B1:

1.9. $\{x_1, \dots, x_n\}$

$\hat{\rho}(h)$: Sample ACF, at lag h.

a, $x_t = a + b.t$. ~~$n-h+k$~~

Tính $\hat{\rho}(h)$:

$$\hat{\rho}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)}$$

$$\hat{\gamma}(h) = \sum_{i=1}^{n-h} (x_i - \bar{x})(x_{i+h} - \bar{x}) = \sum_{i=1}^{n-h} (x_i - \bar{x})(x_{i+h} - \bar{x})$$

$$\hat{\gamma}(0) = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\bar{x} = a + \frac{n+1}{2} b$$

Xét: $$\hat{\gamma}(h) = \sum_{i=1}^{n-h} (-\bar{x} + a + ib)(-\bar{x} + a + (i+h)b)$$

$$= b^2 \sum_{i=1}^{n-h} \left(-\frac{n+1}{2} + i\right)\left(-\frac{n+1}{2} + i + h\right)$$

$k^2 + 2ak + a^2$
$+ hk + ha$

Đặt $a = -\frac{n+1}{2}$, $k = n-h$.

Có: $$\sum_{i=1}^{n-h} \left(-\frac{n+1}{2} + i\right)\left(-\frac{n+1}{2} + i + h\right) = \sum_{i=1}^{k} (i+a)(i+a+h)$$

$$= (1+a)(1+a+h) + (2+a)(2+a+h) + \dots + (k+a)(k+a+h)$$

$(k+a)^2 + h(k+a)$

$$= a^2(1^2+2^2+\dots+k^2) + 2a.(1+2+\dots+k) + k.a^2 + h(1+2+\dots+k) + k.a.h$$

$$= \frac{k(k+1)(2k+1)}{6} + 2a.\frac{k(k+1)}{2} + ka^2 + h.\frac{k(k+1)}{2} + k.a.h.$$

$$= \frac{(n-h)(n-h+1)(2n-2h+1)}{6} + \frac{-(n+1)}{2}.(n-h)(n-h+1) + (n-h).\left(\frac{n+1}{2}\right)^2 + h.\frac{(n-h)(n-h+1)}{2} + (n-h).h$$

$\cancel{\Rightarrow b^2 \frac{13}{12} n^3}$

~~bài 4 với n~~

~~thay h = 0 cx là bài 4~~

Khi $n \to \infty$: $\hat{\gamma}(n) \to \frac{13}{12} b^2 n^2$

$$\to \lim_{n\to\infty} \hat{\rho}(h) = \lim_{n\to\infty} \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)} = 1$$

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1.9b Giả sử $C > 1$, $h \geq 0$

$$\sum_{k=1}^{n} \cos(kx) = \frac{\sin\left(\frac{nx}{2}\right)\cos\left(\frac{(n+1)x}{2}\right)}{\sin\left(\frac{x}{2}\right)} \quad \forall x \in \mathbb{R}, \ \forall n \in \mathbb{N}^*$$

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t = \frac{1}{n}\sum_{t=1}^{n} \cos(\omega t)$$

$$= \frac{\sin\left(\frac{n\omega}{2}\right)\cos\left(\frac{(n+1)\omega}{2}\right)}{n\sin\left(\frac{\omega}{2}\right)} \xrightarrow{n\to\infty} 0$$

$$\hat{\gamma}(h) = \frac{1}{n}\sum_{t=1}^{n-h}(x_{t+h} - \bar{x})(x_t - \bar{x})$$

$$= \frac{1}{n}\sum_{t=1}^{n-h}\left(x_{t+h}x_t - \bar{x}(x_t + x_{t+h}) + \bar{x}^2\right)$$

Xét $\frac{1}{n}\sum_{t=1}^{n-h} x_{t+h}x_t = \frac{1}{n}\sum_{t=1}^{n-h}\cos((t+h)\omega)\cos(t\omega)$

$$= \frac{1}{2n}\sum_{t=1}^{n-h}\left(\cos((2t+h)\omega) + \cos(\omega h)\right)$$

$$= \frac{1}{2n}(n-h)\cos(\omega h) + \frac{1}{2n}\sum_{t=1}^{n-h}\cos((2t+h)\omega)$$

$$\xrightarrow{n\to\infty} \frac{1}{2}\cos(\omega h)$$

$$\frac{1}{n}\sum_{t=1}^{n-h}\bar{x}(x_{t+h} + x_t) = \bar{x}\cdot\frac{1}{n}\sum_{t=1}^{n-h}\left(\cos((t+h)\omega) + \cos(t\omega)\right) \xrightarrow{n\to\infty} 0$$

