بسم الله الرحمن الرحيم

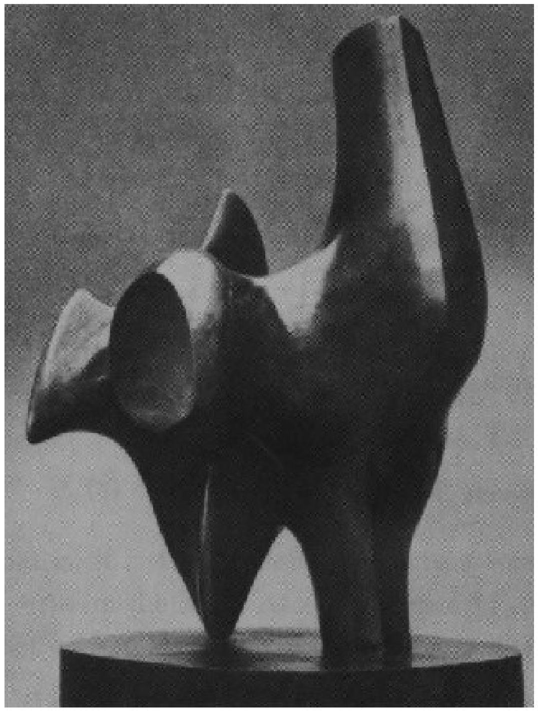
Computer Vision and Image Processing (CSEL-393)

Dr. Qurat ul Ain Akram
Assistant Professor
Computer Science Department (New Campus) KSK, UET, Lahore

Edge detection

- Goal: Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- Ideal: artist's line drawing (but artist is also using object-level knowledge)

Edge Detection



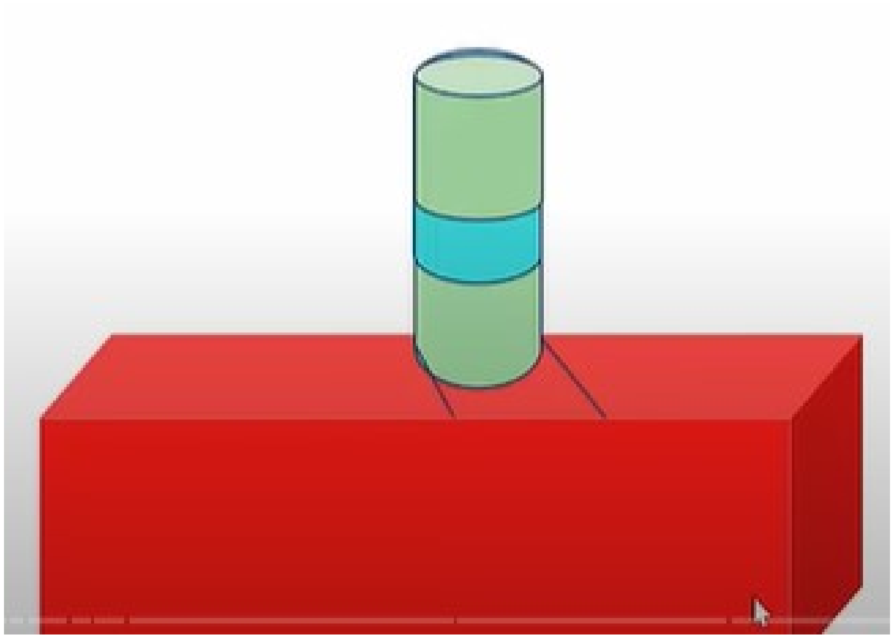
- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

Application

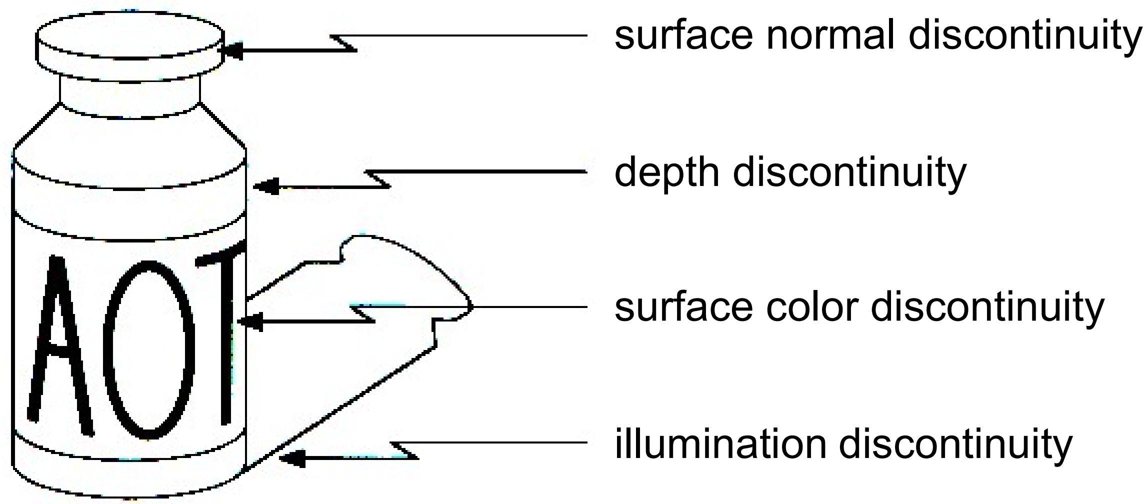
- What is an object
- How can we find it

