Noise Model

| Spatial domain | $g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$ |
| Frequency domain | $G(u, v) = H(u, v)F(u, v) + N(u, v)$ |

Image Restoration using Spatial Filters

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$
$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Assume the noise is only the degradation source
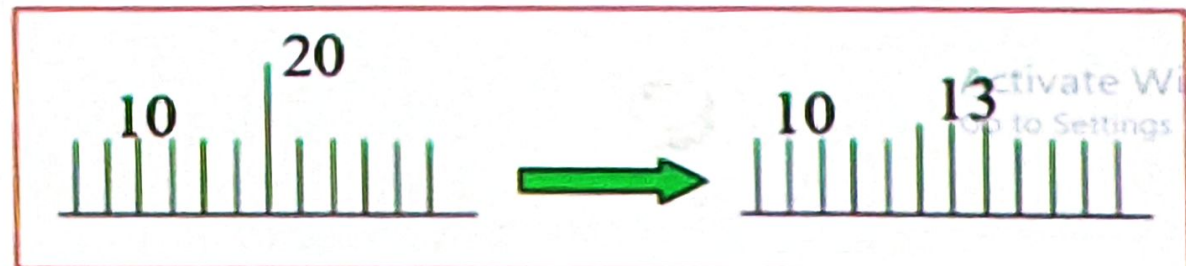
- Spatial Filtering

3x3

Arithmetic mean filter

- The arithmetic mean filter is same as smoothing filter. It removes the noise and blur the image.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

- Change the values of pixels intensities to the nearby range based on the neighboring pixels intensities.

- Disadvantage: Image gets blur means pixels itself intensity value gets loose.
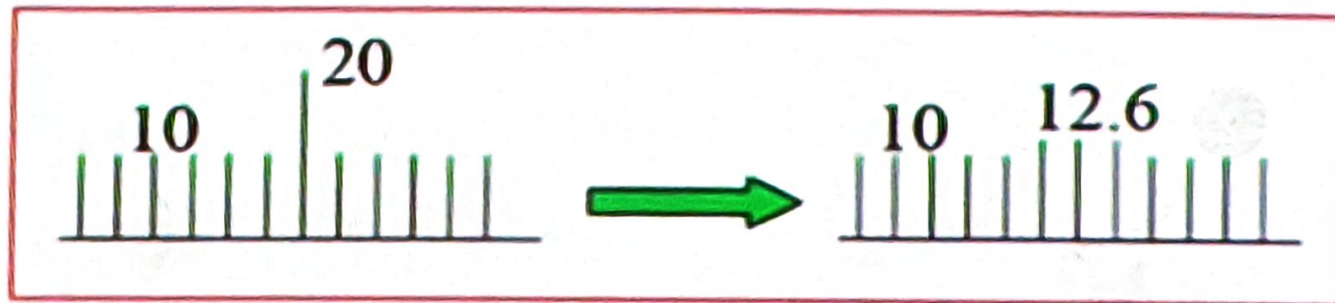
- Example

Types of Mean Filters

- Other types of Mean filters are:
    1. Geometric Mean
    2. Harmonic Mean
    3. Contraharmonic Mean
- Each filter has slightly different behavior

