

Sensors

Computational topology - group project

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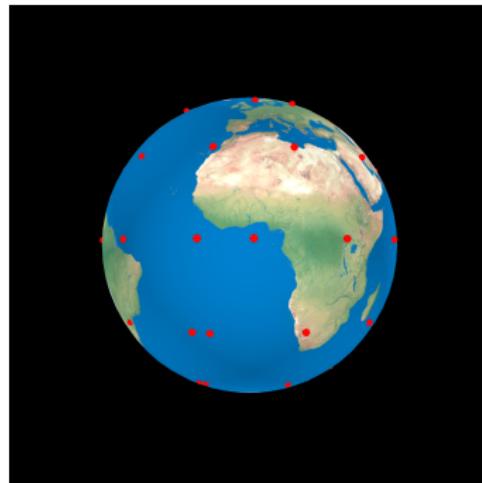
- Vietoris-Rips complex
- Čech complex

3 Results and implementation

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Problem description

Number of sensors on the sphere of radius 1 (Earth):



- each sensor gathers data from the surrounding area in the shape of a circle of radius R ,
- each sensor can communicate with other sensors which are at most r away.

Goals

- ① The sensor network is connected.
- ② The sensor network covers the whole sphere.
- ③ Values of r and R are as small as possible.
- ④ There are no obsolete sensors.
- ⑤ Find optimal distribution of 50 sensors on the sphere.

Vietoris-Rips complex $\longrightarrow r$

Connected sensor network \longrightarrow Such r so that Vietoris-Rips complex $VR_r(S)$ is connected.

- sensors: S ($S_i = (r_i, \phi_i, \theta_i)$),
- sensor connections $\{S_i, S_j\} \subset S; d(S_i, S_j) \leq 2r$,
- $F \subset S$ is a simplex in $VR_r(S)$, if $\text{diam } F \leq 2r$.

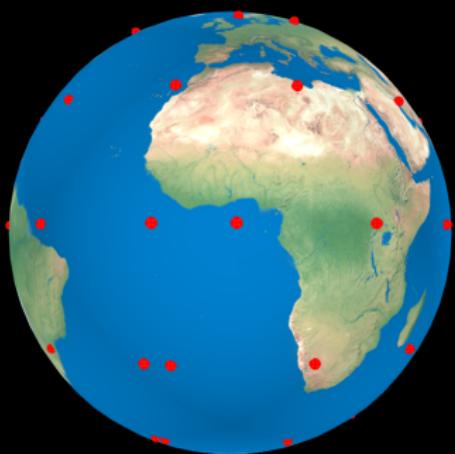
$\check{\text{C}}\text{ech complex} \longrightarrow R$

The sensor network covers the whole sphere \longrightarrow Such R so that Euler characteristic of $\check{\text{C}}\text{ech complex}$ should be that of a sphere.

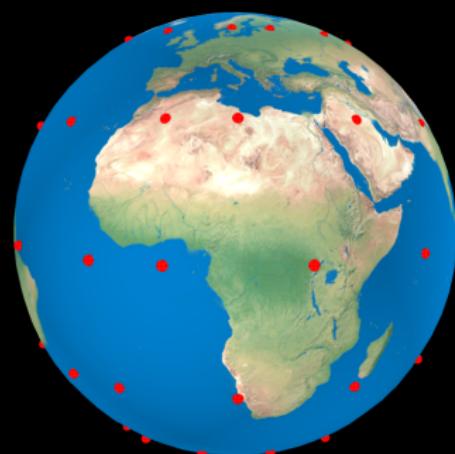
- sensors: S ($S_i = (r_i, \phi_i, \theta_i)$),
- $B_R(x)$ closed ball with radius R around x ,
- $\check{C}_R = \{\sigma \subset S, \cap_{x \in \sigma} B_R(x) \neq \emptyset\}$.

In practice, instead of calculating Euler characteristic we checked first two Betti numbers.

Two different initial distributions of sensors on Earth



(a) Sensors 1.