Edge Detection in images

- At edges intensity or color changes

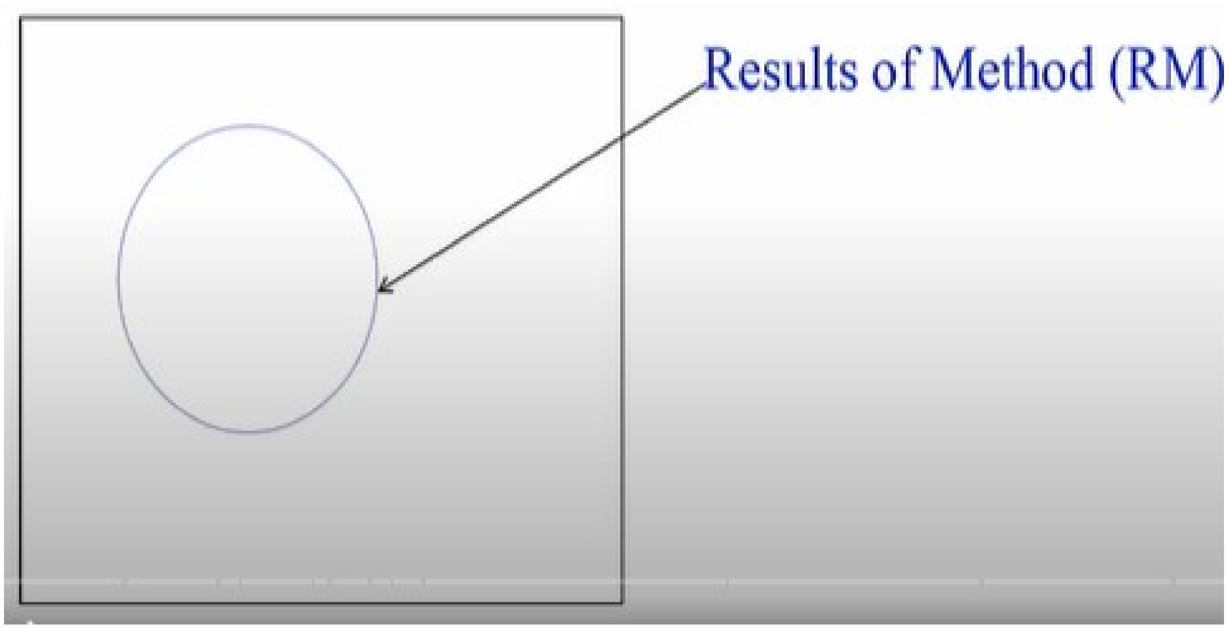


Origin of Edges

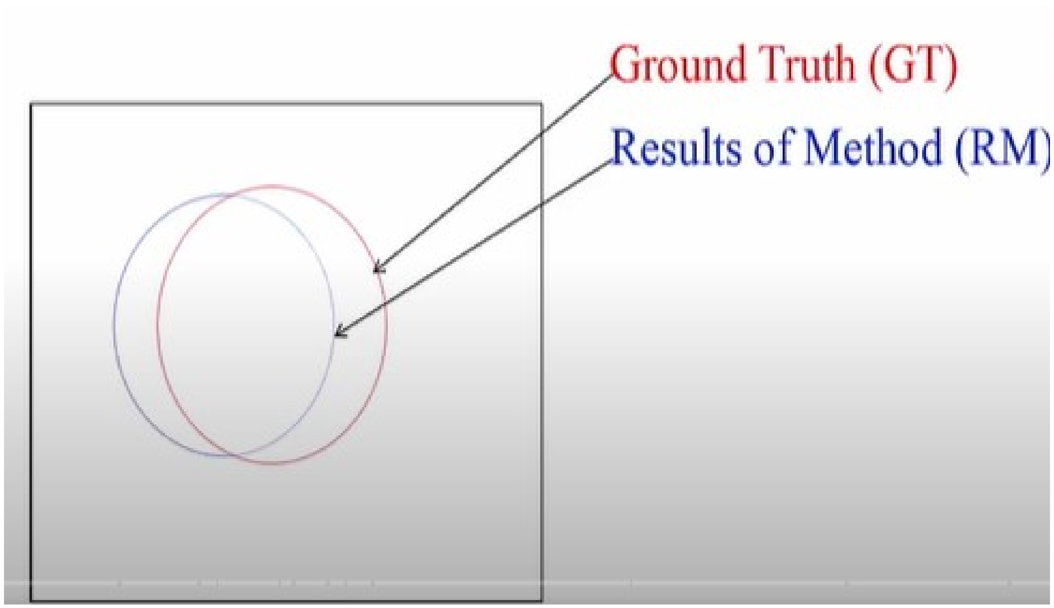


- Edges are caused by a variety of factors

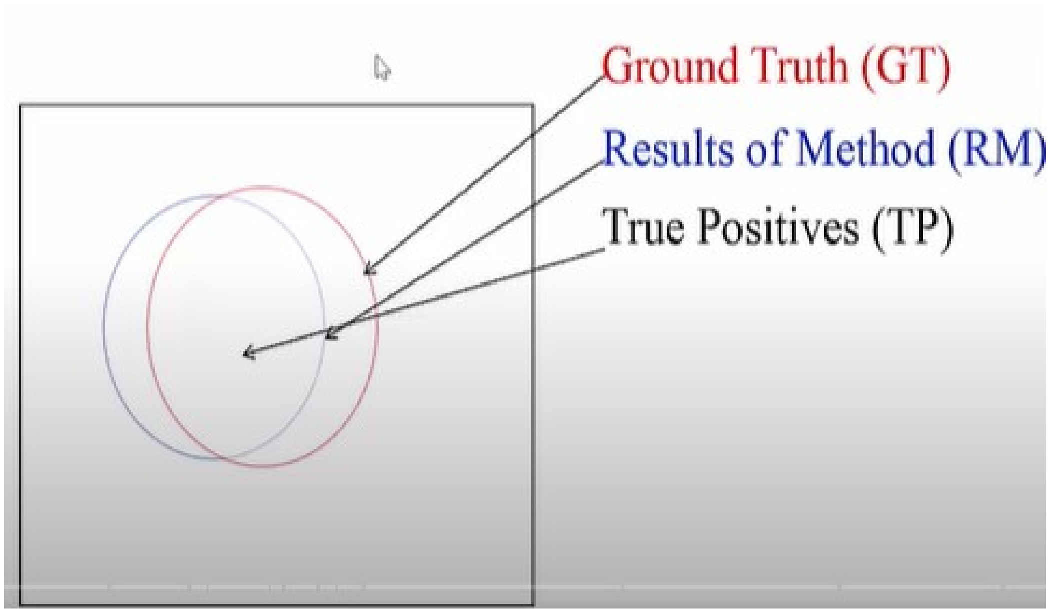
Evaluation Metrics



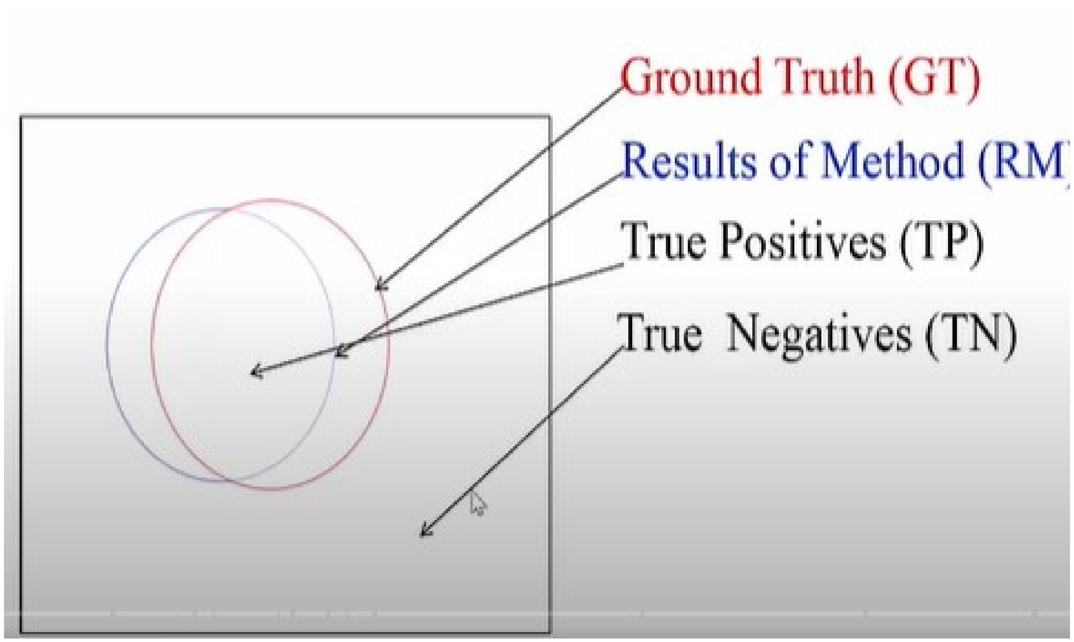
Evaluation Metrics



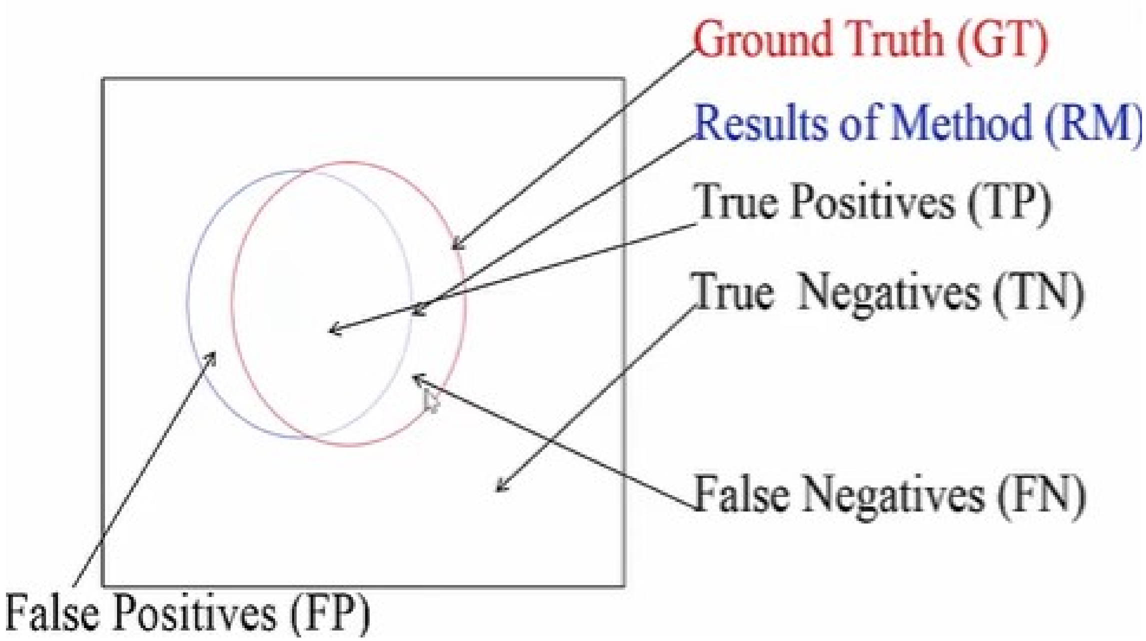
Evaluation Metrics



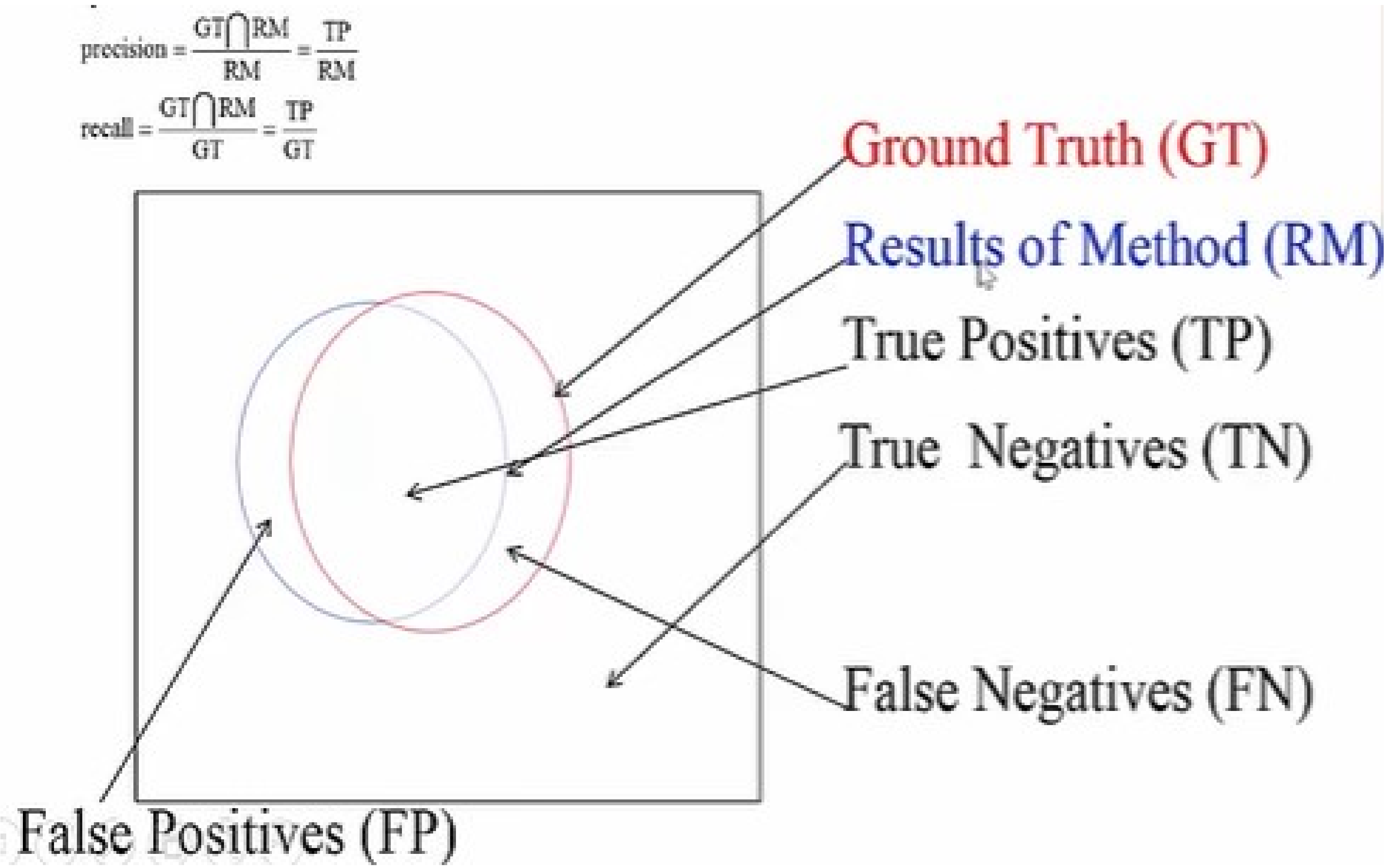
Evaluation Metrics

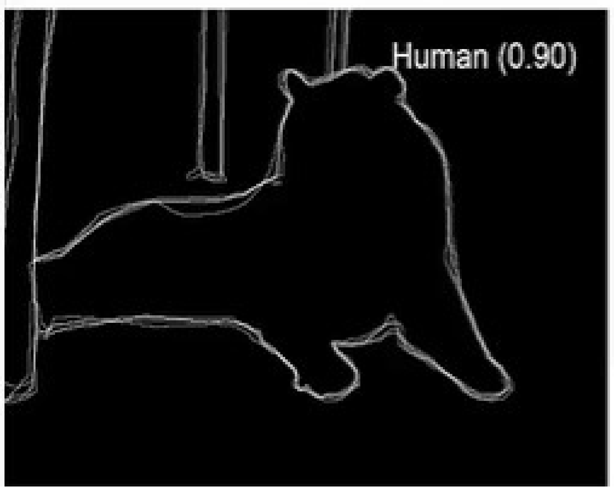


Evaluation Metrics



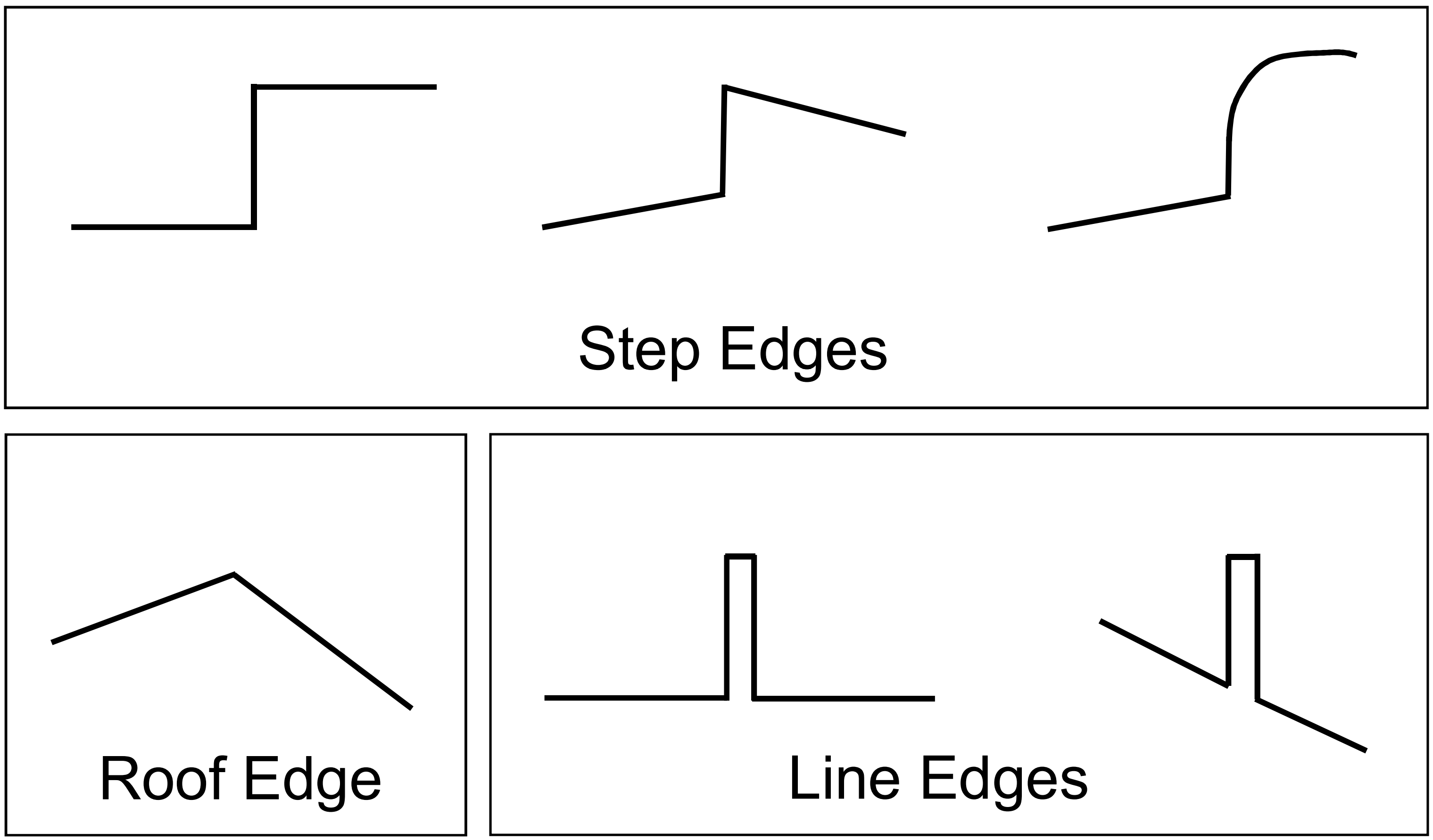