Geometric mean

- Tends to loose less image details as compared to the Arithmetic Mean
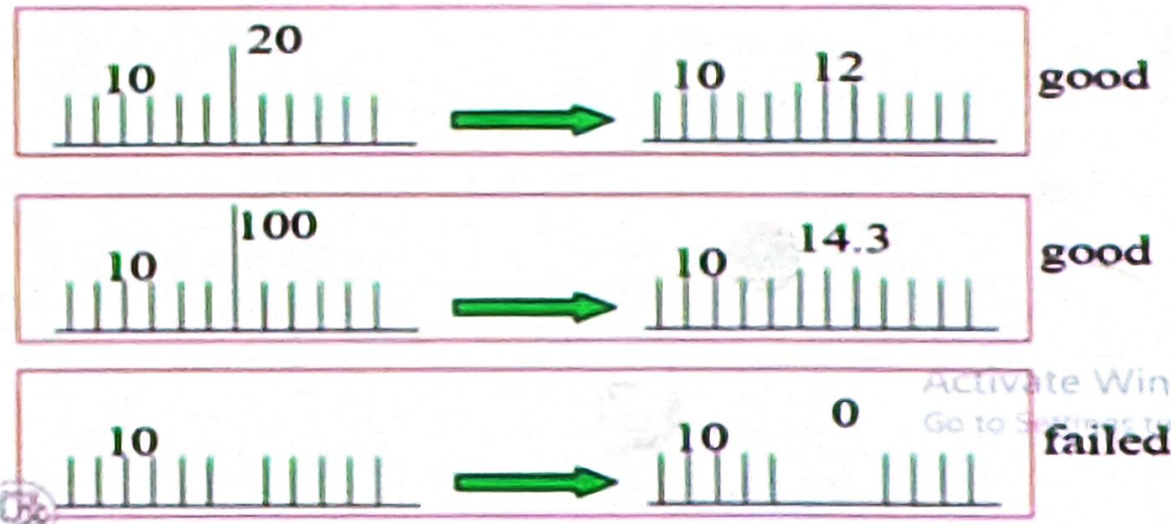
$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

Harmonic Mean

- As division, therefore for missing values it is unable to do perform, whereas Geometric Mean or Arithmetic Mean can handle missing values
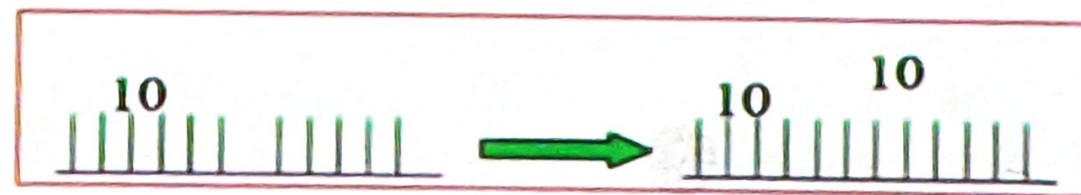
$$\hat{f}(x,y) = \frac{mn}{\displaystyle\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

good

good

failed

Contraharmonic Mean

- Q is the order of the filter.
- Adjusting the value of Q, the filter's behavior changes
- Positive values of Q eliminates pepper noise
- Negative values of Q eliminates salt noise

$$\hat{f}(x, y) = \frac{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\displaystyle\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

Q=1

Other Statistical Filters

- Spatial filters that are based on ordering the pixel values and work in neighboring pixels.

- Useful spatial filters include
  1. Median filter
  2. Max and min filter
  3. Midpoint filter
  4. Alpha trimmed mean filter

Median Filter

$$\hat{f}(x, y) = \underset{(s,t) \in S_{xy}}{median}\{g(s,t)\}$$

- Excellent at noise removal, without adding smoothing effects that can occur with arithmetic mean filter.. Particularly good when **salt and pepper** noise is present

- Better than average filter because, it does not change the intensity value rather select the median value

- Best for both salt and pepper noise as picks the median value

Min and Max Filter

- **Max Filter:**
- Max Filter is best to find the brightest points in the image
- Good in reducing pepper noise. Minimum pixels values will be ignored as max values are towards white
- **Min Filter:**
- Best to find the darkest points in the image.
- Good in reducing salt noise. It selects low intensity values. Low intensities values

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s,t)\}$$

$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s,t)\}$$

Midpoint filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- Middle between minimum and maximum filter.

