Multi-objective Bayesian optimisation for design of Pareto-optimal current drive profiles

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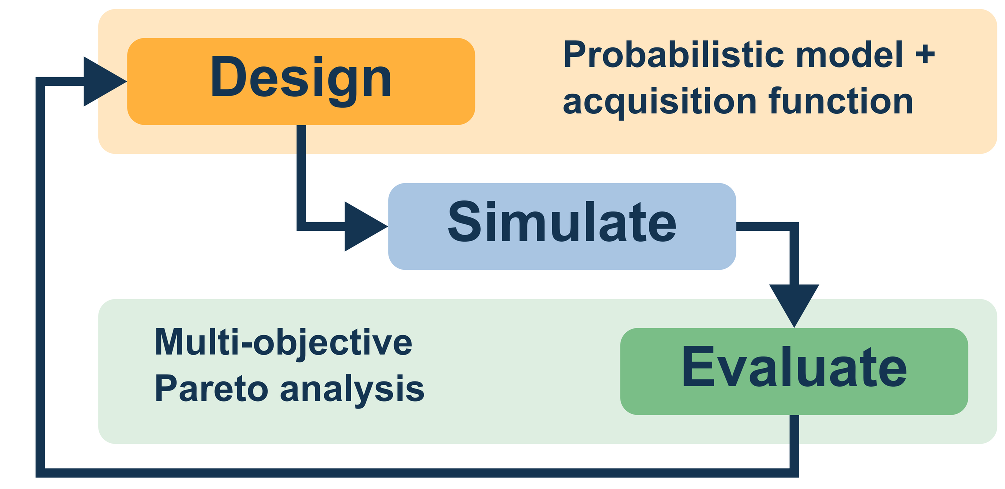
Introduction

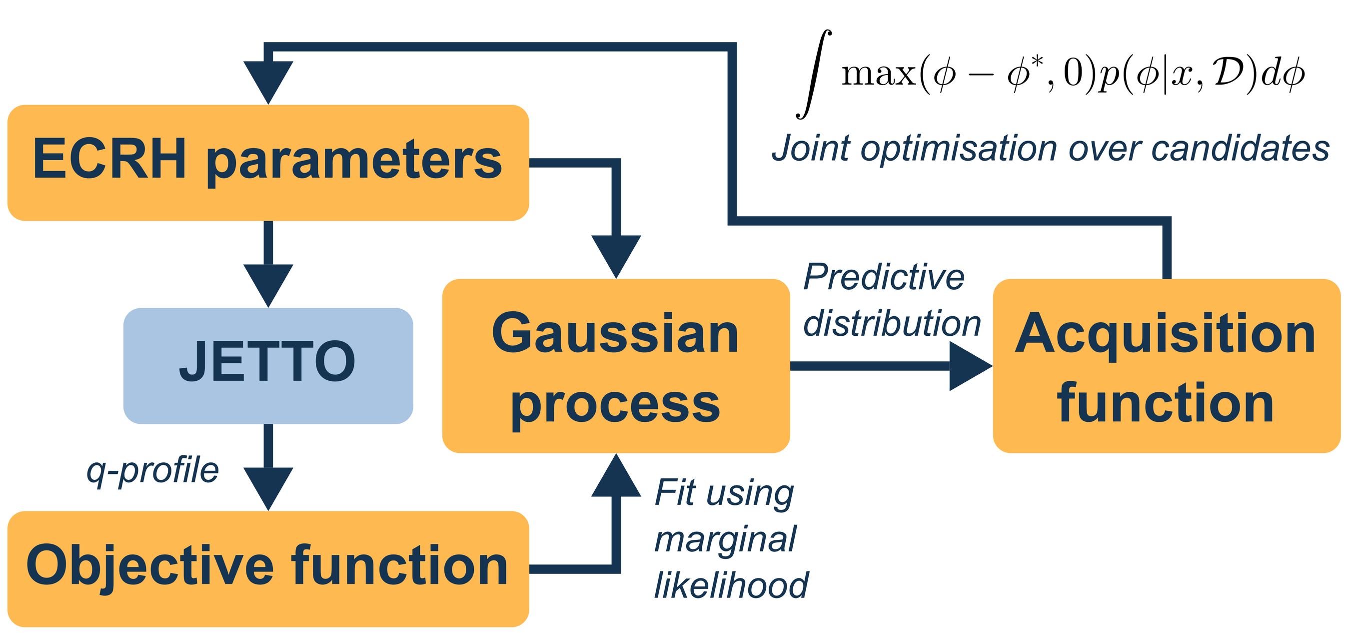
JETTO is a high-fidelity plasma modelling code that solves coupled core transport and equilibrium equations.
It is used to **evaluate plasma scenarios** as part of the STEP design process.
JETTO is initialised with the parameters of a candidate design, and the simulation is run until the plasma reaches a steady state.
The steady-state properties of the plasma are used to assess the **impact of design choices** and the **suitability** of a given design.

