M607 - First Homework **September 16, 2009**

We have seen three methods for solving the discrete Poisson problem
$-\Delta p = f$ in a square with Dirichlet boundary conditions $u = g$ for given functions f and g:

(1) Jacobi's method:
Given $p_{ij}^{(0)},\ 1 \le i \le \mathrm{I}_{\max} - 1,\ 1 \le j \le \mathrm{J}_{\max} - 1$ then
for $n = 0, 1, 2, \ldots$
for $i = 1$ to $\mathrm{I}_{\max} - 1$
for $j = 1$ to $\mathrm{J}_{\max} - 1$

$$p_{ij}^{(n+1)} = \frac{f_{ij} + \dfrac{p_{i+1,j}^{(n)} + p_{i-1,j}^{(n)}}{\Delta x^2} + \dfrac{(p_{i,j+1}^{(n)} + p_{i,j-1}^{(n)})}{\Delta y^2}}{\dfrac{2}{\Delta x^2} + \dfrac{2}{\Delta y^2}}.$$

(2) Gauss - Seidel method with the same initial guess
for $n = 0, 1, 2, \ldots$
for $i = 1$ to $\mathrm{I}_{\max} - 1$
for $j = 1$ to $\mathrm{J}_{\max} - 1$

$$p_{ij}^{(n+1)} = \frac{f_{ij} + \dfrac{p_{i+1,j}^{(n)} + p_{i-1,j}^{(n+1)}}{\Delta x^2} + \dfrac{(p_{i,j+1}^{(n)} + p_{i,j-1}^{(n+1)}}{\Delta y^2}}{\dfrac{2}{\Delta x^2} + \dfrac{2}{\Delta y^2}}$$

(3) Successive Over Relaxation:
Choose a relaxation parameter $\omega \neq 0$ and an initial guess as before. Then
for $n = 0, 1, 2, \ldots$
for $i = 1$ to $\mathrm{I}_{\max} - 1$
for $j = 1$ to $\mathrm{J}_{\max} - 1$

$$p_{ij}^{(n+1)} = (1-\omega) p_{ij}^{(n)} + \omega \frac{f_{ij} + \dfrac{p_{i+1,j}^{(n)} + p_{i-1,j}^{(n+1)}}{\Delta x^2} + \dfrac{p_{i,j+1}^{(n)} + p_{i,j-1}^{(n+1)}}{\Delta y^2}}{\dfrac{2}{\Delta x^2} + \dfrac{2}{\Delta y^2}}.$$

Note that when $\omega = 1$ we have Gauss-Seidel.

Assignment 1a:

I gave you a code called *sov.m*. But the title is misleading! It actually does Gauss-Seidel. Modify the code to implement SOR and run the code for $\omega = 1$, $\omega = 1.7$, $\omega = 2.7$, $I_{max} = J_{max} = 40$, and the boundary data etc from my code. Use 200 iterations.

Which choice of ω decreased the residual faster? Hand in your code and a graph of the residual for all three choices of ω on one axis (one will not converge - only graph a few iterates showing the residual growing, not all iterates).

Assignment 1b:

Let

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}.$$

Write down the iteration matrix G (see p. 325 of the book) when (a) Jacobi, (b) Gauss-Seidel and (c) SOR are used to solve $A\vec{x} = \vec{b}$. In each case calculate $\|G\|_\infty$ (choose $\omega = 1.5$ for SOR). You can use Matlab to help! Which method would converge faster in this case?