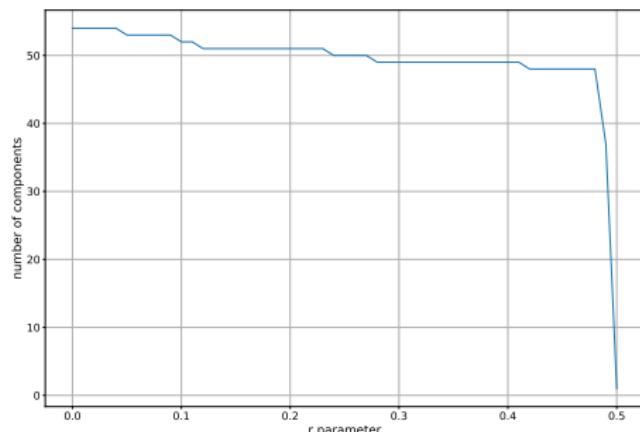


(b) Sensors 2.

Vietoris-Rips \longrightarrow 1 component

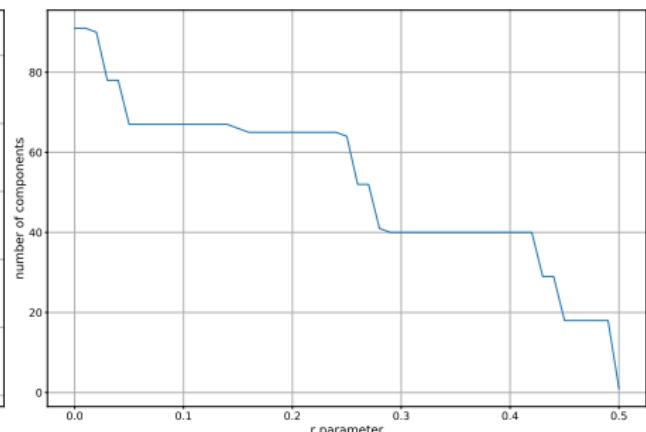
Sensor network must be connected.