Evaluation Metrics



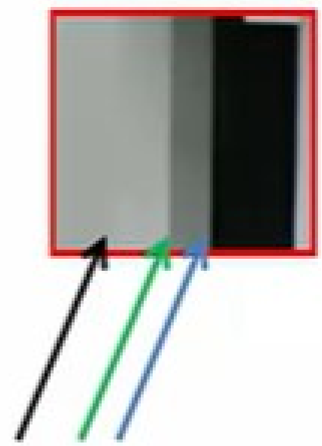


Slide Credit: James Hays

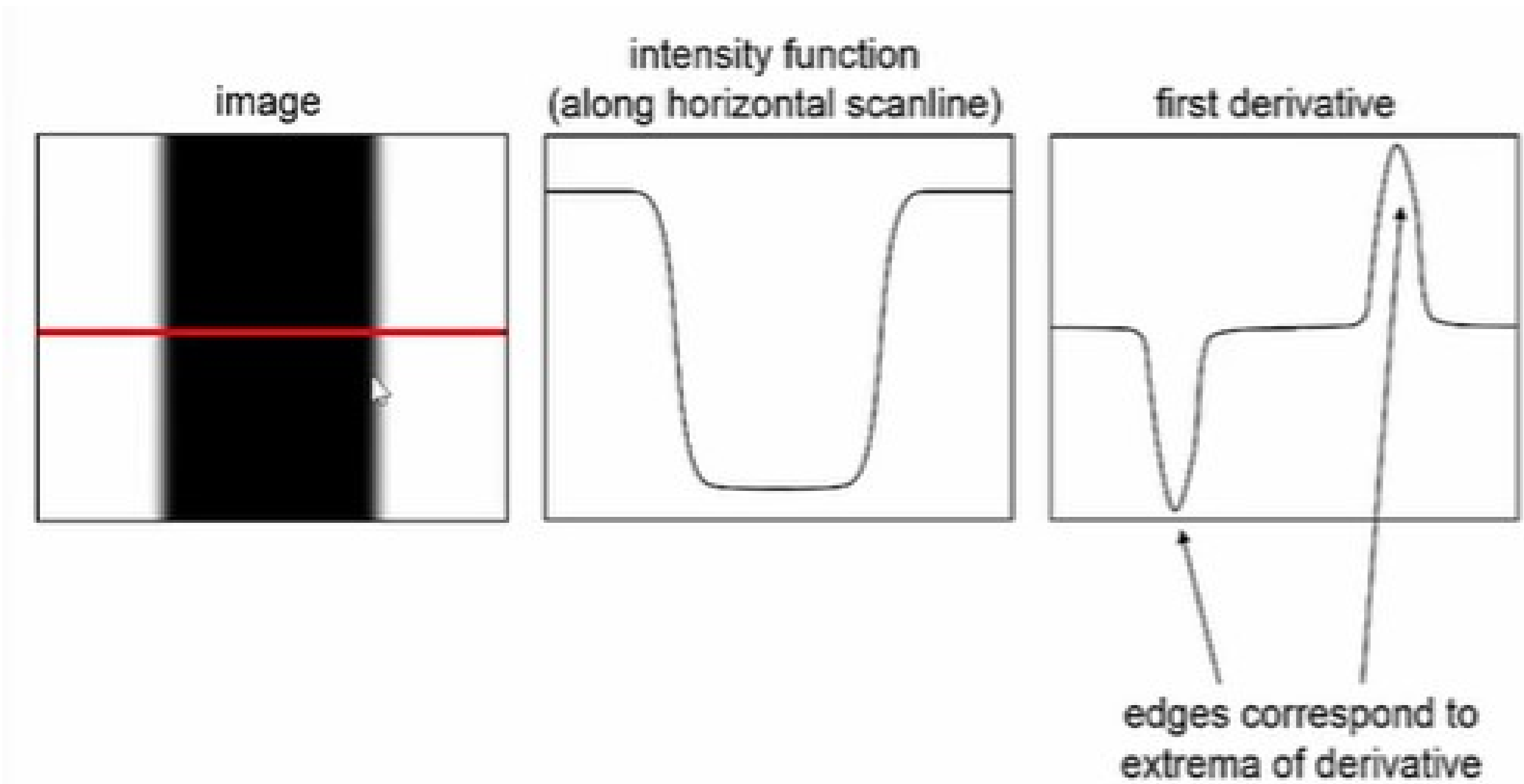
Edge Types



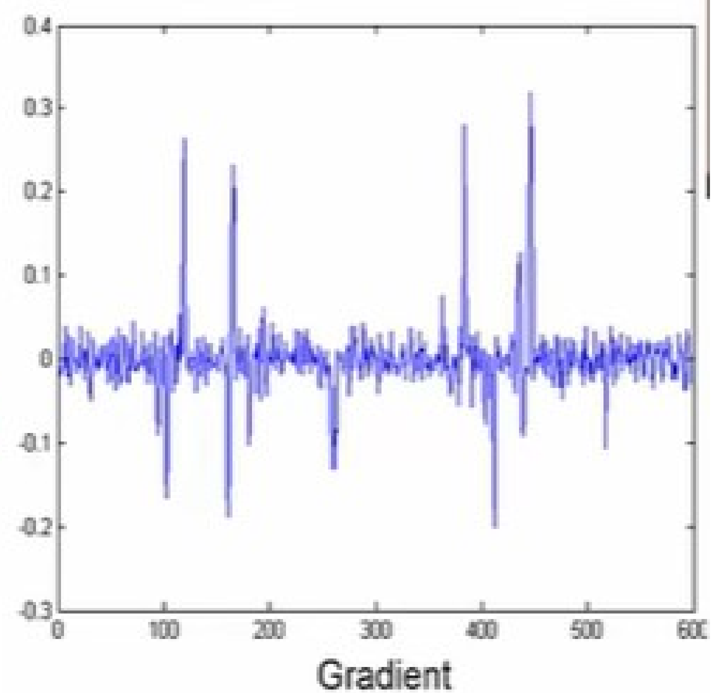
Example of Edges



- An edge is a place of rapid change in the image intensity function

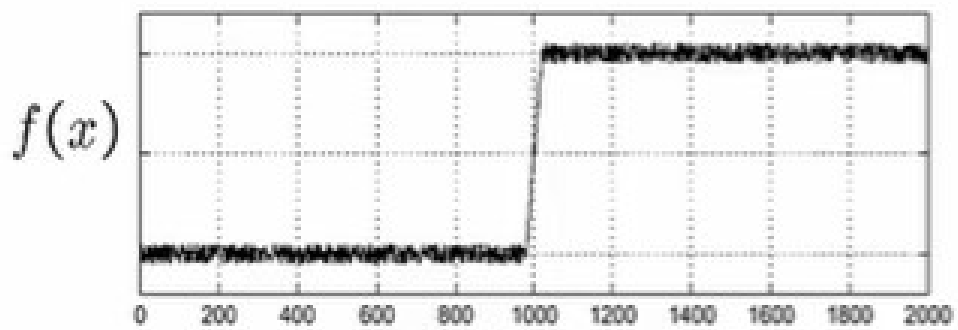


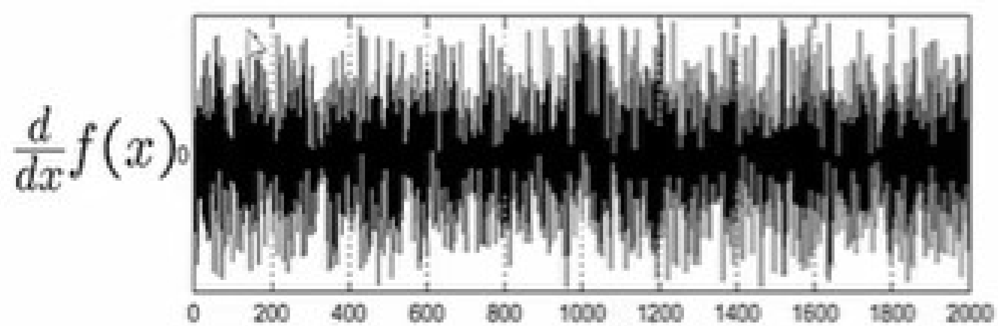
Effect of noise



Effect of Noise

- Consider a single row or column of the image
- Plotting intensity as a function of position gives a signal
- Where is the edge