Do đó $\hat{\gamma}(h) \xrightarrow{n\to\infty} \frac{1}{2}\cos(\omega h)$

$$\hat{\gamma}(0) \xrightarrow{n\to\infty} \frac{1}{2}$$

$$\Rightarrow \lim_{n\to\infty}\hat{\rho}(h) = \cos(\omega h)$$

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1.10 $m_t = \sum_{k=0}^{p} c_k t^k \quad t = 0, \pm 1, \ldots$

$\nabla m_t = m_t - m_{t-1} = \sum_{k=0}^{p} c_k t^k - \sum_{k=0}^{p} c_k (t-1)^k$

$= \cancel{\sum_{k=0}^{p} c_k t^k - \sum_{k=0}^{p} c_k} \sum_{i=0}^{k} \binom{k}{i} t^{k-i} (-1)^i$

$= \cancel{\sum_{k=0}^{p} c_k t^k - \sum_{k=0}^{p}} c_k \sum_{i=0}^{k} \binom{k}{i} t^{k-i} (-1)^i$

$= \sum_{k=0}^{p} c_k \underbrace{\left(t^k - (t-1)^k\right)}_{\text{Đa thức bậc } p-1}$

$\Rightarrow \nabla^{p+1} m_t = \underbrace{\nabla^p m_t - \nabla^p m_{(t-1)}}_{\text{Đa thức bậc } 0} = 0$

$E(a+bt) = a + bE(t)$

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1.11. $a_j = (2q+1)^{-1}$

a, $m_t = c_0 + c_1 t$

CM $\sum_{j=-q}^{q} a_j m_{t-j} = m_t$

$$VT = \sum_{j=-q}^{q} a_j m_{t-j} = \sum_{j=-q}^{q} \frac{1}{2q+1}\left(c_0 + c_1(t-j)\right)$$

$$= c_0 + c_1 t + \sum_{j=-q}^{q} \frac{c_1 \cdot (-j)}{2q+1}$$

$$= c_0 + c_1 t \quad 0 = c_0 + c_1 t$$

b, $$A_t = \sum_{j=-q}^{q} a_j Z_{t-j}$$

$$E(A_t) = \sum_{j=-q}^{q} a_j E(Z_{t-j}) = 0$$

$$Var(A_t) = \sum_{j=-q}^{q} a_j^2 Var(Z_{t-j})$$

$$= \frac{1}{(2q+1)^2}(2q+1) \cdot \sigma^2 = \frac{\sigma^2}{2q+1}$$ rất bé khi q lớn

1.12.

$$m_t = \sum_j a_j m_{t-j}$$

$$\Leftrightarrow \begin{cases} \sum_j a_j = 1 \\ \sum_j j^r a_j = 0 \quad \forall r = \overline{1,k} \end{cases}$$

$$c_0 + c_1 t + \dots + c_k t^k = \sum_j a_j m_{t-j}$$

$$\Leftrightarrow c_0 + c_1 t + \dots + c_k t^k = \sum_j a_j \left(c_0 + c_1 (t-j) + \dots + c_k (t-j)^k\right)$$

VP $$= \sum_j a_j \sum_{r=0}^{k} c_r (t-j)^r$$

$$= \sum_j a_j \sum_{r=0}^{k} c_r \sum_{i=0}^{r} \binom{r}{i} t^i (-j)^{r-i}$$

$$= \sum_{r=0}^{k} c_r \left(\sum_j a_j \sum_{i=0}^{r} \binom{r}{i} t^i (-j)^{r-i} \right)$$

$$= \sum_{r=0}^{k} c_r \left(\sum_{i=0}^{r} \binom{r}{i} t^i \sum_j a_j (-j)^{r-i} \right)$$

$$\Rightarrow t^r = \sum_{i=0}^{r} \binom{r}{i} t^i \sum_j a_j (-j)^{r-i}$$

$$r = 0 \to 1 = \sum_j a_j$$

$\cancel{1} - \cancel{i}$

$$r = 1 \to 1 = \sum_{i=0}^{1} \binom{1}{1} \sum_j a_j (-j)$$

$$= \sum_j a_j + \sum_j a_j (-j)$$

$$\Rightarrow = 0$$

tg tự với $r = 2, 3, \ldots, k$ ta đc $\sum_j j^r a_j = 0$

b, $[a_0, \ldots, a_7] = \frac{1}{320} [74, 67, 46, 21, 3, -5, -6, -3]$

$$\left.\begin{array}{l} \cdot \sum_{j=0}^{7} a_j = 1 \\ \cdot \sum_{j=0}^{7} j^r a_j = 0 \quad \forall r \in \{1, 2, 3\} \end{array}\right\} \Rightarrow \text{ĐPCM}$$

