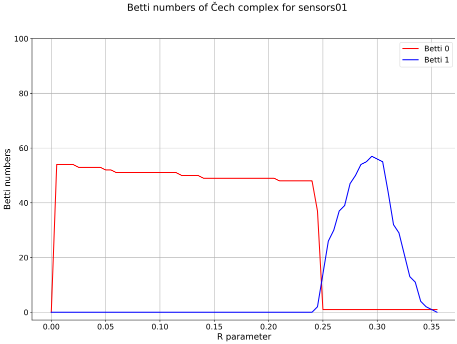
Drawn connections for appropriate parameter r.



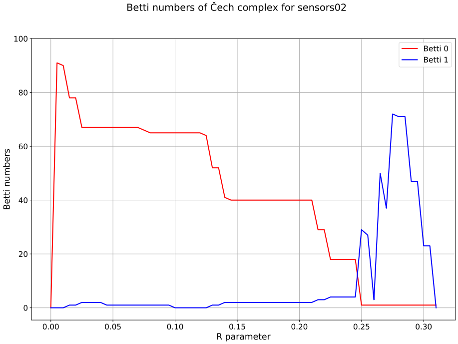
(a) Sensors 1: $r = 0.5$ (b) Sensors 2: $r = 0.5$

Čech complex $\longrightarrow b_0 = 1 \wedge b_1 = 0$

The sensor network should cover the whole sphere.

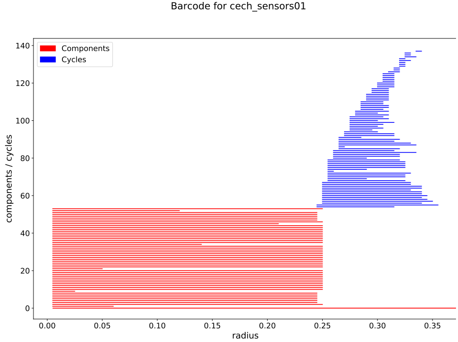


(a) $R = 0.355$

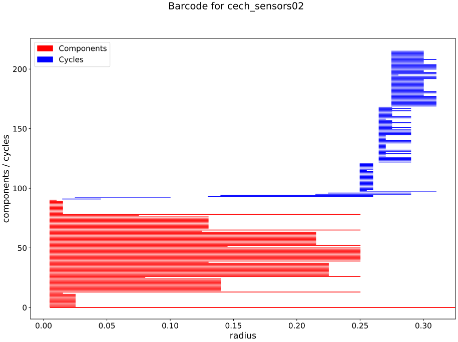


(b) $R = 0.31$

Barcode for Čech

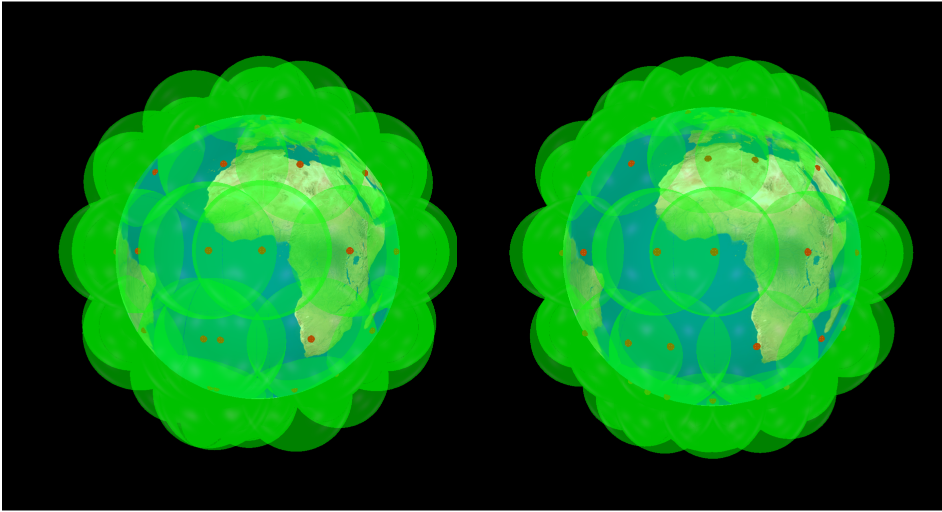


(a) Sensors 1.



(b) Sensors 2.

Coverage in sensor network for appropriate parameter R.



(a) Sensors 1: $R = 0.355$ (b) Sensors 2: $R = 0.31$

Redundant sensors

We looked for redundant sensors with a simple algorithm:

- ignore cut vertices of the VR complex (they separate the network in multiple components),
- ignore points with less than 3 neighbors in radius R (they create holes in Čech complex),
- randomly select points from the remaining list and check if removing them would break our solution
- optionally, check all ways of removing any vertices from the remaining list, if it is small enough (exhaustive search)

In this way we got a compromise between finding the best solution and spent time

Data generator

Distribution of points on the sphere so that parameters r and R are as small as possible.

Coulomb's law

$$|\boldsymbol{F_{ij}}| = k_e \frac{e^2}{|\boldsymbol{r_i} - \boldsymbol{r_j}|^2}$$

Electrostatic potential energy

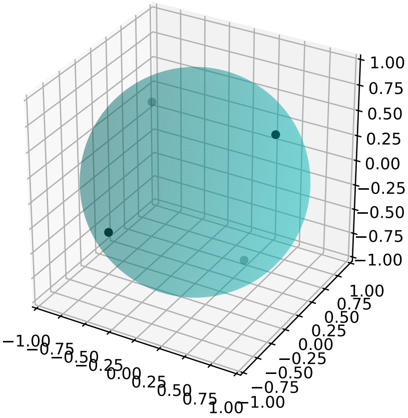
$$V = \sum_{i \neq j} V_{ij} \propto \sum_{i \neq j} \frac{1}{|\boldsymbol{r_i} - \boldsymbol{r_j}|}$$

- Electrons would distribute themselves evenly around the sphere.
- Minimization of V with simulated annealing.

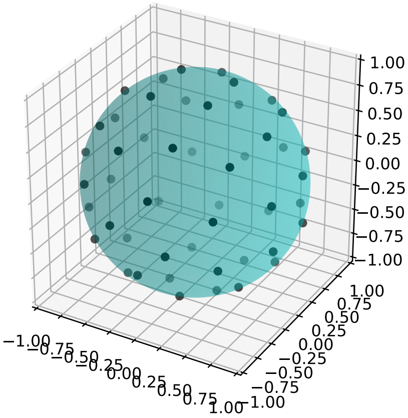
Algorithm for MC simulated annealing

1. Start with random distribution of points on sphere.
2. Set initial temperature of the system T.
3. Choose random point, move it according to Gaussian distribution.
4. Calculate difference in energy ΔE.
5. If $\Delta E < 0$, accept the change.
6. If $\Delta E \geq 0$, accept the change with probability $\exp(\frac{-\Delta E}{T})$
7. If enough changes accepted, decrease the temperature T.
8. Repeat process from 3. $\longrightarrow$

Results for $n = 4$ and $n = 50$



(a) n = 4



(b) n = 50

Summary

Thank you for listening.

Bibliography

Vietoris-Rips. https://en.wikipedia.org/wiki/Vietoris_Rips_complex (5.6.2018).

Čech-complex. https://en.wikipedia.org/wiki/Cech_complex (5.6.2018).

Lecture notes from prof. dr. Neža Mramor Kosta.