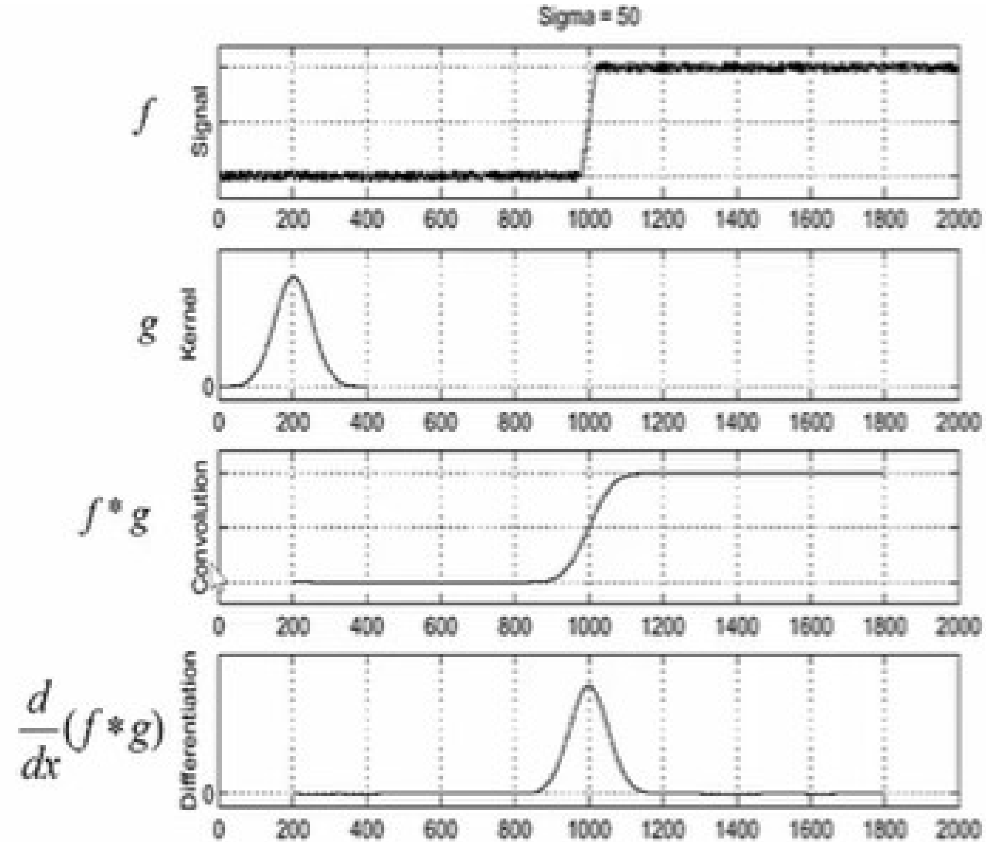


Effects of Noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- What can we do about it?

Solution

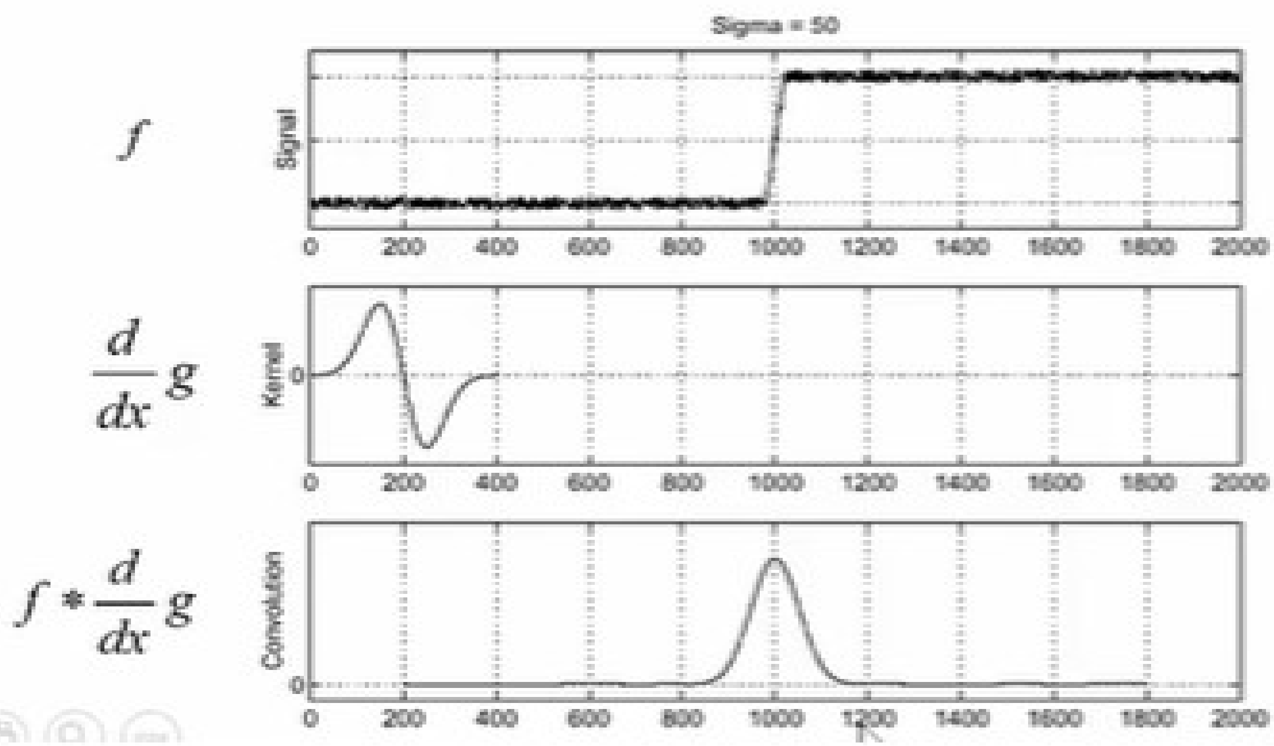
- First Smooth the image



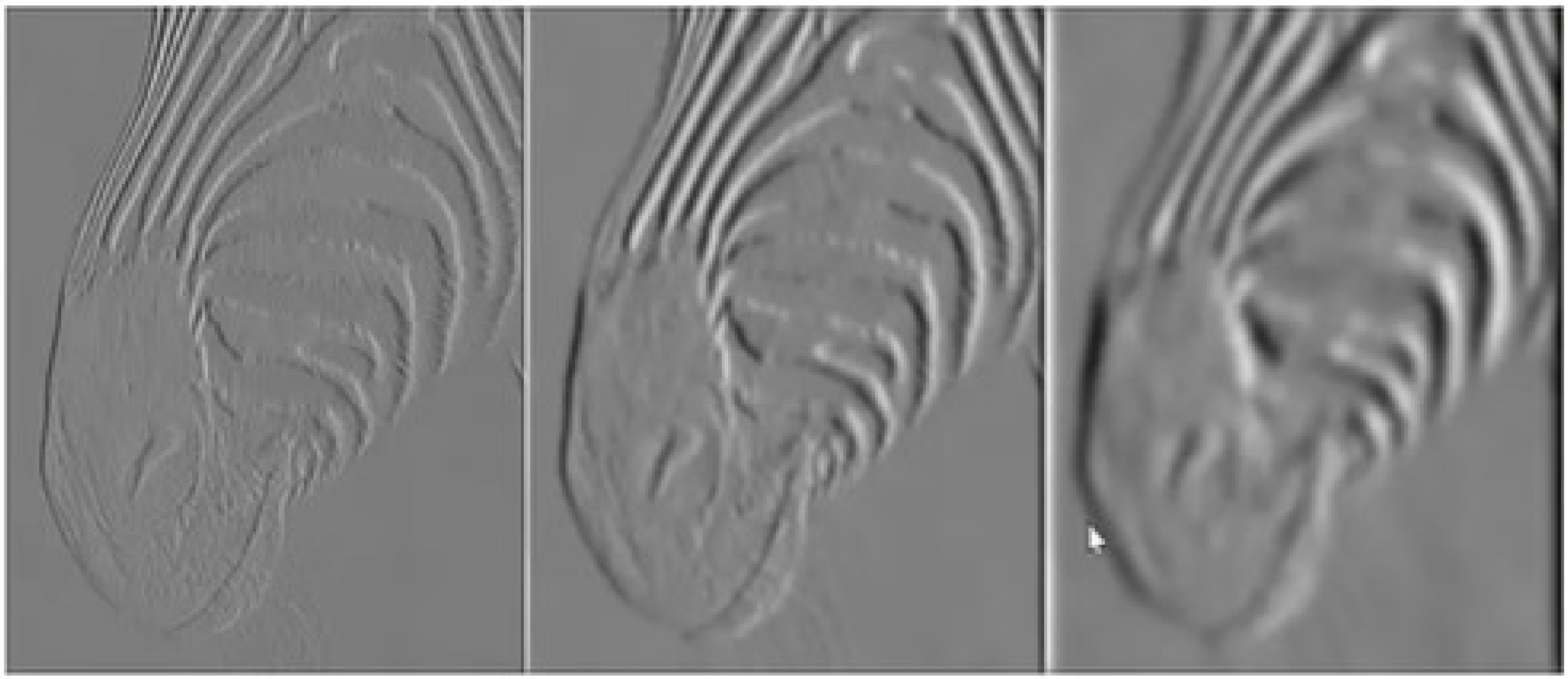
- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Derivative Theorem of Smoothing

- Differentiation is convolution, and convolution is associative: $$\frac{d}{dx}(f*g)=f*\frac{d}{dx}g$$
- This saves us one operation:



Tradeoff between smoothing and localization



1 pixel 3 pixels 7 pixels

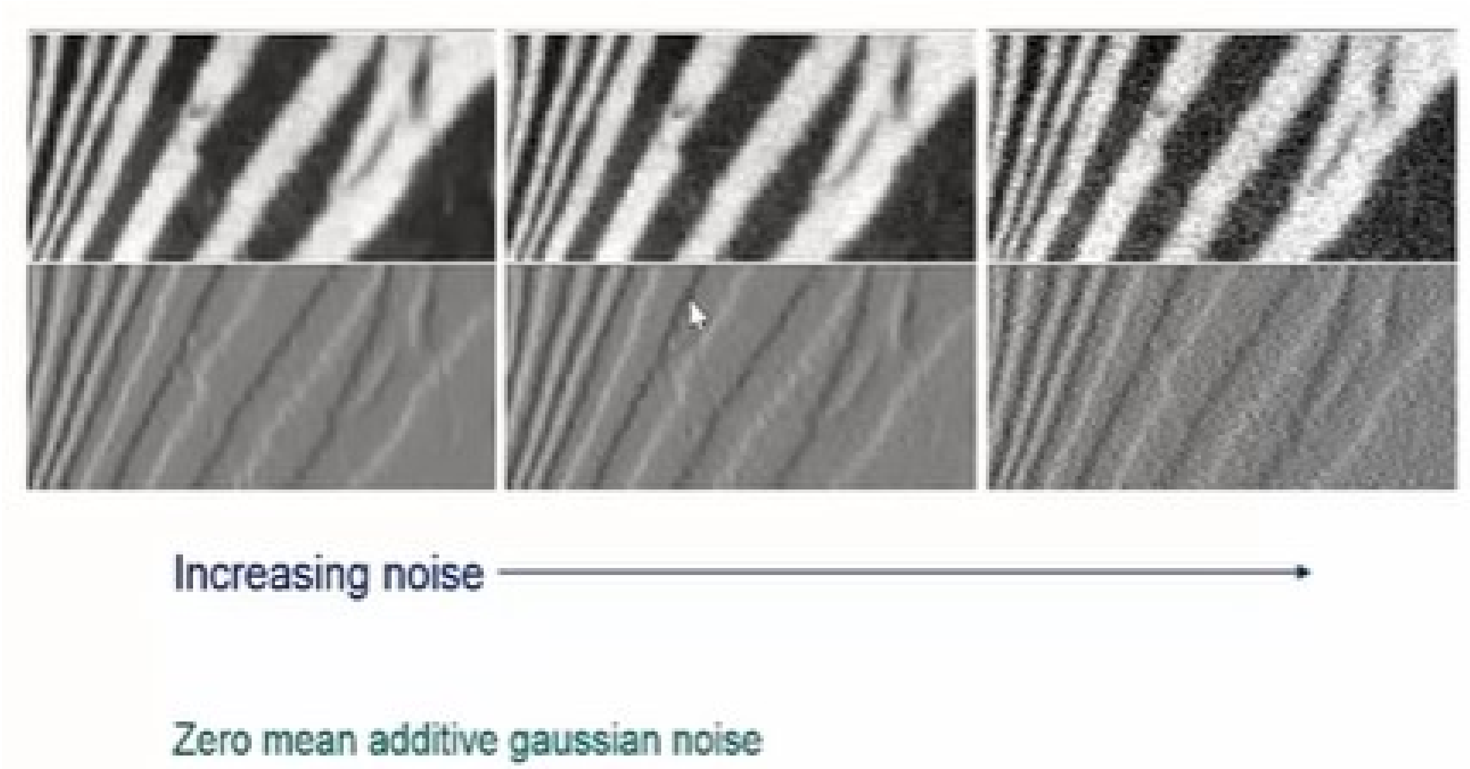
- Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”.