1.13. $m_t = c_0 + c_1 t$ bkỳ (c_0, c_1)

filter pass linear without distortion:

$$m_t = (1 + \alpha B + \beta B^2 + \gamma B^3) m_t \quad \forall t$$

$$\Leftrightarrow \alpha . m_{t-1} + \beta . m_{t-2} + \gamma m_{t-3} = 0$$

$$\Leftrightarrow \alpha (c_0 + c_1 (t-1)) + \beta (c_0 + c_1 (t-2)) + \gamma (c_0 + c_1 (t-3)) = 0$$

$$\Leftrightarrow c_0 (\alpha + \beta + \gamma) + c_1 . (\alpha t + \beta t + \gamma t - \alpha - 2\beta - 3$$

$$\Leftrightarrow \begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha + 2\beta + 3\gamma = 0 \end{cases}$$

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Eliminate arbitary seasonal component of $d = 2$.

$$\Leftrightarrow \begin{cases} (1+\alpha B+\beta B^2+\gamma B^3) S_t = 0 & \forall t. \\ S_{t+2} = S_t, \; S_1 + S_2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} S_t + \alpha S_{t-1} + \beta S_{t-2} + \gamma S_{t-3} = 0 \\ S_{t+2} = S_t, \; S_1 + S_2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} S_t (1+\beta) + S_{t-1} (\alpha + \gamma) = 0 \\ S_{t+2} = S_t, \; S_1 + S_2 = 0 \end{cases}$$

$$\Leftrightarrow (1+\beta) = (\alpha + \gamma)$$

$$\Rightarrow (\alpha, \beta, \gamma) = \left(\frac{1}{4}; \frac{-1}{2}; \frac{1}{4}\right)$$

1.14.

$) = 0$

$$\sum_j a_j = 1$$

$r=1$: $\sum_j j^r a_j = -2a_{-2} + -1 a_{-1} + 0a_0 + 1a_1 + 2a_2$

$r=2$: $= 4a_{-2} + 1a_{-1} + 0a_0 + 1a_1 + 4a_2 = 0$

$r=3$. $\sum_j j^3 a_j = -8a_{-2} - a_{-1} + 0a_0 + 1a_1 + 8a_2 = 0$

$\Rightarrow$ (

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1.15. Y_t, chuỗi dừng, $E=0$, a, b hằng số

$X_t = a + bt + s_t + Y_t$, S_t: period 12.

Show that: $Z_t = \nabla\nabla_{12} X_t = (1-B)(1-B^{12})X_t$ là chuỗi dừng.

Tìm hàm tự hiệp phương sai của chuỗi mới này.

$(1-B^{12})X_t$

$X_t - X_{t-12}$

$Z_t = (1-B)(X_t - X_{t-12}) = X_t - X_{t-12} - (X_{t-1} - X_{t-13})$

$= X_t - X_{t-1} - X_{t-12} + X_{t-13}$

$E(Z_t) = 0$

$= bt - b(t-1) - b(t-12) + b(t-13)$

$+ Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13} = Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}$

$Cov(Z_t, Z_{t+h}) =$

$Cov(Y_t - Y_{t-1} - Y_{t-12} + Y_{t-13}, Y_{t+h} - Y_{t-1+h} - Y_{t-12+h} + Y_{t-13+h})$

$$= \begin{cases} 4\,Var(Y_t) & h = 0 \\ -Var(Y_{t-12}) - Var(Y_{t-1}) & h = 1, 12 \\ - \cdots & h = 13 \end{cases}$$

$+Var(Y_{t-1}) + Var(Y_{t-12}) + Var(Y_{t-13})$

~~h = 13~~

0 5 10 15 20

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1.15 b, $Z_t = \nabla_{12}^2 X_t = (1-B^{12})^2 X_t$

$= \cancel{a+} (1-B^{12})(1-B^{12}) X_t$

$= (1-B^{12})(X_t - X_{t-12})$

$= X_t - X_{t-12} - (X_{t-12} - X_{t-24})$

$= X_t - 2X_{t-12} + X_{t-24}$

$= (a+bt)s_t + Y_t - 2\left[(a+b(t-12))s_{t-12} + Y_{t-12}\right]$

$+ (a+b(t-24))s_{t-24} + Y_{t-24}$

$= Y_t - 2Y_{t-12} + Y_{t-24}$

Làm tg tự.