Number of components in VR complex for sensors01



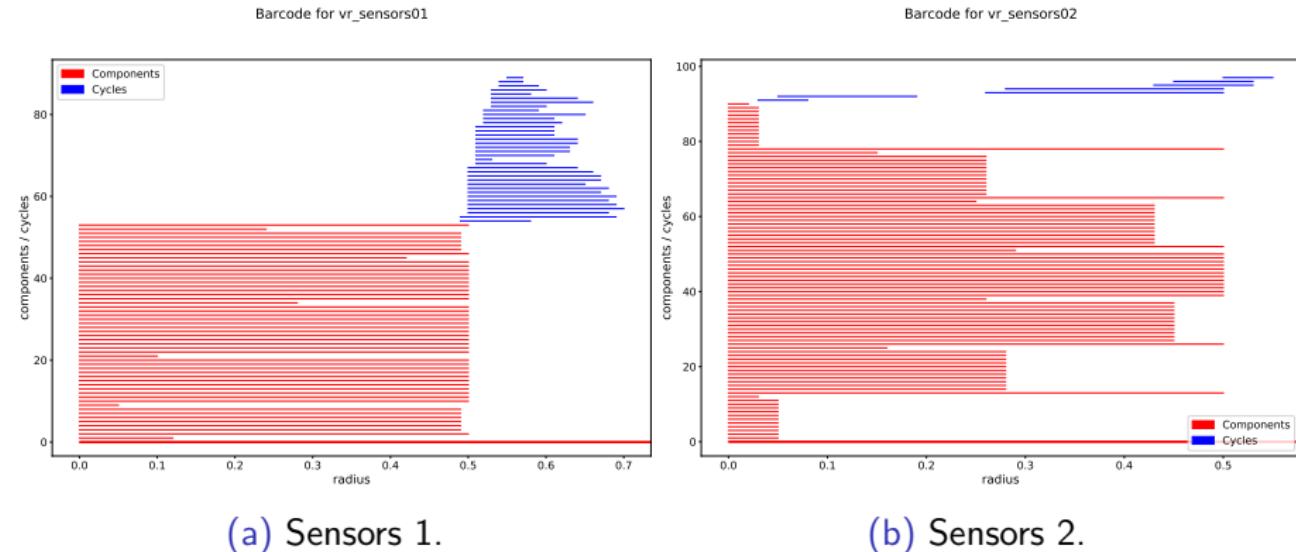
(a) $r = 0.5$

Number of components in VR complex for sensors02



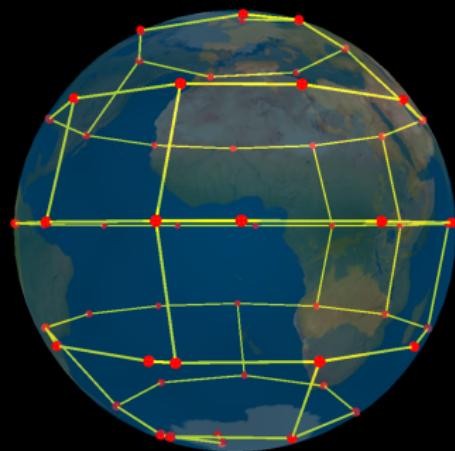
(b) $r = 0.5$

Barcode for Vietoris-Rips

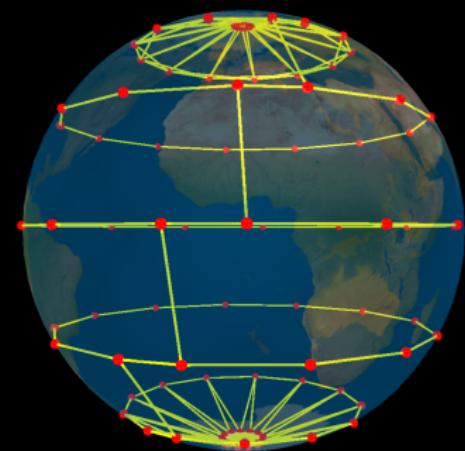


Connections in sensor network

Drawn connections for appropriate parameter r .



