

Sensors

Computational topology - group project

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1 Problem description

- Problem
- Goals

2 Topological solution

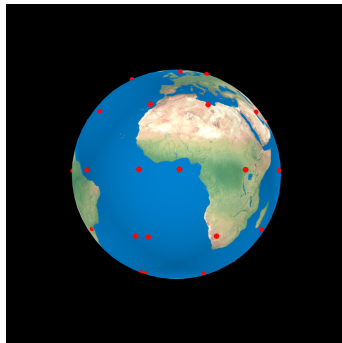
- Vietoris-Rips complex
- Čech complex

3 Results and implementation

- Data generator
- Algorithm for MC simulated annealing

Problem description

Number of sensors on the sphere of radius 1 (Earth):



- each sensor gathers data from the surrounding area in the shape of a circle of radius R ,
- each sensor can communicate with other sensors which are at most r away.

- 1 The sensor network is connected.
- 2 The sensor network covers the whole sphere.
- 3 Values of r and R are as small as possible.
- 4 There are no obsolete sensors.
- 5 Find optimal distribution of 50 sensors on the sphere.

Vietoris-Rips complex $\longrightarrow r$

Connected sensor network \longrightarrow Such r so that Vietoris-Rips complex $VR_r(S)$ is connected.

- sensors: S ($S_i = (r_i, \phi_i, \theta_i)$),
- sensor connections $\{S_i, S_j\} \subset S; d(S_i, S_j) \leq 2r$,
- $F \subset S$ is a simplex in $VR_r(S)$, if $\text{diam } F \leq 2r$.

Čech complex $\longrightarrow R$

The sensor network covers the whole sphere \longrightarrow Such R so that Euler characteristic of Čech complex should be that of a sphere.

- sensors: S ($S_i = (r_i, \phi_i, \theta_i)$),
- $B_R(x)$ closed ball with radius R around x ,
- $\check{C}_R = \{\sigma \subset S, \cap_{x \in \sigma} B_R(x) \neq \emptyset\}$.

In practice, instead of calculating Euler characteristic we checked first two Betti numbers.

Distribution of points on the sphere so that parameters r and R are as small as possible.

Electrostatic potential energy

$$V = \sum_{i \neq j} V_{ij} \propto \sum_{i \neq j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$$

- Electrons would distribute themselves evenly around the sphere.
- Minimization of V with simulated annealing.

Algorithm for MC simulated annealing

- 1 Start with random distribution of points on sphere.
- 2 Set initial temperature of the system T .
- 3 Choose random point, move it according to Gaussian distribution.
- 4 Calculate difference in energy ΔE .
- 5 If $\Delta E < 0$, accept the change.
- 6 If $\Delta E \geq 0$, accept the change with probability $\exp(\frac{-\Delta E}{T})$
- 7 If enough changes accepted, decrease the temperature T .
- 8 Repeat process from 3. \longrightarrow

Summary

Baumchiquabaumbaum.



Vietoris-Rips.

https://en.wikipedia.org/wiki/Vietoris_Rips_complex
(5.6.2018).



Čech-complex. https://en.wikipedia.org/wiki/Cech_complex
(5.6.2018).



Lecture notes from prof. dr. Neža Mramor Kosta.