- Useful for both salt and pepper noise

- Also useful for other types of noise including random Gaussian noise and uniform noise etc

Alpha trimmed filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

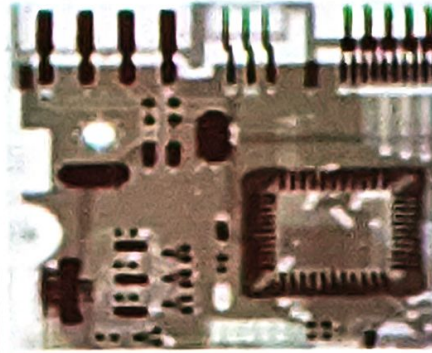1. d = 0 -> arithmetic mean filter
2. d = mn − 1 -> median filter

By tweaking the parameters, we can apply this
   filters to remove multiple types of noise

Noise Removal Examples

Image Corrupted By Salt And Pepper Noise
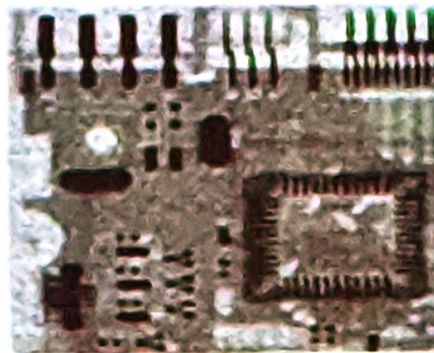
Result of 1 Pass With A 3*3 Median Filter

Result of 2 Passes With A 3*3 Median Filter
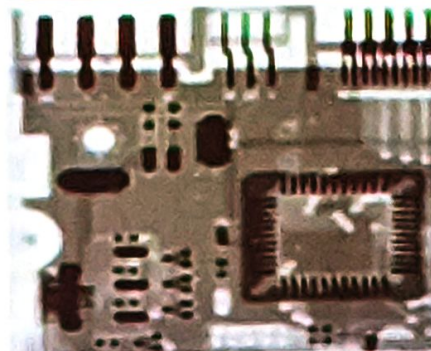
Result of 3 Passes With A 3*3 Median Filter
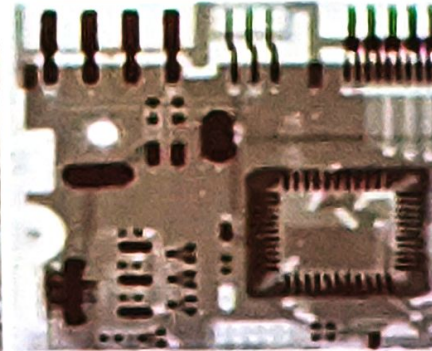
- Repeated passes of median filter tend to blur the image. keep number of passes as low as possible
- Decide whether to apply different types of filter instead of applying same filter multiple times
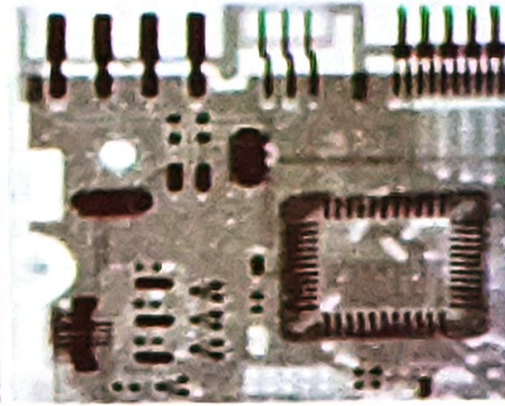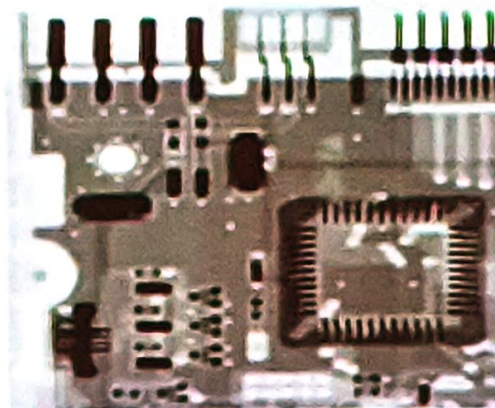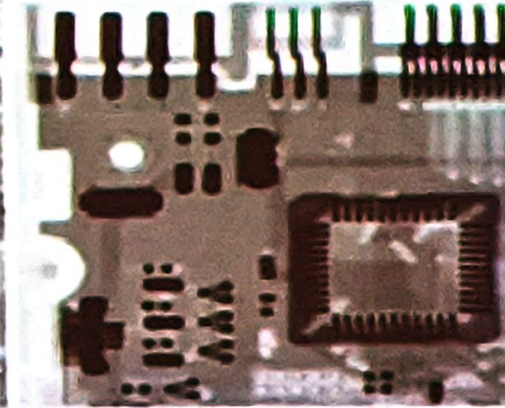
Noise Removal Examples

