Sensors

Computational topology - group project

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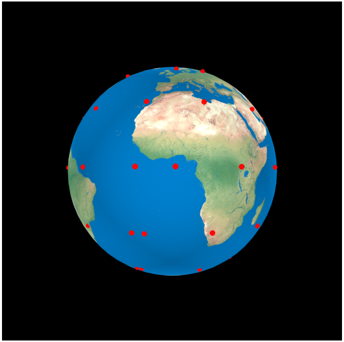
Problem description

Number of sensors on the sphere of radius 1 (Earth):



- each sensor gathers data from the surrounding area in the shape of a circle of radius R,
- each sensor can communicate with other sensors which are at most r away.

Goals

1. The sensor network is connected.
2. The sensor network covers the whole sphere.
3. Values of r and R are as small as possible.
4. There are no obsolete sensors.
5. Find optimal distribution of 50 sensors on the sphere.

Vietoris-Rips complex $\longrightarrow r$

Connected sensor network $\longrightarrow$ Such r so that Vietoris-Rips complex $VR_r(S)$ is connected.

- sensors: S $(S_i = (r_i, \phi_i, \theta_i))$,
- sensor connections $\{S_i, S_j\} \subset S; d(S_i, S_j) \leq 2r$,
- $F \subset S$ is a simplex in $VR_r(S)$, if diam $F \leq 2r$.

Čech complex $\longrightarrow R$

The sensor network covers the whole sphere $\longrightarrow$ Such R so that Euler characteristic of Čech complex should be that of a sphere.

- sensors: S $(S_i = (r_i, \phi_i, \theta_i))$,
- $B_R(x)$ closed ball with radius R around x,
- $\check{C}_R = \{\sigma \subset S, \cap_{x \in \sigma} B_R(x) \neq \emptyset\}$.

In practice, instead of calculating Euler characteristic we checked first two Betti numbers.

Data generator

Distribution of points on the sphere so that parameters r and R are as small as possible.

Coulomb's law

$$|\boldsymbol{F_{ij}}| = k_e \frac{e^2}{|\boldsymbol{r_i} - \boldsymbol{r_j}|^2}$$

Electrostatic potential energy

$$V = \sum_{i \neq j} V_{ij} \propto \sum_{i \neq j} \frac{1}{|\boldsymbol{r_i} - \boldsymbol{r_j}|}$$

- Electrons would distribute themselves evenly around the sphere.
- Minimization of V with simulated annealing.

Algorithm for MC simulated annealing

1. Start with random distribution of points on sphere.
2. Set initial temperature of the system T.
3. Choose random point, move it according to Gaussian distribution.
4. Calculate difference in energy ΔE.
5. If $\Delta E < 0$, accept the change.
6. If $\Delta E \geq 0$, accept the change with probability $\exp(\frac{-\Delta E}{T})$
7. If enough changes accepted, decrease the temperature T.
8. Repeat process from 3. $\longrightarrow$

Summary

Baumchiquabaumbaum.

Bibliography

Vietoris-Rips. https://en.wikipedia.org/wiki/Vietoris_Rips_complex (5.6.2018).

Čech-complex. https://en.wikipedia.org/wiki/Cech_complex (5.6.2018).

Lecture notes from prof. dr. Neža Mramor Kosta.