Derivatives and Noise

- Strongly affected by noise
  - obvious reason: image noise results in pixels that look very different from their neighbors
- The larger the noise is the stronger the response

- What is to be done?
  - Neighboring pixels look alike
  - Pixel along an edge look alike
  - Image smoothing should help
    - Force pixels different from their neighbors (possibly noise) to look like neighbors

Derivatives and Noise

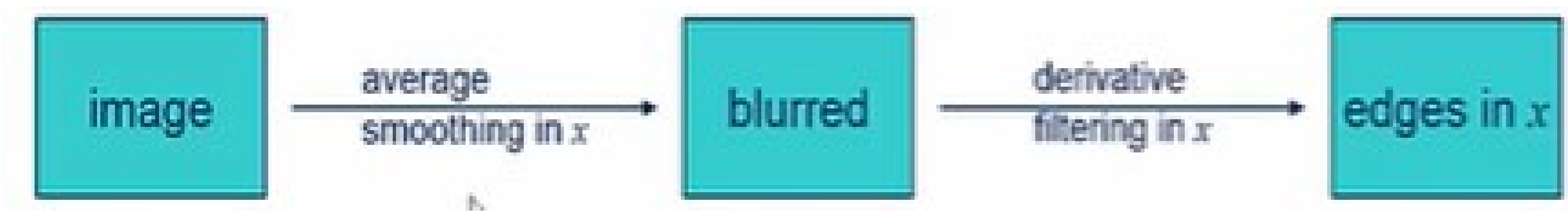


Edge Detectors

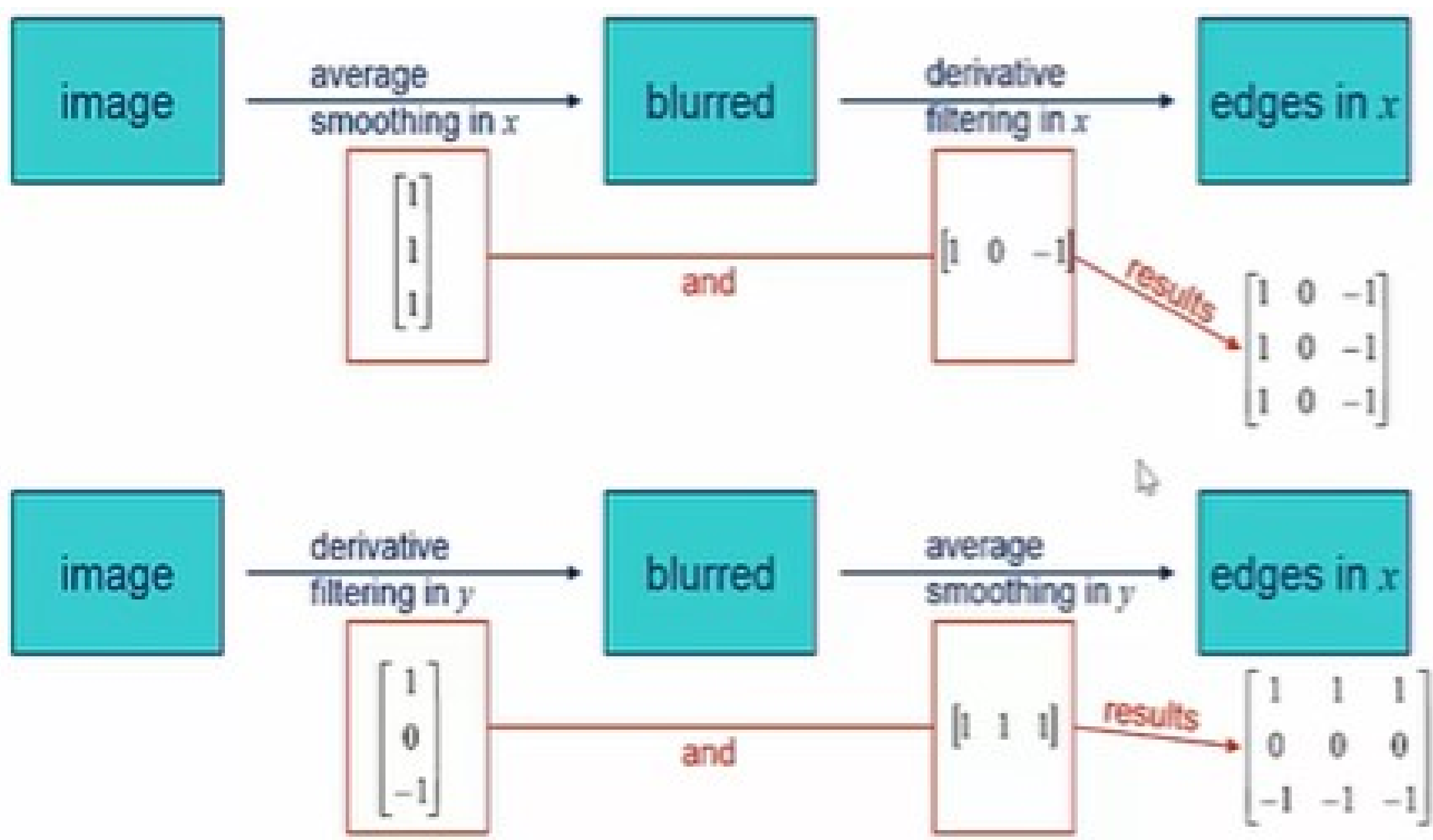
- Gradient Operator
  - Prewitt
  - Sobel
- Laplacian of Gaussian
- Gradient of Gaussian (Canny Edge Detector)

Prewitt and Sobel Edge Detector

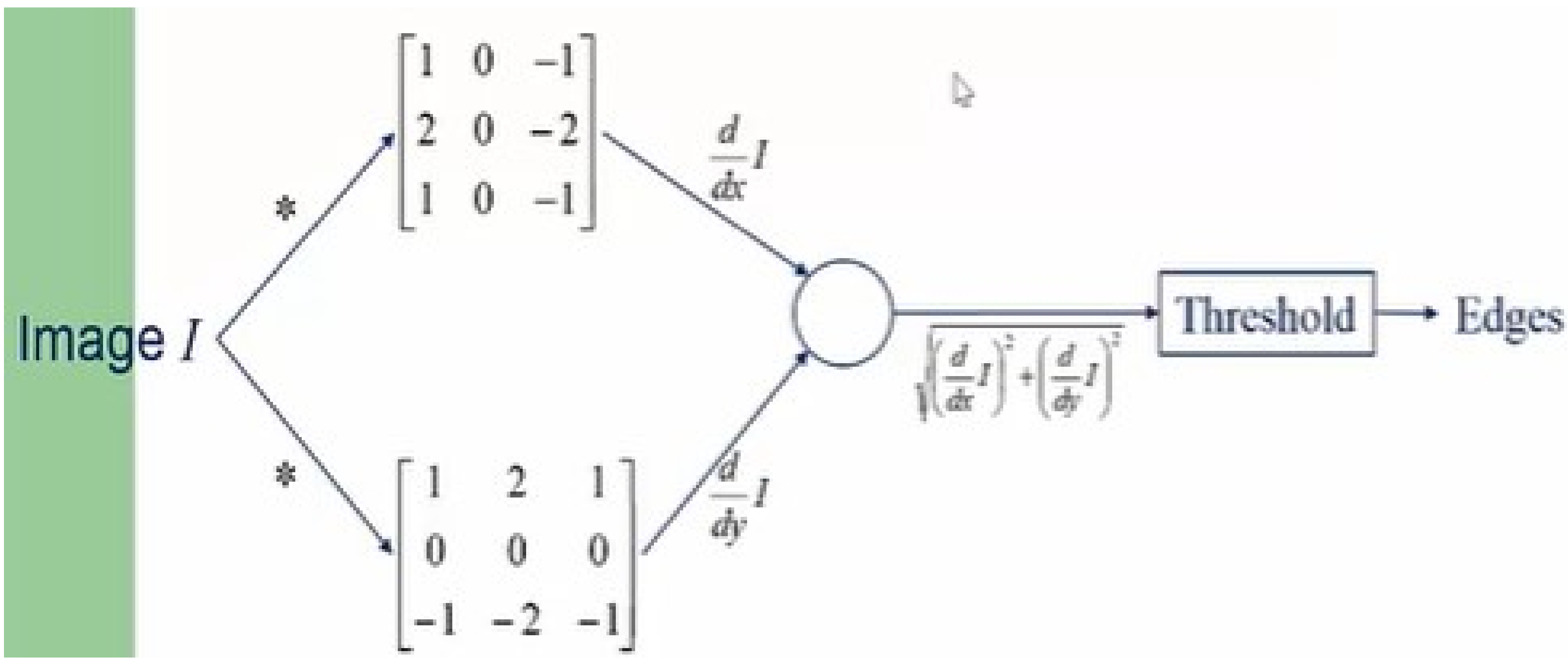
- Compute derivatives in x and y directions
- Find gradient magnitude
- Threshold gradient magnitude