However, JETTO takes **several hours** to run, which severely limits the extent to which the design space can be explored.

We demonstrate an improved method for design optimisation that delivers **higher-quality solutions** in significantly **fewer iterations** than previous methods used by the STEP design team, using techniques from **machine learning** and **surrogate modelling**.
Our approach also offers **improved interpretability**, allowing design engineers to **quantify the tradeoffs** between different objectives.

Our example application is the optimisation of electron cyclotron resonance heating profiles to achieve desirable safety factor properties.





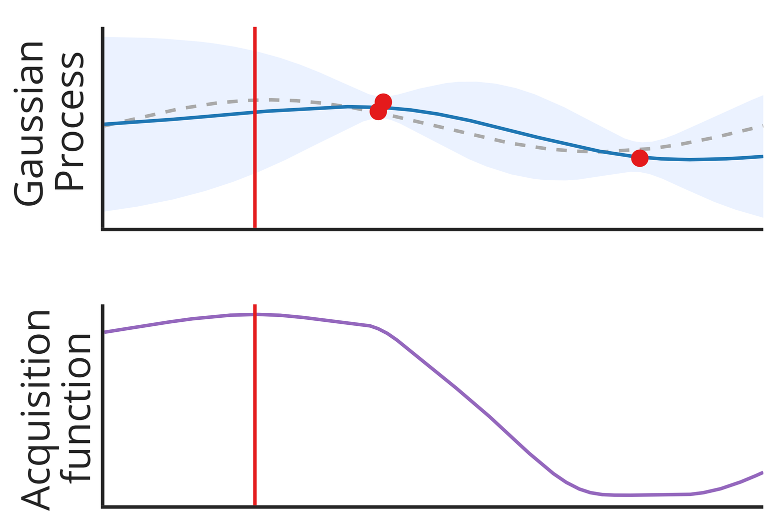
What is Bayesian optimisation?

Bayesian optimisation is built upon a **probabilistic model**. We use a **Gaussian Process** (GP).
The model is fit to the **previous observations** - inputs and the corresponding objective values.
It is used to generate **predictions** about the **performance of unseen points**.
An inner optimisation loop is performed using the surrogate predictions to find the **most promising** set of points to try next.

This method:

a) **reduces** the number of runs **'wasted'** on trying suboptimal points
b) ensures that all the **information** gained at each step is **propagated** to future steps

As a result, we found that BO exhibits **vastly improved performance** compared to stochastic search methods.