Image Corrupted By Pepper Noise
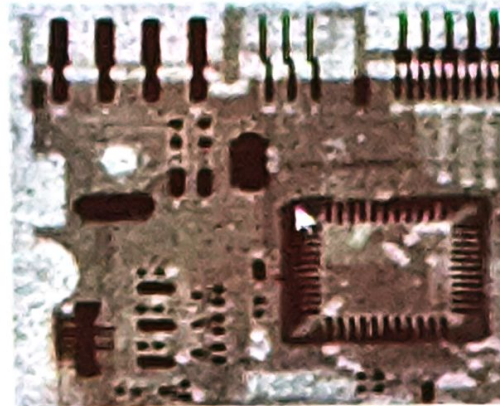
Image Corrupted By Salt Noise

Result Of Filtering Above With A 3*3 Max Filter

Result Of Filtering Above With A 3*3 Min Filter
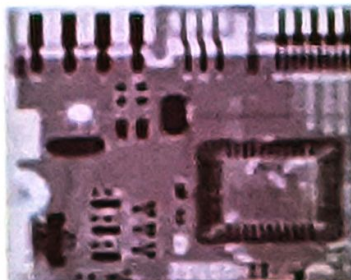
- Application of min filter and max filter
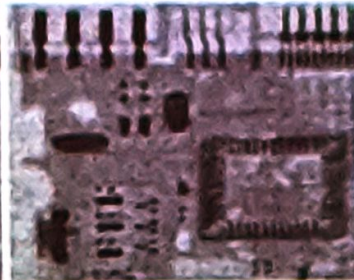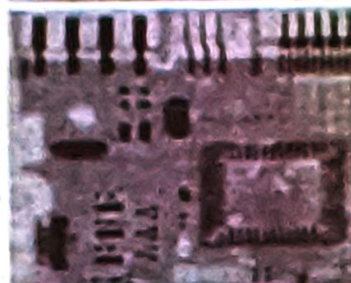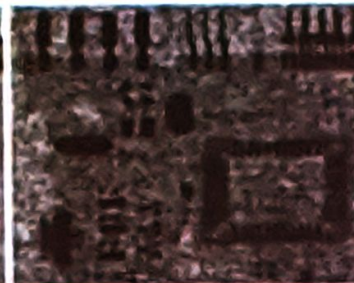
Image Corrupted By Uniform Noise

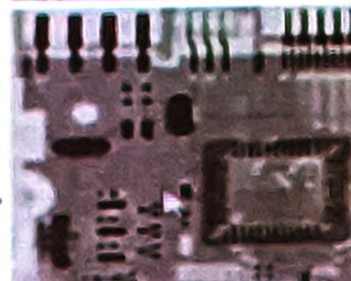Image Further Corrupted By Salt and Pepper Noise
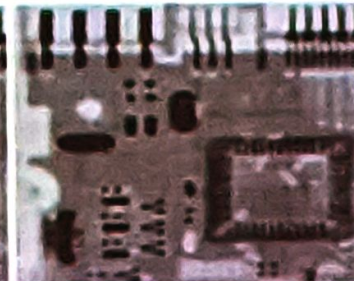
Filtered By 5*5 Arithmetic Mean Filter

Filtered By 5*5 Geometric Mean Filter

Filtered By 5*5 Median Filter

Filtered By 5*5 Alpha-Trimmed Mean Filter

THANK YOU