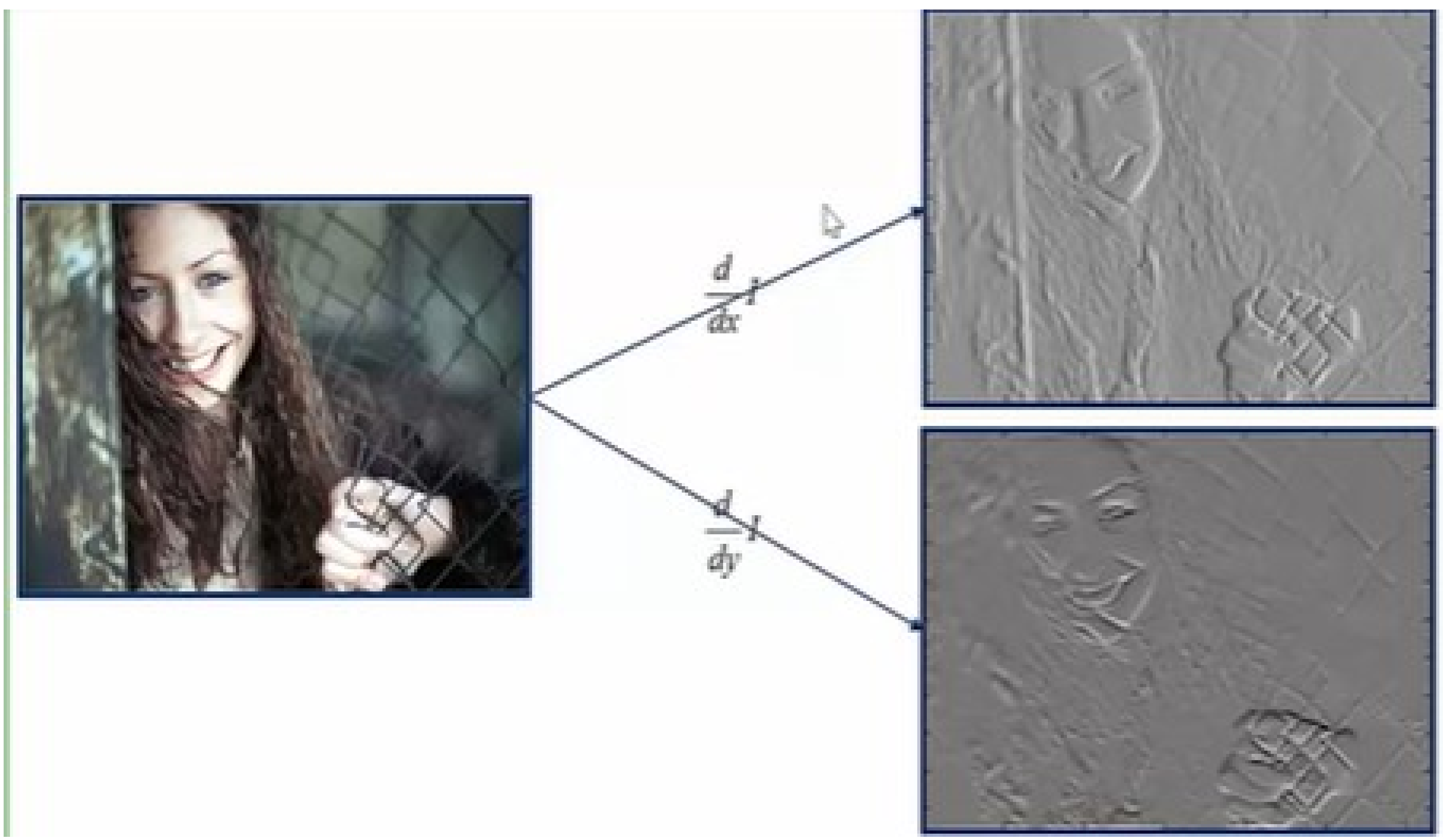


Prewitt Edge Detector

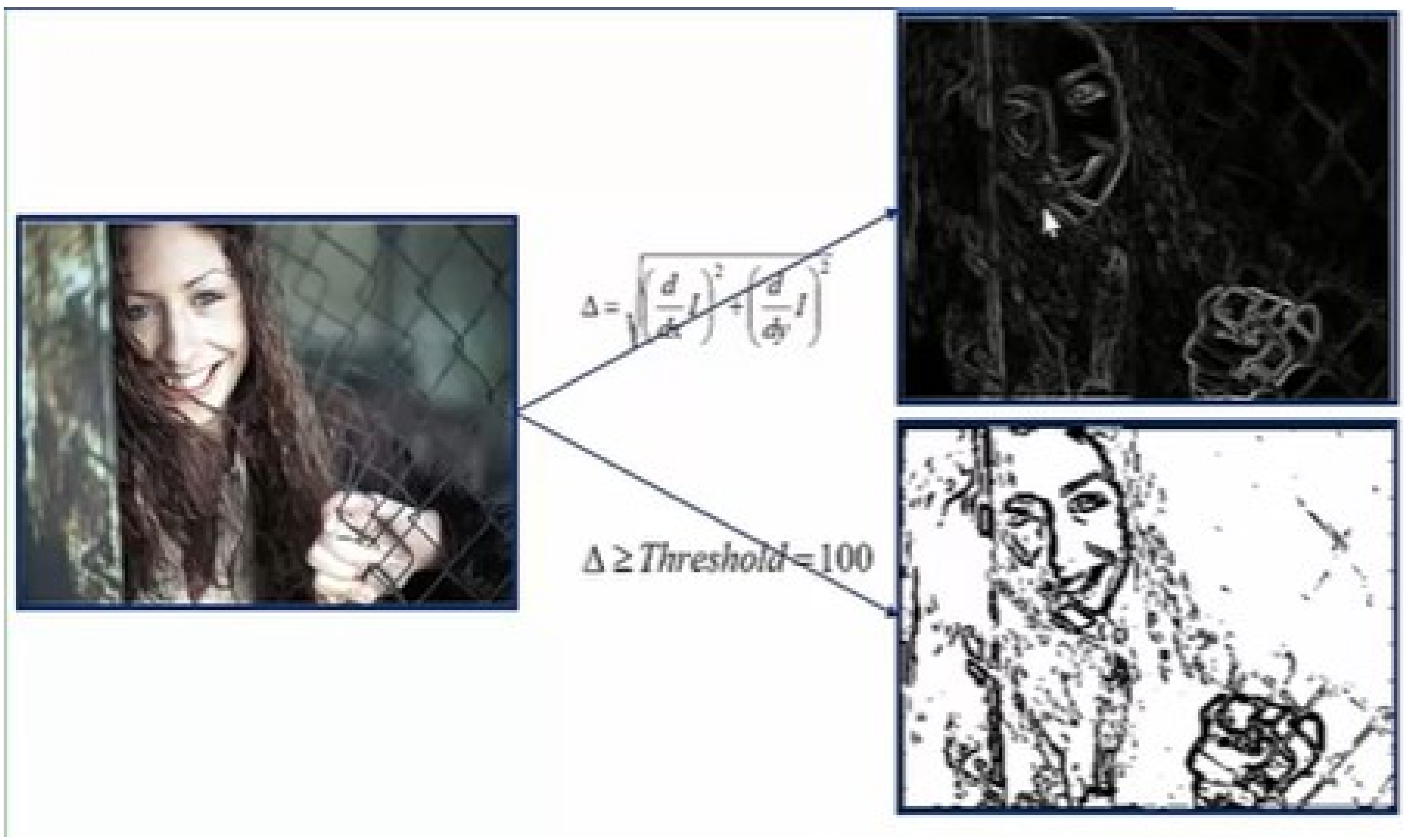


Sobel Edge Detector





$\frac{d}{dx}I$
$\frac{d}{dy}I$



$\Delta = \sqrt{\left(\frac{d}{dx}I\right)^2 + \left(\frac{d}{dy}I\right)^2}$
$\Delta \geq Threshold = 100$

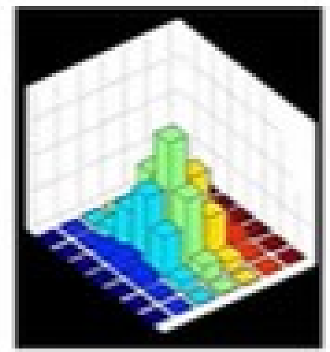
Marr Hildreth Edge Detector

- Smooth image by Gaussian filter → S
- Apply Laplacian to S
  - Used in mechanics, electromagnetics, wave theory, quantum mechanics and Laplace equation
- Find zero crossings
  - Scan along each row, record an edge point at the location of zero-crossing
  - Repeat above step along each column

Marr Hildreth Edge Detector

- Gaussian smoothing

$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}} \qquad g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



- Find Laplacian

$$\Delta^2 S = \overbrace{\frac{\partial^2}{\partial x^2} S}^{\text{secondorder derivative in } x} + \overbrace{\frac{\partial^2}{\partial y^2} S}^{\text{secondorder derivative in } y}$$

- ∇ is used for gradient (first derivative)
- Δ^2 is used for Laplacian (Secondt derivative

Laplacian of Gaussian

- Deriving the Laplacian of Gaussian (LoG)

$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I \qquad g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

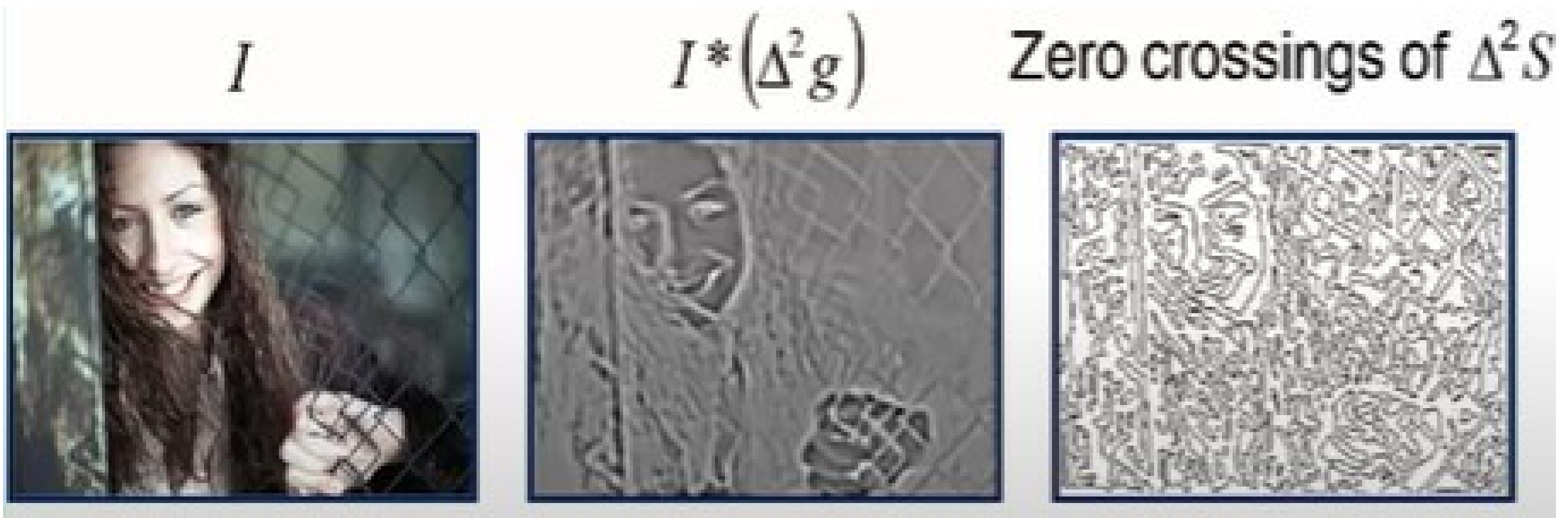
$$g_x = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} \left(-\frac{2x}{2\sigma^2} \right)$$

$$\Delta^2 g = -\frac{1}{\sqrt{2\pi}\sigma^3} \left(2 - \frac{x^2+y^2}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Finding Zero Crossing

