Problem Set 1

CS 6347

Due: 2/18/2024 by 11:59pm

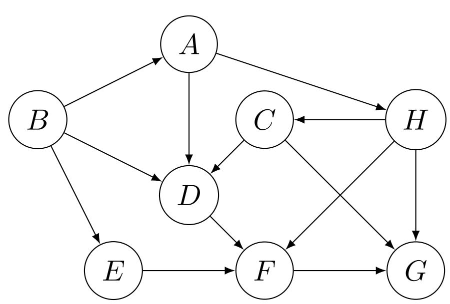
Note: all answers should be accompanied by explanations for full credit. Late homeworks will NOT be accepted.

Problem 1: Separation and Independence (30 pts)

1. Consider the following joint probability distribution, p. Find a directed graph G such that G is a perfect I-map for p. Is G the only directed graph with this property?

A	B	C	$p(A, B, C)$
0	0	0	1/4
0	0	1	1/4
0	1	0	1/24
0	1	1	1/8
1	0	0	1/8
1	0	1	1/8
1	1	0	1/48
1	1	1	1/16

2. Consider the following Bayesian network.



Use the above Bayesian network to answer the following questions. You must explain your answer for full credit.

(a) List all of the local Markov independence relations implied by this graph.

(b) Is G d-separated from A given C, H?

(c) Is B d-separated from H given A?

(d) Is H d-separated from E given A, G?

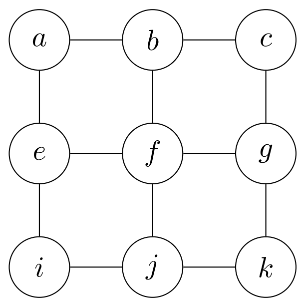
(e) Is E d-separated from A given B, F, H?

(f) Is A d-separated from D given C, B?

3. For an undirected graph $G = (V, E)$, let $X, Y, Z_1, Z_2 \subseteq V$ such that all sets except for possibly Z_1 and Z_2 are mutually disjoint. Argue that if X is graph separated from Y given $Z_1 \cap Z_2$, then X is graph separated from Y given Z_1 and X is graph separated from Y given Z_2. Is the converse also true?

Problem 2: Treewidth (10 pts)

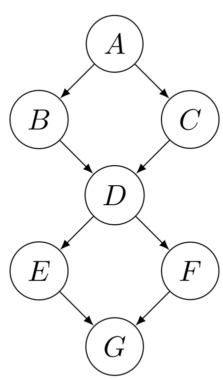
1. Compute the treewidth of the following 3×3 grid graph. Provide an optimal elimination ordering.



2. If the grid is enlarged to $n \times n$ nodes, how does the treewidth scale with n (an asymptotic result is sufficient here)?

3. Explain why the treewidth of a graph is always at least as large as the size of a maximal clique in the graph minus one.

Problem 3: Marginal Bayes Nets (15 pts)



A Bayesian network is a minimal I-map for a distribution if removing any edges from the network will cause it to no longer be an I-map for the distribution. Consider the Bayesian network given above. Construct a new Bayesian network over all of the nodes except node D that is a minimal I-map for the marginal distribution over the remaining variables (A, B, C, E, F, G) for any probability distribution that factorizes over the given Bayesian network.

Be sure that your model contains all of the dependencies from the original network.

Problem 4: Vertex Covers (15 pts)

Consider an undirected graph $G = (V_G, E_G)$. A vertex cover is a subset $S \subseteq V_G$ such that every edge in E_G is incident to at least one vertex in S.

1. Explain how to construct a Markov random field to represent the uniform probability distribution, p, over valid vertex covers of G.

2. Explain why your construction always yields a valid probability distribution.

3. Let G be the graph consisting of a single cycle on four nodes. What is the partition function (normalizing constant) of the MRF for this choice of G?

4. Explain how to construct an MRF to represent a probability distribution over vertex covers such that for all vertex covers S, $p(S) \propto \exp(|S|)$.