Sensors
Computational topology - group project

Nejc Kišek, Žan Klaneček

Faculty of computer and information science
University of Ljubljana

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Outline

Problem description

Number of sensors on the sphere of radius 1 (Earth):

- each sensor gathers data from the surrounding area in the shape of a circle of radius R,
- each sensor can communicate with other sensors which are at most r away.

1. Values of r and R are as small as possible.
2. The sensor network is connected.
3. The sensor network covers the whole sphere.
4. Removal of obsolete sensors.
5. Find optimal distribution of 50 sensors on the sphere.

Vietoris-Rips complex

Connected sensor network $\longrightarrow$ Vietoris-Rips complex $VR_\delta(S)$ is connected.

- sensors: S $(S_i = (r_i, \phi_i, \theta_i))$,
- sensor connections $\{S_i, S_j\} \subset S; d(S_i, S_j) \leq 2\delta$,
- $F \subset S$ is a simplex in $VR_\delta(S)$, if diam $F \leq 2\delta$.

Čech complex

The sensor network covers the whole sphere $\longrightarrow$ Euler characteristic of Čech complex should be that of a sphere.

- sensors: S $(S_i = (r_i, \phi_i, \theta_i))$,
- $B_\delta(x)$ closed ball with radius δ around x,
- $\check{C}_\delta = \{\sigma \subset S, \cap_{x \in \sigma} B_\delta(x) \neq \emptyset\}$.

Data generator

Distribution of points on the sphere so that parameters r and R are as small as possible.

Electrostatic potential energy

$$V = \sum_{i \neq j} V_{ij} \propto \sum_{i \neq j} \frac{1}{|\mathbf{r_i} - \mathbf{r_j}|}$$

- Electrons would distribute themselves evenly around the sphere.
- Minimization of V with simulated annealing.

Algorithm for MC simulated annealing

1. Start with random distribution of points on sphere.
2. Set initial temperature of the system T.
3. Choose random point, move it according to Gaussian distribution.
4. Calculate difference in energy ΔE.
5. If $\Delta E < 0$, accept the change.
6. If $\Delta E \geq 0$, accept the change with probability $\exp(\frac{-\Delta E}{T})$
7. If enough changes accepted, decrease the temperature T.
8. Repeat process from 3. $\longrightarrow$

Summary

Baumchiquabaumbaum.

Literature

Vietoris-Rips.
https://en.wikipedia.org/wiki/Vietoris_Rips_complex
(5.6.2018).

Čech-complex. https://en.wikipedia.org/wiki/Cech_complex
(5.6.2018).

Lecture notes from prof. dr. Neža Mramor Kosta.