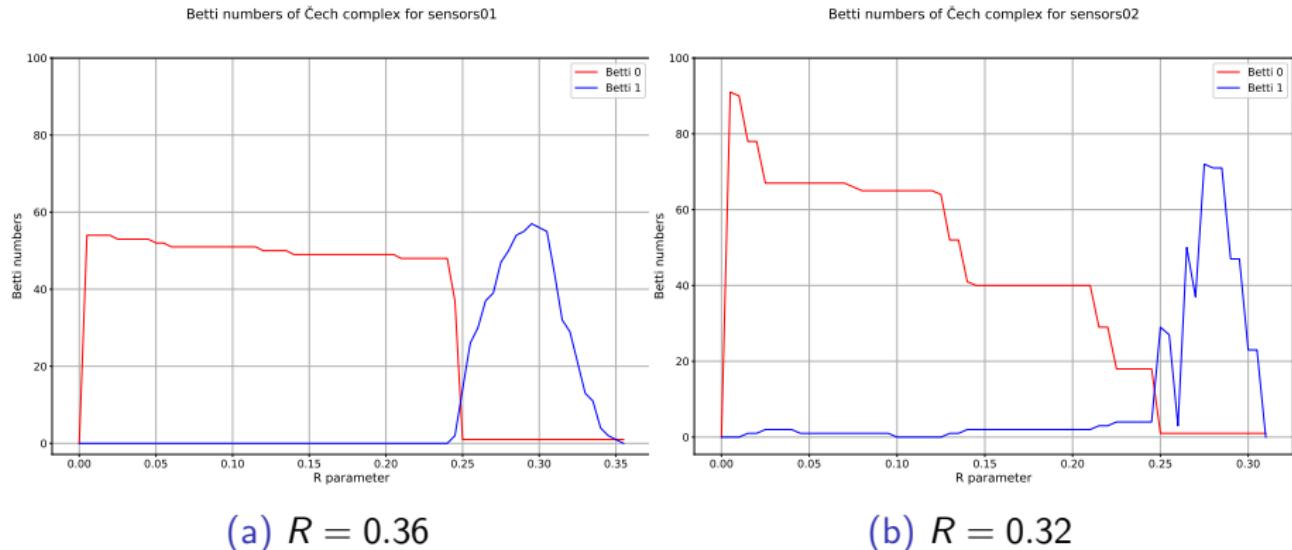
(a) Sensors 1: $r = 0.5$



(b) Sensors 2: $r = 0.5$

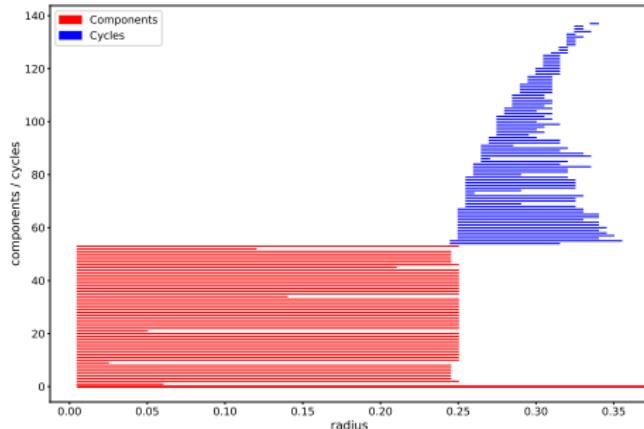
$\check{\text{C}}\text{ech complex} \longrightarrow b_0 = 1 \wedge b_1 = 0$

The sensor network should cover the whole sphere.



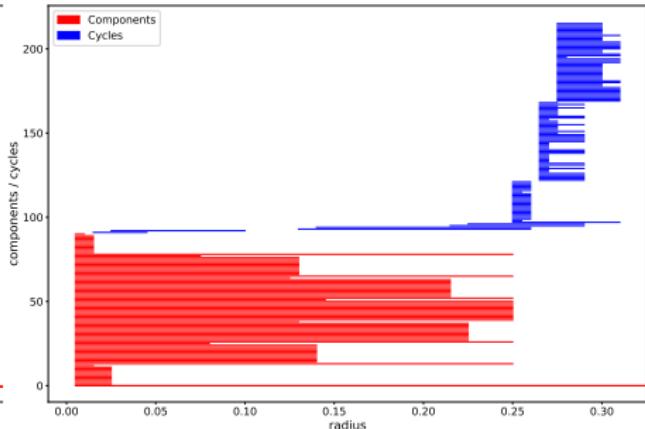
Barcode for Čech

Barcode for cech_sensors01



(a) Sensors 1.