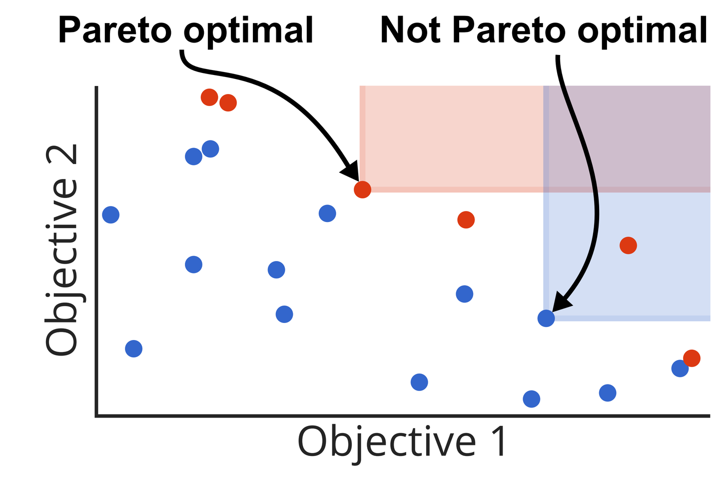


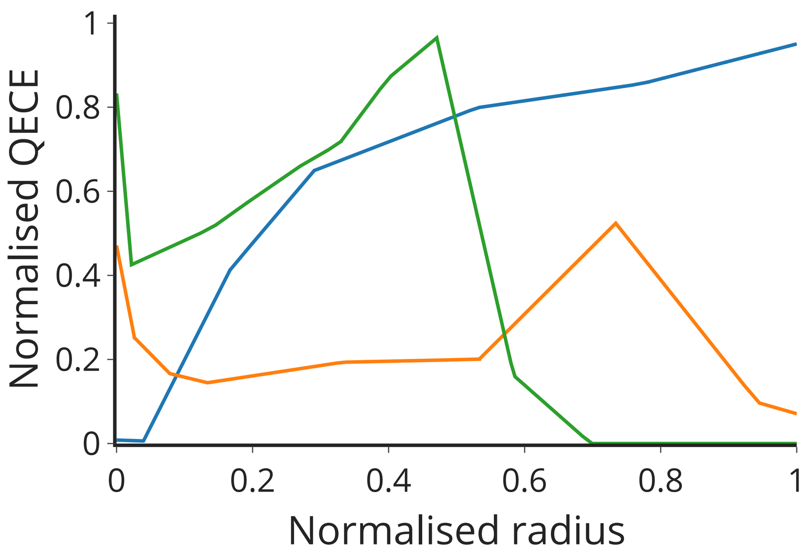
What is Pareto optimality?

Single-objective optimisation tasks involve finding **one solution** that maximises a **scalar objective**.
Comparing solutions is easy, as each solution is either better or worse than the others.

In **multi-objective** settings, there is normally no solution that simultaneously maximises every objective.
Instead, we seek to find a **set of solutions** that represent the **tradeoffs** between each objective.
The set is made up of all points that are **Pareto optimal**.

A point is Pareto optimal if it is impossible to improve its performance under one objective without reducing its performance under another objective.



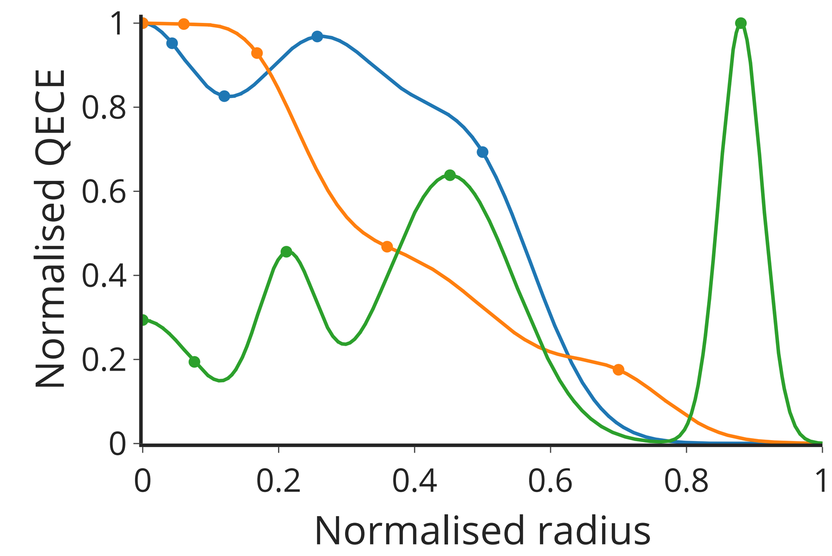


Parameterisation of ECRH profile

The choice of input parameterisation affects the **rate of convergence** and **quality of results**.
Ideally, the parameterisation would be **general**, so that every possible profile can be represented.
However, this can mean that the search space is **too large** to explore effectively.

Previously, the ECRH profile has been represented as a piecewise linear function (left).
We found that using a **sum-of-Gaussians representation** (right) achieves improved performance, while also having **interpretable parameters** - each Gaussian represents an EC beam launcher.

The SoG parameterisation produces smooth ECRH profiles, and leads to a simpler mapping from input space to objective space: **small changes in parameters result in small changes in objective value**. This ensures that the mapping can be well-represented by a GP model.