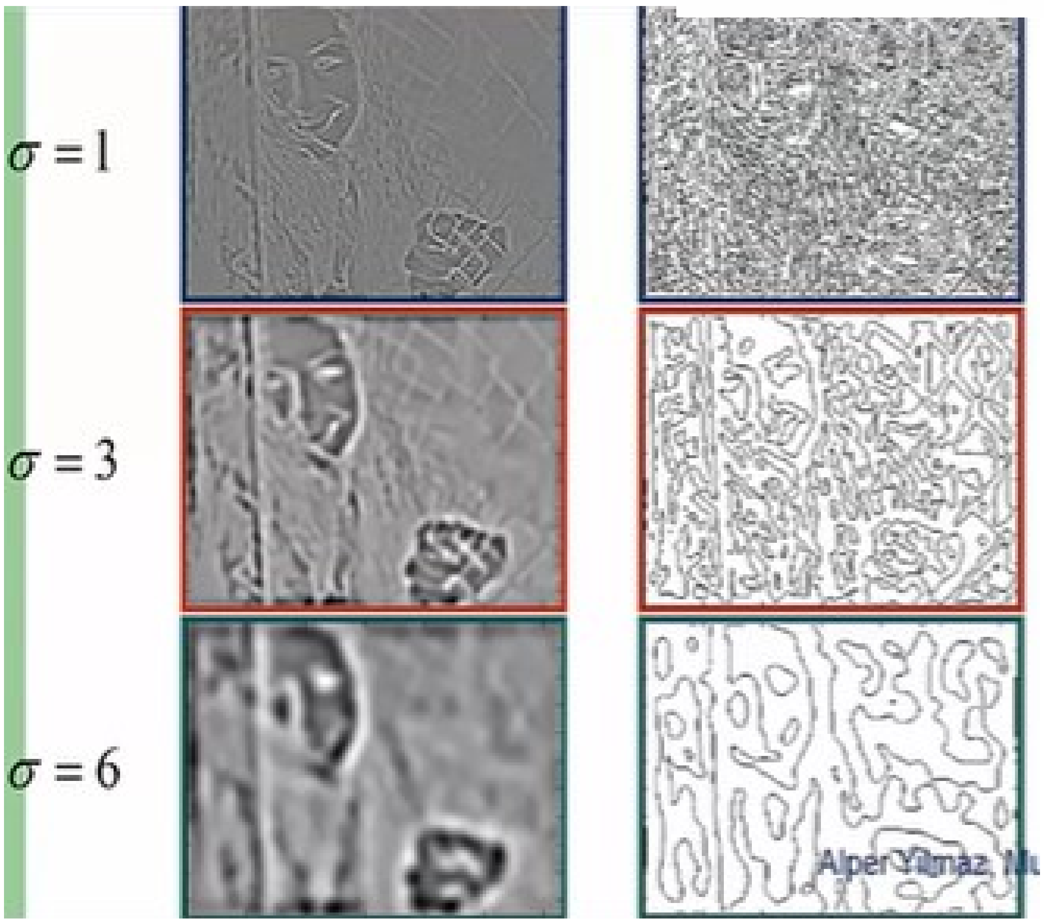
- Four cases of zero-crossings :
  - {+,-}
  - {+,0,-}
  - {-,+}
  - {-,0,+}
- Slope of zero-crossing {a, -b} is |a+b|.
- To mark an edge
  - compute slope of zero-crossing
  - Apply a threshold to slope

Example



Example

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi\sigma^3}}\left(2-\frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$



LOG Algorithm

- Apply LOG to the image
- Find Zero crossings of each row
- Find slope of zero crossing
- Apply threshold to the slope and mark edges

Canny Edge Detection

- Canny Edge Detector Steps
  1. Smooth image with Gaussian filter
  2. Compute derivative of filtered image
  3. Find magnitude and orientation of gradient
  4. Apply "Non-maximum Suppression"
  5. Apply "Hysteresis Threshold" (use range between low a
  6. nd high)

Home assignment

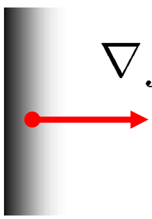
- Write a python code
  - Read an image
  - Find edges using
    1. Prewitt and sobel
    2. Laplacian of Gaussian (LOG)
    3. Canny
- Write image having marked edges on drive

Readings

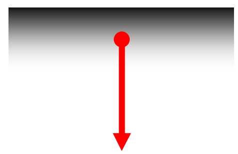
- Chapter
- Richard Szeliski, Computer Vision, Algorithms and Applications, 2nd Ed, https://szeliski.org/Book/

Gradient

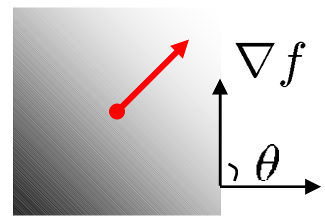
- Gradient equation: $$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$
- Represents direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- Gradient direction: $$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$
- The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

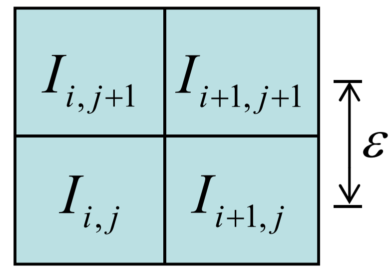
Discrete Edge Operators

- How can we differentiate a ***discrete*** image?

Finite difference approximations:

$$\frac{\partial I}{\partial x} \approx \frac{1}{2\varepsilon}\left(\left(I_{i+1,j+1} - I_{i,j+1}\right) + \left(I_{i+1,j} - I_{i,j}\right)\right)$$

$$\frac{\partial I}{\partial y} \approx \frac{1}{2\varepsilon}\left(\left(I_{i+1,j+1} - I_{i+1,j}\right) + \left(I_{i,j+1} - I_{i,j}\right)\right)$$



Convolution masks :

$$\frac{\partial I}{\partial x} \approx \frac{1}{2\varepsilon} \begin{array}{|c|c|} \hline -1 & 1 \\ \hline -1 & 1 \\ \hline \end{array} \qquad \frac{\partial I}{\partial y} \approx \frac{1}{2\varepsilon} \begin{array}{|c|c|} \hline 1 & 1 \\ \hline -1 & -1 \\ \hline \end{array}$$

Discrete Edge Operators

- First order partial derivatives:

$$\frac{\partial I}{\partial x} \approx \begin{array}{|c|c|}\hline -1 & 1 \\ \hline -1 & 1 \\ \hline\end{array} \qquad \frac{\partial I}{\partial y} \approx \begin{array}{|c|c|}\hline 1 & 1 \\ \hline -1 & -1 \\ \hline\end{array}$$

-

- Second order partial derivatives:

$$\begin{array}{|c|c|c|}\hline I_{i-1,j+1} & I_{i,j+1} & I_{i+1,j+1} \\ \hline I_{i-1,j} & I_{i,j} & I_{i+1,j} \\ \hline I_{i-1,j-1} & I_{i,j-1} & I_{i+1,j-1} \\ \hline\end{array}$$

- **Laplacian** :

$$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

Convolution masks :

$$\nabla^2 I \approx \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline\end{array} \quad \text{or} \quad \begin{array}{|c|c|c|}\hline 1 & 4 & 1 \\ \hline 4 & -20 & 4 \\ \hline 1 & 4 & 1 \\ \hline\end{array}$$

(more accurate)

The Sobel Operators

- Better approximations of the gradients exist
  - The *Sobel* operators below are commonly used

-1	0	1
-2	0	2
-1	0	1

s_x

1	2	1
0	0	0
-1	-2	-1

s_y

Comparing Edge Operators

Gradient: $$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

Good Localization
Noise Sensitive
Poor Detection

Roberts (2 x 2):

0	1
-1	0

1	0
0	-1

Sobel (3 x 3):

-1	0	1
-1	0	1
-1	0	1

1	1	1
0	0	0
-1	-1	1

Sobel (5 x 5):

-1	-2	0	2	1
-2	-3	0	3	2
-3	-5	0	5	3
-2	-3	0	3	2
-1	-2	0	2	1

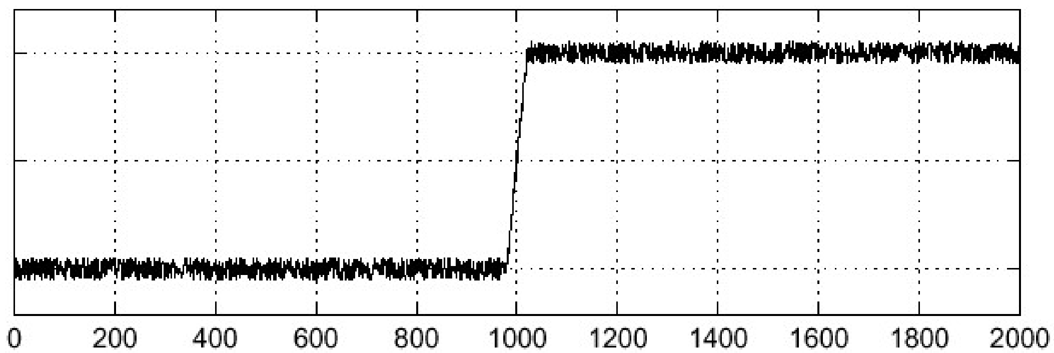
1	2	3	2	1
2	3	5	3	2
0	0	0	0	0
-2	-3	-5	-3	-2
-1	-2	-3	-2	-1

Poor Localization
Less Noise Sensitive
Good Detection

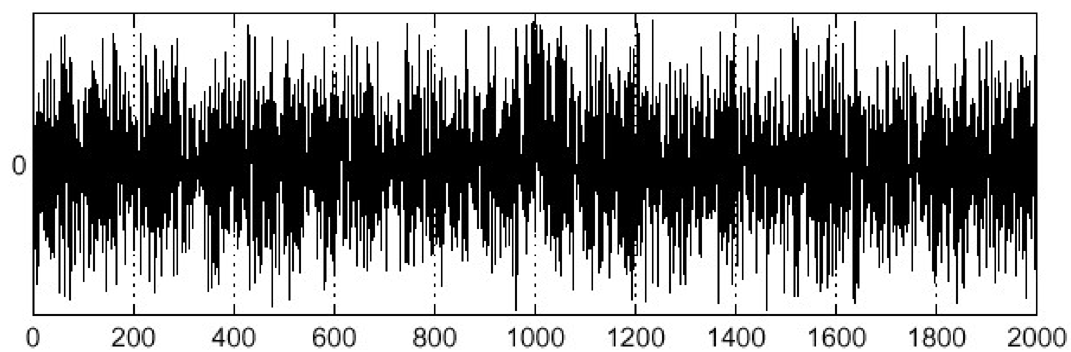
Effects of Noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

$f(x)$



$\frac{d}{dx}f(x)$



Where is the edge??