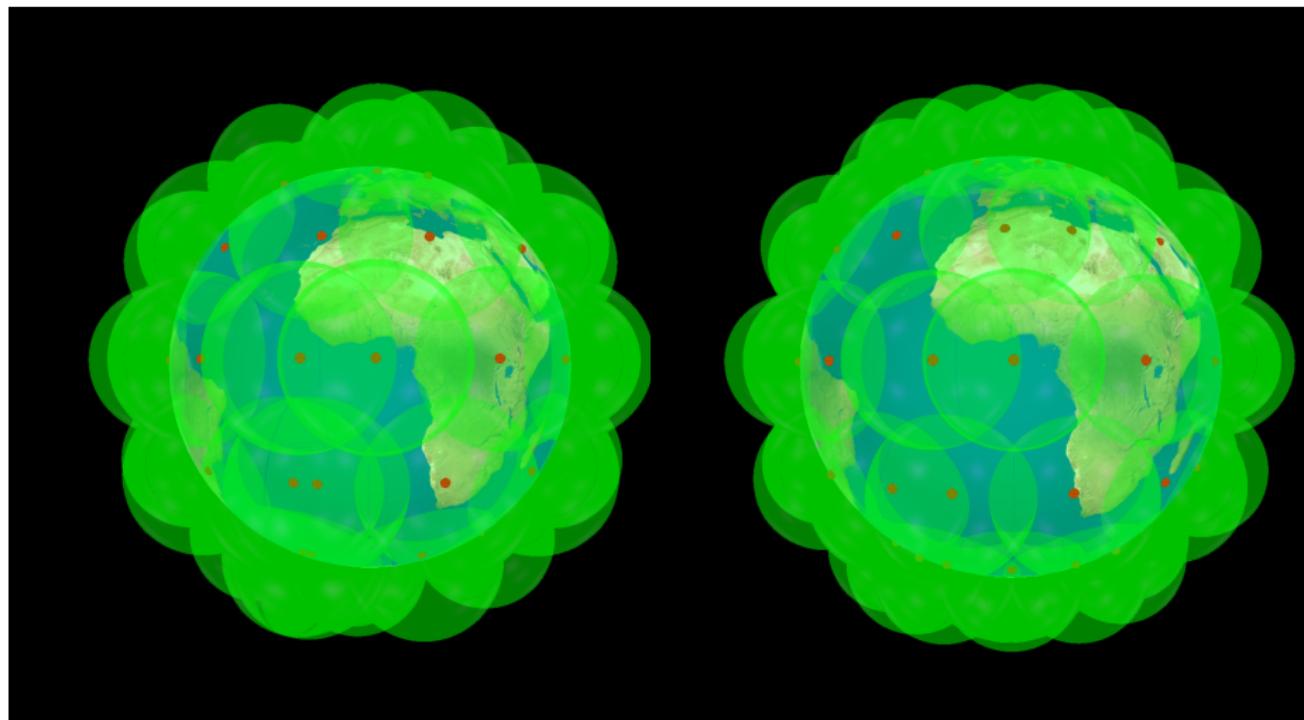
Barcode for cech_sensors02



(b) Sensors 2.

Coverage in sensor network

Coverage in sensor network for appropriate parameter R .



(a) Sensors 1: $R = 0.36$

(b) Sensors 2: $R = 0.32$

Data generator

Distribution of points on the sphere so that parameters r and R are as small as possible.

Coulomb's law

$$|\mathbf{F}_{ij}| = k_e \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|^2}$$

Electrostatic potential energy

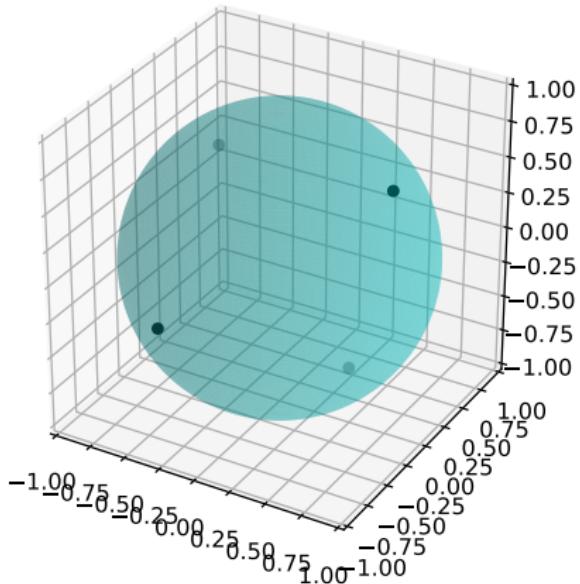
$$V = \sum_{i \neq j} V_{ij} \propto \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

- Electrons would distribute themselves evenly around the sphere.
- Minimization of V with simulated annealing.

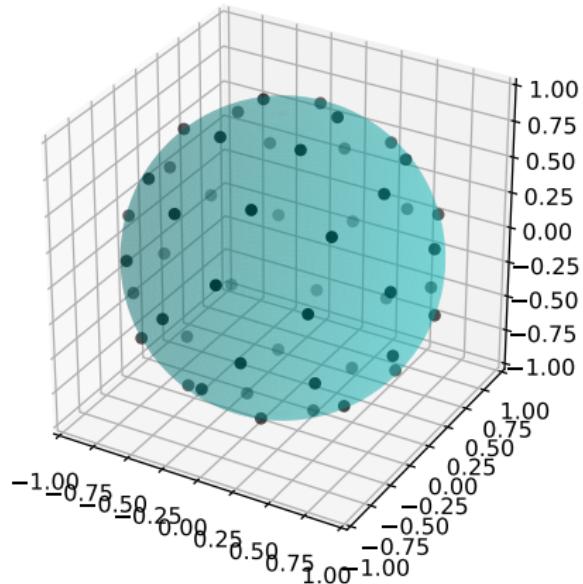
Algorithm for MC simulated annealing

- ① Start with random distribution of points on sphere.
- ② Set initial temperature of the system T .
- ③ Choose random point, move it according to Gaussian distribution.
- ④ Calculate difference in energy ΔE .
- ⑤ If $\Delta E < 0$, accept the change.
- ⑥ If $\Delta E \geq 0$, accept the change with probability $\exp\left(\frac{-\Delta E}{T}\right)$
- ⑦ If enough changes accepted, decrease the temperature T .
- ⑧ Repeat process from 3. →

Results for $n = 4$ and $n = 50$



(a) $n = 4$



(b) $n = 50$

Summary

Thank you for listening.

Bibliography

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- 🌐 Čech-complex. https://en.wikipedia.org/wiki/Cech_complex
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