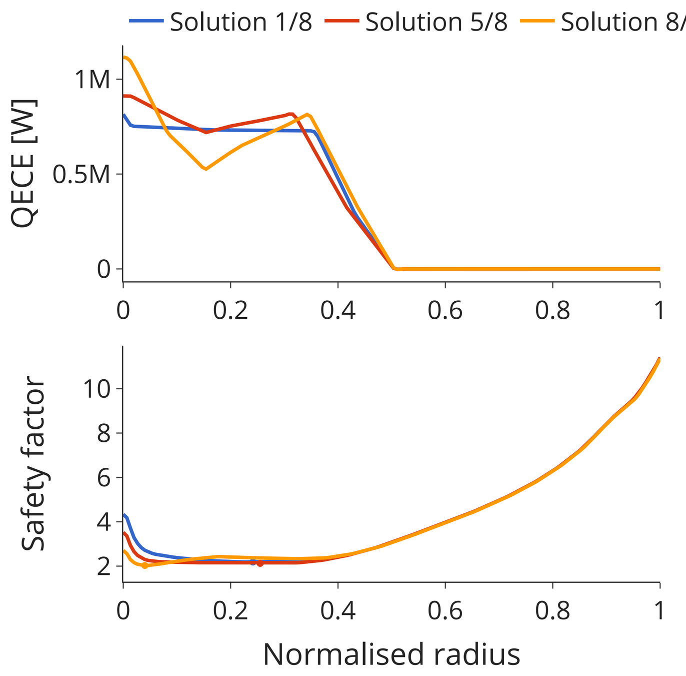


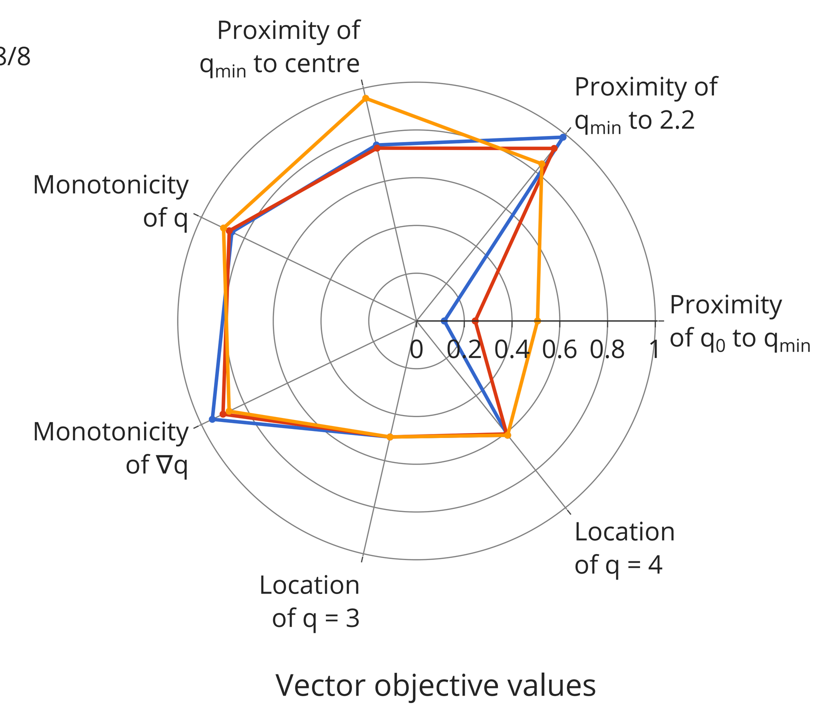
Preliminary results

Running the multi-objective Bayesian optimisation loop using the piecewise linear ECRH function for 5 steps with 5 candidates at each step produces 8 Pareto optimal solutions.

The solutions are split into two groups. The first group contains **monotonically decreasing ECRH** profiles (e.g. Solution 1, below). These solutions resemble those currently used in STEP designs.

The second group contains **double-peaked ECRH** profiles (e.g. Solution 8). The ECRH parameterisation was designed to permit these profiles, but previous optimisations tended to struggle to find good double-peaked solutions.





Target metrics for safety factor profile

Property	Rationale	Formulation
Minimise shear at centre	Improve β	$q(0) - \min q \qquad \arg\min_{\rho} q$
Minimum q above 2	Improve β	$\lVert \min q - (2+\epsilon) \rVert$
Monotonic *q*	Improve β	$\int_0^1 q\,\mathbf{1}\left(\frac{dq}{d\rho} \leq 0\right) d\rho$
Monotonic gradient of *q*	Reduce fast ion losses	$\int_0^1 \frac{dq}{d\rho}\,\mathbf{1}\left(\frac{d^2q}{d\rho^2} \leq 0\right) d\rho$
High shear at integer *q*	Mitigate NTMs	$q^{-1}(n),\ n \in \mathbb{N}$

The exact mathematical formulation of the objectives has a significant impact on the results.
We suggest that **further research is required** to quantify the precise elements of each property that are most important.

Future work

Alongside fine-tuning the ECRH parameterisation and q-profile objectives, we are intending to extend our method to analysis of **ballooning stability.**

We will also investigate **transfer learning**, where the GP model is trained on one design scenario and used as a **model prior** for a different scenario, reducing the number of optimisation steps required.

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This project was funded by the UKAEA/STFC FARSCAPE programme

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