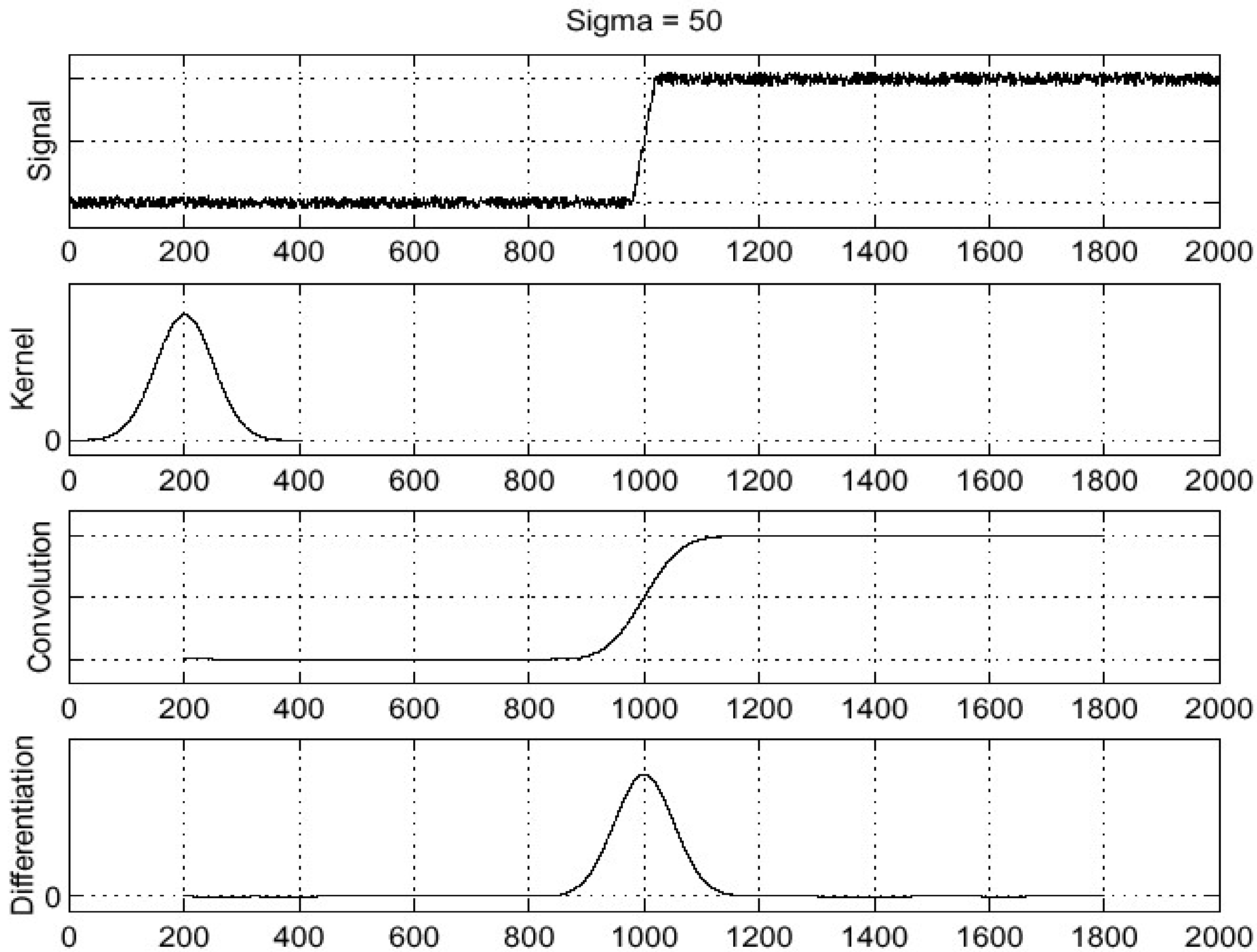
Solution: Smooth First

f

h

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



Where is the edge?

Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative Theorem of Convolution

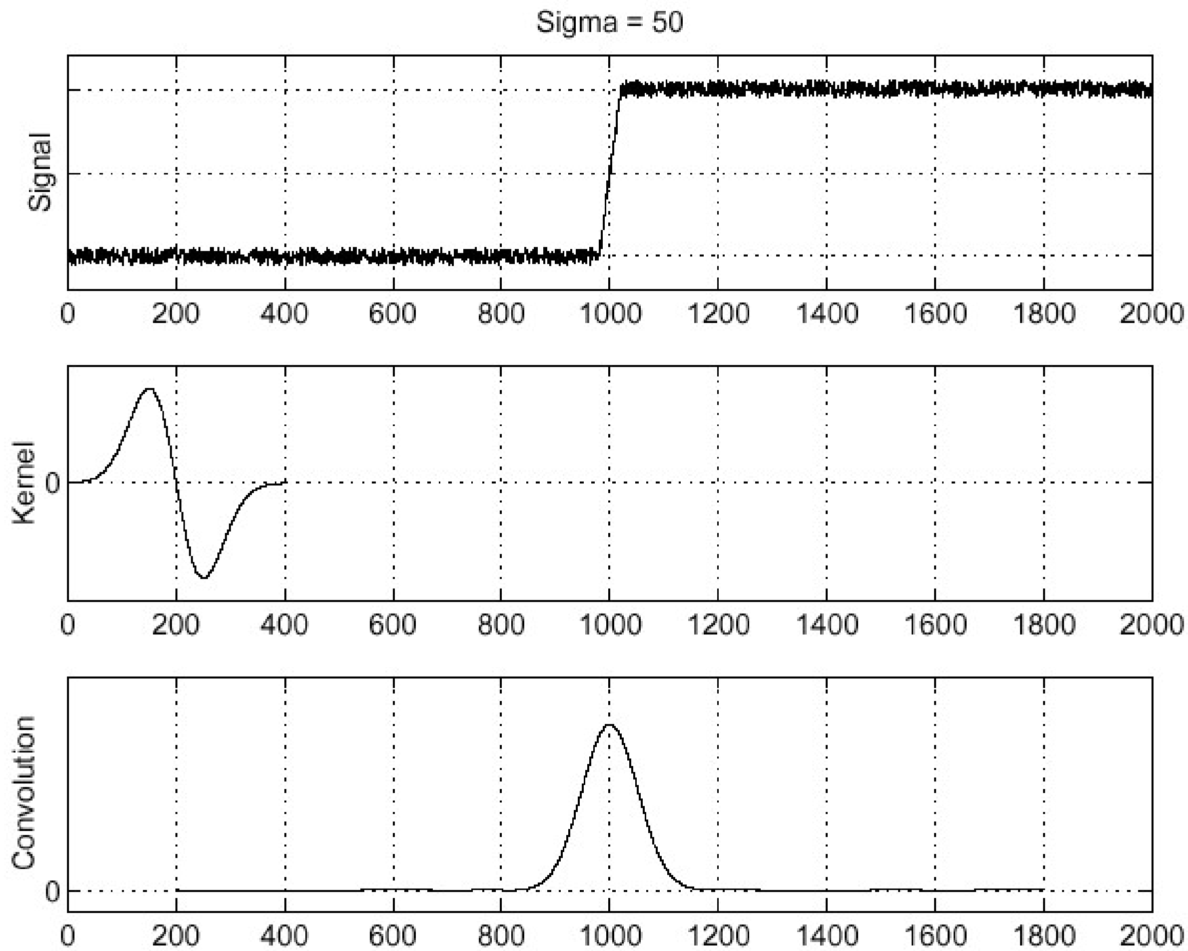
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

…saves us one operation.

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



Laplacian of Gaussian (LoG)

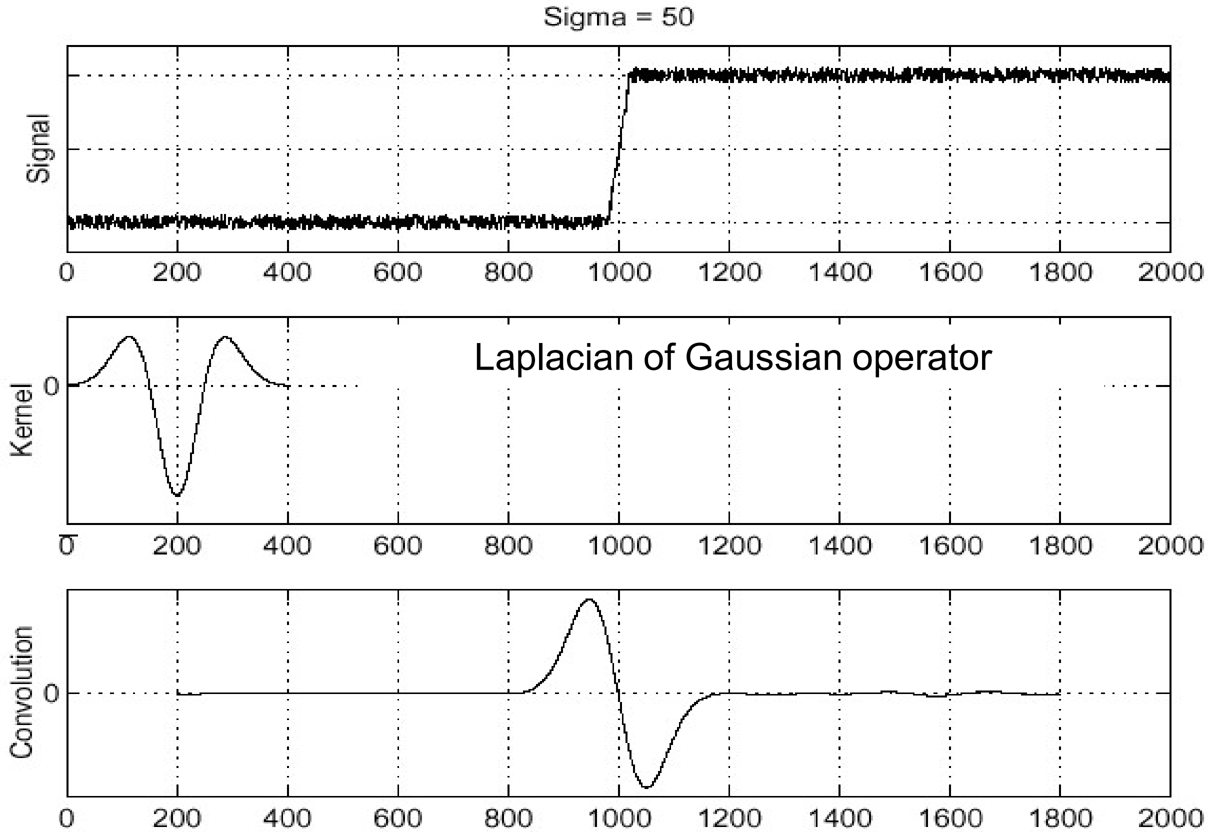
$$\frac{\partial^2}{\partial x^2}(h * f) = \boxed{\left(\frac{\partial^2}{\partial x^2} h\right)} * f$$

Laplacian of Gaussian

f

$\frac{\partial^2}{\partial x^2} h$

$(\frac{\partial^2}{\partial x^2} h) \star f$



Where is the edge?

Zero-crossings of bottom graph !

Canny Edge Operator

- Smooth image *I* with 2D Gaussian: $G * I$
- Find local edge normal directions for each pixel

$$\overline{\mathbf{n}} = \frac{\nabla(G * I)}{\left|\nabla(G * I)\right|}$$

- Compute edge magnitudes $\left|\nabla(G * I)\right|$
- Locate edges by finding zero-crossings along the edge normal directions (**non-maximum suppression**)

$$\frac{\partial^2 (G * I)}{\partial \overline{\mathbf{n}}^2} = 0$$

The Canny Edge Detector

original image (Lena)

The Canny Edge Detector

magnitude of the gradient

The Canny Edge Detector

After non-maximum suppression

Canny Edge Operator

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

- The choice of σ depends on desired behavior
  - large σ detects large scale edges
  - small σ detects fine features